

# Revue d'Histoire des Mathématiques



*Sanskrit versus Greek 'Proofs':  
History of Mathematics at the Crossroads  
of Philology and Mathematics  
in Nineteenth-Century Germany*

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**SANSKRIT *VERSUS* GREEK ‘PROOFS’:  
HISTORY OF MATHEMATICS AT THE CROSSROADS  
OF PHILOLOGY AND MATHEMATICS  
IN NINETEENTH-CENTURY GERMANY**

IVAHN SMADJA

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**ABSTRACT.** — The present paper is intended as a contribution to a critical history of historiography of mathematics, in which history of mathematics is regarded as closely connected to cultural history as a whole. The focus is on analyzing the ways in which a contrast between Sanskrit and Greek mathematics was constructed in nineteenth-century Germany, as Colebrooke’s English translations of Sanskrit mathematical sources spread in both philological and mathematical circles. By keeping track of the shifting significance which this contrast took on for different protagonists operating within different social contexts, one is also provided with a distinctive thread so as to unfold a more encompassing narrative. From a broader perspective, our purpose is to highlight the shaping of history of mathematics at the crossroads of philology and mathematics within nineteenth-century German academia.

**RÉSUMÉ** (Les ‘preuves’ sanskrites comparées aux grecques : l’histoire des mathématiques au croisement de la philologie et des mathématiques en Allemagne du dix-neuvième siècle)

Le présent article se propose de contribuer à une histoire critique de l’historiographie des mathématiques, dans laquelle l’histoire des mathématiques est envisagée en lien étroit avec l’ensemble de l’histoire culturelle. Nous analysons précisément comment une opposition entre mathématiques sanskrites et mathématiques grecques a été construite en Allemagne au dix-neuvième siècle, alors que les traductions par Colebrooke de sources mathématiques sanskrites

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se diffusaient parmi les philologues et les mathématiciens. En retracant comment cette opposition fut successivement investie de sens différents par des acteurs différents, opérant dans des contextes sociaux différents, nous disposons d'un fil conducteur pour développer une histoire croisée de la philologie et des mathématiques en Allemagne au dix-neuvième siècle, mettant ainsi en lumière la formation de l'histoire des mathématiques au confluent de ces disciplines.

## 1. INTRODUCTION

Recent trends in the historiography of mathematical proof in ancient traditions combine a renewed approach to the sources with a reflective stance intent on carefully analyzing the historical processes through which previous historiographical frameworks were shaped.<sup>1</sup> In this regard, a growing awareness on the part of historians of mathematics, that long prevailing views might eventually prove inadequate, leads to the vindication that mathematical proofs should not be deemed the exclusive apanage of Greek mathematics, insofar as varieties of ‘proofs’ may also arguably occur in Akkadian, Chinese and Sanskrit sources. Correlatively, the grids through which these sources have been addressed by scholars at various times, in various historical settings, are subjected to historical scrutiny. The present paper purports to contribute to this critical history of historiography by investigating how and why a contrast between Sanskrit and Greek mathematics was suggested, elaborated and reframed in nineteenth-century Germany, as Colebrooke’s translations of Sanskrit mathematical sources spread in both philological and mathematical circles. By keeping track of the shifting significance which this contrast took on for different protagonists operating within different social contexts, one will be provided with a distinctive thread so as to unfold a more encompassing narrative. From a broader perspective, the goal pursued in the following pages is to highlight the shaping of history of mathematics at the crossroads of philology and mathematics within nineteenth-century German academia. In this connection, emphasis will be laid on the momentous role played at this intersection by the mathematician and historian of mathematics, Hermann Hankel (1839–1873), who achieved an insightful reading of Sanskrit mathematical sources, owing to his conjoining philological rigour and mathematical expertise. Combining the *habitus* and skills of both fields

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<sup>1</sup> See Chemla [2012b] for both a comprehensive overview of such historiographical trends and a unifying research program.

was not unprecedented among nineteenth-century German scholars, although it remained the privilege of a very few, such as Georg Heinrich Ferdinand Nesselmann (1811–1881) or Franz Woepcke (1826–1864). However, before Hankel, none had ever addressed Sanskrit mathematical sources from the standpoint of philology and mathematics. In so doing, he made the contrast between Sanskrit and Greek mathematics into a tool for self-understanding, intended to make sense of modern mathematics, over and above the main guideline for a rewritten history of mathematics. Still, Hankel drew on previous work. In the early 1850s already, Arthur Arneth (1802–1858), a professor of mathematics at the Heidelberg *Lyceum*, had articulated a stark contrast between Indian and Greek mathematics, although quite differently than Hankel would later think of it in the early 1870s. In return, Arneth's naturalistic history of mathematics owed much to the cultural history professed at about the same time by the Heidelberg philosopher Eduard Röth (1807–1858), whose untimely Creuzerian flavor then repelled mainstream German philologists. Our reconstructed narrative aims at making clear the series of contextual shifts which eventually made it possible for mathematics to meet philology. It will be shown along the way that history of mathematics came to be thoroughly reassessed, as those Sanskrit mathematical sources to which Colebrooke had first called the attention of European scholars, were being taken into account in significantly different ways at different stages of that historical process.

A colonial administrator of the East India Company and a Sanskrit scholar, Henry Thomas Colebrooke (1765–1837) marked a turning point in Western writing on India by setting high standards of accuracy, rigour and thoroughness, which fostered the making of Indology as a professional discipline.<sup>2</sup> The outstanding collection of Sanskrit manuscripts he brought back from India constituted a rich fund which he made available for further research.<sup>3</sup> Generations of German scholars who, from Franz Bopp, August Wilhelm Schlegel, Christian Lassen to Friedrich Rosen, established personal connections with him and benefitted from his advice and guidance on Indian matters, contributed in return to his

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<sup>2</sup> Our source here is the important work of Rosane and Ludo Rocher, cf. Rocher & Rocher [2012] and Rocher & Rocher [2013].

<sup>3</sup> Colebrooke's collection of Indian manuscripts on a wide variety of matters ranging from medicine, astronomy, grammar, law and Vedic literature was bequeathed to the library of the East India Company on April 15, 1819, which shifted the center of Western Orientalism from Paris to London, see [Rocher & Rocher 2012, pp. 139–140].

broad reception in Germany.<sup>4</sup> Colebrooke translated two mathematical texts by the twelfth-century Indian astronomer Bhāskara, the *Lilāvati* and the *Bija-ganita*,<sup>5</sup> as well as mathematical chapters from the *Brāhma-sphuṭa-siddhānta*, an earlier astronomical treatise by the seventh-century mathematician Brahmagupta. In making these translations a canonical corpus for Indian mathematics, his German followers proved to be lastingly dependent on Colebrooke's expertise.<sup>6</sup> In particular, they would tend to adopt the interpretation of the base text offered by the ancient Sanskrit commentaries he had selected and from which he occasionally provided extracts in his footnotes. In supplementing the base text with explanations, 'proofs' and procedures, excerpted from those commentaries, he provided his European contemporaries with an editorial artefact which enduringly fashioned the way they would address these Sanskrit sources.

Two aspects of Colebrooke's work should be stressed at the outset. Firstly, the *Dissertation* introducing his translations decisively shaped German scholarship in "inadvertently certif[ying] the boundary line drawn between Indian algebra and Greek geometry"<sup>7</sup>, as a result of his striving to adequately feature Indian mathematics in comparison with Greek and Arab traditions. In this sense, Colebrooke happened to suggest a contrast between Indian and Greek mathematics which he himself never intended as such. Secondly, Colebrooke claimed that there were 'proofs'

<sup>4</sup> Among these, A. W. Schlegel deserves special mention not only for his enduring correspondence with the British Indologist (cf. Rocher & Rocher [2013]), but also for involving him in German academic agendas, see for instance [Rocher & Rocher 2012, p. 202]: "Early German Indologists, who approached Sanskrit as another classical language and wished Sanskrit documents to be treated according to the demanding rules of classical philology, uniformly singled out Colebrooke as the only British scholar who lived up to their expectations."

<sup>5</sup> In the present paper, the international standard ISO 15919 for transliterating Sanskrit into Latin characters is adopted.

<sup>6</sup> Dhruv Raina has shown how, in taking up the working program set up by previous British Indologists for sifting mathematical procedures out of Indian astronomical texts, Colebrooke collated various fragments of the works of Bhāskara and Brahmagupta, owing to elaborate editing practices of his own, in which the information directly supplied by his Brahmin interlocutors was used to sequence and complete those fragments, so as to fashion the text into a finalized form. Cf. [Raina 2012, pp. 239–240].

<sup>7</sup> Cf. [Raina 2012, 246]: "This was not Colebrooke's intention at all, but a consequence of the comparative method he had adopted. Colebrooke's particular comparative method consisted in displaying where India's specific contributions to mathematics resided, and he always contrasted these contributions with the Greek and Arab traditions of mathematics. This attempt to accentuate the contrast certainly revealed the differences, but with the loss of the context of the contrast, it was first transformed into a caricature and then stabilized as a characterization."

in Sanskrit mathematics. He argued that a kind of algebraic analysis could be discerned in Indian mathematical works, an 'analytic art' which "as Hindu writers observe, is merely sagacity exercised", independently of symbols, "calculation attended with the manifestation of its principles", or "a method aided by devices, among which symbols and literal signs are conspicuous"<sup>8</sup>—a method he likened to d'Alembert's conception of analysis, which was, as Karine Chemla points out<sup>9</sup>, the prevailing view of analysis at the turn of the century when rigour had not yet become a central issue.

In the early 1870s, the historiographical landscape had notably changed, as Thomas Edward Colebrooke's biography of his father bears witness. The former's overall appreciation indeed sounded a markedly different note from the latter's. Unlike his father, as Dhruv Raina emphasizes, Colebrooke's son claimed that Indian works would contain "mere rules for practice, and not a word on the path by which they were arrived at", or more pointedly "nothing of the rigour of the ancient geometry"<sup>10</sup>. Within slightly more than half a century, one thus presumably shifted from Colebrooke's case for the existence of demonstrative arguments in Sanskrit mathematics to what Raina called the supposedly hegemonic 'historiography of the absence of proofs'.<sup>11</sup>

However, at about the same time, Hermann Hankel, imbued with the new standards of mathematical rigour ever since his formative years as a student of Weierstrass in Berlin, produced an insightful reading of Indian sources which contrasted Indian supposedly "intuitive proofs" with Greek deductive ones. In so doing, he united two claims which were previously held separately. On the one hand, in agreement with Colebrooke's original approach, he claimed that there were proofs of some kind in Sanskrit sources. But on the other hand, unlike Colebrooke, albeit like Arneth, he contrasted Sanskrit and Greek mathematics. By bringing together these two stances, he refined the cultural contrast between two ways of practicing mathematics into a more definite contrast between Greek and Sanskrit 'proofs', as shown in the following striking passage from his posthumous book *Zur Geschichte der Mathematik in Alterthum und Mittelalter* (1874).

The image of geometry as stepping back far behind arithmetic and algebra, on display with the Brahmins, is entirely different from that of the Euclidean

<sup>8</sup> [Colebrooke 1817, pp. xix-xx].

<sup>9</sup> [Chemla 2012b, p. 6].

<sup>10</sup> [Colebrooke 1873, p. 309].

<sup>11</sup> [Raina 2012, p. 248].

Elements. We find there neither definitions, nor axioms, no series of firmly connected theorems in which each theorem rests upon the previous ones and proves the following ones. There, any proposition stands out independently, as a fact [*wie ein Factum*]. And if the commentators give us some informations on the way the certainty of a proposition must be substantiated, we see with amazement that they do not proceed according to the Greek way, in drawing auxiliary lines first and then in citing many propositions from the logical connection of which the theorem follows; rather the proposition of the hypotenuse is the only one which they apply expressly; intuition teaches them all the others, either directly, or according to a certain directive. The single word "See!" next to the figure being provided with the necessary auxiliary lines, replaces, with the Brahmins, the proof of the Greeks ending with the solemn "Which was to be demonstrated". Everything that an experienced sense could recognize by the sustained consideration of a figure was to be admitted as sure.<sup>12</sup>

Hankel's inspiring analyses interspersed in his work elicited abundant follow-ups from generations of professional historians of mathematics who echoed them in many distorted variations, although accommodating them to different historiographical agendas.<sup>13</sup> As a result of uncritical endorsement, Hankel's philologically informed, contrastive appraisal of Sanskrit *vs.* Greek mathematics, uprooted from its original context and severed from the sound scholarship supporting it, came to be all too often downgraded into a worn-out stereotype opposing more broadly the 'logical Greek' to the 'imaginative Oriental', a shallow view which circulated in the late nineteenth and early twentieth century.<sup>14</sup> As one might expect, the making of that commonplace involved smoothing differences, sidestepping subtleties, rounding off significant edges into a uniform unquestioned belief whose grounds and motivations were ultimately erased. In this process, Hankel's claim that Sanskrit sources contained 'proofs' came to be obliterated in favor of the widespread view that Indian mathematicians were not concerned with proof or logical justification whatsoever, but only with numerical computations. This view which prevailed among historians of mathematics for a long time, mostly because only a few Sanskrit mathematical texts were available, beyond a narrow circle of specialists, gradually subsided in recent years as a growing

<sup>12</sup> [Hankel 1874, p. 205]. Unless otherwise notified, all translations from the German are mine.

<sup>13</sup> Among those historians of mathematics, one may mention Moritz Cantor, Carl Anton Bretschneider, Hieronymus Georg Zeuthen, Paul Tannery, Johan Ludvig Heiberg, Gustav Eneström, Anton von Braunmühl, Siegmund Günther, Friedrich Hultsch, Maximilian Curtze, Heinrich Suter, etc, cf. [Charette 2012, p. 279] and Dauben [2002] for the biographical notes.

<sup>14</sup> For a detailed history of this stereotype, see the work of François Charette, cf. Charette [2012].

number of these texts came to be more widely known. From the viewpoint of recent historical research on the issue of 'proofs' in Sanskrit mathematical sources, Hankel's perceptive elaboration on Colebrooke's work, can only strike the modern reader as anticipatory. Conversely, as Karine Chemla noted, contemporary historians claiming that 'proofs' also occurred in the Eastern sources they dealt with, "[*were*] in a way, [...] partly returning to a past historiography".<sup>15</sup>

Our initial focus in this article was on restoring the complexity of Hankel's original reading of Sanskrit sources by reconstructing the elaborate context in which, at the juncture of philology and mathematics, this reading resulted from the dynamics of both social fields interacting with one another. However, in the course of our research, new issues arose which required changing scales, crossing disciplinary viewpoints, and historicizing categories of analysis (*viz.* 'evidence', 'intuition', 'proof'). When confronting the empirical material thus gathered, we then wondered what kind of narrative should be built out of these data so that it could carry historical knowledge. The methodology which gradually proved to be the most suited to the current pursuit is akin to the approach that Michael Werner and Bénédicte Zimmermann labelled *histoire croisée*.<sup>16</sup> Let us briefly highlight the main aspects thereof that are of interest for our purpose. Being central to this approach, "intercrossings" [*croisements*] are understood as processes through which new meaning arises as a result of historical determinants concretely weaving together in definite situations. "The stress laid by *histoire croisée* on a multiplicity of possible viewpoints [*the authors insist,*] and the divergences resulting from languages, terminologies, categorizations and conceptualizations, traditions and disciplinary usages, adds another dimension to the inquiry. In contrast to the mere restitution of an 'already there', *histoire croisée* places emphasis on what, in a self-reflexive process, can be generative of meaning."<sup>17</sup> In this sense, "intercrossing" does not amount to merely registering a historical situation purportedly spread out in full view to all protagonists, previously to any intervention on their part. "It requires [*on the contrary*] an active observer to construct it and only in a to-and-fro movement between the researcher and object do the empirical and reflexive dimensions of *histoire croisée* jointly take shape. Intercrossing thus appears as a structuring cognitive activity

<sup>15</sup> Cf. [Chemla 2012b, p. 13].

<sup>16</sup> Cf. Werner & Zimmermann [2006]. The approach in terms of *histoire croisée* was originally framed to overcome certain methodological difficulties encountered in comparative history and transfer studies.

<sup>17</sup> Cf. [Werner & Zimmermann 2006, p. 32].

that, through various acts of framing, shapes a space of understanding.<sup>18</sup> In the case at hand, the construction of our object, the shaping of a thoroughly self-reflective contrast between Sanskrit and Greek ‘proofs’ in nineteenth-century Germany, involved an “intercrossing” which may be deemed “intrinsic to the object itself”,<sup>19</sup> namely the production of a specific reading of Sanskrit sources at the crossroads of philology and mathematics—Hankel being himself that “active observer” productive of new meaning, owing to his being immersed in various interlocking historical settings. Correlatively, the role of the historian, as it is conceived here, consists in unfolding those dynamic and generative processes, constitutive of the “intercrossing”, that were largely implicit knowledge for the actors themselves, while at the same time historicizing both the object itself and the views that were taken upon it, so as to reach reflective equilibrium and grasp more effectively the new meaning produced.<sup>20</sup>

In order to make historical sense of Hankel’s reading of Sanskrit sources, it will prove useful, for the sake of orientation, to draw a preliminary map upon which one may situate the main actors of our narrative, with their respective local contexts clearly differentiated (section 2). Since both Arneth’s and Hankel’s views of Sanskrit mathematics significantly depended on Colebrooke’s emphasis on diagrams as alleged visual ‘proofs’, then, for the purposes of consistent historicization, we will first scrutinize Colebrooke’s own editorial operation with regard to diagrams occurring in Sanskrit sources. Our focus will be on assessing the extent to which Colebrooke meant what his followers took him to suggest with respect to diagrammatic ‘proofs’ (section 3). By the same token, particular attention will be paid to the Sanskrit diagrams pertaining to the so-called Pythagorean theorem, for these diagrams offered a paradigm case upon which nineteenth-century European scholars thoroughly reflected. Since all the actors referred to those same diagrams, although reading them in significantly different lights, these will provide us with another thread

<sup>18</sup> Cf. [Werner & Zimmermann 2006, p. 39].

<sup>19</sup> Cf. [Werner & Zimmermann 2006, pp. 39–40].

<sup>20</sup> Cf. [Werner & Zimmermann 2003, p. 33], a previous articulation of Werner’s and Zimmermann’s views, in which the point is more clearly made: “Si elle n’ouvre pas au relativisme historique, l’histoire croisée ne s’inscrit pas davantage dans une logique de régression historique à l’infini. L’historicisation ne s’y confond pas avec une contextualisation qui pousserait toujours plus loin l’investigation historique, afin d’arriver à une représentation plus détaillée du passé et de ses rapports avec le présent. Elle est au contraire construite et circonscrite en fonction d’un objet et d’une problématique permettant d’identifier des temporalités pertinentes et ainsi de borner le processus d’historicisation.”

running throughout, intertwined with our whole narrative, hence yielding a yardstick for comparing various interpretations. On the basis of the preceding discussions, we will analyze the historical and social settings in which Röth's Creuzerian cultural history was both elaborated and antagonized (section 4), then examine Arneth's appropriation of Röth's mainlines within a naturalistic framework so as to construct a specific contrast between Indian and Greek mathematics (section 5). From this standpoint, it will be shown how, through the rise of a professionalized Sanskrit philology modeled upon classical philology (section 6), Hankel gathered various threads into an innovative self-reflective reading of Sanskrit sources (section 7). Another characteristic feature of the approach in terms of *histoire croisée* stipulates that the construction of contexts should be regulated by inner constraints, rooted in the concrete ways in which actors do relate to one another, and to the very object whose historicity is under scrutiny.<sup>21</sup> In this sense, accounting for Sanskrit philology gradually taking over the requirements previously promoted by *Altertumswissenschaft* proves to be necessary, for one can only correctly appreciate the way Hankel connects to Arneth—without however cancelling the historical distance separating them—if both positions are considered against the backdrop of this overall process. Eventually, by drawing upon another hallmark of *histoire croisée*, the distinction between “intermixings” and “intercrossings”,<sup>22</sup> the singularity of Hankel's reading of Sanskrit sources will be underscored. It will be shown that philology and mathematics, once intersecting, later parted ways at the turn of the century. Diagrams first occurring in Colebrooke's translations of Bhāskara II were then addressed in irreconcilable ways by philologists and mathematicians (section 8).

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<sup>21</sup> Cf. [Werner & Zimmermann 2006, p. 47]: “Reliance on specific situations makes it possible to escape the ‘convenient and lazy usage of context’ (*a criticism by Jacques Revel in* [Revel 1996, p. 25]) by rejecting its generic and pre-established nature and integrating a reflection on the principles governing its definition. Such a lazy usage is replaced by an analysis of the manner in which individuals actually connect themselves to the world, ... By focusing on specific situations, it is thus possible to get away from the external, often artificial, nature of the context in order to make it an integral part of the analysis.”

<sup>22</sup> [Werner & Zimmermann 2006, p. 38]: “Intercrossing can be distinguished from intermixing. The latter emphasizes the specificity of the product of hybridization (the interbred) and brings us beyond the original elements, the previously identified constitutive entities of the convergence. In contrast, *histoire croisée* is concerned as much with the novel and original elements produced by the intercrossing as with the way in which it affects each of the ‘intercrossed’ parties, which are assumed to remain identifiable, even in altered form.”

## 2. HISTORY OF MATHEMATICS IN NINETEENTH-CENTURY GERMANY: DIFFERENT ACTORS, DIFFERENT CONTEXTS

In 1872, a separate version in Italian of Hankel's chapter on Arabic mathematics, later to be included in his posthumous book on the history of mathematics, was issued in the *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*. In so contributing to the newly founded journal, edited by Baldassare Boncompagni (1821–1894),<sup>23</sup> Hankel affiliated himself with what he presented as “modern” historiography of mathematics, whose agenda he fully endorsed. While reviewing a book by Heinrich Suter, *Geschichte der mathematischen Wissenschaften* (1872),<sup>24</sup> in the same issue of the *Bullettino*, he blamed the author for ignoring the works of Jean-Jacques Sédillot (1777–1832) the father, Louis-Amélie Sédillot (1808–1875) the son, and Franz Woepcke, hence for failing to consult any of the original sources they recently made available, which in return may have prevented him from merely “repeating Wallis’, Weidler’s and Montucla’s assertions”<sup>25</sup>. The Sédillots, father and son, had strongly put the emphasis on Arabic sources as an integral part of a general history of sciences, thus devising a research program which Woepcke later began to realize.<sup>26</sup> However, one cannot account for Hankel’s special responsiveness to Sanskrit sources by merely referring to his being acquainted with historical work from France. Neither on the German side does it suffice to invoke early romantic folklore fascination for India. With the rise of Sanskrit philology matching up to the scientific standards of a professionalized discipline in the second half of the nineteenth century, it had long ago subsided. Many social and historical factors interwove within this timespan and resulted in shaping Hankel’s thorough interest in Sanskrit mathematics. Yet, as announced above, Hankel built on previous material,

<sup>23</sup> The foundation of Boncompagni’s *Bullettino* in 1868 was a turning point in the institutionalization process of the historiography of the exact sciences. Previously, no other European journal was entirely devoted to this topic, cf. [Charette 1995, pp. 165–166].

<sup>24</sup> In spite of this first imperfect essay, Heinrich Suter (1848–1922) was to become a distinguished historian of Arabic mathematics, working in the line of Sédillot and Woepcke, cf. [Charette 1995, p. 173]: “avant de devenir un historien distingué des mathématiques arabes, il avait commis un ouvrage général, qui fut vite oublié.”

<sup>25</sup> [Hankel 1872a, p. 298].

<sup>26</sup> For a more detailed presentation of both Sédillot and Franz Woepcke, cf. [Charette 1995, chap. 4, 5 and 6].

whatever the amount of rewriting and reconceptualizing his appropriation of it may have required. His main criticism of Suter makes this debt clear.

There is no mention at all of an essential chapter of history in the work in question, [*Hankel went on*,] namely the mathematics of the Indians for which we possess useful documents today, although not in large numbers. By meeting Greek mathematics among the Arabs, Indian mathematics then took the direction in which modern mathematics later developed. Arneth already had seriously drawn attention to the very important position occupied by the Indians in the general development of our science, and in this respect we must consider Mr. Suter's work as a step backward.<sup>27</sup>

Interestingly enough, Hankel added a footnote to his Italian paper on Arabic mathematics, a footnote later removed in the book, in which he “regret[ted] to be obliged to give up the exposition of the ancient development of Indian mathematics, and its characteristic methods compared to those of the Greeks”. Instead, he referred the reader on these matters “to the relevant chapters of *Die Geschichte der reinen Mathematik*, a work by A. Arneth, with which I am generally in agreement [*generalmente parlando, in accordo*]”.<sup>28</sup> Arthur Arneth taught mathematics and physics both at the Heidelberg *Lyceum* where he obtained a professorship in 1838,<sup>29</sup> and at the University where he was appointed to a position as a *Privatdozent* from 1828 until his death. Beside a few textbooks on elementary mathematics, he wrote in the early 1850s a complete history of mathematics which probably qualifies, at least in the German speaking world, as “the first precise formulation of the idea opposing the apodictic rationality of Greek mathematical practice to the more intuitive one of the Indians”.<sup>30</sup> However, Hankel's acknowledgment of his “general agreement” with Arneth should be considered with caution, for, as will be seen, the kind of history of mathematics he practised differed in significant ways from Arneth's. In this connection, one will have to account for reception occurring in spite of historical chasm.

Menso Folkerts, Christoph Scriba and Hans Wussing tentatively depict Arthur Arneth and Georg Heinrich Ferdinand Nesselmann as “opposite twins”, both being representatives of the first generation of nineteenth-century German historians of mathematics occupying antipodal positions

<sup>27</sup> [Hankel 1872a, p. 297].

<sup>28</sup> [Hankel 1872b, p. 347]. The work by Arneth to which Hankel refers is Arneth [1852].

<sup>29</sup> At the Heidelberg *Lyceum*, Arneth counted Heinrich Weber (1842–1913) among his pupils from 1853 to 1860.

<sup>30</sup> [Charette 2012, 279–280].

in the field. “Hankel’s *Zur Geschichte der Mathematik in Alterthum und Mittelalter*, [they claim,] may be regarded, as a kind of synthesis of the approaches taken by Nesselmann and Arneth.”<sup>31</sup> However, as shaped through distinct social trajectories, their respective objectives and methodologies would starkly contrast with one another.

Nesselmann entered the university of Königsberg in 1831, where he first studied mathematics for two years under C. G. J. Jacobi and Friedrich Richelot, before turning to oriental philology with Peter von Bohlen (1796–1840), who had himself learned Sanskrit privately with A.W. Schlegel in Bonn in 1824 and “could not speak highly enough of Colebrooke’s work”<sup>32</sup>. Nesselmann’s main contribution to the history of mathematics, his *Algebra der Griechen* (1842), stands out as the first masterly attempt, in the German speaking world, at a critical history of ancient mathematics based on a firsthand command of the sources.<sup>33</sup> It may also be seen as a milestone in the making of a new moral consensus with regard to the way ancient sources should be dealt with, insofar as some methodological rules were made explicit, so as to shape a new scientific ethos<sup>34</sup> for the scholarly

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<sup>31</sup> [Dauben 2002, p. 123].

<sup>32</sup> [Rocher & Rocher 2012, p. 186]. Although he came to England twice, from May to July 1831 and in July 1837, Bohlen apparently never met Colebrooke, and yet he established a personal connection with him, partly through A. W. Schlegel.

<sup>33</sup> Cf. [Nesselmann 1842, *Vorrede*, pp. ix–x], translated in [Dauben 2002, p. 115]: “I intended to write a *critical* history. I wanted to investigate the history of algebra not as it is taught traditionally, but as it results from a prolonged and conscientious study of the sources. But the basic critical element is doubt. For this reason, I did not accept any fact as such from older historical works until my own inspection had convinced me of their truth and reliability.”

<sup>34</sup> The expression “scientific ethos” is understood here in the sense in which the sociologist Robert Merton takes it, see for instance [Merton 1973, pp. 268–269]: “The ethos of science is that affectively toned complex of values and norms which is held to be binding on the man of science. The norms are expressions in the form of prescriptions, proscriptions, preferences, and permissions. They are legitimatized in terms of institutional values. [...] Although the ethos of science has not been codified, it can be inferred from the moral consensus of scientists as expressed in use and wont, in countless writings on the scientific spirit and in moral indignation directed toward contraventions of the ethos.”

use of citations.<sup>35</sup> Nesselmann interestingly reflected upon the historiographer's dilemma, a particularly acute one with respect to history of ancient mathematics. "Nothing is more usual and natural [*he acknowledged,*] when reading ancient works than our substituting the standpoint of the ancient authors with our own, [...] so that] it is one of the most difficult tasks of the historian to understand and convey the conceptual background not only of each author, but also of each period of time in its individual character, and at the same time to keep the modern viewpoint in mind, without lifting the ancient author from his peculiar sphere of thinking into our present one."<sup>36</sup> As regards Sanskrit mathematics proper, it should be noted however, for our current purpose, that Nesselmann barely touched upon Colebrooke's translations. In reviewing previous significant work on the history of algebra, he only mentioned Colebrooke, although in highly praising terms, among many other names, such as Libri, Chasles, Delambre, etc. "The erudite foreword to this marvelous work [*viz. Colebrooke's Dissertation, Nesselmann pronounced,*] contains very commendable contributions to the history of algebra, in particular with regard to its advancement in Asia. One indeed rarely finds so much informative material concentrated in so little space."<sup>37</sup> Nesselmann also mentioned the *De Algebra Indorum* (1821) by Friedrich Buchner, his former Gymnasium professor at Elbing, to whom his own work was dedicated as well as to C. G. J. Jacobi. In this "interesting and valuable excerpt from the previous work [*viz. Colebrooke's*] the author has the merit to have made the rather incomprehensible rules of the Indian mathematicians—as they are expressed, faithfully to the original, in Colebrooke's translation—available to our mathematicians, by transferring them into understandable formulas".<sup>38</sup>

<sup>35</sup> On Nesselmann's methodological rules, cf. [Nesselmann 1842, p. 37]: "From there, two imperative rules follow for the historiographer. First, he must not only cite exactly the passages upon which he wants to build *ex hypothesen* and from which he wants to draw important consequences, [...] but he must also cite them word for word and in the language of the original. I will follow this rule in my history even for the quotations from oriental authors, but for the sake of a majority of readers, I will add in this case a faithful translation. Second, one must only cite books that one has read by oneself, and never copy a foreign quotation without checking."

<sup>36</sup> Cf. [Nesselmann 1842, pp. 37–38]. His main observation in this regard concerns the various ways of generalization, see [Nesselmann 1842, p. 38]: "As far as subjective representation is concerned, there is for instance a tremendous difference, according as to whether a mathematician today deduces and expresses from the start a proposition, or a formula, in its whole generality, or an Ancient explains the matter through induction out of a few cases and then says, *et sic in infinitum.*"

<sup>37</sup> [Nesselmann 1842, p. 29].

<sup>38</sup> *Ibid.*

On the whole, in spite of his close acquaintance with Colebrooke's work, Nesselmann refrained from comparing Greek with Indian mathematics. In outlining the historical development of algebra, he restricted himself to pointing out "the interesting but in no way unprecedented fact that this science has been invented twice, in Greece (or at least by Greeks, probably in Alexandria) and in India, and apparently by both peoples independently of one another". Further speculations would be vain, he suggested, insofar as "we are unable, neither in Greece nor in India, to trace the stream of this new science back to its sources"<sup>39</sup>. Apart from cursory remarks on the Indian names for algebra<sup>40</sup>, Nesselmann thus sidestepped Sanskrit mathematical sources and focused on Diophantus. An interest in effectively elaborating on Colebrooke's riches hence arose on another side. Among German historians of mathematics, Arneth first took these riches into account in the framework of a wholly reshaped history of mathematics.

However, Arneth's own contribution best makes sense when envisaged in due historical perspective. As Folkerts, Scriba and Wussing already pointed out, he was himself deeply influenced by the Heidelberg philosopher Eduard Röth (1807–1858), whom he praised for having shown him how dualistic schemes could be put to good use for constructing worldwide cultural history. Although probably limited to Heidelberg circles, Röth's influence also extended to Moritz Cantor (1829–1920), whose early works on history of mathematics bear the mark of his mentor. As will be seen, Röth's whole approach to cultural history had a Creuzerian ring to it which, in the German context of the 1850s, met dismissive response from most of his contemporaries as being the badge of a retrograde battle. And yet, he left his imprint on both Arneth and Cantor, who in return contributed to shape history of mathematics as an independent undertaking in Germany. Being colleagues at the Heidelberg faculty of philosophy, where Röth and Cantor<sup>41</sup> respectively joined up with Arneth as fellow *Privatdozenten* in 1840 and in 1853, the three of them indeed rubbed shoulders for years. Although a little older than Röth, Arneth owed him the unifying view which presumably enabled him to grasp history of mathematics as a coherent whole. Cantor, for his part, felt a debt of gratitude toward the man "who showed [him] the way into

<sup>39</sup> [Nesselmann 1842, pp. 30–31].

<sup>40</sup> Cf. [Nesselmann 1842, pp. 44–45].

<sup>41</sup> Florian Cajori and J.E. Hofmann disagree though about whether Cantor was directly influenced by Arneth's work in history of mathematics, see [Cajori 1920, p. 22] and Hofmann [1971].

the historical-mathematical circle of investigations”<sup>42</sup>. Cantor indeed not only wholeheartedly embraced Röth’s methodology in uncritically relying on ancient sources in his first works, but he also took over many of his master’s most daring views, as for instance the teachings presumably drawn from Röth’s purported *tour de force* with regard to the Idalion tablet, which Cantor kept on acknowledging even years after Ewald’s refutation (*see below*)<sup>43</sup>. Since his first publications in the history of mathematics<sup>44</sup>, Cantor focused on the spreading of numeral systems across cultures which he himself later identified as the unifying thread running through all his work. On the basis of a highly questionable literal interpretation of the so-called E manuscript of Boethius’ geometry<sup>45</sup>, he, for instance, notoriously claimed that the Indo-arabic numerals transmitted to the Christian Middle Ages were of Alexandrian origin and could even be traced back from Alexandrian Neopythagorean circles to ancient Pythagoreanism.<sup>46</sup> In formulating the pivotal conjecture of his *Mathematische Beiträge zum Culturleben der Völker* (1863), a view which Hankel later dismissed as a merely “subjective hypothesis”<sup>47</sup>, Cantor still walked in Röth’s footsteps.<sup>48</sup>

<sup>42</sup> [Cantor 1863a, p. 84], cited by [Lützen & Purkert 1994, p. 3] where an overall view of Cantor’s work in history of mathematics is offered.

<sup>43</sup> Cf. [Cantor 1863a, pp. 115–116].

<sup>44</sup> Cf. Cantor [1856], Cantor [1863a].

<sup>45</sup> In this passage of Boethius’ geometry on the Pythagorean abacus, over which Chasles [Chasles 1837, Note XII, pp. 464–476] in the first place and then Nesselmann [Nesselmann 1842, pp. 92–104] had already pondered, one indeed puzzlingly finds written from the right to the left, 9 numerals, very much akin to the Indo-Arabic ones, together with their names. On Cantor’s interpretation of this passage, see [Cantor 1863a, p. 230]: Chapter XV deals with Boethius’ E manuscript, and Chapter XVI with Pythagorean numerals.

<sup>46</sup> Cf. [Lützen & Purkert 1994, p. 5]: “Today we know that the manuscript Geometry II, the earliest Latin work containing Arabic numerals, stems not from Boethius but rather from an unknown scholar who flourished in the first half of the eleventh century.”

<sup>47</sup> [Hankel 1874, p. 331].

<sup>48</sup> While elaborating on Chasles and Nesselmann, Röth noted that if considered authentic, Boethius’ account would conflict with the received view of Indo-Arabic numerals being transmitted to the Christian West by the Arabs in Medieval Spain (see [Röth 1858, 564–565]). But he also added that the names of the numerals are immediately recognized as being “Semitic, or rather Arabic or Chaldean” in a more or less corrupted form. “Since the so-called Chaldean, [Röth then argued,] or more correctly the Aramaic was spoken in Babylon, and from there, as my decipherment of the Cypriot inscription of Idalion [...] makes clear, spread over to Mesopotamia and a great part of the Near East; over to Syria and Phoenicia up to Asia Minor and Cyprus, then the Aramaic, or Babylonian-Phoenician origin of these number names would be explicable, since, on the account of the Ancients, Pythagoras borrowed a great part of his arithmetical knowledge from the Babylonians and the Phoenicians.” [Röth 1858,

Even after the German Sanskritist Georg Thibaut (1848–1914) had drawn attention in 1875 to the fact that ancient Vedic texts on ritual practice, the so-called *Śulba-sūtras*,<sup>49</sup> enforced the conclusion that Indian “peg and cord” procedures for brick altar construction involved knowledge of the Pythagorean theorem<sup>50</sup>, Cantor still persisted in his view that, as regards geometry, the Indians borrowed from the Alexandrians, hence that Pythagoras did not derive his geometry from India, but rather, as Röth had professed, from Egypt.<sup>51</sup> In this respect, it is significant that Cantor only renounced his entrenched conviction much later when he became acquainted with Albert Bürk’s work on Āpastamba’s *Śulba-sūtra*, which, as will be seen below, was to a great extent shaped by Hankel’s reading of Sanskrit sources.<sup>52</sup>

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p. 565]. Whereas Röth remained cautious, Cantor confidently followed his suggestion.

<sup>49</sup> See Thibaut [1875], where Thibaut commented on the best-known *Śulba-sūtras*, *viz.* those attributed to Baudhāyana, Āpastamba and Kātyāyana, which he later partially edited. For an overview of current scholarship on the mathematics of the *Śulba-sūtras*, see [Plofker 2009, pp. 16–28]. See also [Lloyd 1990, pp. 74–75; 98–104] for a critical assessment of the view notoriously held by A. Seidenberg (cf. Seidenberg [1962]) that geometrical knowledge (including the notion of proof) can be traced back from the later *Śulba-sūtras* to the earlier Vedic ritual practices they presumably registered.

<sup>50</sup> Cf. [Thibaut 1875, pp. 233–234].

<sup>51</sup> Thibaut’s work induced Cantor to launch into comparing Greek with Indian sources so as to decide who borrowed from whom, cf. Cantor [1877]. To this end, dating the *Śulba-sūtras* proved essential, and, insofar as Thibaut had left the issue unsettled, Cantor attempted to show that Heron’s geometry diffused in India around 100 BCE. He thus explicitly opposed von Schröder’s view of an Indian origin of Pythagoreanism and made the best of Thibaut’s material so as to counter it.

