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POLYNOMIALS AND MULTILINEAR FORMS ON FULLY NUCLEAR SPACES

by Philipp J. BOLAND (*)

Abstract. - In this paper, we consider the spaces of continuous and hypo continuous m -linear forms and polynomials on fully nuclear spaces. We consider these spaces endowed with the topologies τ_0 (uniform convergence on compact sets) and τ_ω (Nachbin ported topology), and develop some duality relationships for these topologies.

1. Preliminaries.

In this paper, E will represent a fully nuclear space, that is E and E' are complete reflexive nuclear spaces over the field of complex numbers. Any Fréchet nuclear (FN) space or the strong dual of a Fréchet nuclear (QFN) space is fully nuclear. $\mathcal{O}$, $\mathcal{O}'$, $\prod_N \mathbb{C} \times \sum_N \mathbb{C}$ are other examples of fully nuclear spaces.

$\mathcal{L}^{(m)}(E)$ (respectively $\mathcal{L}_{HY}^{(m)}(E)$) is the space of continuous (hypo continuous, i. e. continuous on compact sets) m -linear forms on E . $\mathcal{L}_S^{(m)}(E)$ is the subspace of $\mathcal{L}^{(m)}(E)$ of symmetric m -linear forms. τ_0 is the topology of uniform convergence on compact sets. $P^{(m)}(E)$ (respectively $P_{HY}^{(m)}(E)$) is the space of continuous (hypo continuous m -homogeneous polynomials on E . τ_0 is also defined on $P^{(m)}(E)$ and $P_{HY}^{(m)}(E)$. τ_ω is the topology on $P^{(m)}(E)$ defined by all semi-norms p ported by compact subsets of E (p is ported by K if, for all open U , where $K \subset U \subset E$, there exists $C_U > 0$ such that $p(f) \leq C_U \|f\|_U$, for all $f \in P^{(m)}(E)$). τ_ω is a bornological topology on $P^{(m)}(E)$, and it may also be described as the topology defined by all semi-norms on $P^{(m)}(E)$ which are ported by zero [6].

If E is a K -space [10] (i. e. $f: E \rightarrow X$ is continuous if f is continuous on the compact subsets of E), then $\mathcal{L}_{HY}^{(m)}(E) = \mathcal{L}^{(m)}(E)$ and $P_{HY}^{(m)}(E) = P^{(m)}(E)$. Every metrizable or QFN space is a K -space. $\prod_N \mathbb{C} \times \sum_N \mathbb{C}$ is not a K -space. The following example shows that $\mathcal{O}$ ($\mathcal{O} = \mathcal{O}(\mathbb{R})$ complexified) is not a K -space.

Example 1. - $P_{HY}^{(2)}(\mathcal{O}) \neq P^{(2)}(\mathcal{O})$.

Proof. - Define $p = \sum_{n=1}^{\infty} (\delta^n \delta_0) \delta_n$, where δ_a is the Dirac delta function at a . If K is compact in $\mathcal{O}$, there exists an m such that $K \subset \mathcal{O}_m (= \text{complexification of } \{\varphi \in \mathcal{O} : \text{support } \varphi \subset [-m, m]\})$. Therefore $p|_K = \sum_{n=1}^m (\delta^n \delta_0) \delta_n$ which is continuous on K . Therefore $p \in P_{HY}^{(2)}(\mathcal{O})$. However, one may show that p is not bounded on any neighbourhood of zero, and hence $p \notin P^{(2)}(\mathcal{O})$ (see [5]).

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Note that, from the above $P(^2\mathcal{O})$, τ_0 is not complete. We will now see that, if E is fully nuclear, then $(P_{HY}({}^m E), \tau_0)$ is the completion of $(P({}^m E), \tau_0)$.

2. Density of continuous polynomials.

If U is an open absolutely convex set in E , then $E(U) = E/(p_U^{-1}(0))$, where p_U is the Minkowski functional of U . Similarly, if B is a closed absolutely convex bounded set in E , then $E(B) = \bigcup_n nB$ and $E(B)$ is a normed space with unit ball B . We may identify $(E(U))'$ with $E'(U^0)$, and $E'(B^0)$ is a subspace of $(E(B))'_\beta$ (see [11]).

