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Characterizing a Mathematical school:
Shared Knowledge and Peano's Formulario

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CHARACTERIZING A MATHEMATICAL SCHOOL:
SHARED KNOWLEDGE AND PEANO'S *FORMULARIO*

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ABSTRACT. — This article is devoted to a re-interpretation of the activity of the Peano school linked to the *Formulario di Matematica*, a text that has always been presented as the manifesto par excellence of this group. The choice of a new perspective, buttressing the analysis of this encyclopaedic work with that of the exchanges among the Peanians—reconstructed thanks to new archival sources—has made it possible to replace the clichés produced by historiography and to recover and properly attribute to Peanians a substantial mass of results in logic and foundations of mathematics, which circulated in the school but without ultimately filtering into the *Formulario*. A stark contrast emerges between the usual image of the Peano school (that of a cohesive, hierarchically structured team) and the real practices of collective construction and socialization of mathematics within a network of scholars much looser than one might expect.

RÉSUMÉ (Caractériser une école mathématique : le *Formulario* de Peano et les échanges de savoir)
  Cet article vise à réinterpréter l'activité de l'école de Peano liée à l'édition du texte qui en est généralement considéré comme le manifeste : le *Formulario di Matematica*. Nous proposons d'analyser cette collection encyclopédique à la lumière des échanges entre les membres de l'école de Peano que de nouvelles sources nous ont permis de mettre au jour. Cette nouvelle perspective nous

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amène à porter un regard critique sur l'historiographie de l'école de Peano et à mettre en évidence que de nombreux résultats sur la logique et les fondements des mathématiques ont circulé au sein de cette école sans pour autant se voir publiés dans le *Formulario*. L'image qui a souvent été donnée de l'école de Peano comme présentant une grande cohésion et une forte structure hiérarchique contraste profondément avec la réalité des pratiques collectives de construction et de socialisation du savoir mathématique au sein d'un groupe qui s'avère beaucoup moins rigide qu'on aurait pu l'attendre.

1. INTRODUCTION

The aim of this article is to prove that the interpretive category 'mathematical school' can be applied to what is commonly identified as the 'Peano school' only in a partially coherent way. The approach taken here, which for the first time explicitly traces the exchanges that took place in this group, permits a more complete and nuanced vision of its research and publication practices. In particular, it allows us to:

(1) clarify and explore Giuseppe Peano's leadership and his influence on his students, colleagues and collaborators (to whom we refer collectively as 'Peanians' for convenience) in greater detail, both with regard to his scientific mastery and his authoritarian tendencies;

(2) replace the cliché produced by 19th and 20th-century historiography of the perfect harmony within the Peano school with a more accurate description of the professional and personal relationships of a team, sometimes inevitably in conflict;

(3) prove that the encyclopaedic treatise *Formulario di Matematica*, usually considered the manifesto of this school, did not in fact serve as such;

(4) recover and properly attribute to Peanians of greater or lesser prominence a substantial mass of results, which circulated in the school but without ultimately filtering into the *Formulario*.

The present paper is organized as follows.

First we will examine what is generally meant in the literature by the term Peano school (sometimes also known as the Italian school of mathematical logic), illustrating its structure, aims and project (§ 2).

Next we will focus on the *Formulario*, a text that has always been presented as the result par excellence of teamwork in the Peano school (§ 3 and 4). Taking this treatise as a touchstone, a stark contrast emerges between the image of the Peano school delivered by historians (that of a cohesive, hierarchically structured team) and the facts (a loose network of

scholars, often inclined to work individually, even without informing the *Maestro* and their colleagues about their studies) (§ 5).

In light of this new depiction, a new problem arises: that of tracing the important body of knowledge that was collectively built by the Peano school, but was not completely recorded in the *Formulario* (§ 6.1 and 6.2). The contents of the exchanges that occurred among the Peanians, in the years of their "feverish creativity," [1] can now be reconstructed thanks to a variety of archival sources, recently discovered (§ 7.1 and 7.2).

To document the transition (full or partial, permanent or of limited duration) from a local, mainly oral culture, to a written work intended to be authoritative and even definitive, as was the *Formulario*, we have chosen four examples, related to the area of foundational studies (§ 8.1–4).

Aknowledgments

2. THE CLICHÉS ABOUT THE PEANO SCHOOL

Guido Castelnuovo [1930, p. 615] described the notion of 'school' in a famous passage:

> a scientific school [is] a union or a family of people collaborating in developing and pursuing a well-defined project of research. […] In scientific respects it offers the means of accelerating and deepening the exploration of a given field, penetrating every facet of it, illuminating it from various perspectives. But the school also brings advantages for individuals, since it makes it possible to exploit in the most effective way the various aptitudes, and also to make the most of the work of mediocre scholars […]. Now, to create a school the merit of the Maestro is not sufficient, nor is it sufficient that he knows how to outline a vast plan of research projects that go beyond his own working capacity. It is also necessary that he succeeds in communicating his passion and his faith to his disciples and in demanding and directing their collaboration. [2]

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[1] AVM: A. Padoa to G. Vailati, 5.9.1901.

[2] Una scuola scientifica [è] una riunione o direi quasi una famiglia di persone collaboranti nello sviluppare e proseguire un piano organico di ricerche. […] Nei

Castelnuovo's attitude is emblematic of a widespread trend among Italian mathematicians at the turn of the 20th century to read the evolution of their discipline in terms of the rise, growth, maturation and *decline* of different schools, and to monitor the balance of power and competition among them. This was, for example, the paradigm adopted by Vito Volterra and Francesco Severi—as well as by Castelnuovo himself—when they were charged with making the Italian contributions to analysis and algebraic geometry known in international milieux.

In fact, to give a consistent definition of 'research school', to characterize the essence of the various communities, and to penetrate their inner dynamics are very difficult and intriguing tasks which have been repeatedly tackled by historians of science, both in a theoretical sense [3] and with relation to specific case studies (the Cambridge school of physiology, [4] the Italian school of algebraic geometry, [5] Eliakim H. Moore's and Robert L. Moore's schools, [6] etc.).

In the case of definitions of the Peano school, all the portrayals highlight three components, related to rhetoric and to substance, that became clichés over the course of time: the leadership of this group, the membership of a number of scholars, and the features of the research project carried out by them.

As far as the role of leader is concerned, Peano never presented himself, either publicly or privately, as the master (*Maestro*) or commander (*generale*) of a team. On the contrary, Italian logicians and practitioners of mathematical logic—whether they had been Peano's students or not—identified him as their beloved mentor, and paint him as an "unforgettable friend," "an illustrious head of a school, to be placed in the Pantheon of humanity's luminaries." [7]

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riguardi scientifici essa offre il mezzo di accelerare ed approfondire l'esplorazione di un determinato campo, di penetrarne ogni angolo, di illuminarlo da vari punti di vista. Ma la scuola reca pure vantaggi per quanto concerne gli individui, giacché essa permette di sfruttare nel modo più efficace le varie attitudini, e di adoperare anche l'opera dei mediocri […]. Ora per dar vita ad una scuola non basta il valore del maestro, né basta che egli sappia tracciare un piano di ricerche così vasto da superare la propria forza di lavoro. Occorre altresì che egli riesca a comunicare la sua passione e la sua fede ai discepoli e sappia esigerne e dirigerne la collaborazione.

3   See [Geison 1981]; [Mancosu 2009]; [Rowe 2003].

4   See [Geison 1978]; [Geison & Holmes 1993].

5   See [Brigaglia 2001]; [Brigaglia & Ciliberto 2004]; [Brigaglia et al. 2004]; [Luciano & Roero 2012]; [Giacardi & Conte 2016]; [Luciano & Roero 2016].

6   See [Parshall 2004]; [Lewis 2004].

7   [Padoa 1933a, p. 100]; [Padoa 1933b, p. 15]; [Di Dia 1928, p. 53]; [Boggio 1933a, p. 457]; [Boggio 1933b, p. 446].

Analogously, neither the name nor the interpretive category of school can be ascribed to Peano himself, who defined his collaborators "a society of mathematicians," former distinguished pupils, now colleagues and friends, or at best qualified them, quoting Ernst Schröder, as "experts in mathematical logic, zealous promoters of pasigraphy."[8] Conversely, Alessandro Padoa, Cesare Burali-Forti, Giovanni Vacca, Tommaso Boggio and others not only proudly proclaimed their affiliation to a large and flourishing group,[9] but also recognized their common legacy from Peano and even declared their "veneration" of him. 'Formal' lists of the disciples of the Peano school even appeared. By 1928, on the occasion of Peano's seventieth birthday, Ugo Cassina alluded to Giovanni Vailati, Padoa and Burali-Forti "who belong, so to speak, to the Peano school of the first generation" [Cassina 1928, p. 9]. In the same context, Giuseppe Di Dia spoke of the "foreign representatives of the Peanian group" (Bertrand Russell, Maxime Bôcher, Edward Huntington, Oswald Veblen, Edwill B. Wilson, Louis Couturat) [Di Dia 1928, p. 63] and Caterina Migliorero described the contributions given by the "last Peano school," mentioning in particular "the ladies Comi, De Stefanis, Frisone, Mori-Breda, Vesin, Viglezio, Viriglio, the gentlemen Zavagna, Bersano, Borio and first and foremost Prof. Ugo Cassina" [Migliorero 1928, p. 36]. Some years later, commemorating Peano, Cassina listed 45 scholars, whose papers, "directly inspired by the Maestro," have to be examined in order to fully understand Peano's work [Cassina 1932, p. 124]. Although often repeated in the literature,[10] these various lists are all questionable. In fact, in the majority of them, the membership to the Peano school is credited—more or less explicitly—to a specific editorial venture: the publication of a collection of texts entitled *Formulario di Matematica.* However, from time to time, some protégés like Fiorenzo Chionio, Angelo Pensa and Michele Cipolla, who were not among the co-editors of this encyclopaedia, were added because they had displayed their appreciation for Peano's mathematical and interlinguistic work. Moreover, scholars like Filippo Sibirani, who apparently had no relationship with the Peanians, were listed, while Beppo Levi, Sebastiano Catania and Emilio Artom, who declared themselves alumni of Peano, were excluded. Even more curiously, for example, James

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8   [Peano 1894b, p. 4]; [Peano 1895b, p. 38]; [Peano 1896b, p. 566]; *Préface*, [Padoa 1912, p. 3]; [Peano 1898e, p. 95].

9   [Padoa 1901, p. 309]; [Burali-Forti 1901, p. 288]; [Boggio 1933a, p. 451]; [Cassina 1933, p. 324]; [Padoa 1936, p. VIII.1].

10   [Kennedy 2002a, p. 232]; [Kennedy 2002b, p. 35]; [Luciano & Roero 2010, p. XI-XVIII, 1–212].

Pierpont [1904, p. 158] listed Peano, Veronese, Pieri, Padoa, Burali-Forti, and Levi-Civita as leaders of the Italian critical synthetic school, while Arthur Korselt asserted that Giuseppe Veronese and Federigo Enriques also belonged to the Peano school [Korselt 1932, p. 114–115].

Beyond the identification of its leader and its members, a peculiar image of the Peano school was also established through testimonies and recollections: the portrait of the "small handful of comrades," the "army of staunch and willing co-idealists," who have been trained by the "*duce*" Peano for the intellectual battle. [11] Consolidating this perception is, above all, the French philosopher Couturat who, on several occasions, painted the Peanians as a team in which there reigned the "spontaneous entente of collaborators which in no way diminished their independence or individual originality" [Couturat 1900, p. 401]. In fact, both such a group portrait and the description of Peano as a master who exerted his fascination on his followers without any authoritarianism, [12] merely relying on his prestige and 'Franciscan' charisma were largely fictitious, as we will show.

