ANNALES DE L'I. H. P., SECTION A

ROBERT ALICKI
ALBERTO FRIGERIO

Scattering theory for quantum dynamical semigroups. - II

Annales de l'I. H. P., section A, tome 38, nº 2 (1983), p. 187-197
<http://www.numdam.org/item?id=AIHPA_1983__38_2_187_0>

Ann. Inst. Henri Poincaré,
Vol. XXXVIII, n° 2, 1983, p. 187-197.

Section A :
Physique théorique.

Scattering theory for quantum dynamical semigroups. — II

by

Robert ALICKI (*) and Alberto FRIGERIO
Istituto di Scienze Fisiche, Università di Milano,
Via Celoria 16, 20133 Milano, Italy

ABSTRACT. — For a class of quantum dynamical semigroups, we prove that the scattering matrix on the space of trace class operators exists and can be uniquely decomposed as the sum of an elastic and an inelastic part, which are separately positive.

RÉSUMÉ. — On démontre, pour une classe de semi-groupes dynamiques quantiques, que la matrice de diffusion existe sur l'espace des opérateurs à trace et peut être décomposée de façon unique en somme d'une partie élastique et d'une partie inélastique, dont chacune est positive.

1. INTRODUCTION

This note continues the investigation of the mathematical framework for dissipative scattering in terms of quantum dynamical semigroups [*1*] [*2*] [*3*]. This description can be applied to the phenomenon of dissipative heavy-ion collision, or to the scattering and capture of a neutron by a complex nucleus [*1*]-[*4*]. In this approach, the relevant degrees of freedom are treated as an open system; by elimination of the other degrees of freedom

(*) A. von Humboldt fellow, Sektion Physik, Universität München, BRD. On leave of absence from Institute of Theoretical Physics, Gdańsk University, Gdańsk, Poland. Partially supported by CNR Research Contract n° 79.02337.63.115.2670.

and use of the Markov approximation, one arrives at a reduced description of the dynamics of the open system by means of a quantum dynamical semigroup.

Let $\mathscr{H}$ be the Hilbert space associated with the open system, and let $T(\mathscr{H})$ be the space of trace class operators on $\mathscr{H}$, equipped with the trace norm $\|\cdot\|_1$. We compare a « free » and an « interacting » evolution on $T(\mathscr{H})$, given by a strongly continuous one-parameter group

$$\{ e^{L_0 t} = e^{-iH_0 t} \,.\, e^{iH_0 t} : t \in \mathbb{R} \} \tag{1.1}$$

and by a dynamical semigroup

$$\{ \Lambda_t = e^{Lt} : t \in \mathbb{R}^+ \} \tag{1.2}$$

respectively. By « dynamical semigroup » [*5*] [*6*] we mean here a strongly continuous one-parameter semigroup of completely positive contractions on $T(\mathscr{H})$; we are not assuming that $\mathrm{tr}\,(\Lambda_t \rho) = \mathrm{tr}\,(\rho)$ for all ρ in $T(\mathscr{H})$.

In (1.1), H_0 is a self-adjoint operator in $\mathscr{H}$. The domain

$$D_0 = \mathrm{lin}\,\{ \,|\,\varphi\,\rangle\langle\,\psi\,|\, : \varphi, \psi \in \mathrm{dom}\,(H_0) \}$$

is contained in $\mathrm{dom}\,(L_0)$, and

$$L_0 \rho = -\,iH_0 \rho + i\rho H_0, \qquad \rho \in D_0. \tag{1.3}$$

We shall often denote $-\,iH_0$ by K_0.

