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THE MATHEMATICAL WORK OF JEAN-MICHEL BISMUT: A BRIEF SUMMARY

Jean-Michel Bismut's mathematical career covers two apparently different branches of mathematics: probability theory and global analysis on manifolds. In these two fields, Bismut has made fundamental contributions which have had widespread influence.

In what follows, we will list only a few significant contributions of Bismut in these two fields.

1. From probability theory...

Among many fundamental contributions of Bismut in probability, let us mention the following three:

1.1. — In [**3**], Bismut established a stochastic version of Pontryagin's maximum principle in control theory. As in its deterministic counterpart, the controlled process $x.$ verifies an initial condition, while the corresponding dual process $p.$ satisfies a final condition. Still, both processes have to be adapted to the given filtration. Bismut gave applications of his principle in [**6, 13, 29, 30**]. In [**8**], he applied this principle to mathematical models of economic growth.

Backward stochastic differential equations such as the ones verified by $p.$ have been the starting point of an intensive research in stochastic analysis. Pardoux and Peng, elaborating on Bismut's ideas, built a new theory of such equations, which they applied to mathematical finance. Also Bismut's theory of the hypoelliptic Laplacian (which will be explained later) is partly built on the above considerations.

1.2. — In [**47**], using the the quasi-invariance of the Brownian measure, Bismut gave a new approach to the Malliavin calculus for continuous diffusions. In [**55**] Bismut developed a Malliavin calculus for point processes. This has led to an extension by various authors of Hörmander's theorem to nonlocal operators.

1.3. — In the difficult book [**64**] entitled "*Large deviations and the Malliavin Calculus*", Bismut gave a new development to the Malliavin Calculus. In particular, he established his celebrated integration by parts for the Brownian motion on a Riemannian manifold. In [**64**], Bismut pioneered the relationship between the Malliavin Calculus and the theory of large deviations, and introduced his well-known nondegeneracy condition for subelliptic heat kernels. This led to unexpected results in the study of subelliptic heat kernels which cannot be obtained by the traditional tools of microlocal analysis.

This book also marks a turning point in Bismut's work in the direction of index theory.

2. ... to Index Theory

The starting point of Bismut's work in index theory is Atiyah's article (2) on Witten's heuristic proof of the index theorem for Dirac operators. This article exhibits a formal link between the localization formulas of Duistermaat-Heckman in equivariant cohomology and the index theorem. This led Bismut to give a probabilistic proof of the local version of the Atiyah-Singer index theorem for Dirac operators [**60**], by using the methods developed in [**64**], and also to give a new proof in [**75**] of the localization formulas which is parallel to the heat equation proof of the index theorem for Dirac operators.

Quillen wrote two fundamental papers (3) on superconnections on $\mathbf{Z}_2$-graded vector bundles and on metrics on the determinant line bundle. These two papers have had a dramatic influence on Bismut's subsequent work.

We now list a few other significant contributions of Bismut to index theory.

2.1. Superconnections, Quillen metrics and η-invariants. — In [**74**], Bismut gave a heat equation proof of a local version of the Atiyah-Singer families index theorem for Dirac operators. He introduced the by now well-known "Bismut superconnection" associated naturally to a fibered manifold. The Bismut superconnection plays a central role in modern aspects of index theory. It is used in the construction of the families version of classical spectral invariants, such as the analytic torsion of Ray-Singer and the η-invariant of Atiyah-Patodi-Singer.

In [**78, 79**], Bismut and Freed developed the theory of Quillen metrics on the smooth determinant line bundle associated with a family of Dirac operators, and they

(2) M. F. Atiyah, Circular symmetry and stationary-phase approximation, *Astérisque* **131** (1985), 43–59.

(3) D. Quillen, Superconnections and the Chern character, *Topology* **24** (1985), 89–95; D. Quillen, Determinants of Cauchy-Riemann operators on Riemann surfaces, *Functional Anal. Appl.* **19**, (1985), 31–34.

constructed a canonical unitary connection on this line bundle. Bismut and Freed proved a local curvature formula for this connection, which generalizes the formula obtained by Quillen. As an application, Bismut and Freed obtained a rigorous proof of the holonomy theorem conjectured by Witten, connecting the holonomy of the determinant line bundle to the η-invariant.

In a series of papers with Jeff Cheeger (starting with [**98**]), Bismut studied the adiabatic limit of the eta invariant for a fibered manifold. This result extends his results with Freed to arbitrary fibrations. With Cheeger, he defined the $\tilde{\eta}$ form, which generalizes Atiyah-Patodi-Singer's η invariant to families, as a transgression of the superconnection forms. Bismut and Cheeger also established the extension to families of the Atiyah-Patodi-Singer index theorem for manifolds with boundary.