In the different characterizations of the Peano school, the last cliché is that related to its research project. First of all, one has to underline that, despite Peano's numerous cultural interests, the literature speaks of the existence of a Peano school only with reference to investigations in logic and foundations of mathematics, vector and numerical analysis. [13] Only in recent years have the studies led by the Peanians in history of mathematics [Roero 2009–2011, p. 81–108], mathematical education [14] and linguistics begun to be reappraised. [15] Besides, it must be borne in mind that logic and foundations of mathematics did not exist at the turn of the 20th century, either as institutionalized disciplines in their own right, or as specialised areas of research. For this reason, many authors misinterpreted or overemphasized some aspects of the plan of studies pursued by the Peano school, to the detriment of others. For example,

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[11]   [Padoa 1933c, p. 82]; [Padoa 1902a, p. 186–187]; [Mastropaolo 1932, p. 100].

[12]   [Kennedy 2002a, p. 215, 218]; [Roero 2003, p. 144].

[13]   [Styaazhkin 1969, p. 277–281]; [Rodriguez-Consuegra 1991, p. 127–131]; the volume *Peano e i Fondamenti della Matematica* [*Atti Modena* 1991], the contributions by Allasia, Lolli and Marchisotto in [Lolli 1985, p. 49–83], [Skof 2010, p. 29–45, 63–66, 157–168] and the essays by Guerraggio, Pasini, Dell'Aglio, Smith, Sallent del Colombo and Cantù in [Roero 2010b, p. 263–278, 327–367, 475–492, 493–508, 509–529, 545–561].

[14]   [Luciano 2006, p. 269–303]; [Luciano 2012, p. 31–47].

[15]   See the essays by Gobbo and Minnaja-Paccagnella in [Roero 2010b, p. 563–574, 575–603].

Pierpont surely exaggerated when he affirmed that Peano and his compatriots "had in particular endeavoured to show the *non-contradictoriness* of the axioms of our number system by making them depend on the axioms of logic" [Pierpont 1904, p. 147]. In analogous terms, Arthur Shearman's judgement about the supremacy of the logical system constructed by the Italian school deserves to be reappraised [Shearman 1906, p. 214].

We can nevertheless assert that the line of investigations developed by the Peano school was generally characterized with special reference to ideography, the symbolic language that they formulated and used in most of their publications and teaching, and to a very peculiar view of logic considered "not a science in itself, but a tool" at the service of mathematics [Peano 1913, p. 497]. In fact, the emphasis that E. H. Moore, Felix Klein and many others placed on the ideographical signature of the Peanians' output—an emphasis acknowledged by subsequent literature [16]—should be at least mitigated, since not all members of the school advocated the use of such symbols. For instance, Burali-Forti [1901, p. 289] declared:

> The research that led me to the results contained in this paper were made taking advantage of the powerful means of analysis provided by the logical symbolism [...]. However, to make the reading of my work possible even to those who are not familiar with this symbolism, I have presented everything in the usual forms of natural language. [17]

The same choice was made by Mario Pieri in his publications on the foundations of geometry, Padoa in papers on the principles of algebra, and Angelo Ramorino, Sebastiano Catania, Marco Nassò in their textbooks of arithmetic and geometry for middle-secondary schools.

As far as the utilitarian feature of Italian logic is concerned, it is true that Peano strongly supported this conviction, to such a degree that in 1900 he proposed that the term 'Mathematical Logic' be used to refer to his line of studies in logic applied to mathematics, conceding the names Logistics, Symbolic Logic and Algebraic Logic to the more advanced research

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[16]   [Moore 1902, p. 402–405]; [Wilson 1904, p. 76–78, 80]; [Lalande 1907, p. 285]; [Klein 1909, p. 484–485]; [Enriques 1911, p. 11, 32–33]; [Jourdain 1912, p. 288–289, 295, 313]; [Brouwer 1913, p. 84]; [Stamm 1928, p. 35]; [Richard 1935, p. 18–20]; [Church 1956, n. 539]; [Bourbaki 1960, p. 20–21]; [Freguglia 1981, p. 331–332]; [Mangione & Bozzi 1993].

[17]   Les recherches qui m'ont conduit aux résultats contenus dans ce mémoire ont été faites en tirant parti des puissants moyens d'analyse logique fournis par le symbolisme logique [...]. Cependant, j'ai tout exposé avec les formes usuelles du langage, pour rendre la lecture de mon travail possible même à ceux qui ne sont pas familiers avec ce symbolisme.

in pure logic carried on by Russell and David Hilbert. [18] The same view was inherited by Padoa, Vacca and Burali-Forti, and then transmitted to Clarence I. Lewis, Edward Stamm and many others. [19] However, some of Peano's protégés (Vailati, Pieri, etc.) rejected this usage and did not frame their research activity entirely in the realm of 'methodology', that is to say, logic applied to the analysis and symbolic exposition of mathematics. [20]

3. THE *FORMULARIO DI MATEMATICA*: HISTORY AND STRUCTURE

Peano's ideography, which first appeared in his writings of 1888–89, consisted of around 20 symbols to denote connectives and quantifiers (ε, $\subset$, $\sim$, ι, $\cup$, $\cap$, etc.), and 200 signs to translate classical theories: rational arithmetic, elementary geometry, algebra and the first elements of number theory, infinitesimal and vector calculus, differential geometry, etc. Over time, this formal language, the so-called 'Peanese', was expanded by the members of the school (the aforementioned representatives and many others like Alfredo Arbicone, Filiberto Castellano, Mineo Chini, etc.) in order to render it more flexible and suitable for expressing contents of functional analysis, homography theory, finite difference calculus, measure theory and differential geometry.

The various chapters translated into ideography were brought together in the *Formulario di Matematica*, published in five editions from 1894 to 1908. [21] As Padoa [1933c, p. 190] carefully explained:

> the five volumes of the *Formulario matematico* […] constitute a publication that escapes the common bibliographic classifications: they are not *reprints*, because the discussion becomes increasingly extensive; they are not *sequels*, because the text restarts each time; and they are not subsequent *editions*, each of which cancels the previous ones, because sometimes temporary editorial demands made it

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18 Cf. in APVT the manuscripts by Vacca, *Logica Matematica*, 1939, fol. 1r; *Logica Matematica*, 1939a, fol. 1r; *La Logica Matematica negli ultimi cinquant'anni*, 1939b, fol. 1; *Nel Congresso Internazionale di Filosofia a Parigi 1900*, [post 1939], fol. 1r; [Vacca 1946, p. 31–32].

19 [Padoa 1912, p. 13–14]; [Burali-Forti 1919, p. XXVI, XXX, XXXII, 103, 109, 163, 313, 321, 341, 359, 387, 426, 477]; [Kozlowski 1917]; [Lewis 1918, p. 115]; [Smith 1926]; [Smith 1927]; [Behmann 1931]; [Jörgensen 1931, p. 192–193]; [Zaremba 1926]; Levi in [Terracini 1955, p. 9–21]; Grattan-Guinness and Quine in [*Celebrazioni* 1986, 17–31, 33–43].

20 [Vailati 1899]; [Pieri 1907a] and [Vailati 1907].

21 Here and in what follows, the five editions of the *Formulario* [Peano 1895a; 1899; 1901a; 1903b; 1908] are identified as *F1* (the first), *F2*, *F3*, *F4* and *F5* (the last one).

necessary to highlight the latest results, without reproducing some fundamental contents, considered classic by that time. [22]

Designed as a work in progress and as an instrument of immediate and practical utility, in order to address the problem of the explosion of mathematical production [Peano 1894a, p. 52], the *Formulario* was published in Turin by Bocca, an education-focused publishing house. It was Peano himself who decided on the structure and the long list of rules to govern the compilation of this text:

> Each of the sections must contain all the propositions, theorems and definitions to which reference is made. In that way, once the *Formulario* is sufficiently complete, anyone who wishes to get up to date on science, on a given point already dealt with in the *Formulario*, will only need to check the book, and will find all the known propositions there. […] Quotations and historical information allow the reader to compare original texts where the questions are discussed at greater depth [Peano to Klein, 29.8.1894, in [Luciano & Roero 2012, p. 169–170]]. [23]

In its last edition, the *Formulario* included over 5000 statements and proofs. [24] The language used for mathematical contents was ideography, whilst the annotations were in French for the first four editions and in *latino sine flexione* for the last one, in honour of the ideals of internationalization that marked all of Peano's cultural activity.

The *Formulario* had a paratactic structure, so much so that at first glance it seemed to be a simple collection of 'formulas'. To remedy this, Peano published some ad hoc works to explain how to consult this encyclopaedia, and also took advantage of editorial tricks. In addition to the meta-representations (*, ♣, §, <), which refer to the key conceptual points throughout, the *Formulario* included meta-textual devices consisting of indices of nouns and subjects, prospects for the occurrence of the symbols and concordance tables between the editions. Moreover, the *Formulario*

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[22] Ora i cinque volumi del *Formulario matematico* […] costituiscono una pubblicazione che sfugge alla comune nomenclatura bibliografica: non sono *ristampe*, perché la trattazione diviene sempre più estesa; non sono successivi *libri*, perché ogni volta si ricomincia; e non sono successive *edizioni*, ciascuna delle quali annulli le precedenti, perché talvolta esigenze editoriali momentanee indussero a mettere in luce i risultati più recenti, senza riprodurne alcuni fondamentali, considerati ormai noti.

[23] Ognuna delle parti trattate deve contenere tutte le proposizioni, teoremi e definizioni, che vi si riferiscono. Sicché, quando il *Formulario* sarà alquanto avanzato, chiunque desideri mettersi al corrente della scienza, su un dato punto già trattato nel *Formulario*, non avrà che a confrontarlo; ivi troverà tutte le proposizioni note. […] Citazioni e notizie storiche permettono di confrontare libri ove le questioni sono più diffusamente discusse.

[24] Cassina in [Terracini 1955, p. 71–102]; [Cassina 1955, p. 244–265, 544–574].

was a hierarchical text, i.e., mathematical sections were printed in larger font size, while historical, pedagogical and methodological remarks were in a smaller font. This choice of layout reflected well the dual goal of the treatise, which was conceived from the beginning not only as a catalogue or syllabus of mathematical knowledge, but also as an auxiliary text for teaching and teacher training [Luciano 2008a, p. 65–81].

For more than a decade, Peano devoted all his efforts to the coordination and promotion of the *Formulario* with contributions from himself and his collaborators, even at the risk of leading his school away from more promising trends.[25] Though failing to formulate a wholly efficient strategy for dissemination, the Peanians intertwined a broad web of partnerships with European and American colleagues (Camille Jordan, Robert Montessus de Ballore, Couturat, Klein, Gottlob Frege, Alexander Ziwet, E. H. Moore, Veblen, Huntington, …) through correspondence, vulgarization activity in journals and through attendance at symposia. Meetings devoted to clarifying the meaning of symbols took place, and competitions were organized to find prospective co-authors for the *Formulario*.[26] Further, from 1893 to 1942 the Peanians held a dozen series of lectures and seminars on logic in Italy and abroad, in order to illustrate their 'style and tradition', and explain how to 'read' the *Formulario* (see § 7.2 below). Convinced that prejudice was the only obstacle to the achievement of their plan, Burali-Forti, Padoa and Vacca intended to prove that ideographic code was easier to learn than any other language, and that an editorial experience like the *Formulario* best demonstrated the practical utility of logical studies. The effect of such an attitude was, on one hand, to condemn to silence all the works of Peano's group that did not exploit symbols, and on the other hand, to present the *Formulario* as the *Magna Carta* of the school.