We assume that the dynamical semigroup (1.2) is obtained as follows. Let K be the generator of a strongly continuous contraction semigroup on $\mathscr{H}$, let $B_n : \mathrm{dom}\,(K) \to \mathscr{H}$, $n = 1, 2, \ldots$, be linear operators such that

$$\langle\, K\varphi \,|\, \varphi \,\rangle + \langle\, \varphi \,|\, K\varphi \,\rangle + \sum_{n=1}^{\infty} \langle\, B_n\varphi \,|\, B_n\varphi \,\rangle \leqslant 0 \tag{1.4}$$

for all φ in $\mathrm{dom}\,(K)$. Let $D = \mathrm{lin}\,\{ \,|\,\varphi\,\rangle\langle\,\psi\,|\, : \varphi, \psi \in \mathrm{dom}\,(K) \}$, and let

$$L\rho = K\rho + \rho K^* + \sum_{n=1}^{\infty} B_n \rho B_n^*, \qquad \rho \in D. \tag{1.5}$$

It has been shown by Davies [7] that there exists a dynamical semigroup $\{ \Lambda_t : t \in \mathbb{R}^+ \}$ on $T(\mathscr{H})$ such that

$$\frac{d}{dt}\Lambda_t \rho \,|_{t=0} = L\rho \qquad \text{for all} \quad \rho \text{ in } D, \tag{1.6}$$

which can be canonically constructed with the method of the minimal solution. In the following, we shall denote by L the extension of (1.5) which is the generator of Λ_t.

Suppose that $\lim_{t\to\infty} \langle\, \varphi \,|\, e^{-L_0 t} e^{2Lt} e^{-L_0 t} \rho \,|\, \varphi \,\rangle$ exists for all ρ in $T(\mathscr{H})$

and all φ in $\mathscr{H}$. Then one can easily prove that there is a completely positive contraction S on $T(\mathscr{H})$ such that

$$\lim_{t\to\infty} \langle \varphi \mid e^{-L_0 t} e^{2Lt} e^{-L_0 t} \rho \mid \varphi \rangle = \langle \varphi \mid S\rho \mid \varphi \rangle \tag{1.7}$$

for all ρ in $T(\mathscr{H})$ and all φ in $\mathscr{H}$; S is called the scattering matrix, or the S-matrix. Conditions for the existence of S in terms of wave operators and Cook's criterion may be found in [3], see also [2]. Similarly, assume that

$$\lim_{t\to\infty} \langle \varphi \mid e^{-K_0 t} e^{2Kt} e^{-K_0 t} \psi \rangle = \langle \varphi \mid \Omega \psi \rangle \tag{1.8}$$

exists for all φ, ψ in $\mathscr{H}$, then Ω is a contraction on $\mathscr{H}$. See [8] [9] for existence conditions.

We decompose the scattering matrix S as

$$S\rho = \Omega\rho\Omega^* + T\rho, \qquad \rho \in T(\mathscr{H}); \tag{1.9}$$

then $\Omega\cdot\Omega^*$ and T represent the elastic and the inelastic part of the scattering matrix respectively [1] [10]. In order for this decomposition to be meaningful, it is necessary that T is positive ($\Omega\cdot\Omega^*$ is obviously completely positive) and that the decomposition is unique in some sense. In Section 2 we prove that T is indeed completely positive, and in Section 3 we prove the uniqueness of the decomposition, subject to physically motivated conditions. Finally, in Section 4 we prove the existence of S and Ω for a class of dynamical semigroups whose generators are unbounded perturbations of L_0.

2. POSITIVITY OF THE INELASTIC PART OF THE S-MATRIX

The inelastic part of the scattering matrix, defined by

$$T\rho = S\rho - \Omega\rho\Omega^*, \qquad \rho \in T(\mathscr{H}), \tag{2.1}$$

when S and Ω exist, is easily shown to be completely positive when L is a bounded perturbation of L_0. Here we prove that T is completely positive also in the situation described in the Introduction.

THEOREM 1. — Let $\{ e^{Lt} : t \in \mathbb{R}^+ \}$ be the minimal solution corresponding to an expression L of the form (1.5); suppose that both limits (1.7) and (1.8) exist. Then T, given by (2.1), is completely positive.

