

Revue d'Histoire des Mathématiques



*“Anti-aircraft guns all day long”:
Karl Pearson and computing
for the Ministry of Munitions*

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Tome 21 Fascicule 1

2 0 1 5

SOCIÉTÉ MATHÉMATIQUE DE FRANCE

Publiée avec le concours du Centre national de la recherche scientifique

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Périodicité : La *Revue* publie deux fascicules par an, de 150 pages chacun environ.

Tarifs : Prix public Europe : 87 €; prix public hors Europe : 96 €;
prix au numéro : 43 €.
Des conditions spéciales sont accordées aux membres de la SMF.

Diffusion : SMF, Maison de la SMF, Case 916 - Luminy, 13288 Marseille Cedex 9
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“ANTI-AIRCRAFT GUNS ALL DAY LONG”: KARL PEARSON AND COMPUTING FOR THE MINISTRY OF MUNITIONS

JUNE BARROW-GREEN

ABSTRACT. — In December 1916 Pearson offered the services of his staff from the Drapers' Biometric Laboratory and the Galton Eugenics Laboratory at University College London, to the Ministry of Munitions. The offer was accepted with alacrity by A.V. Hill, head of the Anti-Aircraft Experimental Section, who was eager to liberate his own men from the labours of computation. From January 1917 until March 1918 Pearson worked tirelessly on the often tedious work of computing of ballistic charts, high-angle range tables and fuze-scales. He also made significant contributions to the mathematical theory behind the tables. Pearson's staff consisted of mathematicians, computers and draughtsmen. Women were an important constituent of his work force, not least because the escalating demands of conscription meant that the men were often at risk from the recruiting sergeants. Things did not always go smoothly—Pearson did not take kindly to the calculations of his staff being questioned by the mathematicians producing the data—and Hill sometimes had to work hard to keep the peace.

RÉSUMÉ (“Anti-aircraft guns all day long” : Karl Pearson et l'informatique au Ministère des munitions)

Au mois de décembre 1916, Karl Pearson offrit au Ministère des munitions les services de son personnel du Drapers' Biometric Laboratory et du Galton Eugenics Laboratory, à University College London. L'offre fut très vite acceptée par le chef de la Section expérimentale antiaérienne, A.V. Hill, pressé de

Texte reçu le 5 août 2014, révisé le 29 novembre 2014, accepté le 11 décembre 2014.
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2000 Mathematics Subject Classification : 01A60.

Key words and phrases : Karl Pearson, first world war, exterior ballistics, anti-aircraft gunnery.

Mots clefs. — Karl Pearson, première guerre mondiale, balistique extérieure, artillerie anti-aérienne.

libérer sa propre équipe du travail de calcul. De janvier 1917 à mars 1918, Pearson travailla infatigablement au calcul souvent ingrat des tableaux balistiques, des tables de tir à angle élevé, et des étalonnages de fusée. Il apporta aussi des contributions significatives à la théorie mathématique sur laquelle les tables se fondaient. L'équipe de Pearson incluait des mathématiciens, des calculateurs et des dessinateurs. Les femmes en formaient un contingent important, en particulier parce que les demandes de plus en plus grandes de la conscription rendaient les hommes vulnérables aux sergents-recruteurs. Les obstacles ne manquèrent pas — Pearson n'appréciant pas que les calculs de son équipe soient remis en question par les mathématiciens produisant les données — et Hill eut parfois du mal à maintenir la paix.

