

CAHIERS DE TOPOLOGIE ET GÉOMÉTRIE DIFFÉRENTIELLE CATÉGORIQUES

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Cahiers de topologie et géométrie différentielle catégoriques, tome 36, n° 2 (1995), p. 141-151

http://www.numdam.org/item?id=CTGDC_1995__36_2_141_0

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A NOTE ON \mathbf{r} -EMBEDDINGS
by Luciano STRAMACCIA

Résumé. Dans cet article on étudie la classe des espaces \mathbf{r} -compacts [10,12] et la notion connexe de sous-espace \mathbf{r} -plongé, où \mathbf{r} est un épiréfecteur topologique. Cette notion permet d'obtenir des résultats dans le contexte du problème de la conservation des produits topologiques par \mathbf{r} . De plus, elle donne une caractérisation des réflecteurs totaux, définis en [2,3].

Introduction.

Let $\mathbf{r} : \mathbf{Top} \rightarrow \mathbf{R}$ be a topological epireflector. In [10,12] we have introduced the category $\mathbf{r} - \mathbf{Comp}$ of \mathbf{r} -compact spaces, and the related \mathbf{r} -closure operator.

In this note we go on studying such classes of spaces. Since it turns out that \mathbf{r} -compactness is not an absolute property, then we are concerned, in particular, with a concept that overcomes such a difficulty, namely that of \mathbf{r} -embedded subset. A subset A of a space X is \mathbf{r} -embedded in X whenever $\mathbf{r}(S)$ is a subspace of $\mathbf{r}(X)$.

After presenting the general properties of \mathbf{r} -embeddings, we concentrate on two arguments.

First, we describe the close relation existing between the classes of \mathbf{r} -embeddings in \mathbf{Top} and that of embeddings in \mathbf{R} with respect to which \mathbf{r} is total, in the sense of [2]. Actually, \mathbf{r} is total with respect to the class of open embeddings if and only if every \mathbf{r} -open subset of a given space is \mathbf{r} -embedded there.

Furthermore, we give a characterization of the maximal class $T_{\mathbf{r}}$ of embeddings in \mathbf{R} , such that \mathbf{r} is total with respect to $T_{\mathbf{r}}$. We obtain a somewhat surprising result (Th.2.3) in this direction, by making use

Work partially supported by funds (40%), M.U.R.S.T., Italy.

of the notion of pullback complement [3].

Secondly, the concept of \mathbf{r} -embedding turns out to be useful in connection with the question whether \mathbf{r} preserves products. Given two \mathbf{r} -compact spaces X and Y , and two \mathbf{r} -embedded subspaces $A \subset X$, $B \subset Y$, then $\mathbf{r}(A \times B) = \mathbf{r}(A) \times \mathbf{r}(B)$ holds. Such kind of result applies to a great variety of cases and, in particular, when $\mathbf{r} = \tau$ is the Tychonoff modification functor.

1. \mathbf{r} -Compact Spaces and \mathbf{r} -Embeddings.

In this paper we shall be concerned with an epireflective subcategory \mathbf{R} of the category \mathbf{Top} of all topological spaces and maps, with reflector $\mathbf{r} : \mathbf{Top} \rightarrow \mathbf{R}$.

For every space $X \in \mathbf{Top}$, let $\mathbf{r}_X : X \rightarrow \mathbf{r}(X)$ be its onto reflection map, and let us decompose it as follows

$$\begin{array}{ccc}
 X & \xrightarrow{\mathbf{r}_X} & \mathbf{r}(X) \\
 \searrow \mathbf{j}_X & & \nearrow \mathbf{r}'_X \\
 & X_r &
 \end{array}$$

where X_r is the space having the same underlying set as X , with the initial topology induced by \mathbf{r}_X . Then \mathbf{j}_X acts as the identity, and $\mathbf{r}_X = \mathbf{r}'_X$ as set maps.

