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A Γ-convergence result for optimal design problems

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Abstract. In this paper, we derive the Γ-limit of some optimal material distribution problems as the exponent goes to infinity.

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1. Introduction

Optimal design models describe the optimal distribution of a two phase elastic mixture with respect to some criterion. The energy considered has different properties distribution, such as stiffness or electric resistivity, in different regions of the domain under consideration. Minimizing such energies allows to improve the mechanical or electrical performance by optimizing the distribution of these properties.

Γ-convergence and relaxation are techniques that are useful in the analysis of such models. Such is the case for instance of the works [5, 10] in the context of a dimension reduction process for thin films. Then, among other works, we can cite the work of [6] also dealing with a dimension reduction process involving an additional energy term of perimeter penalization. An adaptation of the last work has been studied in [12] for the Orlicz–Sobolev setting. In [14], the authors obtain the relaxation of an optimal design model involving fractured media which induces an analysis in the space of special bounded variation functions. In [3], in the context of linear growth and still with a perimeter penalization, the authors derive a lower semicontinuity result and a measure representation result for the relaxation of optimal design functionals.

In this work, we consider optimal design models described by functionals of the form

$$J(\chi, u) = \left(\int_\Omega \chi W_1(\nabla u)^p + (1-\chi) W_2(\nabla u)^p \mathrm{d}x\right)^{\frac{1}{p}},$$

where $\chi(x) \in \{0,1\}$ denotes the characteristic function of the first phase, ∇u the gradient of the deformation and W_i^p, $i = 1,2$ models the energy density of the ith phase, then, we proceed with an asymptotic analysis when the exponent p of the energy densities goes to infinity.

We obtain a limit energy of supremal kind that models, for example, dielectric breakdown for double phase composites (see [11] and the references therein) or some simplified models of polycrystal plasticity (see [4]). In the last two references, analogous asymptotic analyses using Γ-convergence techniques for functionals involving single phase elastic density can be found. Also in [1, 2], where the authors obtain limit models under some differential constraints, involving supremal functions and $\mathscr{A}$-quasiconvex envelopes. We mention also [7] where the authors obtained an L^p approximation and a lower semicontinuity result for supremal functionals.

Let $1 < p_0 < \infty$. Consider the sequence of functionals $(I_p)_{p>p_0}$, where p denotes a sequence $p_n \to +\infty$, defined on $L^\infty(\Omega;[0,1]) \times L^{p_0}(\Omega;\mathbb{R}^m)$ by

$$I_p(\chi,u) = \begin{cases} \left(\int_\Omega \chi W_1(\nabla u)^p + (1-\chi)W_2(\nabla u)^p \mathrm{d}x\right)^{\frac{1}{p}} & \text{if } (\chi,u) \in L^\infty(\Omega;\{0,1\}) \times W^{1,p}(\Omega;\mathbb{R}^m), \\ +\infty & \text{otherwise,} \end{cases}$$

where $W_i : \mathbb{M}^{m\times N} \to \mathbb{R}$ are continuous functions verifying linear growth and coercivity hypotheses: there exist $\alpha_i, \beta_i > 0$ such that

$$\beta_i |A| \leq W_i(A) \leq \alpha_i(1+|A|). \tag{1}$$

The functional I_p represents the elastic energy of the solid occupying the domain Ω and undergoing the deformation u, while χ represents the characteristic function of the first phase of stiffness or electric resistivity. Noticing that any function $\chi \in L^\infty(\Omega;[0,1])$ is a weak $\star$-limit in $L^\infty(\Omega;[0,1])$ of a sequence $\chi_n \in L^\infty(\Omega;\{0,1\})$.