PROPOSITION 2. - Let E be fully nuclear. Then $(\mathcal{L}_{HY}({}^m E), \tau_0) = \widehat{(\mathcal{L}({}^m E), \tau_0)}$ and $(P_{HY}({}^m E), \tau_0) = \widehat{(P({}^m E), \tau_0)}$, for all m (where $\hat{F}$ is the completion of F).

Proof. - It suffices to show that $\mathcal{L}({}^m E)$ is dense in $(\mathcal{L}_{HY}({}^m E), \tau_0)$. Therefore, let $\Lambda \in \mathcal{L}_{HY}({}^m E)$, $\epsilon > 0$, and K absolutely convex and compact be given. We want to find $\Lambda' \in \mathcal{L}({}^m E)$ such that $|\Lambda - \Lambda'|_K < \epsilon$.

Without loss of generality, $E(K)$ is Hilbert and therefore $E' \rightarrow E'(K)$ has a dense image as E is reflexive. Now, there exists an absolutely convex compact set K_1 containing K such that $E(K) \rightarrow E(K_1)$ is nuclear. Hence, there exist $(\varphi_n) \subseteq (E(K))'$ and $(y_n) \subseteq K_1$ such that, for all $x \in E(K)$,

$$x = \sum_n \varphi_n(x) y_n \quad (\text{convergence in } E(K_1))$$

and

$$\sum_n |\varphi_n|_K p_{K_1}(y_n) = C < +\infty.$$

Now, Λ is continuous on K_1 (i. e. on ${}^m K_1$), and hence, for $x_1, \dots, x_m \in K$,

$$\begin{aligned} \Lambda(x_1, \dots, x_m) &= \Lambda(\sum_n \varphi_n(x_1) y_n, \dots, \sum_n \varphi_n(x_m) y_n) \\ &= \sum_{n_1, \dots, n_m} \varphi_{n_1}(x_1) \dots \varphi_{n_m}(x_m) \Lambda(y_{n_1}, \dots, y_{n_m}) \\ &= \sum_{n_1, \dots, n_m} \varphi_{n_1} \dots \varphi_{n_m}(x_1, \dots, x_m) \Lambda(y_{n_1}, \dots, y_{n_m}). \end{aligned}$$

As $\sum_n |\varphi_n|_K p_{K_1}(y_n) = C < +\infty$, it follows that there exists a finite set F of indices such that

$$|\Lambda - \sum_F \varphi_{n_1} \dots \varphi_{n_m} \Lambda(y_{n_1}, \dots, y_{n_m})|_K < \frac{\epsilon}{2}.$$

Since E' is dense in $E'(K)$, we can find continuous linear forms $\psi_{n_i} \in E'$ such that

$$|\varphi_{n_1} \dots \varphi_{n_m} \Lambda(y_{n_1}, \dots, y_{n_m}) - \psi_{n_1} \dots \psi_{n_m}|_K < \frac{\epsilon}{2|F|},$$

and therefore

$$|\sum_F \varphi_{n_1} \dots \varphi_{n_m} \Lambda(y_{n_1}, \dots, y_{n_m}) - \sum_F \psi_{n_1} \dots \psi_{n_m}|_K < \frac{\epsilon}{2}.$$

Hence $|\Lambda - \sum_F \psi_{n_1} \dots \psi_{n_m}|_K < \epsilon$, and therefore $\mathcal{L}_f({}^m E)$ (the space of continuous m -linear forms of finite type on E) is dense in $(\mathcal{L}_{HY}({}^m E), \tau_0)$.

Similarly, it follows that $P_f^{(m)}(E)$ (and hence $P^{(m)}(E)$) is dense in $(P_{HY}^{(m)}(E), \tau_0)$.