Proof. — Since the weak limit of a bounded family of completely positive maps is completely positive, and since $e^{L_0 s}$ is completely positive for all real s, it suffices to prove that $e^{Lt} - e^{L_1 t}$ is completely positive for all t in $\mathbb{R}^+$, where

$$e^{L_1 t} = e^{Kt} \cdot e^{K^* t}, \qquad t \in \mathbb{R}^+. \tag{2.2}$$

The proof of this fact is based on the construction of the minimal solution, given by Davies in [7].

By the assumptions made in the Introduction, the map

$$\rho \mapsto \sum_{n=1}^{\infty} B_n(\lambda - L_1)^{-1}(\rho)B_n^*, \tag{2.3}$$

defined on $(1 - K)^{-1}T(\mathscr{H})(1 - K^*)^{-1}$, can be extended to a completely positive contraction A_λ of $T(\mathscr{H})$ for all $\lambda > 0$. We put

$$J = A_1(1 - L_1) : \mathrm{dom}(L_1) \to T(\mathscr{H}), \tag{2.4}$$

then $L_1 + J$ is an extension of (1.5) to $\mathrm{dom}(L_1)$. By definition, the minimal solution is given by

$$e^{Lt} = s\text{-}\lim_{r \uparrow 1} e^{(L_1 + rJ)t}, \qquad t \in \mathbb{R}^+. \tag{2.5}$$

By the resolvent expansion, we have, for $\lambda > 0$, $0 < r < 1$,

$$(\lambda - L_1 - rJ)^{-1} = (\lambda - L_1)^{-1} \sum_{n=0}^{\infty} r^n A_\lambda^n, \tag{2.6}$$

where the series converges in norm. Since A_λ and $(\lambda - L_1)^{-1}$ are completely positive, it follows from (2.6) that the maps

$$(\lambda - L_1 - rJ)^{-1} - (\lambda - L_1)^{-1}, \quad \lambda > 0, \quad 0 < r < 1,$$

are completely positive. Then, for $0 < r < 1$, $t \in \mathbb{R}^+$, $e^{(L_1 + rJ)t} - e^{L_1 t}$ is completely positive, and the same holds for $e^{Lt} - e^{L_1 t}$. ∎

3. UNIQUENESS OF THE DECOMPOSITION

Although L is given by (1.5) in an explicitly decomposed form

$$L\rho = L_1\rho + J\rho,$$

the decomposition is not unique. For example, it is possible to add constants β_n to the operators B_n and redefine K at the same time, in such a way that L remains unchanged, see [*10*]. This arbitrariness might seem to affect the decomposition of the S-matrix into an elastic and an inelastic part.

However, the condition of the existence of both limits (1.7) and (1.8) should restrict the arbitrariness in the decomposition of L, and it might turn out that the elastic and inelastic part of S are uniquely defined, when they exist.

We are unable to prove a direct connexion between existence and uniqueness of Ω and $T = S - \Omega \cdot \Omega^*$, but in Theorem 2 we prove the uniqueness of the decomposition of L (hence of S) under conditions which are physically very close to the existence of both S and Ω. Related results have been proved by Davies in [*10*].

We say that a linear operator K in $\mathscr{H}$ is a *dissipator* for the dynamical semigroup e^{Lt} on $T(\mathscr{H})$ if K is the generator of a strongly continuous contraction semigroup on $\mathscr{H}$, and K and L are related as in (1.5).

We assume that there exists a dense linear manifold $\mathscr{H}_0$ in $\mathscr{H}$, contained in dom (K_0), and a one-parameter unitary group $\{ U_s : s \in \mathbb{R} \}$ on $\mathscr{H}$, leaving dom (K_0) globally invariant. In practical applications, when $\mathscr{H} = L^2(\mathbb{R}^n)$ and $K_0 = i(2m)^{-1}\Delta$, $\mathscr{H}_0$ might be the linear span of Gaussians, and U_s might be the group of space translations in some direction, or also the free evolution $e^{K_0 s}$ itself.