Let $V : [0,1] \times \mathbb{M}^{m\times N} \longrightarrow \mathbb{R}$ be defined by

$$V(q,A) = qW_1(A) + (1-q)W_2(A)$$

and I be defined on $L^\infty(\Omega;[0,1]) \times L^{p_0}(\Omega;\mathbb{R}^m)$ by

$$I(\chi,u) = \begin{cases} \operatorname{ess\,sup} V^\star(\chi,\nabla u) & \text{if } u \in W^{1,\infty}(\Omega;\mathbb{R}^m), \\ +\infty & \text{otherwise,} \end{cases}$$

where

$$V^\star(q,A) := \lim_{p\to+\infty} \inf_{\theta,\varphi} \left\{ \left(\int_Q \left(V(\theta(x), A + \nabla\varphi(x))\right)^p \mathrm{d}x \right)^{\frac{1}{p}}, \right. \\ \left. \varphi \in W_\#^{1,p}(Q;\mathbb{R}^m),\ \theta \in L^\infty(\Omega;\{0,1\}),\ \int_\Omega \theta(x)\mathrm{d}x = q \right\}.$$

The goal of this article is to prove the following Theorem.

Theorem 1. *Let $1 < p_0 < \infty$. The sequence of functionals $(I_p)_{p>p_0}$ Γ-converges to I as p goes to $+\infty$ with respect to the $L^\infty(\Omega;[0,1])$ weak $\star \times W^{1,p_0}(\Omega;\mathbb{R}^m)$ weak topology.*

The minimization problem corresponding to the limit model obtained, describes, for example, the effective behavior of the compatible two-phase mixtures for dielectric breakdown.

In the following section we will present some brief preliminaries on the notions of Γ-convergence and cross-quasiconvexity. Then, the next section will be devoted to the proof of the Γ-convergence result.

2. Preliminaries

2.1. Γ-*convergence*

Let $(G_n)_n$ be a sequence of functionals defined on a topological space X with values in $\mathbb{R}\cup\{+\infty\}$. The Γ-lower limit and Γ-upper limit of $(G_n)_n$ are given by

$$\Gamma\text{-}\liminf G_n(x) := \sup_{U\in\mathcal{N}(x)} \liminf_{n\to\infty} \inf_{y\in U} G_n(y) \text{ and } \Gamma\text{-}\limsup G_n(x) := \sup_{U\in\mathcal{N}(x)} \limsup_{n\to\infty} \inf_{y\in U} G_n(y),$$

where $\mathcal{N}(x)$ denotes the set of all neighborhoods of x in X. If there exist $G : X \to \mathbb{R}\cup\{+\infty\}$ such that $\Gamma\text{-}\liminf G_n = \Gamma\text{-}\limsup G_n = G$, then we say that $(G_n)_n$ Γ-converges to G and we write $G := \Gamma\text{-}\lim G_n$. When X is first countable we have the equivalent definition in terms of sequences, that is, $(G_n)_n$ is said to Γ-converge to the limit functional G with respect to the topology of X if and only if the following two conditions are satisfied for every $x\in X$:

$$\begin{cases} \forall\, x_n \to x, & \liminf_{n\to\infty} G_n(x_n) \geq G(x), \\ \exists\, x_n \to x, & \limsup_{n\to\infty} G_n(x_n) \leq G(x). \end{cases}$$

The main properties of Γ-convergence are first that, up to a subsequence, the Γ-limit always exists and second that if a sequence of almost minimizers stays in a compact subset of X, then the limits of any converging subsequence minimize the Γ-limit. In particular we have that, if G is the Γ-limit of G_n and for every n, x_n is a minimizer of G_n with $x_n \to x$ in X, then x is a minimizer of G. Also, the limit minimization problem has always a solution due to the lower semicontinuity of the Γ-limit with respect to the considered topology (see [8, 9]).

2.2. *Cross-quasiconvexity*

In the limit model, due to the use of Γ-convergence techniques, the energy functional will be lower semicontinuous with respect to the considered topology. Thus, we define the cross-quasiconvex envelope as in [5, 10], for $V : [0,1]\times\mathbb{M}^{m\times N} \to \mathbb{R}$, with

$$V(q,A) = qW_1(A) + (1-q)W_2(A),$$

by

$$V_p^\star(q,A) := \inf_{\theta,\varphi}\left\{\left(\int_Q \big(V(\theta(x), A+\nabla\varphi(x))\big)^p \mathrm{d}x\right)^{\frac{1}{p}},\ \varphi\in W_\#^{1,p}(Q;\mathbb{R}^m),\ \theta\in L^\infty(\Omega;\{0,1\}),\ \int_\Omega \theta(x)\mathrm{d}x = q\right\},$$

where

$$W_\#^{1,p}(Q;\mathbb{R}^m) = \left\{\varphi\in W_{\text{loc}}^{1,p}(\mathbb{R}^N;\mathbb{R}^m) : \varphi \text{ is } Q \text{ periodic}\right\},$$

with Q being the unit cube in $\mathbb{R}^N$. We have the following results that will be useful for the computation of the Γ-limit.

