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# On the instability in Universe

by

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ABSTRACT. — With coordinate conditions which are different from Lifshitz's ones it is investigated the instability in a universe model locally nonhomonogeneous which is the generalization of Shirokov and Fisher's model. The general equation for the density contrast E is obtained. For the gravitational instability in the cases of uniform models containing matter and blackbody radiation and of the oscillating model of Rosen one gets $E \sim t^{4/3}$ and $E \sim t^2$ respectively. The general equation for thermal instability in the later stages of cosmic evolution is also obtained. Basing on different investigations about the thermal conditions of intergalactic matter the numerical values of the growth rate of the density contrast are calculated. All those results show the important role of thermal instability in the galaxy formation.

RÉSUMÉ. — Avec des conditions de coordonnées qui sont différentes de celles de Lifshitz, on a étudié l'instabilité d'un modèle d'Univers localement non-uniforme qui est la généralisation du modèle de Shirokov et Fisher. On a établi l'équation générale pour la variation relative de densité E. Pour l'instabilité gravitationnelle du modèle contenant matière et rayonnement et du modèle oscillant de Rosen, on a montré que $E \sim t^{4/3}$ et $E \sim t^2$ respectivement. L'équation générale pour l'instabilité thermique dans les périodes récentes de l'évolution cosmique est aussi déterminée. On a calculé les valeurs numériques du degré de croissance de la variation relative de densité en se basant sur les différentes investigations concernant les conditions thermiques de la matière intergalactique. Tous ces résultats montrent le rôle important de l'instabilité thermique dans la formation des galaxies.

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## I. INTRODUCTION

In Friedmann's model of isotropic space, as in all relativistic cosmological theories, matter is regarded as being distributed over the universe in the form of a homonogeneous continuous medium. The best assumption to make would be that matter is distributed in the form of individual point (or very small) sources of the gravitational field, being dispersed uniformly through space on the average only, like the molecules of an ideal gas. For cosmological applications, the main interest formerly was in the problem of smoothing out local inhomomogeneities in an average space-time metric. As in the case of an electromagnetic field to receive the equations of the macrofield one must take the averages of the Einstein equations of the microfield. To generalize the averaging method of Shirokov and Fisher [*1*] we obtain the following Einstein equations for our universe model locally nonhomonogeneous

$$R_i^k - \frac{1}{2}\delta_i^k R - C_i^k = \frac{8\pi G}{c^4} T_i^{k\,(\mathrm{macro})} \qquad (1.1)$$

where $C_i^k$, a term depending on the fluctuations of the microfield part of the usual Einstein's equations, appears as an addition to the « macroscopic Einstein equations ». If the metric will assume the form

$$ds^2 = -\, g_{ij} dx^i dx^j = c^2 dt^2 - a^2(t)[dr^2 - \sigma^2(r) d\Omega^2], \qquad (1.2)$$

where $\sigma(r) = \sin r$ ($k = 1$, closed), $r$ ($k = 0$, flat), *or* sh $r$ ($k = -1$, open), and

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\varphi^2,$$

the equation (1.1) becomes

$$\frac{8\pi G}{c^2}\varepsilon = 3\left(\frac{\dot a^2}{a^2} + \frac{kc^2}{a^2}\right) - \frac{8\pi G u}{c^2}, \qquad (1.3)$$

$$\frac{8\pi G}{c^2} p = -\,2\frac{\ddot a}{a} - \frac{\dot a^2}{a^2} - \frac{kc^2}{a^2} + \frac{8\pi G v}{c^2}, \qquad (1.4)$$

where $\varepsilon$ and $p$ denote the energy density and the pressure of the assemblage of matter and radiation respectively, G is the newtonian gravitational constant, the dot denotes the differentiation with respect to $t$, and $u$ and $v$ have the form

$$u = A_1 + B_1 a^{-n}, \qquad v = A_2 + B_2 a^{-n}, \qquad (1.5)$$

where $A_1$, $A_2$, $B_1$ and $B_2$ are constants. We remark that in the equations (1.3) and (1.4) we transfer the terms

$$\frac{8\pi G u}{c^2} \quad \text{and} \quad \frac{8\pi G v}{c^2}$$

to the left hand side: they may be interpreted formally as certain extra-terms to the energy and pressure. It follows from the two equations (1.3) and (1.4) that

$$\dot{\varepsilon}^* + 3\frac{\dot{a}}{a}(\varepsilon^* + p^*) = 0, \tag{1.6}$$

where

$$\varepsilon^* = \varepsilon + u, \qquad p^* = p - v. \tag{1.7}$$

If we put in (1.5) $A_1 = A_2 = 0$, $B_1 = B_2 = \gamma^2$, $n = 4$, we obtain the same equations and the same results (when $p = 0$) as in [1]. If we put $A_1 = A_2 = \lambda$, $B_1 = B_2 = 0$, and, then, $A_1 = A_2 = \lambda$,

$$B_1 = B_2 = -\frac{3c^2C^2}{8\pi G},$$

$n = 6$ we obtain the model with cosmological constant and the oscillating model [18] respectively.

In this paper we shall investigate the instability in our universe model (i. e. taking notice of equations (1.3) à (1.7)).

## II. THE GRAVITATIONAL INSTABILITY

Jeans [2] proposed that galaxies arise from gravitational instability in a uniform gas. Using a physically impossible but convenient uniform cloud as a zero-order state, he showed that the criterion for instability is that the wavelength $\lambda$ of the perturbation obeys the inequality

$$\lambda > \lambda_j = \left(\frac{\pi}{G\rho}\frac{dp}{d\rho}\right)^{1/2}. \tag{2.1}$$