COROLLARY 3. - Let U be a balanced open set in the fully nuclear space E . Then, $H(U)$ is dense in $(H_{HY}(U), \tau_0)$ (where $H(U)$ and $H_{HY}(U)$ are respectively the holomorphic and hypo analytic functions on U).

Proof. - This follows from proposition 2 and the Taylor series expansion of elements of $(H_{HY}(U), \tau_0)$.

Remark 4. - If E is a fully nuclear space with an absolute basis, then it is considerably easier to prove proposition 2 (see [4]).

3. Duality for spaces of polynomials.

There is a natural algebraic isomorphism between $(P^{(m)}(E), \tau_0)'$ and $P^{(m)}(E')$ when E is a fully nuclear space. The isomorphism is via the mapping

$$\beta : (P^{(m)}(E), \tau_0)' \longrightarrow P^{(m)}(E'),$$

where $\beta T(\varphi) = T(\varphi^m)$ (see [2]). Note that, because of proposition 2, $(P_{HY}^{(m)}(E), \tau_0)'$ and $P^{(m)}(E')$ are also isomorphic.

We will now show that, when E is fully nuclear, then $(P^{(m)}(E), \tau_0)' \simeq P_{HY}^{(m)}(E')$. Initially, however we need some definitions and a lemma.

Definition 5. - Let U be an absolutely convex neighbourhood of zero in E . Then

$$\mathcal{L}_N^{(m)}(E(U)) = \{ \Lambda ; \Lambda \in \mathcal{L}^{(m)}(E), \Lambda = \sum_n \langle \cdot, a_n^1 \rangle \dots \langle \cdot, a_n^m \rangle ,$$

$$\text{where each } a_n^i \in (E(U))' \text{ and } \sum_n |a_n^1|_U \dots |a_n^m|_U < +\infty \} .$$

We define the norm π_U on $\mathcal{L}_N^{(m)}(E(U))$ by

$$\pi_U(\Lambda) = \inf \{ \sum_n |a_n^1|_U \dots |a_n^m|_U : \Lambda = \sum_n \langle \cdot, a_n^1 \rangle \dots \langle \cdot, a_n^m \rangle \} .$$

Similarly, we define

$$P_N^{(m)}(E(U)) = \{ p \in P^{(m)}(E), p = \sum_n \varphi_n^m ,$$

$$\text{where each } \varphi_n \in (E(U))' \text{ and } \sum_n |\varphi_n^m|_U < +\infty \} .$$

We define the norm π_U on $P_N^{(m)}(E(U))$ by $\pi_U(p) = \inf \{ \sum_n |\varphi_n^m|_U ; p = \sum_n \varphi_n^m \} .$

Note that, if $p \in P^{(m)}(E)$ and $\mathcal{L}_p$ is the symmetric m -linear form $\in \mathcal{L}_S^{(m)}(E)$ corresponding to p , then

$$p \in P_N^{(m)}(E(U)) \iff \mathcal{L}_p \in \mathcal{L}_N^{(m)}(E(U)) .$$

This follows since if $\Lambda = \sum_n \langle \cdot, a_n^1 \rangle \dots \langle \cdot, a_n^m \rangle \in \mathcal{L}_N^{(m)}(E(U))$, where $\sum_n |a_n^1|_U \dots |a_n^m|_U < +\infty$, then $(a_n^1 \dots a_n^m)_S$ is such that $P_{(a_n^1 \dots a_n^m)_S} \in P_f^{(m)}(E(U))$ and

$$\pi_U(p_{(a_n^1 \dots a_n^m)_S}) \leq \frac{m^m}{m!} \pi_U((a_n^1 \dots a_n^m)_S) \leq \frac{m^m}{m!} \pi_U(a_n^1 \dots a_n^m) = \frac{m^m}{m!} |a_n^1|_U \dots |a_n^m|_U$$

where $(a_n^1 \dots a_n^m)_S$ is the symmetrization of $a_n^1 \dots a_n^m$ and $P_{(a_n^1 \dots a_n^m)_S}$ is the polynomial corresponding to $(a_n^1 \dots a_n^m)_S$. Moreover, we see that

$$\pi_U(p) \leq \frac{m}{m!} \pi_U(\Lambda_p) \leq \frac{m}{m!} \pi_U(p), \text{ for } p \in P_N({}^m E(U)).$$

See [8].