Lemma 2. *The sequence $(V_p^\star)_p$ is an increasing sequence.*

Proof. Let $1<r<p<\infty$. Let $q\in[0,1]$, $A\in\mathbb{M}^{m\times N}$, $\varphi\in W_\#^{1,p}(Q;\mathbb{R}^m)$ and $\theta\in L^\infty(\Omega;\{0,1\})$ verifying $\int_\Omega \theta(x)\mathrm{d}x = q$. Since $W_\#^{1,p}(Q;\mathbb{R}^m)\subset W_\#^{1,r}(Q;\mathbb{R}^m)$, we have that $\varphi\in W_\#^{1,r}(Q;\mathbb{R}^m)$ and thus, using Hölder inequality, we have

$$\left(\int_Q \big(V(\theta(x), A+\nabla\varphi(x))\big)^r \mathrm{d}x\right)^{\frac{1}{r}} \leq \left(\int_Q \big(V(\theta(x), A+\nabla\varphi(x))\big)^p \mathrm{d}x\right)^{\frac{1}{p}},$$

since $|Q| = 1$. Thus, since $W^{1,p}_{\#}(Q;\mathbb{R}^m) \subset W^{1,r}_{\#}(Q;\mathbb{R}^m)$, we obtain

$$\begin{aligned} V^\star_r(q,A) &= \inf_{\theta,\varphi}\left\{\left(\int_Q \big(V(\theta(x), A+\nabla\varphi(x))\big)^r \mathrm{d}x\right)^{\frac{1}{r}}, \varphi \in W^{1,r}_{\#}(Q;\mathbb{R}^m), \theta \in L^\infty(\Omega;\{0,1\}), \int_\Omega \theta(x)\mathrm{d}x = q\right\} \\ &\le \inf_{\theta,\varphi}\left\{\left(\int_Q \big(V(\theta(x), A+\nabla\varphi(x))\big)^p \mathrm{d}x\right)^{\frac{1}{p}}, \varphi \in W^{1,p}_{\#}(Q;\mathbb{R}^m), \theta \in L^\infty(\Omega;\{0,1\}), \int_\Omega \theta(x)\mathrm{d}x = q\right\} \\ &= V^\star_p(q,A), \end{aligned}$$

which gives the result. □

Next, we define $V^\star : [0,1] \times \mathbb{M}^{m\times N} \to \mathbb{R}$ by

$$V^\star(q,A) := \lim_{p\to+\infty} V^\star_p(q,A) = \sup_{p>1} V^\star_p(q,A).$$

The following Lemmas that will be used respectively, for the computation of the lower bound and the upper bound, are a consequence of the dimension reduction studied in [5, 10]. Their proofs follow the same steps as in [5, 10] with simpler arguments since we have no dimension reduction process within it. See also [13].

Lemma 3. *Let $1 < p < \infty$. Suppose $u_n \rightharpoonup u$ in $W^{1,p}(\Omega;\mathbb{R}^m)$ and $\chi_n \overset{*}{\rightharpoonup} \chi$ in $L^\infty(\Omega;[0,1])$, then*

$$\liminf_{n\to\infty} \|V^\star_p(\chi_n, \nabla u_n)\|_p \ge \|V^\star_p(\chi, \nabla u)\|_p.$$

Lemma 4. *Let $1 < p < \infty$. For every $u \in W^{1,p}(\Omega;\mathbb{R}^m)$ and $\chi \in L^\infty(\Omega;[0,1])$, there exist $u_n \in W^{1,p}(\Omega;\mathbb{R}^m)$ and $\chi_n \in L^\infty(\Omega;\{0,1\})$ such that $u_n \overset{*}{\rightharpoonup} u$ in $W^{1,p}(\Omega;\mathbb{R}^m)$ and $\chi_n \overset{*}{\rightharpoonup} \chi$ in $L^\infty(\Omega;[0,1])$, with*

$$\limsup_{n\to\infty} \|V^\star_p(\chi_n, \nabla u_n)\|_p \le \|V^\star_p(\chi, \nabla u)\|_p.$$

3. Proof of Theorem 1

Proof.