Lifshitz and Khalatnikov [3] considered the perturbations of dynamical models of the universe using the theory of general relativity. Their results showed that a condensation due to the gravitational instability cannot grow so fast during the time scale of the universe. Bonnor [4] obtained the same result as Lifshitz by making use of non-relativistic fluid mechanical equations when $p = 0$. Recently Peebles [5] proposed a new approach to the formation of galaxies in close connection with the presence of a cosmic black-body radiation in a evolving universe [*6*]. This approach is similar to Bonnor's [*4*] in the sense that the newtonian gravitational theory was applied to the density fluctuation, while the effect of radiation drag was taken into consideration and he did not necessary postulate that the initial fluctuations were of a statistical origin. It must be noticed, however, that the newtonian theory would not be applicable at the early stage of the cosmic evolution where the radiation drag plays a significant role. In this circumstance one must pay attention to Lifshit's relativistic theory [*3*] of the gravitational instability. In order to eliminate the non-physical wave mode, Lifshitz introduced a particular set of coordinate conditions so as to make the perturbed Einstein equations tractable. Unfortunately however, his coordinate conditions are of such a type that it is difficult to describe Lifshit's formalism in the newtonian language. Many other authors have investigated the density perturbation in different particular cases as in [*7*] ($k = 1$, $p = 0$; $k = 0$, $p = 0$ and $p = \rho c^{2/3}$). Irvine [*8a*] derived general relativistic fluid mechanical equations imposing the generalized de Donder coordinate condition. A similar approach is presented by Silk [*8b*] to the Godel universe. Perturbations of a spatially homogeneous isotropic universe are investigated in [*8c*] in terms of small variations of the curvature. Narai, Tomita et Kato [*9*] have reformulated Lifshitz's relativistic theory for the gravitational instability in an expanding universe in such a way that the rewritten formalism permits them to make a full comparison with Bonnor's approach based on the newtonian theory, and derived the differential equation for the density contrast $K = \delta\varepsilon/\varepsilon_0$ ($\varepsilon_0$ is the unperturbed density of matter and radiation), which is of the third order in general, but is reduced to Bonnor's equation of the second order in the case of dust-filled world models. Field and Shepley [*10*] have studied the same problem by showing that one of the three independant solutions for K (in the case of world models with finite pressure) is not covariantly defined in the sense that it can be eliminated by a suitable infinitesimal transformation of coordinates ([1]). This suggests us that Lifshitz's coordinate condition $h_{0i} = 0$ for the metric perturbation $h_{ij} \equiv \delta g_{ij}$ is unsuitable except in the case of dust-filled world models. Then what coordinate condition is suitable

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([1]) In [*11*] [*13*] it is shown that the equations for the density contrast are in general higher (fourth and even sixth) order differential equations with respect to time which have two physical solutions and fictitions ones. A fictitions solution is a solution that can be eliminated by means of an appropriate transformation of coordinates [*12*] [*13*].

for the description of the density perturbation ? (2). Among three possible wave modes, i. e. the density perturbation, the rotational wave and the gravitational wave, only the first can have a spherical wave. Accordingly we confine our attention to the spherical symmetric perturbation which is specified in general by (3)

$$h_{0\alpha} = 0, \qquad \text{and} \qquad h_{\alpha\beta} = 0 \qquad (\alpha \neq \beta) \tag{2.2}$$

the former set of which, together with $h_{00} = 0$ constitutes Lifshitz's coordinate condition. On the other hand, it is easily seen from Lifshitz's analysis [3] that the perturbed radial 4-velocity $\delta u^1$ is non vanishing except in the case of dust-filled world models (cf. $\delta u^0 = \delta u^2 = \delta u^3 = 0$). This means that the density perturbation $\delta\varepsilon$ in the sense of Lifshitz is not the Lagrangian perturbation in general. As it is shown in [12] [13] [14] the most suitable coordinate condition for dealing with the spherically symmetric perturbation may be the Lagrangian condition such as

$$\delta u^1 = 0. \tag{2.3}$$

Moreover it is of interest to emphasize that in all the above mentioned papers [2] à [15] it was studied only the different particular cases of universe models without cosmological constant (i. e. with assumption $\lambda = 0$).

In this part we shall investigate the gravitational instability in our universe model taking into account the conditions (2.2) and (2.3).

## § 1. Reduction of the perturbation equations.

The metric (1.2) can be written as

$$ds_0^2 = c^2 e^{\nu_0} dt^2 - e^{\lambda_0} dr^2 - e^{\mu_0}(d\theta^2 + \sin^2\theta d\varphi^2), \tag{2.4}$$

where

$$e^{\lambda_0/2} = \frac{e^{\mu_0/2}}{\sigma(r)} = a(t).$$

(2) In [15] Harrison used the coordinate condition

$$h_{0\alpha} = 0, \qquad h^i_{\alpha;0} - \frac{1}{2}\delta^i_\alpha h_{;0} = 0$$

and investigated the solutions of irrotational motion.

(3) This condition is the same as the one employed by Sach and Wolfe [7] but they have put $h_{00} = 0$ at the sacrifice of the diagonality of the perturbed metric, i. e. $h_{0i} \neq 0$.

Let us now assume a spherically symmetric disturbance of the type (2.2). Then the perturbed metric takes the form

$$ds^2 = e^{\nu}c^2dt^2 - e^{\lambda}dr^2 - e^{\mu}(d\theta^2 + \sin^2\theta dy^2), \tag{2.5}$$

where

$$\lambda = \lambda_0 + \delta\lambda = 2\ln a + \delta\lambda, \qquad \mu = \mu_0 + \delta\mu = 2\ln(a\sigma) + \delta\mu,$$
$$\nu = \nu_0 + \delta\nu = \delta\nu. \tag{2.6}$$

Similarly we have

$$u^0 = 1 + \delta u^0, \qquad u^1 = \delta u^1, \qquad u^2 = u^3 = 0. \tag{2.7}$$