Definition 6. - If B is an absolutely convex set in the fully nuclear space E , we define Σ_B on $P_N({}^m E(U))$ by

$$\Sigma_B(\Lambda) = \sup \{ |\sum_n \langle z_1, a_n^1 \rangle \dots \langle z_m, a_n^m \rangle| ; \Lambda = \sum_n \langle \cdot, a_n^1 \rangle \dots \langle \cdot, a_n^m \rangle \text{ and } z_i \in B^{00} \}.$$

Note that, as E is reflexive, $\Sigma_B(\Lambda) = |\Lambda|_B$. Similarly, we may define Σ_B on $P_N({}^m E(U))$. Note that Σ_B may be infinite.

Now, we know that, if E is fully nuclear and U is an absolutely convex neighbourhood of zero in E , then there exists an absolutely convex neighbourhood V of zero such that $V \subset U$, $E(V) \rightarrow E(U)$ is nuclear and $\Lambda \in \mathcal{L}_N({}^m E(V))$, whenever $\Lambda \in \mathcal{L}({}^m E(U))$. See [2].

Lemma 7. - Let U be an absolutely convex neighbourhood of zero in the fully nuclear space E . Then, there exists $C > 0$ and absolutely convex neighbourhoods of zero W, V , where $W \subset V \subset U$ such that, if $\Lambda \in \mathcal{L}({}^m E)$ and $|\Lambda|_U < +\infty$, then $\Lambda \in \mathcal{L}_N({}^m E(W))$ and $\pi_W(\Lambda) \leq C^m \Sigma_V(\Lambda) = C^m |\Lambda|_V$, for all $m \geq 1$.

Proof. - We prove this only for the case $m = 2$. As E is fully nuclear, we may assume that $E(U)$ is pre-hilbert. Now, we can find absolutely convex pre-hilbertian neighbourhoods of zero W and V such that $W \subset V \subset U$ and

$$E(W) \xrightarrow{\text{dual nuclear}} E(V) \xrightarrow{\text{nuclear}} E(U).$$

Hence, there exist $(V_n) \subseteq (E'(V^0))' = \widehat{E(V)}$ and $(\psi_n) \subseteq E'(W^0)$ such that, for all $\varphi \in E'(V^0)$,

$$\varphi = \sum_n \langle \varphi, V_n \rangle \psi_n \quad (\text{convergence in } E'(W^0))$$

and

$$\sum_n \|V_n\|_{V^0} \|\psi_n\|_W = C < +\infty.$$

Without loss of generality, we may assume $C \geq 1$.

Now, if $\Lambda \in \mathcal{L}({}^2 E)$ and $|\Lambda|_U < +\infty$, we know that $\Lambda \in \mathcal{L}_N({}^2 E(V))$ and hence $\Lambda \in \mathcal{L}_N({}^2 E(W))$. Therefore, there exists an $\alpha \in (\mathcal{L}_N({}^2 E(W)))'$, $\pi_W(\Lambda) = \langle \Lambda, \alpha \rangle$ and $|\langle \Lambda', \alpha \rangle| \leq \pi_W(\Lambda')$, for all $\Lambda' \in \mathcal{L}_N({}^2 E(W))$. In particular, if $\varphi_1, \varphi_2 \in E'(V^0)$, then $\varphi_1 \varphi_2 \in \mathcal{L}({}^2 E(W))$ and

$$|\langle \varphi_1 \varphi_2, \alpha \rangle| \leq \|\varphi_1\|_W \|\varphi_2\|_W.$$

Since $E(W) \rightarrow E(V)$ is dual nuclear,

$$\langle \varphi_1 \varphi_2, \alpha \rangle = \sum_n \langle \varphi_1, V_n \rangle \langle \psi_n \varphi_2, \alpha \rangle.$$