<sup>52</sup> See Seidenberg [1962] for a general account of Cantor’s comparative views on Indian and Greek mathematics from 1877 to 1905, when he eventually credited Bürk’s 1901–1902 papers (*viz.* Bürk [1901–1902]) for bringing about “an essential shift” ([Cantor 1905, p. 64]) which required reconsidering the whole problem. Seidenberg wonders why Cantor endowed Bürk’s work with such pivotal significance: “Bürk’s papers are excellent and he does make original points [*he concedes*], but the argument occurs in all its essential aspects already in Thibaut’s paper” ([Seidenberg 1962, p. 510]). This might be true as regards the material Bürk adduced which is very much similar to that which Thibaut commented on, but, in contrast to the latter, Bürk was not deterred by Cantor’s moral authority, from forcefully demonstrating that the ancient Indians independently knew the so-called Pythagorean theorem. In relying on previous work by Thibaut, v. Schröder, Bühler and Garbe, he dated the *Śulba-sūtras* themselves from the fifth or the fourth century BCE, and claimed that such ritual practices involving knowledge of the Pythagorean theorem were already attested much earlier, long before they were codified in written form, that is at the latest in the eighth century. However, what most probably made Cantor change his mind was rather Bürk’s keynote reconstruction of Indian independent mathematical procedures along Hankel’s lines. Bürk for instance first pointed out that such Pythagorean

### 3. COLEBROOKE'S 'SEE'-TROPE: A FAITHFUL ARTEFACT ?

In spite of their differences, Arneth's and Hankel's accounts of intuitive 'proofs' presumably found in Sanskrit sources were both modeled upon a trait of Colebrooke's rendering of the way diagrams displaying certain arrangements of figures were assumedly inserted in the base text of Bhāskara II's *Lilāvati* and *Bija-ganita*.<sup>53</sup> Wherever they occur within the base text as translated by Colebrooke, diagrams are indeed generally preceded by the word "See", as shown, among many other instances, in the excerpt from Colebrooke's translation of *Bija-ganita* § 146, reproduced in facsimile in Fig. 1. In being recurrent as the marker for inserting diagrams within the main text, this word came to gain the status and the semantic value of a type in the German reception of Colebrooke's translations, while conveying the meaning that seeing the figure has probative force. For the sake of clarity, this typical setting for inserting diagrams in the main text as presumably self-contained visual 'proofs', will be referred to here as "Colebrooke's 'See'-trope".<sup>54</sup> This complex designation aims at stressing that the mere textual setting presumably implies that the diagram comes to be endowed with the epistemic value of a 'proof'. Whether or not Colebrooke intended it, at least his German followers read it in this way, which is our main concern in the present contribution. However, this immediately raises further questions. One indeed legitimately wonders whether in putting emphasis on diagrams in this peculiar way, Colebrooke

triples as (8, 15, 17), occurring in the *Śulba-sūtras*, did not fit in the formulas devised for those triples occurring in the Pythagorean corpus, and thus required other (specific) methods for their obtention. In this respect, he complied with the very standards Cantor had set in his 1877 paper. "Even the most resolute advocates of Indian originality in geometry as Hankel [Cantor warned in 1877,] are forced to admit that there is no question of constructive rigorous proofs with the Indians. Computation is the great auxiliary means which they apply wherever the possibility presents itself; the geometric foundation is thus completely left out of sight, and is only regained at the end. Whereas if the geometrical must once be proved completely geometrically, they satisfy themselves with an appeal to the pupil's eyes: See! they tell him, and this seeing must suffice to let congruences, but also similarities, be recognized as true. We cannot see how in this last way, geometrical discoveries were being made, ..." ([Cantor 1877, p. 7]) Bürk's reconstruction precisely intended to make a plausible point about which methods may have led to those specifically Indian Pythagorean triples, which to all appearances convinced Cantor.

<sup>53</sup> There is no such setting for diagrams in Colebrooke's translation of Brahmagupta's *Brāhma-sphuta-siddhānta*, for there diagrams do not appear in the main text, but only in the footnotes in which Colebrooke relies on commentaries.

<sup>54</sup> In rhetoric, a trope is a word or an expression used in a way that is different from its usual meaning in order to produce an effect. The term is also to be understood in this way when embedded in the above syntagma.

was being faithful to his sources, and if so to what extent. Takanori Kusuba warns against misconceptions that might arise from Colebrooke's presentation when one overlooks the crucial differentiation between base text and commentaries, as well as the various intricacies pertaining to the history of the text itself, such as questions of preservation, transmission, and authorship. After recalling that the standard base text in Sanskrit mathematical texts consists of *sūtra* (rule) and *udāharana* or *uddeśaka* (example or exercise), both being versified,<sup>55</sup> whereas the commentary which follows each rule and example of the base text is generally written in prose, he recommends caution as regards figures. "When a figure is given, the text usually refers to a *darśana* (illustration). Colebrooke translated this word [*by*] 'see'. Presumably this is a source of a misconception about Indian mathematics: Indians seldom give proofs. The *mūla* [*base text*] rarely gives 'proofs' because rules are only for being learned by heart. In fact a form of 'proof' is sometimes given in a commentary. The Sanskrit word corresponding to 'proof' is *upapatti*, a kind of demonstration, that shows how the rule can be derived from given rules."<sup>56</sup> As the importance of commentaries in Sanskrit mathematics was being acknowledged and their role better understood, those *upapatti* only to be found there came to be highlighted and carefully studied. M. D. Srinivas, for instance, emphasizes that far from being absent from this tradition,<sup>57</sup> 'proofs' were one of the main purposes of commentarial practice, insofar as results

<sup>55</sup> On the structure of the verses in Sanskrit mathematical texts, see [Plofker 2009, pp. 302–304].

<sup>56</sup> [Kusuba 1993, p. 11]. Kim Plofker also calls for caution. When dealing with verse 128 of Bhāskara II's *Bīja-ganita*, corresponding to § 146 in Colebrooke's translation of *Bīja-ganita*, she renders *kṣetradarśanam* (see [Hayashi 2009, p. 56]) by "Observation of the figure", while making the following comment: "These verses are presumably the ultimate source of the widespread legend that Bhāskara gave a proof of the Pythagorean theorem containing only the square figure shown in figure 4.19 [*viz. the figure also reproduced by Colebrooke in Bīja-ganita* § 146] and the word 'Behold!', cf. [Plofker 2007, p. 477].

<sup>57</sup> In strongly contributing to launch this campaign for reappraising *upapatti* in the Indian commentarial tradition, M. D. Srinivas also interestingly claimed that studying "the Indian epistemological viewpoint on the nature of mathematical knowledge and its validation" might prove to be "of great relevance for the development of mathematics today". "Contemporary mathematics, being rooted entirely in the modern Western tradition, [*he argued,*] does suffer from serious limitations which can be traced to the kind of epistemology and philosophy of mathematics which have governed the development of mathematics in the Western tradition right from the Greek times", see [Srinivas 1987, section IV]. Among those limitations, Srinivas points to the fascination with foundations and absolute certainty, the prevalence of an ideal view of mathematics as a formal deductive system which "causes serious distortion in the very practice of the science of mathematics", and eventually to consequences detrimental

were not to be accepted unless supported by some probative argument (*yukti* or *upapatti*) intended to remove doubts and ensure consent.<sup>58</sup> In his study of the Bakhshāli Manuscript, Takao Hayashi observes, for his part, that “the term *upapatti* stands for the proof or derivation of a mathematical formula, [and that] we find a number of instances of *upapatti* used in that sense in later commentaries such as Gaṇeśa’s *Buddhivilāśī* (A.D. 1545) on the *Līlāvatī* and Kṛṣṇa’s *Navāṅkura* (ca. A.D. 1600) on the *Bija-gaṇita*.<sup>59</sup> With hindsight, it is clear that Colebrooke’s privileged access to both Gaṇeśa’s and Kṛṣṇa’s above mentioned commentaries, as well as Prthūdakasvāmin’s *Vāsanābhāṣya* (ca. 860) on Brahmagupta’s *Brāhma-sphuṭa-siddhānta*, and other commentaries, earned him many valuable insights into the meaning of the texts he edited and provided him with a wealth of material supporting his claim for the presence of ‘proofs’ in Sanskrit mathematics. So as to be able to assess, in the light of what we know today, the extent to which Colebrooke’s ‘See’-trope does, or does not, prove relevant and faithful with regard to the sources he dealt with, one has to clarify the role diagrams played in their relation to ‘proofs’ in Bhāskara II’s *Bija-gaṇita*. More generally, one wonders whether, and if so how, diagrams as such can be envisaged as some sort of visual ‘proofs’. A straightforward and uniform answer to that question for all mathematical sources in Sanskrit is most probably illusory, for only more focused approaches might teach us something. In concentrating on the commentary by the seventh-century astronomer Bhāskara I on the *Āryabhaṭīya*, Agathe Keller for instance analyzed a variety of types of arguments, or ways to provide mathematical justification, distinctively labelled with technical Sanskrit terms corresponding to ‘proofs’ (*upapatti*), ‘verifications’ (*pratyāyakarāṇa*) and ‘explanations’ (*vyākhyāna*, *pradarśana*, *pratipādita*). Significantly enough for our present purpose, she showed how the last ones which consist in ‘reinterpreting’<sup>60</sup> the procedure to explain, may

to the teaching of mathematics since “formal deductive format adopted in most mathematics books and articles greatly hampers understanding”. As regards the last point, it will be seen that Hankel was also anticipatory in connecting sound historiography of mathematical proof with enhanced pedagogy.

<sup>58</sup> Cf. [Srinivas 2005, pp. 231–232]. Moreover, M. D. Srinivas usefully enumerates the main commentaries that have been edited and draws from them a whole range of *upapatti* upon which he comments. See also Srinivas [2008] which presents other examples of *upapatti*, borrowed from different commentaries.

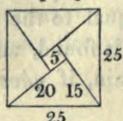
<sup>59</sup> [Hayashi 1995, p. 75]. The Bakhshāli Manuscript which is by far the earliest extant Sanskrit mathematical manuscript was probably written between the eighth and the twelfth centuries, cf. [Plofker 2009, pp. 157–162].

<sup>60</sup> See [Keller 2012b, p. 490]: “A ‘reinterpretation’ does not invalidate the previous interpretation. ... [It] adds a layer of meaning, gives depth to the interpretation of a

146. Example: Say what is the hypotenuse in a plane figure, in which the side and upright are equal to fifteen and twenty? and show the demonstration of the received mode of computation.<sup>2</sup>

[...]

With four such triangles, another figure having four sides, each equal to the hypotenuse,<sup>3</sup> is constructed for the purpose of finding the hypotenuse. See



Thus another interior quadrilateral is framed; and the difference between the upright and side is the length of its side. Its area is 25. Twice the product of the upright and side is the area of the four triangles, 600. The sum of these is the area of the entire large figure; 625. Equating this with the square of *yávat-távat*, the measure of the hypotenuse is found, 25.<sup>4</sup> If the absolute number, however, be not an exact square, the hypotenuse comes out a surd root.

[...]

147. Rule: Twice the product of the upright and side,<sup>5</sup> being added to the square of their difference, is equal to the sum of their squares, just as with two unknown quantities.<sup>6</sup>

Hence, for facility, it is rightly said 'The square-root of the sum of the squares of upright and side, is the hypotenuse?'

[...]

Placing the same portions of figure in another form, see

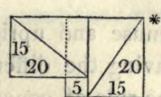


FIGURE 1. Colebrooke's 'See'-trope in the case of Bhāskara II's *Bīja-ganita* § 146, cf. [Colebrooke 1817, p. 222]

occur in the form of a diagram, although it is difficult to tell whether the diagram itself was understood as a fully explanatory or a merely illustrative one. In any case, the explanation would unfold orally from seeing the diagram. "The word *pradarśana* is derived from the verbal root *drś*, 'to see'. It has a similar range of meaning as the English verb 'to show'. It is often

rule. A 'reinterpretation' provides a new mathematical context for the different stages of a procedure which is not modified. Another name for this commentarial technique could be 'rereading' a procedure."

hard to distinguish if the word refers to the visual part of an explanation or to the entirety of the explanation.”<sup>61</sup> No clear-cut answer to our initial questions thus seems to be at hand.

From a more general point of view, the status of diagrams in Sanskrit mathematics should be envisaged in relation to the orality-literacy issue. Graphical figures, however scarce they may be in Sanskrit sources, are attested in mathematical commentaries in their manuscript form. Such commentaries usually present themselves as interpolated prose text between the verses of the main text which, being memorized, were often not even quoted in full but only initiated in order to be recalled from memory. “Thus [as Kim Plofker points out,] they served as a bridge between the oracular pronouncements of the base-texts and the actual manipulations that a mathematician needed to perform in order to solve problems.”<sup>62</sup> To what extent then do the written commentaries help us figure out the actual mental activity supposedly deployed in response to those mnemonic verses? Reasoning with diagrams, being one kind of such mental activity, is a dynamic thought process which can hardly be fully reflected in static drawn figures as found in manuscripts. Although it partakes of the characters of both orality and writing, classical Sanskrit mathematics nevertheless praises the values of the former (terse verses learnt by heart, memorized and mentally used<sup>63</sup>, semantic duality, paronomasia). Pierre-Sylvain Filliozat argues that “the Sanskrit mathematical text is a literary text. It imitates the form and spirit of the poetic text. ... Sanskrit poetry is definitely situated in the structures of orality. It is a poetry of sound emitted and heard, close to music, an inner object of meditation. The ideal mathematical text aims at being that same kind of mental object which speech transmits.”<sup>64</sup> In considering phonocentricity as the unifying feature of Sanskrit culture<sup>65</sup> as a whole, Annette Wilke and Oliver Moebus

<sup>61</sup> [Keller 2012b, p. 501]. See also [Keller 2005, pp. 295–299].

<sup>62</sup> [Plofker 2009, p. 213].

<sup>63</sup> Long before the use of writing in India, Vedic civilization developed elaborate techniques of recitation for preserving the oral text of the Vedas, see [Filliozat 2004, pp. 138–140]. At a later period Indian scientific literature was modeled upon the pattern of the *sūtra*-genre as characterized by “conciseness, emphasis on the essential point, extent of the field of application, links from formula to formula”, cf. [Filliozat 2004, p. 143].

<sup>64</sup> [Filliozat 2004, p. 156].

<sup>65</sup> “Culture” being here understood “in Clifford’s Geertz’ sense as a contextual framework in which symbols, behaviors and forms of thought become comprehensible, a collective construct of meaning that is embodied in texts, behaviors, rituals, festivals, material culture, etc. and in social institutions, and is expressed in values and how people feel about things”, cf. [Wilke & Moebus 2011, p. 15].

claim in the same vein that what they term “sonality” should be envisaged as “a third space habitus” between orality and writing best accounting for their coexistence and combination in India.<sup>66</sup> This sonic culture not only found expression in religious phenomena, but also in Sanskrit scientific traditions which developed a form of mental activity merging abstraction and aesthetic participation. “The poetic cryptic language [Wilke and Moebus argue,] has the function of convincing, and performatively actually creates the state of being convinced because the poetic form of the scientific statement imparts the flair of something important, valuable and mysterious.”<sup>67</sup> Beside the “legitimizing function”<sup>68</sup> of the poetic form by which attention is drawn to the scientific content, brilliance and sophistification on the part of the learned elicit mental exertion on the part of the recipient whose “ambitious prime aim is to interpret correctly the cryptic rules that are woven into every verse”<sup>69</sup> Far from being suppressed, with the spreading of writing in India, the forms of expression and the thought processes characteristic of orality were combined with the tools afforded by writing, which resulted in the twofold nature of Sanskrit mathematical texts described above. Filliozat explains that “the typical composition produced for teachings is the *sūtra*, or a composition in the same kind of style, which the master explains orally in his own way. The general rule is that the disciple memorizes the letter of the *sūtra* and remembers the contents, if not the very wording, of the oral explanation. This oral

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<sup>66</sup> In so doing, they challenge Jack Goody’s and Walter Ong’s theses that only writing made it possible to develop internalization and abstract thought. They emphasize for instance that Pāṇini’s grammar is an orally transmitted and memorized *sūtra* which nevertheless yields a wealth of “audible abstractions”. Hence in their view, any strict dichotomy between the oral and the literal fails to capture the aesthetic dimension of Sanskrit culture, that is first and foremost the kind of phenomena which they named “acoustic piety”, namely such “religious forms in which the act of recitation itself or the devotional attitude range first, whereas the semantic meaning is not necessarily important or known”, forms in which “the religious text [*is seen*] as an icon of the divine charged with special power, an animate reality”, cf. [Wilke & Moebus 2011, p. 12].

<sup>67</sup> [Wilke & Moebus 2011, p. 233]. On this aspect of performativity, see also [Wilke & Moebus 2011, p. 243]: “The imparting of knowledge is celebrated and given in a creative form. The scientific statements are cryptically brief, but also elaborated, because the use of complicated meters and the play with the multiple meanings of signs allows a conscious semioticization and fictionalization going beyond the scientific message. One “stages” the object and also oneself. Making things hermetic is fun, and the beautiful form creates a stronger emotional relationship, and is elitist at the same time. The striking mixture of strictness and ornament is like a spicy snack with sweet tea.”

<sup>68</sup> [Wilke & Moebus 2011, p. 235].

<sup>69</sup> [Wilke & Moebus 2011, p. 243].

explanation may never have been written but always transmitted orally.<sup>70</sup> However, with the emergence of writing, "there also developed a type of commentary composed not only for oral transmission, but chiefly to be couched in writing and so adapted to writing resources. Especially in the field of mathematics the commentator has many occasions to resort to the use of graphic devices, the use of the written place value notation, the graphic arrangement of arithmetical operations, drawing geometrical figures etc. In a general manner we can say that the verses have preserved the style of an oral exposition, and the commentary is an expansion of the memorized knowledge using all the facilities provided by writing."<sup>71</sup>

With these elements of contextualization in the back of our minds, let us resume our case about the role and status of diagrams as they appear in Sanskrit manuscripts and more precisely about their relationship to 'proofs'. One conjectured that oral explanations may have been transmitted for generations before a commentator wrote down his teaching after a presumably regular pattern of exposition.<sup>72</sup> However, caution is required, for, to all appearances, there is no available evidence supporting any general claim on this score. Some commentators indeed report that they transcribed what they learned from the oral teachings of their master, but it is not impossible that some other modes of transmission involving retranscription as a mnemonic device may also have existed. As a matter of fact, we do not know which part of the activity of transmission relied on writing, and which on memory. This remains a puzzling issue. It is true that most Sanskrit mathematical texts as they came to be handed down to us do consist in versified and prose parts, but these parts happen to be arranged in a great variety of ways. There is no definite pattern whatsoever supposedly applying in all cases and at all times. However, for the purposes of orientation, a schema like the following may prove useful, in which the versified stanzas, supposedly learnt by heart, are indicated in boldface, whereas the different parts of the commentary in prose are indented. Still, one should keep in mind that any schema of this kind is to be considered as a mere ideal type in the Weberian sense of the term, that is as an abstract theoretical construct of a fictional nature, only to be used as a measuring rod with respect to which similarities as well as discrepancies can be assessed in concrete cases.

*sūtra* (rule)

<sup>70</sup> [Filliozat 2004, p. 149].

<sup>71</sup> [Filliozat 2004, p. 149].

<sup>72</sup> Cf. [Filliozat 2004, p. 151].

(elucidation of the rule)

*uddesaka, udāharana* (example, enunciation of the problem)

*nyāsa* (setting down the data of the problem)

*karana* (execution)

*yukti, upapatti, etc.* ('proof', verification, explanation)

Whereas graphic arrangements of numeral symbols, whenever they occur, are laid down in the *nyāsa* part of the commentary, diagrams may occur in various places, depending on the various uses they lend themselves to.<sup>73</sup>

Whatever the case may be, an important feature should be underscored at this stage which will shed light on Colebrooke's 'See'-trope as an incentive for further questioning. Explaining a procedure often involves putting down numbers on a writing surface and carrying out manipulations with them. However, Kim Plofker argues, "the details of these manipulations are seldom described in the texts or attested in the manuscripts. In a worked example the given numbers are set down in the prescribed layout and the results of operations are stated, but we do not see the graphical workings of such steps as, for example, taking a square root or multiplying two multi-digit numbers."<sup>74</sup> The same holds for manipulations with figures. We know from sources that in actual practice "the figures were drawn on the ground or on a slab (*phalaka*)"<sup>75</sup> But the commentary did not intend in the least to register all the intermediary steps successively performed on the working surface as a counterpart to reasoning with figures. Only a few pivotal figures were provided. Hence there is no way to reconstruct for sure the dynamic thought processes involved from the merely static diagrams attested in extant manuscripts. However legitimate, this pursuit falls, of necessity, in the blind spot of our sources. On the basis of the available evidence, no such inference can be made without a fair share of guesswork. We do not know which diagrams the recipients of the teaching would draw on their working surface, and in which order, as a way to unfold those intermediary steps and expand memorized knowledge. Being final, this verdict cannot be eluded, which does not mean though that these issues to which Colebrooke first called attention had to remain unaddressed. On the contrary, Colebrooke and the most talented among his followers, such as Chasles or

<sup>73</sup> Michio Yano for instance considers a case in which drawn figures appear as an illustration in the *pratyaya* (verification), where they occur "headed with the word *parilekhaḥ* and numbered", cf. [Yano 2006, p. 156]. See also [Filliozat 2004, p. 154].

<sup>74</sup> [Plofker 2009, p. 213].

<sup>75</sup> [Yano 2006, p. 158].

Hankel, were spurred on to supply, by their ingenuity, that about which the sources were irrevocably silent.

In order to weigh what Colebrooke achieved with respect to the treatment of figures in the process of shaping Bhāskara II's base text, one should bear in mind that not only the many commentaries produced in the course of time, including Bhāskara II's self-commentary, may "differ in the choice of examples and eventual excursus"<sup>76</sup>, and thus in the figures presented, but also that there may be differences on this score between different manuscripts of one and the same commentary. When envisaged as part of the textual setting, diagrams thus do not partake of the core stability of the mnemonic verses. In this respect, it may be interesting to compare Colebrooke's translation with Hayashi's "makeshift edition"<sup>77</sup> of Bhāskara II's *Bijaganita* compiled from seven printed editions, each dealing with a different range of manuscripts. In both cases, the same figure (see facsimile in Fig. 1 above) occurs within the prose part of Bhāskara II's self commentary appended to the *uddeśaka* verse. Colebrooke's version is very much akin to Hayashi's for the layout of the text and the diagrams, with this difference though that the word *kṣetradarśanam* (which Colebrooke renders by 'See') does not appear before the diagram in all the editions of the commentaries Hayashi relies on. This comparison nevertheless proves disappointing, for Hayashi only compiles printed editions, whereas Colebrooke himself dealt with original sources, and in this regard, only a first-hand examination of the manuscripts Colebrooke had access to would of course be decisive for evaluating his editorial decisions on the basis of the textual evidence he had access to. Still, the similarity noted above remains significant all the same. With the textual setting designated as the 'See'-trope, Colebrooke did not starkly depart from later editions of the same texts, while moreover calling attention to the import of a specific kind of diagrammatic reasoning supposedly mentally unfolding from merely seeing pivotal arrangements of figures.

Colebrooke emphasized that there were two kinds of proofs attested in Sanskrit sources, which he *prima facie* contrasted as "geometrical" and "algebraic" proofs. Whereas the Sanskrit word corresponding to "geometrical" is always *kṣetra-gata* in this connection, Colebrooke nevertheless seemed to waver somewhat as regards the proper characterization of the other kind of proofs. For he suggested different translations in different

<sup>76</sup> [Filliozat 2004, p. 155].

<sup>77</sup> [Hayashi 2009, Preface, p. i]. Takao Hayashi uses various commentaries on Bhāskara II's *Bijaganita*, as for instance two different editions of Kṛṣṇa's *Bijapallava*, in order to collect the prose parts of Bhāskara II's self-commentary quoted in these.

places. In a first passage, he pointed to the following opposition, namely in his own transliteration of the Sanskrit: “*Cshétra-gatópapatti*, geometrical demonstration. *Upapatti avyacta-criyayá*, proof by algebra.”<sup>78</sup> But, later on, in a second passage, he further observed, still in connection with proofs: “*Cshétra-gatá*, geometric: *Rási-gatá*, algebraic or arithmetical. (*Varña-gatá*, algebraic exclusively.)”<sup>79</sup> One wonders how Colebrooke exactly understood the Sanskrit term *rāśigatā*, in contrast to *kṣetragatā*, for he sometimes renders it by “algebraic” and, in some other occurrences, by “arithmetical”, in opposition to “algebraic exclusively”.<sup>80</sup> Although this problem certainly deserves clarification,<sup>81</sup> it will suffice here to draw attention to it, while focusing on our main target, namely the meaning of *kṣetra-gata-upapatti*.<sup>82</sup> There are mainly two contexts in which this distinction is put forth in Bhāskara II’s text, namely in connection with the proof of the so-called Pythagorean theorem, in *Lilāvatī*, §134 and *Bijaganita*, §146–147, on the one hand, and with the so-called *bhāvita* problems, or the equations involving a product of two unknown variables, on the other hand. But it is only with respect to the latter context that the relationship between both kinds of proofs is explained. “On the subject of demonstrations, [Colebrooke pointed out,] it is to be remarked that the Hindu mathematicians proved propositions both algebraically and geometrically ; as is particularly noticed by Bhāskara himself, towards the close of his Algebra, where he gives both modes of proof of a remarkable method for the solution of indeterminate problems, which involve a factum of two unknown quantities.”<sup>83</sup> Colebrooke refers here to Bhāskara II’s self-commentary on *Bijaganita*’s verse 204 which poses the problem amounting to solving an equation of

<sup>78</sup> Cf. [Colebrooke 1817, p. 59].

<sup>79</sup> Cf. [Colebrooke 1817, p. 271]

<sup>80</sup> However, in secondary literature, *rāsi* quantity is mostly associated with arithmetic.

<sup>81</sup> One should also clarify the way the contrast between both kinds of proofs in Bhāskara II’s *Lilāvatī* and *Bija-ganita* connects with the well-entrenched distinction between two kinds of mathematics, namely *kṣetraganita* and *rāśiganita*. For it should be kept in mind that this latter distinction can be traced back to much earlier times. See for instance Bhāskara I’s statement in his commentary on the *Āryabhaṭīya*: “Mathematics (*ganita*) is of two kinds: mathematics of fields (*kṣetraganita*) and mathematics of quantities (*rāśiganita*).”, cf. [Keller 2006, vol. 1, p. 8]. In his study of the Bakhshālī Manuscript, Takao Hayashi translates *rāśiganita* by “arithmetic of quantities” and *kṣetraganita* by “arithmetic of geometrical figures”, cf. [Hayashi 1995, p. 62].

<sup>82</sup> Takao Hayashi translates *kṣetra-gata-upapatti* by ‘proof based on figures’, cf. [Hayashi 2009, p. 167].

<sup>83</sup> [Colebrooke 1817, Dissertation, p. xvii].

the form  $4x + 3y + 2 = xy$ ,<sup>84</sup> a problem which, in Bhāskara II's view, can be solved both 'algebraically' and 'diagrammatically',<sup>85</sup> as shown in the following passage<sup>86</sup>: "The demonstration follows. It is twofold in every case: one geometrical, the other algebraic.<sup>87</sup> [...] The algebraic demonstration is next set forth. That also is grounded on figure.<sup>88</sup> [...] The algebraic demonstration must be exhibited to those who do not comprehend the geometric one."<sup>89</sup>

Takanori Kusuba analyses the interplay between such algebraic and geometrical demonstrations in the paradigmatic case of Bhāskara II's discussion of the Pythagorean theorem,<sup>90</sup> and therefore helps shedding light on the significance of Colebrooke's 'See'-trope. The rule given in *Līlāvatī*, § 134, merely stipulates without proof that the diagonal  $d$  of a rectangular triangle of base  $b$  and upright  $u$  is yielded by taking the square root  $\sqrt{b^2 + u^2}$ . In this connection, Colebrooke only mentions in a footnote that "the proof is given both algebraically and geometrically by Gaṇeśa",<sup>91</sup>

<sup>84</sup> Cf. [Colebrooke 1817, § 212–214, pp. 270–272].

<sup>85</sup> In her review of Sita Sundar Ram's study of Kṛṣṇa's *Bījapallava*, cf. Sita Sundar Ram [2012], Clemency Montelle observes that Kṛṣṇa provided the diagrammatic proof announced, but nowhere completely spelled out, by Bhāskara II. Cf. [Montelle 2014, p. 3]: "Kṛṣṇa Daivajña's commentary supplies a worked solution specifically using diagrams, where the products of the constants and unknowns are imagined to be rectangles with yet to be determined sides and various diagrammatic manœuvres produce the unknown 'lengths'. [Sita Sundar Ram's] careful and methodical treatment of Kṛṣṇa Daivajña's account with accompanying diagrams and identification of the various steps of working with the resulting rectangles, gives a sound appreciation of the original mathematical steps of working. [...] of course, this edition may be quite different from the way the scribes presented the text in their manuscripts, however, it gives something of an impression of the layout and aspects invoked when solving this problem." I have been unable to have direct access to Sita Sundar Ram's book.

<sup>86</sup> Cf. [Colebrooke 1817, pp. 271–272]. The transliterated Sanskrit from Hayashi's edition is given in footnotes to be compared with Colebrooke's formulations. As already mentioned, Colebrooke himself only indicated the couple of words *ksetragatā* and *rāśigatā*.

<sup>87</sup> Compare with the Sanskrit, cf. [Hayashi 2009, p. 92]: *asyopapattiḥ/ sā ca dvidhā sarvatra syāt/ ekā ksetragatānyā rāśigateti/*.

<sup>88</sup> Compare with the Sanskrit, cf. [Hayashi 2009, p. 92]: *atha rāśigatopapattir ucyate/ sāpi ksetramūlāntarbhūtā/*.

<sup>89</sup> Compare with the Sanskrit, cf. [Hayashi 2009, p. 93]: *ye ksetragatām upapattim na budhyanti teṣām iyaṁ rāśigatā darśanīyā/*.

<sup>90</sup> [Kusuba 2009, pp. 59–63].

<sup>91</sup> See [Āpaṭe 1937, p. 129]. In the *Buddhivilāśinī*, a commentary on Bhāskara II's *Līlāvatī*, Gaṇeśa elaborates on the figure which Bhāskara II presents in *Bījaganīta*, § 146. M. D. Srinivas puts forward Gaṇeśa's proof as an instance of *ksetragatā upapatti*, and reads the figure in the light of the corresponding algebraic identity.

and refers to Bhāskara II's algebraic proof in the *Bijaganita*. In this respect, Kusuba emphasizes that the *Līlāvatī* belongs to that part of mathematics (*pāṭīgaṇīta*) which merely shows algorithms for calculation and never uses symbols for unknowns, so that the proof of the above rule is best fitted in that other part of mathematics allowing for unknowns (namely *bījaganīta*). In his translation of *Bijaganīta*, § 146 (see facsimile in Fig. 1), Colebrooke only gave the last one of a series of four figures, successively obtained by combining several instances of the same rectangular triangle—a series which is found in its entirety in the *Bijapallava*, a commentary by Kṛṣṇa on the *Bīja-ganīta*, also known as *Navāṇikura* (ca. A.D. 1600), and also a series owing to which Kṛṣṇa presumably unfolded the process of the *kṣetra-gatopapatti*, as shown in Fig. 2.<sup>92</sup> While only inserting the fourth and last drawn figure, together with the word 'See', within Bhāskara II's text, Colebrooke also supplied—in an appended footnote—Kṛṣṇa's description of the manipulation with rectangular triangles leading up to it, albeit without the corresponding diagrams. Now, as Kusuba explains, the reading of that last figure in which the whole process is completed, rests on the algebraic equation (1)  $d^2 = 4(\frac{1}{2}ub) + (u - b)^2$ , where  $d$  is the diagonal of the rectangular triangle of base  $b$  and upright  $u$ , which explains "why the proof is given in the *bījaganīta* text, in the field of the calculation with unknowns."<sup>93</sup> However, one should also account for the way in which the figure would presumably make it clear that the outer quadrilateral delimited by the diagonals, and obtained by juxtaposing the rectangular triangles, is to be acknowledged as a square, as well as the enclosed quadrilateral delimited by the uprights and the bases. And hence how, being geometrical squares, both quadrilaterals can be identified with arithmetical squares, *viz.*  $d^2$  and  $(u - b)^2$ . We will come back to this feature of the diagrammatic *upappatti* below. In any case, at this stage, the proof is not complete, for it was required to "show the demonstration of the received mode of computation",<sup>94</sup> that is to account for the rule yielding the diagonal  $d = \sqrt{b^2 + u^2}$ . A further step is needed which the

<sup>92</sup> The same series of figures appears in all three editions of Kṛṣṇa's *Bijapallava*, although two of them at least are based on different sets of manuscripts, cf. [Pingree 1970–1994, vol. 2, p. 54]. Compare for instance [Āpāte 1930, pp. 147–149], [Radhakrishna Sastri 1958, pp. 198, 201–202], and [Vasistha 1982, pp. 171–173]. The only discrepancy is that Radhakrishna Sastri's edition does not contain the figure of a rhombus obtained in combining the rectangular triangles so that their uprights and bases coincide.

<sup>93</sup> [Kusuba 2009, p. 62].

<sup>94</sup> *Bījaganīta*, § 146, in Colebrooke's translation, cf. [Colebrooke 1817, p. 220]. See [Hayashi 2009, p. 55]: *upapattiś ca rūḍhasya gaṇitasyāya kathyatām/*

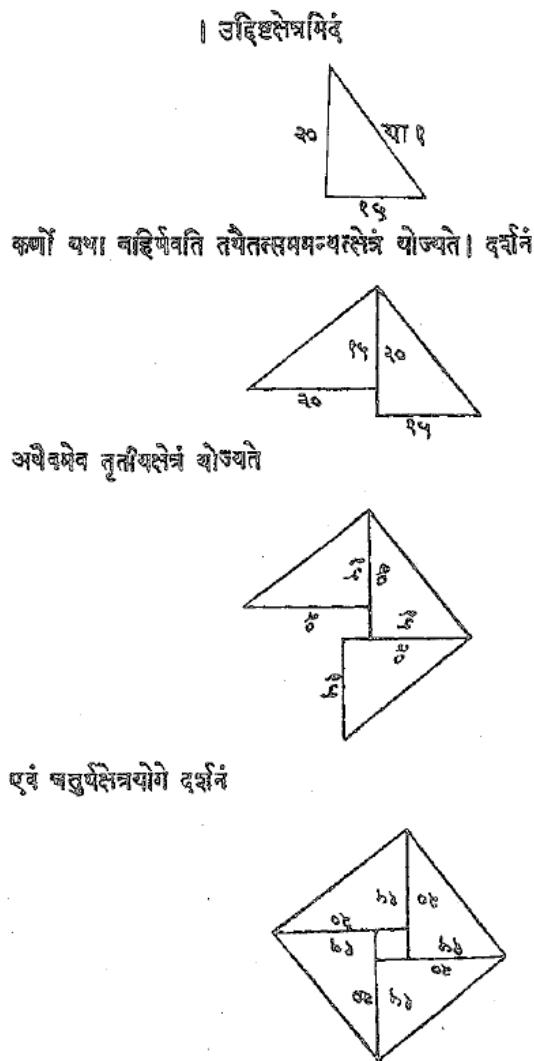
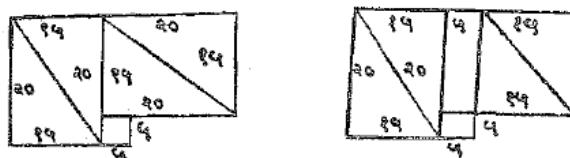


FIGURE 2. The diagrammatic *upapatti* in Kṛṣṇa's *Bījapallava*, cf. [Āpāte 1930, p. 147]

following stanza (§ 147) supplies in the form of an algebraic identity: (2)  $2ub + (u - b)^2 = u^2 + b^2$ , which holds for the base and upright of a

rectangular triangle “just as with two unknown quantities”.<sup>95</sup> But there is a diagrammatic equivalent for this algebraic identity, which Kṛṣṇa exhibits in the series shown in Fig. 3. Two rectangles  $ub$  and the square  $(u - b)^2$ ,

अत्र लघुचतुर्भुजस्य बाह्यभुजेन स्वयाग्नवृद्धेन क्षेत्रं विच्छिद्य दर्शनं



अतो यद्ये रेतामपनीय दृश्यन्तम् ।

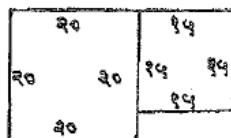


FIGURE 3. The diagrammatic *upapatti* in Kṛṣṇa's *Bījapallava*, cf. [Āpate 1930, p. 149]

whose side is equal to the difference of both sides of the rectangle, can be rearranged so as to form two new squares  $u^2$  and  $b^2$ . In bringing together both series of figures, whose reading corresponds to the algebraic equations (1) and (2), one obtains a diagrammatic proof of the computational rule. Or, if need be, for “those who do not comprehend the geometric [*proof*],” and for whom an algebraic one must be fully spelled out, from

$$(1) \quad d^2 = 4\left(\frac{1}{2}ub\right) + (u - b)^2$$

$$(2) \quad 2ub + (u - b)^2 = u^2 + b^2$$

one demonstrates  $d^2 = u^2 + b^2$ , hence the computational rule  $d = \sqrt{u^2 + b^2}$ . Presumably the diagrammatic *upapatti*, no less belongs to algebra proper than the algebraic proof, for Bhāskara II repeatedly states that algebra

<sup>95</sup> [Colebrooke 1817, § 147, p. 222].

is essentially “spotless understanding” or “sagacity”,<sup>96</sup> whether exercised upon symbols or figures.

One might then wonder how the quadrilateral formed with the four rectangular triangles assembled as in Fig. 2, can be acknowledged as a square. Hence to what extent the figure alone may make a convincing case that it is so. Kusuba observes that this figure “can be traced back to Bhāskara I in his commentary on the *Āryabhatiya*”<sup>97</sup>, where it occurs in connection with a verse by Āryabhaṭa, denoted **Ab.2.3.ab**, “which can be understood in two ways, either as meaning *A square is an equi-quadrilateral and the result which is the product of two identical <quantities>*, or *A square is an equi-quadrilateral and its area is the product of two identical sides.*”<sup>98</sup> The deliberate ambiguity of this verse make it pivotal for connecting both kinds of mathematics, *kṣetragaṇita* and *rāśigāṇita*. Bhāskara I explains why the same name *varga* (square) applies both to the result of the arithmetical operation of squaring and also to a specific geometrical figure. In the form of a succession of objections and responses, the double-sided definition of *varga* is then justified, the connection between both meanings being ensured by the fact that the area of the specific equiquadrilateral to which the name applies is the product of two identical sides. Hence the equiquadrilateral figure which is called *varga* must also be equidiagonal, the rhombus being excluded for in that case the area would not be the product of two identical sides. Pierre-Sylvain Filliozat points out that this way of superimposing one meaning of a term upon another, with a definite purpose in one’s mind, proves to be a standard pattern in Sanskrit linguistic tradition, before being here applied to mathematics.<sup>99</sup> Although

<sup>96</sup> Cf. *Bijaganita*, § 110, in [Colebrooke 1817, p. 195]: “Or the intellect alone is analysis (*vija*). Accordingly it is observed in the chapter on Spherics, ‘Neither is algebra consisting in symbols, nor are there several sorts of it, analysis. Sagacity alone is the chief analysis: for vast is inference.’” See also *Bijaganita*, § 223–224, in [Colebrooke 1817, p. 276]: “The rule of three terms is arithmetic, spotless understanding is algebra. What is there unknown to the intelligent? Therefore for the dull alone, it [*vija*] is set forth.”

