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ASTÉRISQUE 286

**GEOMETRIC METHODS  
IN DYNAMICS (I)**

**VOLUME IN HONOR OF JACOB PALIS**

edited by

**Wellington de Melo**

**Marcelo Viana**

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# GEOMETRIC METHODS IN DYNAMICS (I)

## VOLUME IN HONOR OF JACOB PALIS

edited by Wellington de Melo, Marcelo Viana,  
Jean-Christophe Yoccoz

**Abstract.** — This is the first of two volumes collecting original research articles, on several aspects of dynamics, mostly by participants in the International Conference on Dynamical Systems held at IMPA (Rio de Janeiro), in July 2000, to celebrate Jacob Palis' 60th birthday.

**Résumé (Méthodes géométriques en dynamique (I). Volume en l'honneur de Jacob Palis)**

Ceci est le premier de deux volumes regroupant des articles originaux de recherche concernant des aspects variés de la théorie des systèmes dynamiques, écrits par certains des participants à la Conférence Internationale sur les Systèmes Dynamiques qui s'est tenue à l'IMPA (Rio de Janeiro), en juillet 2000 pour commémorer le 60<sup>e</sup> anniversaire de Jacob Palis.



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## ABSTRACTS

### *On the Mathematical Contributions of Jacob Palis*

SHELDON NEWHOUSE ..... 1

A Conference on Dynamical Systems celebrating the 60th birthday of Jacob Palis was held at IMPA (Instituto de Matemática Pura e Aplicada) in Rio de Janeiro from July 19-28, 2000. This article is a revised and expanded version of a lecture I gave at the Conference. Many additions, including the list of references and the entire sections below on *Homoclinic Bifurcations*, *Cantor Sets and Fractal Invariants*, *Non-Hyperbolic Systems*, and *A Unifying View of Dynamics*, were made later by Marcelo Viana. It was decided to preserve the flavor of the lecture by keeping the narrative in the first person. I am grateful to Marcelo for his contributions to this paper. In my opinion, they greatly improved the presentation of the mathematical scope and influence of Jacob Palis.

### *Random perturbations of nonuniformly expanding maps*

JOSÉ FERREIRA ALVES & VÍTOR ARAÚJO ..... 25

We give both sufficient conditions and necessary conditions for the stochastic stability of nonuniformly expanding maps either with or without critical sets. We also show that the number of probability measures describing the statistical asymptotic behaviour of random orbits is bounded by the number of SRB measures if the noise level is small enough. As an application of these results we prove the stochastic stability of certain classes of nonuniformly expanding maps introduced in [Vi1] and [ABV].

### *The minimal entropy problem for 3-manifolds with zero simplicial volume*

JAMES W. ANDERSON & GABRIEL P. PATERNAIN ..... 63

In this note, we consider the *minimal entropy problem*, namely the question of whether there exists a smooth metric of minimal (topological) entropy, for certain classes of closed 3-manifolds. Specifically, we prove the following two results.

**Theorem A.** *Let  $M$  be a closed orientable irreducible 3-manifold whose fundamental group contains a  $\mathbb{Z} \oplus \mathbb{Z}$  subgroup. The following are equivalent:*

- (1) *the simplicial volume  $\|M\|$  of  $M$  is zero and the minimal entropy problem for  $M$  can be solved;*
- (2)  *$M$  admits a geometric structure modelled on  $\mathbb{E}^3$  or Nil;*
- (3)  *$M$  admits a smooth metric  $g$  with  $h_{\text{top}}(g) = 0$ .*

**Theorem B.** *Let  $M$  be a closed orientable geometrizable 3-manifold. The following are equivalent:*

- (1) *the simplicial volume  $\|M\|$  of  $M$  is zero and the minimal entropy problem for  $M$  can be solved;*
- (2)  *$M$  admits a geometric structure modelled on  $\mathbb{S}^3$ ,  $\mathbb{S}^2 \times \mathbb{R}$ ,  $\mathbb{E}^3$ , or Nil;*
- (3)  *$M$  admits a smooth metric  $g$  with  $h_{\text{top}}(g) = 0$ .*

*Statistical properties of unimodal maps: smooth families with negative Schwarzian derivative*

ARTUR ÁVILA & CARLOS GUSTAVO MOREIRA ..... 81

We prove that there is a residual set of families of smooth or analytic unimodal maps with quadratic critical point and negative Schwarzian derivative such that almost every non-regular parameter is Collet-Eckmann with subexponential recurrence of the critical orbit. Those conditions lead to a detailed and robust statistical description of the dynamics. This proves the Palis conjecture in this setting.

