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UNIVERSALITY OF REVERSIBLE HEXAGONAL CELLULAR AUTOMATA *, **

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Abstract. We define a kind of cellular automaton called a hexagonal partitioned cellular automaton (HPCA), and study logical universality of a reversible HPCA. We give a specific 64-state reversible HPCA H_1, and show that a Fredkin gate can be embedded in this cellular space. Since a Fredkin gate is known to be a universal logic element, logical universality of H_1 is concluded. Although the number of states of H_1 is greater than those of the previous models of reversible CAs having universality, the size of the configuration realizing a Fredkin gate is greatly reduced, and its local transition function is still simple. Comparison with the previous models, and open problems related to these model are also discussed.

AMS Subject Classification. 68Q80, 68Q05.

1. INTRODUCTION

A reversible cellular automaton (RCA) is a special type of CA such that every configuration (*i.e.*, the whole state) of the cellular space has at most one predecessor, *i.e.*, its global transition function is one-to-one. Such a system, as well as other reversible systems (*e.g.* a reversible Turing machine, reversible logic circuits, etc.), has a close connection to physical reversibility, and is known to be very important

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when studying inevitable energy dissipation in computing processes (see *e.g.* [2,10] for general surveys). Besides such problems of energy consumption, these systems are also interesting from a computational viewpoint, because they have relatively rich ability of computing in spite of the reversibility constraint.

Bennett [1] first proved that any (irreversible) Turing machine can be simulated by an equivalent reversible Turing machine (RTM), hence RTMs have computation-universality. Fredkin and Toffoli [3] proposed a theory of reversible and bit-conserving logic circuits in which so-called Fredkin gate was shown to be a universal logic element. In fact, any (even irreversible) logic circuit can be composed of this gate. They also showed that such logic circuits can be realized by the Billiard Ball Model (BBM), an idealized mechanical model having physical reversibility.

As for RCAs, Toffoli [9] showed that any k-dimensional irreversible CA can be simulated by a $k+1$-dimensional reversible CA. From this, computation-universality of two-dimensional RCAs can be derived. Morita and Harao [6] later proved that one-dimensional RCAs are computation-universal in the sense that for any given reversible Turing machine we can construct a one-dimensional RCA that simulates it.

On the other hand, in the two-dimensional case, several very simple models of CAs having logical universality (in the sense that any logic circuit can be embedded in the cellular space) have been shown. Margolus [5] proposed an interesting two-state RCA model in which the BBM (hence a Fredkin gate) can be simulated. Differing from a usual CA, his model uses "block rules" for state transition, which has a little non-uniformity in time and space. Morita and Ueno [8] constructed two simple models of 16-state RCAs by using a framework of partitioned cellular automata (PCA), which can be regarded as a subclass of usual CAs and convenient for designing reversible CAs. These two models can also simulate the BBM, thus they have logical universality. Recently, Imai and Morita [4] showed a logically universal reversible 8-state triangular PCA. This model has a local transition function much simpler than the above RCAs.

In this paper, we introduce a new model of 64-state reversible hexagonal PCA (RHPCA). We show that a Fredkin gate can be embedded in this cellular space. Therefore it has logical universality. Although the number of states of this model is greater than those of the previous ones, the size of the configuration realizing a Fredkin gate is greatly reduced. Furthermore its local transition function is still simple.

In the following, after giving definitions on a HPCA and its reversible version, we propose a new 64-state RHPCA model H_1. Then we describe how basic functions such as signal transmission, delay, and elementary logical operations can be realized. By combining these techniques, we give a configuration that simulates a Fredkin gate. Comparison with the previous models, and open problems related to these model are also discussed.

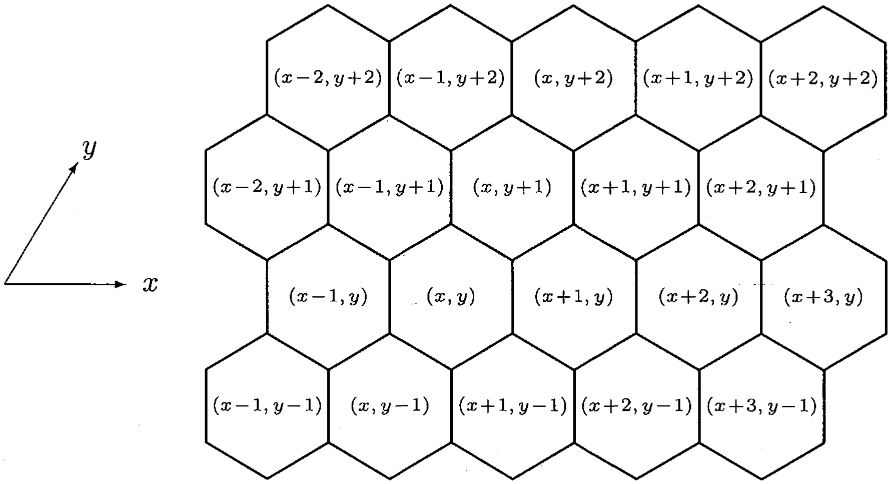


FIGURE 1. A hexagonal cellular space.

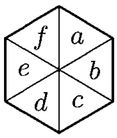


FIGURE 2. A cell of a hexagonal partitioned cellular automaton $((a, b, c, d, e, f) \in A \times B \times C \times D \times E \times F)$.