4. THE *FORMULARIO* AS A COLLECTIVE TEXT

Since its very beginnings, the *Formulario* was depicted as a "beautiful project, whose execution was beyond the forces of a single man" [Peano 1896a, p. 4]. On several occasions, Peano maintained that only a large and well-organized team could complete it and that he was fortunate to have obtained the help of a number of colleagues, and of several young

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[25]   [Borga & Palladino 1992, p. 18, 24, 29, 40–41]; [Borga et al. 1985, p. 7–10].

[26]   Cf. Temi per Concorso, *Il Pitagora*, VI, 1900, 110, 136. See also [Reyes y Prósper 1893, p. 41–43].

graduates, who were enthusiastically designing the various parts. Peano [1898c, p. 88] specified the nature of partnerships he was looking for:

> Whoever is interested in the study of mathematics can collaborate on the *Formulario* and if taken in the right way, it will be a good opportunity both to learn and to have an enjoyable experience. If you are interested in modern publications regarding a specific field of mathematics, you can update the relevant chapter of the *Formulario* in accordance with these new works. […] If you love teaching and are dedicated only to that, you can compare your lesson with the corresponding [paragraph of] the *Formulario*: while at times this comparison may help to improve your lectures, it may also be the case that the *Formulario* will be improved by your lectures, by better ordering the propositions, adding missing proofs or simplifying them. Someone who loves individual study, without the aid of books, can construct entire theories. [27]

In Peano's mind, the members of the 'learned society' editing the *Formulario* were to be ranked on two levels: those like Vailati, Castellano, Burali-Forti, Rodolfo Bettazzi, Francesco Giudice and Gino Fano, who had taken on themselves the compilation of some entries and autonomously proceeded with their structuring, and "the gleaners" [Giusti & Pepe 2001, p. 47], that is, those who were entrusted to submit small additions and corrections. [28] Moreover, according to Peano, not only could *all people* collaborate on the *Formulario* but also, at least in theory, *all knowledge* produced by the team could find a suitable place in it. It is thus quite comprehensible that to see their contributions published in this text became the highest aspiration for Peano's collaborators and, at the same time, the canon according to which they organised their studies.

The *Formulario* was a team effort not only because it was edited by a group around Peano but also because its production entailed a series of collective practices of research and publication. To understand this, it is helpful to describe how a page of this encyclopaedia came into being, and what transformations it underwent from one edition to another.

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[27]   Chiunque s'interessi di Matematiche può collaborare al *Formulario*, e se prende la cosa pel giusto verso, avrà una buona occasione di istruzione e di vero divertimento. Se si interessa alle pubblicazioni moderne, relative ad un ramo della Matematica, può tenere al corrente di queste il capitolo corrispondente del *Formulario*. […] Se ama il proprio insegnamento, e ad esso solo si dedica, può confrontare la sua lezione col corrispondente *Formulario*; se alcune volte questo confronto contribuirà a perfezionare la sua lezione, spesso esso servirà a perfezionare il *Formulario*, mettendone in miglior ordine le Proposizioni, dandone dimostrazioni mancanti, o semplificandole. Chi ama lo studio individuale, senza libri, ha da costruire intere teorie.

[28]   [Peano 1893, p. 1]; [Peano 1896b, p. 568]. Peano himself asked collaborators "to start their work by translating into symbols a simple and limited entry" [*F1*, VII].

A single author, in accordance with Peano or not, produced a first draft on a specific concept, proposition or theory. At this point, his work consisted in examining the background bibliography, both elementary and advanced, in framing the treatise from a hypothetico-deductive perspective, and finally in codifying all the material into symbols. For the *Formulario* this was, properly speaking, the 'constructional' stage, during which new symbols were introduced, eliminated, or changed in form and meaning. Speaking of a 'constructional' stage should not deceive us. Indeed, in answer to the colleagues' doubts on the original aspect of such activity, Peano maintained [1898c, p. 85]:

> The new results contained in the *Formulario* are not the theorems, which are generally due to authors of long ago, but the study of their interdependence; the statement and the proof that a specific idea can or cannot be defined, that a given proposition can or cannot be proven, and the collection of various definitions, proofs and possible theories […]. The results achieved in this way are as important from a theoretical viewpoint as the discovery of any other mathematical truth. [29]

This first stage was followed by a second phase in which Peano, with the help of his staff, fine-tuned the draft, judged whether the translation into symbols was appropriate or not, and corrected mistakes and gaps. It was in view of such revision that Peano [1898c, p. 86] expressly structured the *Formulario* as a project to be carried out by a school, in the larger sense of the term:

> The filling of the gaps is not my responsibility as editor of this publication, busy with various other things and eager to move on, to collect the numerous theories where the same analytical method has already been applied. Nor is it the task of the collaborators, who willingly helped me in this enterprise, despite being so few. Rather, it is the task of the many amateur mathematicians who are found all over the world, but who are particularly numerous in Italy, to carry on and provide an essential contribution to the perfection of the *Formulario*. [30]

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[29] I risultati nuovi contenuti in esso non sono i teoremi, dovuti ad Autori spesso antichissimi, ma lo studio della loro dipendenza; l'affermazione e la prova che una determinata idea si possa, o non si possa definire, che una data proposizione si possa dimostrare o meno, e la raccolta delle varie definizioni, dimostrazioni e teorie possibili […]. I risultati che così si ottengono sono tanto importanti dal lato teorico, quanto la scoperta d'ogni altra verità matematica.

[30] Il colmare queste lacune non spetta a me, editore di questa pubblicazione, assorto in conseguenza in svariati affari, e desideroso di andare avanti, onde raccogliere le numerose teorie cui si è già applicato lo stesso metodo analitico, ed applicarlo ad altre. Né spetta ai Collaboratori, che volenterosi mi aiutano in questa impresa, ma che sono ancora pochi. Spetta invece ai tanti studiosi della Matematica, che si trovano dovunque, e che sono numerosi in Italia, i quali potrebbero portare un utilissimo contributo al perfezionamento del *Formulario*.

At this level, the task for Peano and his co-workers became that of transforming a collection of formulas into a well-articulated exposition, suitable for use in a multiplicity of contexts, and appropriately grounded from mathematical and extra-mathematical points of view.

This second phase was characterized by a constant and rich network of exchanges among the Peanians, both in an abstract sense (exchanges of opinions, information, data, and so forth), and in a concrete one (loans or gifts of books and articles, manuscripts, notebooks, teaching materials, etc.). As a consequence: "every entry of the *Formulario*, though begun by a single Author, would have ultimately been the result of the work of all the collaborators." [31]

5. CIRCULATION OF KNOWLEDGE WITHIN THE PEANO SCHOOL

Since the second phase of compilation of the *Formulario* mobilized a large network of contributors, an analysis of the circulation of knowledge among the members of the Peano school seems appropriate. In this connection, two prevailing types of interactions are to be distinguished. The first took place between Peano and his students at the university and military academy in Turin, in the courses of infinitesimal calculus, higher analysis and complementary mathematics, for which he adopted the *Formulario* as a textbook up to the 1890s. In this case, there were asymmetrical relationships established within a hierarchical context in which Peano, despite his non-aggressive, egalitarian attitude, played the role of the authoritative teacher (*il nostro illustre signor professore* [De Finis 1904, p. 220]), while his interlocutors were in a position of apprenticeship. The absence of genuine reciprocity in this kind of intellectual transfer did not, however, prevent the establishment of lasting and profitable collaborations. For example, Pietro Buffa, a former student of Peano in the courses of infinitesimal analysis for prospective engineers, became an ardent supporter of ideography and a contributor to the *Formulario*, see [Buffa 1901]. An analysis of the relationships between Peano and the students he advised leads to the same conclusion: the theses of Margherita Peyroleri, Maria Gramegna, Vincenzo Mago, Paolina Quarra and Fausta Audisio, were used to revise the last edition of the *Formulario*.

In contrast, the exchanges among the first generation of Peanians, which preceded, accompanied or followed the publication of the *Formulario*, were of a different nature. These were carried out in a milieu

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[31] *F1*, III.

of greater equality between the players but, at the same time, in an atmosphere characterized not only by synergies and affinities, but also by divergences of opinions and debates. [32] The study of this second network of interactions reveals a quite new image of the Peano school: that of a social group in which transparency of relationships was never complete, and in which flattery, friendship and support, rivalries, career ambitions and aspirations to prizes and recognition assumed a role that was anything but marginal, conditioning the very evolution of the teamwork. Hence it is not surprising to see Vacca writing that "despite his diligence, Padoa will not improve the *Formulario* any more than Proclus improved Euclid's *Elements*" [33] or to read, in the letters of Burali-Forti, that "Padoa's little exercises are on fine display, while the important and intelligent work of other collaborators is destined for the waste basket." [34]

Both types of exchanges used oral transmission among other things. The daily relationships within a research group linked to a very precise geographical context—the city of Turin and its institutes—were in fact naturally conveyed via the channels of colloquia and 'luminous' conversations, which took place while huddled in cafés, "walking through the streets of Turin or sitting on some chance bench." [35] Peano was well aware of the importance of these forms of socialization for the advancement of the *Formulario* and, for instance, to satisfy the desire of his co-authors, once a week he met them at his home, where "as in the schools of the ancient philosophers," he spoke about mathematics and humanities, physics and

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[32]   For example, Burali-Forti, Vacca and Vailati often complained about their difficulty in understanding one another and in reaching a shared position on specific instances [AVM: Burali-Forti to Vailati, 28.12.1907]: "It's hard to reach an agreement by letter. If you come to Turin, an hour of conversation will make us thick as thieves. Can you wait to publish? Such a beautiful opportunity for a *complete* and *logical* reform will not happen again, and we should take full advantage of it" (Però per lettera è difficile intendersi. Se tu verrai a Torino, un'ora di conversazione ci porrà d'accordo. Potresti aspettare a far pubblicare? Un'occasione così bella di riforma *radicale* e *logica* non si presenta più e bisognerebbe sfruttarla completamente). Cf. also [AVM: Burali-Forti to Vailati, 5.9.1907]: "If you come to Turin, we will discuss with paper in hand and it will be easier for us to understand each other. Otherwise I will send you a text" (Se verrai a Torino discuteremo con carta in mano e ci intenderemo più facilmente. Altrimenti ti invierò lo scritto).

[33]   Vacca to Vailati, 7.11.1903, in [Lanaro 1971, p. 226].

[34]   Burali-Forti to Vacca, 30.6.1905, in [Nastasi & Scimone 1995, p. 21].

[35]   [AVM: Vacca to Vailati, 16.12.1899, fol. 1r].

philosophy, in other words—as he was accustomed to say—"*de omnibus rebus et de quibusdam aliis.*"[36]

6. THE SEARCH FOR A MANIFESTO FOR THE PEANO SCHOOL

As we have previously mentioned, the description of the *Formulario* as the teamwork par excellence of the Peano school is particularly well attested. However, thanks to an examination of the inner dynamics of this group, a discrepancy between the programmatic declarations and the reality of their commitment emerged.