THEOREM 2. — Let $\mathscr{H}_0$, $\{ U_s : s \in \mathbb{R} \}$ be as above. Assume that

$$\text{dom}(L) \supseteq \text{dom}(L_0), \tag{3.1}$$

for all $\rho = | \psi \rangle \langle \psi |$, $\psi \in \mathscr{H}_0$,

$$\lim_{s \to \infty} \| (L - L_0) U_s \rho U_{-s} \|_1 = 0. \tag{3.2}$$

Then there exists at most one dissipator K for $\{ e^{Lt} : t \in \mathbb{R}^+ \}$ such that

$$\text{dom}(K_0) \qquad \text{is a core for } K, \tag{3.3}$$

for all ψ in $\mathscr{H}_0$,

$$\lim_{s \to \infty} \| (K - K_0) U_s \psi \| = 0. \tag{3.4}$$

Proof. — Let K_1, K_2 be dissipators for e^{Lt}, satisfying (3.3). Then

$$D_0 = \text{lin} \{ | \varphi \rangle \langle \psi | : \varphi, \psi \in \text{dom}(K_0) \}$$

is contained in $D_i = \text{lin} \{ | \varphi \rangle \langle \psi | : \varphi, \psi \in \text{dom}(K_i) \}$ for $i = 1, 2$, and for all ρ in D_0 we have

$$L\rho = K_i \rho + \rho K_i^* + \sum_{n=1}^{\infty} B_n^{(i)} \rho B_n^{(i)*}, \qquad i = 1, 2, \tag{3.5}$$

where $B_n^{(i)} : n = 1, 2, \ldots, i = 1, 2$, are as in the Introduction.

Let ψ, φ be arbitrary vectors in $\mathscr{H}_0$ and in dom (K_0) respectively, and put

$$\rho_s = | U_s \psi \rangle \langle \varphi |, \qquad s \in \mathbb{R}; \tag{3.6}$$

then ρ_s is in D_0 for all real s, by the assumptions made on $\mathscr{H}_0$ and on U_s.

Comparing both decompositions of $L\rho_s$ as in (3.5), we obtain, for all s in $\mathbb{R}$,

$$| U_{-s}(K_1 - K_2)U_s\psi \rangle \langle \varphi | + | \psi \rangle \langle (K_1 - K_2)\varphi | + \sum_{n=1}^{\infty} | U_{-s}B_n^{(1)}U_s\psi \rangle \langle B_n^{(1)}\varphi | - \sum_{n=1}^{\infty} | U_{-s}B_n^{(2)}U_s\psi \rangle \langle B_n^{(2)}\varphi | = 0. \quad (3.7)$$

If $K_1 \neq K_2$ and (3.3) holds, we may choose φ in dom (K_0) such that the vector $\xi = (K_1 - K_2)\varphi$ is of norm one. Acting on ξ with (3.7), and adding and subtracting a term $| U_{-s}K_0U_s\psi \rangle \langle \varphi | \xi \rangle$, we obtain, for all s in $\mathbb{R}$,

$$\psi = | U_{-s}(K_2 - K_0)U_s\psi \rangle \langle \varphi | \xi \rangle - | U_{-s}(K_1 - K_0)U_s\psi \rangle \langle \varphi | \xi \rangle + \sum_{n=1}^{\infty} | U_{-s}B_n^{(2)}U_s\psi \rangle \langle B_n^{(2)}\varphi | \xi \rangle - \sum_{n=1}^{\infty} | U_{-s}B_n^{(1)}U_s\psi \rangle \langle B_n^{(1)}\varphi | \xi \rangle. \quad (3.8)$$

If K_1 and K_2 satisfy (3.4), we have, for all ψ in $\mathcal{H}_0$,

$$\lim_{s\to\infty} \| U_{-s}(K_i - K_0)U_s\psi \| = 0, \qquad i = 1, 2, \quad (3.9)$$

hence also, for all $\rho = | \psi \rangle \langle \psi |$, $\psi \in \mathcal{H}_0$,

$$\lim_{s\to\infty} \| (L_i - L_0)U_s\rho U_{-s} \|_1 = 0, \qquad i = 1, 2, \quad (3.10)$$

where $L_i\rho = K_i\rho + \rho K_i^*$, $i = 1, 2$, $\rho \in D_0 \subseteq D_1 \cap D_2$.