Such a procedure is functorial and gives a decomposition $\mathbf{r} = \mathbf{r}' \cdot \mathbf{j}$ of the reflector \mathbf{r} by means of the two functors $\mathbf{j} : \mathbf{Top} \rightarrow \mathbf{Top}_r$, and $\mathbf{r}' : \mathbf{Top}_r \rightarrow \mathbf{R}$. Here $\mathbf{Top}_r = \{X_r | X \in \mathbf{Top}\}$ is the bireflective hull of \mathbf{R} in \mathbf{Top} with bireflector $\mathbf{j} = \{j_X\}$, while \mathbf{r}' is the restriction of \mathbf{r} to \mathbf{Top}_r , in fact $\mathbf{r}'_X = \mathbf{r}_{X_r}$, for every space X . The topology on X_r is generated by what we called in [12] the \mathbf{r} -closure operator, defined as follows

$$cl_r(M) = \mathbf{r}_X^{-1}(cl(\mathbf{r}_X(M))),$$

where $M \subset X$ and cl denotes ordinary closure. M is \mathbf{r} -closed in X whenever $M = cl_r(M)$. A subset U of X will be called \mathbf{r} -open if

$X - U$ is r -closed there.

EXAMPLES 1.1. (a) When $\mathbf{r} : \mathbf{Top} \rightarrow \mathbf{Top}_o$ is the T_o -identification functor, then $U \subset X$ is \mathbf{r} -open if and only if U is open and $x \in U$ implies $cl\{x\} \subset U$.

In general, for a quotient reflector \mathbf{r} , the \mathbf{r} -open subsets of $X \in \mathbf{Top}$ coincide with those subsets S that are open and saturated with respect to the reflection map \mathbf{r}_X , that is $S = \mathbf{r}_X^{-1}(\mathbf{r}_X(S))$.

(b) Let $\tau : \mathbf{Top} \rightarrow \mathbf{Tych}$ be the Tychonoff modification functor [5,6]. The τ -open subsets of a space X are those which are union of cozero sets of X .

(c) Let $\mathbf{r} : \mathbf{Top} \rightarrow \mathbf{0 - dim}$ be the reflector functor on the category of zero-dimensional spaces; in such a case the \mathbf{r} -open subsets of $X \in \mathbf{Top}$ are those subsets that are union of clopen subsets of X .

(d) Let $\mathbf{R} \subset \mathbf{Top}$ be such that the Salbany closure operator [9](called \mathbf{R} -closure) induced by it, coincides with the ordinary closure on each $X \in \mathbf{R}$. This happens, e.g., for $\mathbf{R} = \mathbf{Haus}$, the category of Hausdorff spaces, $\mathbf{R} = \mathbf{Reg}$, the category of regular spaces, $\mathbf{R} = \mathbf{Tych}$, $\mathbf{R} = \mathbf{0 - dim}$, etc. In such a case the \mathbf{r} -open subsets of X are the same as the \mathbf{R} -open subsets. For the general situation see [11].

A topological space X is said to be \mathbf{r} -compact whenever its reflection $\mathbf{r}(X)$ in \mathbf{R} is a compact space (no separation axiom is assumed); this amounts to say that every cover of X by \mathbf{r} -open subsets has a finite subcover.

The category $\mathbf{r - Comp} \subset \mathbf{Top}$ of \mathbf{r} -compact spaces has been studied in [10,12], where we are concerned with the question whether \mathbf{r} preserves topological products.

Recently, \mathbf{r} -compact spaces have been studied further in [8].

\mathbf{r} -compactness is not an absolute property, as shown by the following example concerning the Tychonoff reflector τ .

EXAMPLE 1.2. Let X be the space having the closed unit interval I as underlying set and a topology which differs from the usual one in that the neighbourhoods of the point 0 do not contain points of the

set $A = \{\frac{1}{n} : n \in \mathbf{N}\}$.

X is a non-compact, τ -compact space, in fact one has $\tau(X) = X_\tau = I$. If we consider the subset $S = A \cup \{0\}$ of X , then S is relatively τ -compact with respect to X , but it is not τ -compact considered as a topological space, with the topology inherited from X .

In order to avoid this kind of situation, it is useful to introduce the notion of \mathbf{r} -embedding, which also turns out to be of interest in connection with other concepts, as we shall see below.

DEFINITION 1.3. A subset S of a space X is said to be \mathbf{r} -embedded in X provided that every \mathbf{r} -open subset of S is the intersection of an \mathbf{r} -open subset of X with S .