*Geometry of Multi-dimensional Dispersing Billiards*

PÉTER BÁLINT, NIKOLAI CHERNOV, DOMOKOS SZÁSZ & IMRE PÉTER TÓTH 119

Geometric properties of multi-dimensional dispersing billiards are studied in this paper. On the one hand, non-smooth behaviour in the singularity submanifolds of the system is discovered (this discovery applies to the more general class of semi-dispersing billiards as well). On the other hand, a self-contained geometric description for unstable manifolds is given, together with the proof of important regularity properties. All these issues are highly relevant to studying the ergodic and statistical behaviour of the dynamics.

*Homoclinic orbits near saddle-center fixed points of Hamiltonian systems with two degrees of freedom*

PATRICK BERNARD, CLODOALDO GROTTA RAGAZZO & PEDRO A. SANTORO SALOMÃO ..... 151

We study a class of Hamiltonian systems on a 4 dimensional symplectic manifold which have a saddle-center fixed point and satisfy the following property: All the periodic orbits in the center manifold of the fixed point have an orbit homoclinic to them, although the fixed point itself does not. In addition, we prove that these systems have a chaotic behavior in the neighborhood of the energy shell of the fixed point.

*On the scaling structure for period doubling*  
 GARRETT BIRKHOFF, MARCO MARTENS & CHARLES TRESSER ..... 167

We describe an order on the set of scaling ratios of the generic universal smooth period doubling Cantor set and prove that this set of ratios forms itself a Cantor set, a Conjecture formulated by Couillet and Tresser in 1977. This result establishes explicitly the geometrical complexity of the universal period doubling Cantor set. We also show a convergence result for the two period doubling renormalization operators, acting on the codimension one space of period doubling maps. In particular they form an iterated function system whose limit set contains a Cantor set.

*Robustly transitive sets and heterodimensional cycles*  
 CHRISTIAN BONATTI, LORENZO J. DÍAZ, ENRIQUE R. PUJALS & JORGE ROCHA ..... 187

It is known that all non-hyperbolic robustly transitive sets  $\Lambda_\varphi$  have a dominated splitting and, generically, contain periodic points of different indices. We show that, for a  $\mathcal{C}^1$ -dense open subset of diffeomorphisms  $\varphi$ , the indices of periodic points in a robust transitive set  $\Lambda_\varphi$  form an interval in  $\mathbb{N}$ . We also prove that the homoclinic classes of two periodic points in  $\Lambda_\varphi$  are robustly equal. Finally, we describe what sort of homoclinic tangencies may appear in  $\Lambda_\varphi$  by studying its dominated splittings.

*Coupled Hopf-bifurcations: Persistent examples of  $n$ -quasiperiodicity determined by families of 3-jets*  
 HENK BROER ..... 223

In this note examples are presented of vector fields depending on parameters and determined by the 3-jet, which display persistent occurrence of  $n$ -quasiperiodicity. In the parameter space this occurrence has relatively large measure. A leading example consists of weakly coupled Hopf bifurcations. This example, however, is extended to full generality in the space of all 3-jets.

*Walks in rigid environments: symmetry and dynamics*  
 LEONID A. BUNIMOVICH ..... 231

We study dynamical systems generated by a motion of a particle in an array of scatterers distributed in a lattice. Such deterministic cellular automata are called Lorentz-type lattice gases or walks in rigid environments. It is shown that these models can be completely solved in the one-dimensional case. The corresponding regimes of motion can serve as the simple dynamical examples of diffusion, sub- and super-diffusion.

*Perverse solutions of the planar  $n$ -body problem*  
 ALAIN CHENCINER ..... 249

The perverse solutions of the  $n$ -body problem are the solutions which satisfy the equations of motion for at least two distinct systems of masses. I contribute



with some simple remarks concerning their existence, a question which curiously seems to be new.

*Chaos versus renormalization at quadratic S-unimodal Misiurewicz bifurcations*  
 EDUARDO COLLI & VILTON PINHEIRO ..... 257

We study  $C^3$  families of unimodal maps of the interval with negative Schwarzian derivative and quadratic critical point, transversally unfolding Misiurewicz bifurcations, and for these families we prove that existence of an absolutely continuous invariant probability measure (“chaos”) and existence of a renormalization are prevalent in measure along the parameter. Moreover, the method also shows that existence of a renormalization is dense and chaos occurs with positive measure.

## RÉSUMÉS DES ARTICLES

### *On the Mathematical Contributions of Jacob Palis*

SHELDON NEWHOUSE ..... 1

Une conférence sur les systèmes dynamiques s'est tenue à l'IMPA (Instituto de Matemática Pura e Aplicada) à Rio de Janeiro, à l'occasion du 60<sup>e</sup> anniversaire de Jacob Palis, du 19 au 28 juillet 2000. Cet article est une version révisée et élargie d'un exposé que j'ai donné lors de la conférence. Plusieurs ajouts, incluant une liste de références et les paragraphes intitulés *Homoclinic Bifurcations*, *Cantor Sets and Fractal Invariants*, *Non-Hyperbolic Systems* et *A Unifying View of Dynamics*, ont été introduits plus tard par Marcelo Viana. Il a été décidé de préserver l'ambiance de l'exposé en conservant une narration à la première personne. Je remercie Marcelo pour ses contributions à cet article. À mon avis, celles-ci ont beaucoup amélioré la présentation de l'envergnure mathématique et de l'influence de Jacob Palis.