If also L satisfies (3.2), we have, for all ψ in $\mathcal{H}_0$,

$$\lim_{s\to\infty} \sum_{n=1}^{\infty} \| U_{-s}B_n^{(i)}U_s\psi \|^2 = \lim_{s\to\infty} \left\| \sum_{n=1}^{\infty} | U_{-s}B_n^{(i)}U_s\psi \rangle \langle U_{-s}B_n^{(i)}U_s\psi | \right\|_1 = \lim_{s\to\infty} \| (L - L_i)U_s\rho U_{-s} \|_1 = 0, \qquad i = 1, 2; \quad (3.11)$$

where we have used (3.2) and (3.10). Now, for ψ in $\mathcal{H}_0$, we have

$$\left\| \sum_{n=1}^{\infty} | U_{-s}B_n^{(i)}U_s\psi \rangle \langle B_n^{(i)}\varphi | \xi \rangle \right\| \leqslant \sum_{n=1}^{\infty} \| U_{-s}B_n^{(i)}U_s\psi \| \cdot | \langle B_n^{(i)}\varphi | \xi \rangle |$$

$$\leq \left(\sum_{n=1}^{\infty} \| U_{-s}B_n^{(i)}U_s\psi \|^2 \right)^{1/2} \left(\sum_{n=1}^{\infty} | \langle B_n^{(i)}\varphi | \xi \rangle |^2 \right)^{1/2}$$

$$= \left(\sum_{n=1}^{\infty} \| U_{-s}B_n^{(i)}U_s\psi \|^2 \right)^{1/2} \langle \xi | (L - L_i)(| \varphi \rangle \langle \varphi |) | \xi \rangle^{1/2} \quad (3.12)$$

($s \in \mathbb{R}$, $i = 1, 2$). Then by letting $s \to \infty$ in (3.8) and by using (3.9), (3.11), (3.12), we obtain $\psi = 0$ for all ψ in $\mathscr{H}_0$, a contradiction. We conclude that $K_1 = K_2$. ■

Remark. — In applications to the case $\mathscr{H} = L^2(\mathbb{R}^n)$, when U_s is the group of space translations in some direction, conditions (3.2) and (3.4) express the vanishing of the perturbations $L - L_0$, $K - K_0$ at large distance. When U_s is taken to be the free evolution $e^{K_0 s}$, conditions (3.2) and (3.4) are related to Cook's criterion for the existence of the wave operators

$$W_1 = s\text{-}\lim_{t\to\infty} e^{Lt}e^{-L_0 t} \quad \text{on} \quad T(\mathscr{H})$$

and

$$M_1 = s\text{-}\lim_{t\to\infty} e^{Kt}e^{-K_0 t} \quad \text{on} \quad \mathscr{H}.$$

4. EXISTENCE OF THE SCATTERING MATRIX

We give a proof of the existence of the scattering matrix for a class of quantum dynamical semigroups which includes the simple model of heavy-ion collision of [*3*] [*4*].

We assume that the expression (1.5) defining L contains only finitely many B_n, $n = 1, \ldots, N$, and that dom(H_0) is contained in the domain of all operators K, K^*, B_n, B_n^*, $n = 1, \ldots, N$. We assume also that there exists a dense linear manifold $\mathscr{H}_0$ in $\mathscr{H}$, contained in dom(H_0), such that the following functions of t are integrable on $[0, \infty)$ for all ψ in $\mathscr{H}_0$:

$$\| (K + iH_0)e^{+iH_0 t}\psi \|, \qquad \| (K^* - iH_0)e^{-iH_0 t}\psi \|, \tag{4.1}$$

$$\| B_n e^{+iH_0 t}\psi \|^2 \quad , \qquad \| B_n^* e^{-iH_0 t}\psi \|^2, \qquad n = 1, \ldots, N. \tag{4.2}$$

Remark. — Integrability of the first quantity in (4.2) follows from that of the first one in (4.1) by means of (1.4). We are indebted to the referee for this observation.