2. HEXAGONAL PARTITIONED CELLULAR AUTOMATA

A two-dimensional hexagonal cellular automaton (HCA) is a system where identical finite automata are placed uniformly on the infinite hexagonal lattice space, and synchronously change their states by communicating with neighbouring cells. Figure 1 shows a hexagonal cellular space, and the coordinates employed here.

We now introduce a six-neighbor hexagonal partitioned cellular automaton (HPCA), which is a subclass of HCAs. A cell of an HPCA is divided into six parts (Fig. 2). They are north-east, east, south-east, south-west, west, and north-west parts, and have their own state sets, say A, B, C, D, E, and F. Therefore the state set of a whole cell is $A \times B \times C \times D \times E \times F$.

Each cell changes its state depending on the states of the six neighboring parts of the adjacent cells (*i.e.*, the south-west part of the north-east-adjacent cell, the west part of the east-adjacent cell, etc.) by a local transition function (Fig. 3). All

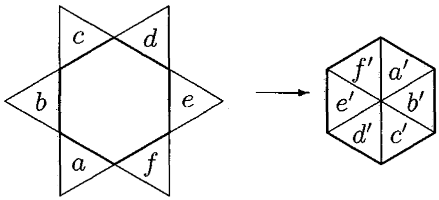


FIGURE 3. A state transition of a cell by a local function g such that $g(d,e,f,a,b,c) = (a',b',c',d',e',f')$.

the cells change their states synchronously. In this way, the whole cellular array changes its configuration.

An HPCA is formally defined as follows.

Definition 2.1. A *deterministic six-neighbor hexagonal partitioned cellular automaton* (HPCA) is defined by

$$H = (\mathbf{Z}^2, (A,B,C,D,E,F), g, (\#,\#,\#,\#,\#,\#)),$$

where $\mathbf{Z}$ is the set of all integers ($\mathbf{Z}^2$ is the set of two-dimensional points at which cells are placed), A,B,C,D,E, and F are non-empty finite sets of states of six parts of a cell, $g: D \times E \times F \times A \times B \times C \to A \times B \times C \times D \times E \times F$ is a local function, and $(\#,\#,\#,\#,\#,\#) \in A \times B \times C \times D \times E \times F$ is a quiescent state that satisfies $g(\#,\#,\#,\#,\#,\#) = (\#,\#,\#,\#,\#,\#)$.

A *configuration* over the set $Q = A \times B \times C \times D \times E \times F$ is a mapping $\alpha : \mathbf{Z}^2 \to Q$. Let $\mathrm{Conf}(Q)$ denote the set of all configurations over Q, *i.e.*, $\mathrm{Conf}(Q) = \{\alpha \mid \alpha : \mathbf{Z}^2 \to Q\}$.

Let $\mathrm{pro}_A : Q \to A$ is a projection function such that $\mathrm{pro}_A(a,b,c,d,e,f) = a$ for all $(a,b,c,d,e,f) \in Q$. We can also define a projection functions $\mathrm{pro}_X : Q \to X$ for each $X \in \{B,C,D,E,F\}$ similarly.

The *global function* $G : \mathrm{Conf}(Q) \to \mathrm{Conf}(Q)$ of H is defined as follows.

$$\begin{aligned}
&\forall (x,y) \in \mathbf{Z}^2 : \\
&G(\alpha)(x,y) \;=\; g(\mathrm{pro}_D(\alpha(x,y+1)), \mathrm{pro}_E(\alpha(x+1,y)), \mathrm{pro}_F(\alpha(x+1,y-1)), \\
&\qquad\qquad\qquad \mathrm{pro}_A(\alpha(x,y-1)), \mathrm{pro}_B(\alpha(x-1,y)), \mathrm{pro}_C(\alpha(x-1,y+1))).
\end{aligned}$$

In the following, an equation $g(d,e,f,a,b,c) = (a',b',c',d',e',f')$ depicted in Figure 3 is called a *rule* of H. We can also write it by

$$[d,e,f,a,b,c] \to [a',b',c',d',e',f'].$$

We regard the local function g as the set of such rules for convenience.

For the special case such that $A = B = C = D = E = F$, we can define the notions of rotation symmetry and reflection symmetry for HPCAs as follows.

Definition 2.2. Let $H = (\mathbf{Z}^2, (A,B,C,D,E,F), g, (\#,\#,\#,\#,\#,\#))$ be an HPCA. H is called a *rotation symmetric* HPCA iff (i) and (ii) hold.

(i) $A = B = C = D = E = F$.
(ii) $\forall (a,b,c,d,e,f), (a',b',c',d',e',f') \in A^6$:
if $g(d,e,f,a,b,c) = (a',b',c',d',e',f')$
then $g(e,f,a,b,c,d) = (b',c',d',e',f',a')$.

H is called a *reflection symmetric* HPCA iff H is rotation symmetric and (iii) holds.

(iii) $\forall (a,b,c,d,e,f), (a',b',c',d',e',f') \in A^6$:
if $g(d,e,f,a,b,c) = (a',b',c',d',e',f')$
then $g(c,b,a,f,e,d) = (f',e',d',c',b',a')$.

Intuitively speaking, a rotation symmetric HPCA is a one that obeies the same local (and global) function even if its space is rotated by 60, 120, 180, 240, or 300 degrees, and thus it is "isotropic". A reflection symmetric HPCA is a one that has the same local (and global) function as its mirror image.

Next, we define the notion of reversibility for HPCAs.

Definition 2.3. Let $H = (\mathbf{Z}^2, (A,B,C,D,E,F), g, (\#,\#,\#,\#,\#,\#))$ be an HPCA. We say H is *globally reversible* iff its global function G is one-to-one, and *locally reversible* iff its local function g is one-to-one.