<sup>97</sup> [Kusuba 2009, p. 62].

<sup>98</sup> Cf. [Keller 2006, vol. 1, p. 13].

<sup>99</sup> In comparing Bhāskara I’s reading of Āryabhaṭa’s verse with Sūryadeva’s—a twelfth-century commentator—Filliozat considers that “both commentators have underlined the fact that Āryabhaṭa identifies the squaring of a number and the finding of the area of a square. In their mind the word *varga* is primarily the technical name of the arithmetical square and secondarily that of the geometrical square. They consider that, when Āryabhaṭa prescribes it, he does a superimposition of the latter on the former. Superimposition (*upacāra*) is a mode of linguistic expression recognized among Indian grammarians and poeticians. The relation between arithmetic and geometry is translated by them in the form of a linguistic fact. In the sophisticated and

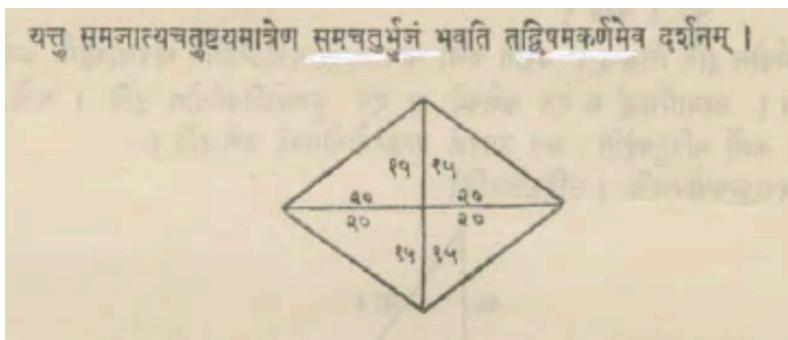


FIGURE 4. The exclusion of the rhombus in Kṛṣṇa's *Bijapallava*, cf. [Āpate 1930, p. 149]

Colebrooke did not refer to Bhaskara I's *Āryabhaṭa*, he summarized in a footnote Kṛṣṇa's reading of the completed figures occurring in Bhaskara II's *Bijaganīta*, § 146–147, a reading which also emphasizes the exclusion of the rhombus as a way to make clear by contrast the meaning of *varga* in a geometrical context. In a nutshell, “by merely joining four rectangular triangles (with the equal sides contiguous,) a quadrilateral having unequal diagonals (that is, a rhomb) is constituted; in which one diagonal is twice the upright; and the other double the side of the triangle [as in Kṛṣṇa's figure reproduced in Fig. 4]; instead of a square comprising five figures (four triangles and a small interior square) [as in Fig. 2]. But, if the upright and side are equal, only a square is framed, which ever way the side is placed, since there is no difference of the upright and the side: and in this case there is no interior square.”<sup>100</sup> Here again, Colebrooke reports in a footnote Kṛṣṇa's comment in the *Bijapallava*, although he omits the corresponding diagram. However, in so doing, he manages to supply the piece of information required to round off the whole argument into a convincing proof. As pointed out above, something more had indeed to be clarified so that one may grasp how one single figure may elicit a diagrammatic *upapatti* from the recipient. The comparison with Hayashi's compilation previously showed that, when inserting a figure within the prose part of *Bijaganīta*, § 146, Colebrooke was probably faithful to the

compelling poetics of Sanskrit pandits, the rule is that any figurative mode of expression should be justified by the intention of the user. [...] For Bhaskara I the purpose of the superimposition is to exclude the rhombus from the present definition of the *varga*.” Cf. [Filliozat 2004, p. 155].

<sup>100</sup> Cf. [Colebrooke 1817, note 1, p. 222].

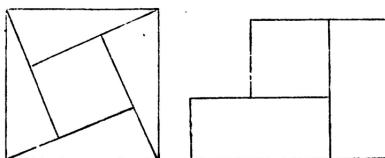
manuscripts of the base text he happened to deal with. But how then, by providing only the last figure made of four rectangular triangles, could the notion be imparted that both the outer and inner quadrilaterals should be justifiably acknowledged as geometrical squares?—hence that they should be connected with arithmetical squares, namely those denoted above  $d^2$  and  $(u - b)^2$ ? Colebrooke's choice to complement the figure with Kṛṣṇa's emphasis on the exclusion of the rhombus suffices to launch the mental process of the diagrammatic *upapatti*. The outer quadrilateral delimited by the diagonals of the four rectangular triangles obviously has all its sides equal, but symmetry also makes it clear that both its diagonals are equal. Indeed, each diagonal connects corresponding vertices of two adjacent rectangular triangles. Since both diagonals of the outer quadrilateral result from identical configurations of triangles, albeit differently oriented, mere reflective inspection of the figure yields the evidence that these diagonals are equal. It will be seen further that in carefully reading Colebrooke, Hankel laid great stress on this kind of symmetry argument as an essential feature of Indian mathematics. In any case, for the time being, one observes that, the exclusion of rhombuses being purportedly recalled in a footnote, Colebrooke makes it clear for the modern reader that only equidiagonal equiquadrilaterals should be acknowledged as geometrical squares, whose area is then obtained by arithmetically squaring the side. Which was presumably needed so as to complete the mental 'proof' process.

One may thus conclude from the previous discussion that the standard textual setting referred to as Colebrooke's 'See'-trope may be viewed with hindsight as a faithful artefact, namely one intended to draw attention to the diagrammatic *upapatti* as a mental process born of the mere inspection of figures. We have seen that, in Sanskrit sources, figures would belong to the prose part of the base text and as such should not be considered on a par with the versified *sūtras* to be memorized, for unlike these, they prove liable to variation from one source to another. From this point of view, significant differences pertaining to the history of text may therefore be somewhat artificially blurred as a result of editorial text-shaping. However, Colebrooke's textual setting as a whole provided all the ingredients that were needed to make a convincing case that seeing the figure had probative force for the recipient, and in that sense proved faithful to the sources.

A subtle reader of Colebrooke, Hankel attempted to understand more thoroughly the thought processes underlying those 'proofs', attested in Sanskrit sources, which, for want of a better term, he provisionally labelled

“intuitive proofs”—a category of ‘proofs’ which Bhāskara II’s ‘proof’ of the so-called Pythagorean theorem best epitomized for a whole generation of scholars.

The basis for all metric and analytic geometry, [Hankel stated,] the proposition of the square of the hypotenuse, has been discovered independently by the Indians. They possess for it two completely natural, typically Indian proofs, which the Greek did not know. Firstly (*Vīja-gan.* 146), they prove the proposition in drawing a perpendicular to the hypotenuse from the vertex of the right angle, and in comparing both triangles thus obtained with the previous one—all three triangles being similar—a thought which Wallis (*De sect. angul. c. VI* in Wallis, *Op. math.* vol. II, 1693) first rediscovered in the Occident.



As for the second proof, the rectangular triangle is being described four times in the square on the hypotenuse, so that a square remains in the middle, whose side is the difference of the cathetes. These four triangles and the inner square, being arranged in a second way, together make up both squares on the cathetes in another decomposition. “See”, writes the author next to the figures—without any further word, leaving everything else to the intuition of the reader.<sup>101</sup>

Hankel was not the first to be struck and intrigued by Colebrooke’s ‘See’-trope. In the above passage, he closely followed Chasles’ account in the *Aperçu historique* (1837),<sup>102</sup> although adding to it a slight but significant twist of his own. Besides, as regards the presumably “typically Indian” character of Bhāskara II’s ‘proof’, Hankel also benefitted from the work

101 [Hankel 1874, pp. 209–210].

102 [Chasles 1837, p. 454]: “Bhāskara donne deux démonstrations du carré de l’hypoténuse. La première consiste à chercher par une proportion, l’expression des segments faits sur l’hypoténuse par la perpendiculaire; et à ajouter ensemble ces deux segments. C’est la démonstration employée par Wallis. (*De sectionibus angularibus*, cap. VI.) La seconde est tout-à-fait d’origine indienne; elle est fort remarquable. Sur les côtés d’un carré, Bhāskara construit intérieurement, quatre triangles rectangles égaux entre eux, ayant pour hypoténuses ces côtés, et il dit *voyez*. En effet, la vue de la figure suffit pour montrer que l’aire du carré égale les aires des quatre triangles (ou quatre fois l’aire de l’un d’eux), plus l’aire d’un petit carré qui a pour côté la différence des deux côtés de l’angle droit de l’un des quatre triangles.” The comparison between Bhāskara’s first proof and the later Western proof by Wallis is taken over from Colebrooke, cf. [Colebrooke 1817, p. xvi]. Hankel endorsed it, as Chasles had previously done, as well as Moritz Cantor in a review of Boncompagni’s work on Leonardo Pisano, cf. [Cantor 1863b, p. 44].

of the German Orientalist and mathematician Franz Woepcke, upon whom he often relied in his manuscript notes in history of mathematics. In his analysis of a Persian translation of an Arabic treatise on geometric constructions—written by one of Abū'l Wafā's disciples after his master's teachings—Woepcke had indeed pointed out two constructions (XI 4 and 8) which he explicitly identified with Bhāskara II's procedure. These con-

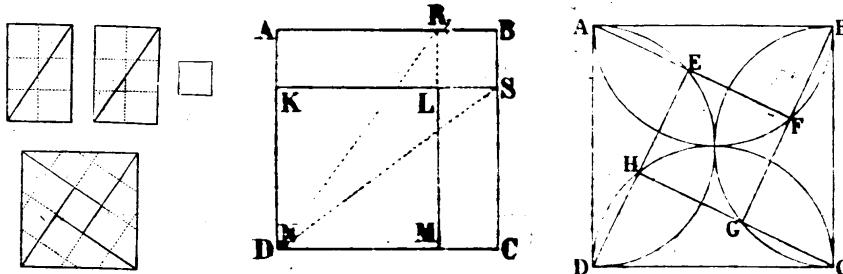


FIGURE 5. Abū'l Wafā's constructions after Woepcke,  
cf. [Woepcke 1855, pp. 346–351]

structions occur in the context of problems arising from practical needs in Arabic ornamental architecture, namely problems of composition and decomposition of squares by juxtaposition of unit squares. So as to frame a square out of a certain number  $n$  of unit squares, this number being the sum of two unequal square numbers, *viz.*  $n = a^2 + b^2$ , the author of the treatise starts with two rectangles whose sides are those of both unequal squares to be added, splits them along their diagonal in four rectangular triangles, and reassembles them after the same pattern as Bhāskara II's, in enclosing a smaller square in their middle (See Fig. 5, left diagram). While observing that the construction of the Arab author is based on an algebraic identity,<sup>103</sup> Woepcke emphasizes the “striking resemblance” with the Indian procedure.<sup>104</sup> The previous construction is then extended to a general one which can now be applied to any two arbitrary given

<sup>103</sup> [Woepcke 1855, p. 347]: “On voit que l'auteur fonde son procédé sur la formule  $a^2 + b^2 = 2ab + (a - b)^2$ ;  $(a - b)^2$  est le carré qu'il va placer au milieu, et  $4 \times \left(\frac{ab}{2}\right)$  sont les quatre triangles rectangles qu'il place autour.”

<sup>104</sup> Cf. [Woepcke 1855, p. 235], see also [Woepcke 1855, p. 347]: “Comparer avec Colebrooke (1817), p. 222; où l'on trouve que les géomètres indiens se servent du même procédé pour démontrer le théorème du carré de l'hypoténuse. En effet, le carré composé des quatre triangles  $\left(\frac{ab}{2}\right)$  et du carré  $(a - b)^2$  est le carré de l'hypoténuse dont les deux cathètes sont  $a$  et  $b$ .”

squares (See Fig. 5, middle diagram)<sup>105</sup>, which, in Woepcke's view, yields "a most elegant proof of the Pythagorean theorem, a construction which as such is found in Bhāskara's algebra."<sup>106</sup> However, chronology raised a problem which Woepcke confronted head on. How indeed could one validate Chasles' judgment about the above method of construction being "tout-à-fait d'origine indienne",<sup>107</sup> since twelfth-century Bhāskara II was blatantly posterior to tenth-century Abū'l Wafā? Woepcke argued that the main criterion enabling one to settle issues pertaining to scientific borrowings between different peoples could be no other, in the last instance, than "the conformity or the difference with respect to the spirit of methods".<sup>108</sup> In the case at hand, he thus claimed that Abū'l Wafā's constructions XI 4 and 8 prove closer to other Indian constructions, "whereas they distance themselves very noticeably from the spirit of Arab geometry, always faithful, with respect to its form, to its Greek models."<sup>109</sup> Abū'l Wafā's next construction XI 9 may bear witness to such anchoring in Greek practice, for, in that instance, when applied to the problem of dividing a square into two squares, Bhāskara's pattern is applied in combination with Greek geometric methods of construction (See Fig. 5,

<sup>105</sup> Both squares *ABCD* and *KLMN* being superposed, one extends the segment *KL* into *KS*, and *ML* into *MR*, so as to obtain two rectangles *ARMN* and *KSCD*, and the smaller square *RBSL*. Then one divides both rectangles along their diagonals *RN* and *DS*, in order to get four rectangular triangles and a small square which are reassembled after the prescribed pattern.

<sup>106</sup> Cf. [Woepcke 1855, pp. 351–352]: "On aura remarqué que cette solution forme en même temps une démonstration très-élégante du théorème de Pythagore. C'est comme telle qu'on trouve cette construction dans l'algèbre de Bhascara (Voir Colebrooke, p. 223)."

<sup>107</sup> [Chasles 1837, p. 454].

<sup>108</sup> Cf. [Woepcke 1855, p. 237]: "Or, dans la discussion des emprunts scientifiques faits d'un peuple à un autre, le critérium, qui doit figurer en première ligne, et qui l'emporte de beaucoup sur tous les autres, est la conformité ou la différence de l'esprit des méthodes, et dans le cas actuel, ce critérium décide ...en faveur de l'origine indienne des deux constructions d'Aboûl Wafâ."

<sup>109</sup> Cf. [Woepcke 1855, pp. 235–236]: "Si l'on s'en tient seulement à la circonstance que Bhascara est postérieur à Aboûl Wafâ, et que, dans l'intervalle de temps qui les sépare l'un de l'autre, la conquête musulmane de l'Inde vient faciliter entre les Indiens et les Arabes l'échange de leurs connaissances respectives, on peut être porté à croire que c'est Bhascara qui emprunte à Aboûl Wafâ. Mais si l'on considère que les deux constructions en question présentent la ressemblance la plus intime et la plus prononcée avec d'autres constructions indiennes (Colebrooke, *Vija-Ganita*, § 148, 149, 150, 212, 214, *Lilavati*, § 203, 3<sup>e</sup> note), tandis qu'elles s'éloignent très sensiblement de l'esprit de la géométrie arabe, toujours fidèle, sous le rapport de la forme, à ses modèles grecs, on ne peut s'empêcher de dire, avec M. Chasles, que ces deux constructions, ou (puisque elles n'en font qu'une au fond) que cette construction est tout-à-fait d'origine indienne."

right diagram).<sup>110</sup> Significantly enough, Woepcke cites Chasles' “brilliant restitution of the true meaning of Brahmagupta's geometry”<sup>111</sup> as the ultimate reason to defer to his expertise with regard to the issue of Indian methods. No surprise then that Hankel may have been induced to also focus on Brahmagupta, while putting Chasles' reading to the test. Still, his originality already surfaces here with regard to Bhāskara II. Chasles considered that, in the *Bīja-ganīta*, “one finds several questions of geometry solved by computation, and several rules of algebra demonstrated by geometry”, and also that algebraic identities “are demonstrated by figures which speak to the eye and to the mind, with no further explanation”.<sup>112</sup> However, he abstained from specifying the kind of geometrical proof process operating with the figures *per se*, other than by merely referring to their algebraic counterparts. By contrast, Hankel put the emphasis on the figures themselves with the aim of grasping, according to its own norms of justification, the diagrammatic process they supposedly embodied. With hindsight, one observes that both Bhāskara II's first and second proof of the Pythagorean theorem respectively foreshadow the so-called principles of similarity and congruence, upon which Hankel's characterization of Sanskrit mathematics would ultimately hinge. But more specifically, his account of Bhāskara II's second proof bears witness to his being receptive to Colebrooke's insight that, in the case of diagrammatic *upapatti*, the probative process implies nothing more than bringing together certain figures with their appropriate reading.

#### 4. A WATERSHED IN METHODOLOGY

Hankel's interpretive account of Sanskrit mathematical sources drew on material previously elaborated by Arneth, although distinctively distancing

<sup>110</sup> Cf. [Woepcke 1855, p. 351]: “Diviser un carré en deux carrés, le côté de l'un de ces deux derniers carrés étant donné.” Abū'l Wafā's construction runs as follows. One describes half-circles on each of the four sides of the square *ABCD*, then one takes the chords *AE*, *BF*, *CG*, *DH*, all equal to the side of one of the squares in which *ABCD* is to be decomposed. The points *A*, *E*, *F* are shown to be aligned, as well as *B*, *F*, *G*; *C*, *G*, *H*; and *D*, *H*, *E*. In this way, one obtains, a square *EFGH* and four rectangular triangles out of which one forms the two squares into which *ABCD* is decomposed.

<sup>111</sup> Cf. [Woepcke 1855, p. 236].

<sup>112</sup> Cf. [Chasles 1837, p. 454]: “Les formules d'analyse  $2ab + (a - b)^2 = a^2 + b^2$ ,  $(a + b)^2 - (a^2 + b^2) = 2ab$ ,  $(a + b)^2 - 4ab = (a - b)^2$ , sont démontrées par des figures qui parlent aux yeux et à l'esprit, sans qu'il soit besoin d'aucune explication (§ 147, 149 et 150).”

itself from the kind of history of mathematics his predecessor practiced. Arneth in return was largely indebted to Röth's cultural history, as would also be Moritz Cantor. Hankel's approach on the contrary involved his breaking loose from presumably outdated scholarship with regard to cultural history, as represented by the Heidelberg trumvirate, *viz.* Röth, Arneth and the young Cantor. Considering Röth's cultural history in historical context will thus help highlighting a process that started there.

Eduard Maximilian Röth<sup>113</sup> studied at the University of Giessen where he most probably met Johann August Vullers (1803–1881) who, like many other German orientalists at the time, had been trained in Paris by Silvestre de Sacy (1758–1838)<sup>114</sup>. He was appointed in 1833 to a chair of oriental languages in Giessen. Röth's first interests were with the historical roots of Christianity in Judaism, which led him thereafter to study rabbinic literature with a Jewish scholar in Frankfurt<sup>115</sup>. His first published work in 1835 thus dealt with the dogmas of the early Christian communities through investigating the author and the addressees of the Epistle to the Hebrews. In so doing, he came to be convinced that “the [*proto-Christian*] circle of ideas was not an original, but a derived one, [*and*] a collection of hints pointed out further to the Orient and to Egypt”<sup>116</sup>. In order to deepen his historical knowledge in these matters, he came to Paris in 1836 to study oriental languages with Silvestre de Sacy (maybe on Vullers' advice), Eugène Burnouf (1801–1852) and the sinologist Stanislas Julien (1797–1873). He is also said to have taken advantage of his staying there for four years to widen his scope in attending the public lectures of such leading scientists as Arago, Biot, Dulong and Dumas. But his main interest focused on the Zend Avesta and hieroglyphics.<sup>117</sup> What he learned in Paris with the French orientalists shaped his view, later to be embodied in his

<sup>113</sup> Very little is known about Röth's life. All the information available in the literature (cf. [Lützen & Purkert 1994, p. 3], [Dauben 2002, p. 116]) on this score apparently stems from a short notice in the *Allgemeine Deutsche Biographie*, dated 1889 and merely authored by “L.”, which in return condenses a notice in von Weech's *Badische Biographieen*, II, 210 (1875).

<sup>114</sup> Cf. Espagne [2008] and [Mangold 2004, pp. 38–41]

<sup>115</sup> Studying under Jewish scholars was not an uncommon practice among German orientalists at the time. Heinrich Ewald for instance and a few others who were to become authorities in Semitic languages, also cultivated such rabbinic connections, cf. [Marchand 2009, note 70, p. 77].

<sup>116</sup> [v. Weech 1875, p. 210].

<sup>117</sup> Cf. [Röth 1846, *Vorrede*, p. vii]: “...I recognized in the beliefs of the Egyptians and the Persians the common sources of Greek philosophy and Judeo-Christian ideas. Now a new resolution was ahead of me. I had to seek information also on these remote domains. The hieroglyphs and the Zend provided me with the key”.

1846 book on the history of occidental philosophy, that “the roots of our modern knowledge are not to be sought in India nor China, but rather in the doctrine of the Egyptians and Zoroaster”<sup>118</sup>. After habilitating at Heidelberg, Röth continuously lectured there from 1840 until his death in 1858, first as *Privatdozent*, then as *Extraordinarius* in 1846, eventually as “full professor of philosophy and Sanskrit”<sup>119</sup> in 1851. Röth’s second monograph, published in 1858, focused on the history of Greek philosophy, more specifically on the doctrines of the Ionian philosophers and the Pythagoreans, with the aim of showing, as the title page of the book advertized, how these resulted from “the transfer of Oriental ideas into Greece”. Significantly enough, Röth’s opus was dedicated to the “jewels of Germany”, Alexander von Humboldt, “who encouraged a global natural science and revived the cosmos idea”, August Boeckh, “who founded a realistic science of Antiquity and a more correct appreciation of the Pythagorean remains”, and eventually, although rather disruptively, to Philipp Fallmerayer (1790–1861),<sup>120</sup> “who victoriously challenged narrow

118 *Ibid.* Röth’s *Geschichte unserer abendländischen Philosophie* is composed of two volumes: the first one was published in 1846 under the title *Die Ägyptische und die Zoroastrische Glaubenslehre als die ältesten Quellen unserer spekulativen Ideen*, and the second one followed in 1858 under the title *Geschichte der Griechischen Philosophie. Die Übertragung der orientalischen Ideenkreise nach Griechenland und ihre Fortbildung durch die ältesten Ionischen Denker und Pythagoras*.

119 [v. Weech 1875, p. 210]. As the registration list of the lectures given at the Ruprecht-Karls Universität of Heidelberg during the whole period 1840–1858 shows, Röth indeed began lecturing on Sanskrit and comparative grammar, before gradually extending the range of his lectures to various philosophical topics such as logic, metaphysics, psychology, philosophical encyclopedia and history of ancient and modern philosophy. In the winter semester 1840–1841, Röth gave for the first time a lecture course on “Sanskritgrammatik mit grammatischer Interpretation des Nalus”, which he gave regularly up to the summer semester 1852. Later, Sanskrit grammar was exclusively taught by Adolf Holtzmann (1810–1870), a professional Sanskritist who officially held the chair of Sanskrit and German literature at Heidelberg from 1852 onwards. This noticeable shift bears witness to the process of professionalization of oriental studies which was then under way in Germany; see Mangold [2004], Rabault-Feuerhahn [2008] and *infra*.

120 Jakob Philipp Fallmerayer was an Austrian Lyceum professor, historian, publicist and politician who held controversial historical views so as to counter what he considered to be misguided European politics. He argued that the originally Hellenic people of the South Balkans had been replaced by Slavic peoples during European migrations. In so doing, his aim was to induce European powers to renounce supporting the Greek Independence War against the Ottoman Empire, for, he claimed, the purported philhellenism on behalf of which these powers sided with the modern Greeks was utterly pointless. On the role of Fallmerayer’s ideas in the context of nineteenth-century debates on Antiquity, ethnogenesis and identity in the Balkans, see [Klaniczay et al. 2011, pp. 219–220]. In praising Fallmerayer’s ideological intrigues conjoining politics and science, Röth thus provocatively reenacted a well-known German debate

Hellenomania".<sup>121</sup> The oddity of bringing together the first two dedica-tees with the third, already hints at Röth's peculiar mix of eclecticism and untimeliness, which will be best delineated in contrast to an alternative stance in the same slot within the German academic field.

At about the same time, Eduard Zeller (1814–1908), professor of philosophy at Marburg<sup>122</sup>, also brought into question the supposed debt of Greek philosophy with regard to oriental speculation in his monumental *Philosophie der Griechen in ihrer geschichtlichen Entwicklung* (1844–1852), but his methods gradually proved themselves incompatible with Röth's. Not only did he markedly dissent from Röth's overall conception<sup>123</sup>, but he also sharply criticized the methodology<sup>124</sup> which presumably flawed Röth's purported contribution to the study of ancient Pythagoreanism<sup>125</sup>. Despite the fact that Röth extensively drew on Boeckh's scholarship on Pythagorean sources, which Zeller also highly praised,<sup>126</sup> the deep

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of the 1820s, when philhellenic Protestant liberals had begun to consider Creuzer's work not only as scientifically disputable, but also as politically dangerous, cf. [Mar-chand 2009, p. 70].

<sup>121</sup> Cf. [Röth 1858, p. vi].

<sup>122</sup> Before being eventually called to the University of Berlin in 1872, Zeller moved from Marburg—where he had shifted from theology to philosophy—to Heidelberg in 1862. Röth had then been dead for four years.

<sup>123</sup> Cf. [Zeller 1856, p. 21]: "If, indeed Röth were correct in asserting, as he does ([Röth 1846, pp. 74, 241]), that Philosophy did not spring from the cultural con-ditions and the spiritual life of the Greek people, but was transplanted [*verpflanzt*] among them as something foreign, and that the whole circle of notions lying at its root came ready made from without, then, and then only, we might derive Greek philosophy absolutely from the Orient. But if, on the other hand, it was the immediate product of the Greek philosophers' own reflection, in that case it has essentially a na-tive origin, and the question can no longer be whether, as a whole, it came from the Orient, but whether Oriental doctrines had any share in its formation, how far this for-eign influence extended, and to what extent we can still recognize in it the Oriental element proper, as distinct from the Hellenic element."

<sup>124</sup> See for instance Zeller's severe pronouncement on Röth's unbridled use of groundless etymologies in historical reasoning, cf. [Zeller 1856, pp. 28–29].

<sup>125</sup> For most scholars of that period, the problem of assessing Greek indebtedness to the Orient focused on Pythagoras, and in particular on the extent to which his mathe-matics and the theorem bearing his name, were derived from Egyptian, Persian or Indian sources. In the mid 1880s, the German indologist Leopold von Schröder (1851–1920) sidestepped the intricacies of extracting the supposed core of historical truth from traditional accounts of Pythagoras' eastbound travels—in this regard he never-theless tipped the scale in favor of Zeller's interpretation vis-à-vis Röth's (see [von Schroeder 1884, p. 4]),—and chose a more circumspect approach, namely probing Pythagoras' doctrines and comparing them with those of India so as to gradually strengthen the conviction of an Indian influence.

<sup>126</sup> Cf. [Zeller 1856, p. 211]: "The greater number of the fragments of Philolaos, as Boeckh has shown in his excellent monograph [*viz.* Boeckh [1819]], must certainly be

methodological break between them may be traced back to divergent practices in historiography. In his early twentieth century *Geschichte der neueren Historiographie* (1911), Eduard Fueter presented Zeller as “one of the most distinguished representatives of Hegel’s school” of German historians, although one embodying “the reaction against the master’s schematism … [*insofar as*] he rejected the old method of mere compilation as decidedly as Hegel, but still did not infer from this that history should be constructed logically.”<sup>127</sup> Anthony Grafton has shown when, why and how, in the history of scholarship, modern historians came to adopt what he called the “standard double form”,<sup>128</sup> which is distinctive of modern historical narrative. In providing room for the historians to systematically scrutinize and compare the available evidence, argue over their sources so as to hierarchize them in proportion to their reliability, footnotes gradually became “the outward and visible signs of this kind of history’s inward grace—the grace infused into history when it was transformed from an eloquent narrative into a critical discipline”<sup>129</sup>. But, as Grafton also emphasized, “the footnote’s rise to high social, if not typographical, position took place when it became legitimate, after history and philology, its parents, finally married”<sup>130</sup>. Significantly enough, Zeller entrusted the kind of “middle course” historiography he pursued<sup>131</sup> to those footnotes, whose regime of critical scholarship Röth utterly ignored, while, on another front, he kept estranging himself from the increasingly professionalized practice of German orientalists.

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considered genuine, not merely on the score of external testimony, but also, and far more, because in content and mode of expression they agree with one another, and are in harmony with all that we know from well authenticated sources as Pythagorean.”

<sup>127</sup> [Fueter 1911, p. 442]. Zeller’s charge against Hegel and his distinction between “*Geschichtschreibung*” and “*Geschichtsconstruction*” (see [Zeller 1856, pp. 7–17]) bears witness to his being more attuned to the German school of historians (cf. *infra*) than to Hegelian orthodoxy.

<sup>128</sup> Cf. [Grafton 1999, p. 23]: “Modern historians, by contrast [*with historians of previous periods*], make clear the limitations of their own theses even as they try to back them up. The footnotes form a secondary story, which moves with but differs sharply from the primary one. In documenting the thought and research that underpin the narrative above them, footnotes prove that it is a historically contingent product, dependent on the forms of research, opportunities, and states of particular questions that existed when the historian went to work.”

<sup>129</sup> Cf. [Grafton 1999, p. 24].

<sup>130</sup> *Ibid.*

<sup>131</sup> Cf. [Zeller 1856, p. vii]: “In the treatment of my subject I have constantly kept in view the task which I proposed to myself in my first approaches to it: *viz.* to maintain a middle course between erudite enquiry and the speculative study of history: neither,

A controversy that started in 1855 may help to delineate more precisely how such marked differences in methodology running through the whole field of historical sciences confronted with ancient sources, also affected the shaping of history of mathematics in Germany. In his second 1858 volume, Röth reaffirmed the Egyptian origin of ancient Greek philosophy, which he claimed to have proved with irrefutable evidence in spite of the dismay these views aroused in many of his fellow scholars, whose “restricted school opinions”<sup>132</sup> were thus blatantly antagonized. In this connection, he alluded to “malicious and slanderous attacks, namely those of Ewald from Göttingen against the author’s decipherment of the Cypriot language”<sup>133</sup>. Röth had indeed also ventured in the domain of ancient epigraphy in publishing a tentative deciphering of the bronze tablet of Idalion, whose inscription had been reproduced in print three years earlier by the French *savant antiquaire*, collector and numismatist Honoré d’Albert, duc de Luynes (1803–1867).<sup>134</sup> This was holy bread for Röth, who thought he could make the best of this new material so as to establish his views on solid historical grounds. His goal was to prove that Greek culture could be traced back to the so-called Pelasgians,<sup>135</sup> who supposedly immigrated from Egypt into Greece in the nineteenth century BCE. The

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on the one hand, to collect facts in a merely empirical manner; nor, on the other, to construct *a priori* theories; but through the traditions themselves, by means of critical sifting and historical combination, to arrive at a knowledge of their importance and interdependence. ... In order that the clearness of the historical exposition, however, might not be thereby impaired, I have consigned these discussions as much as possible to the notes, where the testimonies and references respecting the authorities find a fitting place.”

<sup>132</sup> [Röth 1858, Foreword, p. x].

<sup>133</sup> [Röth 1858, Foreword, p. xi].

<sup>134</sup> Cf. d’Albert duc de [1852]. This bronze tablet—now known as ICS 217 (cf. Masson [1983]) and preserved at the Bibliothèque nationale de France, Musée du cabinet des médailles—had been found near Dali (the ancient Idalion) and bought by Honoré d’Albert, duc de Luynes to the French consul in Beirut. Both faces of the tablet were covered by an inscription in an unknown writing which also appeared in coins found in the same area. In his *Numismatique et inscriptions chypriotes* (1852), de Luynes published all these inscriptions and suggested that they might correspond to an unknown ancient Cypriot language.

<sup>135</sup> The name “Pelasgians” was used by ancient Greek authors, for instance Herodotus, to refer to populations that were supposed to be the ancestors, or the predecessors, of the Greeks. However, modern scholarship strikes a cautious note: cf. [Myres 1907, p. 170]: “Few peoples of the ancient world have given rise to so much controversy as the Pelasgians; and of few, after some centuries of discussion, is so little clearly established. ... they have been a peg upon which to hang all sorts of speculation.” For our purpose, one should discern here a distinctively Creuzerian feature of Röth’s enterprise, since Creuzer maintained that the mythology of Homer and Hesiod came from an Eastern source through the Pelasgians.

Idalion tablet would thus afford the means to reinforce Herodotus against “the short-sighted prejudices of the last philological schools”<sup>136</sup>, insofar as presumably instantiating that ancient Pelasgian writing which, on the account of ancient Greek sources, would preexist Phoenician characters in Cyprus. “Although closely related to Phoenician writing, according to the form, [Röth argued,] this [Idalion] writing still wholly connects up with the hieroglyphic writing owing to its polysematic characters—in the few monuments that have come down to us now, there are already 120 signs for the 22 sounds of the usual Phoenician alphabet—and therefore forms the missing link [*Mittelglied*] between the even richer hieroglyphic writing and the absolutely simple Phoenician one.”<sup>137</sup> Röth’s point was indeed twofold. In assuming on the one hand that the language of the inscription was a mix of Chaldean and Hebrew<sup>138</sup>, he astutely produced what he confidently took to be the translation of an address to the people of Cyprus, supposedly pronounced by the Egyptian king Amasis, who was known to have conquered the island in the middle of the sixth century. Though a misguided historical artefact, Röth’s deciphering thus told a story aptly consistent with both ancient sources<sup>139</sup> and modern scholarship<sup>140</sup>, namely the story of a civil war opposing a pro-Chaldean aristocratic party and a pro-Egyptian democratic one.<sup>141</sup> But on the other hand, he

136 [Röth 1858, p. 10].

137 *Ibid.* While assuming that the introduction of the Phoenician alphabet into Greece dated back to the sixteenth century BCE, Röth stated that the Cypriot language of the Idalion tablet must have originated even earlier, from which he eventually inferred what he took to be the probable date of the installation of the Pelasgians in Greece.

138 We now know that the language of the Idalion tablet is in fact a Greek dialect, cf. Masson [1983], Egetmeyer [2010].

139 Cf. [Röth 1855, p. 98] for the references to Herodotus II, 182 and Diodorus I, 68.

140 Cf. [Röth 1855, p. 104], Röth specifically referred to the work of the German Roman catholic cleric and orientalist, Franz Karl Mövers (1806-1856), professor of theology at Breslau from 1839 until his death, who wrote a great monograph *Die Phönizier* (1841-1850). Röth also relied on some results published by the German classical archeologist Ludwig Ross (1806-1859) in *Hellenika*, Halle, 1846.

141 The true content of the inscription is now known to be somewhat different. The Idalion tablet was effectively deciphered more than twenty years later, owing to a bilingual inscription—in Cypriot and Phoenician—found in 1871 by Robert Hamilton Lang, British consul in Cyprus. The deciphering process began in 1872 as a joint international enterprise which involved the British assyriologist George Smith (1840-1876), the British egyptologist Samuel Birch (1813-1885), the German polymath Johannes Brandis (1830-1873), as well as the German philologists Moritz Schmidt, Wilhelm Deecke, Justus Sigismund and Heinrich L. Ahrens. As soon as 1877, the French linguist Michel Bréal (1832-1915), professor of comparative grammar at the Collège

tried to establish the Egyptian origin of the Cypriot writing he thought he had cracked, in putting forward two properties, supposedly common to Phoenician alphabet and hieroglyphic writing, *viz.* “akrophonia”<sup>142</sup> and “polysematism”<sup>143</sup>. At this critical juncture, Röth’s argument ultimately rested on the authority of the German philologist and orientalist Wilhelm Gesenius<sup>144</sup> (1786-1842), who had pointed out that the written characters also arose as pictograms in the Phoenician alphabet.<sup>145</sup> Röth’s step further consisted then in shifting from Gesenius’ merely comparative remark to the assumption of an effective historical continuity, which beguiled him into illusorily searching for an *Ur-Alphabet* between the Egyptian and Phoenician writings. In pointing out that his narrative accorded with the expected “course of development” from the more complex and cumbersome forms of writing to the simpler and the more appropriate ones, he jumped to the conclusion that the emergence of Phoenician writing from the Egyptian one was to be regarded as a “historically documented fact”. In his view, the purported deciphering of the Idalion tablet he prided himself on proved to be “the brightest triumph of historical research over the empty negative criticism of the modern schools”<sup>146</sup>.

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de France, and translator of the work of Franz Bopp with whom he had studied Sanskrit in Berlin, published a note in which he recapitulated these various contributions and summarized the content of the inscription, see [Bréal 1877, p. 186], and [Masson 1983, p. 238] for a more complete account. The text proved to be a decree from the Greek king of Idalion, Stasikypros, around 478-470 BCE, about an agreement with a physician named Onasilos and his brothers, so that they would take care of the wounded at the siege of the city by the Persians and the people of Kition which was under Phoenician rule at the time.

<sup>142</sup> To pin this property down, Röth formulated it “according to Champollion’s astute conception: ‘Les Égyptiens voulant exprimer soit une voyelle, soit une consonne, soit une syllabe, se sont servis d’un signe exprimant ou représentant un objet quelconque dont le nom, en langue parlée, contenait ou dans son entier ou dans sa première partie le son de la voyelle, de la consonne, ou de la syllabe qu’il s’agissait d’écrire.’ (Lettre à Dacier, p. 34, in accordance with Gramm. égyptienne, p. 28). Gesenius now proves that the same principle in the formation of the writing also applies to the Phoenician alphabet.” [Röth 1855, pp. 108-109].

<sup>143</sup> Röth coins the term “Polysematik” to designate the property of a writing in which one sound may be associated with many signs. Cf. [Röth 1855, p. 109].

<sup>144</sup> Wilhelm Gesenius, who held a chair of theology at the university of Halle, may be considered as the initiator in Germany of critical semitic philology as an autonomous field of inquiry, emancipated from the prejudices of religion, cf. [Benfey 1869, p. 685]. It is noteworthy that he counted among his students Peter von Bohlen who later initiated Heinrich Nesselmann into oriental philology in Königsberg.

<sup>145</sup> In this connection, Röth referred to Gesenius’ *Geschichte der hebräischen Sprache und Schrift* (1815) and to the entry “Paläographie” in Ersch and Gruber’s widely known *Allgemeine Encyklopädie der Wissenschaften und Künste*, Brockhaus, Leipzig, 1837, pp. 287-316.

<sup>146</sup> [Röth 1858, p. 12].