THEOREM 3. — Under the above assumptions, the scattering matrix S on $T(\mathscr{H})$ and the scattering matrix Ω on $\mathscr{H}$ exist.

Proof. — By Cook's criterion, the integrability of the functions (4.1) implies the existence of the wave operators

$$M_1 = s\text{-}\lim_{t\to\infty} e^{Kt}e^{iH_0 t}, \qquad M_2 = s\text{-}\lim_{t\to\infty} e^{K^* t}e^{-iH_0 t} \tag{4.3}$$

on $\mathscr{H}$; then $\Omega = M_2^* M_1$. Similarly, it follows from the integrability of the functions (4.1) and (4.2) that

$$\int_0^\infty \| (L - L_0)e^{-L_0 t}\rho \|_1 dt < \infty \tag{4.4}$$

for all ρ in the dense domain $\tilde{D}_0 = \text{lin}\{ | \varphi \rangle \langle \psi | : \varphi, \psi \in \mathscr{H}_0 \}$; then the wave operator

$$W_1 = s\text{-}\lim_{t \to \infty} e^{Lt} e^{-L_0 t} \tag{4.5}$$

exists on $T(\mathscr{H})$ (cf. [3], Propositions 1 and 2). In order to conclude the proof, we must show that also

$$\int_0^\infty \| (L^* - L_0^*) e^{-L_0^* t} a \|_\infty dt < \infty$$

for all a in $\tilde{D}_0$, where $\| \cdot \|_\infty$ denotes the operator norm: then the wave operator

$$W_2 = s\text{-}\lim_{t \to \infty} e^{L^* t} e^{-L_0^* t} \tag{4.7}$$

exists as a map from the space $C(\mathscr{H})$ of compact operators on $\mathscr{H}$ to the space $B(\mathscr{H})$ of all bounded operators on $\mathscr{H}$; then $S = W_2^* W_1$ [3].

The proof of (4.6) from the assumptions on the functions (4.1) and (4.2) is easy (cf. [3], Proposition 2), provided one shows preliminarily that $D_0 = \text{lin}\{ | \varphi \rangle \langle \psi | : \varphi, \psi \in \text{dom}(H_0) \} \supseteq \tilde{D}_0$ is contained in dom(L^*) and that

$$L^* a = K^* a + aK + \sum_{n=1}^{N} B_n^* a B_n, \qquad a \in D_0. \tag{4.8}$$

While « formally obvious », (4.8) must be proved, since the domain of L is not given explicitly. We give the proof in two Lemmas. The assumptions made in the beginning of this Section will be understood.

LEMMA 4. — Let J: dom$(L_1) \to T(\mathscr{H})$ be defined by (2.4). Then $D_0 \subseteq$ dom (J^*) and

$$J^* a = \sum_{n=1}^{N} B_n^* a B_n, \qquad a \in D_0. \tag{4.9}$$

Proof. — We have $J = A_1(1 - L_1)$, where A_1 is the contraction on $T(\mathscr{H})$ which extends the map

$$\rho \mapsto \sum_{n=1}^{N} B_n (1 - L_1)^{-1}(\rho) B_n^* = \int_0^\infty e^{-t} \sum_{n=1}^{N} B_n e^{Kt} \rho e^{K^* t} B_n^* dt,$$

defined on a dense domain. By assumption, dom$(H_0) \subseteq$ dom(B_n^*) for all n. Then, for all a in D_0, the expression

$$\tilde{a} = \int_0^\infty e^{-t} \sum_{n=1}^{N} e^{K^* t} B_n^* a B_n e^{Kt} dt = (1 - L_1^*)^{-1} \sum_{n=1}^{N} B_n^* a B_n \tag{4.10}$$

is a well defined operator in $C(\mathscr{H})$, and

$$tr[\tilde{a}\rho] = tr[aA_1\rho] \qquad \text{for all} \quad \rho \text{ in a dense domain.}$$