The first problem consists in establishing whether the *Formulario* is or can be considered as the *only* publication expressing the activity of the Peano school *as a whole*. In principle, alternatives could be taken into consideration, starting with the journals *Rivista di Matematica*[37] and *Schola et Vita*[38] directed or supervised by Peano. In a certain sense, the *Rivista di Matematica* could in fact be considered the journal of a school,[39] both because its 88 authors included 23 Peanians out of the 45 listed by Cassina and because, over the years, it became an instrument for promoting both ideography and the *Formulario*, which was published as its supplement. Analogously, *Schola et Vita* can be understood as a 'programmatic journal' both because it involved 22 of Peano's protégés and because this periodical makes it possible to reconstruct the contributions of the team in the sectors of mathematical education and recreational mathematics, after the final edition of the *Formulario*. However, neither the *Rivista di Matematica* nor *Schola et Vita* constitutes a comprehensive manifesto of the Peano school, since their editorial boards were formed, respectively, by representatives of only the first or the second generation. This is a significant clue inasmuch as it shows that the two generations were like watertight compartments: except in sporadic cases, their memberships were mutually exclusive, even where physical or cultural proximity of their

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[36] [Gliozzi 1932, p. 255]. Analogous testimonies can be found in [Vacca 1933, p. 99]; [Cassina 1933, p. 377]; [Ascoli 1958, p. 266]; Geymonat in [*Celebrazioni* 1986, p. 7, 9, 13] and in many letters preserved in APVT.

[37] The *Rivista di Matematica* (1891–1906) edited by Peano with a small group of cofounders: F. Porta, F. Castellano and F. Porro, published articles and debates on the foundations of mathematics and on hypothetico-deductive teaching.

[38] *Schola et Vita* (1926–1939), co-edited by Peano and Nicola Mastropaolo, with the collaboration of Gaetano Canesi, Ugo Cassina and Mario Gliozzi, deserved to be considered "the most important scientific and pedagogic journal of our time written in Interlingua."

[39] [Peano 1892, p. 76]; [Natucci 1928, p. 73]; [Ziwet 1891, p. 43].

members would have made it natural to interact. [40] A further potential manifesto might be represented by the *Dizionario di Matematica*, which Peano planned to publish jointly with the *Formulario*. Also the *Dizionario* (collecting terms that occur in contemporary mathematics, together with their etymology, history and definition) had been expressly conceived as a collective publication [Peano 1901b, p. 3–4]:

> It is supremely useful […] that, before it goes to print, the *Dizionario* be seen by a large number of people. This is so that any form of individualism can be removed from the work, because a single author necessarily imprints a one-sided style to his work. Furthermore the collection will turn out to be more complete […]. The book will thus avoid any of the errors of negligence to which such a work is generally liable [41].

In this case, the main problems arose in relation to the fragmentary character of the *Dizionario*, the drafting of which stopped at the first entries [Padoa 1902d], because it was impossible for Peano, Vacca, Vailati, Padoa, and some 'minor' collaborators like Catania, Tommaso Vannini, Umberto Ceretti and Corrado Ciamberlini to reach agreement on the epistemological tenets on which to base it.

However, even admitting the *Formulario* to be the *only* publication that adequately testifies to the activity of the Peano school does not completely answer the questions raised by this treatise, both as a textual phenomenon in itself, and in its teamwork perspective.

First, it is necessary to determine which of the editions to judge as normative: the fifth, *F5*, which Peano himself defined as 'complete and definitive', or one of the previous four, which were instead richest from the point of view of extra-mathematical issues. Furthermore, we have to establish whether to consider only the volumes of the *Formulario* or to also include 11 papers by Peano published between 1884 and 1910, deemed by Cassina (and many historians afterward) to be preparatory chapters,

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[40] It was indeed Peano himself who, when he had to recruit the editorial team for *Schola et Vita*, preferred to turn to a new group of collaborators rather than to his 'historic' team, even while recognizing that the Peanians of the second generation were less well trained than his first companions of the *Rivista di Matematica*. Cf. [APM: Peano to Mastropaolo, 6.3.1923, 21.4.1925, 2.5.1925, 7.12.1925].

[41] È sommamente utile […] che prima della stampa definitiva esso sia visto da un gran numero di persone. E ciò per togliere ogni carattere individuale al lavoro; perché necessariamente un solo autore imprime al suo lavoro un carattere unilaterale. Inoltre la raccolta riuscirà più completa, poiché ognuno potrà aggiungere quel nuovo termine, o quel nuovo significato d'un termine che crederà più opportuno. […] Il libro riuscirà emendato dalle sviste in cui si cade necessariamente, in un lavoro di questo genere.

though used in a very limited way. [42] Besides, recent studies have shown that the pool of writings linked to the *Formulario* should be expanded to comprise various teaching materials (in particular, lecture notes of Peano's university courses) from the period 1889–1919, many parts of which were expressed in symbols and earmarked for insertion in this encyclopaedia. [43]

Second, even more serious questions arise regarding the collective authorship. Is it really valid to present the *Formulario* as the manifesto of the Peano school when only some of its members took part in its writing? While Vacca, Vailati, Padoa, and Giuliano Pagliero participated in this project for decades, continually devoting the best fruits of their research to it, some (Pieri, Boggio, Enrico Novarese, Matteo Bottasso, Alberto Tanturri, etc.), only contributed sporadically, while others (Fano, Vivanti, Bettazzi, and Giudice) stopped collaborating after the first edition.

In the third place, it is possible to ascertain that a substantial part of the shared knowledge constructed by the Peanians did not find a place in the *Formulario* and that many contents "were lost" [Natucci 1932, p. 54]. This failure of some content to filter into the publication affected the *Formulario*'s mathematical, as well as historical and educational, contents.

With regard to the school's historical studies, what passed from their exchanges into the *Formulario* was merely a set of *Notes*, juxtaposed to various propositions, which included biographical and bibliographic references and extracts from printed and manuscript sources [Roero 2010a]. What was left out was all the rest, that is, the historiographical assumptions underlying the compilation of these entries and the bulk of the secondary literature used to write them. [44] Concerning the pedagogical and educational stances, indeed, no traces of attempts at the didactic transposition of the *Formulario* were recorded in the text. Nor is there any allusion to the criticisms of the hypothetico-deductive teaching, patterned after this work, that was undertaken by some of Peano's supporters (Ciamberlini, Giuseppe Sforza, Alpinolo Natucci, and Vincenzo Cavallaro, among others). Lastly, the gap between the knowledge circulating in the Peano school and its codification in the *Formulario* is remarkable in the realm of meta-mathematical contents. Based on a common culture and enhanced

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[42] Cassina in [Terracini 1955, p. 76–77]; [Cassina 1955, p. 244–245]; [Cassina 1960, p. X-XI]; [Grattan-Guinness 2000, p. 219–267]; [Kennedy 2002a, p. 63–70].

[43] Cf. [Luciano 2008b, vol. I, 60–92, vol. II, 67–114].

[44] Cf., in APVT, Vacca's personal card index, used in order to compile *Formulario*'s historical annotations: *Storia della Matematica I, II, fascicolo anni 1000 a.C.-500 a.C.*; *anni 500–1500*; *anni 1500*; *anni 1600*; *anni 1700*; *anni 1800*; *anni 1830–1900*; *Storia della matematica Torino 1892-Genova 1904.*

through a body of readings shared by the Peanians, these contents are much broader than can be imagined from browsing the references in the *Formulario*. [45] In fact there was a circulation of volumes and articles considered to be 'reference works' that were exchanged among Peano's followers, thanks to the rich library held by the *Rivista di Matematica* and placed at the disposal of its readers [Peano 1898c, p. 88]. It was no coincidence that the same publications by Couturat, Huntington, E. H. Moore, Russell, Shearman and Alfred North Whitehead (among the few contemporary authors to be cited in the *Formulario*) were studied by Peano, Vacca, Vailati, Padoa, Cassina, etc. and commented on together. There was also a set of readings (Frege, Ernest Naville, Hilbert, Enriques, and others) that were recommended by individual members of the school, but that were appreciated by only a part of the team. This second type of reference little by little cross-fertilized the works of Peanians, enriching them with quotations from authors outside the school. However, these works were accepted by Peano in only a few cases and they reached the *Formulario* even more rarely. [46] As far as meta-mathematical issues are concerned, two concomitant factors hindered the transition from the shared culture to the definitive written text. On one hand the Peano school never reached an agreement on the epistemological framework of its studies. [47] In the attempt to consolidate the image of a cohesive and unified group, Peano usually chose to gloss over controversial stances in the *Formulario*, and entrusted their dissemination to other vectors of socialization (for example, the seminars on mathematical logic). On the other hand, the division of roles that characterised the group should not be underestimated. According to this division of labour, Vailati was 'the philosopher', Vacca 'the historian'. This meant that some of Peano's collaborators, like Padoa, intentionally remained silent about the epistemological tenets of their production, leaving to their fellows the task of dealing with them. [48]

6.1. *Two levels of decision-making*

In the face of this evidence, new questions emerge: why is some remarkable material hardly mentioned in the *Formulario*, or not included at all,

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[45] Cf., in APVT, the manuscript by Vacca, *Bibliografia*, 1903, attached to his copy of *Letture di Logica*, fol. 1r and the *marginalia* by Peano to *F5*, *Bibliographia de Logica Mathematica post 1908*, p. XIV.

[46] Cf. for example [APVT: Peano to Vacca, 16.11.1906].

[47] Cf. [APVT: Peano to Vacca, 24.2.1906].

[48] Cf. Padoa to Vacca, 13.9.1912, in [Nastasi & Scimone 1995, p. 135–136].

even though it would have fit well into the treatise? Why were other results promptly included, passing unchanged through the various editions? On the basis of thousands of comments by Peanians on how and where to integrate or redistribute the materials, we conclude that two levels of decision-making can be distinguished.

The first one was constituted by macro-decisions on the overall structure of the *Formulario*, and at this stage the organigram of the school was undoubtedly pyramid-based with Peano at the apex. The *Maestro* not only monitored the overall co-ordination of the work (typesetting, publicity, promotion, etc.) but was ultimately the only one to wield power on the features and contents of the text. So for example, though saying that he aimed at rendering the *Formulario* "independent from any personal will," in 1898 Peano candidly informed the readers that he had chosen to group the propositions according to the symbols that occurred in their statements [Peano 1898c, p. 82]. Similarly, in 1910, Peano admitted that the *Formulario* was still lagging behind in the initial parts, having made more notable progress in differential and integral calculus, the sections "which interested him most." [49] In this perspective, it is not thus surprising that, in the transition from *F1* to *F2* (1894 and 1897–99), and even more in the passage from *F4* to *F5* (1903 and 1908), whole pages were removed by the *Maestro* because he maintained them to be—respectively—unoriginal and/or too advanced for university courses.

In principle, Peano was quite prepared to explain the choices for such modifications and he asserted more than once that this was necessary, because the *Formulario* was the result of a large team project. [50] However, in spite of such intentions, it should be acknowledged that he focused attention almost exclusively on the additions, while failing to comment on what sections had been left out, and why. So for instance, in 1896 and 1898 Peano dedicated several pages to the new theories reduced to symbols. He also gave an accurate list of concordances among the chapters II, III, and V of *F1* and the corresponding paragraphs of *F2*, and indicated the propositions of his *Arithmetices Principia* that had newly merged into the work. However, the reader looking for information on the sections deleted in this period will be disappointed. After saying that "the reasons for the cuts would have to be given somewhat in brief, because of the long journey made from *F1* to *F2*" [Peano 1898c, p. 76], Peano omitted the motives

49   [APVT: Peano to Vacca, 28.1.1910].
50   [Peano 1898a, p. 48]. Cf. also *F2*, 4; *F3*, VI-VII; *F4*, VII-VIII; *F5*, XIII and [APVT: Peano to Vacca, 2.9.1905, 27.12.1904].

that had prompted him to completely eliminate the section on algebraic numbers (part IX of *F1*) and to salvage just a small group of formulas on set theory (part VI of *F1*). With regard to the suppression of chapter IX, entrusted to Fano, perhaps through largely incidental circumstances (his sojourn in Göttingen and the fact that he had attended there a course by Heinrich Weber on number theory), there are the following few lines of justification [Peano 1898c, p. 87]:

> The Propositions on degree and divisibility of polynomials, their greatest common divisor, etc. have not been retained. Something is done in F_1 IX; but in order to develop this theory we need new notations; and I think that they are mistakenly prescribed for middle and secondary teaching. [51]

With respect to the removal of most of chapter VI on set theory, prepared by Giulio Vivanti, Peano is a bit more specific. After declaring that some of its propositions have been also addressed in the new edition, he noticed that an overabundance of notations and two incomplete definitions had been found there. As a result, the main part of the section "cannot be reprinted until someone has reordered it, giving purely symbolic definitions that do not require ordinary language in order to be understood" [Peano 1898c, p. 82].

