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ALGEBRAIC K-FLOWS

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The algebraic approach to the study of Statistical Mechanics suggests an expansion of classical ergodic theory to a noncommutative ergodic theory. We indicate here how one can proceed in this spirit to generalise the classical concept of a Kolmogorov-Sinai flow.

Firstly we recall that a classical K-flow is constituted by: a probability space (Ω, Σ, μ), a measurable group $\{T(t) \mid t\in R\}$ of measure preserving transformations of (Ω, Σ, μ), and a partition $\xi \subset \Sigma$ such that: (i) $\xi \leqq T(t)[\xi]$ $\forall t \geqslant 0$; (ii) $\bigcap_{t\in R} T(t)[\xi] = \hat{0}$; (iii) $\bigvee_{t\in R} T(t)[\xi] = \hat{1}$.

Secondly we construct from these elements: a separable Hilbert space $\mathcal{H} = L^2(\Omega,\mu)$, a maximal abelian von Neumann algebra $\mathcal{R} = L^\infty(\Omega,\Sigma)$ acting on $\mathcal{H}$, a cyclic and separating vector $\Phi(\omega)=1$ $\forall \omega\in\Omega$ for $\mathcal{R}$ in $\mathcal{H}$, a faithful normal state on $\mathcal{R}$ $\phi : f \in \mathcal{R} \mapsto \langle\phi; f\rangle = (f\Phi, \Phi) = \int f(\omega) d\mu(\omega)$, a continuous group of automorphisms of $\mathcal{R}$ $\alpha(t) : f \in \mathcal{R} \mapsto \alpha(t)[f] = f\circ T(t)$, and a von Neumann subalgebra $\mathcal{A}$ of $\mathcal{R}$ $\mathcal{A} = \{\chi_\Delta \mid \Delta \in \xi\}''$ with the properties: (i) $\mathcal{A} \subseteq \alpha(t)[\mathcal{A}]$ $\forall t \geqslant 0$; (ii) $\bigcap_{t\in R} \alpha(t)[\mathcal{A}] = \mathbb{C}I$; (iii) $\bigvee_{t\in R} \alpha(t)[\mathcal{A}] = \mathcal{R}$.

Thirdly we notice that a standard representation theorem allows to get back to the original definition from $(\mathcal{R}, \phi, \alpha, \mathcal{A})$ where: $\mathcal{R}$ is an <u>abelian</u> von Neumann algebra acting on a separable Hilbert space $\mathcal{H}$, ϕ is a faithful normal state on $\mathcal{R}$, α: R $\longrightarrow$ Aut$(\mathcal{R}, \phi)$, and $\mathcal{A}$ is a completely selfrefining, generating von Neumann subalgebra of $\mathcal{R}$ . .

Fourthly the <u>generalisation to the noncommutative domain</u> now consists exactly in taking the above as an alternative definition of a classical K-flow, and in dropping from this definition the condition that $\mathcal{R}$ be abelian. For reasons which would be too long to make explicit here we also impose that $\mathcal{A}$ be stable under the modular automorphism group $\{\sigma_\phi(t) \mid t\in R\}$ canonically associated to ϕ, and that every maximal abelian subalgebra $\mathcal{X}$ of the centralizer $\mathcal{R}_\phi$ of $\mathcal{R}$ be already maximal abelian in $\mathcal{R}$. (Notice that both of the last two conditions are redundant when $\mathcal{R}$ is abelian since $\sigma_\phi(t) = id$ $\forall$ $t\in R$ in this case.)

We now can emphasize [1] that algebraic proofs can be given to several theorems which are well-known in the classical case, and which

thus do generalize to the new situation just defined. For instance the system $(\mathcal{N},\Phi,\alpha,\mathcal{A})$ is ergodic (i.e. $N\in\mathcal{N}$ and $\alpha(t)[N]=N \ \forall t\in R \Rightarrow N=\lambda I$ with $\lambda\in C$); it is mixing (i.e. $<\Phi;N\alpha(t)[M]>\to<\Phi;N><\Phi;M>$ as $t\to\pm\infty$ for all $N,M\in\mathcal{N}$); it has Lebesgue spectrum (i.e. $\alpha(t)$ is spatial and the generator H of the corresponding unitary group on $\mathcal{H}$ has the property $Sp(H) = Sp_d(H)\cup Sp_{ac}(H)$ with $Sp_d(H) = \{0\}$ simple and $Sp_{ac}(H) = R$ has countable multiplicity). Furthermore a noncommutative entropy can be defined [2] which is strictly positive for all such systems.

We next remark that the generalisation is genuine in the sense that, in addition to the classical case where $\mathcal{N}$ is abelian, there exist [1,3,4] K-flows where $\mathcal{N}$ is of type II_1 , III_λ $(0<\lambda<1)$, or III_1.

Finally a link has been established [3] between certain quantum transport phenomena, governed by an evolution equation of the diffusion type, and the generalised K-flows presented here.

References

[1] Nonabelian Special K-flows
Journ. Funct.Analysis 19 (1975) 1-12.

[2] Positivity of the K-entropy on Non-Abelian K-Flows
Z.Wahrscheinlichkeitstheorie verw. Gebiete 29 (1974) 241-252.

[3] The Minimal K-Flow associated to a Quantum Diffusion Process
in Physical Reality and Mathematical Description,Enz & Mehra,Eds.,
D.Reidel Publishing Company, Dordrecht-Holland, 1974, 477-493.

[4] in preparation

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