### *Random perturbations of nonuniformly expanding maps*

JOSÉ FERREIRA ALVES & VÍTOR ARAÚJO ..... 25

Nous donnons des conditions suffisantes et des conditions nécessaires pour la stabilité stochastique de transformations non uniformément dilatantes, avec ou sans ensembles critiques. Nous prouvons aussi que le nombre de mesures de probabilité qui décrit le comportement statistique asymptotique des orbites aléatoires est borné par le nombre de mesures de SRB si le niveau de bruit est assez petit. Comme application de ces résultats nous prouvons la stabilité stochastique de certaines classes de transformations non uniformément dilatantes présentées dans [Vi1] et [ABV].

### *The minimal entropy problem for 3-manifolds with zero simplicial volume*

JAMES W. ANDERSON & GABRIEL P. PATERNAIN ..... 63

Dans cet article, nous considérons le *problème de l'entropie minimale*, c'est-à-dire la question de l'existence d'une métrique lisse d'entropie (topologique)

minimale, pour certaines classes de variétés fermées de dimension 3. Précisément, nous montrons les deux résultats suivants.

**Théorème A.** *Soit  $M$  une variété fermée de dimension 3, orientable et irréductible, dont le groupe fondamental contient un sous-groupe  $\mathbb{Z} \oplus \mathbb{Z}$ . Les propriétés suivantes sont équivalentes:*

- (1) *le volume simplicial  $\|M\|$  de  $M$  est nul et le problème de l'entropie minimale pour  $M$  peut être résolu;*
- (2)  *$M$  admet une structure géométrique modelée sur  $\mathbb{E}^3$  ou Nil;*
- (3)  *$M$  admet une métrique lisse  $g$  avec  $h_{\text{top}}(g) = 0$ .*

**Théorème B.** *Soit  $M$  une variété fermée de dimension 3, orientable et géométrisable. Les propriétés suivantes sont équivalentes:*

- (1) *le volume simplicial  $\|M\|$  de  $M$  est nul et le problème de l'entropie minimale pour  $M$  peut être résolu;*
- (2)  *$M$  admet une structure géométrique modelée sur  $\mathbb{S}^3$ ,  $\mathbb{S}^2 \times \mathbb{R}$ ,  $\mathbb{E}^3$ , ou Nil;*
- (3)  *$M$  admet une métrique lisse  $g$  avec  $h_{\text{top}}(g) = 0$ .*

*Statistical properties of unimodal maps: smooth families with negative Schwarzian derivative*

ARTUR AVILA & CARLOS GUSTAVO MOREIRA ..... 81

Nous montrons que l'ensemble des familles d'applications unimodales telles que presque tout paramètre non régulier est Collet-Eckmann avec récurrence sous-exponentielle de l'orbite critique est résiduel. Ceci nous amène à donner une description statistique détaillée et robuste de la dynamique. Nos résultats démontrent la conjecture de Palis dans ce contexte.

*Geometry of Multi-dimensional Dispersing Billiards*

PÉTER BÁLINT, NIKOLAI CHERNOV, DOMOKOS SZÁSZ & IMRE PÉTER TÓTH 119

Dans cet article, on étudie les propriétés géométriques des billards dispersifs multi-dimensionnels. D'une part, on découvre un comportement non régulier dans les variétés singulières du système (cette découverte concerne aussi la catégorie plus générale des billards semi-dispersifs). D'autre part, on donne une description géométrique cohérente pour les variétés instables, puis on démontre d'importantes propriétés de régularité. Toutes ces questions sont particulièrement en rapport avec l'étude du comportement ergodique et statistique de la dynamique.

*Homoclinic orbits near saddle-center fixed points of Hamiltonian systems with two degrees of freedom*

PATRICK BERNARD, CLODOALDO GROTTA RAGAZZO & PEDRO A. SANTORO SALOMÃO ..... 151

On étudie une classe de systèmes hamiltoniens sur une variété symplectique de dimension 4 qui admettent un point fixe de type selle-centre et vérifient la propriété suivante: chaque orbite périodique de la variété centrale du point fixe

a une orbite homocline, mais le point fixe lui-même n'a pas d'orbite homocline. On montre de plus que ces systèmes ont un comportement chaotique au voisinage de la surface d'énergie du point fixe.