Let us assume further that the linear approximation is appropriate. Then it follows from equation (2.7) and $u_iu^i = -1$ that

$$\delta u^0 = -\frac{1}{2}\delta\nu. \tag{2.8}$$

Moreover, the variation of the energy-momentum tensor $T_i^j$ for a perfect fluid leads us to

$$\delta T_0^0 = -\delta\varepsilon, \qquad \delta T_1^1 = \delta T_2^2 = \delta T_3^3 = \delta p, \qquad \delta T_0^1 = -(\varepsilon + p)\delta u^1, \tag{2.9}$$

by the use of equations (2.7), and, in general, the variations of $u$ and $v$ are $\delta u$ and $\delta v$. On inserting Eqs (2.6) and (2.9) into the perturbed Einstein field equations (cf. (1.1))

$$\delta R_i^k - \frac{1}{2}\delta_i^k\delta R + \delta C_i^k = \frac{8\pi G}{c^4}\delta T_i^k, \tag{2.10}$$

we obtain

$$-\frac{8\pi G}{c^4}\delta\varepsilon^* = c^2a^{-2}\left\{\left(\delta\mu'' + 3\delta\mu'\frac{\sigma'}{\sigma} + \frac{\delta\mu}{\sigma^2}\right) - \left(\delta\lambda' + \delta\lambda\frac{\sigma'}{\sigma}\right)\frac{\sigma'}{\sigma}\right\}$$
$$- \frac{\dot a}{a}(\delta\dot\lambda + 2\delta\dot\mu) + 3\left(\frac{\dot a}{a}\right)^2\delta\nu + \frac{2kc^2}{a^2}\delta\lambda, \tag{2.11}$$

$$\frac{8\pi G}{c^2}\delta p^* = c^2a^{-2}\left\{\frac{\sigma'}{\sigma}\delta\mu' + \frac{\sigma'}{\sigma}\delta\nu' + \frac{\delta\mu}{\sigma^2} - \left(\frac{\sigma'}{\sigma}\right)^2\delta\lambda\right\}$$
$$+ \left(\frac{\dot a}{a}\right)\left\{\delta\dot\nu + 2\left(\frac{\ddot a}{\dot a} + \frac{\dot a}{a}\right)\delta\nu\right\} - \left(\delta\ddot\mu + 3\frac{\dot a}{a}\delta\dot\mu\right), \tag{2.12}$$

$$\left(\delta\nu'' - \frac{\sigma'}{\sigma}\delta\nu'\right) + \left(\delta\mu'' - \frac{\sigma'}{\sigma}\delta\lambda'\right)$$
$$+ \frac{2}{\sigma^2}(\delta\lambda - \delta\mu) - c^{-2}a^{-1}\{a^3(\delta\dot\mu - \delta\dot\lambda)\}^{\cdot} = 0, \tag{2.13}$$

$$-\frac{8\pi G}{c^3}(\varepsilon + p)\delta u^1 = a^{-2}\left\{\left(\delta\dot{\mu}' + \frac{\sigma'}{\sigma}\delta\dot{\mu}\right) - \frac{\sigma'}{\sigma}\delta\dot{\lambda} - \frac{\dot{a}}{a}\delta v'\right\}, \quad (2.14)$$

where

$$\delta\varepsilon^* = \delta\varepsilon + \delta u, \qquad \delta p^* = \delta p - \delta v. \quad (2.15)$$

The above four equations are the perturbation equations which hold under any coordinate condition.

On imposing the Lagrangian coordinate condition (2.3), it follows from Eq. (2.14) that

$$\frac{\dot{a}}{a}\delta v' = \delta\dot{\mu}' + \frac{\sigma'}{\sigma}(\delta\dot{\mu} - \delta\dot{\lambda}). \quad (2.16)$$

On the other hand, Lifshitz's original analysis [3] for the density perturbation suggests us that $h_1^1 = \delta\lambda$ and $h_2^2 = \delta\mu$ can be represented by

$$\begin{aligned}\delta\lambda &= (n^2 - k)^{-1}\mathrm{Y}\mathrm{Q}'' + \mathrm{YQ}/3\\ \delta\mu &= (n^2 - k)^{-1}\mathrm{X}(\sigma'\mathrm{Q}'/\sigma) + \mathrm{YQ}/3\end{aligned} \quad (2.17)$$

where X and Y are functions of $t$ only, and Q is the function of $r$ such as

$$\nabla^2\mathrm{Q} = \mathrm{Q}'' + 2\frac{\sigma'\mathrm{Q}'}{\sigma} = -(n^2 - k)\mathrm{Q},$$

or

$$\mathrm{Q} = \sin(nr)/n\sigma(r), \quad (2.18)$$

in which $n$ is a positive constant corresponding to the wave number. On inserting Eq. (2.17) into Eq. (2.16) and by making use of Eq. (2.18) we obtain (cf. $h_{00} = -\delta v$)

$$\delta v = -2\delta u^0 = \left(\frac{a}{\dot{a}}\right)\{\dot{\mathrm{Y}}/3 - (n^2 - k)^{-1}k\dot{\mathrm{X}}\}\mathrm{Q}, \quad (2.19)$$

where we have used the reasonable requirement that $\delta v = 0$ if $\delta\lambda = \delta\mu = 0$. An insertion of Eqs (2.17) and (2.19) into Eq. (2.13) permits us to derive

$$\ddot{\mathrm{X}} + 3\left(\frac{\dot{a}}{a}\right)\left(1 + \frac{k}{3\dot{a}^2}\right)\dot{\mathrm{X}} = (n^2 - k)(\mathrm{Y} + a\dot{\mathrm{Y}}/\dot{a})/3a_2. \quad (2.20)$$

Similarly we can reduce Eqs (2.11) and (2.12) to

$$\frac{8\pi G}{c^2}\delta\varepsilon^* = (n^2 - 4k)\left\{\frac{\mathrm{Y}c^2}{3a^2} - \frac{\dot{a}}{a}(n^2 - k)^{-1}\dot{\mathrm{X}}\right\}\mathrm{Q}, \quad (2.21)$$

and
$$\frac{8\pi G}{c^2}\delta p^* = \left(\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} - \frac{ka^2}{a\dot{a}}\right)\{\dot{Y}/3 - (n^2 - k)^{-1}k\dot{X}\}\,Q, \qquad (2.22)$$

where we have used Eq. (2.20) in the derivation of Eq. (2.22). If we introduce the density contrast

$$E = \frac{\delta\varepsilon^*}{\varepsilon^*} \qquad (2.33)$$

we can derive from the perturbed equations (2.21) and (2.22) together with Eqs. (2.2), (2.4) and (2.20) the following differential equation for E