THEOREM 1.4. For a subset S of a space X , the following are equivalent:

- (1) S is \mathbf{r} -embedded in X .
- (2) S_τ is a subspace of X_τ .
- (3) $\mathbf{r}(S)$ is a subspace of $\mathbf{r}(X)$.
- (4) $cl_{r,S}(M) = cl_{r,X}(M) \cap S$, for every $M \subset S$.

PROOF: The equivalence of (1) and (2) follows directly from the definitions.

(2) \rightarrow (3). Let s be the embedding of S in X . We have the following situation: $\mathbf{r}(s_r) \cdot \mathbf{r}_{S_r} = \mathbf{r}_{X_r} \cdot s_r$, where s_r is the embedding of S_r in X_r and $\mathbf{r}_{X_r} \cdot s_r, \mathbf{r}(s_r)$ are initial maps. Our first task is to show that $\mathbf{r}(s_r)$ is also initial. To this end, let $V \subset \mathbf{r}(S)$ be an open subset; then there exists an open subset $T \subset \mathbf{r}(X)$ such that

$$\mathbf{r}_{S_r}^{-1}(V) = (\mathbf{r}_{S_r} \cdot s_r)^{-1}(T) = (\mathbf{r}(s_r) \cdot \mathbf{r}_{S_r})^{-1}(T) = \mathbf{r}_{S_r}^{-1}(\mathbf{r}(s_r)^{-1}(T)).$$

Since \mathbf{r}_{S_r} is onto, we obtain that $V = \mathbf{r}(s_r)^{-1}(T)$. For $\mathbf{r}(s_r)$ to be an embedding it remains to show that it is injective. In order to do this let us recall [7] that \mathbf{R} is either bireflective in \mathbf{Top} or it is contained in the category \mathbf{Top}_o of T_o -spaces; in the former case $\mathbf{r}(s)$ must be injective, hence an embedding. In the latter case $\mathbf{r}(S)$ is a T_o -space. If x and y are two distinct points in $\mathbf{r}(S)$ then there is an open subset U of $\mathbf{r}(S)$ such that $x \in U$ and $y \notin U$. By the first part of the proof there

exists an open set $V \subset \mathbf{r}(X)$ such that $U = \mathbf{r}(s)^{-1}(V)$. It follows that $\mathbf{r}(s)(x)$ and $\mathbf{r}(s)(y)$ must be different.

(3) \rightarrow (4). Let us recall that $cl_{r,S}(M)$ is given by the intersection of all \mathbf{r} -closed subsets F of S that contain M . Every such F is of the form $F = \mathbf{r}_S^{-1}(F')$, being F' a closed subset of $\mathbf{r}(S)$. Note also that, by assumption, F' is of the form $F' = \mathbf{r}(s)^{-1}(F'')$ for some closed subset F'' of $\mathbf{r}(X)$. Then one has $\mathbf{r}_S^{-1}(\mathbf{r}(s)^{-1}(F'')) = s^{-1}(\mathbf{r}_X^{-1}(F'')) = \mathbf{r}_X^{-1}(F'') \cap S$. It follows

$$\begin{aligned} cl_{r,S}(M) &= \bigcap \{ \mathbf{r}_S^{-1}(F') : M \subset F \} = \bigcap \{ \mathbf{r}_S^{-1}(\mathbf{r}(s)^{-1}(F'')) : M \subset F \} = \\ &= \bigcap \{ s^{-1}(\mathbf{r}_X^{-1}(F'')) : M \subset F \} = \bigcap \{ \mathbf{r}_X^{-1}(F'') \cap S : M \subset F \} = \\ &= \bigcap \{ \mathbf{r}_X^{-1}(F'') : M \subset F \} \cap S = cl_{r,S}(M) \cap S. \end{aligned}$$

(4) \rightarrow (1). This is immediate.

COROLLARY 1.5. *Let S be an \mathbf{r} -embedded subspace of X . Then:*

- (1) $\mathbf{r}(S) = \mathbf{r}_X(S)$ and $\mathbf{r}_S = \mathbf{r}_{X|S}$.
- (2) S is \mathbf{r} -compact iff it is \mathbf{r} -compact relatively to X .