Since A_1 is a contraction, this implies $A_1^*a = \tilde{a}$. It is clear from (4.10) that $\tilde{a}$ is in dom (L_1^*) and

$$J^*a = (1 - L_1^*)A_1^*a = (1 - L_1^*)\tilde{a} = \sum_{n=1}^{N} B_n^* a B_n,$$

which proves (4.9). ■

LEMMA 5. — Let $\{ e^{Lt} : t \in \mathbb{R}^+ \}$ be the minimal solution corresponding to (1.5). Then $D_0 \subseteq \text{dom}(L^*)$ and (4.8) holds.

Proof. — The right-hand side of (4.8) is well defined for a in D_0 since $\text{dom}(H_0) \subseteq \text{dom}(K^*) \cap \text{dom}(B_n^*)$ for all n. We must prove that

$$\frac{d}{dt}\,\text{tr}\,[a\,e^{Lt}\rho] = \text{tr}\left[\left(K^*a + aK + \sum_{n=1}^{N} B_n^* a B_n\right)e^{Lt}\rho\right], \qquad t \in \mathbb{R}^+, \quad (4.11)$$

for all ρ in $T(\mathscr{H})$, a in D_0. Since dom (L_1) is dense in $T(\mathscr{H})$ and $\{ e^{Lt}: t \in \mathbb{R}^+ \}$ is a strongly continuous contraction semigroup, it suffices to prove that (4.11) holds for all ρ in dom (L_1).

Let a be in D_0, ρ in dom (L_1). For $0 < r < 1$, let

$$f_r(t) = \text{tr}\,[a\,e^{(L_1+rJ)t}\rho], \qquad t \in \mathbb{R}^+.$$

By definition of the minimal solution, we have

$$\lim_{r\to 1} f_r(t) = \text{tr}\,[a\,e^{L_1}\rho] \equiv f(t), \qquad t \in \mathbb{R}^+,$$

uniformly on compacts in t.

Since dom $(L_1 + rJ) = $ dom (L_1) for $0 < r < 1$ [*7*], we have

$$\frac{d}{dt} f_r(t) = \text{tr}\,[a(L_1 + rJ)e^{(L_1+rJ)t}\rho], \qquad t \in \mathbb{R}^+.$$

Clearly, $D_0 \subseteq \text{dom}(L_1^*)$ and $L_1^*a = K^*a + aK$ for all a in D_0. Using also Lemma 4, we obtain

$$\frac{d}{dt} f_r(t) = \text{tr}\,[(L_1^* + rJ^*)(a)e^{(L_1+rJ)t}\rho], \qquad t \in \mathbb{R}^+. \qquad (4.12)$$

In the limit as $r \to 1$, (4.12) tends to

$$g(t) = \text{tr}\,[(L_1^* + J^*)(a)e^{Lt}\rho], \qquad t \in \mathbb{R}^+,$$

uniformly on compacts in t. Hence $\frac{d}{dt} f(t) = g(t)$ for all t in $\mathbb{R}^+$, which proves (4.11), taking into account Lemma 4. ■

