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BLANKA KOLIBIAROVÁ

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ON THE CONSTRUCTION OF SOME SEMIGROUPS

by Blanka KOLIBIAROVÁ

We study the properties and the construction of semigroups S , each left ideal of which contains a unique right identity.

Denote by $I(S)$ the set of all idempotents of S (elements of $I(S)$ by e , with indices if needed), by $L(x)$ ($R(x)$) the set of all elements of S generating left (right) principal ideal $(x)_L$ ($(x)_R$). Then, $I(S)$ is a commutative subsemigroup of S , $I(S)$ is a well ordered set according to the relation

$$e_k \leq e_i \iff e_i e_k = e_k e_i = e_k.$$

In S , each element x belongs to some $L(e_i)$ and to some $R(e_k)$ class. Denote

$$L^*(e) = \{L(e) \cap R(e_i) ; e_i \leq e\}.$$

Then $L^*(e)$ is a subsemigroup of S with the two-sided identity e . Further, $L(e) = \{L(e_i) ; e_i \leq e\}$ is a subsemigroup of S with the two-sided identity e . The same for $R^*(e)$, $R(e)$.

Let S be a finite semigroup. Then S is a finite well ordered set of finite groups.

Every semigroup S , with $I(S)$ of the type ω , and with $L(e_i) \cap R(e_k) = D_{ik}$ containing just one element for each e_i, e_k , is isomorphic to the bicyclic semigroup. If S has the classes D_{ik} containing at least one element, then S can be homomorph mapped into bicyclic semigroup (preserving D_{ik}).

The semigroups of required properties, if D_{ik} contain at most one element, can be constructed in a simple manner if $I(S)$ is of the type ω . Firstly, we define in $I(S)$

$$e_i e_k = e_k e_i = e_k \text{ if } e_k \leq e_i.$$

Then we assign to each $e_i \in I(S)$ two well ordered sets $L^*(e_i)$ and $R^*(e_i)$ orderisomorphic if both are non empty, having the property: Let e be the first element of $I(S)$ with a non empty $L^*(e)$. We begin to define the multiplication between $L^*(e)$ and $I(S)$; for $x \in L^*(e)$, and e_i where $e \leq e_i$, let

$$xe_i = e_i x = x.$$

To each x , we choose a fixed $e_k < e$, this choice is to be orderpreserving (the

mapping of $L^*(e)$ into $I(S)$ denote by α), and $e_m x = x$ for $e_k \leq e_m$. Now we define the multiplication in $L^*(e)$:

$$x_1, x_2 \in L(e), \quad \alpha x_1 = e_1 < \alpha x_2 = e_2,$$

then

$$x_1 x_2 = x_2 x_1 = x_3 \in L^*(e) \quad \text{with} \quad \alpha x_3 = e_3 \quad \text{where} \quad \langle e_1, e_3 \rangle \simeq \langle e, e_2 \rangle.$$

Further to each $e_i < e$, we assign $L(e_i)$ orderisomorphic to $L^*(e)$ (the mapping φ). We continue in the definition of the multiplication between $L^*(e)$ and $L^*(e_i)$, using α and φ . Similarly, for $R^*(e)$, $R^*(e_i)$ (mappings β and ψ), at last between $R^*(e)$ and $L^*(e_i)$, using α , β , φ , ψ .

Remark. - If we have choosen $L^*(e)$ for some e , and in $L^*(e_i)$ would exist some element x which is not an image of an element of $L^*(e)$, this x requires, for each $y \in L^*(e)$, $\alpha y = e_1$, the existence of $y' \in L^*(e)$ where, if $\alpha x = e_k$, then $\alpha y' = e_2$ with $\langle e_1, e_2 \rangle \simeq \langle e_i, e_k \rangle$. Respecting this, we can construct each semigroup of required property.

This construction can be adapted to the other cases.

Blanka KOLIBIAROVÁ
Ass. Prof. Techn. Univ.
Gottwaldovo nám., 2
BRATISLAVA (Tchécoslovaquie)

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