$$\ddot{E} + 2\left(\frac{\dot{a}}{a}\right)\left\{1 + \frac{3}{2}\left(\frac{dp^*}{d\varepsilon^*} - 2\frac{p^*}{\varepsilon^*}\right)\right\}\dot{E} + \left\{(n^2 - 4k)\left(\frac{v_s}{a}\right)^2 - \frac{4\pi G\varepsilon^*}{c^2}\left(1 + 8\frac{p^*}{\varepsilon^*} - 3\frac{p^{*2}}{\varepsilon^{*2}} - 6\frac{dp^*}{d\varepsilon^*}\right) + 3k\frac{c^2}{a^2}\left(1 + 5\frac{p^*}{\varepsilon^*} - 3\frac{dp^*}{d\varepsilon^*}\right)\right\}E = 0, \qquad (2.24)$$

or

$$\mathscr{P}(E) + (n^2 - 4k)\left(\frac{v_s}{a}\right)^2 E = 0, \qquad (2.24a)$$

where

$$\mathscr{P} \equiv \frac{\partial^2}{\partial t^2} + 2\frac{\dot{a}}{a}\left\{1 + \frac{3}{2}\left(\frac{dp^*}{d\varepsilon^*} - 2\frac{p^*}{\varepsilon^*}\right)\right\}\frac{\partial}{\partial t} - \frac{4\pi G\varepsilon^*}{c^2}\left(r + 8\frac{p^*}{\varepsilon^*} - 3\frac{p^{*2}}{\varepsilon^{*2}} - 6\frac{dp^*}{d\varepsilon^*}\right) + 3k\frac{c^2}{a^2}\left(1 + 5\frac{p^*}{\varepsilon^*} - 3\frac{dp^*}{d\varepsilon^*}\right), \qquad (2.25)$$

and

$$v_s = c\left(\frac{\delta p^*}{\delta\varepsilon^*}\right)^{1/2}. \qquad (2.26)$$

The differential equation (2.24) is of second order (4) and it is reduced to the Bonnor's newtonian equation [*4*] when $k = 0$, $p = 0$ and $u = v = 0$.

## § 2. **Solving the obtained equation for density contrast** E.

To solve equation (2.24) exactly is not easy. In order to deal with the evolution of the universe in a realistic manner, we must take account of respective behaviours of matter (specified by $\varepsilon_m$ and $p_m$) and radiation

(4) Thus under Lagrangian coordinate condition, fictitious (non physical) solutions are eliminated automatically.

(specified by $\varepsilon_r = 3p_r = aT_r^4$) (5) at various stages of the cosmic evolution. It is generally believed that the discovery of the cosmic black-body radiation of T = 3 °K (at present) has supported the big-bang cosmology in the sense of Gamov and Dicke [*6*]. In this part, however, we are not concerned with such a realistic evolution. We use the Zeldovich's equation of state [*17*]

$$p = (\nu - 1)\varepsilon \equiv \alpha\varepsilon \tag{2.27}$$

and in many cases of physical interest in which the pressure is appreciable, $\nu$ has a constant value ($1 \leqslant \nu \leqslant 4/3$ [*15*]). To estimate approximately the growth rate of E we assume that

$$u, v \ll \varepsilon \tag{2.28}$$

and we consider the density perturbation such as (6)

$$c^2, \qquad (n^2 - 4k)v_s^2 \ll a^2. \tag{2.29}$$

Then Eq. (2.24) can be approximated so that (7)

$$\frac{d^2E}{da^2} + \frac{3(1 - 3\alpha)}{2a}\frac{dE}{da} - \frac{3}{2a^2}(1 + 2\alpha - 3\alpha^2)E = 0, \tag{2.30}$$

the solution of which is of the form

$$E = A_1 a^{(1+3\alpha)} + A_2 a^{3(\alpha-1)/2}. \tag{2.31}$$

Therefore if we assume that

$$a \sim t^{2/3(1+\alpha)}, \tag{2.32}$$

we obtain

$$E = A_1 t^{\frac{2(1+3\alpha)}{3(1+\alpha)}} + A_2 t^{\frac{(\alpha-1)}{(1+\alpha)}}. \tag{2.33}$$

When $\alpha = 1/3$ (radiation-filled model) we have

$$E = A_1 t + A_2 t^{-1/2}. \tag{2.34}$$

At the later stages of the universe evolution, when $\alpha \approx 0$

$$E \approx A_1 t^{2/3} + A_2 t^{-1}. \tag{2.35}$$

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(5) And even of cosmic rays (see part III).

(6) This approximation is well satisfied [*8b*].

(7) In this case $E = \dfrac{\delta\varepsilon}{\varepsilon}$.

For uniform model universes containing matter and blackbody radiation [16] we have

$$a = \frac{cA}{2\mu}(\zeta - 1)^{1/2}(3\zeta + 5)^{5/6}, \tag{2.36}$$

where $\zeta = (1 + \mu t)^{1/2}$.

Then $a \sim t^{2/3}$, and, therefore, from (2.31) we get

$$E \sim t^{4/3} \qquad \text{when} \qquad \alpha \approx 1/3 \tag{2.37}$$

and

$$E \sim t^{2/3} \qquad \text{when} \qquad \alpha \approx 0. \tag{2.38}$$

For oscillating universe [18], in the case $k = 1, \lambda = 0, \alpha = 1/3$ and assuming $\varepsilon \ll B$ [18], we get

$$a^2 \sim t^2 \tag{2.39}$$

and, therefore,

$$E \sim t^2. \tag{2.40}$$

Thus we have investigated the gravitational instability in our universe model and we have obtained the general differential equation (2.24) for the density contrast E. Moreover we have the expression for E in different cases. Some of our results have showed that the density contrast grows with time with a velocity greater than Lifshitz's. Especially in the case of oscillating universe, in the expression for E, there is a term which is proportional to $t^2$. Thus our universe model appears to offer more advantage over other nonstatic models in the problem of gravitational instability.

