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D. PAVÓN
D. JOU
J. CASAS-VÁZQUEZ

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On a covariant formulation of dissipative phenomena

by

D. PAVÓN, D. JOU and J. CASAS-VÁZQUEZ
Departamento de Termologia. Facultad de Ciencias.
Universidad Autónoma de Barcelona, Bellaterra (Barcelona), Spain

ABSTRACT. — In this article, a covariant non-stationary theory of continuum irreversible thermodynamics is proposed. The specific entropy and the entropy flux as well as the thermodynamical forces are expanded up to second order in the dissipative fluxes. As a consequence, the speed of propagation of perturbations is finite and the phenomenological laws are non-linear in the dissipative fluxes. In the non-relativistic limit, these equations reduce to those of extended irreversible thermodynamics.

RÉSUMÉ. — Dans cet article on propose une théorie covariante et non stationnaire de la thermodynamique irréversible des milieux continus. L'entropie spécifique et le flux d'entropie, d'une part, ainsi que les forces thermodynamiques d'autre part, sont développés jusqu'au deuxième ordre dans les flux dissipatifs. Comme une conséquence, on obtient des perturbations se propageant à vitesse finie et des lois non-linéaires dans les flux. Dans la limite non-relativiste ces équations se réduisent à celles de la thermodynamique irréversible généralisée.

1. INTRODUCTION

In the last deçade, the interest of physicits about relativistic thermodynamics has increased, perhaps, as noted by Israel [*1*], due in part to possible applications to astrophysical and cosmological problems where relativistics effects can play a main role. In particular, the problem of detection of gravitational waves has oriented the work of some authors [*2*]-[*5*] toward the dissipative solid which, under suitable conditions, could be utilized for detecting such waves.

The common feature of some recent covariant theories describing dissipative processes in a fluid is their non-stationary, —causal—, character [*6*]-[*12*]. All of them avoid the paradox of instantaneous propagation of disturbances by allowing the inclusion of the dissipative quantities (heat flux and dissipatives stresses), in the expression of the entropy density and the entropy flux. Indeed, conventional thermodynamical approaches [*13*]-[*16*] postulate vanishing contribution of dissipative fluxes to the entropy and to the entropy flux. It is to say, they maintain the equilibrium equations of state for the entropy density and assume moreover a linear relation between the entropy flux and the heat flux. Actually, this approximation is valid in quasi-stationary situations only, where the space-time gradients of dissipative quantities are negligible.

All the above referred causal theories have Newtonian counterparts. Thus, those of Israel [*6*] and Israel and Stewart [*7*] as well as the Dixon's [*8*] one, rest on the Muller's [*17*] classic approach, the Bampi and Morro's [*9*] [*10*] one extends the method of hidden variables [*18*] to the relativistic framework, whereas our approach to heat and electric conduction [*11*] [*12*] generalizes their Newtonian treatment in extended irreversible thermodynamics [*19*] [*20*]. Recently, the latter theory has been used in the non-relativistic context in the analysis of several problems, as the thermodynamic description of second order fluids [*21*], the fluctuations of dissipative fluxes [*22*] [*23*], and the fluctuations of the heat flux near a critical point [*24*].

Our main purpose in this paper is to develop a phenomenological causal theory for a relativistic dissipative simple fluid, which can be understood as the covariant version of the afore-mentioned classical extended thermodynamics. In this connection we get a set of equations describing the evolution of dissipative variables of that fluid along the world line by means of the expansion of the specific entropy as well as the entropy flux and the thermodynamical forces up to second order in the dissipative fluxes. As a consequence, the constitutive equations obtained are more general than those of earlier relativistic formulations of continuum thermodynamics.