2.2. Analytic torsion and complex geometry. — In [**90, 91, 92**], Bismut, Gillet and Soulé established a curvature theorem for the Quillen metric on the holomorphic determinant of a direct image by a holomorphic proper submersion, and they also established anomaly formulas for such metrics, which extend Polyakov's anomaly formulas for Riemann surfaces. Here the Quillen metric on the determinant of the cohomology is the product of the L_2-metric by the Ray-Singer analytic torsion. They also defined higher holomorphic analytic torsion forms, which are a families version of the holomorphic analytic torsion of Ray-Singer.

If $i : Y \to X$ is an embedding of compact complex manifolds, if E is a holomorphic vector bundle on Y, and if F is a complex of holomorphic vector bundles on X which gives a projective resolution of i_*E, Bismut and Lebeau [**115**] established an *embedding formula*, which compares the Quillen metric on the determinant of the cohomology of E on Y with the Quillen metric on the determinant of the cohomology of F on X. Bott-Chern classes appear in the final formula, as well as the R-genus of Gillet and Soulé. A crucial ingredient is the paper [**103**], in which Bismut obtained the R genus in terms of the fibrewise harmonic oscillator of a vector bundle. The results of Bismut and Lebeau play a crucial role in the proof by Gillet and Soulé of an *arithmetic Riemann-Roch theorem* in Arakelov theory, in which analytic torsion is an essential ingredient in the definition of the direct image. They are also important for other questions of arithmetic geometry.

The techniques used in the above papers are formidable. They have had many applications to other problems in index theory. For example, Bismut and Zhang [**123**] gave a new proof of the Cheeger-Müller theorem, which relates the (real) Ray-Singer analytic torsion of a flat vector bundle with the Reidemeister torsion. This proof uses the Witten deformation of the de Rham complex, with ideas partly inspired from [**115**]. Bismut and Zhang's result is also valid for flat vector bundles which are not unitarily flat. Also Bismut and Lott [**136**] proved a Riemann-Roch-Grothendieck formula for flat vector bundles, and defined analytic Ray-Singer torsion forms which generalize the (real) Ray-Singer analytic torsion.

2.3. From loop spaces to the hypoelliptic Laplacian. — We have mentioned that Atiyah's article on Witten's heuristic loop space proof of the index theorem for Dirac operators was the starting point of Bismut's work in index theory. Bismut [**67**] developed a complete analogy between formal equivariant cohomology of the loop space and his work in index theory (see also [**169**]).

More recently, he started a courageous program to understand the Hodge theory of the loop space of a Riemannian manifold. This led him to develop a theory of the hypoelliptic Laplacian in de Rham theory in [**160**], and to construct a hypoelliptic deformation of the Dirac operator in [**164**]. The hypoelliptic Laplacian, an operator acting on the cotangent bundle of the considered manifold, interpolates between a standard Hodge Laplacian on the original manifold and the generator of the geodesic flow on the cotangent bundle.

One motivation was to extend to loop spaces the Witten deformation of the Hodge Laplacian in de Rham theory as in the proof by Bismut-Zhang of the Cheeger-Müller theorem. Bismut found that, at least formally, a conjecture by Fried, which relates analytic torsion to special values of dynamical zeta functions, should be thought of as an infinite dimensional version of the Cheeger-Müller theorem, in which the Morse function defining the Witten deformation would be the energy. The hypoelliptic Laplacian appears as a semiclassical version of the Witten deformation of the Hodge Laplacian on the loop space.

The rigorous analytic theory is developed in a recent book of Bismut and Lebeau [**168**]. In their book, Bismut and Lebeau give a comparison formula relating the elliptic and hypoelliptic Ray-Singer metrics on the determinant of the cohomology of a flat vector bundle. In [**164**], similar results were obtained by Bismut for the Dirac operator and for Quillen metrics.

Bismut gave a simple application of the hypoelliptic Laplacian in [**165**] to compact Lie groups. In his original paper, Atiyah had suggested that the formula for the heat kernel on a compact Lie group in terms of the lattice of coroots could be possibly interpreted as the consequence of localization formulas over coadjoint orbits for the associated Kac-Moody algebra. Bismut shows that this is indeed the case, the hypoelliptic Laplacian providing the requested link with the localization formulas.

A more recent application to the Selberg trace formula for locally symmetric spaces of noncompact type was found by Bismut in [**170**]. This work gives a direct link between the trace formula and index theory. There are many potential applications of this result to other areas of mathematics.

3. Conclusion

Several other papers of Bismut have been left aside, such as his paper on excursions of Brownian motion [**70**], his work with Gillet and Soulé on Bott-Chern currents [**110**], his proof with Labourie [**144**] of the Verlinde formula, and his papers with Goette on torsion forms [**146, 148, 157**]. We hope that the above description of Bismut's work offers a few glimpses of the depth and diversity of Bismut's mathematical contributions.

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