Step 1. The lower bound. Let $(\chi, u) \in L^\infty(\Omega;[0,1]) \times L^{p_0}(\Omega;\mathbb{R}^m)$ and $(\chi_p, u_p) \in L^\infty(\Omega;\{0,1\}) \times L^{p_0}(\Omega;\mathbb{R}^m)$ such that $\chi_p \overset{*}{\rightharpoonup} \chi$ in $L^\infty(\Omega;[0,1])$ and $u_p \rightharpoonup u$ in $W^{1,p_0}(\Omega;\mathbb{R}^m)$. We will prove that

$$\liminf_{p\to\infty} I_p(\chi_p, u_p) \ge I(\chi, u).$$

We can suppose that $M = \liminf_{p\to\infty} I_p(\chi_p, u_p) < \infty$, which implies that $\chi_p \in L^\infty(\Omega;\{0,1\})$ and $u_p \in W^{1,p}(\Omega;\mathbb{R}^m)$. Using (1) we have

$$\begin{aligned} I_p(\chi_p, u_p) &= \left(\int_\Omega |\chi_p W_1(\nabla u_p) + (1-\chi_p) W_2(\nabla u_p)|^p \mathrm{d}x\right)^{\frac{1}{p}} \\ &\ge \left(\int_\Omega |\chi_p \beta_1 |\nabla u_p| + (1-\chi_p)\beta_2 |\nabla u_p\|^p \mathrm{d}x\right)^{\frac{1}{p}} \ge \left(\int_\Omega \beta^p |\nabla u_p|^p \mathrm{d}x\right)^{\frac{1}{p}}, \end{aligned} \tag{2}$$

where $\beta = \min(\beta_1, \beta_2) > 0$. Thus, there exist $p_1 \ge p_0$ such that (∇u_p) is bounded in $L^p(\Omega;\mathbb{M}^{m\times N})$ for every $p \ge p_1$. Next, using Hölder's inequality, we have that for every $p_1 \le r < p$

$$\left(\int_\Omega |\nabla u_p|^r \mathrm{d}x\right)^{\frac{1}{r}} \le |\Omega|^{\frac{p-r}{pr}} \|\nabla u\|_p \le |\Omega|^{\frac{p-r}{pr}} \frac{M+1}{\beta}$$

Thus, for every $p_1 \le r < p$, we have

$$\|\nabla u_p\|_r \le \max(|\Omega|^{\frac{1}{r}}, 1) \frac{M+1}{\beta}$$

and thus $(\nabla u_p)_{p>r}$ is uniformly bounded in $L^r(\Omega;\mathbb{M}^{m\times N})$ which gives, using Poincaré's inequality, that $(u_p)_{p>r}$ is uniformly bounded in $W^{1,r}(\Omega;\mathbb{R}^m)$ and thus, up to a subsequence, it converges weakly in $W^{1,r}(\Omega;\mathbb{R}^m)$ to $u\in W^{1,r}(\Omega;\mathbb{R}^m)$ for every $r\geq p_1$. Next, we prove that $u\in W^{1,\infty}(\Omega;\mathbb{R}^m)$. Indeed, we have, for every $x_0\in\Omega$ and $t>0$ such that the open ball $B_t(x_0)\subset\Omega$,

$$\frac{1}{|B_t(x_0)|}\int_{B_t(x_0)}|\nabla u|\mathrm{d}x\leq |B_t(x_0)|^{-\frac{1}{r}}\|\nabla u\|_r$$
$$\leq |B_t(x_0)|^{-\frac{1}{r}}\liminf_{p\to\infty}\|\nabla u_p\|_r\leq |B_t(x_0)|^{-\frac{1}{r}}\max\big(|\Omega|^{\frac{1}{r}},1\big)\frac{M+1}{\beta}.$$