## III. THE THERMAL INSTABILITY

The importance of thermal instability as another possible mechanism for galaxies formation has been emphasized. In [19] Bonnor has shown that in the world models with zero cosmological constant, the nebulae cannot have resulted from gravitational instability following perturbations of magnitude predicted by ordinary statistical theory. Therefore Bonnor considered that models of general relativity with long time-scales require $\lambda \neq 0$. In [20] Gamow has suggested that the large fluctuations required might have arisen from turbulence of the cosmic medium. Another suggestion is one of Terletsky [21] that ordinary gas theory may be quite inadequate to deal with very large masses of gravitating gas and Terletsky considered that such large masses are liable to much larger fluctuations than those predicted. In [22] the gravitational instability is studied on the basis of statistical physics method and it is shown the possibility of large fluctuations; it is found also some values of masses of galaxies in the newtonian cosmology but with the assumption that $\lambda \neq 0$. Attemps to apply thermal instability to galaxy formation were made by Hoyle [*23*] and Gold and Hoyle [*24*] and Field [*25*]. Field examined thermal instability under the condition that the incoming and outgoing energies balance each other in the imperturbed state. However he neglected the self-gravitation of disturbances (as in [*26*]) and also the pressure perturbation for thermal instability (see also [*27*]) in an expanding universe. In [*28a*] it is investigated the stability of perturbations in the thermodynamics parameters of an isotropic expanding universe which contains matter and radiation in equilibrium but the effects of gravitation are neglected. In [*15*] [*28b*] the thermal instability is studied in an expanding medium in the newtonian world model. In [*29*] the stability of a uniform medium is discussed including thermal and magnetic effects.

To investigate the thermal instability we must well know the thermal conditions of the imperturbed universe model. Although the thermal history of the universe and the temperature of intergalactic matter is not yet well known, several theoretical investigations have been made. For generality we assume that our universe is composed of three components: gaseous matter, radiation and cosmic rays ([8]). At early stages of the universe the coupling among these three components are strong and energy exchanges among them take place in the form of diffusion. In the present part, we are concerned only with the later stages of the universe, when gas, radiation and cosmic rays are nearly independant and the heat-loss functions of the medium are determined only by the local properties of the medium. A general hot intergalactic medium at $10^9$ °K was envisaged by Gold and Hoyle [*24*]. However in [*31*] it has been shown that this model must be incorrect. Another estimate for the temperature of intergalactic matter was given by Field [*25*] who obtained a temperature of $5.10^4$ °K from considerations of thermal instability leading to the formation of galaxy clusters. In [*34*] it is given a brief review of the problem of the temperature of intergalactic matter and favored a temperature of about $10^5$ °K, maintained by a cosmic-ray heating which balances losses by bremsstrahlung and recombination radiation in the plasma, assumed to be pure hydrogen. Gould and Ramsay considered the various processes which determine the temperature of intergalactic matter and considered

([8]) In [*28a*] [*30*] it is proposed a model of universe filled by radiation and dustlike matter. But one must take into account even the role of cosmic rays (see, for example [*32*] [*33*]).

the possible existence of a quasi-equilibrium state whereby a thermal balance arises in a time less than the cosmic expansion time. A somewhat different and more general approach to the problem is taken by Ginzburg and Ozernoy [33] who considered the variation in the temperature of the intergalactic medium as the universe expands.

In this part we shall investigate the growth rate of thermal instability in our universe model at the later stages under the thermal conditions which are consistent with the results of Ginzburg and Ozernoy and others.

## § 1. The equations characterizing the unperturbed state.

In our universe model, matter conservation is expressed by the equation of continuity as

$$(\rho_m u^i + \rho_c w^i)_{;i} = 0, \tag{3.1}$$

where the semicolon indicates covariant differentiation, $u^i$ and $w^i$ are the four-velocities for gaseous matter and cosmic rays whose number densities are $\rho_m$ and $\rho_c$ respectively. The energy-momentum tensor $T_i^k$ will be decomposed conveniently into three components associated with gaseous matter ($m$), radiation ($r$) and cosmic rays ($c$) as

$$T_i^k = T_{(m)i}^{\ k} + T_{(r)i}^{\ k} + T_{(c)i}^{\ k}. \tag{3.2}$$

Because of homogeneity and isotropy, the energy-momentum tensor $T_{(m)i}^{\ k}$ in the unperturbed state will be given by the same expression as that of a perfect gas

$$T_{(m)i}^{\ k} = \varepsilon_m u_i u^k + p_m(g_i^k + u^k u_i), \tag{3.3}$$

where $\varepsilon_m$ and $p_m$ are the energy density and pressure of gaseous matter. As we are interested only in the behaviour of gaseous matter, the explicit expressions for $T_{(r)i}^{\ k}$ and $T_{(c)i}^{\ k}$ are unnecessary.

If $L^+$ is the heating rate and $L^-$ is the cooling rate per unit volume and unit time in the rest frame, then from the equation

$$T_{i;k}^k = 0, \tag{3.4}$$

the equation of energy balance for the matter is written as

$$L^+ - L^- = cu^i(T_{(r)i}^{\ k} + T_{(c)i}^{\ k})_{;k} = - cu^i T_{(m)i;k}^{\ k}. \tag{3.5}$$

From (3.3) and (3.5) we get

$$(\varepsilon_m u^k)_{;k} + p_m(g_i^k + u^k u_i)u^i_{\ ;k} = \frac{1}{c}(L^+ - L^-) \tag{3.6}$$

Moreover from (1.2) and (3.2) and Einstein's field equations in our universe we get (cf. part II)

$$\frac{8\pi G}{c^2}(\varepsilon_m^* + \varepsilon_r + \varepsilon_c) = 3a^{-2}(\dot{a}^2 + kc^2), \tag{3.7}$$

$$\frac{8\pi G}{c^2}(p_m^* + p_r + p_c) = -\, a^{-2}(2a\ddot{a} + \dot{a}^2 + kc^2), \tag{3.8}$$

where the dot denotes the time derivative and

$$\varepsilon_m^* = \varepsilon_m + u, \qquad p_m^* = p_m - v. \tag{3.9}$$

In the imperturbed state, we can reduce equation (3.6) to (cf. part II)

$$\frac{(\varepsilon_m^* a^3)^{\cdot}}{a^3} + 3\frac{\dot{a}}{a}p_m^* = \mathrm{L}^+ - \mathrm{L}^-, \tag{3.10}$$

and, equation (3.1) expressing matter conservation is reduced to

$$(\rho_m + \rho_c)a^3 = \text{const.} \tag{3.11}$$

Because we shall be interested only in the later stages of the universe, we can suppose