We show the following proposition on HPCA. It is proved in a similar manner as in the one-dimensional case [6].

Proposition 2.4. *Let $H = (\mathbf{Z}^2, (A,B,C,D,E,F), g, (\#,\#,\#,\#,\#,\#))$ be an HPCA. H is globally reversible iff it is locally reversible.*

Proof. Let G be the global function of H. We first show the "if" part. Assume H is locally reversible but not globally reversible. Then, there are two different configurations α_1 and α_2 such that $G(\alpha_1) = G(\alpha_2)$. Thus, for any $(x,y) \in \mathbf{Z}^2$, $G(\alpha_1)(x,y) = G(\alpha_2)(x,y)$. Hence, by the definition of a global function

$$\begin{aligned} & g(\mathrm{pro}_D(\alpha_1(x,y+1)), \mathrm{pro}_E(\alpha_1(x+1,y)), \mathrm{pro}_F(\alpha_1(x+1,y-1)), \\ & \quad \mathrm{pro}_A(\alpha_1(x,y-1)), \mathrm{pro}_B(\alpha_1(x-1,y)), \mathrm{pro}_C(\alpha_1(x-1,y+1))) \\ &= g(\mathrm{pro}_D(\alpha_2(x,y+1)), \mathrm{pro}_E(\alpha_2(x+1,y)), \mathrm{pro}_F(\alpha_2(x+1,y-1)), \\ & \quad \mathrm{pro}_A(\alpha_2(x,y-1)), \mathrm{pro}_B(\alpha_2(x-1,y)), \mathrm{pro}_C(\alpha_2(x-1,y+1))) \end{aligned} \tag{1}$$

holds for all $(x,y) \in \mathbf{Z}^2$. On the other hand, there must be some $(x',y') \in \mathbf{Z}^2$ that satisfies the following condition, since $\alpha_1 \neq \alpha_2$.

$$\begin{aligned} & \mathrm{pro}_D(\alpha_1(x',y'+1)) \neq \mathrm{pro}_D(\alpha_2(x',y'+1)) \\ \vee\ & \mathrm{pro}_E(\alpha_1(x'+1,y')) \neq \mathrm{pro}_E(\alpha_2(x'+1,y')) \\ \vee\ & \mathrm{pro}_F(\alpha_1(x'+1,y'-1)) \neq \mathrm{pro}_F(\alpha_2(x'+1,y'11)) \\ \vee\ & \mathrm{pro}_A(\alpha_1(x',y'-1)) \neq \mathrm{pro}_A(\alpha_2(x',y'-1)) \\ \vee\ & \mathrm{pro}_B(\alpha_1(x'-1,y')) \neq \mathrm{pro}_B(\alpha_2(x'-1,y')) \\ \vee\ & \mathrm{pro}_C(\alpha_1(x'-1,y'+1)) \neq \mathrm{pro}_C(\alpha_2(x'-1,y'+1)). \end{aligned}$$

This contradicts the assumption of local reversibility, since the equation (1) must hold for x' and y'.

We next show the "only if" part. Assume H is globally reversible but not locally reversible. Then, there are $a_1, a_2 \in A$, $b_1, b_2 \in B$, $c_1, c_2 \in C$, $d_1, d_2 \in D$, $e_1, e_2 \in E$, $f_1, f_2 \in F$ that satisfy the following condition.

$$\begin{array}{ll} & g(d_1, e_1, f_1, a_1, b_1, c_1) = g(d_2, e_2, f_2, a_2, b_2, c_2) \\ \wedge & (a_1 \neq a_2 \ \vee \ b_1 \neq b_2 \ \vee \ c_1 \neq c_2 \ \vee \ d_1 \neq d_2 \ \vee \ e_1 \neq e_2 \ \vee \ f_1 \neq f_2). \end{array}$$

Let α_1 and α_2 be two configurations defined by

$$\begin{array}{ll} \mathrm{pro}_A(\alpha_1(0,-1)) = a_1, & \mathrm{pro}_A(\alpha_2(0,-1)) = a_2, \\ \mathrm{pro}_B(\alpha_1(-1,0)) = b_1, & \mathrm{pro}_B(\alpha_2(-1,0)) = b_2, \\ \mathrm{pro}_C(\alpha_1(-1,1)) = c_1, & \mathrm{pro}_C(\alpha_2(-1,1)) = c_2, \\ \mathrm{pro}_D(\alpha_1(0,1)) = d_1, & \mathrm{pro}_D(\alpha_2(0,1)) = d_2, \\ \mathrm{pro}_E(\alpha_1(1,0)) = e_1, & \mathrm{pro}_E(\alpha_2(1,0)) = e_2, \\ \mathrm{pro}_F(\alpha_1(1,-1)) = f_1, & \mathrm{pro}_F(\alpha_2(1,-1)) = f_2, \end{array}$$
$$\begin{array}{l} \mathrm{pro}_A(\alpha_1(x,y)) = \mathrm{pro}_A(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (0,-1), \\ \mathrm{pro}_B(\alpha_1(x,y)) = \mathrm{pro}_B(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (-1,0), \\ \mathrm{pro}_C(\alpha_1(x,y)) = \mathrm{pro}_C(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (-1,1), \\ \mathrm{pro}_D(\alpha_1(x,y)) = \mathrm{pro}_D(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (0,1), \\ \mathrm{pro}_E(\alpha_1(x,y)) = \mathrm{pro}_E(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (1,0), \text{ and} \\ \mathrm{pro}_F(\alpha_1(x,y)) = \mathrm{pro}_F(\alpha_2(x,y)) = \# \ \text{ for all } (x,y) \neq (1,-1). \end{array}$$

Apparently $\alpha_1 \neq \alpha_2$. Furthermore, the equation (1) holds for all $(x,y) \in \mathbf{Z}^2$ because of the above conditions. Thus $G(\alpha_1) = G(\alpha_2)$ is concluded, and this contradicts the assumption of global reversibility of H.