COROLLARY 1.6. *Every retract of X is \mathbf{r} -embedded in X .*

EXAMPLES 1.7. (a) Let \mathbf{r} be a quotient reflector with $\mathbf{r}_X : X \rightarrow \mathbf{r}(X) = X / \sim$, for every $X \in \mathbf{Top}$. A subset $M \subset X$ is then \mathbf{r} -embedded exactly when M / \sim is a subset of X / \sim . It follows that a sufficient condition in order that M be \mathbf{r} -embedded in X is that M is either open or closed in X and saturated with respect to the quotient map \mathbf{r}_X .

If \mathbf{r} is the T_0 -identification functor, then every closed subset of X is \mathbf{r} -embedded in X , as one can easily verify.

(b) Recall that a subset S of a space X is z -embedded [1] whenever every cozero set in S is the intersection with S of a cozero set in X . Hence it is clear that every z -embedded subset is also τ -embedded. Moreover, since every cozero set is z -embedded, it follows that every τ -open subset of X is τ -embedded.

REMARK 1.8. *In the non-surjective case of the Hewitt realcompactification functor $\nu : \mathbf{Top} \rightarrow \mathbf{r-Comp}$, the concept of ν -embedding coincides with that defined in [1].*

The situation described in 1.7.(b) above is not typical of τ , but it is fairly general, since it is shared by all those reflectors that are *total* (in the sense of [2]), with respect to the class of open embeddings. Besides τ , other examples are the reflectors onto the categories **Haus**, **Reg**, **0 – dim**.

Let us recall that the epireflector \mathbf{r} is said to be *total* [2,3] with respect to a class \mathbf{S} of topological embeddings, whenever, given an embedding $s : S \rightarrow \mathbf{r}(X)$, with $s \in \mathbf{S}$, then the restriction of \mathbf{r}_X to $\mathbf{r}_X^{-1}(S)$, that is the map

$$\mathbf{r}_{X|\mathbf{r}_X^{-1}(S)} : \mathbf{r}_X^{-1}(S) \rightarrow S$$

is uniquely \mathbf{R} -extendable, hence a reflection map.

PROPOSITION 1.9. *The epireflector \mathbf{r} is total with respect to the class of open embeddings iff, for every $X \in \mathbf{Top}$, every \mathbf{r} -open subset of X is \mathbf{r} -embedded in X .*

PROOF: Let \mathbf{r} be total with respect to the class of open embeddings and let A be an \mathbf{r} -open subset of X . Then $\mathbf{r}_X(A)$ is open in $\mathbf{r}(X)$ and $A = \mathbf{r}_X^{-1}(\mathbf{r}_X(A))$. It follows that $\mathbf{r}_{X|A} : A = \mathbf{r}_X^{-1}(\mathbf{r}_X(A)) \rightarrow \mathbf{r}_X(A)$ is the reflection map for A , hence $\mathbf{r}(A) = \mathbf{r}_X(A)$ and $\mathbf{r}_A = \mathbf{r}_{X|A}$, that is A is \mathbf{r} -embedded in X .

Conversely, assume that, for any space X , every \mathbf{r} -open subset of X is \mathbf{r} -embedded. Let $s : S \rightarrow \mathbf{r}(X)$ be an open embedding; then $\mathbf{r}_X^{-1}(S)$ is \mathbf{r} -open in X . It follows that $\mathbf{r}_{X|\mathbf{r}_X^{-1}(S)} : \mathbf{r}_X^{-1}(S) = \mathbf{r}_X^{-1}(\mathbf{r}_X(\mathbf{r}_X^{-1}(S))) \rightarrow S$ is a reflection map. This shows that \mathbf{r} is total with respect to the class of open embeddings.

2. Further Properties of \mathbf{r} -Embeddings.

Let the epireflector $\mathbf{r} : \mathbf{Top} \rightarrow \mathbf{R}$ be given and let us denote by $T_{\mathbf{r}}$ the maximal class of topological embeddings such that \mathbf{r} is total with respect to it.

A natural question that arises is then to characterize the class $T_{\mathbf{r}}$.

The next results deal with this problem.

PROPOSITION 2.1. *Let $X \in \mathbf{Top}$. An embedding $s : S \rightarrow \mathbf{r}(X)$ is an element of $T_{\mathbf{r}}$ if and only if there exists a subset A of X such that:*

- (1) $S = \mathbf{r}_X(A)$.
- (2) A is saturated with respect to \mathbf{r}_X .
- (3) A is \mathbf{r} -embedded in X .