$$\frac{\varepsilon_* + \varepsilon_r}{\varepsilon_m} \ll 1, \qquad \frac{p}{\varepsilon_m} \ll 1, \qquad \frac{\delta p_m}{\delta \varepsilon_m} \ll 1. \tag{3.12}$$

and, as in part II, let us assume that

$$u,\, v \ll \varepsilon_m,\, p_m \tag{3.13}$$

Furthermore, for gas, we have the equation

$$\varepsilon_m = \rho_m(m_{\mathrm{H}} + m_e)c^2 + \frac{p_m}{\gamma - 1}, \tag{3.14}$$

where $\gamma$ is the effective value of the ratio of specific heats ($\gamma \approx 5/3$). Then from (3.11)-(3.14) the equation (3.10) can be written as [9]

$$\frac{d}{dt}\ln(a^{3\gamma}p_m) = \frac{d}{dt}\ln(a^{3(\gamma-1)}\mathrm{T}_m) = (\gamma - 1)\frac{(\mathrm{L}^+ - \mathrm{L}^-)}{p_m}. \tag{3.15}$$

The solution of this equation describes the thermal history of the universe.

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[9] We can deduce this equation from the fundamental equation of thermodynamics.

## § 2. The equation describing the growth of density contrast.

Now let us assume that there has occured a density perturbation in our universe as in part II. Then the perturbed part of the equation (3.6) can be written as (taking notice of (3.13))

$$(\varepsilon_m + p_m)\delta[u^k{}_{;k}] + \frac{(a^3\delta\varepsilon_m)^{\cdot}}{a^3} + 3\frac{\dot{a}}{a}\delta p_m = \delta(\mathrm{L}^+ - \mathrm{L}^-). \tag{3.16}$$

In practical application, matter conservation given by equation (3.1) can be written approximately as (cf. (3.12))

$$(\rho_m u^i)_{;i} = 0,$$

and the perturbed part of this equation is

$$(a^3\delta\rho_m)^{\cdot}/a^3 + \rho_m\delta[u^i{}_{;i}] = 0 \tag{3.17}$$

From (3.16) and (3.17) we get

$$\frac{(a^3\delta\varepsilon_m)^{\cdot}}{a^3} + 3\frac{\dot{a}}{a}\delta p_m - (\varepsilon_m + p_m)\frac{(a^3\delta\rho_m)^{\cdot}}{\rho_m a^3} = \delta(\mathrm{L}^+ - \mathrm{L}^-). \tag{3.18}$$

Furthermore if we use the relation (3.14), the equation (3.18) reduces to the simple form

$$\frac{1}{a^{3\gamma}}\frac{\partial}{\partial t}(a^{3\gamma}\delta p_m) - \gamma\frac{p_m}{\rho_m a^3}\frac{\partial}{\partial t}(a^3\delta\rho_m) = (\gamma - 1)\delta(\mathrm{L}^+ - \mathrm{L}^-). \tag{3.19}$$

As is shown in part II the general equation for the density contrast is (cf. (2.24))

$$\mathscr{P}\left(\frac{\delta\varepsilon^*}{\varepsilon^*}\right) + (n^2 - 4k)\left(\frac{v_s}{a}\right)^2\frac{\delta\varepsilon^*}{\varepsilon^*} = 0. \tag{3.20}$$

Because

$$v_s = c\left(\frac{\delta p^*}{\delta\varepsilon^*}\right)^{1/2}$$

then

$$(n^2 - 4k)\frac{v_s^2}{a^2}\frac{\delta\varepsilon^*}{\varepsilon^*} = (n^2 - 4k)\frac{c^2 p^*}{a^2\varepsilon^*}\left(\frac{\delta p^*}{p^*}\right) \tag{3.21}$$

Taking notice of (3.21) and (3.12)—(3.14) we can reduce equation (3.20) to the form (in the linear approximation)

$$\mathscr{P}\left(\frac{\delta\rho_m}{\rho_m}\right) + (n^2 - 4k)\frac{c^2 p_m}{a^2\varepsilon_m}\left(\frac{\delta p_m}{p_m}\right) = 0. \tag{3.22}$$

From Equations (3.19), (3.22) and the equation of state which is

$$\delta p_m/p_m = \delta\rho_m/\rho_m + \delta T_m/T_m, \tag{3.23}$$

we eliminate $\delta p_m$ and $\delta T_m$ and obtain the following equation for the density contrast

$$\left\{\frac{\partial}{\partial t} + (3\gamma - 1)\frac{\dot{a}}{a} - (\gamma - 1)\frac{T_m}{p_m}\left[\frac{\partial(L^+ - L^-)}{\partial T_m}\right]_{\rho_m}\right\}\mathscr{P}\left(\frac{\delta\rho_m}{\rho_m}\right)$$
$$+ \frac{(n^2 - 4k)c^2 p_m}{\varepsilon_m a^2}\left\{\gamma\frac{\partial}{\partial t} - (\gamma - 1)\frac{T_m}{p_m}\left[\frac{\partial(L^+ - L^-)}{\partial T_m}\right]_{p_m}\right\}\left(\frac{\partial\rho_m}{\rho_m}\right) = 0, \tag{3.24}$$

where

$$\left[\frac{\partial(L^+ - L^-)}{\partial T_m}\right]_{\rho_m} \quad \text{and} \quad \left[\frac{\partial(L^+ - L^-)}{\partial T_m}\right]_{p_m}$$

denote the differentiation of $(L^+ - L^-)$ with respect to $T_m$ at constant $\rho_m$ (isochoric) and at constant $p_m$ (isobaric) respectively. In the derivation of Eq. (3.24) we used (cf. (3.11), 3.12))

$$\rho_m a^3 \sim \text{const.} \tag{3.25}$$

Equation (3.24) is of the third order with respect to time. This property is the same as the equation describing perturbations in a homogeneous medium in newtonian hydrodynamics (see [*15*]). Two among the three orders come from the acoustic mode (the gravitational instability is related to this mode) and the additional order comes from the thermal mode (the thermal instability is related to this mode).