Now, as $A \in \mathcal{L}_N(^2E(V))$, A may be represented in the form

$$A = \sum_p \varphi_r \bar{\varphi}_r, \text{ where } \varphi_r, \bar{\varphi}_r \in E'(V^0) \text{ and } \sum_r |\varphi_r|_V |\bar{\varphi}_r|_V < +\infty.$$

Therefore

$$\begin{aligned} \pi_\omega(A) &= \langle A, \alpha \rangle = \langle \sum_r \varphi_r \bar{\varphi}_r, \alpha \rangle = \sum_r \langle \varphi_r \bar{\varphi}_r, \alpha \rangle \\ &= \sum_r \sum_n \langle \varphi_r, V_n \rangle \langle \bar{\varphi}_r, \alpha \rangle = \sum_n \langle \bar{\varphi}_r, \sum_r \langle \varphi_r, V_n \rangle \bar{\varphi}_r \rangle, \alpha \rangle \\ &\leq \sum_n |\bar{\varphi}_r|_W \left| \sum_r \langle \varphi_r, V_n \rangle \bar{\varphi}_r \right|_W \leq \sum_n |\bar{\varphi}_r|_W \left| \sum_r \langle \varphi_r, V_n \rangle \bar{\varphi}_r \right|_V \\ &\leq \sum_n |\bar{\varphi}_r|_W |V_n|_{V^0} \Sigma_V(A) = C \Sigma_V(A) \leq C^2 \Sigma_V(A). \end{aligned}$$

COROLLARY 8. - Let U be an absolutely convex neighbourhood of zero in the fully nuclear space E . Then, there exist $C' > 0$ and absolutely convex neighbourhoods of zero W and V , where $W \subset V \subset U$ such that, if $p \in P(^mE)$ and $|p|_U < +\infty$, then $p \in P_N(^mE(W))$ and

$$\pi_W(p) \leq (C')^m \epsilon_V(p) = (C')^m |p|_V, \text{ for all } m \geq 1.$$

Proof. - Let p be given, and let A_p be the symmetric m -linear form corresponding to p . Let C, W, V be as in lemma 7. Then, $A_p \in \mathcal{L}_N(^mE(W))$ and

$$\pi_W(p) \leq \frac{m^m}{m!} \pi_W(A_p) \leq C^m \frac{m^m}{m!} \epsilon_V(A_p) \leq C^m \left(\frac{m^m}{m!}\right)^2 \epsilon_V(p).$$

Hence, by choosing $C' = \sup_m (C^m (m^m/m!)^2)^{1/m}$,

$$\pi_W(p) \leq (C')^m \epsilon_V(p).$$

PROPOSITION 9. - Let E be a fully nuclear space. Then, $(P(^mE), \tau_\omega)' \simeq P_{HY}(^mE')$.

Proof. - We define $\beta : (P(^mE), \tau_\omega)' \rightarrow P_{HY}(^mE')$ by $\beta T(\varphi) = T(\varphi^m)$, for $T \in (P(^mE), \tau_\omega)'$ and $\varphi \in E'$. It is clear that β is well defined, and moreover as $P_f(^mE)$ is dense in $(P(^mE), \tau_\omega)$ (see [4]), it follows that β is $[-]$.

We now show that β is surjective. Let $p' \in P_{HY}(^mE')$. We define $T_{p'}$ on $P(^mE)$ as follows. If $p \in P(^mE)$, then $p \in P_N(^mE(U'))$, for some U' . Hence, $p = \sum_n \varphi_n^m$, where $|\varphi_n^m|_{U'} < +\infty$. We define $T_{p'}(p) = \sum_n p'(\varphi_n)$. $T_{p'}$ is well defined on $P(^mE)$ (see [8]), and we show that $T_{p'}$ is τ_ω -continuous. In fact, we show that $T_{p'}$ is ported by zero. Hence, let U be an absolutely convex neighbourhood of zero, and let C', V and W be as in corollary 8. Then, if $p \in P(^mE)$ and $|p|_U < +\infty$, we know p may be represented in the form $p = \sum_n \varphi_n^m$, where $\sum_n |\varphi_n^m|_W < +\infty$.