2. CONSERVATION LAWS AND THERMODYNAMIC ASSUMPTIONS

We start introducing the notation and some useful relations. Let S_4 be the four-dimensional space-time of the special or general relativity and $g_{\mu\nu}$ the metric tensor with signature $(-, +, +, +)$. Comma and semicolon represent partial and covariant derivative respectively. Symmetrization is indicated by round parentheses around the corresponding indexes. The dimensionless hydrodynamical velocity u^μ fulfills the normalization condition $u^\mu u_\mu = -1$. Differentiation along the world line is defined by $\dot{A}\ldots = u^\mu A\ldots_{;\mu}$ where $A\ldots$ is a generic tensorial quantity. In particular $\dot{u}_\mu = u^\nu u_{\mu;\nu}$ stands for the acceleration vector, which satisfies $\dot{u}^\mu u_\mu = 0$. The symmetric spatial projector $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$ projects any vectorial or tensorial quantity in the tri-space orthogonal to u_μ, i. e. $u_\mu \Delta^{\mu\nu} = \Delta^{\mu\nu} u_\nu = 0$. With its help any vector V_μ can be split uniquely into a « temporal part » $| V_\mu |_\parallel$ parallel to u_μ and another « spatial » $| V_\mu |_\perp$ orthogonal to u_μ, $V_\mu = | V_\mu |_\parallel + | V_\mu |_\perp$, $| V_\mu |_\parallel$ and $| V_\mu |_\perp$ being respectively $| V_\mu |_\parallel = - u_\mu u^\nu V_\nu$ and $| V_\mu |_\perp = \Delta_\mu{}^\nu V_\nu$. Likewise, following to Bressan [*25*], any tensor of rank two $A_{\mu\nu}$ can be decomposed into temporal, mixed and spatial parts according to

$$A_{\mu\nu} = A u_\mu u_\nu + A'_\mu u_\nu + u_\mu A''_\nu + | A_{\mu\nu} |_\perp$$

with

$$A = u^\mu u^\nu A_{\mu\nu}\,, \qquad A'_\mu = - \Delta^\rho{}_\mu A_{\rho\lambda} u^\lambda$$

$$A''_\nu = - u^\lambda \Delta^\sigma{}_\nu A_{\lambda\sigma}\,, \qquad | A_{\mu\nu} |_\perp = \Delta^\lambda{}_\mu \Delta^\rho{}_\nu A_{\lambda\rho}$$

Finally, we denote the trace-free part, or deviator of $A_{\mu\nu}$ by

$$\langle A_{\mu\nu} \rangle = A_{(\mu\nu)} - \frac{1}{3} g_{\mu\nu} A^\lambda{}_\lambda$$

Let us consider a heat conducting viscous simple fluid, with momentum-energy tensor $T^{\mu\nu}$ symmetric and conserved. Let ρ be the mass density of the system in its proper frame and $J^\mu = \rho u^\mu$ the mass flux which is submitted to the restriction $J^\mu{}_{;\mu} = 0$ provided that chemical reactions as well as creation and annihilation of matter are excluded. From that relation the continuity equation

$$\rho \dot{\upsilon} = u^\mu{}_{;\mu} \tag{1}$$

follows immediately, υ being the specific proper volume.

Such as Bressan has shown [*25*] the expression proposed by Eckart [*13*] for $T^{\mu\nu}$

$$T^{\mu\nu} = \varepsilon u^\mu u^\nu + 2c^{-1} q^{(\mu} u^{\nu)} + P^{\mu\nu} \tag{2}$$

is adequate to describe our fluid. In (2) the quantities ε, q^μ and $P^{\mu\nu}$ stand respectively for the specific internal energy, the heat flux and the pressure tensor; these two later quantities are of spatial type, i. e. $q^\mu = | q^\mu |_\perp$ and $P^{\mu\nu} = | P^{\mu\nu} |_\perp$.

Usually, the pressure tensor is decomposed in their equilibrium and viscous parts according to

$$P^{\mu\nu} = p\Delta^{\mu\nu} - \overline{W}^{\mu\nu} \tag{3}$$

p being the hydrostatic pressure. In turn, $\overline{W}^{\mu\nu}$ admits the following decomposition

$$\overline{W}^{\mu\nu} = - \Pi \Delta^{\mu\nu} + W^{\mu\nu} \tag{4}$$

where $\Pi = (-1/3)\overline{W}^{\mu}{}_{\mu}$ is the bulk viscosity and $W^{\mu\nu}$ the traceless shear viscosity tensor. Both $\overline{W}^{\mu\nu}$ and $W^{\mu\nu}$ are of spatial type, and if moreover, as in our case, the fluid is structureless they are also symmetric.