In most cases, Peano did not enter into such details and justified his decisions by invoking the desire to shorten as much as possible the time between editions:

> Seven sextodecimos are drafted. I will see to the other corrections and gradually it will be finished. The work is long: I will finish it by refusing any addition that may delay its completion [APVT: Peano to Vacca, 31.12.1900]. [52]

In other circumstances Peano referred, more or less vaguely, to the wish to overcome difficulties of various kinds (notably, to collect and order the mass of material, in such a way that research becomes easy, in a system entirely independent of personal opinions and established habits) that prevented him from accepting suggestions by his fellows. Sometimes he went further to admit that his choices were largely fortuitous because, contrary

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[51]   Non sono raccolte le P sul grado e divisibilità dei polinomii, sul loro massimo comun divisore, ecc. Qualche cosa è fatta in F_1 IX; ma per sviluppare questa teoria occorrono nuove notazioni; e ritengo che per errore esse trovano posto nell'insegnamento secondario.

[52]   Sette fogli sono tirati. Faccio eseguire le altre correzioni, e poco alla volta si terminerà. Il lavoro è lungo; lo termino sopprimendo ogni aggiunta che ne ritarda il compimento.

to other encyclopaedic projects, the *Formulario* was not required to "conform to any pre-set deadline or program but simply publishes the propositions gradually, as someone becomes interested in them, gathers them together and sends them off to be published" [Peano 1898c, p. 87]. This kind of explanation is obviously rather weak, so it is not surprising that often there were followers of Peano who attempted to tried to guess at the policies of their *Maestro*.[53] For instance, Padoa and Vacca justified some of his decisions *a posteriori* by invoking his "tacit self-criticism"[54] or his aesthetic canon, according to which Prof. Peano "did not publish in the *Formulario* everything he had found, but only the system that he deemed more beautiful."[55] Padoa, indeed, imputed the fact that some parts, despite their importance, had been dropped in the passage from *F4* to *F5*, to the frantic intensity of the work carried out in the years 1904–1908:

> independent of the omissions that have to do with me, and which you kindly pay more attention to than they deserve, I have often noticed that the haste to compile a new edition caused some questions from previous versions to be omitted which ought not to have been left out. I believe that a complete revision of the various editions would be useful […] [APVT: Padoa to Peano, 31.1.1910].[56]

If it is true that Peano was solely responsible for the major lines of the work, the situation was completely different regarding the finishing touches given to an already-existing entry, putting in the historical and

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53 Cassina in [Terracini 1955, p. 81–93].

54 Manuscript by Padoa kept in [APVT, *Evoluzione dell'ideografia logica di G. Peano prima del Formulario Matematico*, 1932, fol. 11]: "In the brief interval of time between *F4* and *F5*, a tacit self-criticism, surely in accordance with my § 23, convinced Peano to exclude the three axioms quoted above from primitive arithmetical propositions without explaining why" (Nel breve tempo trascorso fra *F4* e *F5*, un'autocritica tacita, ma certo conforme al mio § 23, ha indotto l'autore [Peano] ad escludere le tre proposizioni citate dalle Proposizioni aritmetiche, senza motivarne l'esclusione). Similarly, Vacca annotated, in his copy of the *F5* (at p. 6): "here there is a worsening with respect to previous editions. There Peano had recognized that one of the primitive ideas (;) is not necessary. And indeed that is so! Later he forgot this subtle reasoning and he reintroduced this entry, perhaps on the suggestion of Schröder" (qui c'è un peggioram. sulle antichissime edizioni. P[eano] allora non riconosceva la necessità dell'idea primitiva; E difatti essa non occorre. Poi si è dimenticato del sottile ragionamento e l'ha reintrodotta forse su suggestione di Schröder).

55 Vacca to Vailati, 7.12.1903, in [Lanaro 1971, p. 226] and [APVT: Vacca to Peano, April-May 1910].

56 Indipendentemente dalle omissioni che mi riguardano, e cui gentilmente dai più peso di quel che non meritino, più volte ho notato che la fretta di comporre una nuova edizione faceva lasciar da parte qualche cosa delle precedenti, che non era da abbandonarsi. Credo che una revisione completa delle varie edizioni sarebbe utile […].

bibliographical information, and sometimes compiling new paragraphs. Small groups of collaborators who commented on the drafts of the *Formulario* comprised a secondary level of decision-making. Little by little, the stream of corrections and proposals submitted by these authors extended, following not a rectilinear trajectory in an outwards-inwards direction, but rather a model of circulation 'in convection cells'. The contents entered into an upward-rising chain of shared knowledge and, if they received the "coveted consent" [Padoa 1933b, p. 21] of the *Maestro*, were recorded in the *Additions et Corrections* to the *Formulario*, periodically published in the *Rivista di Matematica* [Peano 1898b; Peano, Arbicone et al. 1901; Peano, Cantoni et al. 1901; Vailati 1903]. In some cases, after passing through this process, they would enter into the *Formulario* itself. Often, they would survive there permanently; other times they would disappear from one of the ensuing editions.

The outcome of the entire process was very uncertain and only a part of the results collectively obtained by the school actually made it into the *Formulario*. This dynamic of construction-circulation-filtering of knowledge proceeded differently according to the period in question. It was more rapid and efficient in the years 1892–1899, when Peano was much more willing to listen to his co-workers, while it slowed down and jammed more frequently after the fourth edition (1904). [57] From this date on, the letters of the Peanians are full of allusions to the difficulty of contacting Peano, to the fact that "no one has heard from him for a long time" and to his giving no news of what he is up to and what he has in mind for the *Rivista* and the *Formulario*. [58] After Peano's death, Cassina finally affirmed that Peano took advantage of the work of his disciples only for the first and second editions of the *Formulario*, while from the third version onwards the project became more and more a one-man operation [Cassina 1933, p. 366]. Also heavily conditioning the process was the nature of the partnerships intertwined among Peanians. With respect to both macro- and micro- decisions, there was no lack of tensions within the school. Hence it is not surprising to find Vailati deploring Peano's "bad habits," [59] or to read, in a letter from Russell to Philip Jourdain, that Peano "is not touchy […] tho' he does not himself readily adopt improvements made by others." [60]

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57   Cf. [APVT: Peano to Vacca, 14.1.1904, 28.1.1904, 7.6.1906, 11.1.1907].

58   [APVT: Vailati to Vacca, 21.2.1905; 3.9.1906; 24.4.1907].

59   [APVT: Vailati to Vacca, 20.7.1902].

60   Russell to Jourdain, 15.4.1912, in [Grattan-Guinness 1977, p. 132].

The most emblematic episode of such inner conflicts is perhaps one concerning the destiny of the theory of functions worked out in 1899. As Vacca noted in his copy of the *Formulario*:

> Given the continuous exchange of ideas, in those days, between Burali and Peano, it is difficult to say which of the two had the idea. The symbol F [by Peano] probably precedes those of Burali [APVT: marginalia by Vacca to *F5*, 79]. [61]

Partially published in *F2, F3* and *F4*, this entry was subsequently deleted by Peano, who justified as follows his decision: the theory of functions *f* was incomplete, but it had been kept in the *Formulario*, up to the last edition, because of its applications in mathematics. Peano had drafted another theory, based on the properties of the sign F, 'definite function', which did not present the preceding difficulties. His theory of the symbols *f* and F had then been corrected by Burali-Forti but "due to lack of time," it could not be included in the *Formulario*. [62] On that occasion, Burali-Forti vented his bitterness towards 'Our Father Peano', writing to Vacca:

> Given Peano's temperament it is inevitable that all the intelligent and fruitful work of some collaborators will disappear, and only that of those who have numerically enriched the *Formulario* on the basis of a concept or formula of Peano have remained. Here is another example: the pages in which a return is made to the erroneous definition of F are already stenotyped, and this is only because that of the 1903 edition is mine and not Peano's. He wasn't able to find another and he suppressed the good one because not a single thing can be said that goes beyond ordinary and material compilation, by anyone other than Peano [Burali-Forti to Vacca, 30.6.1905, in [Nastasi & Scimone 1995, p. 21]. [63]

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[61] Dato lo scambio continuo di idee di quei giorni tra Burali e Peano è difficile dire di chi dei due sia stata l'idea. Il simbolo F [di Peano] probabilmente precede quelli di Burali.

[62] [Jourdain 1912, p. 293, note †† by Peano].

[63] Dato il carattere di Peano è inevitabile che tutto il lavoro intelligente e proficuo di alcuni collaboratori sparisca, e rimanga solo quello di coloro che in base ad un concetto o ad una formula di Peano hanno arricchito numericamente il *Formulario*. Eccone un altro esempio: sono già stenotipate le pagine nelle quali si ritorna alla definizione sbagliata di F e questo solo perché quella della edizione 1903 è mia e non di Peano. Non è riuscito a trovarne un'altra ed ha soppresso la buona perché non si possa dire che una cosa, che esce dall'ordinaria e materiale compilazione, è stata fatta da persona che non è Peano.

7. NEW ARCHIVAL SOURCES

In the preceding sections, we have alluded to archival sources that have allowed us to recover the contributions of the Peanians, and to track the pieces of the research carried out by the school in issuing the *Formulario* with an abundance of details never before attained. In particular, in addition to a small group of manuscripts by Peano, Vailati and Vacca from the period 1897–1939, the correspondence between Peano, Vacca, Vailati, Pieri, Padoa, Burali-Forti and Boggio, and the transcripts of excerpts from talks and reunions, contained in the notebooks of Vacca, [64] we have based our analysis on two main sources:

– the marginalia added by Peano and by some of his protégés to their copies of the *Formulario*;

– the lithographed notes of lectures in logic given by the Peanians in Italy and abroad.