However, as mentioned above, Röth's proposal immediately met with scathing criticism. In the *Göttingische gelehrte Anzeigen* dated November 5, 1855, the leading German orientalist Heinrich Ewald<sup>147</sup> (1803–1875), who reluctantly reviewed what he characterized as a "half-erudite publication"<sup>148</sup>, stated that the whole deciphering could not be trusted insofar as it built on unsupported conjectures, which he did not even care to dismiss, apart from pointing out that "the essence of an alphabetic writing is cancelled out, if it allows completely different signs for the same sound".<sup>149</sup> It should be noted that the author of this judgment occupied a central position in the community of German orientalists, which he had largely contributed to develop and organize, providing it with its own institutional organs and networks. Together with Christian Lassen (see *infra*), the first German scholar who obtained a few years later an independent chair in indology at Bonn, Ewald had founded in 1837 the *Zeitschrift für die Kunde des Morgenlandes* (ZKM) in hopes of advancing oriental studies by unifying the forces dispersed all over Germany. The next year, he took an active part in the creation of the *Verein deutscher Philologen und Schulmänner* which, being modeled upon the *Gesellschaft deutscher Naturforscher und Ärzte*, was meant to gather all German classical philologists each year in a different city. Eventually the foundation of the *Deutsche Morgenländische Gesellschaft* (DMG) in 1845 in Darmstadt on the occasion of one of these meetings would foster the structuration and professionalization of German orientalists.<sup>150</sup> Not only did Ewald play an important role in the process of disciplinary differentiation by which oriental studies separated from classical philology, but he was also an energetic proponent of the introduction

<sup>147</sup> Heinrich Ewald may be described as "the greatest of those [*protestant theologians*] who attempted to embrace the whole Orient in their research and knowledge. His main strengths lay in the domain of Hebraic and Arabic languages but he had also acquired a good knowledge in Sanskrit." ([Windisch 1917–1920, II, p. 210]) He held the chair of oriental languages in Göttingen, where he married C. F. Gauss' daughter Wilhelmina and taught all his life, albeit with an interruption of a few years due to his being, together with the Grimm brothers and the physicist Wilhelm Weber, one of the so-called *Göttingen Seven*, who protested against the politics of the king of Hannover in 1837.

<sup>148</sup> [Ewald 1855, p. 1761].

<sup>149</sup> [Ewald 1855, p. 1763]. With this simple remark, Ewald refuted Röth's fundamental insight. Not only was Röth's postulation of polysematism, reducing the 61 signs of the Idalion tablet to the well-known 22 sounds of the semitic alphabet, denounced as an arbitrary decision, but his use of Gesenius' remark about the so-called "akro-phonía" of Phoenician alphabet was censured as detrimental to the very notion of an alphabetic writing.

<sup>150</sup> On the relation between the orientalists of the DMG and the philologists of the *Verein*, cf. [Mangold 2004, Chap. V, pp. 176–225].

of historical methods in the emerging academic field.<sup>151</sup> In this context, Röth appeared as an outsider amidst the German orientalists of his time. Although the *DMG* would gather scholars of various status, from university professors to Gymnasium teachers, along with independent scholars, and various disciplines, from theology, biblical philology, oriental languages, Sanskrit literature to philosophy, there is no trace of Röth, nor Arneth, in the official record of the members which was yearly updated in the annual reports of the society. By contrast, such professionalized Sanskrit scholars, well versed in history of mathematics, as Hermann Brockhaus or Georg Heinrich Nesselmann, sooner or later joined the *DMG*.<sup>152</sup> Publishing in the *Zeitschrift der DMG*, the journal that took over from the *ZKM* after the creation of the *DMG*, might also be another good indicator of the degree of sharing in the values and methods of the unifying German orientalist community. Obviously, Röth did not see himself as belonging to what he scornfully called the last short-sighted *Philologenschulen*, while reciprocally, from the viewpoint of those professionalized orientalists, his allegedly uncritical practice ruled him out, all the more so as questions of method were being increasingly focused on.

The controversy over the Idalion tablet may thus be regarded as a prism which helps us separating different, though intermingled, scholarly practices at a given time. However, what appears synchronically as a methodological watershed in the middle of the 1850s may also be thought of diachronically. One may see it as resulting from the long term transformation of German classical scholarship from the erudite practices of the polyhistorians of the late eighteenth century to the rise of nineteenth-century historical philology and textual criticism. In focusing on classical scholarship—the matrix from which oriental studies grew—Anthony Grafton<sup>153</sup> has shown how both kinds of accounts—external and internal—should be combined into one coherent picture, if one were to make sense of that change: namely not only the story of professionalization and social demand as told by historians of education from sociological and institutional evidence, but also the internal history of philology, as made by the practitioners themselves, highlighting the shift

<sup>151</sup> Cf. [Mangold 2004, pp. 103–104]. As emphasized by Sabine Mangold, in a conclusive note to the third issue of the journal he founded, Ewald urged his colleagues to “turn to history, much more than has been done until now, in all parts of oriental knowledge” *ZKM* (1840), 491.

<sup>152</sup> Brockhaus was a founding member of the *DMG* right from the start in 1845, while Nesselmann did not join in until 1853.

<sup>153</sup> Cf. Grafton [1983].

in the very content of philology. In Grafton's view, what the Humboldtian reform with its so-called "research imperative"<sup>154</sup> brought about was "less a revolution in teaching [*viz. with regard to the previous practices of textual elucidation and emendation taught in philological seminars*<sup>155</sup> such as Heyne's in Göttingen, or Wolf's in Halle,] than a gradual increase in the attention given to controverted problems and technical methods for their solution—and a gradual decrease in the attention given to the traditional objects of humanistic studies"<sup>156</sup>. The tendency toward specialization and expertise resulted in technicalities being so overemphasized that any attempt to grasp classical culture as a whole would be jeopardized. "Humboldtian ideals of research [*thus*] came to contradict [...] Humboldtian ideals of *Bildung*"<sup>157</sup>. As a result, "the Humboldtian system subverted itself, [*insofar as*] *Wissenschaft* and *Bildung* could not fit easily in one bed or one curriculum"<sup>158</sup>. Furthermore, Grafton suggests that the received view of nineteenth-century German philology being divided in competing schools, Hermann's *Wortphilologie* *vs.* Boeckh's *Sachphilologie*,<sup>159</sup> should not conceal such shared underlying features as the common inclination to dissolve ancient texts so as to recreate something lost, whether the lost manuscript from which the extant ones supposedly proceeded, the lost meaning of an ancient practice or a lost dialect. However, in dissecting and reconstructing texts for all kinds of purposes, philologists gradually lost their footing inasmuch as evidence could be indefinitely carved up. Consequently, "the professors of philology could not possibly present their students with that unified picture of the development of nations and cultures which would have engendered *Bildung*".<sup>160</sup> In his controversy with the philologists, Röth clearly took the side of cultural history on a grand scale, in his view unduly neglected, whereas his opponents put forward

<sup>154</sup> Cf. Turner [1981].

<sup>155</sup> On the Göttingen philological seminar and the role of Christian Gottlob Heyne (1729–1812) in the emergence of philological discourse in Germany, cf. Leventhal [1986]; on Friedrich August Wolf and his view of philology as *Alterthumswissenschaft*, see *infra*.

<sup>156</sup> [Grafton 1983, pp. 167–168].

<sup>157</sup> [Grafton 1983, p. 169].

<sup>158</sup> [Grafton 1983, p. 176].

<sup>159</sup> On the methodological controversy between the Berlin classicist August Boeckh and his Leipzig opponent Gottfried Hermann, and the polarizing tension between *Sachphilologie* and *Wortphilologie*, see Vogt [1979], and *infra*. Whereas the supporters of the latter focused exclusively on language, textual criticism and editing techniques, the adherents of the former considered textual sources on a par with other *realia* from the past (monuments, coins, inscriptions, ...) in giving access to ancient cultures.

<sup>160</sup> [Grafton 1983, p. 183].

the new standards of scientific canonicity. In ignoring the specific “double form” narrative of nineteenth-century critical historiography, he certainly displayed external attributes more akin to a polyhistor’s than a philologist’s, but in deliberately doing so around the middle of the nineteenth century, he also meant to distance himself from what he stigmatized as the failure of the new philology. “According to all signs, [Röth indeed warned in 1846,] our spiritual *Bildung* is now going through one of those crises which mark an epoch in the course of human development.”<sup>161</sup> While holding that “history of philosophy forms an inner, essential part of the whole history of human culture”<sup>162</sup>, Röth set out to delineate “where our present philosophical *Bildung* begins”<sup>163</sup> and “sought the elucidation of our present in the past”<sup>164</sup>, namely in Egypt and Persia. In this connection, Röth’s second 1858 volume can be read as an even sharper manifesto against what he referred to as the *Hellenomania*<sup>165</sup> of the philosophers and the philologists of his time.<sup>166</sup> Not only did he dismiss as completely erroneous the “quite universally prevailing view [...according to which] the first blossom of Ionian cities [...were] like Oases in the desert, like isolated lights in the darkness of the preceding and the subsequent centuries”<sup>167</sup>, but interestingly enough, he also yielded a rather accurate diagnosis of such a misconception. In considering it a general law of history that “a people must have covered the greatest part of its development before the need for knowledge and science can only be felt”<sup>168</sup>, he held that the

<sup>161</sup> [Röth 1846, Introduction, p. 1].

<sup>162</sup> [Röth 1846, p. 38]. Röth saw the philosophical turmoil of his time as “a proof that our spiritual *Bildung* feels the need of a genuine and appropriate expression for its *Weltanschauung*, [and he considered that] all the shudders of our present philosophical crisis are the pangs of this new spiritual birth”[Röth 1846, p. 16].

<sup>163</sup> [Röth 1846, p. 21].

<sup>164</sup> [Röth 1846, Foreword, p. v].

<sup>165</sup> Compare with David Pingree’s views in Pingree [1992], where the term “Hellenophilia” is endowed with more or less the same meaning as that which Röth attached to “Hellenomania” one century and a half earlier. In both cases, one finds a similar denunciation of the myth of a Greek miracle arising out of nowhere.

<sup>166</sup> Cf. [Röth 1858, p. 753]: “Quite apart from their *Gräkomania* and their prejudices against the Orient, the Egyptian *Bildung* and literature was a complete *terra incognita* for the philosophers and the philologists”. The issue of the presumably Egyptian origin of Greek culture was in particular very much discussed at the time in philologists’ circles; see for instance the heated debate between Hermann and Gerlach at the tenth meeting of the *Verein deutscher Philologen* in 1847 in Basel, cf. [Grafton 1983, pp. 174–175].

<sup>167</sup> [Röth 1858, p. 3].

<sup>168</sup> [Röth 1858, p. 4].

beginnings of Greek culture should have preceded the advent of philosophy proper by a millennium. For the Greeks, as for any other people, such beginnings are shrouded in darkness, which is only natural, at least in part, Röth conceded, since they go back to the very beginnings of historical tradition itself. However, in the case of the Greeks, "this darkness is also in part an artifical one"<sup>169</sup>. In analyzing the process by which such darkness was created as a historiographical artefact, Röth then articulated an assumedly less naive stance.

The addiction to doubting together with the eagerness for denying which are currently so predominantly practised ...—this transitional disease of our time—has preferentially applied its negativity precisely in this field of ancient history, with the last philological schools; for it is easier to doubt than to understand, to destroy than to build. It has thus made such a terrible mess out of the few accounts from the Ancients that have come down to us, so corroded and volatilized them that the gaps that were previously there in this domain have become a complete wasteland which turned the twilight into a complete darkness. But an *horror vacui* is so deeply entrenched in the human mind that one attempted to fill this empty space with forged fantasies, ...whereas, with a comical fear, one tried hard to get rid of all the historical traces of cultural influences from the highly cultivated kingdoms of the Orient, whether in Egypt or Asia Minor. Owing to these efforts, one has then succeeded in making Greek prehistory into a complete fable for most of our contemporaries, and into a region even more remote and fabulous than *ultima Thule*.<sup>170</sup>

In this context, returning to the Ancients clearly appeared as a deliberate move against the philologists' alleged ideological forgeries.

### 5. **KULTURGESCHICHTE NATURALIZED**

A few years before the controversy between Röth and the philologists broke out, Arthur Arneth had already appropriated his Heidelberg colleague's anti-Hellenomania motto for devising a new way to envisage the history of mathematics. As recounted in the foreword to his *Geschichte der reinen Mathematik* (1852), Arthur Arneth first learned in the beginning of the 1820s, while reading Playfair's works, about the marked difference between Greek and Indian mathematics. His interest in the cultural contexts of mathematics being aroused, he later engaged in a series of investigations which resulted in scattered memoirs that were not originally bound to be published. "It is only [Arneth explained,] as I read Röth's *Geschichte der Philosophie* that I recognized the connection, the thread that runs through

<sup>169</sup> [Röth 1858, p. 6].

<sup>170</sup> [Röth 1858, pp. 6–7].

the whole, the dualism in mankind”<sup>171</sup>. However, Arneth had a purpose of his own, insofar as he meant to reap the fruit of Röth’s *Kulturgeschichte* sown into the soil of the natural sciences. His original conception of history of mathematics thus grew from his accommodating Röth’s main insights to a naturalistic framework. In his view, history of mathematics could indeed be told in two essentially different ways. The first one consists in “investigating and faithfully representing the facts, *viz.* when, how and by whom single propositions or whole parts of this science were introduced, [...] so that [...] the main thing, the scientific element comes to the fore.”<sup>172</sup> This kind of narrative which proves best fitted to the mathematician who can grasp the abstract contents had been presumably favored to the detriment of another one, which Arneth claimed to herald. “One only endeavored to show how mathematics rose up to its present state, and one has almost completely failed to investigate the why, the grounds for the characteristic development of mathematics in the different great groups of peoples.”<sup>173</sup> In acknowledging that knowledge of Oriental cultures had significantly increased in recent times, Arneth advocated a renewed overall picture accounting for the differences between the main peoples with regard to their fundamental “mindset” [*Geistesrichtung*]. In so doing, he also viewed the abstraction process leading to mathematical content as being conditioned by cultural factors.

Abstract science only arose at the end of a very long period of time, during which mathematics only related to reality. People started with reality and then, painfully and slowly, rose up to abstraction. If only we had a complete history of mathematics, we would have a history of pure thought or rather a representation of the endeavors of mankind to rise up to pure thought. But such a history is basically nothing else than a history of the development of the human mind, which expresses itself in the whole life of a people, in intellectual as well as in material culture, in public as well as in private states of affairs, but especially in its religious *Anschauungsweise* which forms the foundation for everything else. Therefore where traditions in the domain of mathematics are lacking, one will be able to draw inferences from the totality of all phenomena in a people’s life, so as to explain further productions, and fill the gaps with the greatest likelihood in order to produce a whole of the same kind.<sup>174</sup>

<sup>171</sup> [Arneth 1852, p. iii]. In addition to the first 1846 volume of Röth’s *Geschichte unserer abendländischen Philosophie*, Arneth also referred to the second volume which was only published with an important delay, in 1858, that is, as Röth explained, after the period of political turmoil which started in 1848. Hence Arneth took cognizance of its content before 1852.

<sup>172</sup> [Arneth 1852, p. 1].

<sup>173</sup> *Ibid.*

<sup>174</sup> [Arneth 1852, p. 4].

This pronouncement is not to be understood though as a license given to the historian to proceed to more or less arbitrary reconstructions, but rather the vindication that pure thought should be envisaged as a cultural product, entrenched in the concrete background of practices, habits and beliefs from which it emerges as an illusorily self-contained content. Therefore, history of mathematics is much more than a mere history of mathematical content; it is rather the cutting-edge of cultural history: it is where cultural history ends up in a concentrated form. But in return, in Arneth's view, cultural history only makes sense within a broad naturalistic outlook, since its crux is to account for human diversity, which only the natural sciences may eventually put in the right light. In referring to the mind-body problem, he considered that not only is the body subject to physical laws, but also, to a certain extent, the mind, insofar as it depends upon the body as its organ of expression. Cultural history is therefore ultimately founded in the laws of nature, for these encompass in the end the bodily roots of all mental life. However, Arneth's point here is far from being a hard-nosed reductionist one. Knowledge of the laws of nature should allow one to grasp "a higher law for the whole world process, in which therefore the mind also takes part, insofar as it is connected to the body."<sup>175</sup> From the standpoint of this higher law then, the reasons why different forms of mental development occur among different groups of peoples would presumably become transparent. Gaining such an insight into why such various mindsets emerged would therefore enable us to carry out a genuine history of mathematics.

Arneth's monograph is thus divided into three parts which embody this scheme. While the first part presents the laws according to which world life presumably unfolds, the second and the third ones deal with history of mathematics proper, respectively in ancient traditions up to the Arabs, and in modern times up to the nineteenth century. A review of the book published in 1854 in the *Göttingische gelehrte Anzeigen* bears witness to the way Arneth's approach to history of mathematics was received in German mathematical circles. The author of the note, Heinrich Christian Schnuse (1808-*ca.* 1878) had studied at Göttingen in the first half of the 1830s where he attended Gauss' lectures, then moved to Marburg to study under Gauss' friend, Christian Ludwig Gerling. After obtaining his doctorate, he did not

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<sup>175</sup> [Arneth 1852, p. 5].

find a position and earned his living as one of the most prolific translators, reviewers and textbook writers of the period.<sup>176</sup> While considering that Arneth's views were "in general correct and objective—at least in respect to mathematical matters"<sup>177</sup>, Schnuse was nevertheless much more cautious as regards the grand narrative that opens the book. "As is well-known, [Schnuse warned,] most often in these matters, there is no question of positive knowledge as in pure mathematics—Often these are only more or less likely opinions (hypotheses) which frequently diverge and stand in opposition to one another. An in-depth appraisal of the author's views can all the more be omitted here as these objects do not belong at all to mathematics."<sup>178</sup> Notwithstanding, Schnuse provided, although in a sceptical way, a concise summary thereof, which sheds light on Arneth's peculiar attempt to embed history of mathematics into an overarching vitalism. Two fundamental forces are assumed at the outset, a "binding force" [*bindende Kraft*], *viz.* matter, and a "breaking force" [*lösende Kraft*], *viz.* life. Although originally bound in matter and gradually freeing itself from it by the process of world formation, life only reaches free self-determination with the creation of man, which in return appears as the ultimate purpose of all natural processes. Arneth thus sharply contrasted the vitalistic world view he promoted with the presumably prevailing materialistic-reductionist one which he resisted.

According to this [*last*] system, [*he explains,*] matter is the only real being and apart from it, there is nothing. Matter is forever existent, imperishable and equipped with certain properties and forces. As a result of these forces, the material particles are forced to enter into combinations and to unite into new bodies, which possess new and higher properties. The properties so increase in each new combination that at a certain stage, organic matter and life arise. By these the development is pursued up until man, as the goal of the whole process. Life is a mere natural life, an expression of the activity of matter. As for what is called mind, operating in us through matter, but thought of as different from it, there is no such thing as that. The function of the so-called mind comes from those parts of organic matter which one calls brain and nervous system. ... This view, with the most diverse variations, but still with the same foundations, has come to be the dearest child of our time [*das Schoßkind unserer Zeit*] ... Anyone who does not embrace it runs the risk of being considered an ignorant or at least a good-natured *Schwärmer*.<sup>179</sup>

<sup>176</sup> On the role of Heinrich Christian Schnuse as a translator in Germany, cf. [Reich 2003, pp. 448–453], [Hunger Parshall & Rice 2002, p. 22].

<sup>177</sup> [Schnuse 1854, p. 1672].

<sup>178</sup> [Schnuse 1854, pp. 1655–1656].

<sup>179</sup> [Arneth 1852, p. 15].

However, Arneth willfully ran that risk insofar as vitalism proved to be pivotal in his attempt to establish cultural essentialism with regard to human diversity, as it is displayed by the variety of those entrenched "mindsets". Still, Schnuse denounced on the spot the rhetorically enforced premiss supposedly starting the whole teleological argument.

When the author affirms: one necessarily comes to these fine views [*viz. Arneth's own*], if one examines the natural phenomena without a preconceived opinion, this is certainly too much to say—and it lays bare that the opposite view, which the author discusses, and according to which matter is the only real being &c. &c. could just as well make similar claims (? [*sic!*]). We also find the author's criticism of this second view, which he rightly calls the dearest child of our time, much too hard (?).<sup>180</sup>

Against those he called "the defenders of the unicity of mankind"<sup>181</sup> who ascribe human diversity merely to local influences, among whom Alexander von Humboldt (1769–1859)<sup>182</sup> probably stands out as the most representative, Arneth elaborated a theory of races which he viewed as some sort of conjectural model intended to save the phenomena along combinatorial lines. "Races [*he claimed,*] are the elements, the components of the combinations which occur according to the same laws as all formations on earth; their goal is the higher configuration, the refinement of men with respect to body and mind."<sup>183</sup> In taking for granted that mankind first arose in two different locations on earth, *viz.* in Africa "not far from the Equator", and in Asia "in the vicinity of the 40th degree of latitude", Arneth considered that "the assumption of the double origin of mankind is necessary, but also sufficient, for the explanation of the phenomena"<sup>184</sup>, or rather, as he would state more precisely, the assumption of a quadruple origin, since in each of these "primordial seats" [*Ursitzen*], mankind would

<sup>180</sup> [Schnuse 1854, p. 1657]. The question marks between parentheses are Schnuse's.

<sup>181</sup> [Arneth 1852, p. 4].

<sup>182</sup> Cf. Alexander von Humboldt's *Kosmos* I (1845), for instance [von Humboldt 1845, p. 379]: "As long as one dwelled only on the extremes in the variation of colour and form and abandoned oneself to the vividness of the first impressions of the senses, one could indeed be inclined to consider the races not as mere varieties, but as originally different human stocks [*Menschenstämme*]. The permanence of certain types amidst the most adverse action of external powers, in particular climatic ones, seemed to favor such an assumption, in spite of the shortness of the period of time of which we have achieved historical knowledge. In my opinion, however, there are more powerful reasons in support of the unity of mankind, as, for instance, the many intermediate gradations in the colour of the skin and in the form of the skull, which have been made known to us in recent times by the rapid progress of geographical knowledge."

<sup>183</sup> [Arneth 1852, p. 31].

<sup>184</sup> [Arneth 1852, p. 28].

presumably be instantiated in two different stocks, a superior race “with respect to the body and mind”<sup>185</sup>, bound to a pattern of both north- and westward, or south- and eastward migrations, depending on the case, and one or several inferior races, supposedly remaining in their original location.<sup>186</sup> Since it is presumably a fundamental law of the world process that only the conflict between opposites can yield something of a higher order, these migration trends would bring the hypothetical *Stammvölker* in contact with one another in as many combinations as required to account for the historically documented peoples.

The difference in the climatic influences of both regions [*viz. in Asia and Africa*], and in the other circumstances thereby conditioned, enforce the conclusion [*Arneth claimed*,] that in both places, men must have arisen with unequal corporeal as well as mental dispositions [*Anlage*]. With the original creation [*Urzeugung*], the external conditions [*Arneth assumed*,] have done everything to give to the being to be created the only possible form which was thereby conditioned. But in this form were also retained the conditions which first created it, and later changing influences exerted upon it were not capable of modifying again its whole essence. Climatic influences produce changes, but they never lead a race into another.<sup>187</sup>

In Arneth's attempt to articulate a theory of races supposedly accounting for human variety in a non reductionist way, that is, without wholly ascribing it to merely external causes such as climatic conditions, one may find distorted echoes of late eighteenth-century German debates surrounding the emergence of an allegedly scientific discourse on race.<sup>188</sup>

<sup>185</sup> Cf. [Arneth 1852, p. 22]. Arneth not only did subscribe to the common view of a racial hierarchy, but on occasion also unrestrainedly drew on the most abhorrent racist stereotypes; see for instance his portrait of the “Niggers” as “hard at work, but idle if unconstrained”, “addicted to terrestrial pleasures”, “showing no trace of mental activity”, etc., cf. [Arneth 1852, pp. 36–43].

<sup>186</sup> Cf. [Arneth 1852, p. 22]: “The higher races left their original seats in both places, either on their own free will, or forced by the others; they thereby came into direct contact with one another and gave the impetus toward a higher form of life.”

<sup>187</sup> [Arneth 1852, p. 21].

<sup>188</sup> For detailed studies unfolding some of the complexities of this emerging racial discourse, and more broadly for analyses of the subsequent nineteenth-century debates on race straddling borders among disciplines, from physiology and geography to aesthetics and philology, see Eigen & Larrimore [2006].

It is well-known that both the Göttingen comparative anatomist, natural historian and anthropologist Johann Friedrich Blumenbach (1752–1840)<sup>189</sup> and the Königsberg philosopher Immanuel Kant (1724–1804)<sup>190</sup> played a decisive role in the shaping of this discourse. They attempted—first independently, then conjointly—to steer a middle course between mere preformationism and a rival form of epigenesis, tainted by hylozoism, which was represented by the physiologist Caspar Friedrich Wolff (1735–1794).<sup>191</sup> However, in his theory of races, Arneth perplexingly combined

<sup>189</sup> In trying to conceive of organized structures as primary, hence unaccountable in terms of mechanical causes, Blumenbach had devised, already in the 1780s—see Blumenbach [1781]—the concept of a “formative drive”, or *Bildungstrieb*, which he conceived as an actual drive present in all living organisms, and not as a regulative construct, although he considered it as a mere *qualitas occulta* only to be known, like Newtonian attraction, through bringing its effects under general laws. Blumenbach’s original contribution amounted less to his experiments on the regeneration of polyps, than to the formulation of a new version of epigenesis setting in stark contrast the “formative drive” exclusively pertaining to organisms with the “formative forces” supposedly structuring inorganic matter, see [Bernaconi 2006, p. 76]. The *Bildungstrieb* was thus thought as an agent inseparable from organic matter as such, and responsible for its organization, or a receptivity to external stimuli intimately linked with an ability to respond to them in setting the organs in motion. More importantly for our present concern, Blumenbach endowed his “formative drive” with functional adaptation. He argued that the variations in the “formative drive”, brought about by environmental differences such as climatic ones, “could take root in the generative fluid itself thus becoming a permanent structural feature of the organism”, cf. [Lenoir 1980, p. 85].

<sup>190</sup> In his 1785 paper, *Die Bestimmung des Begriffs einer Menschenrasse*, Kant had sought to account for the emergence of different races within one single human species. He postulated “one single generative stock” [*Stamm*], in which were originally laid all the seeds [*Keime*] for the specific characteristics, supposedly bound to be triggered in appropriate environments, together with certain intrinsic predispositions [*Anlage*], understood as “adaptive mechanisms for the preservation of species”, cf. [Lenoir 1980, pp. 90–92], see also [Zammito 2006, pp. 36–43], [Zammito 2012, pp. 122–124] for a detailed account of Kant’s theory of races. For Kant, the color of the skin was such an adaptive mechanism [*Anlage*] resulting in morphological differences between races. A black skin, for instance, would correspond to an organization of the flesh designed to remove from the blood the excess of phlogisticon in regions where there is an abundance of it; cf. [Kant 1785, p. 93]. In Kant’s view, the latent adaptive capacities that would not be prompted by environmental causes, for a long enough period of time, would eventually disappear, thus making it impossible for a given race to develop them once they were extinguished.

<sup>191</sup> Recent studies have focused on differentiating Blumenbach and Kant’s respective contributions, while providing a refined account of their complex interactions from the mid 1780s; see for instance Zammito [2006] and Bernasconi [2006]. In a pioneering work on German biology at the end of the eighteenth century, Timothy Lenoir claimed that Blumenbach’s “material vitalism”, hence the research program of the so-called “Göttingen school” in the 1790s (Christoph Gitaner, Carl-Friedrich Kielmeyer, etc.), resulted from Blumenbach’s endorsement of Kant’s methodological guidelines, most notably the constitutive-regulative distinction with regard to the use

anti-hylozoism and polygeneticism, whereas both Kant and Blumenbach conjoined no less a virulent opposition to hylozoism with monogeneticism, in strong opposition to polygeneticism. The channels through which Arneth may have received late eighteenth-century naturalistic views on races remain unclear.<sup>192</sup> However, within this timespan, Blumenbach's views spread from Göttingen to Heidelberg where Karl Cäsar von Leonhard (1779–1862) who had studied in Göttingen under Blumenbach, held a professorship in mineralogy from 1818 until his death. If not through his elder colleague, Arthur Arneth may have become acquainted with these ideas through the former's son, Gustav Leonhard (1816–1878)—also a mineralogist—whose *Privatdozent* years overlapped with his. In any case, Arneth did not limit himself to anchoring his theory of races in the life sciences. In knitting together disparate bits and pieces, he also elaborated on racialist views derived from comparative linguistics, reminiscent of August Friedrich Pott's at about the same time.<sup>193</sup> Language considered as “the image of both the natural dispositions [*Anlage*] and the mindset”<sup>194</sup> would thus occupy a mediating position in his construction at the juncture of natural and cultural history.

A few years earlier though, Alexander von Humboldt had already denounced all the geographical speculations on the so-called cradle of mankind as being “of a mythical character”<sup>195</sup> In relying on such authorities as the physiologist Johannes Müller (1801–1858) and his own brother, Wilhelm von Humboldt (1767–1835), he pointed out that neither can one learn from experience whether the human races descended from several or one *Urmensch*, nor can it be decided historically whether the division of mankind into families of peoples was original or occurred at a later

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of teleological principles in the life sciences. However, Robert Richards first, then recently John Zammito, challenged Lenoir's thesis in making clear why “Blumenbach's affiliation with Kant is best understood as a misunderstanding”, cf. [Zammito 2012, p. 127], see also [Richards 2002, pp. 221–237]. Blumenbach and Kant both opposed the notion of a continuity from the inorganic to the organic, that is, material naturalism or hylozoism. But they professed anti-hylozoism for very different reasons. Therefore Blumenbach's outward agreement with Kant is most aptly seen as resulting from his endeavour to distance himself from C. F. Wolff's world-view in which, beyond certain thresholds, chemical attraction-repulsion forces are responsible for matter achieving new levels of organization in living organisms.

<sup>192</sup> On the overall influence of Blumenbach's vitalism on the development of the natural sciences in Germany, see Reill [2005].

<sup>193</sup> On the “linguistic physiology of August Friedrich Pott”, see [Benes 2008, pp. 204–211].

<sup>194</sup> [Arneth 1852, p. 28].

<sup>195</sup> [von Humboldt 1845, p. 381].

time.<sup>196</sup> In contrast to Blumenbach, he even denied the very concept of race. "Distribution of mankind [Humboldt stated,] is only a distribution in varieties, which one designates though by the somewhat indefinite word *races*. ... Whether we follow the old classification in five races of my master Blumenbach, ... or, with Pritchard, assume seven races, ... we always fail to recognize a typical sharpness, a natural principle of division carried out in these groupings."<sup>197</sup> More decisively, "in affirming the unity of mankind, we also repel [Humboldt claimed,] any unpleasant assumption of superior and inferior races of men. There are more shapable, more highly cultivated families of peoples [Volkstämme], but none nobler than others. All are in like degree designed for freedom."<sup>198</sup> A man of another generation, Arneth was all the less imbued with such francophile commitment to universal liberty and cosmopolitanism, as his naturalistic views culminated in a cultural essentialism which Humboldt would have abhorred. As Arneth's story goes, different sets of mental dispositions would indeed have their seat in the different physical constitutions, presumably distinctive of races, that were supposedly brought about by the diverse climatic conditions prevailing in the places where mankind first arose. History was thus envisaged in continuity with the conjectural unfolding of nature, "for history must be the other side of natural sciences, which grasps the products of mankind's mental activity".<sup>199</sup> Arneth stipulated four original stocks [Völkerstämme], the "Semitic and Egyptian peoples" and the "Niggers" in Africa, the "Aryans" and the "Mongols" in Asia. Great migrations supposedly drove the "Semitic and Egyptian peoples" to the North and the East, whereas the "Aryans" would go South and West, so that eventually both parts of mankind came into contact with one another from Western Europe to South Asia. As a result, both Greece and India were to be seen as essentially mixed cultures. In both cases, Africans settled first, Aryans came next and found there an already existing material culture which they embraced, although reshaping it along their own lines. Arneth constructed an opposition between Greece and India in contrasting two ways in which the so-called "undetermined and fluctuating Aryan element" presumably operated on the substratum of a preexisting Egyptian culture, which was held to enhance forms to the point of petrifying in them. In

<sup>196</sup> Cf. [von Humboldt 1845, pp. 280–381] for the complete citation from Johannes Müller's *Physiologie des Menschen* II, p. 768, and the excerpt from Wilhelm von Humboldt's then unpublished manuscript *Über die Verschiedenheit der Sprachen und Völker*.

<sup>197</sup> [von Humboldt 1845, p. 382].

<sup>198</sup> [von Humboldt 1845, p. 385].

<sup>199</sup> [Arneth 1852, p. 32].

one case, Aryan plasticity was changed into “the clarity and determination which makes the mental greatness of the Greek people, as expressed in their mathematics”<sup>200</sup>, whereas in the other, forms came to be utterly dissolved. Furthermore, external conditions fostered these contrasted evolutions. The Greek world being small, partitioned and scarcely supplying the needs, “everything conspired [*there*] to make the body powerful and the mind down-to-earth [*nüchtern*]”. Whereas, in being immense, highly fertile and profuse in resources, the Indian world presumably awakened flights of unconstrained reflection.

The conception of the external world was more magnificent among the Indians than among the Greeks [*Arneth explained*]; they had already achieved the dissolution of the bodies of this world, the earthly configurations, into the infinite, the inconceivable, the unformed, the world itself as the changing, the transitory; the form, the figure [*Gestalt*] lost all value in front of the substance, the divine. All this came to be imprinted in the mathematics of the Indians, just as well as with the Greeks, although in an opposite way. The Greeks sought to recognize that which is given [*das Gegebene*], that which has a form [*das Gestaltete*], as it is necessarily so; the Indians investigated, created configurations [*Gestaltungen*] and contented themselves to know that something is, without caring to know how it is. These directions were one-sided, but both necessary; modern mathematics owes its quick development to their association. Whereas the Greeks connected everything to the form [*Form*], so that purely arithmetical propositions would also be presented geometrically, the Indians only knew the number, and their geometry consisted in a part of their arithmetic.<sup>201</sup>

Poetic form and proof would then represent mutually exclusive possibilities for codifying knowledge. In Greece, mathematics presumably freed itself from the fetters of a science of priests; being intended from the outset for the common good of everyone, proof prevailed over poetic form. Whereas in India, mathematics never emancipated itself from priesthood and thus retained poetic form as a vehicle for empirically found propositions, which both helped memorizing and facilitated transmission. “When the investigator had found a proposition, [*Arneth claimed*], he would at once put it in this form and show its correctness in examples. Such discoveries were considered as prompted by divine inspiration, in which naturally all proof was superfluous.”<sup>202</sup> Although strongly emphasizing the absence of proofs in the Greek sense, Arneth sought to delineate what would stand in their stead in Indian mathematics.

<sup>200</sup> [Arneth 1852, p. 140].

<sup>201</sup> [Arneth 1852, p. 141].

<sup>202</sup> [Arneth 1852, p. 142].

An impressive phenomenon is the complete lack of proofs [*in Sanskrit sources*] ... Here we only have a series of instructions and rules saying what one has to do to reach a given goal, without the least hint about the way in which one had come to the rule, and the Indians seem to have put the proof only in the coincidence of the results with the requirement. If a rule teaches how to find the roots of an equation, the proof consists in showing in a given case that the roots found in this way really satisfy the equation.<sup>203</sup>

Arneth remarks that one can nevertheless find here and there in Bhāskara an attempt at providing some kind of explanation or proof, but that, when it occurs, it does not suit the whole style of the treatise so that “[*Bhāskara*] moves there in a foreign element and deviates from the way in which Brahmagupta freely proceeds”<sup>204</sup>, that is, providing mere sequences of instructions.

Furthermore, the propositions of the Indians are of a peculiar kind: they rarely express that something is so and so, but rather how something can be found, that is how it can be computed; they do not deal with the properties of numbers, as those of the ancient Greeks, but with the operations on numbers, in which in particular the metrical element [*in the sense of mensuration*] comes to the fore. But how did the Indians come to their remarkable discoveries ...? One cannot assume that they did it through blind search and experimentation, for that would never have led them to these results; therefore we are left with the assumption that they proceeded in their investigations and their developments so that what comes first always served as a support for what comes next.<sup>205</sup>

Arneth characterizes this last procedure as being the opposite of Greek analysis, for, with the Indian method, “no investigator could use the experiences of his predecessors”,<sup>206</sup> as the Greeks did with analysis. Since only numbers would be used in Indian mathematics, conducting a general proof would presumably be difficult, for the procedure would adhere to the result. All this supposedly constituted a hindrance to the development of mathematics. As for geometry proper, there was no such thing in India, according to Arneth. Not only did Indian mathematicians compute with lines, areas and bodies “as we do with kilograms or pounds”, but they disregarded properties of spatial magnitudes no less than properties of numbers.

<sup>203</sup> [Arneth 1852, p. 175].

<sup>204</sup> *Ibid.*

<sup>205</sup> [Arneth 1852, p. 175].

<sup>206</sup> [Arneth 1852, p. 176].

Here too, one dealt with how to find, that is how to compute, certain things. There was no mention of parallel lines, angles, congruence or similarity of figures; [*the Indians*] did not develop all these auxiliary means without which a geometry cannot be thought of. In this way, it came out that, even where a determination of magnitudes was necessary, they often lacked the means. When the case required something more than the rule of three or the Pythagorean theorem, they turned to empirical formulas, which most of the time only gave a poor approximation.<sup>207</sup>

In contrasting Indian computational geometry, which he envisioned as an “*Urgeometrie*” arising from practical needs, with Greek theoretical geometry, Arneth assumed that there were certainly reciprocal influences in the long run, most probably through Babylon. But he denied any direct influence from Greece over India, or at least averred that none exerted itself soon enough to contribute in any significant way to the shaping of Indian mathematics.

One easily sees that a Brahmin would not have known what to do, even with Euclid's Elements. [...] an Indian commentator drew a figure so as to explain a proposition and said: “See the figure”; infinitely much lies in these three words and the solution to the whole enigma. What an Indian would have been supposed to do with the whole apparatus of Greek scholarship!—Not only the propositions, but also the whole kind of presentation would have appeared incomprehensible to him. Considering only the proposition: in any triangle, two sides together are greater than the third. To want to prove this proposition must have appeared to the Indian as the greatest absurdity [*die größte Lächerlichkeit*]. Why all this, I see well that it is so. [...] Should one want to provide an entry to Greek mathematics among the Indians, one would not only have to wake their sense for geometry, one should do more than that and change their whole mindset [*Geistesrichtung*].<sup>208</sup>

It should be incidentally remarked that, no less than Röth, Arneth generally sidestepped the rising regime of critical scholarship, and in particular the rules, first heralded by Nesselmann, regarding quotations. Here, for instance, he obviously drew on Colebrooke's ‘See’-trope without even citing the evidence he was relying upon so as to make his own point. “Since I never had the intention to claim anything for me, [*he rather bluntly warned beforehand*], this relieved me of the citations which would not accord with the goal of the work and for which, besides, I had no room.”<sup>209</sup>

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<sup>207</sup> [Arneth 1852, p. 176].