Letting $r\to\infty$, we obtain

$$\frac{1}{|B_t(x_0)|}\int_{B_t(x_0)}|\nabla u|\mathrm{d}x\leq\frac{M+1}{\beta}.$$

Then, letting $t\to 0^+$, we obtain for every Lebesgue point $x_0\in\Omega$

$$|\nabla u(x_0)|\leq\frac{M+1}{\beta}.$$

Thus, since Ω is bounded, we obtain for a.e. $x_0\in\Omega$

$$|u(x_0)|\leq C(\Omega)\frac{M+1}{\beta}$$

and thus $u\in W^{1,\infty}(\Omega;\mathbb{R}^m)$. On the other hand, following the same steps as in (2), we have, since $\chi_p=0$ or $(1-\chi_p)=0$ a.e. in Ω, for every $r\geq p_1$ and every $p>r$

$$\begin{aligned}
I_p(\chi_p,u_p)&=\left(\int_\Omega\chi_pW_1(\nabla u_p)^p+(1-\chi_p)W_2(\nabla u_p)^p\mathrm{d}x\right)^{\frac{1}{p}}\\
&=\left(\int_\Omega|V(\chi_p,\nabla u_p)|^p\mathrm{d}x\right)^{\frac{1}{p}}\\
&=\|V(\chi_p,\nabla u_p)\|_p\\
&\geq|\Omega|^{\frac{r-p}{pr}}\|V(\chi_p,\nabla u_p)\|_r\\
&\geq|\Omega|^{\frac{r-p}{pr}}\|V_r^\star(\chi_p,\nabla u_p)\|_r.
\end{aligned}$$

Next, since $u_p\rightharpoonup u$ in $W^{1,r}(\Omega;\mathbb{R}^m)$ and $\chi_p\overset{*}{\rightharpoonup}\chi$ in $L^\infty(\Omega;[0,1])$, we obtain using Lemma 3

$$\liminf_{p\to\infty}I_p(\chi_p,u_p)\geq|\Omega|^{\frac{-1}{r}}\|V_r^\star(\chi,\nabla u)\|_r.$$

Thus, we have for every $p_1\leq q\leq r$,

$$\liminf_{p\to\infty}I_p(\chi_p,u_p)\geq|\Omega|^{\frac{-1}{r}}|\Omega|^{\frac{q-r}{qr}}\|V_r^\star(\chi,\nabla u)\|_q=|\Omega|^{\frac{-1}{q}}\|V_r^\star(\chi,\nabla u)\|_q.$$

Making $r\to\infty$ we obtain

$$\liminf_{p\to\infty}I_p(\chi_p,u_p)\geq|\Omega|^{\frac{-1}{q}}\|V^\star(\chi,\nabla u)\|_q,$$

then, making $q\to\infty$ we obtain

$$\liminf_{p\to\infty}I_p(\chi_p,u_p)\geq\|V^\star(\chi,\nabla u)\|_\infty=\operatorname{ess\,sup}\,V^\star(\chi,\nabla u).$$

Step 2. The upper bound. We need to prove that converse inequality stating that

$$\Gamma\text{-}\limsup I_p(\chi,u)\leq I(\chi,u)$$

for every $(\chi,u)\in L^\infty(\Omega;[0,1])\times L^{p_0}(\Omega;\mathbb{R}^m)$. If $u\notin W^{1,\infty}(\Omega;\mathbb{R}^m)$ then there is nothing to prove. Then, let $(\chi,u)\in L^\infty(\Omega;[0,1])\times W^{1,\infty}(\Omega;\mathbb{R}^m)$ and since Ω is bounded we have that $(\chi,u)\in L^\infty(\Omega;[0,1])\times W^{1,p}(\Omega;\mathbb{R}^m)$ for every $p\geq 1$. Notice that using Lemma 3 and Lemma 4, the lower semicontinuous envelope of I_p with respect to the $L^\infty(\Omega;[0,1])$ weak $\star \times W^{1,p}(\Omega;\mathbb{R}^m)$ weak topology is given by

$$\bar{I}_p^p(\chi,u) := \left(\int_\Omega V_p^\star(\chi,\nabla u)^p \mathrm{d}x\right)^{\frac{1}{p}}.$$