7.1. *The marginalia*

Cassina, in the 1950s, was the first to highlight Peano's habit of annotating his books, making notes about points that could prove useful for future reprints or further editions. [65] After Peano's death, Cassina, Gaetano Canesi and Nicola Mastropaolo inherited his personal library (which Cassina would later sell to the Mathematical Library of the University of Milan) and thus came into the possession of Peano's books with his marginalia. However, Cassina failed to make any systematic use of this patrimony, even when the Unione Matematica Italiana assigned him the task of publishing Peano's *Selecta*. Considering the manuscript notes of his *Maestro* as

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[64] Cf. APVT: *I Adversaria Mathematica Dr. G. Vacca*, fols. 2r (14.9.1900), 2v (14.9.1900), 9r (23.11.1900-January 1901), 15r (2.2.1901), 28v (24.3.1901), 69r (6.3.-20.4.1902); *Adversaria Matematica 1905–1916–1922–1944–1946*, fols. 4v (8.2.1905-November 1908), 155r (18.9.1946–28.9.1946), 161v (28.9.1946–6.1.1947), 165v (8.1.1947); *Adversaria Mathematica Roma 1914–1915*, fols. 26r (17.7.1914), 27r (17.7–14.11.1914), 46r (20.6–20.9.1915), 71r (9.11–17.11.1915); *Adversaria Mathematica incipit 24 Ottobre 1915–30 Maggio 1946*, fols. 45v (7.7.1946), 87r (1.8.-7.8.1946), 90r (1.8–7.8.1946); *Adversaria Mathematica incipit 14 Gennaio 1916, explicit 1925*, fols. 139r (3.6–13.11.1919); *Adversaria Mathematica incipit 25 Luglio 1916*, [25.7.1916–29.5.1930], fols. 1–97; *Adversaria Matematica 11 Maggio 1927–20 Febbraio 1931*, fols. 113v (20.2–6.3.1929); *Adversaria Mathematica, Calcoli numerici e tentativi vari, incipit 17 Febbraio 1932*, fols. 35v (11.6–28.12.1932), 67r (24.1–23.10.1933), 150r (1.10.1937-April 1938).

[65] Cassina in [Terracini 1955, p. 93–94]; [Cassina 1955, p. 252, 259–260, 558–559].

something of a curiosity, he simply mentioned some of them and published just a few others in the anastatic reprint of *F5*. [66]

The lack of further information on the fate of Peano's personal library led to fears that his annotated copies of the *Formulario* and of many other volumes may have been lost. That was the situation when in 2007 a copy of the book *Lezioni di Analisi infinitesimale*, sprinkled with autograph notes, was found in the Civic Library of Cuneo. A systematic search for Peano's 'lost library' was then initiated and concluded with its virtually complete reconstruction [Luciano 2008b, vol. II, 141–172]. With reference to the *Formulario*, this included 36 copies of *F1-F5*, with marginalia by Peano, Vacca, Vailati, Cassina, Boggio and Peano. [67] To these should be added the annotated proofs of some of Peano's preparatory works for the *Formulario* and of 466 other texts used to draft or update individual entries.

Such handwritten notes consist of intra-line marginalia (generally used for minor corrections and misprints), extra-textual worksheets (in margins, on covers, on dustcovers or back pages, mostly referring to historical additions and references but also to alternative proofs, exercises and examples related to the university teaching by Peano, where the *Formulario* was used as textbook), annotations on fly leaves and—only for Peano's own volumes—entries on white papers, bound with the printed pages of the *Formulario*, during the correction of the galleys, the blueprint stage or in test print copies.

For our purposes, the marginalia by Peano and members of his school have been a source of great relevance inasmuch as they have contributed to supply information on collective research by Peano, Vacca and Vailati on topics like Leibniz's contributions to logic, recursive definitions, the principle of induction, and the primitive ideas of logic and arithmetic. [68] Moreover, these marginalia have documented all the contents excluded from the *Formulario* for mere chronological reasons, that is the contributions in

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[66]   [Cassina 1960, p. XXV-XXVI, XXXV-XLVIII]. In the 1980s, Bottazzini [1985, p. 185] examined Peano's marginalia to the volume *Calcolo geometrico secondo l'Ausdehnungslehre di Hermann Grassmann, preceduto dalle operazioni della logica deduttiva.*

[67]   The copies of *F1-F5*, with marginalia by Peano are available, in electronic form, in [Roero 2008]; 7 copies of *F1-F5* with marginalia by Vacca and Peano were found in 2009 and are now kept in APVT. Four copies of the *Formulario* annotated by single scholars such as Eugenio Maccaferri, Francesco Gerbaldi, Marco Nassò, Florentano, according to their different roles (author, reader, proof-reader), are kept, respectively, in the Library of the Department of Mathematics at the University of Milan and in the Historical Library of the Istituto Val Salice of Turin.

[68]   Cf. in particular the marginalia by Vacca at *F2*, p. 34, 55, 61, 86; the marginalia by Vacca at *F5*, p. 11–12, 13 16.

numerical analysis, variational calculus and number theory received from the second generation of the Peano school in the years 1910–1929, after the last edition of the *Formulario.*

7.2. *The lithographs of the lectures on logic*

From the 1890s, the Peanians organized series of lectures on mathematical logic, addressed to people who had "not yet had the opportunity to benefit from an oral presentation" of ideography. [69] This tradition of schooling was begun by Burali-Forti in 1893–94 with a course on "scientific readings" held at the university of Turin and repeated by him in Augusta, Sicily (1894–95). Series of seminars in foundations of mathematics and logic were then held by Peano in Turin (1906, 1909, 1925–32), by Padoa in Brussels (1898), Pavia (1899), Rome (1900), Cagliari (1907), Geneva (1911) and Genoa (1932–37), by Vacca in Genoa (1903–04) and by Alessandro Terracini and Beppo Levi in Tucumán ([Terracini 1939]; [Levi 1942]). [70]

These lectures constitute a very unusual form of mathematical sociability. Positioned as they were at the crossroads between common teaching practice, chats among friends and lectures for a large audience, the Peanians saw them as moments of production (and not only of re-production) of knowledge. The public, composed of mathematicians, philosophers, teachers, students and amateur mathematicians, was invited to work in synergy with the lecturer, [71] formulating suggestions, objections, and so forth. From our perspective, the examination of the outlines, the summaries and the hand-outs of such 'courses', has been intriguing because it allowed us to brought to light new partnerships, such as those formed with Luigi Certo, Edmondo de Amicis, Carlo Garibaldi, and Italo Zignago (none of whom were ever listed among the members of the Peano school), who enrolled in the entourage of the *Formulario* after having attended the lectures on mathematical logic delivered by Burali-Forti, Padoa and Vacca.

The following four examples will offer a concrete explanation of how these archival sources contributed to our reappraisal of the scientific and publication practices of the Peano school.

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[69]   *F4*, X.
[70]   These experiences of dissemination of logic are illustrated in [Luciano 2010, p. 279–315].
[71]   [APVT: Padoa to Vacca, 30.12.1932].

8. EXAMPLES

8.1. *Example: The Padoa-Beth method*

Among the meta-mathematical topics to which the Peano school attributed the greatest importance is that of ordered and absolute independence of the axioms of a deductive system. The problem was faced and solved by Padoa, thanks to the development of a method, today known as the Padoa-Beth method, anchored in the concepts of interpretation and model. This result achieved its definitive status as written knowledge in the *Formulario* after being the object of exchanges among the Peanians for a very short period of time.

In effect, when in 1891 Peano published the article "Sul concetto di numero," he dealt with the problem of the independence of arithmetical postulates, but in *F1* he barely hinted at it [Peano 1891a, p. 93–94]. Between 1895 and the spring of 1899, Vailati and Padoa worked out a technique for proving the ordered and absolute independence of a system of axioms and they illustrated their attempts in three articles appearing in the *Rivista di Matematica*,[72] and in two lecture courses on logic given in Brussels and in Pavia.[73] The information about Padoa's results circulated among the members of the Peano school, so it is not surprising that the proof of the absolute independence of the arithmetical axioms given by Padoa was noted by Peano and Vacca in their marginalia[74] and a reference to Padoa's lectures was inserted before the definitive printing of *F2*.[75] Meanwhile Padoa refined his method, exploiting it for the presentation of a general deductive theory in a new series of lectures on logic given in Rome between December 1899 and February 1900 [Padoa 1900, p. 4]. A few months later, in August, Padoa's contributions became the object of two papers [Padoa 1901; 1902b] that he presented at the international congresses of philosophy and of mathematicians held in Paris. At the same time, Peano was working on *F3*, in which he once again inserted the models given by Padoa, in order to prove the independence of arithmetical axioms.[76] References to Padoa's method remained also

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[72] [Vailati 1895, p. 183]; [Padoa 1896, p. 35].

[73] [Padoa 1898, p. 50–51 (conférence 3.11.1898)]; [Padoa 1899].

[74] APVT: marginalia by Vacca to [Peano 1898d, p. 2].

[75] *F2*, 30.

[76] *F3*, 2, 8, 41–44.

in *F4* and *F5*, where the lithographs of Padoa's Rome lectures were cited among the best works in logic that had ever appeared. [77]

8.2. *Example: Peano Arithmetic*

From *F2* on, rational arithmetic was presented according to the deductive system developed by Peano in [Peano 1889a] and [Peano 1891a], and founded on three primitive concepts and on five postulates. Over the years, as is well known, this arrangement underwent some variations. Specifically, in *F3* the number of postulates was increased to six, with the addition by Padoa of the axiom NεCls. [78] This foundational scheme of arithmetic was reproduced without further changes in *F4* and *F5*. [79]

Moreover, two theories, respectively promoted by Vacca and Pieri, were proposed as alternatives. At the end of 1902 Vacca worked out a new deductive system for arithmetic beginning with three primitive ideas $(0, N_1, +)$ and four postulates, and he attempted to prove their compatibility. Aware of the importance of this result, Vacca discussed it with Peano, who expressed appreciation for the outline: "that is great. Prepare a version for print." [80] At the same time Vacca also explained his arrangement to Vailati, concluding:

> … a Pp [postulate] is saved without complicating the system, like Padoa does. I hope to publish this as well in the *Formulario* [APVT: Vacca to Vailati, 15.2.1903]. [81]

Three years later, Pieri, in his turn, conducted an independent study of the principles of arithmetic, hinging his system on two primitive ideas and four axioms. This was the object of two articles, [Pieri 1906] and [Pieri 1907b]. Once again, Peano was quite familiar with these contributions, so much so that he mentioned them [82] in his marginalia to *F5* and declared that, after Pieri's works, his own studies "had only historic value" [Peano

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[77] *F4*, 14, 33–34, 36; [Peano 1906, p. 27–28]; *F5*, 28.

[78] *F3*, 39–44. About this addition cf. the following manuscripts preserved in APVT: Vacca, *La Logica Matematica negli ultimi cinquant'anni*, 1939b, fols. 10–11; Padoa, *Evoluzione dell'ideografia logica di G. Peano prima del Formulario Matematico*, 1932, fols. 11–12.

[79] *F4*, 31–36; *F5*, 27–31.

[80] [APVT: Peano to Vacca, 14.2.1903].

[81] Si risparmia una Pp senza complicare il sistema, come fa Padoa. Spero di pubblicare pure questo sul *F*[*ormulario*].

[82] Marginalia by Peano in [Peano1906*, 26–27]. When a symbol * is added to a reference, it refers to the copy of the corresponding work with Peano's *marginalia*, preserved in the Library of the Dept. of Mathematics 'F. Enriques', University of Milan,

1916, p. 3]. Compared to the system devised by Peano and Padoa, Vacca's and Pieri's arrangements presented some advantages, first of all that of being minimal. Moreover, substituting them in the place of the theory published in the *Formulario* would have involved only slight adjustments to the remaining parts of the treatise.

Vacca's and Pieri's results circulated widely within the Peano school and were several times the focal point of their discussions. [83] For example, eight years after the last edition of the *Formulario*, Boggio still asked Vacca:

> You […] told me, I seem to recall, that you had succeeded in eliminating the proofs by induction or something to that effect. I beg you to write me regarding this, because it would be extremely useful to my friend Burali who is writing the second edition of his volume on mathematical logic. And if you have other observations to communicate regarding the *Formulario*, send those to me as well [Boggio to Vacca, 9.12.1916, in [Nastasi & Scimone 1995, p. 13]]. [84]

However, the arithmetical systems of both Vacca and Pieri were relegated to the sphere of shared knowledge: not only were they not included in *F4* and *F5*, but they were not even alluded to as alternatives to the 'classical' theory. [85]

8.3. *Example: Pieri's and Padoa's systems for Euclidean geometry*

For mathematical schools, national and international congresses constitute very propitious opportunities for promoting their styles of research. The Peano school was aware of this. For example, on many occasions

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coll. Op. I 46, Op. A 138, Op. A 139, Op. A 141. Note that all the published and unpublished works by Peano are now available on a DVD, [Roero 2008] (for the classification of the items on this DVD, see the bibliography).