<sup>208</sup> [Arneth 1852, p. 182].

<sup>209</sup> [Arneth 1852, Foreword, p. iv].

As Schnuse emphasized, Arneth “expressly and repeatedly remark[*ed*] how incorrect it is to ascribe everything to the Greeks—that only the conflicting can have a reorganizing and formative effect.”<sup>210</sup> Furthermore, he diagnosed that what he took to be the prevailing attitude of his time<sup>211</sup> was being entrenched in nineteenth-century German education, according to which “we are taught from our youth that there is nothing higher, nor will ever be, than classical Antiquity”<sup>212</sup>. Hence he claimed that “the prejudices [*thus*] planted in us exert a noxious influence, [*insofar as*] we learn thereby to disregard the present, as well as the non classical past, the national, as well as the foreign”<sup>213</sup>. Escaping the one-sidedness of an exclusively classical education therefore appeared as a requisite for self-understanding. In this connection, Arneth’s most influential point was to use this interpretive key to make sense of the state of the mathematical art in the early 1850s.

The geometry of the Greeks [*Arneth explained*,] was wholly grounded in the consideration of the figure, in sense intuition. This characteristic which corresponds to the essence of the method got lost with the application of algebra [*in modern times*]; one then recognized the properties of the figure from the relations between magnitudes developped by computation, without reproducing them pictorially or conceiving them in their spatiality.<sup>214</sup>

With the adoption of projective methods, synthetic geometry later received a new impetus from the end of the eighteenth century to the mid nineteenth century. In acknowledging the importance of the works of Monge, Carnot, Brianchon, Poncelet, Gergonne and Steiner, Arneth nevertheless pointed out that “pure geometry, with its great collection of new and beautiful theorems, remained alien to most [*contemporary*] mathematicians, who operated on the well-known terrain of analytic geometry and preferred assured successes to uncertain ones”<sup>215</sup>. Yet, there were noteworthy exceptions. Among the few German mathematicians who attempted to make progress in synthetic geometry, while purposefully turning their back on analytic geometry, Arneth mentioned Carl

<sup>210</sup> [Schnuse 1854, p. 1662].

<sup>211</sup> Cf. [Arneth 1852, p. 179]: “We are so used to bringing everything back to the Greeks that, if we find something worthy of attention among other peoples, we are inclined to look for the sources equally in Greece, whereas conversely we reject any influence from abroad on the Greeks, whom we consider as the sole originators of whatever great mankind ever achieved. The previous considerations have shown how false this view proves to be.”

<sup>212</sup> [Arneth 1852, p. 141].

<sup>213</sup> *Ibid.*

<sup>214</sup> [Arneth 1852, p. 284].

<sup>215</sup> [Arneth 1852, p. 286].

Adams (1811–1849), a teacher of mathematics at a vocational school in Winterthur (Switzerland), whose writings were “universally acclaimed for they met a real need”.<sup>216</sup> Adams set out to extend Euclidean geometry by incorporating into it new material from modern synthetic geometry. His 1843 theory of transversals applied to planimetry,<sup>217</sup> for instance, contains “in the form of aphorisms, and after the manner of Euclid, the most important theorems of elementary geometry which can be obtained from Steiner’s fundamental elements”.<sup>218</sup> Arneth saw Adams’ work as representative of a significant trend within current mathematics, whose overall situation was then read through the lens of cultural history.

The separation in which the subject matter [*viz. synthetic geometry as split up from analytic geometry*] appears here, looks very much like a step backwards, namely as to the way the dualism is sublated [*aufgehoben*]—a dualism which essentially belongs to these investigations, lies in the nature of spatial forms [*Raumgebilde*] and corresponds to both their attributes of magnitude and position. But this is only an appearance: the subject matter was new, the isolation was necessary, if it was to be transfused into the flesh and blood of mathematics. We encounter here *im Kleinen* the law of mental development which holds *im Grossen*; within a few years, the conversion will be completed.<sup>219</sup>

Both shifts were thus to take effect concomitantly. “The mathematician did not suspect that the seeds of the most beautiful branches of his science had arisen millennia ago in the remote Orient, but could only develop into a fruitful organism insofar as curbed by the influence of the Greek mind. ... Since the so-called restoration of the sciences, one has only dealt with Greece; but this time is fortunately over, and also here the German mind has taken another course”<sup>220</sup> One would, on the one hand, counterbalance classical Antiquity with a thorough investigation of Sanskrit sources so as to grasp the true meaning of Indian mathematics, while striving, on the other hand, to combine both orientations in blending synthetic and analytic geometry, so as to fulfill the integrative programme of modern mathematics.

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<sup>216</sup> *Ibid.*

<sup>217</sup> Cf. Adams [1843]

<sup>218</sup> Cf. [Arneth 1852, p. 286].

<sup>219</sup> [Arneth 1852, p. 287].

<sup>220</sup> [Arneth 1852, p. 289]. It is a significant fact that, at the very end of his book, in connection with the supposedly forthcoming unification of synthetic and analytic geometry, Arneth reiterates his indebtedness to Röth’s dualism in cultural history.

## 6. FROM *ALTERTUMSWISSENSCHAFT* TO SANSKRIT PHILOLOGY

Against the backdrop of the positivistic mid-century generational stance<sup>221</sup>, Röth's unwavering urge for expelling the Greeks from their fictitious cultural-historical insularity, as well as Arneth's attack on the noxious exclusiveness of a too classical education, would appear as reminiscences of past heated debates. In his monumental *Symbolik und Mythologie der alten Völker* (1810–1812), the famous Heidelberg idiosyncratic classicist Friedrich Creuzer (1771–1858) had indeed claimed decades earlier that Greek myths and symbols had their origin in oriental lore.<sup>222</sup> A friend of Clemens Brentano, Achim von Arnim and Joseph von Görres, Creuzer had strong ties with the Heidelberg romantic group, and like Friedrich Schlegel in his *Sprache und Weisheit der Inder* (1808), he sought sparks of a lost primeval revelation in the Orient. His main reason for anchoring western religions in the East was mainly his pursuing a grand narrative that would historically unfold Christianity from its eastern sources. In so doing, he intended to reinstate it as the foundation for European culture against the challenge posed by the post-revolutionary secularizing trend which tended to replace it by classical antiquity. In an epistolary controversy with Creuzer,<sup>223</sup> which historians of philology characterize as

<sup>221</sup> On the “post-romantic, fact-seeking, theory-adverse” frame of mind of German scholars in the 1850s, see [Marchand 2009, pp. 74–75]: “[they] preferred facts to systems and feared that conjectures would undermine the legitimacy of those who did seek truth. … It was clearly a reaction to the overweening ambitions of the romantics. … This generation did not want to dream; rather, it wanted to make itself uncontroversial and to draw too few conclusions rather than say too much.”

<sup>222</sup> So as to shed light on Homeric and Hesiodic myths, Creuzer assumed the existence of a primeval wisdom, common to all mankind, which took the form of a natural monotheism corresponding to an original form of consciousness all of a piece with divinity and nature. This wisdom presumably broke up at a later stage into many polytheistic religions due to the symbolizing operation of oriental priests, who encapsulated it in concise and striking statements as in flashes of insight. Later their original meaning gradually got lost and myths were produced so as to regain it in linguistic expressions. Creuzer's approach to ancient religions was thus based on the central distinction between symbol and myth. Whereas the symbol is to be understood as the “embodied idea made perceptible to the senses”, and as such “an instantaneous totality” ([Creuzer 1819, p. 70]) instantiated in primitive cults, myths are elaborated narratives belatedly born of the need to explain symbols in words. In Creuzer's views then symbols and cults are more important than myths, hence “syntax is less significant than semantic” ([Judet de La Combe 1995, p. 62]), which epitomizes Creuzer's dissent with more traditional philological practices centered on textual analysis.

<sup>223</sup> On his own initiative, Creuzer edited his correspondence with Hermann in 1818, under the title *Briefe über Homer und Hesiod, vorzüglich über die Theogonie*. Hermann, who refused to leave him the last word, published a rejoinder the following year: *Über das Wesen und die Behandlung der Mythologie*, Leipzig, 1819.

one of the major “founding conflicts” of nineteenth-century philology, the leading Leipzig classicist Gottfried Hermann attempted to refute his Heidelberg adversary’s view of a historical process of differentiation from a single original monotheistic religion. He also rejected Creuzer’s idea of a presumably lost symbolic meaning as opposed to belated myths which, in return, would supposedly remain inscrutable without the appeal to oriental symbols. Contrary to Creuzer, Hermann held that Greek myths were to be considered in themselves as a pure product of the Hellenes and that the problems raised by their interpretation should be resolved solely on the basis of a grammatical analysis of the linguistic expression in which they were handed down to us. Shortly afterwards, the classicizing reaction to Creuzerianism took a more vehement turn in the 1820s and gradually increased in intensity all through the decade. Johann Voß, the translator of Homeric epics, struck the first severe blow in vigorously attacking Creuzer in an uncommonly harsh review of the *Symbolik*, which launched the public *Creuzer Streit*.<sup>224</sup> In a first phase, Creuzer’s ideas gained recognition and support from different sides; Hegel, the historian of law Friedrich von Savigny, Friedrich Schlegel and Wilhelm von Humboldt<sup>225</sup> were among his admirers, while the theologian C. F. Baur, or the geographer Carl Ritter initially worked along his lines in their respective fields. However, his opponents progressively took the lead and teamed up under the banner of the anti-*Symboliker*. In his *Prolegomena zu einer wissenschaftlichen Mythologie* (1825) for instance, the classical philologist Karl Otfried Müller (1797–1840) called for a scientific treatment of mythology banishing all speculations on presumably oriental origins of Greek culture which he claimed was to be considered as a separate *Stammeskultur*.<sup>226</sup> In a letter of

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<sup>224</sup> For a detailed account of the unfolding of the *Creuzer Streit*, see Howald [1926], Judet de La Combe [1995], [Marchand 2009, pp. 70–71].

<sup>225</sup> Wilhelm von Humboldt’s assessment of Creuzer’s *Symbolik* was in fact a balanced one, as he wrote to his friend, the classicist and archaeologist Friedrich Gottlieb Welcker (1784–1868), in a letter dated December 15, 1822, cf. [von Humboldt 1869b, p. 80]: “It will be difficult to bring together more scholarship and wide reading than Creuzer possesses; in every page of his book, his spirit is visible, a deep feeling and a rare gift for intuition; one often recognizes clear sparks of a true genius. But on the whole, his book proves more dispiriting than uplifting and informative. No chapter gives satisfaction as regards clarity and positiveness.”

<sup>226</sup> Cf. [Müller 1825, p. 281]: “I wished to treat merely of the mythology of the Greek as a distinct historical science. To say that it could not be handled separately in this way, would amount to saying, and indeed would even be more than saying, that one could not learn the Greek language without the Sanskrit and Hebrew.” Creuzer persisted in resisting this historicising trend according to which mythology ought to be shaped in the mold of history, insofar as, in his view, both disciplines would sharply

August 1825 to Friedrich von Savigny, Creuzer deplored the increasing narrowing of German philology which “no longer partakes of the larger perspectives on scholarly inquiry as a whole.”<sup>227</sup> However, by the 1830s, the *Symboliker* had given way to their adversaries<sup>228</sup> and the controversy was finally quenched.<sup>229</sup>

In this context, it is highly significant that in the 1860 annual review of the *ZDMG*, the organ of the *DMG*, the orientalist Richard Gosche (1824–1889) who held a chair of Semitic languages in Halle and was also a member of the executive committee of the *DMG*, explicitly connected Röth’s work with Creuzer’s despite the fact that the whole intellectual landscape had completely changed in the meantime.

The old Creuzer [Gosche wrote,] departed this world on February 16, 1858, and Ed. Röth followed him on July, 7 ... Both do not only stand in connection from the topographic point of view—they were both related to Heidelberg—but also in a more fundamental way insofar as they both knotted together the Orient and the Occident, either mysteriously or by force, with ties whose solidity is questionable for they are neither enduringly woven by philosophical threads, nor by philological ones. What Wilhelm v. Humboldt already designated in 1822 as a dispiriting characteristic of Creuzer’s mythological writings—namely that “no chapter gives satisfaction as regards clarity and positiveness”—also characterizes Röth’s wearisome writings, let us say that Röth himself had already begun his career in 1836 in dealing with history of religion, with a brief presentation of the Jewish doctrine of faith.<sup>230</sup>

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differ in their methodologies; cf. [Creuzer 1848, pp. 64–65]: “What makes a mythologist rests on a completely different mental activity from that underlying any historical operation; it rests on a kind of apperception which one can neither learn, nor prescribe; but one which is conditioned by a mental organization not dissimilar with that which makes a poet.” Wilhelm von Humboldt may be seen as steering a middle course in this respect, insofar as sharing Creuzer’s views on cross-cultural influences, while promoting at the same time a sober historical scientificity.

<sup>227</sup> Creuzer to Savigny, August 17, 1825, in *Briefe Friedrich Creuzers an Savigny: 1799–1850*, ed. Hellfried Dahlmann, Schmidt, Berlin, 1972, p. 351; quoted by [Marchand 2009, p. 71].

<sup>228</sup> Voß’ *Antisymbolik* (1824) and Christian Lobeck’s *Aglaophamus, sive de theologiae mysticae Graecorum causis* (1829) forcefully contributed to bring Creuzerianism into disrepute.

<sup>229</sup> In recapitulating the whole Creuzer debate for the French readers of the *Revue des deux mondes*, Ernest Renan averred in 1853 that all German scholars then “concurred in recognizing, against M. Creuzer, the originality of Greek mythology. All agreed in rejecting that blasphemy, that Greece was ever a province of Asia, that the Greek spirit, so free, so objective, so limpid, could contain any element of the vague and obscure spirit of the Orient.”, cf. [Renan 1853, p. 835], quoted by [Marchand 2009, p. 74].

<sup>230</sup> [Gosche 1860, p. 138].

Significantly enough, Gosche obliterated Humboldt's words in praise of Creuzer, insofar as everything recalling what he referred to as "the worst Creuzerian fusionist epoch"<sup>231</sup> would then be turned down as a misguided view of the past. However, as seen above, through Röth's cultural history, Creuzerian ideas had resurfaced at the confines of philology and mathematics, in contributing to shape Arneth's naturalistic history of mathematics. At the end of the 1860s however, although outwardly endorsing some of Arneth's comparative analyses on Indian and Greek mathematics, Hankel nevertheless gave them an anti-essentialist historicist twist. For he aptly transposed them into the standard setting of Orientalist philology, which by then had resulted from the complex social and institutional dynamics unfolding within German academia since the beginning of the nineteenth century.

Recent studies have substantiated a nuanced historical picture of German orientalism resolutely at variance with Edward Said's all too sweeping, although highly influential, interpretation, according to which European scholarly engagement with the so-called "Orient" should be thought of as part of the colonial enterprise.<sup>232</sup> If obviously it is to be conceded that, unlike the British and the French, German scholars were not directly involved in setting overseas domination, they would however, in Said's view, contribute to colonial power in effectively shaping an authoritative discourse on constructed otherness.<sup>233</sup> Sheldon Pollock described this presumably German stance as "inward colonization", in contrast to more conventional "outward colonization", namely as "a complex of knowledge-power ...to be seen as vectored not outward to the Orient but inward to Europe itself, to constructing the conception of a historical German essence and to defining Germany's place in Europe's destiny"<sup>234</sup>. Furthermore, critical inquiries call attention to the role philological ideas have played, ever since Friedrich Schlegel's famous distinction between inflecting and agglutinating languages (cf. *infra*), in

<sup>231</sup> [Gosche 1860, p. 165].

<sup>232</sup> See [Mangold 2004, p. 111], [Rabault-Feuerhahn 2008, pp. 13–14], [Marchand 2009, pp. xviii–xx] and [Meßling 2012, pp. 166–168].

<sup>233</sup> Cf. [Said 1978, p. 19]: "What German Oriental scholarship did was to refine and elaborate techniques whose application was to texts, myths, ideas, and languages almost literally gathered from the Orient by imperial Britain and France. Yet what German Orientalism had in common with Anglo-French ... Orientalism was a kind of intellectual authority over the Orient within Western culture. This authority must in large part be the subject of any description of Orientalism."

<sup>234</sup> [Pollock 1993, p.83]; see also [Rabault-Feuerhahn 2008, pp. 13–14] on Pollock's reading of Said's thesis in the perplexing case of German orientalism.

the emergence of nineteenth-century racialist doctrines up to twentieth-century deadly ideologies.<sup>235</sup> However, historians increasingly felt the need for a more balanced and elaborate account. Whenever what one reconstructs as a hegemonic discourse is presented as an unchallenged homogeneous trend, and not, as it should be, as a mainstream path in the midst of other alternative paths that were also available within the concrete historical situation, when every voice striking a different note is downplayed, then one is dangerously exposed to "reverse teleology".<sup>236</sup> "We need, instead, [*as Suzanne Marchand declares*,] a synthetic and critical history, one that assesses oriental scholarship's contributions to imperialism, racism, and modern anti-Semitism, but one that also shows how modern orientalism has furnished at least some of the tools necessary for constructing the post-imperialist worldviews we cultivate today."<sup>237</sup> In the case of German Indology and Sanskrit studies which are our focus here, Pascale Rabault-Feuerhahn convincingly stresses the fact that the shaping of a German knowledge interest in Indian culture is a multifaceted historical phenomenon which should not be reduced to an undifferentiated and assumedly entrenched romantic fascination with India.<sup>238</sup> On the contrary, indiscriminately embracing the falsely unifying view of a German Indomania supposedly reflecting the German quest for self-definition, would assuredly block the detailed historical analysis which ought to account for the process by which this knowledge interest came to be constituted in the form of an independent discipline within the German philological tradition.

In spite of marked differences, such internal histories of German orientalism and Sanskrit philology as Theodor Benfey's (1869) and Ernst Windisch's (1917) concur in emphasizing broadly the same historical

<sup>235</sup> See for instance Oelnder [1989], Harpham [2009], Meßling [2012]. For a detailed account of this intricate historical process, see also Benes [2008]. "While comparative philology was an important venue for the German invention of race, reducing the field to a precursor of National Socialism, [*Tuska Benes claims*,] . . . , ignores its complexities. . . . Nineteenth-century philologists set the terms of a discourse whose relevance only increased as their exclusive control over it ceded to more dominant cultural figures within the newly established nation-state. . . . The tale philologists told of a pure linguistic community that had primordial roots in Asia and the Germanic past influenced how political leaders, educators, artists, and authors defined the nation until the ultimate perversion of these ideas in the Aryan theory and Germanophilia of the Third Reich." [Benes 2008, pp. 17–18]

<sup>236</sup> This point is persuasively made by Markus Meßling who takes this notion of "reverse teleology" over from Wulf Oesterreicher, see [Meßling 2012, pp. 164, 167].

<sup>237</sup> [Marchand 2009, p. xx].

<sup>238</sup> Cf. [Rabault-Feuerhahn 2008, pp. 17–19].

pattern. While Henry Thomas Colebrooke is presented as “the founder of Sanskrit philology”<sup>239</sup>, or at least as “the first one who dealt with Sanskrit and its literature in a truly philological sense and thereby laid the secure foundations for a Sanskrit philology”<sup>240</sup>, in a German context, Friedrich Schlegel (1772–1829), “the profound and spirited pioneer of the new science” and Franz Bopp (1791–1867), “the ingenious founder of the comparative method”<sup>241</sup> stand out in a formative complementary opposition. Back in 1784, the Anglo-Welsh scholar William Jones (1746–1794), known for pointing out similarities between Sanskrit, Latin, Greek and Persian, had indeed founded the *Asiatic Society of Bengal* and launched a journal devoted to Indian culture, the *Asiatick Researches*, reissued in London with the label “Printed *verbatim* from the Calcutta edition”. In being produced with the help of pandits, English translations of Sanskrit texts thereby flowed from Calcutta to Europe and, by the end of the eighteenth century, German translations from the English began to circulate in Germany. However, the romantic passion for Sanskrit was only sparked at the turn of the century in the Jena-Weimar circle around the Schlegel brothers, when Friedrich Majer, one of Herder’s disciples and the translator of Charles Wilkins’ English translation of the *Bhagvat-Geeta* (1785), began lecturing on Indian philosophy at Jena in 1796. In the ensuing years, German scholars learned Sanskrit by connecting with Parisian networks crystallizing around the manuscripts at the Bibliothèque impériale. Friedrich Schlegel directly benefitted from the teachings of Alexander Hamilton, a former British naval officer of the East India Company who had been trapped in Paris by the Anglo-French war flaring up again in 1803. At that time, Hamilton was cataloging the French Sanskrit collections, but could still freely proceed owing to Volney’s protection. A few years later, when Franz Bopp also made the trip to Paris, Hamilton had left, so that he first studied with Antoine Léonard de Chézy—a follower of Silvestre de Sacy, who held the newly created chair of Sanskrit at the Collège de France—before pursuing further as an autodidact. The two great works that shaped the beginnings of German indology, namely Friedrich Schlegel’s *Über die Sprache und Weisheit der Inder* (1808) and Franz Bopp’s *Über das Conjugationssystem der Sanskritsprache* (1816) therefore resulted

<sup>239</sup> [Windisch 1917–1920, I, p. 26].

<sup>240</sup> [Benfey 1869, p. 348].

<sup>241</sup> [Benfey 1869, p. 15].

from a cultural transfer of English expertise to the German states through the mediation of French scholarly circles.<sup>242</sup>

Since the turn of the century, Friedrich Schlegel had claimed that Indian sources would provide “the pinnacle of Romanticism [*das höchste Romantische*]”<sup>243</sup>, that is the ultimate remedy against the cultural fragmentation and disharmony resulting from the dessicated mechanical rationality of the *Aufklärung*. His study of Sanskrit led him to the momentous distinction between two main families of languages depending on whether modifications of meaning are produced by internal variation of a primitive root or by particles being adjoined to it. Languages with inflection like Sanskrit or Greek, are then characterized as “organic”, insofar as all inflected words proceeding from common roots—described as “living germ[s]”—“bear the stamp of affinity, all being connected in their simultaneous growth and development by community of origin”<sup>244</sup>. In presumably being the source of all inflected languages, Sanskrit would thus offer the key to the spiritual regeneration of Europe. By way of contrast, the so-called languages with affixes, best epitomized by Chinese, would be unproductive and artificial, inasmuch as being exclusively agglutinative. Although disclaiming any superiority for inflective languages, Schlegel nevertheless kept emphasizing the “clearest and deepest sense [*die klarste und innigste Besonnenheit*]”<sup>245</sup> which mankind presumably achieved from the start with Sanskrit. In response, Wilhelm von Humboldt denied as illegitimate any such sharp demarcation between inflective and non inflective languages. “I confess [*he later wrote to A. W. Schlegel*] that I could never share the opinion professed by your brother who wanted to establish in this way two families of languages, [*insofar as*] until now, no language devoid of agglutination ever occurred to me.”<sup>246</sup> Furthermore, he did not accept Schlegel’s shift from purely inflective languages to peoples presumably instantiating their pristine perfection, which he considered a speculative assumption, a mere “postulate of reason”, on a par with Creuzer’s superposition of an original monotheism.<sup>247</sup> More fundamentally, Humboldt’s well-known conception of languages as national world-views should not be

<sup>242</sup> For detailed accounts of the spread of Sanskrit studies in Europe by diffusion of British expertise, cf. [Halbfass 1990, chap. 5] and [Rabault-Feuerhahn 2008, Chap. 1].

<sup>243</sup> Cf. *Athenäum* 3 (1800), 103, quoted in [Halbfass 1990, p. 75].

<sup>244</sup> [Schlegel 1808, pp. 50–51].

<sup>245</sup> [Schlegel 1808, p. 63].

<sup>246</sup> Humboldt to A. W. Schlegel, November 1, 1821, [Hum 1908, p. 32].

<sup>247</sup> Humboldt to A. W. Schlegel, December 30, 1822, [Hum 1908, p. 32].

understood simply as a form of cultural essentialism<sup>248</sup>, inasmuch as the formal structure of languages does not univocally determine or bound the cultural capacities of the corresponding peoples, but only shapes the uses to which language lends itself.<sup>249</sup> In this connection, the case of Chinese is highly significant. In taking up the gauntlet thrown down by the French sinologist Jean-Pierre Abel-Rémusat (1788–1832) in a context of polarization between Sanskritists and Sinologists in the Société Asiatique de Paris, Humboldt laid stress on the distinction between the structure of thought and the linguistic form, so as to account for the intellectual excellence of Chinese in spite of its being non inflective.<sup>250</sup> Insofar as “grammar essentially exists in the mind”<sup>251</sup>, all languages are on a par as regards their ability to express each and every relation of ideas. However, whereas some, like Chinese, restrict themselves to conveying such logical combinations clearly and precisely, others, like Sanskrit, provide linguistic means to articulate them perceptibly.<sup>252</sup> Besides, Humboldt’s suspicion that Schlegel’s theory was flawed had already been strengthened by Bopp’s initial research on Sanskrit verbs which had shown that in certain cases apparent inflections resulted from previous agglutinations. As they both stayed in London in 1819, Bopp had taught him Sanskrit, while in return

248 Since languages are supposed to reflect in their structure the mental individualities of nations, Ruth Römer does not equivocate and points out what she takes to be Humboldt’s “moral flaw” in the *Kawi* opus, von Humboldt [1836–1839], inasmuch as the classification of languages advocated there is being infused with value judgments, cf. [Römer 1985, p. 109]. However, Markus Messling and others have recently argued that this accusatory reading was misguided, cf. Meßling [2008].

249 See [Meßling 2008, pp. 486–487]; in support of this interpretation, Markus Meßling persuasively comments on the following excerpt from Humboldt’s letter to F. G. Welcker, dated March 12, 1822, cf. [von Humboldt 1869b, p. 63]: “There are apparently two principal manners of treating language, one in order to merely express thoughts and fulfil material purposes, the other to shape thoughts into a distinct form and, thus, to expand thinking, and to let this view accompany any use of language, even the most material one. I believe that a high level of culture may be related to the first manner, such as technical and mathematical knowledge, and to a certain degree, also knowledge of natural history; but poetry, philosophy, eloquence in the strict sense will never be related to it, as their purpose is the form itself.”

250 See Joseph [1999] for a detailed analysis of Humboldt’s view of Chinese in response to Abel-Rémusat’s challenge; cf. also [Meßling 2008, pp. 491–494].

251 [von Humboldt 1827, p. 9].

252 In a Lockean way, Humboldt considers that “every mental judgment is a comparison of two ideas which we pronounce compatible or incompatible, [so that it] can be reduced to a mathematical equation” [von Humboldt 1827, p. 11]. In this connection, John E. Joseph explains that, in Humboldt’s view, “judgment begins as *pure thought or logic* and remains in this form in Chinese, just as it does in a mathematical equation; whereas other languages do not leave thought in this ‘pure’ form but incorporate it into *their particular logic*—‘linguise’ it, we might say”; see [Joseph 1999, p. 110].

Humboldt provided incentive for further in-depth investigations so as to decide whether the difference between inflective and agglutinative languages was an original one or simply the upshot of a sequence of obliterations.<sup>253</sup> Bopp soon replied in a letter dated March 5, 1820, that "Friedrich Schlegel's division between organic and mechanical languages completely collapses, and I will always strive to prove the contrary".<sup>254</sup> Furthermore, he challenged Schlegel's view of Sanskrit as the source from which Greek, Latin and all other European languages were derived<sup>255</sup>, and claimed that it was only one among many other kindred languages.<sup>256</sup> In contrast to Schlegelian romantic fantasies about lost origins, Bopp's hard-nosed comparative grammar would turn Indo-European linguistics into a sound scientific discipline.

The institutionalization of oriental studies within German universities may be characterized as going through a phase of establishment in the 1820s and 1830s, then through a phase of consolidation from 1840 to 1875.<sup>257</sup> However, as Sanskrit gradually succeeded making its way within university curricula, Sanskrit studies as a whole came to challenge the received partition of traditional disciplines. Until the end of the eighteenth century, both classical and oriental studies were the prerogative of theologians, inasmuch as being thought of as propaedeutics to biblical studies.<sup>258</sup> The wide-ranging secularization process of German society

<sup>253</sup> Humboldt's letter to Bopp, dated February 9, 1820, in [Lefmann 1891–1897, vol. 2, p. 5]; see also [Rabault-Feuerhahn 2008, pp. 79–89] for an overall account of the interactions between Humboldt and Bopp.

<sup>254</sup> [Lefmann 1891–1897, vol. 2, p. 7]. See [Bopp 1833, § 108] for the *locus classicus* for Bopp's refutation of Schlegel's conception.

<sup>255</sup> Cf. [Bopp 1820, pp. 10–11].

<sup>256</sup> In a letter to F. G. Welcker, dated November 6, 1821, Humboldt emphasized the need to learn Sanskrit for anyone who wants to engage seriously in linguistic studies, although still ranking it lower than Greek with regards to the way language fosters the use of ideas: "[Sanskrit] is in my view a center from which one can proceed backwards in the direction of less developed languages, so as to appraise the mechanism of language, and forwards in the direction of more developed languages so as to assess the ability of language for forming ideas", cf. [von Humboldt 1869b, p. 54].

<sup>257</sup> Cf. [Mangold 2004, chap. 4].

<sup>258</sup> In being rooted in biblical studies, German orientalism laid stress on the study of those ancient languages that were useful for the exegesis of sacred texts (Hebrew, Aramaic, Arabic in connection with Hebrew). According to Michael Werner, the strong ties which link philology with theology in Germany are as important as their increasing opposition to one another from the end of the 18th century, cf. [Werner 2006, p. 173]: "By insisting on the believer's need to find the truth of the divine word through incessantly renewed textual work, Protestantism favored the pivotal role of philology in German culture."

brought about the reappraisal, heralded by Kant, of the formerly lower faculty (philosophy) with regard to theology<sup>259</sup> and found expression in the movement of academic reforms. In granting social recognition and career perspectives to Greek and Latin teachers trained in philological seminaries and philosophy faculties, the classicizing Gymnasium contributed to emancipate philology from theology. In the midst of this process of disciplinary differentiation, Sanskrit studies at first appeared as a perplexing case between orientalist studies which were still annexed to theology and classical philology which was growing hegemonic. Bopp's appointment as an *Extraordinarius* in "oriental literature and general science of language" at the university of Berlin in 1821, under the aegis of Wilhelm von Humboldt, may be interpreted as a first step in the direction of granting Sanskrit studies academic autonomy within the framework of comparative grammar. But another, distinct, trend was already under way. August Wilhelm Schlegel (1767–1845), who studied philology in Göttingen under Heyne and eventually came to devote himself, more earnestly than his younger brother Friedrich, to the study of Indian sources, indeed traced a different path to institutional recognition. After learning Sanskrit with Bopp in Paris in 1815<sup>260</sup>, he began teaching it in Bonn from 1819 on, first *privatissime et gratis*, then with the declared intention of making the newly created Rhenish university into one of the main centers for Indology in Germany. He launched a journal, the *Indische Bibliothek*, whose first issue opened up with a programmatic state of the art<sup>261</sup> and remained active throughout the 1820s. In contrast to Bopp, A. W. Schlegel set out to graft Sanskrit studies onto philology by taking advantage of the dynamics of institutional and social forces coming into play within the newly prevailing model. However, the road to scientificity was thorny. In the wake of the Creuzer affair, the shaping of "Sanskrit

259 Cf. [McClelland 1980, pp. 42, 77].

260 Cf. A. W. Schlegel's letter to Guillaume Favre, dated February 4, 1815, quoted in [Benfey 1869, p. 372]: "je n'ai pu résister au désir d'apprendre la langue sanscrite; ... me voilà depuis deux mois écolier zélé des Brahmes. Je commence à débrouiller assez facilement les caractères; je m'orienté dans la grammaire, et je lis même déjà, avec le secours d'un Allemand, que j'ai trouvé ici, l'Homère de l'Inde, Valmiki." The German alluded to was none other than Bopp, as his letter to Windischmann, dated February 24, 1815, bears witness, cf. [Windisch 1917–1920, I, p. 75]: "I give him [Schlegel] lectures in Sanskrit, and he does not attend Chézy's collegium."

261 *Über den gegenwärtigen Zustand der indischen Philologie* (1820), cf. [Schlegel 1820, p. 22]: "Should the study of Indian literature thrive, then the principles of classical philology, namely with its scientific rigor, must be definitely applied to it."

philology" required gradual disengagement from romantic *Schwärmerei* and wholehearted endorsement of the ruling values among philologists.

The rise of philology as a dominant discipline in the first half of the nineteenth century has been already thoroughly analyzed as a complex historical phenomenon interweaving both internal and external factors. Michael Werner pointed out recurrent oppositional patterns in the way German philologists, contradictorily reflected upon their own practices, whether it be a polarization between narrowly textual and broadly cultural scholarship, or between philology as a propaedeutics focusing on establishing sources or as a totalizing science encompassing all aspects of ancient written cultures.<sup>262</sup> However, for our purpose, it will prove useful to simply mark out the main stages in this process so as to spell out how Sanskrit philology progressively gained academic credentials. In the first place, Friedrich August Wolf (1759–1824) who had been Heyne's student in Göttingen, although rather reluctant and autodidactic, and who later founded a philological seminary in Halle in 1787 according to his own principles, before joining the university of Berlin in 1809, may be credited with two important steps. He effectively severed philology from theological training and defined it anew as a consistent disciplinary field.<sup>263</sup> In his *Darstellung der Alterthumswissenschaft nach Begriff, Umfang, Zweck und Wert* (1807), he conceived philology as the "science of Antiquity" which, in addition to philology in the narrow sense, included, in his view, a host of subdisciplines ranging from archaeology, history, geography, epigraphy, numismatics, to art history and (notably) history of sciences. It is significant in this connection that Wolf emphasized the need for such cross-disciplinary skills as were later epitomized by Hankel. "Anyone [Wolf proclaimed,] who intends to deal with the inner history of an art or a science must not be a mere literary scholar or critic well-versed in Antiquity; he must also get hold of the art or the science itself, should

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262 See Werner [2006].

263 A close friend of Wilhelm von Humboldt, whose neo-humanist ideals he implemented in his teachings, F. A. Wolf set high goals for philology, insofar as it should aim at "the knowledge of human nature in Antiquity, a knowledge which comes from the observation of an organically developed, significant national culture, founded on the study of the ancient remains" ([Wolf 1807, p. 883]). His method implied, as Michael Werner emphasized, going "back and forth between the macro and the micro levels of the historical-philological inquiry" ([Werner 2006, p. 176]). Wolf first applied it in his *Prolegomena ad Homerum* (1795) so as to prove his contention that the Homeric epics only took on a written form in the mid-sixth century when various independent parts were assembled into a single whole. For a detailed account of F. A. Wolf's contributions to philology, see Grafton [1981], [Werner 2006, pp. 176–177], [Marchand 2009, pp. 17–24], [Judet de La Combe 2011, pp. 73–83].

he want to draw its development, or appreciate its successes and results. Therefore it is a deplorable experience that we still read few thorough historical investigations about those sciences in which the profundity and the astuteness of the Greeks have discovered so much with poor means, as for instance in mathematics.”<sup>264</sup> However, Wolf’s *Altertumswissenschaft* markedly excluded Oriental peoples in order to focus exclusively on the Greeks and, to a lesser extent, the Romans. The justification for separating the former from the latter was that “the Egyptians, the Hebrews, the Persians and other nations of the Orient ... barely, if ever, elevated themselves above that kind of culture [*Art von Bildung*] which one should call a *policed civility or civilization* [*bürgerliche Policirung oder Civilisation*], in contrast to a *genuinely higher culture of the mind* [*höhere eigentliche Geistescultrur*]”<sup>265</sup>. Whereas civilization solely amounts to producing the conditions granting a people’s life with security, political order and material comfort, genuine culture mainly implies the existence of a literature begotten by the nation as a whole. “Asians and Africans [*Wolf for instance claimed,*] will be safely excluded as literarily uncultivated, only civilized peoples, from our boundaries.”<sup>266</sup> Antiquity would thus be understood less as a period in world history than as a normative model for German culture.<sup>267</sup> Appropriating Greek culture through the philological learning process was expected to shape “new modernity against old modernity”<sup>268</sup>. Insofar as being implied in the project of building a *Kulturnation*, classical philology then

<sup>264</sup> Cf. [Wolf 1807, pp. 846–847]; in contrast to such compilations as Montucla’s and Bossut’s histories of mathematics, Wolf remarked that particular monographs on the works of Archimedes or Apollonius were still lacking and should be launched in the future.

<sup>265</sup> [Wolf 1807, p. 817]. Although Wolf did not mention explicitly the Indians, his restrictive definition of *Altertumswissenschaft* constituted, as Pascale Rabault-Feuerhahn emphasized, a serious impediment to the shaping of Sanskrit philology, see [Rabault-Feuerhahn 2008, p. 108].

<sup>266</sup> [Wolf 1807, p. 819].

<sup>267</sup> It should be remarked that Wolf’s definition of *Altertum* fits in with Norbert Elias’ well-known analyses on the formation of the antithesis *Kultur* vs. *Zivilisation* in Germany, cf. [Elias 1978, p. 4]: “In German usage, *Zivilisation* means something which is indeed useful, but nevertheless only a value of second rank, comprising only the outer appearance of human beings, the surface of human existence. The word through which Germans interpret themselves, which more than any other expresses their pride in their own achievement and their own being, is *Kultur*.”

<sup>268</sup> This suggestive formulation is taken from Pierre Judet de la Combe, cf. [Judet de La Combe 2011, p. 73]: “Modern societies submit themselves unto the rule of civilization: material goods prevail over men, the masses over individuals. No real individuality can exist in such a context. It has to be restored. ... If the goal of public education is to guarantee individual autonomy, the best object to be taught is the most

succeeded under the auspices of Wolf's former student, August Boeckh (1785–1867), in establishing its supremacy as an all-encompassing science of culture which would challenge both philosophy and history.<sup>269</sup> In spite of various internal tensions, classical philology nevertheless set the standard for a whole range of neighboring disciplines, such as for instance Sanskrit studies which gradually conquered dedicated chairs in German universities in the 1840s.<sup>270</sup> In his opening allocution at the eleventh *Versammlung Deutscher Philologen, Schülmänner und Orientalisten*, held in Berlin on September 30, 1850, Boeckh eventually ratified the outcome of that whole process of legitimization and acknowledged oriental philologies as being on a par with classical philology within the framework of one overarching philology.