By Proposition 2.4, a globally or locally reversible HPCA is called simply "reversible" and denoted by RHPCA. By this, if we want to construct a reversible HCA, it is sufficient to give an HPCA whose local function g is one-to-one. This makes it easier to design a reversible HCA.

3. A UNIVERSAL 64-STATE RHPCA

Here, we give a specific RHPCA model H_1 that has logical universality. Each of six parts of a cell in H_1 has two states, hence a cell has 64 states in total. It is defined as follows.

$$H_1 = (\mathbf{Z}^2, \{0,1\}^6, g_1, (0,0,0,0,0,0))$$

H_1 is a rotation symmetric RHPCA, and its local function g_1 is defined as the set of rules shown in Figure 4. We can easily verify that H_1 is reversible and rotation symmetric. But, because of the existence of the rule (4), H_1 is not reflection symmetric. Note that only the rules (1)–(4), (8), (9), and (12) are used in the following construction of a Fredkin gate.

In order to show that H_1 is logically universal, it suffices to show that signal propagation, routing, signal delay, and a Fredkin gate are realizable in it.

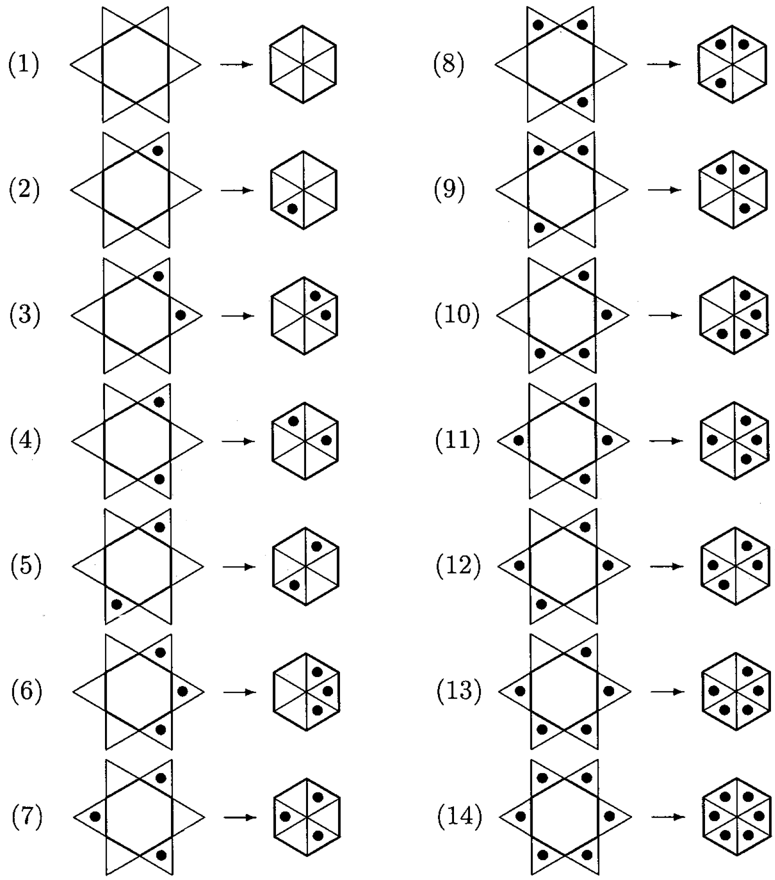


FIGURE 4. The set of rules of an RHPCA H_1, where 0 and 1 are represented by a blank and •, respectively (since H_1 is rotation symmetric, rotated rules are omitted here).

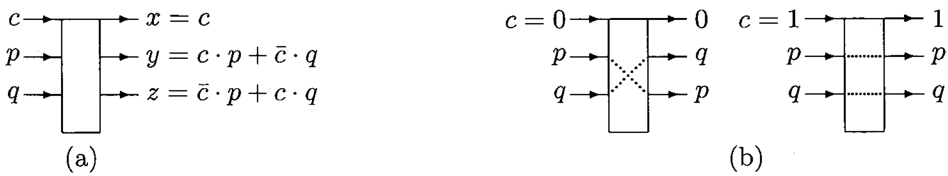
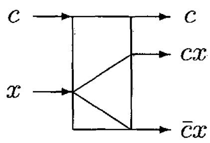


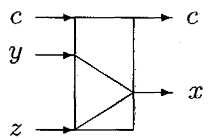
FIGURE 5. (a) A Fredkin gate, and (b) its function.

A Fredkin gate (F-gate) [3] is a reversible (*i.e.*, its logical function is one-to-one) and bit-conserving (*i.e.*, the number of 1's is conserved between inputs and outputs) logic gate shown in Figure 5. It has been known that any combinational logic



c	x	c	cx	$\bar{c}x$
0	0	0	0	0
0	1	0	0	1
1	0	1	0	0
1	1	1	1	0

(a)



c	y	z	c	x
0	0	0	0	0
0	0	1	0	1
1	0	0	1	0
1	1	0	1	1

(b)

FIGURE 6. (a) An S-gate, and (b) an inverse S-gate (the logical function of an inverse S-gate is not totally defined on $\{0,1\}^3$ as shown in the above table).