When the momentum-energy conservation equation $T^{\mu\nu}{}_{;\nu} = 0$ is split in their temporal and spatial parts, it gives rise on the one hand to the linear momentum conservation $\Delta^{\lambda}{}_{\mu}T^{\mu\nu}{}_{;\nu} = 0$, and on the other hand to the energy conservation $u_{\mu}T^{\mu\nu}{}_{;\nu} = 0$. By using (1) and (2), a brief calculation shows that these relations can be written respectively as

$$\varepsilon\dot{u}^{\lambda} + \frac{1}{c}\{\dot{q}^{\lambda} - u^{\lambda}q_{\mu}\dot{u}^{\mu} + q^{\lambda}u^{\nu}{}_{;\nu} + u^{\lambda}{}_{;\nu}q^{\nu}\} + \Delta^{\lambda}{}_{\mu}P^{\mu\nu}{}_{;\nu} = 0 \tag{5}$$

$$\rho(\dot{\varepsilon} + p\dot{\upsilon}) = -\frac{1}{c}(q^{\mu}{}_{;\mu} + q^{\mu}\dot{u}_{\mu}) - (\Pi\Delta^{\mu\nu} - W^{\mu\nu})u_{(\mu;\nu)} \tag{6}$$

This latter is the relativistic version of the first law of thermodynamics, which we will use later.

Following the lines of extended irreversible thermodynamics, we postulate the existence of a quantity s, called non-equilibrium specific entropy, which is a function not only of equilibrium variables but also of the dissipative quantities, namely q^{μ}, Π and $W^{\mu\nu}$,

$$s = s(\varepsilon, \upsilon, q^{\mu}, \Pi, W^{\mu\nu}) \tag{7}$$

This means that the thermodynamic state of the system under consideration is completely specified with the knowledge of these variables in every point of space-time occupied by the fluid, and then expression (7) plays the role of the fundamental equation in thermostatics.

The evolution of s along u^{μ} is given by means of a generalized Gibbs relation. In order to write $\dot{s}$ up to second order in the dissipative variables we adopt the following equations of state

$$\left.\frac{\partial s}{\partial \varepsilon}\right|' = \frac{1}{T}; \quad \left.\frac{\partial s}{\partial \upsilon}\right|' = \frac{p}{T}; \quad \left.\frac{\partial s}{\partial q^{\mu}}\right|' = \frac{\alpha_1}{\rho T} q^{\mu} \tag{8a}$$

$$\left.\frac{\partial s}{\partial \Pi}\right|' = \frac{\alpha_2}{\rho T}\Pi; \quad \left.\frac{\partial s}{\partial W^{\mu\nu}}\right|' = \frac{\alpha_3}{\rho T} W^{\mu\nu} \tag{8b}$$

where an upper prime indicates that all quantities but the one subject to derivation are to be kept constant. Obviously all these derivatives are also functions of ε, υ, q^{μ}, Π and $W^{\mu\nu}$.

The first two relations of (8*a*) are similar to the classical equations of thermostatics defining the absolute temperature T and the thermodynamic pressure respectively, while (8*b*) and the latter of (8*a*) establish the dependence between s and the dissipative variables and vanish identically at equilibrium. We have moreover introduced in (8) three parameters α_i functions of ε and υ which as we will show later are related to the relaxation proper times. Note that in our approach, T and p are non-equilibrium quantities which can be related to the corresponding thermostatic or local equilibrium variables by means of appropriate expansions in series around their equilibrium values.

Taking into account the above equations of state differentiation of (7) yields the covariant and second-order Gibbs equation

$$T\dot{s} = \dot{\varepsilon} + p\dot{v} + v\alpha_1 q^\mu \dot{q}_\mu + v\alpha_2 \Pi\dot{\Pi} + v\alpha_3 W^{\mu\nu}\dot{W}_{\mu\nu} \tag{9}$$

which differs largely of that used in stationary theories.