It has been a long time [Boeckh claimed,] that the study of oriental languages and oriental literature is no longer related to our sacred books, hence to theology; our view of the whole Orient gradually evolved: next to the Semitic languages ... we have learnt to know a richly segmented family of languages, closely related to classical language and German, whose noblest branch is Sanskrit ... Leaving aside the *Streitfrage* about which influence the Orient, and in the first place Egypt, may have exerted on the classical peoples of Antiquity, it must in any case be admitted that not only the later history of the Orient ... is interwoven with the history of classical peoples, but also that, as for languages, the most ancient representations of the above mentioned oriental and classical peoples have many points of contact, notwithstanding the highly developed, strictly hellenic singularity. ... I would like to claim that, in the same way as comparative linguistics took shape, a comparative cultural history of the whole of Antiquity should emerge as one of the main tasks of philological science.<sup>271</sup>

In 1852, the Leipzig Sanskrit scholar, Hermann Brockhaus (1806–1877) presented before the Royal Society of Sciences of Saxony a transliteration

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autonomous culture we know, Greek culture, which owes almost nothing to more ancient cultures because it is the only one which could achieve, by means of its language, the reconciliation of religion, art, philosophy and society.”.

<sup>269</sup> Cf. [Werner 2006, p. 178]; in contrast to philosophy, philology would emphasize the historicity of both knowledge and the known, while, in contrast to history, it would lay stress on the scientific establishment of the sources as well as on the need for extending hermeneutical methods beyond mere textual analysis.

<sup>270</sup> Christian Lassen obtained an *Ordinariat* in Bonn in 1840, Hermann Brockhaus was called as an *Extraordinarius* to Leipzig in 1841, then confirmed as a full professor in 1848, both of *Altindische Sprache und Literatur*. Some others, like Rudolf Roth in Tübingen, or Theodor Benfey in Göttingen shifted, also around 1848, from positions as *Privatdozenten* in oriental languages to more specialized chairs in Sanskrit philology, whereas others, like G. H. F. Nesselmann in Königsberg, proposed Sanskrit only as a part of their orientalist teachings.

<sup>271</sup> A. Boeckh, “Von der Philologie, insbesondere der klassischen, in Beziehung zur morgenländischen, zum Unterricht und zur Gegenwart”, in [Boeckh 1859, pp. 183–199].

of the Sanskrit original text, and a German translation of a sample, of Bhāskara II's algebra, after Colebrooke's English translations which he praised as "the most learned and thorough work in the whole field of Indian philology"<sup>272</sup>. His purpose was mainly to draw the attention of his contemporaries to that Indian part of a universal cultural heritage. In his view, "there could hardly be any task more worthy to be engaged with than making the Indian sources of these sciences [*viz. mathematics*] available for the European public"<sup>273</sup>. In emphasizing that Indian mathematics is written in the most sophisticated verses, Brockhaus remarked that "one may smile at such a bizarrerie to read the rules of addition and multiplication expressed in Sapphic, Alcaic<sup>274</sup> or any other meter only used in higher poetry, [*but that*] one must [*still*] marvel at the rare power of language [*that is required*] to represent such a rough stuff in such a challenging form; a problem which, as far as I can judge, Bhāskara seems to have solved in a masterly way."<sup>275</sup> Furthermore, Brockhaus explicitly threw into relief the indispensability of commentaries for understanding Bhāskara II's works insofar as "they [*not only*] give the proofs (*utpatti*) for the mathematical theorems of the master", but also "introduce us to the methods of Indian

<sup>272</sup> [Brockhaus 1852, p. 9]. In using the system of transliteration of Sanskrit into the Latin alphabet he had himself set up, Brockhaus only dealt with the first four sections of the first chapter of Bhāskara II's *Bīja-ganita*, that is, only a small part of even Colebrooke's translation.

<sup>273</sup> [Brockhaus 1852, p. 17]. In addressing a request in 1844 to the Saxon ministry of culture for the advancement of his career, Brockhaus recalled that Sanskrit studies were, in his view, to gradually deliver the solution to "the deepest enigmas about the historical development of the whole mental culture of mankind in language and religion, in art and science, in morals and law", cf. [Mangold 2004, pp. 110–111] for Brockhaus' quote from unpublished archival document.

<sup>274</sup> By referring to those Indian meters as analogous to the Greek ones used in higher poetry, Brockhaus implicitly bolstered the view of Sanskrit philology being a legitimate counterpart of classical philology. In the same vein, in a short note published in *ZDMG* 6 (1852) under the title "Ueber Homer's Ilias in Sanskrit", he reproduced as a "literary curiosity", the translation into Sanskrit verses of the opening lines of the Iliad, made by a learned pandit at the request of Charles Philipp Brown to be inserted in his book *The Prosody of the Telugu and Sanskrit languages explained*, Madras, 1827, p. 44. Brockhaus was in particular interested in the modifications brought about by translation, as for instance the way the Indian translator strove to "nationalize the Iliad", for instance in making Achilles into a saint whose devotions are disturbed by the Acheans, rather than an outraged hero.

<sup>275</sup> [Brockhaus 1852, p. 12]. Brockhaus pointed out that the wealth of synonyms in Sanskrit both for numerals and for the arithmetical operations may at least partially account for the possibility of such versified mathematics. Still, he noted that this form could also prove a constraint in limiting further mathematical developments.

computation”<sup>276</sup> However, Brockhaus was no mathematician, and thus felt the need to translate, “with the help of a younger friend of mathematics”<sup>277</sup>, the Indian text of Bhāskara into algebraic formulae, contrary to Colebrooke who could afford to scorn the use of modern algebra thanks to his presumably intimate acquaintance with Indian methods. “If I, a complete layman in the lofty sciences taught by Bhāskara, nonetheless dared to read his algebra, [Brockhaus further pleaded,] the curiosity of a philologist will provide an excuse for seeing how the author has solved a problem which, according to all our ideas, seems almost impossible”<sup>278</sup>, namely doing mathematics in verse. As Wolf had claimed decades earlier for Greek mathematics, Brockhaus acknowledged that Sanskrit philology should meet mathematics so as to account for these sources of European culture. “Time is over for dilettantism revelling only in Indian poetry, [he contended,] rigorous science asserts its right, and I believe that in this domain there are many materials, until now still unused, to extract from India for the history of the development of the human mind.”<sup>279</sup>

By the late 1840s already, Sanskrit philology as envisioned by A. W. Schlegel had become a reality. Bonn would challenge Berlin in promoting an approach to Sanskrit sources markedly different from Bopp’s, insofar as embodying an extended view of *Alterthumswissenschaft* rather than one centered on linguistic analysis and comparative grammar. The young Friedrich Nietzsche’s philological notes from 1867–1868 bear witness to

276 [Brockhaus 1852, p. 15], see also further: “Namely the commentaries compute all the examples from the beginning; for any further operation, the rule is recalled according to which it must be conducted, until eventually the result is obtained. It gives me the [same] impression, [Brockhaus goes on,] when I read this part of the commentaries, as if I were in an Indian school, and the teacher would compute an example at the blackboard”. Although crediting Colebrooke with relying on commentators in the clarifying footnotes to his translation, Brockhaus nevertheless insisted that “if an edition of the mathematical and the astronomical works of Bhāskara was ever to be launched in Germany, it should also contain commentaries, since these often are, strictly speaking, more important than the text they explain”; cf. [Brockhaus 1852, p. 15].

277 [Brockhaus 1852, p. 16]. One may wonder who this “younger friend of mathematics” might be. Although there is no indication about it whatsoever, it is not absolutely improbable that Brockhaus may have heard from his colleague at the Leipzig University, the physicist Wilhelm Hankel, about the outstanding mathematical and philological capacities of his thirteen-years-old son, Hermann, then a pupil at the *Nicolai-Gymnasium*.

278 [Brockhaus 1852, p. 15].

279 [Brockhaus 1852, p. 19]. Brockhaus’ well-known conclusive pronouncement is often underscored as marking a turning point in the history of Sanskrit philology, see [Windisch 1917–1920, II, p. 213], [Rabault-Feuerhahn 2008, p. 143], [Marchand 2009, p. 132].

the influence these developments were to exert in the long run even on the way classical philologists considered their own practice.<sup>280</sup> In thoroughly discussing Wolf's view of classical studies, Nietzsche indeed incidentally tempered his praise with the following significant reservation: "The concept of Antiquity is vague. Indians, Hebrews, Egyptians?"<sup>281</sup> A. W. Schlegel's student, Christian Lassen (1800–1876), who settled in Bonn after his years in Paris where he had been trained by Eugène Burnouf, wrote a monumental five-volume *Indische Alterthumskunde* (1847–1862) in which he laid stress on geography and ethnography as providing the key to understanding the cultural history of the Indian peoples.<sup>282</sup> With this work, new influential trends were under way. Lassen's ethnographic overview of India embodied the gradual shift from linguistic kinship between Indo-European languages as substantiated by comparative grammar to the unsupported assumption of a presumably ethnic kinship between the peoples natively speaking those languages, a propensity eventually turning into a widely shared tenet by the 1860s.<sup>283</sup> Lassen indeed distinguished two main branches among the Indian peoples, the southern Deccan or *Dravida-Nishâda* peoples on the one hand, and on the other hand the so-called "Aryans" who came from the North in waves of migration and were, in his eyes, to be considered as India's genuine *Kulturvolk*, "the true subject of Indian history"<sup>284</sup>. In claiming that "through their language, the Aryan Indians attest to an original and deep kinship with the peoples now called Indo-Germanic"<sup>285</sup>, Lassen both endorsed and strengthened a view tainted with nationalistic longings which Humboldt

<sup>280</sup> Nietzsche had been trained in classical philology under Friedrich Wilhelm Ritschl, whom he followed from Bonn to Leipzig in October 1865. There, he became acquainted with Hermann Brockhaus and his wife who, being Wagner's sister, offered him the opportunity to encounter the great composer, as Nietzsche recounted in a famous letter to his friend Erwin Rohde (November 9, 1868, cf. *Digitale Kritische Gesamtausgabe*).

<sup>281</sup> Quoted in [Brobjer 2007, p. 165].

<sup>282</sup> For a detailed account of the content of the *Indische Alterthumskunde*, see [Rabault-Feuerhahn 2008, pp. 131–134]. In claiming the primacy of an overall assessment of Indian culture, Lassen appears as being in line with Boeckh's *Sachphilologie*.

<sup>283</sup> Georges Dumézil clearly identified this shift as the characteristic lapse of scientific rigor commonly shared by nineteenth-century Indo-Europeanists. By contrast, he warned in his 1949 *Leçon inaugurale* at the Collège de France that a mere working hypothesis for making sense of structural homologies should not be turned into a fictitious narrative: "la reconstruction vivante, dramatique, de ce qu'était la langue ou la civilisation des ancêtres communs est impossible, puisqu'on ne remplace par rien les documents, et qu'il n'y a pas de documents", cf. [Dumézil 1950, p. 15].

<sup>284</sup> [Lassen 1847–1861, I, p. 410].

<sup>285</sup> [Lassen 1847–1861, I, p. 400].

and Bopp resisted.<sup>286</sup> Furthermore, while outlining the cultural history of the Indians, Lassen strove to grasp the characteristic features of Sanskrit works in such varied fields as grammar, epic poetry, philosophy, fine arts, architecture, sciences and mathematics<sup>287</sup>. "Indian Antiquity" would thus supposedly emerge from these comprehensive accounts as an alternative to classical Antiquity, thereby discarding the normative claims previously attached to the latter, in compliance with the increasingly prevailing views of nineteenth-century German historism (*Historismus*).<sup>288</sup> Ulrich Muhlack emphasized how in Germany, the "idealistic-historical view of history" stemming from the momentous revolutionary experience that history is ultimately man-made, first took hold of classical philology through Humboldt's reflections on the concept of individuality. In Humboldt's view, the task of the historian would indeed consist in interpreting historical events as the expression of definite individualities, by which he neither meant hypostatized world-historical personalities, nor solipsistic subjectivities, but "the impetus for historical action stemming from the inner human being, which in return does not exist in the abstract, but as a concrete individual, although always engaged in close communication with other concrete individuals".<sup>289</sup> In this sense, Humboldt and Wolf put the premium on Greek Antiquity as the foremost epoch in world-history as regards individuality, hence historicity, and accordingly considered classical philology as the first genuinely historical discipline, one presumably affording its

286 In the preface to the second 1857 edition of his *Comparative Grammar*, Bopp rejected the increasingly in vogue denomination "Indo-Germans" in favor of the more dispassionate "Indo-Europeans", mainly "because [he did] not know any reason why in the name of the comprehending linguistic stock [*Sprachstamm*], the Germans should precisely stand out as the representatives of the rest of the originally related peoples of that part of the Earth, in the present as well as in the past" [Bopp 1833, 2nd ed., p. XXIV] In this connection, Bopp invoked Humboldt's authority, while recalling that the denomination he had given his preference in the *Kawi* work, the so-called "Sanskrit stock, [*proved*] very much appropriate [*insofar as*] emphasiz[ing] no nationality".

287 On Indian mathematics, see [Lassen 1847–1861, II, pp. 1114–1146]; Lassen's focus is mainly on the relationship between mathematics and astronomy, on Indian numerals and on Āryabhaṭa.

288 On the history and meaning of the term *Historismus* as a label for the central tenet of the nineteenth-century German conception of history, practiced for instance by Leopold Ranke, Johann G. Droysen and others, see Iggers [1983]; cf. also [Iggers 1995, p. 130]: "*Historismus* signified a historical orientation which recognized individuality in its 'concrete temporal-spatiality' (Prantl), as pursued for example by the Historical School of Law (Savigny and Eichhorn), distinct from a fact-oriented empiricism as well as from the system-building philosophy of history in the Hegelian manner which ignores factuality."

289 [Muhlack 1979, p. 232].

practitioners historical experience *par excellence*. However, in the long run, that normative view of Greek Antiquity came into conflict with the all-encompassing historical thinking it had released, thereby leading, with Boeckh, to “consistent historicization and therefore final rejection of any normative view”<sup>290</sup>. Sanskrit philology would then eventually fulfill the requirements, first met by classical philology, for qualifying as a fully historical science.

## 7. LOOSE ENDS TIED UP

Hermann Hankel’s innovative reading of Sanskrit mathematical sources, as it took shape from 1869 to his premature death in August 1873, attests to his being imbued with the methods of historical criticism that permeated professionalized German Sanskrit philology at the time. A fair share of the volume *Zur Geschichte der Mathematik in Alterthum und Mittelalter*, published posthumously by his father in 1874, is devoted to Indian mathematics and draws extensively on the work of such Sanskrit scholars as Christian Lassen, Hermann Brockhaus, Theodor Benfey, Albrecht Weber and the Leiden professor Hendrik Kern (1833–1917), whom he was indebted to for letting him know about unpublished manuscripts of Āryabhaṭa, whose editing process was then under way.<sup>291</sup> As his friend Wilhelm v. Zahn<sup>292</sup> later explained, Hankel’s outstanding combination of mathematical gift and philological expertise may be traced back to his youthful years at the Nicolai Gymnasium in Leipzig. There, exceptionally,

290 [Mühlack 1979, p. 238].

291 Cf. [Hankel 1874, p. 179]: “Only in recent times has one discovered that under the name Ārya-siddhānta, two different works are referred to, of which the first, often designated by the prefix *Vṛiddha* (= ancient) or *Laghu* (= little), is this Āryabhaṭīya, whereas the other, designated as *Mahā* (= great) -siddhānta, cannot have been written before the twelfth century. Already, Indian commentators no longer distinguished the two works and thereby they brought into the history of Indian mathematics a confusion which I could partially escape owing to the fact that Herr Prof. Hendrik Kern (see also H. Kern, Preface to Varāhamihira’s *Bṛhat Sanhitā*, *Bibl. indica*, Calcutta 1865, p. 55.) in Leiden was good enough to make some appreciable communications to me out of the manuscripts of the ancient Aryabhaṭa.” Let us mention that Kern’s edition of the Āryabhaṭīyam with a commentary by Paramādīsvara, out of two recent copies from Calcutta (1820 and 1863) owned respectively by A. C. Burnell and C. Wish, was eventually published in 1874, cf. Kern [1874].

292 Together with Hankel, Wv. v. Zahn (1839–1904) studied mathematics and physics at the university of Leipzig where he took part in Gustav Fechner’s experimental work and specialized in physics. Most of what we know of Hankel’s life comes from von Zahn’s obituary published in *Mathematische Annalen* in 1874.

he was allowed by the rector to indulge in his spare time in the writings of the mathematicians of Antiquity, instead of the otherwise mandatory classics ("a Greek drama", as a rule, according to J. E. Hofmann), "so as to fulfill in a higher measure the philological requirements of the school and his thirst for knowledge in mathematics"<sup>293</sup>. Besides, as J. E. Hofmann also emphasized, Hankel most probably benefitted from stimulating influences at home, for, all through his formative years, his father prepared, under the auspices of Alexander von Humboldt, the 16-volume German edition of the works of François Arago. However, in the long run, Hankel's "vivid interest in the history of mathematics [*v. Zahn pointed out,*] was all the more awakened as the kind of thoroughness which was peculiar to him would lead him to consider the connections tying together the elements of knowledge he had obtained, and to find real satisfaction only when he knew how to round them off by integrating them into a higher unity"<sup>294</sup>. For his part, Hankel's father also emphasized that his son "had pursued the historical development of the mathematical sciences with a particular predilection right from the beginning of his studies, and that, owing to a thorough and extended investigation of the sources, he had sought as far as possible to gain a clear insight into its course and an unbiased and firm judgment on the achievements of single men and peoples."<sup>295</sup> At the Leipzig University which he entered in 1857, Hankel studied mathematics under Möbius and Drobisch, and physics under his father who held a professorship since 1849. In 1860, he moved to Göttingen where he mastered the theory of functions with Riemann, then in 1861 to Berlin where he studied under Weierstrass and Kronecker, before submitting his dissertation to Drobisch and Möbius back in Leipzig in 1862. After habilitating in 1863, he occupied teaching positions in Leipzig, Erlangen and eventually in 1869 in Tübingen, where he set up a mathematical seminar modeled upon the already existing philological one. As a mathematician, he was mostly productive in two main fields, the theory of complex and higher complex numbers and the theory of functions. Among his mathematical achievements, one should mention his *Theorie der complexen Zahlensysteme* (1867) in which he surveyed within one single theoretical framework both Hamilton's theory of quaternions and Grassmann's *Ausdehnungslehre*, whose reception he thus fostered. As for history of mathematics, he is reported to have envisioned early on "writing a comprehensive critical

<sup>293</sup> [von Zahn 1874, p. 583].

<sup>294</sup> [von Zahn 1874, p. 584].

<sup>295</sup> [Hankel 1874, Foreword, p. IX].

history of mathematics out of the material he gathered with historical thoroughness on the occasion of his mathematical studies”<sup>296</sup>. Still, he would postpone it so as to devote himself entirely to creative mathematics while still in his fruitful years. However, in yielding to a request, he nonetheless decided to jot down an outline of the grand work in progress whose completion he planned for a later stage, and in so doing, “he set himself the goal to subject the traditional conception to rigorous criticism”<sup>297</sup>. For three semesters ranging from 1870 to 1871, he lectured on history of mathematics proper<sup>298</sup>. In the summer of 1872, he contracted meningitis which he hardly overcame, and died from a stroke the following year, leaving his work unpolished, which accounts for the inaccuracies Moritz Cantor later pointed out in addition to praising his insightful analyses written down “with a kind of curtness and acerbity”<sup>299</sup>.

How and why should the scholarly stance commonly labelled as *Kritik*<sup>300</sup> in German philological and historical circles, have a direct bearing on history of mathematics in Hankel’s view, is explained in the opening pages supposedly intended as an introduction to his book.

The history of a science [*Hankel affirmed,*] can itself become a science, if it attempts to bring that which is decisive out of the infinite wealth of the particulars, to recognize the necessary in the real. Whatever the extent to which the freedom of the particular real may range, still, it does not abolish the law of the whole current; the waterfall retains its form, however accidental the course of

<sup>296</sup> [von Zahn 1874, p. 589].

<sup>297</sup> *Ibid.*

<sup>298</sup> See the digitalized list of lecture courses at the *Universitätsarchiv Tübingen*. The following titles for Hankel’s lectures are registered: in the spring term of 1870, “Übersicht über die Geschichte der Mathematik: 3 mal”; in the winter term of 1870–1871, “Geschichte der Mathematik”; eventually in the spring term of 1871, “Geschichte der griechischen Mathematik mit Erklärung ausgewählter Stücke aus Euklid, Archimedes, Apollonius, Diophant, u.s.w.: 3 mal um 6 Uhr.”

<sup>299</sup> See Cantor’s review of Hankel’s History of Mathematics, as quoted by Hofmann in [Hankel 1874, p. XII].

<sup>300</sup> On the meaning of *Kritik* in this context, cf. [Turner 1983, pp. 473–474]: “In its broadest sense the popularity of *Kritik* as a category represented the philologists’ response to the philosophical atmosphere of epistemological reassessment that prevailed in post-Kantian Germany. In this sense it denoted little more than the precept that philological and historical scholarship presupposed a constant, sceptical evaluation of the consistency, reliability, and authenticity of sources. [...] But *Kritik* came to mean still more. [...] according to the rhetoric of the day, the exercise of critical method must be preconditioned by the grasp of the Idea, by the insight that comes from exhaustive study and devotion to the discipline. It is inseparable from interpretation and both merge into the act of understanding or recreation at which all philology aims. *Kritik* therefore connoted both method and intuitive insight, and so passed well beyond the negative or destructive implications of previous usage.”

each drop may be. If already, with such a purely mechanical phenomenon, the human mind is incapable of determining the laws of the movement, how should it be able to construct theoretically, from scratch, the progress of history, to conceive the development according to the categories of cause and effect, or even to dress it up in general concepts! Nothing else remains to be done then than to provide the image of the times themselves in their main outlines, to present these necessary laws of mental development more in the representation of the significant—in the sharper characterization of the turning points—than in general pragmatic reflections.<sup>301</sup>

Hankel may have implicitly referred here to the historiographical debate launched by Johann Gustav Droysen (1808–1884) in a widely influential paper on “The Elevation of History to the Rank of a Science” (1863).<sup>302</sup> Droysen himself replied to Henry Thomas Buckle’s empiricist pronouncement on the way “scientific history” should be pursued. In emphasizing the fruitfulness of increasingly powerful statistical methods, the British historian had considered that the split between moral and exact sciences ought to be denounced as an artificial one, hence that the same standards of scientificity should hold for both realms. Since human actions result from the joint workings of outer phenomena upon our inner selves and vice versa, it would presumably be the historian’s business to “discover the ‘laws’ of this double agency”<sup>303</sup>, as Droysen formulated the core of Buckle’s view. Accordingly, history would only be raised to a science by showing how to prove historical facts deductively out of general laws, which in return would be inductively extracted out of the mass of the particular facts heretofore collected. Although dismissing Buckle’s approach as being “erroneous, artificial and inadequate”<sup>304</sup> in the way the sources were dealt with, Droysen nevertheless retained as worthy of further consideration the issue of clarifying the standards of scientificity for history. “We in Germany [*he countered*,] have least of all a reason to misjudge the high value of the enhanced technique in our studies, of the increasing practice and certainty in the handling of historical *Kritik*, of the results which have been obtained by these means.”<sup>305</sup> In Droysen’s view, inductive laws are but mere external generalizations failing to provide the

<sup>301</sup> [Hankel 1874, p. 1].

<sup>302</sup> Cf. Droysen [1863]. Intended as a critical review of Henry Thomas Buckle’s *History of Civilization in England* (1858), this paper was originally published in 1863 in Heinrich von Sybel’s journal, *Historische Zeitschrift*, then reedited as an appendix in Droysen’s well-known *Grundriss der Historik* in 1867.

<sup>303</sup> [Droysen 1868, p. 43]

<sup>304</sup> [Droysen 1868, p. 43].

<sup>305</sup> [Droysen 1868, pp. 44–45].

historian with a real understanding of what history is all about. Hence it is only through interpretation, *viz.* through immersion into the mental world of the actors of history, that one can grasp the meaning and the coherence of historical events, owing to the so-called “moral powers” [*sittlichen Mächte*], those “laws of an entirely different kind and energy” which arise in critical retrospect as being all at once “factors and products of the historical life”.<sup>306</sup> Unsurprisingly, Droysen thus praised Wilhelm von Humboldt as “a Bacon for the historical sciences”<sup>307</sup>, Buckle being denied that same qualification for his having misguidedly placed history among the natural sciences.<sup>308</sup> Hankel therefore sided with the German historians in rejecting in principle all forms of naturalized history, hence, significantly enough for our current purpose, Arneth’s cultural history of mathematics.

In this respect, it should also be noted that Hankel justified the emphasis placed on Indian mathematics in his approach to history of mathematics by subscribing to the “anti-normativity” leitmotiv of German historism.<sup>309</sup>

Insofar as it is the task of historiography, [*Hankel claimed*,] by describing different peoples and different times, to expand our intuition so that it does not

<sup>306</sup> [Droysen 1868, p. 58]. Here Droysen closely follows W. v. Humboldt’s hermeneutical view of history; see for instance [von Humboldt 1822, pp. 1–4]: “The task of the historiographer is to present what happened. [...] But what happened is only partially visible in the world of the senses; the rest which is added must be felt, inferred, guessed [*empfunden, geschlossen, errathen*] [...] Historical truth is, as it were, rather like the clouds which take shape for the eye only at a distance [...] Thus two ways have to be followed simultaneously in the approach to historical truth; the first is the exact, impartial, critical investigation of what has happened; the second is the connecting of what is investigated, the intuitive understanding of what could not be reached by the first means.” Droysen’s “moral powers” then correspond to Humboldt’s *Ideen*, or “creative forces”, which “emerge from the wealth of events themselves, or, to be more precise, originate in the mind through consideration of these events undertaken with a truly historical sense”, see [von Humboldt 1822, p. 13].

<sup>307</sup> [Droysen 1868, p. 6].

<sup>308</sup> [Droysen 1868, pp. 45, 47]. Humboldt struck a similar note, cf. [von Humboldt 1822, p. 18]: “we do not believe the understanding of events to be completely achieved by explanations taken from the realm of nature [...] The guiding force governing events, although situated outside of the process of nature, nevertheless reveals itself in those events [*viz. provided they are aptly interpreted by the historian*]”.

<sup>309</sup> Georg Iggers pinpoints “anti-normativity” [*Antinormativität*] as a catchword for one of the main tenets of the German conception of history, cf. [Iggers 1983, pp. 8–9]: “No individual, no institution, no historical deed can be judged by standards external to the situation in which it arises, but rather must be judged in terms of its own inherent values. There are thus no rational standards of value applicable to a diversity of human institutions. Instead all values are culture-bound, but all cultural phenomena are emanations of divine will and represent true values.”

take, in a narrow-minded way, the state of a certain time and a certain people for the absolute norm—insofar as I hold as the task of the historiographer of mathematics especially to remove the prejudice that there is only one kind of historical development for mathematics and only one form of scientific development for it—then the section [*on Indian mathematics*] belongs to the most instructive ones.<sup>310</sup>

Not only did Hankel deliberately take over Wilhelm v. Humboldt's exact phrase in his well-known discourse “On the task of the historiographer”, delivered before the Prussian Academy of Sciences on April 12, 1821, but, in accommodating it to the special case of history of mathematics, he also endorsed Humboldt's views on the “mental individuality” of peoples and nations as being more enduring than their original bearers, hence liable to transference to new ones<sup>311</sup>.

Accustomed from our early youth [*Hankel continues*,] to the rigorous, Greek form of geometry, imbued with respect for the classical literature of the Greek people, we are raised in the opinion that this form is the absolutely necessary and the only scientific one. We hardly notice that not only the form but also the spirit of our arithmetic and algebra, actually of the whole of modern mathematics, utterly differ from the form and spirit of ancient geometry. It will not elude the reader how close the spirit of contemporary science comes to the spirit displaying itself in the mathematics of the Indians; the following will show that the development of the modern peoples has been influenced by the Indians through the mediation of the Arabs. Under these circumstances, the mathematics of our relatives on the Ganges [*unserer Stammverwandten am Ganges*] becomes highly interesting.<sup>312</sup>

Although reminiscent of Arneth, Hankel's point was nevertheless differently meant insofar as being entirely devoid of essentialist undertones. To all appearances, in incidentally borrowing from the registered lexicon

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<sup>310</sup> [Hankel 1874, p. 219].

<sup>311</sup> Humboldt's point is articulated at the end of his 1821 discourse, cf. [von Humboldt 1822, pp. 21–23]: “The idea can entrust itself only to an individual mental force, but the fact that the seed which the idea implants in the force develops in its own way, that this way remains the same whatever other individual it is transferred to, [...] this shows that it is the independent nature of the idea which completes its course in the realm of phenomena. [...] Every human individuality is an idea rooted in the phenomenon, and this idea shines forth so brilliantly from some individuals that it seems to have assumed the form of an individual merely to use it as a vehicle for expressing itself. [...] It is the same with the individuality of nations [...] In the midst of the peoples' deeds driven by need, passion and apparent chance, the mental principle of individuality therefore continues to have an effect, and more powerfully than these elements. [...] Beside the direction which they impart to mankind by their actions, both peoples and individuals leave behind them forms of mental individuality which are more enduring than occurrences and events.”

<sup>312</sup> [Hankel 1874, p. 219].

of Indo-Germanic studies, Hankel only jocularly alluded to voguish folklore, in much the same way as, on occasion in his correspondence with the French, Alexander von Humboldt puckishly endorsed such branded expressions, thus conferring upon them a meaning of self-mocking irony.<sup>313</sup> For in his view, mathematics “[*did*] not know of national boundaries”<sup>314</sup>. Greeks and Indians both contributed, although differently, to “our cosmopolitan science which certainly makes the most of national characteristics, but eventually surmounts [*aufhebende*] them”<sup>315</sup>. In developing pure arithmetic as an object of independent interest and in shaping a number system designed for theoretical and practical applications, the Indians imparted a new direction to mathematics, *viz.* “a purely arithmetical-algebraic one, which is so completely different in form and content from Greek mathematics and arises from such an essentially new intuition that it cannot be considered as the continuation of the previous [*geometric*] direction, but rather as its opposite”<sup>316</sup>. In so doing, they contributed to shape “the culture of our modern Europe” and exerted “an influence which, although outwardly not as dazzling as that of classical Antiquity, is still secretly and powerfully active; let us only think that any European, at least during his school years, learns to know and to apply every single

<sup>313</sup> See for instance Humboldt’s letter to Arago, dated July 6, 1847, cf. [von Humboldt 1869a, p. 354]: “Mon cher et excellent ami, les paroles me manquent pour t’exprimer combien j’ai été touché de ton admirable lettre … J’en ai été touché jusqu’aux larmes, car tu sais que la race indo-germanique a le défaut d’une sensibilité pleureuse.” One may also mention Humboldt’s letter to Lettrone, dated September 8, 1837, cf. [von Humboldt 1869a, p. 142]: “…mais ce M. Forchhammer est de ma race (germanique), et lorsque nous ne sommes pas froidement ennuyeux, comme je le suis dans mon *Examen critique*, nous sommes parfois un peu fols”; or his letter to Guizot from 1840, cf. [von Humboldt 1869a, p. 203]: “Je me suis logé à deux époques dans la maison de madame de Rumford, pour exempter sa maison des visites et logements des Slaves et de la race indo-germanique.” For a broader view on Humboldt’s cosmopolitanism, see Ette [2001].

<sup>314</sup> [Hankel 1874, p. 1].

<sup>315</sup> [Hankel 1874, p. 221].

<sup>316</sup> [Hankel 1874, p. 2]. Hankel’s history of mathematics embodies some Humboldtian notion of the individuality of nations as being entrusted to “mental forces” or energies; see [Hankel 1874, p. 172]: “When a people loses the ability and the strength for mathematical research, another immediately comes up to take over further progress for the following centuries. …as the scientific energy of the specifically Greek intellect was exhausted and all mental strength still present was devoted to the lofty task of establishing historically, defining dogmatically and understanding speculatively the holy facts and the holy truths of Christianity, with a dedication and an exclusiveness which made all other studies appear empty and pointless, and even harmful, mathematics, exiled from the Occident, found an asylum far in the East, beyond the Indus.”

day in his computations, the wisdom of the ancient Indians”<sup>317</sup>. In W. v. Humboldt’s terms, the “mental individuality” first brought to fruition by a historically remote people thus outlives its original carrier insofar as it attaches itself to elaborate intellectual practices (computations) which can be inherited by disparate successors (the Arabs, the Europeans), who in return may combine them in unsuspected ways with other practices. “The meeting of Greek and Indian minds among the Arabs [*Hankel accordingly claimed,*] and the influence of the latter upon the shaping of the scientific mind in modern Europe, constitute the most remarkable and the most momentous event in the domain of our science, which, like no other, is capable of making good use of the dispositions of the most different peoples so as to advance its objective content.”<sup>318</sup>

In characterizing Indian mathematics by the acute “sense of numbers” entrenched in cultural habits combined with the immediate intuition prevailing in geometry, Hankel sharply contrasted it, as already mentioned, with Greek construction of propositions by means of concepts. Insofar as drawing on Lassen’s ethnography and history of India, whose theory of migrations he appropriated, Hankel displayed a great sensitivity for nuances in assessing those traits of Indian culture standing in opposition to Greek ones. Among the most significant scientific achievements of the Indians, he for instance laid stress on grammar.

[Whereas] with the Greeks, [*Hankel contended,*] grammar began with aphorisms of philosophy of language and proceeded with a sharply logical and well polished syntax; on the contrary, the Indians have devoted their work almost exclusively to the formative etymological side of language and by incessant toil and a surprising talent for observation, they have empirically established their laws—with a success shown by that judgment pronounced about Pāṇini’s grammar written a few centuries BCE: “*It is a grammar so complete that there is no match for it, with any other language of the world, apart from Sanskrit. The task of a true scientific grammar, which is to deal with all linguistic forms from the grammatical point of view and to represent them, is at least attempted without exception, and it succeeds, if not down to the detail, yet in the whole.*” [Theodor Benfey, *Geschichte der Sprachwissenschaft und Orientalischen Philologie in Deutschland* (1869), cf. [Benfey 1869, vol. I, p. 77]]<sup>319</sup>

In attempting to grasp what he held to be the “national disposition [*Nationalanlage*] characteristic of the Aryan-indian people”<sup>320</sup>, Hankel understood it, as seen above, along non essentialist lines, as being inseparable

<sup>317</sup> [Hankel 1874, p. 2].

<sup>318</sup> [Hankel 1874, p. 3].

<sup>319</sup> [Hankel 1874, pp. 173–174].

<sup>320</sup> [Hankel 1874, p. 183].

from a broad spectrum of cultural practices ranging from a socially ritualized predilection for riddles in numbers to the invention of various positional numeral systems. “A particular disposition for building formal auxiliary means in word and writing”<sup>321</sup> would thus be effective in different sciences, but presumably first and foremost in grammar.

[*In this connection*], I mention a procedure by which the great grammarian Pāṇini made it possible to achieve an almost unbelievable conciseness in his rich work and which can be seen out of an example [*which, as it stands, is directly taken from Theodor Benfey, see [Benfey 1869, vol. I, pp. 75–76]*] The rule is: “any affix has the acute accent on its first syllable”, for instance the first person of the plural *dvishmás* (we hate) with the affix *mas*. But now there are exceptions, for instance *mi* (affix of the first person singular) has not the accent, therefore *dvéshmi*. Pāṇini expresses this in such a way that he calls this affix not *mi*, but *mīp*, and here as in other similar exceptions appends the *p* as precisely the sign of the exception. Philologists tend to call this procedure an algebraic one and they are not wrong in doing so. This sense for an abbreviated and concise formalism which makes itself known in algebra even arises herein.<sup>322</sup>

While the scarcity of the available sources, as regards mathematics proper, would presumably debar him from writing a comprehensive history of Indian mathematics, Hankel opted for synchronic transverse sections as the only fruitful approach. Accordingly, he distinguished four main stages associated with the works of Āryabhāṭa, Brahmagupta, Śrīdhara and Padmanābha, and eventually Bhāskara, about whom he gathered biographical material mostly from Bhāu Dājī’s 1865 paper<sup>323</sup>. Apart from a few passages from the *Āryabhaṭīya* which Kern translated for him as he did not read Sanskrit, Hankel only studied two astronomical treatises, the *Sūrya-siddhānta*, in both translations by Bāpū Deva Śāstrin<sup>324</sup> and Ebenezer Burgess,<sup>325</sup> and Bhāskara’s *Siddhānta-śiromāṇi* in Lancelot Wilkinson’s translation<sup>326</sup> on the one hand, and the mathematical chapters of the *Brāhma-sphuṭa-siddhānta* of Brahmagupta, together with the *Līlāvatī* and the *Bīja-ganīta* of Bhāskara II, both in Colebrooke’s translation. Although he thus mostly relied on Colebrooke, Hankel also referred to Brockhaus’ partial German translation of *Bīja-ganīta*, and like him, he emphasized the effects induced by versification on the way mathematics

<sup>321</sup> *Ibid.*

<sup>322</sup> [Hankel 1874, pp. 183–184]. Hankel does not say who those philologists are. However, Benfey speaks of “algebraic signs” used to spare the grammarian a wealth of exceptional rules.

<sup>323</sup> Cf. Bhāu Dājī [1865].

<sup>324</sup> Cf. Śāstrin [1861].

<sup>325</sup> Cf. Burgess [1860].

<sup>326</sup> Cf. Wilkinson [1861].

was conveyed in Sanskrit sources, as well as the importance of commentaries for getting hold of the underlying mathematical procedures.

Like all scientific works in Sanskrit [*Hankel indeed pointed out*], the astronomical and mathematical ones are also written in verses of the most diverse, often very elaborate meters, to which one appended remarks and examples in prose. But all the rules are given with utmost brevity in almost oracular verses which, without the examples, often resist their being unriddled, but, after one has understood them, may be admittedly suited to be fixed in memory and to be easily applied. Their intelligible translation into a modern language is, as Colebrooke's work bears witness, often impossible; Latin is more suitable for this because through its flexions, etc., it allows a more precise relation between words. An example may suffice to lay before the reader's eyes the difficulty of understanding.

It is about the solution to the equation

$$x \pm b\sqrt{x} = c$$

which must be given by the formula [*viz. Lilavati III, section V, § 62–63, see [Colebrooke 1817, p. 29]*]

$$x = \left( \sqrt{\left(\frac{b}{2}\right)^2 + c} \pm \frac{b}{2} \right)^2.$$

Bhāskara describes this process in words in the following way [N.B. *Hankel significantly shifts here from Colebrooke's English translation*<sup>327</sup> *to Friedrich Rosen's Latin presumably better rendering as proposed in his Algebra of Muhammad Ben Musa (1831) ([Rosen 1831, p. 189])*]:

Per multiplicatam radicem diminutae vel auctae quantitatis  
Manifestae, additae ad dimidiati multiplicatoris quadratum  
Radix, dimidiato multiplicatore addito vel subtracto  
In quadratum ducta—est interrogantis desiderata quantitas.

