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ON THE COMPUTATION OF THE ORDER OF JANKO'S FIRST SIMPLE GROUP $\mathfrak{F}_1$

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Janko's computation of the order of $\mathfrak{F}_1$ contains a mistake as the generalized character φ'_4 of [1; p. 156] does not vanish on non special classes of $N(U)$ in case (i) of [1; Lemma 3.1]. The purpose of this note is to correct this mistake.

We use terminology and notation of [1].

Case (i) of Lemma 3.1.

Here $N(U) \subset C(t)$. Thus, $T = \langle t, t_1 \rangle$ is a S_2 -subgroup of $N(U)$, where t_1 is an involution of F which acts invertingly on U .

We have $U\langle t_1 \rangle \times \langle t \rangle = N(U)$.

The group $N(U)/\langle t \rangle$ has two linear characters: ξ'_0 (the principal character), β' , and two irreducible characters ξ'_1, ξ'_2 of degree 2.

Let ξ' be the nontrivial linear character of $N(U)/(N(U) \cap F)$. Put $U = \langle \mu \rangle$. We get the following character-table of $N(U)$.

Let the set of « special classes » of $N(U)$ in the sense of Suzuki [2] consist of the conjugate classes of $\mu, \mu^2, t\mu, t\mu^2$ in $N(U)$.

The module (over the ring of integers) of the generalized characters of $N(U)$ vanishing on non special classes of $N(U)$ has the following basis:

$$\begin{aligned}\varphi'_1 &= \xi'_2 - \xi'_1 \\ \varphi'_2 &= \xi' \cdot \xi'_2 - \xi' \cdot \xi'_1 \\ \varphi'_3 &= \xi'_0 + \beta' - \xi'_1 \\ \varphi'_4 &= \xi' + \xi' \cdot \beta' - \xi' \cdot \xi'_1.\end{aligned}$$

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	ξ'_0	β'	ξ'_1	ξ'_2	ξ'	$\xi' \cdot \beta'$	$\xi' \cdot \xi'_1$	$\xi' \cdot \xi'_2$
1	1	1	2	2	1	1	2	2
μ	1	1	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	1	1	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$
μ^2	1	1	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	1	1	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$
t_1	1	-1	0	0	1	-1	0	0
t	1	1	2	2	-1	-1	-2	-2
$t\mu$	1	1	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	-1	-1	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$
$t\mu^2$	1	1	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	-1	-1	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$
tt_1	1	-1	0	0	-1	1	0	0

We have the following decomposition of the induced characters φ_i^{**} :

$$\varphi_1^{**} = \overline{\varepsilon_1 x_1} + \overline{\varepsilon_2 x_2}$$

$$\varphi_2^{**} = \overline{\eta_1 y_1} + \overline{\eta_2 y_2}$$

$$\varphi_3^{**} = 1_G + \overline{\varepsilon_1 x_1} + \overline{\zeta_2 z_2}$$

$$\varphi_4^{**} = \overline{\eta_1 y_1} + \overline{\theta_2 v_2} + \overline{\theta_3 v_3}$$

where $\bar{x}_i, \bar{y}_i, \bar{z}_2, \bar{v}_i$ are nontrivial, different irreducible characters of G and $\bar{\varepsilon}_i, \bar{\eta}_i, \bar{\zeta}_2, \bar{\theta}_i$ are all ± 1 .

If χ is an irreducible character of G which appears in φ_i^* with the multiplicity $n_i (1 \leq i \leq 4)$, then

$$\chi(\sigma) = n_1 \xi'_2(\sigma) + (n_2 - n_4) \xi'_1(\sigma) \cdot \xi'_2(\sigma) + n_3 - n_4 \xi'_1(\sigma) \cdot \xi'_1(\sigma)$$

where σ is an arbitrary element of the set of « special classes » of $N(U)$.

Hence we get:

$$\bar{x}_1(t\mu) = \frac{1}{2} \bar{\varepsilon}_1(1 + \sqrt{5}) \text{ and } \bar{z}_2(t\mu) = \bar{\zeta}_2$$

$$\bar{x}_1(t\mu^2) = \frac{1}{2} \bar{\varepsilon}_1(1 - \sqrt{5}) \text{ and } \bar{z}_2(t\mu^2) = \bar{\zeta}_2.$$

In Case (1) of [1; p. 155] we get $\bar{x}_1 \in \{y_1, y_2, y_3, y_4\}$ and $\bar{z}_2 \in \{x_3, v_1, v_2\}$. Since $0 = \varphi_3^*(t)$, we get $\bar{x}_1 \in \{y_1, y_2, y_3, y_4\}$ and $\bar{z}_2 \in \{v_1, v_2\}$.

In Case (2) of [1; p. 156] we get $\bar{x}_1 \in \{x_1, x_2, y_3, y_4\}$ and $\bar{z}_2 \in \{z_4, v_1, v_2\}$. Since $0 = \varphi_3^*(t)$, we get $\bar{x}_1 \in \{y_3, y_4\}$ and $\bar{z}_2 \in \{v_1, v_2\}$.

Therefore, in both cases we get: $\bar{x}_1(t) = \pm 3$ and $\bar{z}_2(t) = \pm 4$. We shall apply the Suzuki order formula of [2] for the generalized characters $\varphi'_3 = \xi'_0 + \beta' - \xi'_1$, $\varphi_3^* = 1_G + \bar{\varepsilon}_1 \bar{x}_1 + \bar{\zeta}_2 \bar{z}_2$ and the subgroup $N(U)$ of G .

Put $f = \bar{\varepsilon}_1 \bar{x}_1(1)$, thus $f + 1 = -\bar{\zeta}_2 \bar{z}_2(1)$ as $0 = \varphi_3^*(1)$.

Denoting $|G|$ by g we obtain

$$\frac{1}{g} \left\{ \frac{g^2}{120^2} \left(1 + \frac{3^2}{f} - \frac{4^2}{f+1} \right) \right\} = \frac{1}{20} \left\{ \frac{11^2}{1} + \frac{9^2}{1} - \frac{2^2}{2} \right\} = 10.$$

Hence, $g \cdot (f-3)^2 = 2^7 \cdot 3^2 \cdot 5^3 \cdot f \cdot (f+1)$.

We know that both f and $f+1$ divide g and that f and $f+1$ are coprime. This implies that $f-3$ is a divisor of $2^3 \cdot 3 \cdot 5$.

On the other hand a S_2 -subgroup of G has order 8 and U is a S_5 -subgroup of G . This implies $2^2 \cdot 5$ divides $f-3$.

Therefore we have the following possibilities:

$$f-3 = \pm 20, \pm 40, \pm 60, \pm 120.$$

It is easy to see that all eight possibilities lead to a contradiction. For example

$$f-3=20 \text{ implies } g \cdot 20^2 = 23 \cdot 24 \cdot 2^7 \cdot 3^2 \cdot 5^3$$

which is impossible as 7 divides g .

Similarly $f-3=-60$ implies $g=2^6 \cdot 3 \cdot 5 \cdot 7 \cdot 19$ which is against the order of a S_2 -subgroup of G .

Thus, we have ruled out Case (i) of Lemma 3.1.

REFERENCES

- [1] JANKO, Z.: *A new finite simple group with abelian Sylow-2-subgroups and its characterization*, Journal of Algebra 3 (2) (1966), 147-186.
- [2] SUZUKI, M.: *Applications of group characters*, Proc. Symp. Pure Math. 1 (1959), 88-89.

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