[83] Cf. [Vacca 1946, p. 37–44] and the notebooks by Vacca, in APVT: *I Adversaria Mathematica Dr. G. Vacca*, fol. 62v (29.1.1902); *Adversaria Mathematica 1902–1903, pp. 1–30 mediocri*, fol. 68v (28.8.1929); *Adversaria Matematica 1905–1916–1922–1944–1946*, fols. 9v (8.2.1905-November 1908), 75r (October 1912–29.12.1912), 111r (6.1.1916); *Adversaria Mathematica Roma 1914–1915*, fol. 69r (28.10–9.11.1915); *Adversaria Mathematica incipit 14 Gennaio 1916, explicit 1925*, fol. 65r (16.5–24.6.1916); *Adversaria Mathematica, Calcoli numerici e tentativi vari, incipit 17 Febbraio 1932*, fol. 198r (August 1943).

[84] Tu mi parlasti pure […] dicendomi, mi pare, che sei riuscito ad eliminare o le dimostrazioni per induzione o qualcosa di simile. Ti pregherei di volermi scrivere diffusamente a questo riguardo, perché ciò sarebbe utilissimo all'amico Burali, il quale sta scrivendo la 2ª edizione del suo volumetto di Logica matematica. E sei hai altre osservazioni da comunicarmi riguardo al *Formulario*, mandamele pure.

[85] Peano alluded to these contributions on just a single occasion, in a note addressed to Jourdain [1912, p. 295, n. ¶], mistakenly attributing them to Padoa!

Vailati advised his fellows "not to miss important dates" [86] and he pressed upon them the need to have something ready for disseminating the *Formulario.* [87] However, the fact that this text included only a part of the knowledge produced by the Peano school was at the origin of curious false steps and some missed opportunities, as the following episode shows.

Peano, motivated by teaching aims, published two essays on the foundations of geometry, both written with ideographic signs [Peano 1889b; 1894a]. Starting in 1894 Vacca also proposed an analysis of some propositions of elementary geometry by means of symbols, to be included in *F1*. According to Peano:

> … the idea is good. But it appears to me that nowadays it is easier to translate the works of calculus than those of geometry, since as yet no well-ordered system of symbols has been devised to represent geometric entities [APVT: Peano to Vacca, 12.7.1894]. [88]

However, in the Peano school it was Pieri who, since 1894, had been considered the specialist in the field of the foundations of geometry, and who would naturally be involved in compiling these entries for the *Formulario*. Pieri took as his starting point the concept of motion already investigated by Peano [Peano 1894a, p. 75–80]. In April 1899, he became aware that it was possible to deduce all of elementary geometry from only two primitive concepts: the point and a relation between three points that can be interpreted with the phrases 'c is as far from a as b', or 'c belongs to the sphere of b, of centre a' and 'the pair (a, b) is congruent to the pair (a, c)'. The historical archetype of this intuition was identified in Leibniz's works on *characteristica geometrica*, known to Pieri through Moritz Cantor's *Geschichte der Mathematik*. The excessive complications of the development of such a system gave rise, however, to "the desire for new studies and ulterior research" [Pieri 1899, p. 175], which Pieri undertook in view of the International Congress of Philosophy in Paris, in summer 1900. The organizer of the session dedicated to logic and history of science was Couturat, who extended an invitation to Pieri along with all those collaborating on the *Formulario.* [89] Pieri was unable to take part in the congress but he sent his paper [Pieri

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[86]   Vailati to Vacca, 22.3.1904, in [Lanaro 1971, p. 234].

[87]   Vailati to Vacca, 7.12.1903, in [Lanaro 1971, p. 231] See also [APVT: Peano to Vacca, 12.8.1904; 27.8.1904, 26.11.1904].

[88]   L'idea è buona. Però parmi al giorno d'oggi più facile tradurre i lavori di analisi che quelli geometrici, poiché finora non si è ancora formato un sistema ben ordinato di simboli per rappresentare gli enti geometrici.

[89]   Couturat to Peano, 8.7.1899, in [Luciano & Roero 2005, p. 28]; Couturat to Pieri, in [Arrighi 1997, p. 42–43].

1901], which was read by Couturat. Among those present was Padoa who, without informing Peano, had worked out, with the same Paris Congress in mind, an axiomatic system for Euclidean geometry. Like Pieri, and partially unaware of his results, [90] Padoa had assumed the point and the congruence of two pairs of points as primitive concepts. Padoa asked Couturat's permission to consult his colleague's manuscript, and thus came to possess an autograph copy of Pieri's paper, which he kept until 1901. [91] Upon his return to Italy, and before the proceedings of the congress were printed, Padoa illustrated his and Pieri's systems in Rome in a series of lectures on logic, [92] and published his results [Padoa 1902c]. Meanwhile, in view of the publication of *F4*, and again independently, Vacca and Peano were also discoursing on the possibility of constructing a system of Euclidean geometry, with the objective of overcoming the "serious difficulties" that rendered previous attempts by Pieri "still not at all didactic." [93] In November 1902 Peano communicated to Vacca that he had completed a note which developed geometry beginning with the concepts of point and distance "as was discussed several times." [94] It was in this same period that Vacca taught his course on mathematical logic in Genoa, and among those attending was Italo Zignago, a secondary-school teacher already known to Peano for having published an article in the *Rivista di Matematica* [Zignago 1894] and for having contributed some additions to *F3*. [95] In February 1903 Vacca announced to Vailati that he had been able, with Zignago's help, to reduce the number of postulates in Peano's system to thirteen, and that he was examining the writings of Leibniz, in order to complete the vague references on *characteristica geometrica* provided by Pieri, and thus be able to insert an historical note on this topic in *F4*. [96] In autumn of that year, Padoa too was still studying the principles of geometry, extending his work. [97]

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[90]   Cf. Padoa to Pieri, 25.7.1899, in [Arrighi 1997, p. 84].

[91]   Cf. Padoa to Pieri, 6.1.1901, in [Arrighi 1997, p. 85]. A manuscript by Padoa concerning neutral geometry, drafted in this period, is reproduced in [Borga et al. 2009] and in [Borga 2011].

[92]   [AVM: Padoa to Vailati, 12.1.1901].

[93]   [APVT: Vacca to Vailati, 30.9–6.11.1902].

[94]   [APVT: Peano to Vacca, 20.11.1902]. Peano's work [1903a] appeared shortly afterwards.

[95]   Cf. *F4*, 332 and [APVT: Zignago to Vacca, 22.5.1893, 24.5.1893, 8.1.1900; Peano to Vacca, 14.2.1903].

[96]   Cf. [APVT: Vacca to Vailati, 15.2.1903, the notebook *I Adversaria Mathematica*, fol. 113 (23.4.1903) and the handouts by Vacca 1903, c. 20r].

[97]   Cf. [AVM: Padoa to Vailati, 23.11.1903].

The work by Padoa, Pieri, Vacca and Zignago all shared the same fate. Although each hoped to see their own contributions inserted in the *Formulario*, in the end in *F4* that of Peano was presented, and completed with Vacca's historical note. [98] The results obtained by Vacca and Zignago did not appear; those by Pieri, developed in two essays published independently (1899, 1901), were cited as bibliographical references. [99] Even more surprisingly, half of the definitions published in *F4* in the section on *point* and *distance* were attributed to Padoa, although they were in fact derived from Pieri's suggestions. [100] The situation worsened with *F5*: Peano's point-distance system was re-published; [101] Vacca's historical note vanished. [102] Pieri was in any case cited in the preface (never in the text) as the author of the best system of axioms for elementary geometry. [103] Padoa too was cited in the preface, and twice in the text, but for marginal contributions. [104] All of Pieri's and Padoa's meta-mathematical considerations about the nature of geometric entities, and about the arrangement of geometry as an abstract hypothetico-deductive system—considerations largely developed in their papers [105]—never surfaced in the *Formulario*. The sources that we have examined show that it was not Peano who created a link *a posteriori* between the two constructions by Pieri and Padoa, thus presenting them as a product of the Peano school, but an external commentator: Couturat. Moreover, the same sources prove that Peano's choices really perplexed his followers, who tried to find different explanations. For instance, discussing his later system in correspondence with Pieri, Vacca wrote:

> Your [work] […] has called to my mind some ideas that I conceived many years ago and that were included just in a minimal part in the published text (APVT: Vacca to Pieri, 12.9.1912). [106]

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[98] *F4*, 264–265, 332–335.

[99] *F4*, 14, 253, 264.

[100]   *F4*, 264.

[101] *F5*, 165–167, 177.

[102] Cf. marginalia by Peano to *F4*, [Peano1903b*, 264–265].

[103] *F5*, XI, XIV. Cf. also [Peano 1915, p. 170–171].

[104] *F5*, IX, XIV, 79, 103.

[105] [Pieri 1899, p. 173–177]; [Pieri 1901, p. 367–390]; [Padoa 1902c, p. 353–356]; [Pieri 1908, p. 345–347, 446–448].

[106] Il suo [lavoro] […] mi ha richiamato alla memoria alcune riflessioni da me fatte molti anni fa, le quali non furono se non in minima parte allora pubblicate.

Many years later, in a handwritten note dated 12 October 1942 and inserted in his copy of the *Formulario,* Vacca returned to the failed inclusion of his and Pieri's contributions into the last edition of the *Formulario*:

> Why didn't Peano dare to begin with point and distance? That's how it should be done, because in this way you immediately have Euclid's propositions 23.1, 23.2, 23.3 … none of which are proven here. The explanation lies in the fact that this theory arrived late, in the October of 1902. Proposition 24.2 which defines the vector, i.e., the parallelogram, is mine, while Peano was originally a Grassmannian. Euclid (geometry) was felt by him later. The point-distance system leads immediately to solid geometry (Pythagoras, etc.). Cf. Pieri. [Manuscript by Vacca at *F5*, post p. 177, 12.10.1942–26.2.1945]. [107]

8.4. *Example: Logic Variables and Proper Names*

From *F1*, the chapter *Logica* opens with a section about the meaning of variable letters, [108] gathered for the most part from Peano's youthful works. [109] The first to criticise it strongly was Vailati who, upon proofreading *F3*, explained to Vacca:

> Why not also do away with the ambiguous designation of real variables, calling them, for example, effective variables? These are small changes, but they have their effect, especially in facilitating access for beginners [Vailati to Vacca, 27.1.1901, in [Lanaro 1971, p. 179]]. [110]

In the following days, Vacca pointed out that there was an analogy between variables and proper names, and urged his friend to clarify what that consisted in, and how far it could be pushed. A month later, Vailati showed Vacca the conclusions he had reached, encouraging him in turn to carry on with the analysis. [111] One characteristic of proper names, according to Vailati and Vacca, is that they do not have any meaning, properly speaking, that is, they are signs whose application to an individual does not require

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[107] Perché Peano non ha avuto il coraggio di incominciare con punto e distanza? è così che bisogna fare perché, in tal modo, si hanno subito le proposizioni di Euclide 23.1, 23.2, 23.3 … nessuna delle quali qui è dimostrata. La ragione è che questa teoria è venuta tardi, nell'ottobre del 1902. È mia la proposizione 24.2 che definisce il vettore, ossia il parallelogrammo, mentre Peano era in origine Grassmanniano. Euclide (geometria) fu sentito da lui più tardi. La teoria punto, distanza, conduce subito alla geometria solida (Pitagora, …). Cfr. Pieri.