Although it was possible to lie down by force the rules themselves upon this bed of Procrustes, by which they may have lost much in completeness and sharpness, still it is clear that a methodical development of the propositions and their logical proof could never be brought into this form. But precisely such developments and such ways to carry out the proof are not much to be found among the Indians. Only here and there a commentator adds some remarks to the rules and propositions which can provide a way to derive them.<sup>328</sup>

<sup>327</sup> Cf. [Colebrooke 1817, p. 29]: "The sum or difference of a quantity and of a multiple of its square-root being given, the square of half the coefficient is added to the given number; and the square root of their sum [is extracted: that root] with half the coefficient added or subtracted, being squared, is the quantity sought by the interrogator." Let us simply remark that Colebrooke is constrained to add something between square brackets so as to render the relations which Sanskrit (or Latin) can convey through flexions without breaking the continuous thread of the procedure.

<sup>328</sup> [Hankel 1874, p. 182].

However, those external constraints imposed by Sanskrit versification should not obliterate the strengths of Indian mathematics which Hankel kept highlighting in spite of its otherwise alleged weaknesses, nor should they suffice to account for the contrasting features of Indian and Greek mathematics. Both should be comprehended more thoroughly, in Hankel's view, as rooted in different cultural contexts.

Just as well as it is no accident that the Euclidean method happens to be the method of the Greek mathematicians, [Hankel argued,] [the] intuitive method with the Brahmins also had a more general significance beyond geometry. Their metaphysics, cosmology and theology did not spring as the philosophy of the Greeks from a reflective activity, which decomposed the given representations, formed concepts and sought by means of their logical systematic connection to get to the knowledge of the truth; their method is rather the method of immediate intuition [*die der unmittelbaren Intuition*], of devoting oneself sustainedly to deepening one thought, of absorbing oneself mystically in the highest ideas, in which the mind, forgetting itself, means to behold in one image [*in einem Bilde*], in their essential connection, the thoughts spreading out of this center. Perhaps, so as to show that this geometrical method of the Indians had been tied with invisible threads with their overall disposition [*Gesamtanlage*], I may point out that the philosopher of Germany who felt himself so strongly attracted to the metaphysics of the ancient Brahmins, that Schopenhauer has been one of the first who fought against the Euclidean method, and without knowing about Indian geometry, recommended an intuitive development essentially coinciding with it.<sup>329</sup>

This passage which was later hailed as “golden words”<sup>330</sup> by the Sanskritist Albert Bürk who edited and translated Āpastamba's *Śulba-sūtra*, sheds light on the way Hankel connected his interest in Sanskrit sources with methodological issues that were of increasing concern in Germany at the time. In order to strengthen his point, Hankel aptly made a persuasive case of the fact that both features of Schopenhauer's philosophy, though disconnected in the latter's view, should in fact be thought of in conjunction. On the one hand, the Frankfurt hermit indeed highly praised Indian philosophy which he considered the highest human wisdom, while on the other hand he characterized the Euclidean method in geometry as “a brilliant piece of wrongheadedness”.<sup>331</sup> However, he himself failed to grasp the connection between his two recurrent themes, leaving it to others to later take up these loose ends and tie them together. To all appearances,

<sup>329</sup> [Hankel 1874, p. 220].

<sup>330</sup> [Bürk 1901–1902, p. 559]. Bürk inserts Hankel's whole quote in his commentary on Āpastamba's *Śulba-sūtra* as a guideline for his own reading of those Sanskrit sources.

<sup>331</sup> [Schopenhauer 2010, p. 96].

in drawing on Schopenhauer's fame<sup>332</sup>, which had grown overwhelming at the end of the 1860s in Germany, Hankel intended to perform some kind of a *captatio benevolentiae*.<sup>333</sup> Unlike the Schlegel brothers or Wilhelm von Humboldt, Schopenhauer never mastered Sanskrit, hence he became acquainted with Indian tradition through reading the *Upaniṣads* in Anquetil du Perron's Latin translation from a Persian version, which, together with the knowledge of Buddhism he later acquired, definitively shaped his Indian views.<sup>334</sup> On the whole, his interest in Indian sources remained focused on religion and philosophy, and would be kindled only insofar as these fitted with his own metaphysical views. Accordingly, there is no hint that Schopenhauer ever read Colebrooke's, nor any other's, translation of Indian mathematical works, so that his sharp criticism of the Euclidean method in geometry came up as completely free of Indian associations. In referring to Schopenhauer's charge against Euclid, Hankel most probably had in mind the *locus classicus* for it, the well-known § 15 of the philosopher's main opus, *The World as Will and Representation*, which went through three successive editions in 1818, 1844, and 1859.

<sup>332</sup> For a sociological account of Schopenhauer's late fame, see [Collins 2002, pp. 636–638]. After his failed attempt to establish himself as a *Privatdozent* in Berlin in 1820, when his lectures were almost unattended while Hegel's next-door ones attracted more than a hundred listeners, Schopenhauer (1788–1860) retired to Frankfurt to live out his days away from the academic world. He only regained credit in his old age when Hegelianism and *Naturphilosophie* no more monopolized public attention. The breakthrough occurred in the 1850s and increased beyond all expectations to his own amazement. On May 4, 1857, in a conversation with Friedrich Hebbel, he wryly described what he called the "comedy of [his] fame": "I feel strange with my present fame. No doubt you will have seen how, before a performance, as the house-lights are extinguished and the curtain rises, a solitary lamplighter is still busy with the footlights and then hurriedly scampers off into the wings—just as the curtain goes up. This is how I feel: a latecomer, a leftover, just as the comedy of my fame is beginning." Cf. [Safranski 1989, p. 347] for the quote.

<sup>333</sup> See also Hankel's slightly ironical note in [Hankel 1874, p. 173]: "The brilliant intuitions of Indian philosophy, although often fantastic, but always deeply thought and grounded in varied ways, are too well-known to need further comment; after all, the most modern German philosopher has believed he had found in these speculations on the Nirvana, that in which all existence and nothingness is universally grounded, the true and lofty solution to all the enigmas of human life."

<sup>334</sup> See [Halbfass 1990, p. 106]: "Throughout his life, he clung to the belief that [Anquetil's Duperron's translation] was a definitive achievement and the key to a philosophical understanding of the *Upaniṣads*. [...] He greeted the subsequent direct translations by H. Th. Colebrooke and H. E. Röer, and especially those of Rammohan Roy, with suspicion and dismissal. He found theistic and Europeanizing corruptions in these works, and was not ready to accept them as a basis for a reexamination or revising of his own opinion of the *Upaniṣads*."

[W]e cannot help finding that [*mathematics as Euclid has organized it*] has taken a peculiar route, indeed that it is going the wrong way. We expect every logical ground to be reduced to an intuitive one; but mathematics had devoted itself wholeheartedly to casting its own easily accessible intuitive evidentness wilfully aside and replacing it with logical evidentness. We ought to treat this as akin to someone cutting off his legs so that he can go on crutches, or to the prince in *The Triumph of Sensibility* [viz. *Goethe's drama*] who flees from the actual beauty of nature to enjoy a theatre set that imitates nature.<sup>335</sup>

In his dissertation *On the Fourthfold Root of the Principle of Sufficient Reason* (1813, 1847), Schopenhauer distinguished the merely cognitive ground of a mathematical proof, the so-called *ratio cognoscendi*, from its ground of being, its *ratio essendi*. Whereas the former which is being provided through logical deduction may indeed yield knowledge *that* something is as it is, only the latter presumably gives insight into *why* it is so through direct intuitive grasp. In referring to Aristotle's well-known passage from *Posterior Analytics* I, 27, so as to recall that a science proves to be all the more perfect as it brings out simultaneously what something is together with why it is so—which mathematics presumably instantiates first and foremost—Schopenhauer laid stress on the Euclidean method constantly striving to separate these two sides completely.

[*With the Euclidean proof, Schopenhauer thus contended*], we are in the same position as someone who is shown the various effects of an ingenious machine, whose inner workings and mechanism are withheld. The principle of non-contradiction compels us to admit that everything Euclid demonstrates is true; but we do not find out *why* it is so. We have almost the same uncomfortable sensation people feel after a conjuring trick, and in fact most of Euclid's proofs are strikingly similar to tricks. The truth always emerges through a back door, the accidental result of some peripheral fact.<sup>336</sup>

Accordingly, Schopenhauer focused on Pythagoras' theorem as best epitomizing his point.

As in Pythagoras' theorem, [*he argued*,] lines are often drawn without any indication of why: later they show themselves to be traps that spring unexpectedly to capture the assent of students, who must admit in astonishment what remains completely incomprehensible in its inner workings, so much so that they can devote themselves to a thorough study of Euclid in its entirety without obtaining

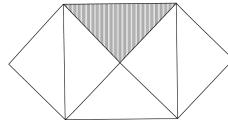
<sup>335</sup> [Schopenhauer 2010, p. 95]. One finds a wealth of such similar suggestive comparisons whose disillusioning strength no doubt partly lay in their aphoristic sharpness, as for instance: Euclid's logical way to deal with mathematics "is like a night traveler who, mistaking a clear and solid path for water, takes care not to tread on it and instead walks along the bumpy ground beside it, happy all the while to keep to the edge of the supposed water." ([Schopenhauer 2010, p. 97]); or "[it] is like wanting to award feudal rights over an estate to its immediate lord" ([Schopenhauer 2010, p. 100]).

<sup>336</sup> [Schopenhauer 2010, pp. 95–96].

any insight into the laws governing spatial relations—instead of the laws themselves, they only learn by rote a few of their consequences.<sup>337</sup>

More specifically, Schopenhauer's dismissal of the Euclidean proof in favor of a diagram-based one presumably forged for the sake of the argument, obviously suggested, at least for someone acquainted with Sanskrit mathematical literature as Hankel was, that what came as separate threads in Schopenhauer could be consistently tied together in a unified contrastive account, *viz.* Indian intuitive insights versus Euclid's "mousetrap" proofs.

Pythagoras' theorem tells us about an occult quality of the right-angled triangle [*Schopenhauer continued*]: Euclid's stilted, indeed underhand, proof leaves us without an explanation of the why, while the following simple and well-known figure yields more insight into the matter in one glance than the proof, and also gives us a strong inner conviction of the necessity of this property and of its dependence on the right angle:



even when the sides at the right angle are unequal we must still be able to achieve this intuitive conviction [*anschauliche Überzeugung*], as we can generally with every possible geometrical truth.<sup>338</sup>

All geometrical truths presumably proceed from such intuitions and their proof is only made up afterwards, so that the analysis of the thought processes that first led to their discovery should prevail over logical reconstruction. As Schopenhauer's figure conspicuously shows, the whole intuitive proof hinges on foldings and symmetries, which will also prove essential in Hankel's interpretation of Indian geometrical proofs. One can indeed see how in folding the small squares along their symmetry axes, one covers up the big square. However, the Frankfurt philosopher did not rest content with idle criticism, he was also interested in implementing new ways to teach geometry in the German school system.

Here and there in Germany, [*he flattered himself in the 1859 edition of his book,*] a start is already being made in changing the way mathematics is taught, and this analytical method is being increasingly adopted. The most decisive step in this direction has been taken by Herr Kosack a teacher of physics and mathematics at the Nordhausen Gymnasium, who had added a thoroughgoing attempt to treat

<sup>337</sup> [Schopenhauer 2010, p. 96].

<sup>338</sup> [Schopenhauer 2010, pp. 98–99].

geometry according to my principles to the schedule for the school examination on the 6th of April 1852.<sup>339</sup>

Only a few days after Carl Rudolf Kosack (1823–1869) sent him his *Beiträge zur einer systematischen Entwicklung der Geometrie aus der Anschauung* (1852), Schopenhauer replied on May 2, 1852: “In sending me your essay, you have greatly rejoiced me, and I have thoroughly studied it from the beginning to the end with the greatest interest. What a satisfaction should it not give me to see the principles I had made public since 1813 already, now at last recognized by a mathematician and carried out, and in a rational as well as an original way! Would you like to continue your presentation through the whole of elementary geometry, even though the difficulties increase at each step, you could earn yourself a name with this.”<sup>340</sup> Schopenhauer then spread the word among his most fervent supporters, his “apostles and evangelists”, as he used to call them, so as to draw their attention to “a very successful sample”<sup>341</sup> which he praised as a harbinger of the long awaited reception of his ideas underway. To Julius Frauenstädt, he wrote on June 10, 1852: “There it is explained how one had attempted for a long time already to change the Euclidean method, until eventually the matter was settled irrefutably by me forever; then follow the key passages from the chapter on the ground of being [*viz. the ratio essendi*] in the Fourthfold Root, word for word as I wrote them down in 1813 [...], and then a detailed sample of a presentation of geometry in my sense, in the whole 30 pages in-4° and copperplate engravings. You must absolutely read that!”<sup>342</sup> Schopenhauer’s first circle of admirers would then take care to circulate Kosack’s program as a major contribution to their cause. Frauenstädt not only reviewed it in print, but even strove to emulate Kosack in devising simpler intuitive proofs along the same lines, hence prompting Schopenhauer’s conciliatory words: “Your

<sup>339</sup> [Schopenhauer 2010, p. 99]. In the 1818 edition, Schopenhauer had greeted, although much less enthusiastically, the attempts made by two German professors of mathematics to depart from the Euclidean method, namely Bernhard Friedrich Thibaut (1775–1832) in Göttingen, who “performed a great service in his *Grundriss der reinen Mathematik* [1801], although I would like a much more decisive and thorough substitution of the evidentness of intuition for logical proof.”, and Ferdinand Schweins (1780–1856) in Heidelberg, one of the last representatives of the German Combinatorial School, who, in his *Mathematik für den ersten wissenschaftlichen Unterricht* (1810), “also had declared himself against the Euclidean treatment of mathematics and attempted [although quite insufficiently in Schopenhauer’s view,] to move away from it.” In the last edition, these remarks were replaced by the passage on Kosack.

<sup>340</sup> [Schemann 1893, p. 335].

<sup>341</sup> Schopenhauer to Adam von Doss, May 10, 1852, cf. [Frauenstädt 1863, p. 240]

<sup>342</sup> [Frauenstädt 1863, pp. 538–539].

mathematical proofs are good and correct: but Kosack's are not to be scorned"<sup>343</sup> Rotations for instance would be put to good use in Kosack's approach. He would not only consider that any rotation of a straight line about one of its points describes equal angles at that point, but he would also prove intuitively that the sum of the angles of a triangle is equal to two right angles by aptly combining rotations.<sup>344</sup> Let three distinct straight

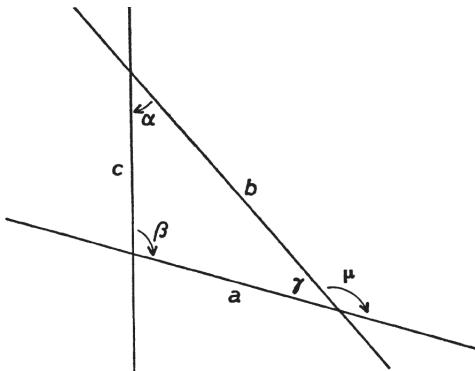


FIGURE 6. Kosack's proof

lines  $a$ ,  $b$ ,  $c$  be given (cf. Fig. 6) Rotating the straight line  $b$  about its intersection with  $c$  so as to describe the angle  $\alpha$ , and after coincidence with  $c$ , rotating it again about its intersection with  $a$  until it coincides with  $a$ , thereby describing the angle  $\beta$ , amounts on the whole to rotating  $b$  from its initial position to its final position  $a$ , through the angle  $\mu$  in the same sense. Therefore the exterior angle  $\mu = \alpha + \beta$ , and since  $\mu + \gamma = \pi$ , then  $\alpha + \beta + \gamma = \pi$ , which can presumably be read off the figure through direct intuition.

In referring to Schopenhauer's argument, Hankel's point was in no way to join in his crusade against Euclidean geometry, but rather to suggest

<sup>343</sup> [Frauenstädt 1863, p. 545].

<sup>344</sup> Kosack's proof is commented on from a different perspective and with a different agenda by the Austrian physicist Ernst Mach in his *Erkenntnis und Irrtum* (1905), see [Mach 1905, pp. 393–394]: "The first geometry [as presumably instantiated in Kosack's sample,] was naturally not based on purely metric concepts, but made significant concessions to the physiological factor of intuition. [...] Doubts as to whether successive rotations about several points is really equivalent to a rotation about one point, [...] which are immediately justified as soon as a surface of curvature differing from zero is substituted for the Euclidean plane, could naturally not dawn at that stage on the happy naive discoverer of these relationships."

close relationships between Sanskrit mathematics and Sanskrit culture as a whole. Spurred by Colebrooke's 'See'-trope, he was interested in figuring out how to account for 'proofs' in Sanskrit sources. As previously seen, Chasles and Woepcke had already pondered Bhāskara II's diagrams supposedly 'proving' the Pythagorean theorem. Hankel investigated further along these lines in analyzing more in depth the interplay, presumably attested in Sanskrit mathematics, between diagrammatic processes and algebraic identities governing them. Rather than with the Pythagorean theorem, his main concern at the time was with a more difficult topic, Brahmagupta's rules on quadrilaterals, as presented in *Brāhma-sphuṭasiddhānta* XII. 21–38. This intriguing set of rules had aroused a great interest in German mathematical circles ever since the 1840s, when Colebrooke's translations were helpfully completed by Chasles' *Mémoire sur la géométrie des Hindous* (1836), shortly afterwards inserted as note XII in the widely known *Aperçu sur l'origine et le développement des méthodes en géométrie* (1837).<sup>345</sup> At Alexander v. Humboldt's request, Johann Peter Gustav Lejeune Dirichlet (1805–1859), for instance, wrote a memorandum<sup>346</sup> on Chasles' essay and, in his correspondence with Humboldt of April 1847, he discussed the issue of the purported specificity of Indian mathematics with regard to Greek mathematics. At about the same time, Ernst Eduard Kummer (1810–1893) got so smitten with Brahmagupta's rules on quadrilaterals that he submitted a paper to Crelle's Journal in which he reelaborated the whole matter in a different way than Chasles' presumably misguided approach.<sup>347</sup> It will be shown in a forthcoming contribution that Hankel's innovative reading of Brahmagupta's rules incorporated Kummer's criticism of Chasles' account, although with a significant twist of his own aiming at making a suitable interpretation of these rules the cornerstone of a fine-grained philologically informed characterization of Sanskrit mathematics. In so doing, he consistently combined the flair of a working mathematician with the rigorous scholarship gathered from Sanskrit philology. Elaborating on the sources that were available in Colebrooke's translations, Hankel delineated two fundamental principles presumably ruling Sanskrit geometry, which he called the principles of

<sup>345</sup> Cf. [Chasles 1837, pp. 416–456]. A German translation of Chasles' *Aperçu* by Ludwig Adolph Sohncke (1807–1853), professor of mathematics at Halle, was issued in 1839 under the title *Geschichte der Geometrie, hauptsächlich mit Bezug auf die neueren Methoden*, cf. Chasles [1839]. German mathematicians mostly knew about Chasles through Sohncke's translation, Kummer and Hankel extensively quoted from it.

<sup>346</sup> See [Lejeune Dirichlet 1897, pp. 345–346].

<sup>347</sup> Cf. Kummer [1848].

congruence and similarity, respectively. While the first principle which “simply consists in the intuition that equal constructions lead to the same figure”<sup>348</sup> first and foremost applies in the many symmetries which Sanskrit geometrical ‘proofs’ exploit, the second principle may be characterized as a spatialized rule of three to be read off the figure. So as to depict this last principle in action, Hankel, as will be shown in detail elsewhere, perceptively drew on the commentary on Brahmagupta’s rules on quadrilaterals by the ninth-century commentator Pr̥thūdakasvāmin, as partially quoted by Colebrooke in the footnotes appended to his translation. Leaving aside technicalities for another occasion, let us merely emphasize here that Hankel’s account amounted to contrasting Greek and Sanskrit mathematics with the aim of taking the best of both in the context of the above mentioned nineteenth-century German debates on mathematical pedagogy.

If, in uniting the proper merits of the Greek and the Indian methods, one would sharply define these general principles [*viz. the so-called “principle of congruence” and “principle of similarity”*] and place them in front of geometry, then there is no doubt that, following the direction of the Indians, the conglomerate which the Elements of geometry consist in with Euclid, could be reworked into a system in which essential ideas, not accidental ones, lead the way forward. The few propositions of this kind which I derived in this way [*viz. the interpretation of Brahmagupta’s rules on quadrilaterals*], may serve as an example of what may be achieved in this connection. If, as one has already begun here and there, such a system were established as the basis of education, the pupil would derive from his geometrical instruction the real benefits he should derive from it, whereas now he anxiously gathers the trivial propositions on congruence and similarity, but rarely achieves any free geometric intuition.<sup>349</sup>

The Grassmannian resonances of the above passage should not pass unnoticed. In his *Ausdehnungslehre*, Hermann Grassmann (1809–1877) had indeed called for a form of presentation of mathematics aptly combining rigor and overview<sup>350</sup>. “The scientific presentation in essence is an

<sup>348</sup> [Hankel 1874, p. 206].

<sup>349</sup> [Hankel 1874, p. 208].

<sup>350</sup> Cf. [Lewis 1977, pp. 130–133] for a translation and an analysis of §§ 13–16 of Grassmann’s 1844 *Ausdehnungslehre*; see for instance [Grassmann 1844, § 14, p. 30], where Grassmann elaborates in his own way on a theme akin to Schopenhauer’s criticism of Euclidean “mousetrap” proofs: “We add scientific quality to a method of treatment when the reader is, on the one hand, led by it necessarily to the recognition of each individual truth, and, on the other, is put in the position at each point of the development of seeing the direction of further progress. The indispensability of the first requirement, namely scientific rigor, anyone will grant. As for the second, that is another matter, not yet properly recognized by most mathematicians. Proofs often occur, in which at first, if it were not for the statement [of the theorem] standing above

interlocking of two series of developments, [*Grassmann claimed*,] of which one consistently leads from one truth to another and makes up the essential content, while the other governs the process itself and determines the form. In mathematics, both these series flow from one another in the most rigorous way. It has been the practice in mathematics for a long time, and Euclid himself set the precedent, to allow only that one series of development to predominate which forms the essential content; as for the other, it was left for the reader to make it out between the lines.”<sup>351</sup> Although Grassmann himself was an eminent Sanskrit scholar<sup>352</sup>, more famous in his lifetime for his expertise in Sanskrit linguistics and Vedic studies than for his mathematical work, one finds no trace in his writings of a connection between his methodological claim regarding mathematics and his interest in Sanskrit sources and Indian culture. He began learning Sanskrit in 1849<sup>353</sup>, apparently as a distraction from mathematics after being repeatedly turned down for a professorship in Germany, and carefully studied Bopp’s Sanskrit grammar as well as his Comparative grammar of Indo-Germanic languages.<sup>354</sup> He only started the translation in German of the *Rgveda*<sup>355</sup> in the early 1860s and prepared, at first for his own use, a

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[the proof], one would not know at all to where it is supposed to lead. Consequently after one has blindly and haphazardly followed each step for quite some time, finally, before you expect it, that truth which was to be proven is suddenly attained. Such a proof can perhaps leave nothing to be desired in rigor, but it is not scientific; the second requirement is lacking, the provision of an overview [*Übersichtlichkeit*].”

<sup>351</sup> [Grassmann 1844, § 16, p. 31]. We depart from Lewis’ translation in only one respect, when it comes to both series of developments “flowing from one another [*treten dies … aus einander*] in the most rigorous way”.

<sup>352</sup> Grassmann is presented as an insider in all internal histories of Sanskrit-philology, see for instance [Windisch 1917–1920, II, pp. 365–366], [Stache-Rosen 1981, pp. 38–39].

<sup>353</sup> See [Engel 1911, p. 155].

<sup>354</sup> On Grassmann’s linguistic studies, see [Engel 1911, pp. 244–246; pp. 298–302], [Petsche et al. 2009, pp. 343–352].

<sup>355</sup> Cf. Grassmann [1873–1875b]

reasoned glossary of the terms occurring in the Vedic hymn,<sup>356</sup> which gradually grew over the years into the monumental *Wörterbuch zum Rig-Veda*,<sup>357</sup> completed in 1872, but published from 1873 to 1875, while the translation itself was eventually issued in two parts in 1876–1877. When Hankel first came into contact with Grassmann at the end of 1866, he was interested in the generalization of the concept of a complex number which he found in the *Ausdehnungslehre*, and the few letters they exchanged between November 1866 and September 1867 exclusively revolved around mathematical topics pertaining to the *Ausdehnungslehre*.<sup>358</sup> However, unlike most of Grassmann's contemporaries, the young Hankel was particularly responsive to his elder's central insight with regard to mathematics. As Friedrich Engel, his editor and biographer, would later point out, ever since his early mathematical works, Grassmann "wanted to avoid the co-ordinates which have nothing to do with the matter," and "instead of

<sup>356</sup> In his *History of Indian Literature*, Moriz Winternitz (1863–1937), an Austrian Orientalist at the German University in Prague, gives some elements of context which are helpful to grasp the meaning of Grassmann's enterprise. Because of their great age, the Vedic hymns had become partly unintelligible, already in very early times, to the Indian scholars themselves. So as to interpret the *Rgveda*, these had to prepare glossaries (*nighantu*), collections of rare and obscure words occurring in the hymns. The earliest extant commentary of this kind, based on a glossary, is by Yāska, "who doubtless is older than Pāṇini", cf. [Winternitz 1927, p. 69]. However, the main comprehensive Indian commentary, which explains the *Rgveda* word by word, dates back to the fourteenth century and is due to Sāyaṇa. Nineteenth-century debates raged on the question whether one should use these ancient traditions. Prior to Grassmann, the English scholar Horace Hayman Wilson (1786–1860) had published a complete translation of the *Rgveda*, which entirely depended upon Sāyaṇa's commentary. But other Vedic scholars took another stance, among them the Sanskritist Rudolf Roth and his follower Hermann Grassmann. "They denied, [Winternitz explains,] that a commentator, who lived more than two thousands years after the composition of the book explained by him, could know anything which we Europeans, with our philological criticism and with the modern resources of linguistic science, could not fathom and understand better." Cf. [Winternitz 1927, p. 71].

<sup>357</sup> Cf. Grassmann [1873–1875a]. On Grassmann's lexicographic studies on the *Rgveda*, see [Engel 1911, pp. 302–310]. See also Maria Kozianska's contribution to the proceedings of the Graßmann's Bicentennial Conference held in September 2009, cf. [Petsche et al. 2009, pp. 353–361]. Vedic language shows a wealth of differentiated forms. Nouns and adjectives admit of eight cases and, as regards number, there is the dual in addition to the singular and the plural. Verbs are also characterized by a great many forms. One of Grassmann's main goals was to account for this variety of forms. In the case of verbs, for instance, he classified verbal forms according to the tense stems: the present, the perfect and the aorist, which resulted in a tree-like systematic organization in which the flectional forms of a verb are rooted in the individual stems. Besides, Grassmann was driven by the urge to do justice to the many compositional types of Vedic, for which he tried to find German equivalent ones.

<sup>358</sup> On this correspondence, see [Engel 1911, chap. 29, pp. 269–278].

these wanted to compute with the geometrical objects themselves".<sup>359</sup> His new theory of extensive magnitudes hence provided the means to grasp the relationships between both kinds of operations, on magnitudes and on numbers, more transparently and more thoroughly than analytic geometry could ever afford. Being himself attuned to Grassmann's approach, Hankel expressed his feeling of "daunting respect for [his] mathematical-philosophical mind"<sup>360</sup> While admitting that Grassmann's readers may have been "disheartened by the philosophical clothing of his presentation", he nevertheless considered it "completely suited to the topic."<sup>361</sup> Indeed, in one of those striking passages of philosophical tenor, Grassmann very clearly articulated his point. "With the usual method [*viz. analytic geometry, he argued*], the idea was completely obscured by the introduction of arbitrary coordinates which have nothing to do with the matter, and the computation consisted in a mechanical development of formulas which present the mind with nothing and therefore kills it. Here however, [*viz. in the Ausdehnungslehre*,] where the idea, no more obfuscated by anything foreign, shone in complete clarity through the formulas, the mind was also engaged in the process of developing the idea with each and every development of the formulas."<sup>362</sup> Hankel most probably had similar opinions about the shaping of ongoing mathematics, in the back of his mind, when, a few years later, he addressed Sanskrit sources, as a historian of mathematics interested in questioning the relationship between algebra and geometry in ancient traditions. Although he did not derive his primary incentive for studying Sanskrit mathematics from his direct connection with Grassmann, part of his motivation for doing so may have stemmed from concerns he shared with his elder.

## 8. A PARTING OF THE WAYS

At the turn of the century, German Sanskrit philologists significantly departed from their fellow mathematicians in showing how Schopenhauer's insights on the nature of geometric proof could be put to good use for the interpretation of those newly exploited sources known as the *Śulba-sūtras*. Far from dismissing the philosopher's views, they held on the contrary that these might provide useful guidelines for the philologist. With hindsight,

<sup>359</sup> Cf. [Engel 1909, p. 350].

<sup>360</sup> Cf. [Engel 1911, pp. 273–274].

<sup>361</sup> Cf. [Hankel 1867, p. 16].

<sup>362</sup> Cf. [Grassmann 1844, Foreword to the first edition, p. 9].

the mutual estrangement of both communities, around 1900, as regards Schopenhauer's legacy, namely philologists on the one side and mathematicians on the other side, in return highlights the specificity of Hankel's position at the junction of both worlds, when, three decades earlier, he had sought to articulate some of the prevailing mathematical and philological concerns of his time.

As mentioned above, the *Śulba-sūtra* of Āpastamba was edited in European transliteration, translated and commented in 1901–1902 under the supervision of the Tübingen Sanskrit scholar Richard Garbe (1857–1927), who suggested to one of his students, Albert Bürk, to engage in this work, for which he would offer him guidance and advice. Garbe who had been Hermann Grassmann's pupil at the Stettin Gymnasium and who, as he himself later acknowledged<sup>363</sup>, embraced the career of an Indologist owing to the decisive influence of the great mathematician,<sup>364</sup> studied in Tübingen under the Vedic scholar Rudolf Roth (1821–1895) whose chair he later took over in 1895. Over the years he became an authority on Indian philosophy which he had studied under Indian pandits during his 1885 journey in India undertaken with the financial support of the Prussian *Kulturministerium*. Owing to his connections with the English Sanskrit scholar Arthur Coke Burnell (1840–1882), Garbe had access to

<sup>363</sup> Richard Garbe to Friedrich Engel, July 14, 1909, in [Engel 1911, p. 310]: "It has certainly come to your knowledge that I was a pupil of Grassmann's. But you will not know what decisive influence Grassmann's personality exerted on my whole life. Without him, I would never have become an Indologist, nor even eventually professor of Indology at Tübingen. Grassmann was my teacher in religion for two years, and in mathematics for three. Shortly before my end examination, Grassmann had to give a replacement class and took advantage of this occasion to explain a verse of the Rigveda on the basis of our knowledge of Greek and Latin. This hour exerted such a strong attraction on me that I went to him afterwards and asked him: 'Herr Professor, where can I best learn this?' To this, Grassmann replied: 'In Tübingen, with professor Roth. I will give you a letter for him.' This little event decided my fate." Friedrich Engel received this letter from Garbe as he was preparing a biographical volume in the context of the edition of Grassmann's complete works he conducted from 1894 to 1911.

<sup>364</sup> Valentina Stache-Rosen nevertheless indicates that "Garbe went to Tübingen to study mathematics, [*and that*] it was R. v. Roth who aroused his interest in Indology", see [Stache-Rosen 1981, p. 141]. In the light of Garbe's above mentioned letter to Engel, it seems likely that Garbe moved to Tübingen to learn Sanskrit with Roth, but still had the intention to major in mathematics, before making the decision to shift to Indology. In that case, Grassmann may have had more than one good reason to suggest Tübingen, namely, in addition to the scholarship of Rudolf Roth, the teachings of Hermann Hankel, whom he was indebted to for greatly contributing to the reception of his mathematical work. If the young Garbe already attended Tübingen University in the summer semester 1873, then he may have studied mathematics under Hermann Hankel (who died on August that year).

a number of manuscripts relating to the *Sūtra* of Āpastamba,<sup>365</sup> as for instance the *Śrauta-sūtra* belonging to the Black *Yajur Veda Saṃhitā*, which he edited in the *Bibliotheca Indica* from 1882 to 1903. Since Burnett had pointed out as early as 1880 the interest of the *Śulba-sūtra* with respect to the beginnings of Indian geometry,<sup>366</sup> Garbe in return induced his student to launch into editing these sources. Bürk thus worked on various manuscripts, some of which were handed out to him directly by Garbe, or were sent to him by Thibaut.<sup>367</sup> The resulting edition and translation of

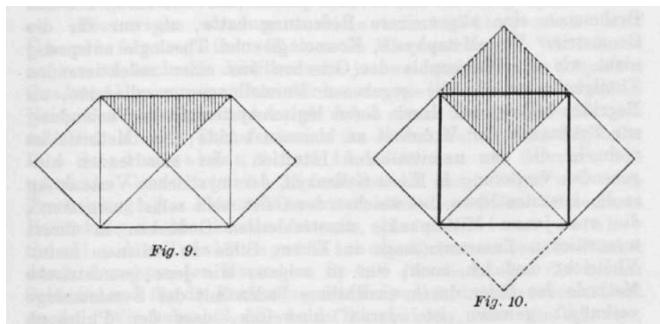


FIGURE 7. Albert Bürk's commentary on Āpastamba's *Śulba-sūtra*, cf. [Bürk 1901–1902, p. 559]

Āpastamba's *Śulba-sūtra* was preceded by a highly perceptive commentary which drew on Schopenhauer and Hankel, while striving to make clear those mathematical justifications which, although presumably borne out

<sup>365</sup> See [Garbe 1885, p. 3]: “In the autumn 1882, [Garbe explained,] some weeks before his deplorable death, Dr. A. Burnett placed in my hands a number of manuscripts relating to the Āpastamba Sūtra which he requested me to make over to the Imperial Library of Strassburg, after I had done with them.”

<sup>366</sup> See [Burnell 1880, p. 17]: “Of the Āpastamba sūtras but little has been published. [...] Perhaps the most interesting section of the whole is the Āulva chapter, which treats of the construction of altars; this involves [...] considerable geometrical knowledge, and must throw much light on the beginnings of Indian geometry. Dr Thibaut has recently taken up this subject, and it is to be hoped that he will be able to bring out an edition. The Āpastamba and Baudhāyana sūtras differ much in this respect, and a comparison of the two is requisite [...]”

<sup>367</sup> For his edition, Bürk consulted three manuscripts for the main text of the *Śulba-sūtra* of Āpastamba (a copy of MS. D made by Garbe, MS. S, and MS. Gr), and four other manuscripts for the commentaries of Sundararāja, Karavindasvāmin (in total three manuscripts “that Thibaut kindly sent to me on loan so that I could use them”), and Kapardisvāmin (one manuscript belonging to the Sanskrit College in Benares of which a copy was made for him); cf. [Bürk 1901–1902, pp. 576–577].

by the sources, Thibaut overlooked in his previous account.<sup>368</sup> In contrast to Thibaut, Bürk's main concern was to abstain from preconceptions so as to find in the sources themselves how the "proposition of the square of the hypotenuse"<sup>369</sup> may have been found by the Indian priests. As in the other *Śulba-sūtras*, one also encounters in Āpastamba two propositions which Bürk envisaged in this order:

– Āp. Śulb-S. I. 5: "The diagonal of a square produces (when a square is constructed upon it) an area twice as large (as the original square is)."

– Āp. Śulb-S. I. 4: "The diagonal of a rectangle produces (when a square is constructed upon it) both what the longer and the shorter sides produce, each by itself."

As regards the former, "the authors of the sūtras [Thibaut had argued,] do not give any hint as to the way in which they found their proposition regarding the diagonal of the square; but we may suppose that they, too, were observant of the fact that the square on the diagonal is divided by its own diagonals into four triangles, one of which is equal to half the first square. This is at the same time an immediately convincing proof of the Pythagorean proposition as far as squares or equilateral rectangular triangles are concerned."<sup>370</sup> However, Bürk countered, Thibaut's explanation does not give satisfaction because "it finds no anchoring point in the sources, [for] it would be difficult to say what must have induced the Indian priests, after they had drawn a square, to construct a new square upon the diagonal of the first one".<sup>371</sup> On the contrary, Bürk suggested, one should start from what Thibaut himself presented as "the most ancient and primitive form", the so-called *Caturaśraśyēnacit*, that is the pattern of the falcon-shaped altar, whose body (*ātman*) consisted in 4 adjacent square bricks assembled in a larger square, "a figure from which it was not difficult to find the proposition in question"<sup>372</sup> (Ap. Śulb-s. I, 5), provided that the diagonals were drawn. Moritz Cantor's remark that "one looks in vain for

368 For a general assessment of Thibaut's historiography, see Keller [2012a].

369 Bürk complied with Hankel in preferring this designation for the proposition known as the Pythagorean theorem, although, as Datta suggests (see [Datta 1993, pp. 104–105]), "it would be more in keeping with the form and the spirit of the early Hindu geometrical terminology" to call it 'the proposition of the square of the diagonal'.

370 [Thibaut 1875, p. 234].

371 [Bürk 1901–1902, p. 558].

372 [Bürk 1901–1902, p. 557]. Āpastamba teaches two methods for constructing the *Caturaśraśyēnacit*. Since only the most ancient of these methods does not presuppose the proposition of the square of the hypotenuse, Bürk conjectured that the latter was read off that well-known pattern.

a proof”<sup>373</sup> in Baudhāyana, then afforded the perfect foil for the point Bürk intended to make, for “who knows more in depth the *Śulba-sūtras* will hardly look for a proof therein, no more than Schopenhauer for instance would have sought a proof therein”<sup>374</sup>. Bürk subtly argued in connecting Schopenhauer’s well-known figure (*viz. Fig. 7-nº 9*), “the mere sight of [which] without words conveys ten times more conviction of the truth of the Pythagorean theorem than Euclid’s mousetrap proof”,<sup>375</sup> with the *Caturaśraśyēnacit* pattern (*viz. Fig. 7-nº 10*).

If we slightly complement the figure praised by Schopenhauer, then we obtain exactly the figure from which the Indians, in my conjecture, discovered the proposition of the square [*constructed*] on the diagonal of a given square, that is the intuitive conviction of the geometrical truth obtained in this connection for the first time. And the Brahmins obviously satisfied themselves with this intuitive conviction. But we are far from requiring of them a proof modeled upon the Euclidean one. In this, we rather keep pace with Schopenhauer and Hankel, the latter being a man who immersed himself so sensitively and affectionately in the characteristics of foreign peoples in his bright book *On the History of Mathematics in Antiquity and Middle Ages*, whose chapter on the Indians he concluded by the following golden words [*cf. supra*] (which [*Bürks emphasized*,] it is necessary to quote here).<sup>376</sup>

Bürk intended to work out a similar account for the way in which the Indians found the proposition of the square on the diagonal of an oblong, *viz. Āp. Śulb-s. I*, 4. Here too, he started with Moritz Cantor’s view that the Pythagoreans first obtained that proposition as an arithmetical one while stumbling upon the numerical example  $3^2 + 4^2 = 5^2$ . Insofar as it was already known to them as an “experiential fact”, as it was presumably to the Egyptians, the Babylonians and the Chinese, that a right angle can be formed out of a cord with marked lengths 3, 4, 5, “the geometric and arithmetic truths [*Cantor claimed*,] then united into a joint proposition in Pythagoras’ consciousness”<sup>377</sup> A rule would henceforth be devised to ascertain other Pythagorean triplets. Although endorsing Cantor’s assumption of a widely known “experiential fact” regarding the triplet 3, 4, 5, Bürk nevertheless levelled serious objections against his account. These objections in return paved the way to Bürk’s strong denial that the *Śulba-sūtras*

<sup>373</sup> [Cantor 1877, p. 13].