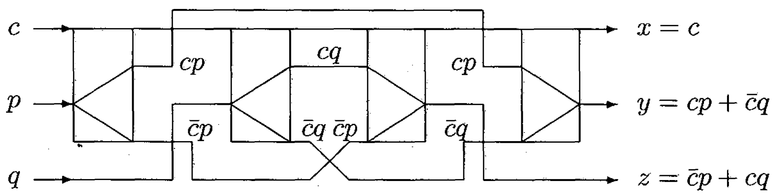


FIGURE 7. Construction of an F-gate by two S-gates and two inverse S-gates.

element (especially, AND, OR, NOT, and fan-out elements) can be realized only with F-gates [3]. Thus, any sequential circuit can be constructed from F-gates and delay elements. Furthermore, any reversible finite automaton, and reversible cellular automaton (hence reversible Turing machine) can be constructed only from F-gates and delays without generating garbage signals [7].

It is known that an F-gate can be composed of much simpler gates called switch gate (S-gate) [3]. An S-gate is also a reversible and bit-conserving gate (Fig. 6). An F-gate is constructed from two S-gates and two inverse S-gates as shown in Figure 7. Note that an inverse S-gate is a gate that realizes inverse logical function of the former.

In the following, we show how to realize (i) signal propagation and routing, (ii) a delay element, (iii) an S-gate and an inverse S-gate, and (iv) an F-gate, in the cellular space of H_1.

(i) Signal propagation and routing: a signal "1" is represented by a single dot "•" in this model. Such a dot goes straight ahead into the cellular space by the rule (2) as shown in Figure 8 if no obstacle exists. On the other hand, a signal "0" is represented by no existence of a dot (hence, some clock is assumed here).

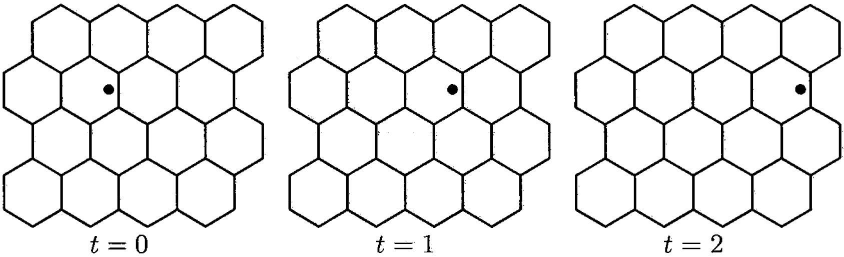


FIGURE 8. Signal propagation in the cellular space of H_1.

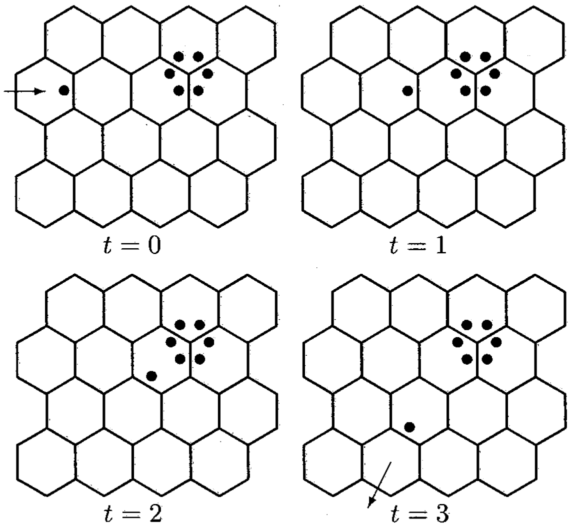


FIGURE 9. A right turn of a signal (by 120 degrees clockwise) at a block.

A signal "1" can make a right or left turn by 120 degrees by using a special pattern called "block". Figure 9 shows the process of right turn. A block is a pattern consisting of six dots, and is stable due to the rule (3). If a single dot reaches a block as in Figure 9, it is reflected by the rule (9), resulting in a right turn. A left turn is realized similarly (in this case the rule (8) is used). Note that, in H_1, a 60 degrees turn is not possible. Hence, a signal can travel only in three directions rather than six directions. But it is sufficient in order to reach any point in the plane. In this model crossing of two signals is easy (though delay elements may be needed to avoid a collision in some cases). By above, any signal routing is possible in this cellular space.

(ii) A delay element: It is realized by combining blocks as shown in Figure 10. By this method, delay elements whose delays are multiple of three units of time are available.

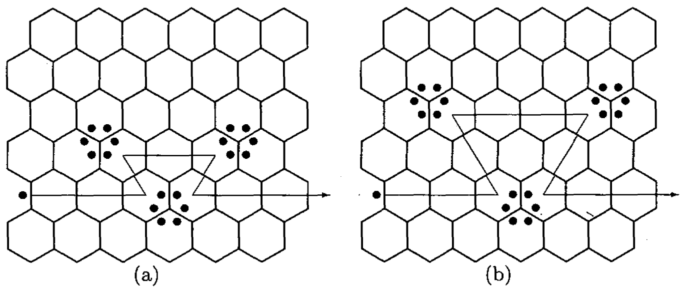


FIGURE 10. Delay elements (a) of three units of time, and (b) of six units of time.