Hence,

$$|T_{p'}(p)| = \left| \sum_n p'(\varphi_n) \right| \leq |p'|_{W^0} \sum_n |\varphi_n^m|_W.$$

Therefore

$$|T_{p'}(p)| \leq |p'|_{W^0} \pi_W(p) \leq |p'|_{W^0} (C')^m \epsilon_V(p) \leq |p'|_{W^0} (C')^m \epsilon_U(p).$$

Hence $T_{p'}$ is ported by zero. Since $\beta T_{p'} = p'$, this complete the proof.

COROLLARY 10. - Let E be a fully nuclear space. Then

$$(P_{HY}({}^mE), \tau_0)'_{\beta} = (P({}^mE), \tau_w)'$$

Proof. - $(P_{HY}({}^mE), \tau_0)$ is a complete nuclear space and hence, semi-reflexive (see [3]). By proposition 9, $(P({}^mE), \tau_w)'$ is isomorphic to $P_{HY}({}^mE)$. Let τ_{β} denote the topology on $P({}^mE)$ induced by $(P_{HY}({}^mE), \tau_0)'_{\beta}$. Then $(P({}^mE), \tau_w)$ and $(P({}^mE), \tau_{\beta})$ have the same dual. τ_w is bornological and hence Mackey. Since τ_0 is semi-reflexive on $P_{HY}({}^mE)$, τ_{β} is barrelled and hence also Mackey. Therefore, $\tau_{\beta} = \tau_w$ on $P({}^mE)$, completing the proof.

PROPOSITION 11. - Let E be fully nuclear. If τ_w bounded subsets of $P({}^mE)$ are locally uniformly bounded, then $(P({}^mE), \tau_w)'_{\beta} = (P_{HY}({}^mE), \tau_0)$.

Proof. - Let U an absolutely convex neighbourhood of zero in E and $\epsilon > 0$ be given. We will show that there exists a τ_w bounded subset B of $P({}^mE)$ such that $B^0 \subset \{p' ; p' \in P_{HY}({}^mE), |p'|_U \leq \epsilon\}$. Let $B = \{p \in P({}^mE), |p|_U \leq \frac{1}{\epsilon}\}$. Then, if $T \in B^0$, $|\beta T(\varphi)| = |T(\varphi^m)| \leq \epsilon$, for all $\varphi \in U^0$. Hence

$$B^0 \subset \{p' ; p' \in P_{HY}({}^mE), |p'|_U \leq \epsilon\}.$$

Conversely, let B be a given bounded subset of $(P({}^mE), \tau_w)$. By assumption, there exists an absolutely convex neighbourhood of zero U and $\alpha > 0$ such that $B \subset \{p ; p \in P({}^mE), |p|_U < \alpha\}$. Let C', W, V be as in corollary 8, and let $\gamma = \{p' ; p' \in P_{HY}({}^mE), |p'|_W \leq (1/(C')^m \alpha)\}$. Then

$$|T_{p'}(p)| \leq |p'|_W \pi_W(p) \leq \frac{1}{(C')^m \alpha} (C')^m |p|_U,$$

for all $p \in P({}^mE)$ such that $|p|_U < +\infty$. Hence $\gamma \subset B^0$ and this completes the proof.

4. Some examples.

Remark 12. - If E is a Fréchet nuclear space or a $\mathcal{QFN}$ space, then τ_0 and τ_w bounded subsets of $P({}^mE)$ are locally uniformly bounded (see [1]). In this case, as τ_w is bornological, we have that $\tau_{0,b} = \tau_w$, where $\tau_{0,b}$ is the bornological topology associated with τ_0 . However, since a Fréchet nuclear space or a dual of Fréchet nuclear space is a K -space, it follows from corollary 10 and proposition 11 that $(P({}^mE), \tau_0)$ is reflexive and hence $\tau_0 = \tau_w$ on $P({}^mE)$, whenever E is a Fréchet nuclear or dual of Fréchet nuclear space.