Let $\bar{I}_p^{p_0}$ be the lower semicontinuous envelope of I_p with respect to the $L^\infty(\Omega;[0,1])$ weak $\star \times W^{1,p_0}(\Omega;\mathbb{R}^m)$ weak topology. Since the dual of $L^{p_0}(\Omega;\mathbb{R}^m)$ is a subset of the dual of $L^p(\Omega;\mathbb{R}^m)$, we have

$$\bar{I}_p^{p_0}(\chi,u) \le \bar{I}_p^p(\chi,u). \tag{3}$$

Indeed, by the definition of $\bar{I}_p^p$ and using Lemma 4, there exist $(\chi_n) \subset L^\infty(\Omega;\{0,1\})$, $(u_n) \subset W^{1,p}(\Omega;\mathbb{R}^m)$ such that $\chi_n \overset{*}{\rightharpoonup} \chi$ in $L^\infty(\Omega;[0,1])$ and $u_n \rightharpoonup u$ in $W^{1,p}(\Omega;\mathbb{R}^m)$ with

$$\limsup_{n\to\infty} I_p(\chi_n,u_n) \le \bar{I}_p^p(\chi,u).$$

Since $u_n \rightharpoonup u$ in $W^{1,p}(\Omega;\mathbb{R}^m)$ implies that $u_n \rightharpoonup u$ in $W^{1,p_0}(\Omega;\mathbb{R}^m)$, we obtain (3). Finally, since $(V_p^\star)$ is an increasing sequence, we have

$$\begin{aligned}\Gamma\text{-}\limsup I_p(\chi,u) &= \Gamma\text{-}\limsup \bar{I}_p^{p_0}(\chi,u)\\ &\le \Gamma\text{-}\limsup \bar{I}_p^p(\chi,u)\\ &= \lim_{p\to+\infty} \bar{I}_p^p(\chi,u)\\ &\le \lim_{p\to+\infty} |\Omega|^{\frac{1}{p}} \operatorname{ess\,sup} V_p^\star(\chi,\nabla u)\\ &= \operatorname{ess\,sup} V^\star(\chi,\nabla u)\end{aligned}$$

and thus the result. □

We have the following Corollaries that are useful in the context of optimal design.

Corollary 5. *The functional I is lower semicontinuous with respect to the $L^\infty(\Omega;[0,1])$ weak $\star \times W^{1,p}(\Omega;\mathbb{R}^m)$ weak topology for every $1 < p < \infty$.*

Proof. This lower semicontinuity result is a direct consequence of the last Theorem since the Γ-limit is always lower semicontinuous with respect to the considered topology. □

Corollary 6. *Let $1 < p_0 < \infty$ and $(\chi_p,u_p)_{p>p_0}$ a diagonal minimizing sequence for I_p, i.e. : $(\chi_p,u_p) \in L^\infty(\Omega;\{0,1\}) \times W^{1,p}(\Omega;\mathbb{R}^m)$ such that*

$$I_p(\chi_p,u_p) = \inf_{\substack{u\in W^{1,p}(\Omega;\mathbb{R}^m)\\ \chi\in L^\infty(\Omega;\{0,1\})}} I_p(\chi,u) + \varepsilon\left(\frac{1}{p}\right),$$

with $\varepsilon(x) \to 0$ when $x \to 0$. Then, $(\chi_p,u_p)_{p>p_0}$ is uniformly bounded in $L^\infty(\Omega;[0,1]) \times W^{1,p_0}(\Omega;\mathbb{R}^m)$ and its limit points for the weak topology of $L^\infty(\Omega;[0,1]) \times W^{1,p_0}(\Omega;\mathbb{R}^m)$ minimizes I on $L^\infty(\Omega;[0,1]) \times W^{1,\infty}(\Omega;\mathbb{R}^m)$.

Proof. The proof is a consequence of Theorem 1 and the compactness part in its proof. □

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