Our next basic assumption concerns to the spatial vector entropy flux I^μ_s. It states that this quantity must be a function of the same variables entering in the specific entropy, i. e.

$$I^\mu_s = I^\mu_s(\varepsilon, v, q^\mu, \Pi, W^{\mu\nu}) \tag{10}$$

whose most general expression up to second order in these variables reads

$$I^\mu_s = \beta_1 q^\mu + \beta_2 \Pi q^\mu + \beta_3 W^\mu{}_\nu q^\nu \tag{11}$$

where the β_i are scalar functions of ε and v, in particular the coefficients β_2 and β_3 couple heat flux to bulk and shear viscosity respectively. A comparison of (11) with eq. (6) from ref. [*11*] allows us to identify β_1 as $1/c\mathrm{T}$. Evidently the r. h. s. of (11) coincides formally with the usual expression of the entropy flux only if viscosity is neglected.

3. ENTROPY PRODUCTION AND PHENOMENOLOGICAL RELATIONS

In order to derive the phenomenological laws governing the behaviour of Π, q^μ and $W^{\mu\nu}$, it is necessary to obtain previously the entropy production σ in terms of these variables. This quantity is related to s and I^μ_s by means of the entropy balance equation, whose standard form reads

$$\rho\dot{s} + I^\mu_{s;\mu} = \sigma \tag{12}$$

By resorting to (9) and (11) σ can be expressed as

$$\sigma = \frac{1}{T}\{ q^\mu \,|\, {}^1X_\mu \,|_\perp + \Pi\, {}^0X + W^{\mu\nu} \,|\, {}^2X_{\mu\nu} \,|_\perp \} \tag{13}$$

with

$$|\, {}^1X_\mu \,|_\perp = \Delta_\mu{}^\nu \left\{ -\frac{1}{cT}(T_{,\nu} + T\dot{u}_\nu) + \alpha_1 \dot{q}_\nu + T\beta_2 \Pi_{,\nu} + \eta T \Pi \beta_{2,\nu} + T\beta_3 W^\lambda{}_{\nu;\lambda} + \gamma T W^\lambda{}_\nu \beta_{3,\lambda} \right\} \tag{14}$$

$${}^0X = -\, u^\mu{}_{;\mu} + \alpha_2 \dot{\Pi} + T\beta_2 q^\mu{}_{;\mu} + \eta^* T q^\mu \beta_{2,\mu} \tag{15}$$

$$|\, {}^2X_{\mu\nu} \,|_\perp = \Delta^\lambda_{(\mu}\Delta^\rho_{\nu)} \{ u_{(\lambda;\rho)} + \alpha_3 \dot{W}_{\lambda\rho} + T\beta_3 q_{(\lambda;\rho)} + \gamma^* T \beta_{3(,\lambda} q_{\rho)} \} \tag{16}$$

Here η, η^*, γ and γ^* denote dimensionless factors with no other restriction that $\eta + \eta^* = \gamma + \gamma^* = 1$. In the deduction of (13) we have used the relations $q^\mu\, {}^1X_\mu = q^\mu \,|\, {}^1X_\mu \,|_\perp$, and $W^{\mu\nu}\, {}^2X_{\mu\nu} = W^{\mu\nu} \,|\, {}^2X_{\mu\nu} \,|_\perp$ implying that only the spatial parts of ${}^1X_\mu$ and ${}^2X_{\mu\nu}$ contribute to the increase of entropy.