Example 13. - The space $E = \prod_N C \times \sum_N C$ is a fully nuclear space which is neither a FN or $\mathcal{QFN}$ space. Although τ_0 bounded subsets of $P({}^mE)$ are not necessarily locally uniformly bounded.

Therefore

$$((P({}^mE), \tau_0)'_{\beta})'_{\beta} = (P_{HY}({}^mE), \tau_0)$$

and $(P_{HY}({}^mE), \tau_0)$ is reflexive. Note that $\tau_0 \neq \tau_w$ on $P({}^mE)$, but

$$(P(^mE), \tau_0)'_\beta = (P(^mE), \tau_\omega) \quad \text{and} \quad (P(^mE), \tau_\omega)'_\beta = (P_{HY}(^mE), \tau_0) .$$

Example 14. - Let $E = \mathbb{Q}$. Then, E is a fully nuclear space and $P(^m\mathbb{Q}) \neq P_{HY}(^m\mathbb{Q})$ while $P(^m\mathbb{Q}') = P_{HY}(^m\mathbb{Q}')$, for $m > 1$. Also $\tau_0 = \tau_\omega$ on $P(^m\mathbb{Q})$ while $\tau_{0,b} = \tau_\omega \neq \tau_0$ on $P(^m\mathbb{Q}')$. τ_0 bounded subsets of $P(^m\mathbb{Q}')$ are locally uniformly bounded. However, τ_0 bounded subsets of $P(^m\mathbb{Q})$ are not locally uniformly bounded. Otherwise, by corollary 10 and proposition 11, τ_0 would be reflexive and this would imply $\tau_0 = \tau_{0,b}$ on $P(^m\mathbb{Q}')$.

BIBLIOGRAPHY

- [1] BARROSO (J. A.), MATOS (M. G.) and NACHBIN (L.). - On bounded sets of holomorphic mappings, "Infinite dimensional holomorphy [Lexington, 1973]", p. 123-134. - Berlin, Springer-Verlag, 1974 (Lecture Notes in Mathematics, 364).
- [2] BOLAND (P. J.). - Malgrange theorem for entire functions on nuclear spaces, "Infinite dimensional holomorphy [Lexington, 1973]", p. 135-144. - Berlin, Springer-Verlag, 1974 (Lecture Notes in Mathematics, 364).
- [3] BOLAND (P. J.). - An example of a nuclear space in infinite dimensional holomorphy, Arkiv für Mat., t. 15, 1977, p. 87-91.
- [4] BOLAND (P. J.) and DINEEN (S.). - Holomorphic functions on fully nuclear spaces, Bull. Soc. math. France, t. 106, 1978, p. 311-336.
- [5] BOLAND (P. J.) and DINEEN (S.). - Duality theory for spaces of germs and holomorphic functions on nuclear spaces (to appear).
- [6] DINEEN (S.). - Holomorphic functions on locally convex topological vector spaces, I : Locally convex topologies on $H(U)$, Ann. Inst. Fourier, Grenoble, t. 23, 1973, fasc. 1, p. 19-54.
- [7] DINEEN (S.). - Surjective limits of locally convex spaces and their applications to infinite dimensional holomorphy, Bull. Soc. math. France, t. 103, 1975, p. 441-509.
- [8] GUPTA (C.). - Convolution operators and holomorphic functions on a Banach space, Séminaire d'analyse moderne, n° 2, Université de Sherbrooke.
- [9] HORVATH (J.). - Topological vector spaces and distributions, Vol. 1. - Reading, Addison-Wesley, 1966 (Addison-Wesley Series in Mathematics).
- [10] KELLEY (J.). - General topology. - Princeton, Van Nostrand, 1955 (The University Series in higher Mathematics).
- [11] PIETSCH (A.). - Nuclear locally convex spaces. - New York, Springer-Verlag, 1972 (Ergebnisse der Mathematik, 66).