[108] *F1*, V-VI; *F2*, 5–6; *F3*, 1–6; *F4*, 3–4; *F5*, 6–9.

[109] Cf. [Peano 1894b, p. 16–17]. Apropos this, cf. also [Jourdain 1912, p. 297–299].

[110] Perché non abolire anche l'ambigua designazione di variabili reali, chiamandole, per esempio, variabili effettive? Sono piccoli cambiamenti, ma che hanno il loro effetto, specialmente per facilitare l'accesso a chi comincia.

[111] Vailati to Vacca, 7.2.1901, in [Lanaro 1971, p. 181–182].

(and does not express) the possession on the part of that individual of any particular quality that distinguishes him from one to whom the name is not applied. In the same way, variable letters have no meaning, and when they are found repeated in the same formula, they serve no purpose other than to identify a single object in the various situations in which it occurs. In the meantime, Vailati wrote to Peano, explaining how he intended to modify the *incipit* of the *Formulario*. He related to Vacca the outcome of the dialogue with the *Maestro*.[112] First of all Vailati remarks that

> the proposal to use the term *effective variable* instead of that of *real variable* can be carried out even without changing the correlative term *apparent variable*, to which no similar kind of objection can be made [Vailati to Vacca, 17.2.1901, in Lanaro 1971, 184].[113]

But he would make some other proposals, more substantial, and he again wrote to Peano about them. In particular, Vailati believed it was necessary: 1) to indicate the distinction between symbols that have a meaning and symbols that simply stand to indicate the place where symbols with a meaning must be placed; 2) to characterise the distinction between equations and logical identities; 3) to show the importance of this distinction; 4) to establish that the distinction between conditional and categorical propositions depends on the fact that conditional propositions represent a set of (categorical) propositions, that is, all those that can be obtained by giving all possible meanings to a given sign that they contain (variable letter). The question of whether it is necessary to define the variable letters first and then the conditional propositions or vice versa, thus appears moot to Vailati, since it is not possible to define either of the two without defining the other *ipso facto*. Vailati therefore concluded:

> I have formulated in a slightly different way (to remedy the drawbacks of my exposition in the previous letter to Peano which you have seen) the proposed modifications to that blessed page 2, which, as it stands, seems to me to be a true scandal for anyone beginning to read the *Formulario*. To me it appears absolutely necessary for it to be modified, in one way or another, and that greater clarity in it would reverberate in successive chapters as well, especially that on definitions.

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[112]   Vailati to Vacca, 17.2.1901, in [Lanaro 1971, p. 183].

[113]   Quella proposta di porre, invece del termine *variabile reale*, quello di *variabile effettiva*, mi sembra possa realizzarsi anche senza cambiare il termine correlativo di *variabile apparente*, al quale non si possono fare obiezioni dello stesso genere.

[…] That entire short chapter would need to be completely rewritten [Vailati to Vacca, 17.2.1901, in [Lanaro 1971, p. 183]]. [114]

Vailati returned to the topic on 13 March 1901, saying that,

… the place indicating symbols could be called *pro-symbols* (fabricating that word on the model of pronouns) and such designations would be free of ambiguities to which the term apparent variables gives rise, as well as sufficiently suggestive of the meaning which would be attributed to them [G. Vailati to G. Vacca, 13.3.1901, in [Lanaro 1971, p. 186]]. [115]

Vacca recorded the outcomes of his exchanges with Vailati about the opening section of *F3* in his notebooks. Peano too noted this last linguistic observation of Vailati in his marginalia, but left the treatment in *F3* unchanged. [116] Receiving the galleys of *F4*, in December 1903, Vailati appreciated the insertion of Vacca's new studies "on the theory of theories," but was dissatisfied with the fact that the *Formulario* continued to

give greater importance to propositions than to the relationships that exist among them. The study of the criteria aimed at creating a preference for a given theory over another regarding a given subject has not even been approached [Vailati to Vacca, 18.12.1901, in [Lanaro 1971, p. 195–196]]. [117]

From this point on, Vailati stopped sending comments about the *Formulario*. Vacca, however, did not forget his friend's criticisms, as can be seen from his marginalia, [118] and in November 1905 he once again brought it up with Peano, in order to convince him to change the offending section. Peano responded with a letter sent to both Vacca and Vailati, concluding:

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[114] Ho formulato in modo un po' diverso (per rimediare a inconvenienti della mia esposizione nella precedente lettera al Peano che tu hai vista) le proposte modificazioni a quella benedetta pagina 2, la quale, così com'è, a me sembra un vero scandalo (nel significato etimologico di pietra d'inciampo) per chi comincia a leggere il *Formulario*. A me pare assolutamente necessario che essa venga modificata, in un modo o in un altro, e che una maggior chiarezza in essa si riverbererebbe anche sui capitoli successivi, specialmente quello sulle definizioni […] Quel capitoletto andrebbe completamente rifuso.

[115] Tali simboli indicatori di posto potrebbero esser chiamati pro-simboli (fabbricando tale parola sul tipo di pronomi) e tale designazione sarebbe libera dagli equivoci cui dà luogo la parola variabili apparenti, e abbastanza suggestiva del senso che le si attribuirebbe.

[116] Marginalia by Peano to *F3*, p. 2.

[117] Dare più importanza alle proposizioni che alle relazioni che sussistono tra loro. Lo studio dei criteri atti a far preferire una data teoria a un'altra su un dato soggetto non è finora stato neppur mai abbordato.

[118] [APVT: marginalia by Vacca to *F5*, p. 3].

Regarding the observation of Vailati, which was already discussed another time, and to which I responded, and then the letters were lost, it is needless to say that I do not share the opinion. The existence of a real or apparent variable in a formula is a property that results from the examination of the value of the formula, and not from the signs that accompany it. In the system of symbols adopted by the *Formulario*, it is possible to recognise the apparent letter from its position with respect to three signs. But in another ideography—and there are several possible ones—and I would be happy to see others arise—which would not at all cause damaging competition—it might be that the variable letter is accompanied by only one sign. But of course Vailati produces no reason for his thesis. And thus it is time to be silent [APVT: Peano to Vacca and Vailati, 29.11.1905]. [119]

Peano would essentially repeat this opinion in one of his notes to Jourdain's essay "The development of the theories of mathematical logic and the principles of mathematics." [120] Vailati's proposals for changes to be made to the first pages of the *Formulario*, though interesting in themselves, never crossed the frontier between shared and published. No trace of these attempts would have survived until today if it had not been possible to reconstruct the conversations and correspondence it gave rise to. These interactions were sometimes resolved under the influence of the personality of the leader Peano and sometimes in opposition to it.

9. CONCLUSION

In light of the examples given, it appears legitimate and effective to attempt a re-interpretation of the activity of the Peano school linked to the *Formulario*. The choice of a new perspective, buttressing the analysis of this encyclopaedic work with that of the exchanges among the Peanians, has made it possible to individualise to a great extent the research collectively carried out by the school and 'the life that was lived' in it.

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[119] Riguardo all'osservazione di Vailati, che già fu fatta altra volta, e cui io ho risposto, e poi le lettere andarono perdute, è inutile il dire che io non ne divido l'opinione. L'essere una variabile reale od apparente in una formula è proprietà che risulta dall'esame del valore della formula, e non dai segni che l'accompagnano. Nel sistema di simboli adottati dal *Formulario*, risulta la possibilità di riconoscere la lettera apparente dalla sua posizione rispetto a tre segni. Ma in un'altra ideografia—e ne sono possibili più—e io ne vedrei volentieri sorgere altre—che non si farebbero punto concorrenza dannosa—può essere che la lettera variabile sia accompagnata da un solo segno. E invero, modificando leggermente le notazioni, possiamo fare in modo che accompagni solo il segno di inversione. Ma del resto Vailati adduce nessuna ragione della Sua tesi. Ed è quindi tempo di tacere.

[120] [Jourdain 1912, p. 279 (n. †† by Peano)], 297–299.

In particular, new aspects of the effects of Peano's leadership have been discovered. What emerges is that, on one hand, Peano's authority over his followers was loosely exerted, to the point where he was not completely informed about their studies. On the other hand, he truly conditioned the success or the failure of the filtering process of contents into the *Formulario*, inasmuch as he alone dictated the policies for this editorial project.

Information derived from archival sources has likewise contributed to prove how Peano was naive in his attempt to safeguard a true spirit of shared decision-making in the work aimed at completing the *Formulario*, and to dispel the stereotype that a perfect community of views reigned within his school. Indeed, the fragile equilibria that characterized the relationships among the members of this 'solidarity group' have been clearly shown. In actual fact, Peano's tendency to include and maintain in the *Formulario* all and only those contributions that entered into the spirit of the treatise as *he* had conceived it, chagrined the Peanians, fomented jealousies among colleagues and acrimony towards the *Maestro*, who was judged guilty of having preferences among his protégés [121] and of "forcefully maintaining the *Formulario* in the state of the Koran!" [122]

Further, we have documented that there is no direct causal connection between the production and circulation of a certain result within the school and its insertion in the *Formulario*. Thus, dozens of suggestions and proposals were recovered from the archival sources which came from Peanians of greater and lesser importance, and sometimes also from scholars who had never been listed among the members of the school. As far as this last aspect is concerned, we feel that a reconstruction of the contents of the long daily conversations within the 'invisible college' of Peano would be interesting: such forms of constructing and exchanging mathematics led to the working out and circulation of a rich patrimony of tacit knowledge [123] which often carried more weight than information conveyed in a written text like the *Formulario*.

To conclude, a 'Peano school' effectively existed, but it had neither the configuration nor many of the characteristics that have been attributed to it over time. Peano's role as *Maestro*, the lists of members, the mirage

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[121] Burali-Forti to Vacca, 30.6.1905 and 9.6.1919 in [Nastasi & Scimone 1995, p. 21, 24]. Cf. *F2*, 28; *F3*, 37–38; *F4*, 126–127; marginalia by Peano to *F3*, p. 37; marginalia by Peano to *F4*, p. 127; marginalia by Cassina to *F4*, p. 127; marginalia by Peano to [Peano 1906, p. 79–80]; marginalia by Vacca to *F5*, p. 79; [APVT: Peano to Vacca, 28.12.1902, 20.1.1910 and 28.1.1910].

[122] Burali-Forti to Vacca, 20.6.1919, in [Nastasi & Scimone 1995, p. 24–25].

[123] Cf. [Rowe 2004].

of their *entente cordiale*, and the presentation of the *Formulario* as the mani-
festo of this group, have all been substantially revealed as misleading
stereotypes.

What we do find is the existence of a fleeting network of scholars that
forms and reforms around Peano and his enterprises, a network which
shared some features of the phenomena of research schools and some
strategies for the collective construction and sociability of mathematics.
A key solution for tracking down these features and strategies consists in
reversing the traditional direction of historical analysis and in replacing
a Peano-centric and *Formulario*-centric perspective with a new approach
anchored in the examination of the exchanges between the Peanians.

LIST OF ABBREVIATIONS

APVT: Peano-Vacca's Archive, Library of the Dept. of Mathematics, University of
Turin.

AVM: Vailati's Archive, Library of the Dept. of Philosophy, University of Milan.

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