*On the scaling structure for period doubling*

GARRETT BIRKHOFF, MARCO MARTENS & CHARLES TRESSER ..... 167

Nous décrivons un ordre sur l'ensemble des rapports d'échelle de l'ensemble de Cantor du doublement de période générique universel lisse, et montrons que cet ensemble de rapports forme lui-même un ensemble de Cantor, ce qui est une conjecture formulée par Couillet et Tresser en 1977. Ce résultat établit explicitement la complexité géométrique de l'ensemble de Cantor du doublement de période universel. Nous montrons aussi un résultat de convergence pour les deux opérateurs de renormalisation du doublement de période, agissant sur l'espace de codimension 1 des applications de doublement de période.

*Robustly transitive sets and heterodimensional cycles*

CHRISTIAN BONATTI, LORENZO J. DÍAZ, ENRIQUE R. PUJALS & JORGE ROCHA ..... 187

On sait que les ensembles robustement transitifs non hyperboliques possèdent une décomposition dominée et contiennent génériquement des points périodiques de différents indices. Nous montrons que, sur une partie  $C^1$ -ouverte et dense de difféomorphismes  $\varphi$ , les indices des points périodiques d'un ensemble  $\Lambda_\varphi$  robustement transitif forment un intervalle dans  $\mathbb{N}$ . Nous montrons aussi que les classes homoclines de deux points périodiques de  $\Lambda_\varphi$  sont robustement égales. Finalement, nous décrivons les types de tangences homoclines qui peuvent apparaître dans  $\Lambda_\varphi$ , en analysant les différentes décompositions dominées de  $\Lambda_\varphi$ .

*Coupled Hopf-bifurcations: Persistent examples of  $n$ -quasiperiodicity determined by families of 3-jets*

HENK BROER ..... 223

Dans cet article, on présente des exemples de champs de vecteurs dépendant de paramètres et déterminés par leur 3-jet, qui présentent une  $n$ -quasipériodicité persistante. Dans l'espace des paramètres, ce phénomène apparaît sur un ensemble de mesure relativement grande. Les bifurcations de Hopf couplées en sont l'exemple principal. On étend cet exemple en toute généralité à l'espace de tous les 3-jets.

*Walks in rigid environments: symmetry and dynamics*

LEONID A. BUNIMOVICH ..... 231

Nous étudions des systèmes dynamiques engendrés par le mouvement d'une particule sur un ensemble de dispersions distribuées dans un réseau. Ces automates cellulaires déterministes sont appelés gaz de réseau de type Lorentz

ou marches en environnements rigides. Nous démontrons que ces modèles peuvent être complètement résolus en dimension 1. Les régimes de mouvement peuvent servir d'exemples dynamiques simples de diffusion, sous-diffusion et supra-diffusion.

*Perverse solutions of the planar  $n$ -body problem*

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Les solutions perverses du problème des  $n$  corps sont celles qui satisfont aux équations du mouvement pour au moins deux systèmes distincts de masses. Je fais quelques remarques simples sur la question de leur existence, question qui curieusement semble nouvelle.

*Chaos versus renormalization at quadratic  $S$ -unimodal Misiurewicz bifurcations*

EDUARDO COLLI & VILTON PINHEIRO ..... 257

Nous étudions des familles  $C^3$  d'applications unimodales de l'intervalle avec une dérivée de Schwarz négative et un point critique quadratique, qui déploient transversalement une bifurcation de Misiurewicz, et nous démontrons, pour ces familles, que l'existence d'une mesure de probabilité invariante absolument continue ("chaos") et l'existence d'une renormalisation sont prévalentes en mesure dans l'espace des paramètres. D'autre part, la méthode montre aussi que l'existence d'une renormalisation est dense et le chaos a lieu avec une mesure positive.

## PREFACE

These two volumes collect original research articles submitted by participants of the International Conference on Dynamical Systems held at IMPA, Rio de Janeiro, in July 19-28, 2000 to commemorate the 60th birthday of Jacob Palis.

These articles cover a wide range of subjects in Dynamics, reflecting the Conference's broad scope, itself a tribute to the diversity and influence of Jacob's contributions to the mathematical community worldwide, and most notably in Latin America, through his scientific work, his role as an educator of young researchers, his responsibilities in international scientific bodies, and the efforts he has always devoted to fostering the development of Mathematics in all regions of the globe.

His own mathematical work, which extends for more than 80 publications, is described in Sheldon Newhouse's opening article. It is, perhaps, best summarized by the following quotation from Jacob's recent nomination for the French Academy of Sciences: "sa vision, en constante évolution, a considérablement élargi le sujet".

As Jacob does not seem willing to slow down, we should expect much more from him in the years to come...

Rio de Janeiro and Paris,

May 20, 2003

Wellington de Melo, Marcelo Viana, Jean-Christophe Yoccoz

### Jacob Palis Mathematical Tree



Location of present institution indicated.

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