(iii) An S-gate and an inverse S-gate: an S-gate can be simulated by a single cell of H_1. Figure 11a shows the input-output relation, and we can easily verify it. For example, if the input channels c and x both receive signal "1"s, then both the output channels c and cx give signal "1"s by the rule (4). An inverse S-gate can be also simulated by a single cell. It is shown in Figure 11b.

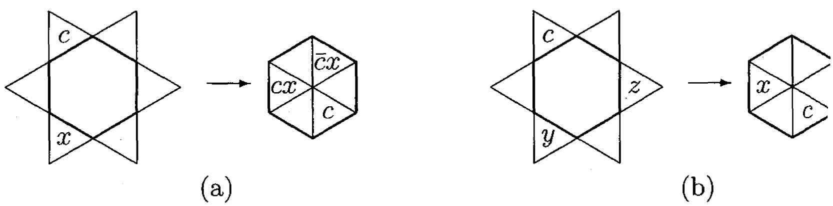


FIGURE 11. A single cell of H_1 can simulate both of (a) an S-gate, and (b) an inverse S-gate.

(iv) An F-gate: an F-gate can be embedded in the cellular space of H_1 by connecting two S-gates and two inverse S-gates appropriately. In order to synchronize signals at the gates, several delay elements are needed. Figure 12 shows the configuration of an F-gate. The size of the configuration is 28×17, and the delay of the gate is 58 units of time.

By above, logical universality of H_1 is concluded.

4. COMPARISON WITH OTHER RPCA MODELS

In this section, we compare the RHPCA model H_1 with other models of reversible PCAs having logical universality. There are two models on square grid

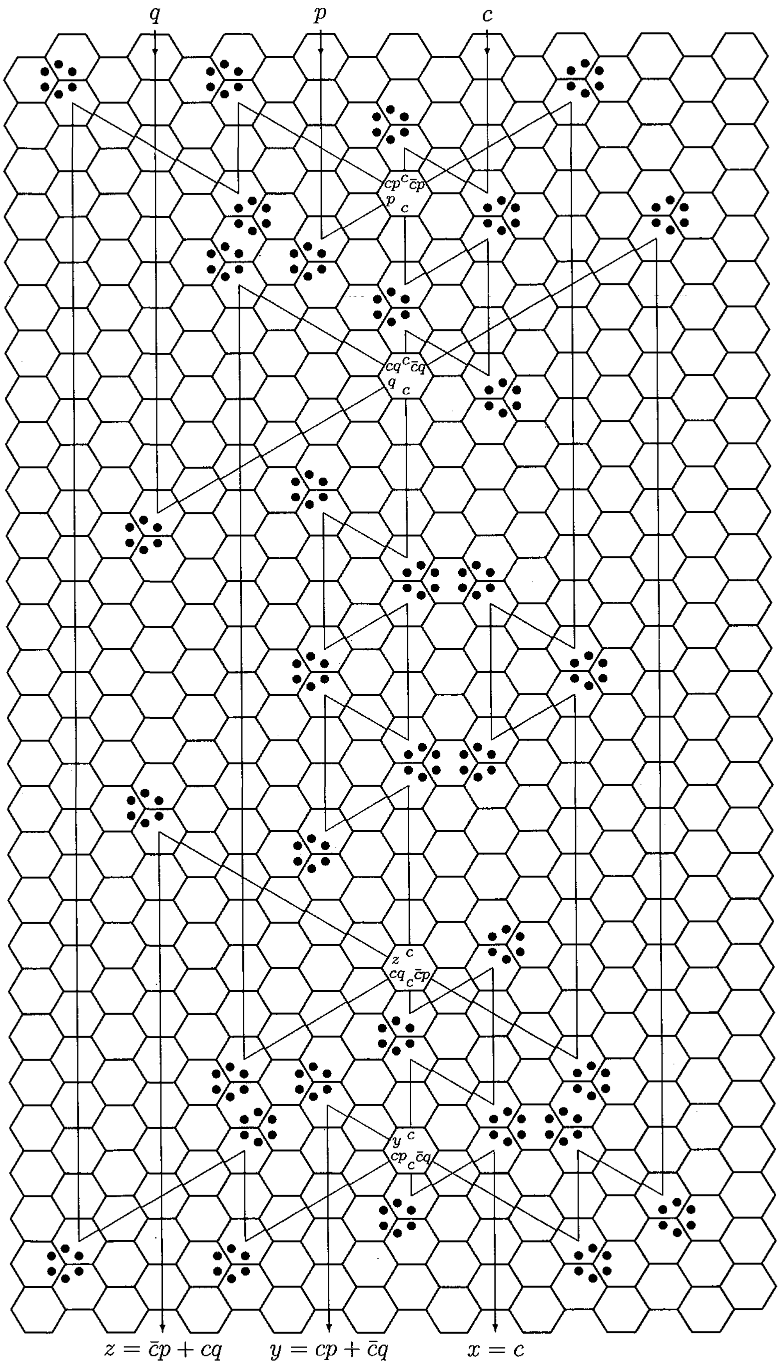


FIGURE 12. An F-gate embedded in the cellular space of H_1.

proposed in [8] (we call them S_1 and S_2 here), and a model (call it T_1) on triangular grid proposed in [4]. We discuss their features, especially how the realization methods of basic functions (*i.e.*, signal propagation, primitive logical operation, etc.) vary depending on the symmetry of the local function, and the shape of the tessellation.

The models S_1 and S_2 are 16-state RPCAs on a square grid. Both these models can simulate the Billiard Ball Model (BBM). The BBM is a kind of computation model in which a signal "1" is represented by an ideal ball, and logical operations can be performed by their elastic collisions and reflections by mirrors. For example, an S-gate can be realized in the BBM as shown in Figure 13 [3].