## § 3. The growth of the thermal instability.

To solve equation (3.24) exactly is not easy. Therefore we shall be satisfied with studying the characteristics of the thermal instability in equation (3.24) by an approximate treatment as in part II (cf. (2.29) and (3.12), (3.13)). Then we have, approximately ([10])

$$\mathscr{P}(D) = 0, \qquad \text{where} \qquad D = \frac{\delta\rho}{\rho}. \tag{3.26}$$

([10]) Equation (3.26) describes the gravitational instability considered in part II. For convenience we shall omit the indice $m$ at $\rho_m$, $T_m$, $p_m$.

Therefore from (3.24) we obtain the following equation for thermal instability ([11])

$$\left\{\frac{\partial}{\partial t} - \frac{(\gamma-1)}{\gamma}\frac{T}{p}\left[\frac{\partial(L^+ - L^-)}{\partial T}\right]_p\right\} D = 0, \qquad (3.27)$$

The integration of which is formally written as

$$D = D_0 \exp\left\{\int^t \frac{(\gamma-1)}{\gamma}\frac{T}{p}\left[\frac{\partial(L^+ - L^-)}{\partial T}\right]_p dt\right\}, \qquad (3.28)$$

where the lower limit of integration is the time when the perturbation has occured. As it is easy to see from (3.28) the numerical values of the growth rate of density contrast depend on the thermal conditions of matter in universe. We shall calculate the growth rate of density contrast basing on different investigations about the thermal history of the universe during the time intervals, in the first case from $t_1 = 8.10^7$ years ($\rho_1 = 1{,}6.10^{-1}$ cm$^{-3}$) to $t_2 = 10^8$ years ($\rho_2 = 10^{-1}$ cm$^{-3}$) and, in the second case from $t_1 = 8.10^7$ years to $t_3 = 2.10^8$ years ($\rho_3 = 2.10^{-2}$ cm$^{-3}$) ([12]).

1) We shall first study the growth rate of density contrast on the basis of Ginzburg and Ozernoy's investigations [*33*]. As it is shown in [*33*]

$$L^+ = L_c^+ + L_g^+ + L_j^+, \qquad (3.29)$$

$$L^- = L_{ff}^- + L_{fb}^-, \qquad (3.30)$$

where

$$L_c^+ \simeq 8{,}0.10^{-29}\ \rho\ \text{erg/cm}^3.\text{sec},$$

$$L_g^+ = 3.10^{-33}\ \text{erg/cm}^3.\text{sec},$$

$$L_j^+ = 10^{-36}\left(\frac{10^6}{T}\right)^{3/2} \cdot \text{erg/cm}^3.\text{sec},$$

$$L_{ff}^- = 1{,}4.10^{-27}\ T^{1/2}\ \rho^2\ \text{erg/cm}^3.\text{sec}$$

and

$$L_{fb}^- = 5{,}4.10^{-22}\ T^{-1/2}\ \rho^2\ \text{erg/cm}^3.\text{sec}$$

In [*33*] it is assumed the wide temperature range $10^4$ °K-$10^6$ °K at the epoch when $\rho = 10^{-3}$ cm$^{-3}$. We shall calculate the growth rate of D at the radiatively cooling stage i. e. assuming that ([13]) $(L^+ - L^-) \approx (-L^-)$

([11]) The equation (3.27) can be reduced *exactly* by assuming isobaric conditions throughout intergalactic space (see, for example [*24*] [*25*] [*27*]), i. e. $\delta p = 0$. In this case from (3.22) we have *exactly* $\mathscr{P}(D) = 0$. But here we do not prefer this procedure.

([12]) Here we adopt $\rho = 10^{-5}$ cm$^{-3}$ and $t = 10^{10}$ years as the density and the age of the present universe.

([13]) For simplicity, the growth rate of density contrast was calculated either only in the case $L^- = L_{ff}^-$ (see [*24*] [*25*] [*26*] [*28b*] [*29*]), or in two separate cases $L^- = L_{ff}^-$ and $L^- = L_{fb}^-$ (see [*27*]). This simplification, in our standpoint, is unreasonable.

and that $T = 10^4$ °K at the epoch $\rho = 10^{-3}$ cm$^{-3}$. For the sake of numerical estimation, let us assume that ([14])

$$a \sim t^{2/3} \tag{3.31}$$

and integrate equation (3.15). From (3.15), (3.25) and (3.30) we get

$$(\alpha T)^{1/2} - \text{arc tg}\,(\alpha T)^{1/2} - \beta \rho^{1/2} = \text{const}, \tag{3.32}$$

where $\alpha = 2{,}6.10^6$, $\beta = 2{,}88$ and const $= -\,0{,}09$. From (3.28) and (3.32) with complicated calculations we get

$$D = D_0 \left\{ \left(\frac{T_0}{T}\right)^3 \left(\frac{1 + \alpha T}{1 + \alpha T_0}\right)^{6/5} \right\}, \tag{3.33}$$

and, therefore,

$$\frac{D_3}{D_1} = 28. \tag{3.34}$$

Now we shall take into account even the heating processes and assume that $T = 1{,}2.10^5$ °K at the epoch $\rho = 2.10^{-2}$ cm$^{-3}$ ([15]). Moreover to make another rough estimation, let us assume that the unperturbed state expands nearly adiabatically under balance between the heating and the cooling, i. e., from (3.15),

$$a^{3(\gamma - 1)}T = a^2 T = T\rho^{-2/3} \simeq \text{const} \tag{3.35}$$

where const $= 1{,}63.10^6$. Then from (3.28), (3.30) and (3.35) we get

$$\frac{D_3}{D_1} = \exp\left\{ 6{,}4.10^3\,(t_1^{-1/3} - t_2^{-1/3}) + 6{,}1.10^{-8}(t_2 \to t_1) \right\} = 4.10^5. \tag{3.36}$$

Here it should be noticed that during the time interval considered here the density of the unperturbed universe decreases by a factor 8.

2) Now we shall calculate the growth rate of thermal instability on the basis of Gould and Ramsay's investigations about the temperature of intergalactic matter [32]. Gould and Ramsay make quite different assumptions for the effective heating and cooling processes in comparaison with Ginzburg and Ozernoy's investigations. Especially they showed that the energy loss due to inelastic collisions of electrons with He and especially $He^+$ is more than two orders of magnitude larger than the

([14]) The adopted form of a is exact for the case of flat universe ($k = 0$) and is applicable to the case of closed and open universes ($k = \pm 1$) during non very long time intervals.