PROOF: The proof is straightforward and depends essentially on the definition of \mathbf{r} -embedding.

The following theorem indicates how one must choose the subset A of X , involved in the previous proposition. It make use of the notion of pullback complement [3] which we recall for sake of completeness.

DEFINITION 2.2. *Let $f : A \rightarrow U$ and $s : U \rightarrow Y$ be morphisms in a category \mathbf{C} . A pullback complement of the pair (f, s) is a pair (s', f') in such a way that*

$$\begin{array}{ccc}
 A & \xrightarrow{f} & U \\
 s' \downarrow & & \downarrow s \\
 P & \xrightarrow{f'} & Y
 \end{array}$$

is a pullback square with the property that for every other pullback square

$$\begin{array}{ccc}
 V & \xrightarrow{g} & U \\
 t \downarrow & & \downarrow s \\
 T & \xrightarrow{g'} & Y
 \end{array}$$

and any morphism $h : V \rightarrow A$ with $f \cdot h = g$, there is a unique $h' : T \rightarrow P$ such that $f' \cdot h' = g'$ and $s' \cdot h = h' \cdot t$.

THEOREM 2.3. *Let $a : A \rightarrow X$ be an embedding that is saturated with respect to \mathbf{r}_X . Then a is an \mathbf{r} -embedding if and only if the pair $(\mathbf{r}_A, \mathbf{r}(a))$ is a pullback complement of (a, \mathbf{r}_X) .*

PROOF: Assume that a is a saturated \mathbf{r} -embedding with respect to \mathbf{r}_X ; then the following diagram is a pullback:

$$\begin{array}{ccc}
 A & \xrightarrow{a} & X \\
 \mathbf{r}_A \downarrow & & \downarrow \mathbf{r}_X \\
 \mathbf{r}(A) & \xrightarrow{\mathbf{r}(a)} & \mathbf{r}(X)
 \end{array}$$

Suppose there is another pullback square

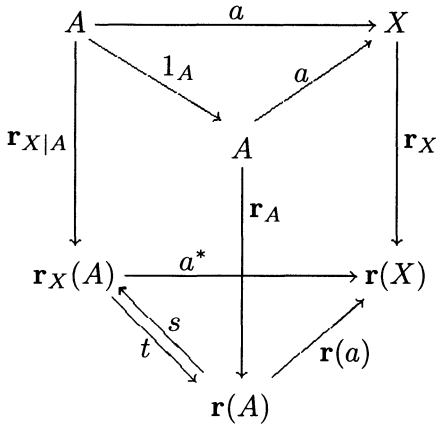
$$\begin{array}{ccc}
 V & \xrightarrow{g} & X \\
 t \downarrow & & \downarrow \mathbf{r}_X \\
 T & \xrightarrow{g^*} & \mathbf{r}(X)
 \end{array}$$

and a map $h : V \rightarrow A$ with $a \cdot h = g$.

By commutativity it follows $g^* \cdot t = \mathbf{r}_X \cdot g = \mathbf{r}_X \cdot a \cdot h = \mathbf{r}(a) \cdot \mathbf{r}_A \cdot h$. Because of Theorem 1.4(3) we know that $\mathbf{r}(a)$ is an embedding. Moreover, t as a pullback of the onto map \mathbf{r}_X is onto as well. Now the fact that onto maps and embeddings form a factorization system implies the existence of the required $h^* : T \rightarrow \mathbf{r}(A)$; it arises as a "diagonal fill-in". This concludes the first half of the proof.

Conversely, assume that $a : A \rightarrow X$ is a saturated embedding such that the first square is a pullback complement.

Consider the following diagram



where a^* is the inclusion.

By the universal property of the reflection there is a unique $s : r(A) \rightarrow r_X(A)$ such that $s \cdot r_A = r_{X|A}$; by the universal property of the pullback complement, there is a unique $t : r_X(A) \rightarrow r(A)$ such that $r(a) \cdot t = a^*$. Then $r(a) \cdot t \cdot r_{X|A} = a^* \cdot r_{X|A} = r_X \cdot a$; by the universal property of the right outer pullback square, it follows that 1_A must be the unique map which renders the left outer square commutative, that is, $r_A = t \cdot r_{X|A}$. Finally, the composition $t \cdot s$ must be the identity, hence s is an onto map having a left inverse; s is a homeomorphism, and this concludes the proof.