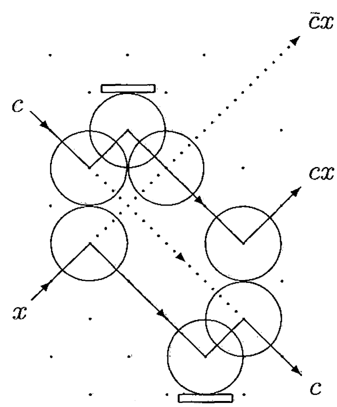


FIGURE 13. A switch gate in the BBM.

The model S_1 is a rotation and reflection symmetric RPCA having the set of transition rules as shown in Figure 14. (Note that this set of rules is essentially the same as the two-state "block cellular automaton" of Margolus [5], in which the BBM can be simulated, though the frameworks of these automata are very different.) In S_1, a ball of BBM is represented by two dots, and a collision of balls, which has finite non-zero radius, can be simulated by this (Fig. 15). Figure 16 shows a reflection of a ball by a mirror. In S_1, an F-gate configuration of size 34×58 has been obtained.

The model S_2 is a rotation symmetric but not reflection symmetric RPCA having the set of transition rules as shown in Figure 17. In S_2, the shape of a mirror and reflection by it are different from those of S_1, but the other features are similar. Figure 18 shows a configuration of an S-gate. In S_2, there is an F-gate configuration of size 30×62.

The model T_1 is an 8-state RPCAs on a triangular grid. It is a rotation symmetric but not reflection symmetric RPCA. Its local function is extremely

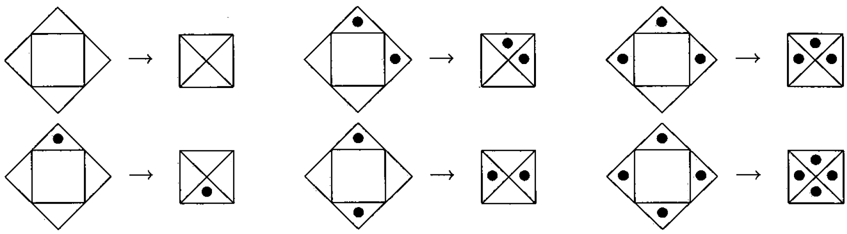


FIGURE 14. The local function of the rotation and reflection symmetric 16-state RPCA S_1.

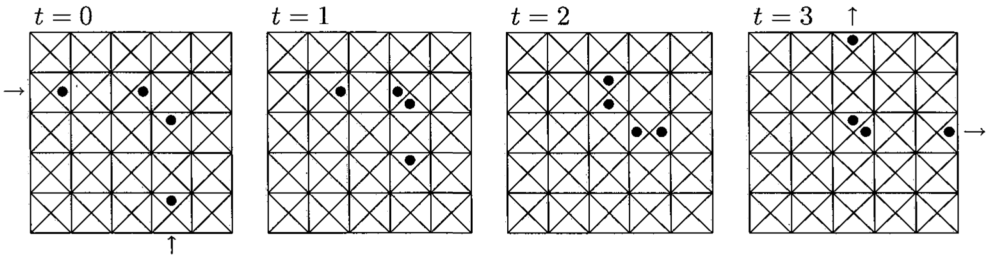


FIGURE 15. Collision of two balls in the 16-state RPCA model S_1.

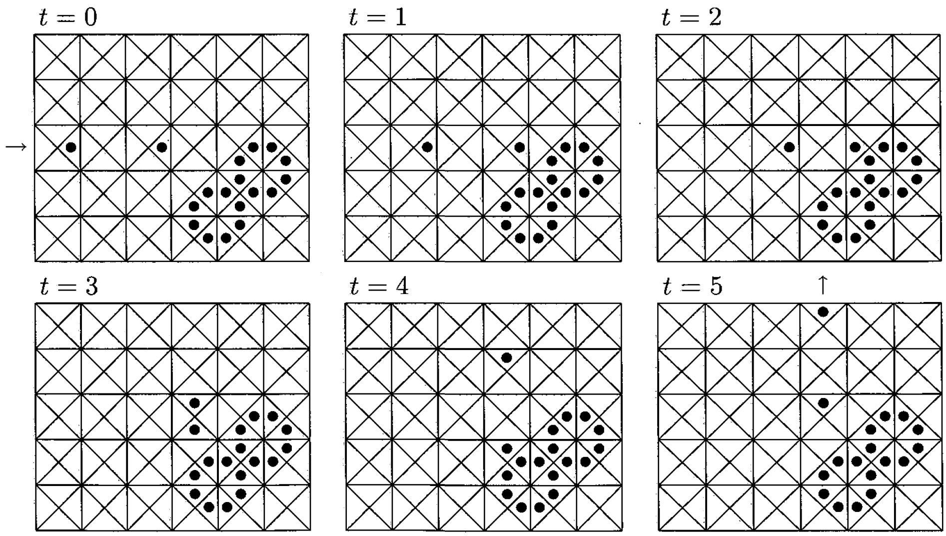


FIGURE 16. Reflection of a signal by a mirror in the 16-state RPCA model S_1.