In the current literature phenomenological relations are usually obtained assuming a linear relation between fluxes and forces. However, we will follow a different procedure since we are interested in the obtention of a more general set of constitutive equations which, as we shall see later, may be interpreted as evolution equations of the dissipative variables. With this in mind, we expand $|\, {}^1X_\mu \,|_\perp$, ${}^0X$ and $|\, {}^2X_{\mu\nu} \,|_\perp$ up to second order in terms of the thermodynamical variables; thus, taking into account that $|\, {}^1X_\mu \,|_\perp$ and $|\, {}^2X_{\mu\nu} \,|_\perp$ are spatial quantities, we write down

$$|\, {}^1X_\mu \,|_\perp = a_{10} q_\mu + a_{11} \Pi q_\mu + a_{12} W_{\mu\nu} q^\nu \tag{17}$$

$$ {}^0X = a_{00} + a_{01}\Pi + a_{02}\Pi^2 + a_{03} q^\mu q_\mu + a_{04} W^{\mu\nu} W_{\mu\nu} \tag{18}$$

$$|\, {}^2X_{\mu\nu} \,|_\perp = a_{20}\Delta_{\mu\nu} + a_{21} W_{\mu\nu} + a_{22} W_{\mu\lambda} W^\lambda{}_\nu + a_{23} q_\mu q_\nu \tag{19}$$

the a_{ij} coefficients being functions of the equilibrium variables.

These developments are more consistent with equations (9) and (11) than the conventional linear relations between fluxes and forces. It is to say, if both $\dot{s}$ and I_s^μ have been developed up to second order in the dissipative quantities also $|\, {}^1X_\mu \,|_\perp$, ${}^0X$ and $|\, {}^2X_{\mu\nu} \,|_\perp$ must be expanded up to that order, because the importance of (17)-(19) in this approach is analogous to that of equations (9)-(11). Note that all these developments could be carried out to some order higher than two, but the inherent formal complications would not be compensated by a deeper physical insight.

When (17)-(19) are introduced into (13) the second law of thermodynamics imposes on the a_{ij} coefficients the following restrictions

$$\begin{aligned} & a_{10} \geqslant 0\,, \qquad a_{21} \geqslant 0\,, \qquad a_{01} \geqslant 0 \\ & a_{22} = a_{04} = a_{00} = a_{02} = 0 \\ & a_{11} = -\,a_{03}\,, \qquad |\, a_{12} + a_{23} \,| \leqslant \frac{1}{2}(a_{10} + a_{21}) \end{aligned} \tag{20}$$

since $q^\mu q_\mu$ as well as Π^2 and $W^{\mu\nu}W_{\mu\nu}$ are non-negative quantities, while Π, $W^{\mu\nu}W_{\mu\nu}W^\lambda{}_\nu$ can be both positive or negative and

$$|\, q^\mu W_{\mu\nu} q^\nu \,| \leqslant (1/2)(q^\mu q_\mu + W^{\mu\nu} W_{\mu\nu})\,.$$

By equating the r. h. s. of (14), (15) and (16) with the corresponding r. h. s. of (17), (18) and (19) and taking moreover into account the definition of the projector tensor as well as the set of restrictions (20) we have the phenomenological relations

$$\alpha_1 \dot{q}_\mu = a_{10} q_\mu + a_{11} \Pi q_\mu + a_{12} W_{\mu\nu} q^\nu + Q_\mu + \alpha_1 u_\mu q_\nu \dot{u}^\nu \tag{21}$$

$$\alpha_2 \dot{\Pi} = a_{01}\Pi - a_{11} q^\mu q_\mu + T\beta_2 q^\mu{}_{;\mu} + \eta^* T q^\mu \beta_{2,\mu} + u^\mu{}_{;\mu} \tag{22}$$

$$\alpha_3 W_{\mu\nu} = a_{21} W_{\mu\nu} + a_{23} \langle q_\mu q_\nu \rangle + \langle Z_{\mu\nu} \rangle + \alpha_3 \langle W_{\mu\rho} u_\nu \dot{u}^\rho + u_\mu W_{\lambda\nu} \dot{u}^\lambda \rangle \tag{23}$$

with

$$Q_\mu = -\Delta_\mu{}^\nu \left\{ -\frac{1}{cT}(T_{,\nu} + T\dot{u}_\nu) + T\beta_2\Pi_{,\nu} + \eta T\Pi\beta_{2,\nu} + T\beta_3 W^\lambda{}_{\nu;\lambda} + \gamma T W^\lambda{}_\nu \beta_{3,\lambda} \right\} \quad (24)$$

$$Z_{\mu\nu} = -\Delta^\lambda_{(\mu}\Delta^\rho_{\nu)} \{ u_{(\lambda;\rho)} + T\beta_3 q_{(\lambda;\rho)} + \gamma^* T\beta_{3(\lambda,} q_{\rho)} \} \quad (25)$$