([15]) We used this value of temperature for the sake of comparaison with the results obtained below.

contribution from electron-proton bremsstrahlung and radiative recombination at the temperature around $10^5$ °K. As in [32], see fig. 1) in the temperature interval from $1{,}1.10^5$ °K to $5{,}5.10^5$ °K we get ([16])

$$L^- \approx (2{,}41.10^{-27}\,T^{1/2} + 1{,}09.10^{-21}\,T^{-1/2} + 10^{-2}\,T^{-4})\rho^2, \quad (3.37)$$

and

$$L^+ \approx 2.10^{-28}\rho + 3.10^{-25}\rho^2. \quad (3.38)$$

with the assumption as in 1), from (3.28), (3.31), (3.35), (3.37) and (3.38) we get

$$D = D_0 \exp\{\,1{,}27.10^4(t_0^{-1/3} - t^{-1/3}) + 1{,}21.10^{-7}(t - t_0) + 8{,}48.10^{-46}(t^{17/3} - t_0^{17/3})\,\}, \quad (3.39)$$

and, therefore,

$$\frac{D_2}{D_1} = 6{,}3.10^7, \qquad \frac{D_3}{D_1} = 6.10^{86}. \quad (3.40)$$

Here it should be noted that during the time intervals considered here the density of the unperturbed universe decreased only by factor 1,6 and 8.

3) Recently in [35] it is shown that the matter in an expanding universe cannot be heated and kept at temperatures higher than $10^5$ °K. If this result is admissible, we can use the Michie's expression for the cooling rate [29a]

$$L^- = (7{,}75.10^{-17}\,T^{-3/2} + 6{,}8.10^{-22}\,T^{-1/2} + 1{,}42.10^{-27}\,T^{1/2})\rho^2, \quad (3.41)$$

and this formula is correct for temperature interval between $1{,}5.10^4$ °K and $10^6$ °K. In this case assuming $T = 8.10^4$ °K at the epoch $\rho = 1{,}6.10^{-1}$ cm$^{-3}$ ([17]), we get

$$D = D_0 \exp\{\,1{,}82.10^4(t_0^{-1/3} - t^{-1/3}) + 3{,}48.10^{-18}(t^{7/3} - t_0^{7/3})\,\}, \quad (3.42)$$

and, therefore,

$$\frac{D_2}{D_1} = 4.10^{71}, \qquad \frac{D_3}{D_1} = 4{,}8.10^{87}. \quad (3.43)$$

4) Finally we remark that for generality we can take into account the heat loss due to thermal conductivity (see, for example, [15] [28a] [28b]).

([16]) The expression (3.37) is extrapolated from figure 1 in [32].

([17]) We used this value of temperature so that in all time interval considered here, the temperature $T \leqslant 8.10^4$ °K, i. e. is smaller than $10^5$ °K corresponding to the result in [35].

In this case, in analogy to newtonian cosmology (see [*15*]), equation (3.27) becomes (cf. Eqs (40) and (41) in [*15*])

$$\left\{ \frac{\partial}{\partial t} - \frac{(\gamma - 1)}{\gamma} \frac{\mathrm{T}}{p} \left[ \left( \frac{\partial(\mathrm{L}^+ - \mathrm{L}^-)}{\partial \mathrm{T}} \right)_p - \frac{n^2 \mathrm{K}}{a^2} \right] \right\} \mathrm{D} = 0 \qquad (3.27a)$$

where K is the coefficient of thermal conductivity and $\mathrm{K} \sim 10^{-6}\,\mathrm{T}^{5/2}$ (see [*28b*]). Then generally we get

$$\mathrm{D} = \mathrm{D}_0 \exp \left\{ \int^t \frac{(\gamma - 1)}{\gamma} \frac{\mathrm{T}}{p} \left[ \left( \frac{\partial(\mathrm{L}^+ - \mathrm{L}^-)}{\partial \mathrm{T}} \right)_p - \frac{n^2 \mathrm{K}}{a^2} \right] dt, \right. \qquad (3.28a)$$

and, therefore, in general the thermal conductivity diminishes the density perturbation. But it is easy to verify that in all cases considered here the contribution of the thermal conductivity to the growth of density contrast is insignificant. But assuming the expression (3.28*a*) we can discuss approximately about the minimum size of the density contrast which can grow by the thermal instability. Indeed the number $n$ in equation (3.28*a*) denotes the wave number of perturbations normalized by $a$, that is, $a/n$ is the size of perturbations. Thus the minimum size of perturbation is given by

$$\left\{ \frac{\mathrm{K}}{\left( \frac{\partial(\mathrm{L}^+ - \mathrm{L}^-)}{\partial \mathrm{T}} \right)_p} \right\}^{1/2}$$

From that, for example for $\rho = 1{,}6.10^{-1}\ \mathrm{cm}^{-3}$ and $\mathrm{T} = 1{,}88.10^6\ {}^\circ\mathrm{K}$ (from (3.30)) we have the minimum size of $4{,}6.10^{20}$ cm, or we have

$$\mathrm{M}_{\min} \approx 10^5\ \mathrm{M}_\odot$$

for minimum masses. This result coincides with those calculated in newtonian cosmology by other methods (see, for example [*5*]).

## § 4. CONCLUSION

We have investigated the thermal instability and have obtained the general equation for the growth rate of density contrast (see Eqs (3.28) and (3.28*a*)). We have calculated the numerical values of the growth rate of density contrast in different cases. We remark that the reliable estimation of the rate of the density perturbation requires more detailed information about the thermal history of the unperturbed universe, because the results depend strongly upon the models (see Eqs (3.34), (3.36), (3.40), (3.43)) ([18]). However we can conclude that the fact that thermal instability can suppress the absolute decrease of density by expansion as shown in those results will be sufficient for the formation of galaxies, because then the gravitational instability will be followed in the case of large scale disturbances (see [29]).

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([18]) It is difficult to compare our results with those obtained by other authors because their methods and assumptions as well as universe models used are different from ours. But we can remark that the growth rate of density contrast for thermal instability obtained by us is many orders of magnitude larger than those obtained by others.

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