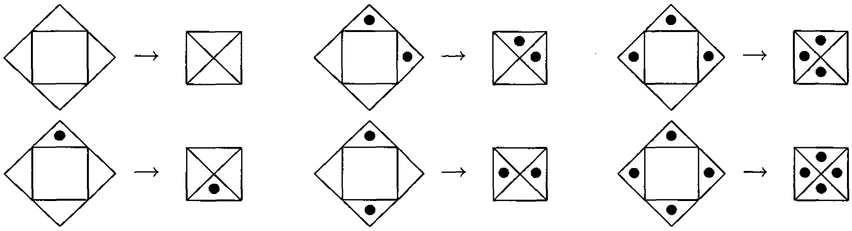


FIGURE 17. The local function of the rotation symmetric 16-state RPCA S_2.

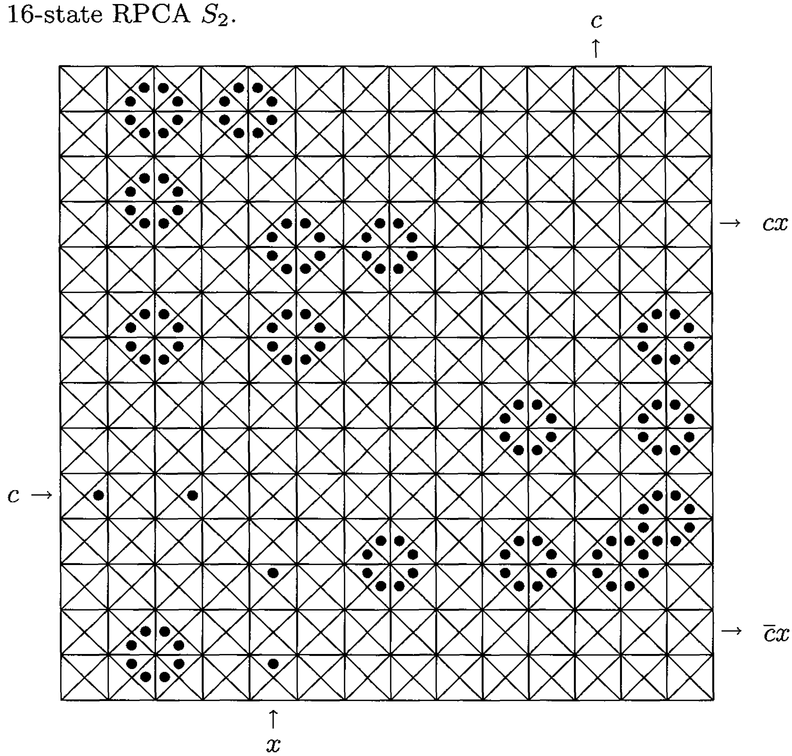


FIGURE 18. Realization of an S-gate in the 16-state RPCA model S_2.

simple as shown in Figure 19. As in H_1, a single cell of T_1 can directly simulate an S-gate and an inverse S-gate as in Figure 20. Therefore, there is no need to simulate BBM, hence a signal "1" can be represented by a single dot. But signal routing, crossing, and delay are very complex to realize, because a kind of "wall" is necessary to make a signal go straight. So the size of an F-gate configuration is very large (26×220).

These features of the four models are summarized in Table 1. If we use the framework of PCA, and assume rotation symmetry, then 64, 16, and 8 are the minimum number of states for each grid type, except the trivial 1-state PCAs.

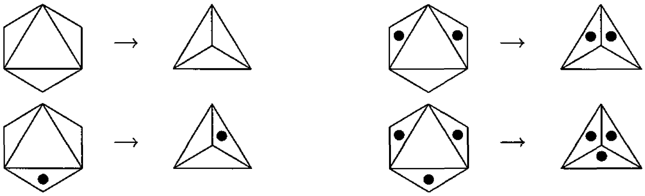


FIGURE 19. The local function of a rotation symmetric 8-state 3-neighbor triangular RPCA T_1.

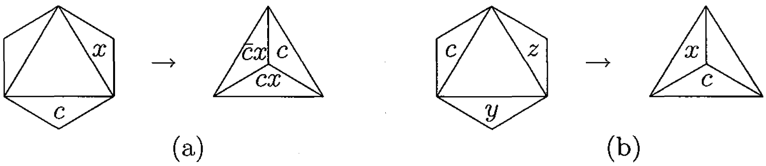


FIGURE 20. Realization of (a) an S-gate, and (b) an inverse S-gate by a single cell of T_1.

TABLE 1. Features of four universal RPCA models. (∗ means that the answer seems "no", but no formal proof has been obtained.)

	H_1	S_1	S_2	T_1
Grid type	hexagonal	square	square	triangular
Number of states	64	16	16	8
Rotation symmetric?	yes	yes	yes	yes
Reflection symmetric?	no	yes	no	no
Can embed the BBM?	∗	yes	yes	∗
Can simulate an S-gate by one cell?	yes	no	no	yes

Therefore, under the above assumption, these four are minimum state models having logical universality. But it is unknown whether there is a rotation-asymmetric universal RPCA having a smaller number of states for each grid type.

5. CONCLUDING REMARKS

In this paper, we proposed a new 64-state model of reversible HPCA H_1 having logical universality. There still remain various open problems as listed below.

1. Is there another model of 64-state reversible HPCA having logical universality? Especially, is there a rotation and reflection symmetric 64-state model?
2. Is there a model of 64-state reversible HPCA that can simulate the BBM? (Of course, the BBM on hexagonal grid is a little different from the usual one.)
3. Is there another model of logically universal 16-state reversible PCA on square grid? Especially, is there a model such that a signal "1" is represented by a single dot (rather than two dots)?
4. Is there a rotation-asymmetric (and simple) model having logical universality for each grid type?

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