In the deduction of (23) we have used the restriction $a_{20} = 0$, which arises from the requirement $\dot{W}_{\mu\nu} = \langle \dot{W}_{\mu\nu} \rangle$. The relativistic temperature gradient $T_{,\nu} + T\dot{u}_\nu$ [*13*] [*25*] [*26*] occurring in (21) through (24) follows in a natural way from the inertia of heat [*27*]. The vector $\alpha_1 \dot{q}_\mu$ appears in (22) split into a purely spatial part $a_{10} q_\mu + a_{11}\Pi q_\mu + a_{12} W_{\mu\nu} q^\nu + Q_\mu$, and another purely temporal part $\alpha_1 u_\mu q_\nu \dot{u}^\nu$. This phenomenological relation constitutes a generalization of the issue already obtained by the authors [*11*], which when viscosity is disregarded have identified the coefficient a_{10} as $1/kcT$, k ($\geqslant 0$) being the thermal conductivity of the fluid. Consequently, a_{10} is a positive quantity such as was stated above. Likewise the relaxation coefficient α_1 is easily identified as $-\tau_1/kT$ where τ_1 is the proper relaxation time of the heat conduction process.

In the local instantaneous rest frame one has $\Delta^\mu{}_\nu = \text{diag}\,(0, 1, 1, 1)$, $u_{k;j} = c^{-1} v_{k;j}$, $\dot{u}_k = c^{-2} a_k$, $q_\mu = (0, q_k)$, $W_{0\nu} = 0$, ($j, k = 1, 2, 3$), v_k and a_k being the Cartesian components of the ordinary three-velocity and the ordinary three-acceleration respectively. If in that frame and in the absence of heat flux (22) and (23) are compared with the corresponding relaxational constitutive equations [*22*] $\Pi + \tau_2\dot{\Pi} = \xi v_{k,k}$ and $W_{kj} + \tau_3 \dot{W}_{kj} = 2\mu \langle v_{(k,j)} \rangle$, the following identifications arise:

$$a_{01} = 1/c\xi\,, \qquad a_{21} = 1/2c\mu\,, \qquad \alpha_2 = -\tau_2/\xi\,, \qquad \alpha_3 = -\tau_3/2\mu\,,$$

where ξ and μ are respectively the bulk and shear viscosity coefficients, and τ_2 and τ_3 stand for the proper relaxation times of the involved processes. Recently, equations of that kind have been used in connection with cosmological problems [*28*]. The above identifications confirm two important points of the theory, on the one hand the positive semidefinite character of a_{01} and a_{21} and on the other hand the negativeness of α_2 and α_3. Indeed, $\alpha_i < 0$ ($i = 1, 2, 3$) implies a finite speed whereby the causality of the present theory is guaranted, and if moreover τ_i is assumed to be of the order of a molecular mean collision time, the speed of the corresponding dissipative disturbance results not only lower than c but moreover comparable to the velocity of sound in the fluid under consideration.

Relation (23) exhibits the tensor $\alpha_3 \dot{W}_{\mu\nu}$ split into a purely spatial tensor $a_{21} W_{\mu\nu} + a_{23} \langle q_\mu q_\nu \rangle + \langle Z_{\mu\nu} \rangle$ and another one $\alpha_3 \langle W_{\mu\rho} u_\nu \dot{u}^\rho + u_\mu W_{\lambda\nu} \dot{u}^\lambda \rangle$ which is not purely spatial nor temporal but mixed.