<sup>374</sup> [Bürk 1901–1902, p. 558]

<sup>375</sup> In addition to this quote from the § 39 of *Fourthfold Root of the Principle of Sufficient Reason*, Bürk also quoted from the § 15 of the *World as Will and Representation*, see [Bürk 1901–1902, p. 558].

<sup>376</sup> [Bürk 1901–1902, p. 559]. Bürk inserts here the whole quotation from Hankel’s book, commented on in the previous section, cf. [Hankel 1874, p. 220].

<sup>377</sup> [Cantor 1880, pp. 153–154].

should derive from Alexandrian knowledge, as Cantor had unwaveringly professed until then. Bürk opposed the view that it was not a geometrical figure, but the arithmetical truth  $9 + 16 = 25$ , that must have provided the starting point for the discovery of the Pythagorean theorem—a view which went along with Cantor's conviction that immediate intuition could not lead to the discovery of new propositions, and which Bürk resisted on behalf of Schopenhauer. Besides, Bürk also questioned the assumption that,

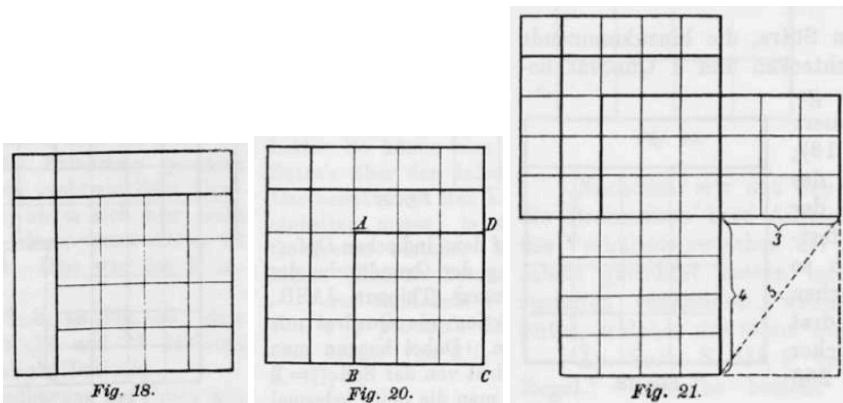


FIGURE 8. Bürk's reconstruction, cf. [Bürk 1901–1902, pp. 567–568]

starting from only one case fusing the arithmetical and the geometrical truths, further triplets would be found with the help of a mere formula, without relying anymore on geometrical figures. In any case, as mentioned above, he showed that the triplets attested in the *Śulba-sūtra* tradition did not fit any of the formulas historians of mathematics ever worked out for them. Another account was therefore needed. As for the way the Indians may have originally discovered their rational rectangular triangles, Bürk felt much more congenial to Thibaut's conjecture, although he would have to reshape it so as to make it cling more closely to the sources. "The way in which the Sútrakáras found the cases enumerated above, [Thibaut claimed,] must of course be imagined as a very primitive one. Nothing in the sútras would justify the assumption that they were expert in long calculations"<sup>378</sup>, so that they more likely kept their footing in geometric figures throughout the whole process, rather than relying on abstract formulas to find their triplets.

<sup>378</sup> [Thibaut 1875, p. 238].

Or, if we suppose a more convenient mode of trying [*Thibaut went on*], they might have found that twenty-five pebbles or seeds, which could be arranged in one square, could likewise be arranged in two squares of sixteen and nine. Going on in that way, they would form larger squares, always trying, if the pebbles forming one of these squares could not as well be arranged in two smaller squares. So they would form a square of 36, of 49, of 64, etc. Arriving at the square formed of  $13 \times 13 = 169$  pebbles, they would find that 169 pebbles could be formed in two squares, one of 144, the other of 25. Further on 625 pebbles could again be arranged in two squares of 576 and 49, and so on. The whole thing required only time and patience, and after all the number of cases which they found is only a small one.<sup>379</sup>

Whereas Thibaut suggested that the Indians started from a greater square which they split up into two smaller ones, Bürk based his account on a rule enunciated in Āpastamba's text, *viz.* Āp. Śulb-s. III, 9, so as to show that they rather started from a smaller square and only found, through increasing it, that a greater square turned up to be the sum of two smaller ones. Whereas the way through decomposition was difficult, Bürk emphasized, and all the more so as the square to decompose grew, so that it might have depended upon chance whether anything would ever come out of it, the way through increasing a given square, he pleaded, would on the contrary be easy and more straightforward. Being introduced by the words: "There follows a general rule", the *sūtra* III, 9, teaches how to increase a given square when both of its sides are increased in the same length. The rule thus prescribes adding two oblongs and a square, the oblongs being placed next to the original square along its upper, respectively its left side, and the added square being placed at its upper left vertex (see Fig. 8-n° 18, where the square  $4^2$  is increased by the two oblongs  $4 \times 1$  and the square  $1^2$ ). This rule, Bürk pointed out, was entrenched in ancient Indian practice, since altar construction proceeded by increasing the squares, through gradually adding bricks to the length of their sides. If one now applies the *sūtra* III, 9 twice starting from the square  $3^2$ , then one obtains two further squares, each extending the previous one, *viz.*  $4^2 = 3^2 + (2 \times 3 + 1)$  and  $5^2 = 4^2 + (2 \times 4 + 1)$ . Bürk then suggests that insofar as they presumably already knew the "experiential fact" about the cord with lengths 3, 4, 5, the Indians may have been led to place the three squares  $3^2$ ,  $4^2$ ,  $5^2$ , side by side as in Fig. 8-n° 21, instead of one inside another as in Fig. 8-n° 20, so that in combining both insights, the proposition of the square on the diagonal would be immediately grasped in that particular case. The same procedure would then be generalized to further cases. As one increased the square upon the side 12, Bürk

<sup>379</sup> *Ibid.*

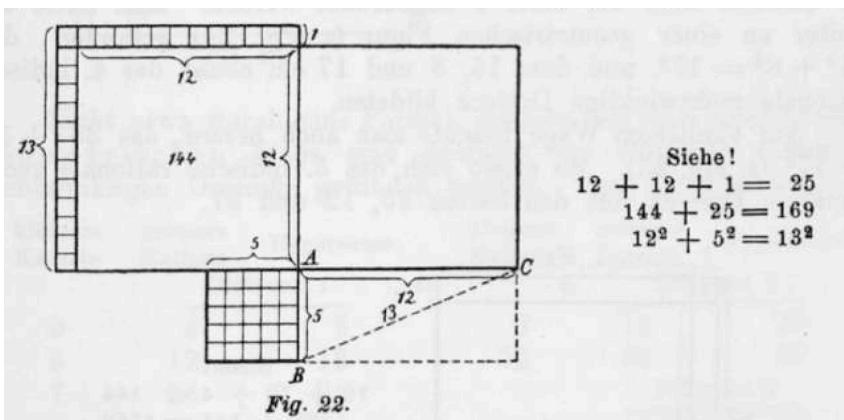


FIGURE 9. Bürk's reconstruction, cf. [Bürk 1901–1902, p. 569]

continued, one found that  $25 = 2 \times 12 + 1$  small squares were added. Since one knew from a previous stage that these formed a square of side 5, and since the new square had side 13 and an area composed of 169 small squares, one had therefore found that the squares on the sides 12 and 5 together were as large as the square upon the side 13 (see Fig. 9-n° 22). "But since one had previously made the experience with  $3^2 + 4^2 = 5^2$ , that the sides of the squares 3, 4, 5 resulted in a rectangular triangle, then it was suggested to investigate whether if  $AC = 12$  and  $AB = 5$ , then  $BC = 13$ ."<sup>380</sup> Significantly enough, Bürk insisted that his reconstruction complied with the 'See'-trope, which, as he recalled, Hankel had suggestively thrown into relief. In each and every case considered in Bürk's commentary<sup>381</sup>, as for instance in Fig. 9-n° 22, the diagram appears coupled with the imperative followed by an exclamation mark: "Siehe!", and complemented by a few arithmetical equations presumably to be read off the figure. Now, Bürk went on, the Indians would proceed in a similar way, when increasing the length of the side of the given square not only by one, but by two (brick squares), as for instance when shifting from  $15^2$  to  $17^2 = 15^2 + (30 + 30 + 2^2)$ , that is eventually  $17^2 = 15^2 + 8^2$ , or by three, as in increasing  $36^2$  into  $39^2 = 36^2 + 108 + 108 + 9$ , *viz.*  $39^2 = 36^2 + 15^2$ .

<sup>380</sup> [Bürk 1901–1902, 569].

<sup>381</sup> It should be noted that there is of course no such thing in Bürk's translation of Āpastamba's text itself, where diagrams occur unheralded by any kind of "See"-apparatus.

Bürk hence reorganized the Pythagorean triplets attested in the *Śulba-sūtras*. As the difference between the greater side and the hypotenuse successively amounted to the length in side of 1, 2 and 3 small square bricks juxtaposed in a row, the whole process culminated in 15, 36, 39, which the Indians held in high esteem since the measurement of the most important of their altars rested upon it.

difference 1	difference 2	difference 3
3    4    5	8    15    17	15    36    39
5    12    13	12    35    37	
7    24    25		

However, Bürk suggested, the Indians might have recognized afterwards that some of these results were related to one another. Āpastamba for instance saw the connection between 5, 12, 13 and 15, 36, 39. “But this was only possible [*he emphasized*], after one had learned to know them individually on geometrical figures, [*so that*] the Indian rational rectangular triangles were not found by means of a formula, but in the geometrical way described above.”<sup>382</sup> Not only did Bürk’s reconstruction clearly show how intuition of figures may lead to discovering new truths, which finally convinced Cantor to renounce his previous dogma of a supposedly Alexandrian origin of Indian geometry, but it also instantiated how the Indians perhaps came to endow the proposition of the square of the diagonal of an oblong with some kind of generality presumably involved in their ritual knowledge. “Owing to the repetition of the same geometrical figure, [*Bürk contended*], the Indians 1. found several times three squares, two of which taken together would amount to the third, and 2. discovered rational rectangular triangles which can be built precisely out of the sides of these squares; in this way, the geometrical truth which they first took notice of in one case, revealed itself to them as generally valid [*allgemeingültig*] through an increasing number of cases: *the diagonal of an oblong produces both what the longer and the shorter sides produce each for itself*:”<sup>383</sup> Bürk’s argument mainly amounted to emphasizing that Āpastamba’s *Śulba-sūtra* displays certain “general” rules of construction for adding (Āp. Śulb-S. II. 4), or subtracting (II. 5, see Fig. 10-n° 11) squares,<sup>384</sup> or for turning a

<sup>382</sup> It should be noted that there is presumably no such thing in Āpastamba’s text itself. To all appearances, Bürk himself added the word “See”, together with the equations, to the original text.

<sup>383</sup> [Bürk 1901–1902, pp. 571–572].

<sup>384</sup> Cf. [Bürk 1901–1902, p. 333]. To subtract a square *ECFG* from a larger square *ABCD* (see Fig. 10-n° 11 above), one cuts off the rectangle *HBCE* having one of its

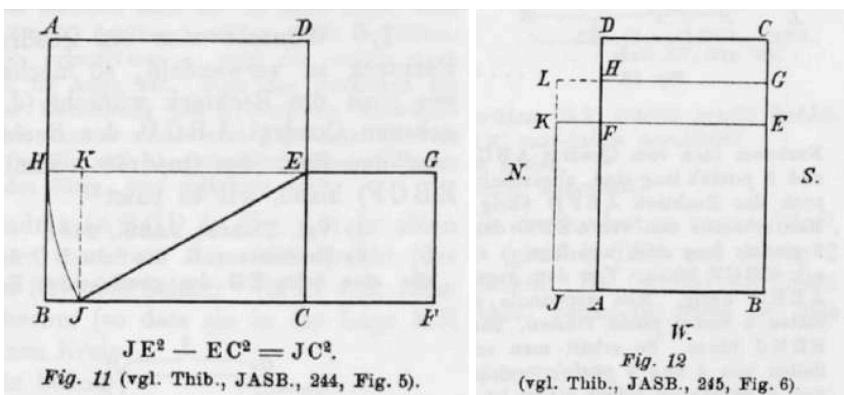


FIGURE 10. Āpastamba's "general" rules Āp. Śulb-S. II. 5 and II. 7, cf. [Bürk 1901–1902, p. 569]

rectangle into a square (II. 7, see Fig. 10–n° 12),<sup>385</sup> all of which rest upon an unrestricted use of the Pythagorean theorem. Being presupposed in those “general” rules, the latter proposition would also presumably partake of their general validity. Furthermore, Bürk implied that the Indians had also grasped the notion of irrationality, his evidence for such a claim being that Āpastamba’s *Śulba-sūtra* gives a formula for the approximate value of  $\sqrt{2}$ , hence “obviously, [in Bürk’s view,] after they had discovered the irrational”.<sup>386</sup>

Bürk's contribution almost immediately elicited responses on the part of mathematicians. Within the timespan of a year, both Hieronymus Georg Zeuthen (1839–1820), the renowned Danish mathematician and historian of mathematics, and Heinrich Vogt, a less prominent figure based in Breslau, reviewed it and, independently of one another, came

sides equal to the side of the square  $ECFG$ ; then one takes the length  $JE$  equal to the side  $BC$ , with the point  $J$  lying on the side  $BC$ . In applying the Pythagorean theorem to the rectangular triangle  $EJC$ , the square constructed upon the side  $JC$  solves the question.

385 Cf. [Bürk 1901–1902, p. 333]. To convert a rectangle  $ABCD$  into a square (see Fig. 10–n° 12 above), one cuts off the square  $ABEF$ , bisects the rectangle  $DCEF$  left over into two equal rectangles  $HGEF$  and  $DCGH$ , then moves the latter so as to adjust it along the side  $AF$ , namely so as to shift it into the position of the rectangle  $AFKJ$ . Then the initial rectangle  $ABCD$  is equal to the difference between both squares  $BGLJ$  and  $HLKF$ . One then applies the previous rule for subtracting squares.

386 Cf. [Bürk 1901–1902, p. 575].

to similar conclusions.<sup>387</sup> As Moritz Cantor before them, they admitted being rather convinced by Bürk's chronological point about the *Śulba-sūtras* bearing witness of ancient Indian knowledge, prior to any influence from the Greeks. But they both clearly denied that Bürk's reconstruction, which they acknowledged though as a sound one, should be interpreted as showing that the Indians had either proved the Pythagorean theorem in full generality, or in the least discovered irrationality *per se*. However, in making clear why Bürk's presumably excessive claims should be discarded, they both articulated what they held as *bona fide* proofs, which in both cases were Greek ones. Zeuthen for instance observes that in comparing the *Śulba-sūtras* with what we know of Greek geometry, "we learn to know the geometrical level at which the discovery of the 'Pythagorean theorem' occurred."<sup>388</sup> In this regard, "*simple intuition*", as involved in the practical knowledge of craftsmen, beyond which the *Śulba-sūtras* presumably never ventured, is to be contrasted with the level of abstract thinking embodied in what Zeuthen labels "proper (or scientific) geometry". For his part, Vogt analyzes more in depth the underlying thought of the *Śulba-sūtras*, as reconstructed by Bürk, and wonders "which degree of insight in the validity of the Pythagorean theorem"<sup>389</sup> might legitimately be ascribed to the Indians. "The ancient Indians have enunciated and applied [Vogt argued,] the proposition of the oblong [viz. *the so-called Pythagorean theorem*] in a completely general way and without restriction; but the grounds for their knowledge is none other than the realization that in some cases both the property of the right angle and the equality of the sum of squares on the sides and the square on the diagonal coincide. From this, it is inferred by an incomplete induction that the property of the sum of squares always implies the property of the right angle, and conversely."<sup>390</sup> Although

<sup>387</sup> Compare Zeuthen [1905] with Vogt [1906]. Heinrich Vogt observes in a postscriptum to his review, that he only took cognizance of Zeuthen's, after his own submission, when he was given notice of it by the editor of the *Bibliotheca mathematica*.

<sup>388</sup> [Zeuthen 1905, p. 838].

<sup>389</sup> Cf. [Vogt 1906, p. 7].

<sup>390</sup> Cf. [Vogt 1906, p. 10]. A few years later, Thomas Heath would draw similar conclusions, cf. [Heath 1908, p. 363]: "The [Pythagorean] theorem is, it is true, enunciated as a general proposition [*in Āpastamba's Śulba-sūtra*], but there is no sign of anything like a general proof; there is nothing to show that the assumption of its universal truth was founded on anything better than an imperfect induction from a certain number of cases, discovered empirically, of triangles with sides in the ratio of whole numbers in which the property (1) that the square on the longest side is equal to the sum of the squares on the other two sides was found to be always accompanied by the property (2) that the latter two sides include a right angle."

useful as a tool for finding, Vogt continues, within the realm of proper geometry, incomplete induction is irrevocably denied any claim to yielding the required "grounds for knowledge". Neither would it be justified, in Vogt's view, to suggest that the Indians ever discovered irrationality,<sup>391</sup> for "practical measurement which, according to F. Klein's fortunate expression, belongs to the 'geometry of approximation', yields no irrationality whatsoever, [*whereas*] the irrational only arises in the soil of the 'geometry of precision' (still after Klein), and the proof of the impossibility of rationality can be conducted in no other way than by means of abstractions".<sup>392</sup> Eventually, both Zeuthen and Vogt observed, Bürk's reconstruction had to confront the same assaults as those launched against Schopenhauer, "the ennemy of Greek discursive thought, the friend of Indian intuition"<sup>393</sup>. Although conceding that "together with Schopenhauer, Hankel and von Schroeder, one should admire the intuitive flair of the ancient Indians, [Vogt nevertheless judged that] they were very far from the essence of both geometrical thinking and knowledge."<sup>394</sup> Zeuthen for his part emphasized that Schopenhauer had failed to grasp the necessity of abstract thinking for want of clear evaluation of what the intuitive appropriation of geometrical truths he so strongly advocated would amount to, beyond the simple paradigmatic case of the isosceles rectangular triangle.<sup>395</sup>

In a broader context, Schopenhauer's guidelines for reshaping the teaching of geometry were to play a central role in the early twentieth-century German debates prompted by the reform movement against neohumanism and pure mathematics, led by the schoolteachers, the

<sup>391</sup> On this score, Vogt concurred with Zeuthen, see for instance [Zeuthen 1905, p. 851]: "Mais les Indiens ne se sont sans doute jamais posé la question plus abstraite de savoir s'il était en vérité absolument impossible de trouver une fraction numérique égale à  $\sqrt{2} - 1$ ."

<sup>392</sup> Cf. [Vogt 1906, p. 12]. For the distinction between *Approximationsgeometrie* and *Präzisionsgeometrie*, Vogt refers to Felix Klein's *Anwendung der Differential- und Integralrechnung auf Geometrie, eine Revision der Prinzipien*, Leipzig, 1902.

<sup>393</sup> Cf. [Vogt 1906, p. 10].

<sup>394</sup> Cf. [Vogt 1906, p. 11].

<sup>395</sup> Cf. [Zeuthen 1905, p. 834]: "Schopenhauer recommande la même appropriation intuitive des vérités géométriques qui a été possible avant la géométrie propre; mais il n'a pas su évaluer la portée de cette appropriation. Voilà ce qui le porte à reprocher à Euclide de ne pas rendre sa démonstration du théorème général de Pythagore aussi simple que celle de son application à un triangle rectangle et isocèle. ... Euclide s'en serait sans doute servi s'il ne s'était agi que de ce cas particulier. Les artifices de la démonstration générale d'Euclide lui étaient nécessaires, parce qu'il avait besoin du théorème avant d'avoir parlé de la similitude, et sa démonstration est à cet égard un chef d'œuvre."

so-called *Oberlehrer*. Ernst Mach, for instance, one of the first declared reformists among university professors, obtained a copy of Kosack's program through Friedrich Pietzker, an *Oberlehrer* at the Nordhausen Gymnasium and an active member of the reform movement. Lewis Pyenson has shown how some university professors attempted to deflect the reform movement so as to counter the threat posed to their own corporate privileges. In promoting a reorientation of mathematical instruction in the direction of applied mathematics while still preserving the leading role for pure mathematics, Felix Klein (1849–1925) first and foremost strove to surmount the increasing gulf between engineers and research-oriented mathematicians. As is well-known, intuition was his main propitiatory motto in the midst of this contest<sup>396</sup>, but his prescriptions elicited hostile responses on both sides. In a discourse held before the Munich Academy of Sciences in 1904, one of Klein's opponents from inside the university,<sup>397</sup> Alfred Pringsheim (1850–1941) defended the opposite strategy to withstand the rising tide of the “anti-mathematical movement”, which presumably gathered under the banner of Schopenhauer. In deplored that “many pure mathematicians [*were*] thought to be, if not ‘pure fools’, then at least utterly superfluous representatives of a conceited and abstruse Brahmin-like wisdom”,<sup>398</sup> he set out to contribute to a fairer estimation of mathematics by counterattacking. “As is well known, Schopenhauer turned against mathematics in different parts of his writings. That was quite long ago already, nevertheless, to my knowledge, [Pringsheim added,] no one ever refuted his accounts [...] But since, until most recent times, namely in writings and essays which put the case for a restriction of mathematical instruction in middle schools, it is attempted with an almost unfailing regularity to put Schopenhauer's authority in the balance as a particularly weighty one, it seems to me urgently desirable to submit for once Schopenhauer's arguments [...] to a public examination.”<sup>399</sup> Pringsheim then discussed Schopenhauer's views on geometric proof from the

<sup>396</sup> On Klein's activism and his emphasis on intuition as the proper antidote against the alleged blindness to reality on the part of pure mathematicians, see [Pyenson 1983, chap. 6].

<sup>397</sup> On the Klein-Pringsheim controversy in the 1890s, see [Pyenson 1983, pp. 66–67].

<sup>398</sup> Cf. [Pringsheim 1904, p. 358]. See also [Pyenson 1983, p. 80] for a comment on Pringsheim's line of defense. One may also incidentally note that, along with his criticism of Schopenhauer's intuitive alternative to Euclid's proof, Pringsheim meant “Brahmin-like wisdom” as a proverbially dismissive qualification, thus showing, unlike Hankel, no interest in Indian mathematics.

<sup>399</sup> [Pringsheim 1904, p. 358].

standpoint of Hilbert's axiomatic geometry.<sup>400</sup> "If one may also concede that, as Schopenhauer emphasized, the Euclidean method of proof is inappropriate from the didactic point of view, [*in Pringsheim's view*,] the far more essential flaws of the Euclidean theoretical structure lie much deeper, namely in the fundamental definitions and axioms, which is precisely what Schopenhauer does not in the least understand."<sup>401</sup> However, Pringsheim claimed, even if, for the sake of the argument, one adopted Schopenhauer's pre-Hilbertian stance, the philosopher's point would still remain misguided. "In most Euclidean proofs, [*he argued*,] what makes the insight difficult for the learner is in no way the *content*, but merely the *purely synthetic form* of the presentation which any skilled teacher can easily replace by a more analytic-genetic and at the same time a geometrically more intuitive one. A telling example of this [*he went on*,] is provided precisely by the Euclidean proof of the Pythagorean theorem, which Schopenhauer characterized as 'contrived and deceitful', but which by an insignificant change of the form of presentation appears as a brilliant example of a faultless proof of elementary geometry, whereas that which Schopenhauer dares to offer as an *Ersatz* must be designated as extremely naive, to say it mildly."<sup>402</sup> Even in the "miserable special case" to which he restricted himself, as Pringsheim disparagingly put it, namely in the case of an isosceles rectangular triangle (cf. Fig. 11-nº I), Schopenhauer failed to disclose the presumably underlying ground of being, and gave "not a whit more than Euclid",<sup>403</sup> that is merely the cognitive ground. Interestingly, while trying to work out an equivalent of Schopenhauer's (or, as he suggests, rather Plato's) intuitive proof in the general case of an arbitrary rectangular triangle, Pringsheim falls back on Bhāskara II's proof in *Bija-ganita* V, 146, although reading it through the lens of the key concepts of Hilbert's theory of areas.<sup>404</sup>

<sup>400</sup> Pringsheim referred to the second edition of Hilbert's *Grundlagen der Geometrie*, Leipzig, 1903.

<sup>401</sup> [Pringsheim 1904, p. 359].

<sup>402</sup> *Ibid.*

<sup>403</sup> *Ibid.* Strictly speaking, in Pringsheim's view, the congruence of triangles by which the areas of both squares constructed on the cathetes are proved to be equal to the area of the square constructed on the hypotenuse (cf. Fig. 11-nº I.) ultimately rests on the axioms of congruence, which should be seen as what properly captures, if ever, the so-called "ground of being" which neither Euclid's, nor Schopenhauer's proof articulates.

<sup>404</sup> In his theory of areas (see chapter 4 of his *Grundlagen der Geometrie*), Hilbert distinguished two different concepts for the equality of polygonal areas, the "equidecomposability" [*Flächengleichheit* (1st ed.) or *Zerlegungsgleichheit* (2nd to 10th ed.)] on the one hand, and the "equicomplementarity" [*Inhaltsgleichheit* (1st to 6th ed.) or

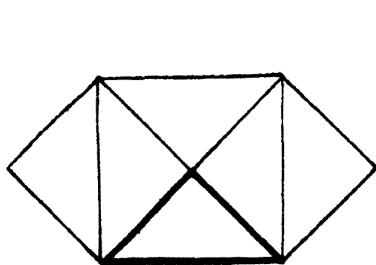


Fig. I.

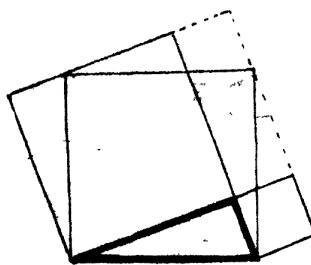


Fig. II.

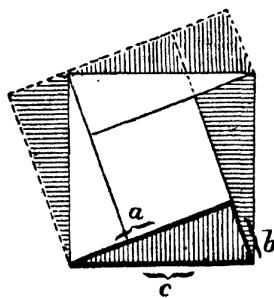


Fig. III.

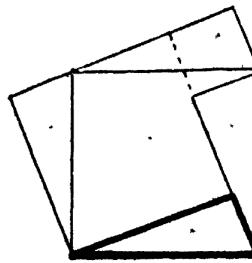


Fig. IV.

FIGURE 11. Pringsheim's working out of Schopenhauer's figure,  
cf. [Pringsheim 1904, pp. 359–360]

If one tries to transfer this proof [*viz. Fig. 11–nº I*] to the case of an arbitrary rectangular triangle, it takes on an essentially less satisfactory character, [...] for one does *not* succeed in this way to prove that the relevant figures are “equidecomposable” [“*zerlegungsgleich*”], but only that they are “equicomplementary”

*Ergänzungsgleichheit* (7th to 10th ed.)]. Cf. [Hilbert 1903, pp. 39–40]: “Two polygons are said to be *equidecomposable* [*zerlegungsgleich*] if they can be decomposed into a finite number of triangles which are respectively congruent to one another in pairs”, whereas “two polygons are said to be of *equal content* [*inhaltsgleich*] [*or, as Hilbert will later say, equicomplementary* [*ergänzungsgleich*]], if it is possible, by the addition of equidecomposable polygons, to obtain two resulting equidecomposable polygons”. Although referring to the second edition of Hilbert’s work, Pringsheim uses the term *ergänzungsgleich*.

[“*ergänzungsgleich*”] (see Fig. 11–nº II). Besides, in early Middle Ages, the Indians and the Arabs<sup>405</sup> already knew a proof by “decomposition” [“*Zerlegungs-Beweis*”] which rests on the double equation  $c^2 = (a-b)^2 + 2ab = a^2 + b^2$ . If one arranges the cutting in parts and the reallocation of these parts so as to make this double equation geometrically intuitive [zur geometrischen Veranschaulichung] in the way indicated in Fig. 11–nº III, then, after leaving aside the cuts which prove to be superfluous (see Fig. 11–nº IV), one gets the simple and elegant proof by cutting [Zerschneidungsbeweise] of An-Nairizi.<sup>406</sup>

Pringsheim knew about this last proof by the late ninth-century mathematician Al-Nayrīzī, from Johannes Tropfke (1866–1939) who had suggestively commented upon it in his *Geschichte der Elementar-Mathematik* (1902–1903).<sup>407</sup> Tropfke had himself picked out Al-Nayrīzī’s diagram (cf. Fig. 12 left) from Maximilian Curtze’s 1899 edition of the Latin translation by Gerard of Cremona of Al-Nayrīzī commentary on Euclid’s *Elements*, based on a manuscript found in Cracow.<sup>408</sup> In his account, Tropfke nevertheless innovated in comparing Al-Nayrīzī’s figure to a proof by decomposition (cf. Fig. 12 right) published by Adolph Göpel (1812–1847) in Grunert’s journal in 1844.<sup>409</sup> Göpel had worked out his own proof along the lines of Paul Gerwien’s first attempts to account for polygonal congruence by means of decomposition procedures,<sup>410</sup> an approach which would later be consistently taken over and generalized by Hilbert in his theory of areas. For his part, Pringsheim endorsed Tropfke’s comparison and

<sup>405</sup> Pringsheim refers here to Cantor’s *Vorlesungen über Geschichte der Mathematik I* (1880), namely to the passage in which Cantor steps back behind Colebrooke’s translation of Bhāskara II’s proof in *Bija-gaṇita* V, 146, together with the figure and ‘See’ trope, cf. [Cantor 1880, p. 557]

<sup>406</sup> [Pringsheim 1904, p. 360].

<sup>407</sup> Tropfke’s book was significantly enlarged from the first to the later editions. The passages from the second volume of the first edition to which Pringsheim refers, are reworked in the fourth volume (devoted to geometry) of the third edition. References are given in both editions.

<sup>408</sup> The philologist and historian of mathematics Maximilian Curtze (1837–1903), a Gymnasium professor in Thorn, East Prussia, recounted how he had found this manuscript during a study trip made from August to October 1896, cf. [Curtze 1898, pp. 454–455]. Curtze’s edition of Al-Nayrīzī commentary on Euclid’s *Elements* was published as a supplement to the edition of Euclidis *Opera Omnia* (1883–1916) by Johan Ludvig Heiberg and Heinrich Menge. On Al-Nayrīzī’s diagram, see [Curtze 1899, p. 85]. In the third edition of his book, Tropfke remarks that the proof should rather be attributed to Thābit ibn Qurra (826–901), as testified by Cremona’s Latin text, see Curtze’s edition, [Curtze 1899, p. 84]: “*Quod sequitur addidit Thebit*”.

<sup>409</sup> Cf. Göpel [1844]. In the third edition of his book, Tropfke went further and affirmed that Al-Nayrīzī’s proof and Göpel’s proof were “essentially identical”, cf. [Tropfke 1940, p. 194].

<sup>410</sup> Cf. Gerwien [1833].

regarded Bhāskara II's and Abū'l Wafā's diagrams in the light these later concepts would in his view irrevocably shed upon them, yet at the cost of obfuscating the meaning of the original proofs. Accordingly, unlike Hankel, he did not in the least suspect that Schopenhauer's vindication that intuitive insight should prevail over logic in geometrical proofs, might suggest ways in which Sanskrit proofs may have been obtained, or principles which may have governed them. Although acknowledging Bhāskara II's proof as correct, he nevertheless interpreted it in the framework of Hilbertian axiomatic geometry, and simply ignored the issue which Hankel had primarily dealt with as a historian of mathematics, namely how to characterize the very procedures of Indian mathematics. As regards Indian and Arabic proofs of the Pythagorean theorem, Pringsheim only referred to Cantor and Tropfke,<sup>411</sup> who in return entirely relied on three main sources, namely Colebrooke's translation of Bhāskara II's *Bija-ganita*, Franz Woepcke's translation of a treatise on Abū'l Wafā's geometric constructions, and Curtze's edition of the Latin translation of Al-Nayrīzī's commentary on Euclid's Elements. As seen above, the first two were also available to Hankel more than thirty-odd years earlier, but Pringsheim exploited them in a completely different way than his predecessor, in unison with the defense of pure mathematics he was advocating at the turn of the century.

A few years later, partly in response to Pringsheim, Klein espoused a more balanced appreciation of Schopenhauer's views. He admitted that the philosopher's claim that "beside logical deductions, there [was] another mathematical method which directly extracts mathematical truth from intuition" could in no way be agreed with, "for, although intuition may be granted a great role in mathematics as a heuristic principle [...], still logical proof should always ensue as the last and only decisive instance"<sup>412</sup>. "However, [Klein argued,] if Schopenhauer had merely attacked the broken *staccato form* of presentation in Euclid, if he had wished

<sup>411</sup> Whereas Johannes Tropfke held that the intuitive proof of the Pythagorean theorem occurring in the treatise on Abū'l Wafā's geometric constructions, was not only akin to Bhāskara II's proof, but had even been "borrowed by the Arabs from the Indians" (see [Tropfke 1903, p. 73], also [Tropfke 1940, p. 193]), Cantor still claimed in 1880 that, although it had been later appropriated by the Indians and worked out along their own lines, the proposition nevertheless resulted from a transfer of knowledge from Alexandria, inasmuch as mere intuition presumably does not lead to new propositions. As already mentioned, Cantor would only change his mind after reading Bürk's commentary of Āpastamba's *Śulba-sūtra*. Pringsheim, for his part, mostly overlooked this historical issue and merely took over Al-Nayrīzī's figure (*viz.* Fig. 11–n° IV) from Tropfke so as to make his own point.

<sup>412</sup> [Klein 1909, p. 257].

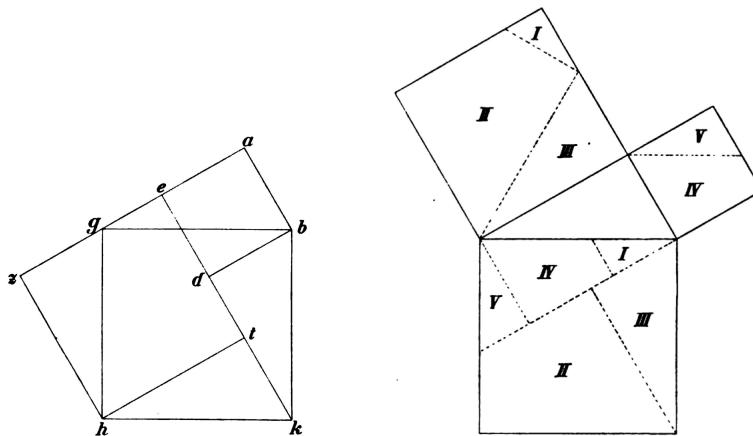


FIGURE 12. Comparing Al-Nayrizī's figure with Göpel's proof by decomposition, cf. [Tropfke 1903, pp. 73–74], [Tropfke 1940, pp. 193–194]

the ideas underlying every proof process to be worked out in a more transparent way, if he had claimed that, besides logic, a more far-reaching consideration for intuition should be granted, one could definitely agree with him.”<sup>413</sup> But in targeting Euclid’s proof of the Pythagorean theorem, Schopenhauer presumably jeopardized even the justifiable aspect of his point, for, in Klein’s eyes, the Euclidean proof should be understood as providing precisely what was needed to extend to the general case what is directly read off the figure in the particular case of the isosceles rectangular theorem. “Both proofs, [Klein concluded,] are equally a mix of logic and intuition”<sup>414</sup>.

With hindsight, Bürk’s interpretation of Āpastamba’s *Śulba-sūtra* thus appears as being pivotal at the turn of the century. On the one hand, Moritz Cantor’s surrender to his views about ancient Indian knowledge predating Greek influences significantly fostered his reception in mathematical circles. More broadly, mathematicians and historians of mathematics discussed his contribution with reference to the above mentioned early-twentieth century German debates on mathematical pedagogy revolving round Schopenhauer’s views on geometrical proof. But on the other hand, Bürk’s reconstruction, later to be acknowledged as a turning

<sup>413</sup> [Klein 1909, pp. 257–258].

<sup>414</sup> [Klein 1909, p. 259].

point in the historiography of ancient mathematics<sup>415</sup>, had originally been intended to unfold Hankel's core insights about Sanskrit intuitive proofs in showing both their correctness and fruitfulness for sound philological work, although Hankel himself did not know anything of the *Sulba-sūtras* in the early 1870s. Our purpose in the previous pages was to shed light on the singularity of Hankel's contrastive appraisal of Sanskrit and Greek mathematics, as the respective dynamics of philology and mathematics intersected in the second half of the nineteenth century. Although drawing on some of Arneth's ideas modeled upon dualistic patterns for a world history of mathematics, Hankel nevertheless worked them out in a completely different way insofar as repudiating his predecessor's naturalistic framework. So as to account for the complex filiation presumably connecting both stances, we had to analyze the social and historical processes unfolding from Arneth's naturalistic appropriation of Röth's Creuzerian cultural history in the early 1850s to Hankel's Humboldtian historicized history of mathematics two decades later. In that instance, keeping track of influences proved inseparable from measuring historical distance.

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415 Whereas Bürk had sought, along Hankel's lines, to substantiate the view that Indian (or at least Vedic) mathematics developed independently of Greek mathematics and thus laid stress on their differences with regard to the way they carried out proofs, his contribution eventually turned out to prompt new questions in the twentieth century. In 1928, Neugebauer opposed Bürk's conclusions in pointing out that "the difficulties [*involved in the view*] of a direct borrowing by the Greeks from India fall away on the assumption of a common origin in Babylonia" ([Neugebauer 1928, p. 47]). Seidenberg replied that many of the aspects common to Greek and Vedic mathematics were unknown to the Old Babylonians, cf. [Seidenberg 1978, p. 307]. In distinguishing two aspects in Pythagoras' Theorem, one in which the side of a square equal to the sum or difference of two squares is constructed, and one in which the diagonal of a rectangle is computed, he for instance remarked that only the *Sulba-sūtras* knew both aspects, whereas the Greeks cultivated the first and the Old-Babylonians the second aspect. Frits Staal eventually investigated more in depth the linguistic and archaeological grounds in favor of such a common origin of Greek and Vedic geometries, see Staal [1999].

seminars held in Paris, at the Laboratoire SPHERE, UMR 7219, Université Paris Diderot, where I presented part of the material of this research project.

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