Let now X and Y be two topological spaces. One says that the epireflector r preserves their product and writes $r(X \times Y) = r(X) \times r(Y)$, whenever the unique map $t_{X \times Y} : r(X \times Y) \rightarrow r(X) \times r(Y)$ such that $t_{X \times Y} \cdot r_{X \times Y} = r_X \times r_Y$, which is induced by the universal property of the reflection, is a homeomorphism. This happens, e.g., whenever X and Y are r -compact spaces.

In [12] it was shown that r preserves the product of X and Y if and only if every r -open subset of $X \times Y$ is union of subsets of the form

$A \times B$, where $A \subset X$ and $B \subset Y$ are \mathbf{r} -open.

THEOREM 2.4. *Let X and Y be topological spaces \mathbf{r} -embedded in \mathbf{r} -compact spaces X^* and Y^* , respectively. Then $\mathbf{r}(X \times Y) = \mathbf{r}(X) \times \mathbf{r}(Y)$ iff $X \times Y$ is \mathbf{r} -embedded in the product space $X^* \times Y^*$.*

PROOF: Let us assume that $X \times Y$ is \mathbf{r} -embedded in the product $X^* \times Y^*$. Let $P \subset X \times Y$ be an \mathbf{r} -open subset, then there is a \mathbf{r} -open subset $Q \subset X^* \times Y^*$ such that $P = (X \times Y) \cap Q$. For every $q \in Q$, there exists a rectangular \mathbf{r} -open set $A_q \times B_q$ in $X^* \times Y^*$ such that $q \in A_q \times B_q$. It follows that $Q = \cup_{q \in Q} A_q \times B_q$, therefore $P = \cup_{q \in Q} (X \times Y) \cap (A_q \times B_q) = \cup_{q \in Q} ((X \cap A_q) \times (Y \cap B_q))$, where $(X \cap A_q)$ and $(Y \cap B_q)$ are \mathbf{r} -open subsets in X, Y , respectively. Hence P can be written as a union of rectangular \mathbf{r} -open sets of $X \times Y$.

Conversely, assume that $\mathbf{r}(X \times Y) = \mathbf{r}(X) \times \mathbf{r}(Y)$, then every \mathbf{r} -open subset of $X \times Y$ is a union of rectangular \mathbf{r} -open subsets. Let $P \subset X \times Y$ be an \mathbf{r} -open subset; there are \mathbf{r} -open subsets $A_i \subset X$ and $B_i \subset Y, i \in I$, such that $P = \cup_{i \in I} A_i \times B_i$. By assumption there are \mathbf{r} -open subsets $A_i^* \subset X^*$ and $B_i^* \subset Y^*$ with $A_i = X \cap A_i^*, B_i = Y \cap B_i^*$. Now $P^* = \cup_{i \in I} A_i^* \times B_i^*$ is an \mathbf{r} -open subset of $X^* \times Y^*$ with the property that $P = (X \times Y) \cap P^*$.

COROLLARY 2.5. *Assume that \mathbf{r} is total with respect to the class of open embeddings and let X, Y be \mathbf{r} -compact spaces. For any pair of \mathbf{r} -open subsets $A \subset X$ and $B \subset Y$, we have $\mathbf{r}(A \times B) = \mathbf{r}(A) \times \mathbf{r}(B)$.*

REMARKS 2.6. (a) *The category of w -compact spaces, defined and studied in [5,6], is contained in τ -Comp. Since τ is total with respect to the open embeddings, because of Proposition 1.9, Corollary 2.5 applies to any two (unions of) cozero sets $A \subset X$ and $B \subset Y$, whenever X and Y are w -compact.*

The same is true, e.g., when A, B are Lindeloff subspaces of X, Y , respectively, since then they are z -embedded [1], hence τ -embedded.

(b) *The corollary above applies to most of the usual situations; see, in fact, [2] for a list of topological epireflectors that are total with respect to the class of open embeddings.*

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