Israel and Stewart [7] have succeded in relating some coefficients (α_1, α_2, α_3, β_2, β_3, a_{10}, a_{01} and a_{21}) appearing in our constitutive equations to kinetic quantities for a relativistic quantum gas, by means of the Grad's method of the 14-moment. For the time being, and analogous determination of the remaining a_{ij} coefficients, —although desirable—, has not been made, at least to our knowledge.

Except for the term $c^{-1}\dot{u}^\nu$ occurring in Q_μ, the spatial parts of (21) and (23) reduce in the classical limit to the Newtonian phenomenological relations obtained by Jou *et al.* [*20*], the temporal and mixed parts take into account purely relativistic acceleration effects, and therefore they have not classic counterparts. Indeed, in the local instantaneous rest frame, (21)-(23) differ from the referred Newtonian equations by terms of the order c^{-2} only.

Extended irreversible thermodynamics concerns with the evolution of the thermodynamic quantities. In our case, this evolution means evolution with respect to the proper time, and consequently we interpret the constitutive relations (21)-(23) as those giving the evolution of the dissipative fluxes along the world line of each material point of the fluid. The evolution of the equilibrium variables υ and ε is governed by the continuity equation (1) and by the energy conservation law (5) respectively, whereas the evolution of u^μ is furnished by the linear momentum conservation (6).

The non-linear terms Πq_μ, $W_{\mu\nu}q^\nu$, $q^\mu q_\mu$ and $\langle q_\mu q_\nu \rangle$ appearing in our constitutive equations do not occur in other causal phenomenological approaches [*6*] [*10*]. Indeed, these terms may be negligible in ordinary situations, but their importance increase as the thermodynamic system is separated from equilibrium and consequently they can contribute with new phenomena, qualitatively different from the linear ones. They may have interest in cosmological and astrophysical problems where rapid fluctuations, fast rotations and strong gravitational fields can occur [*7*]. Thus, Novello and d'Olival have analysed the Bianchi type-I Universe filled with a non-linear Stokesian fluid [*29*]. According to these authors, in a highly compressed early epoch of the Universe the large scale anisotropy could be so important as to induce a non-linear response. In fact they find several new results, as for instance, a very sensitive dependence of the stability of the Friedmann solution on the values of the quadratic coefficient of viscosity and a non-symmetric behaviour of the Universe under time inversion. Likewise, the high values of the phenomenological coefficients k, ξ and μ obtained by De Groot for some components of the cosmic fluid in the lepton era [*30*] suggest that the afore-mentioned non-linear terms must not be ruled out in such an extreme situation of the Universe.

Concerning both the temperature T and the pressure p our approach differ likewise from other causal theories. Thus, whereas on the one hand Dixon [*8*] and Israel [*6*] use their local equilibrium values $- T_0, p_0 -$, it is to say the same ones used in stationary approaches [*13*]-[*16*], and on the other hand Bampi and Morro [*9*] [*10*] use their so-called empirical values, the temperature and the pressure introduced by us —in eqs (8)—

differ from the equilibrium ones by terms of second order in the dissipative fluxes. These second order terms could be found from the equality of the second derivative of the entropy, as obtained from (8) and (9), [*31*]. As a consequence the identifications $T = T_0$, $p = p_0$ are valid to first order only as Israel and Stewart [*7*] have also noted.

4. CONCLUSIONS

On the basis of extended irreversible thermodynamics, we have developed a phenomenological non-stationary relativistic theory for heat-conducting viscous simple fluids. This has been done by including the heat flux and the shear and bulk viscosity in both the Gibbs equation and the entropy flux and expanding moreover the thermodynamic forces up to second order in the dissipative variables.

We have presented a complete set of equations governing the evolution of the thermodynamic variables along the world line: the equilibrium ones from the conservation laws and the dissipative ones by means of the phenomenological relations. In this account, the evolution of the specific entropy is set through the Gibbs equation.

Usual extended irreversible thermodynamics [*19*] [*20*] appears to be the non-relativistic limit of the present approach. Three new coefficients, namely a_{11}, a_{12} and a_{23} are introduced with respect to Israel and Stewart [*7*] in the phenomenological laws, which are no longer linear in the dissipative quantities.

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