The result of Theorem 3 can be applied to the model of heavy-ion collision of [3]-[4]. There $\mathscr{H} = L^2(\mathbb{R}^3)$, and

$$L\rho = K\rho + \rho K^* + \sum_{n=1}^{3} B_n \rho B_n^*,$$

where

$$K = -i(H_0 + V) - \frac{1}{2}\sum_{n=1}^{3} B_n^* B_n, \tag{4.13}$$

and where

$$(H_0\psi)(x) = -\frac{1}{2m}\Delta\psi(x), \tag{4.14a}$$

$$(V\psi)(x) = V(x)\psi(x), \tag{4.14b}$$

$$(B_n\psi)(x) = W(x)\left(x_n + \alpha\frac{\partial}{\partial x_n}\right)\psi(x), \quad n = 1, 2, 3, \quad \alpha > 0. \tag{4.14c}$$

We assume that

$$\left.\begin{array}{l} V \in L^2(\mathbb{R}^3) + L^\infty(\mathbb{R}^3),\ \operatorname{Im} V(x) \leqslant 0, \\ W(x),\ x_n W(x),\ \dfrac{\partial}{\partial x_n} W(x),\ n=1,2,3, \text{ are continuous and bounded,} \\ \sup\limits_{x \in \mathbb{R}^3} |W(x)|^2 < (m\alpha^2)^{-1}. \end{array}\right\} \tag{4.15}$$

Then $K - K_0$ is relatively bounded with respect to $K_0 = -iH_0$ with relative bound smaller than 1 (use [*11*], p. 302), and K is the generator of a strongly continuous contraction semigroup on $\mathscr{H}$, with dom (K) = dom (K_0) ([*11*], p. 500). Similarly, dom (K^*) = dom (K_0) and also the operators B_n, B_n^*, $n = 1, 2, 3$, are well defined on dom (K_0).

Assuming moreover that

$$|V(x)|,\ |W(x)|,\ |x_n W(x)|,\ \left|\frac{\partial}{\partial x_n} W(x)\right|,\quad n=1,2,3, \quad \text{are } O(|x|^{-1-\varepsilon})$$

as $|x| \to \infty$, one can prove the existence of S and Ω. The linear manifold $\mathscr{H}_0$ can be taken to be the linear span of Gaussians, and the integrability of the functions (4.1), (4.2) can be shown with the usual methods ([*11*], p. 535). Note also that the assumptions of Theorem 2 concerning uniqueness of the decomposition are satisfied.

ACKNOWLEDGMENT

One of the authors (R. A.) is grateful to Prof. V. Gorini for a warm hospitality at the Institute of Physics in Milan, and to A. von Humboldt-Stiftung for a research scholarship.

REFERENCES

[1] A. BARCHIELLI, L. LANZ, *Nuovo Cimento*, t. **44 B**, 1978, p. 241-264; A. BARCHIELLI, *Nuovo Cimento*, t. **47 A**, 1978, p. 187-199.
[2] E. B. DAVIES, *Ann. Inst. Henri Poincaré*, t. **XXXII A**, 1980, p. 361-375.
[3] R. ALICKI, *Ann. Inst. Henri Poincaré*, t. **XXXV A**, 1981, p. 97-103.
[4] R. ALICKI, Quantum Dynamical Semigroups and Dissipative Collisions of Heavy Ions, *Z. Phys.*, t. **A 307**, 1982, p. 279-285.
[5] E. B. DAVIES, *Quantum Theory of Open Systems*. Academic Press, 1976.
[6] V. GORINI, A. FRIGERIO, M. VERRI, A. KOSSAKOWSKI, E. C. G. SUDARSHAN, *Rep. Math. Phys.*, t. **13**, 1978, p. 149-173.
[7] E. B. DAVIES, *Rep. Math. Phys.*, t. **11**, 1977, p. 169-188.
[8] Ph. A. MARTIN, *Nuovo Cimento*, t. **30 B**, 1975, p. 217-238.
[9] E. B. DAVIES, *Commun. math. Phys.*, t. **71**, 1980, p. 277-288.
[10] E. B. DAVIES, *Rep. Math. Phys.*, t. **17**, 1980, p. 249-255.
[11] T. KATO, *Perturbation Theory for Linear Operators*, Springer, 1966.

(Manuscrit reçu le 3 mai 1982)