1. INTRODUCTION

On 4 August 1914, the day Britain declared war on Germany, Karl Pearson offered the services of his Laboratory staff at University College London to the Government, and from then on and for the duration of the War he spent much of his time working in support of the war effort.¹ Between August 1914 and July 1917, he produced unemployment charts for the Board of Trade, calculated the torsional strain in the blades of aeroplane propellers for the Royal Aircraft Factory, and calculated bomb trajectories for the Admiralty Air Department. Meanwhile in January 1917 he began working for the Ministry of Munitions on ballistic computations for anti-aircraft guns, as an aid to the Anti-Aircraft Experimental Section (AAES), and it is this work, which occupied him throughout the whole of 1917 and the early months of 1918, that is the subject of this article.²

¹ According to Pearson's account, written two years after the event, Pearson offered the services of the Laboratory on the 3 August 1914. K. Pearson to T.G. Foster (Provost of University College London) *Confidential Report on the Work of the Laboratory Staff*, 23 February 1916. [Pearson Papers, 600] But Pearson's son, Egon Pearson, gives the date as the 4 August which is more likely. [Pearson 1938, 85]

That Pearson responded so immediately to the declaration of war may have been to leave no doubt about his patriotism. (During 1879/1880 Pearson had spent a year at university in Germany and had developed a strong attachment to German culture, and it was during this period that he started to spell his first name with a K instead of a C, possibly to reflect this attachment. [Haldane 1957, 304])

For biographical information about Pearson, see [Porter 2004], which largely focuses on Pearson's personal and intellectual development, and [Pearson 1938]. For discussions of Pearson's scientific work, see the many publications of M.E. Magnello, for example [Magnello 2009], [Magnello 1999], [Magnello 1998], [Magnello 1994].

² For discussions of WW1 ballistic practices outside Britain, see [Aubin & Goldstein 2014] (in particular [Aubin 2014] and [Nastasi & Tazzioli 2014]), [Gluchoff 2005; 2011], [Grier 2001].

2. FORMATION OF THE ANTI-AIRCRAFT EXPERIMENTAL SECTION

On 8 September 1915 Adelaide Davin, one of Karl Pearson's computing assistants at University College London (UCL), saw a Zeppelin for the first time. It was a momentous event and she gave Pearson a vivid description:

The whole of London is in a state of subdued excitement today as a result of the raid last night. We are all congratulating ourselves that we have seen a Zeppelin at last [...] from all accounts the damage appears to have been greatest just at the back of the College. A bomb fell in the centre of Queen's Square [...]. I was coming home in a tram just before 11 o'clock when the driver called out that there had been a Zep, and that it had been fired at twice—then the tram stopped, and the lights went out, whereupon several women began to shriek. I got out walked home to find all the neighbours in the street gazing heavenwards. Nobody obeyed the instructions to seek shelter. We could see the flashes of the anti-aircraft guns, but they all went very wide of the mark.³

The Zeppelin raid was the worst in London during 1915, not only in terms of the number of people killed and injured, but also with regard to the damage to property.⁴ But the enemy escaped unscathed. This was not an isolated incident. Zeppelins, despite being large and slow targets, were difficult to bring down. The theory and practice of anti-aircraft gunnery was in the early stages of development—the British, for instance, did not have any anti-aircraft guns until 1914—and anti-aircraft guns were not yet reliable. But as the Germans took to the air in increasing numbers, for reconnaissance and for bombing, in Zeppelins and in aeroplanes, the need to improve air defences became ever more pressing.⁵

In early 1916, in response to this need, the Ministry of Munitions⁶ set up the AAES within the Munitions Inventions Department (MID),⁷ and appointed as its head A.V. Hill, a physiologist who had graduated as third

³ A. Davin to K. Pearson, 9 September 1915. [Pearson Papers, 674/9]

⁴ 20 people were reported killed and 86 injured, and the damage to property was estimated at half a million pounds. *The Times* 11 September 1915. The first Zeppelin attack of the war took place over Norfolk on 19 January 1915. [Hogg 1978, 33–37]

⁵ There was also a problem dealing with Zeppelins from the air. A simple penetration of their fabric was not enough to destroy them and it was only after the development of the incendiary bullet that Zeppelins were brought down on English soil, the first such event occurring only days before the raid witnessed by Miss Davin.

⁶ Within three years of its formation, the Ministry of Munitions had become the "largest government department the country had ever seen with as many as 1,600,000 men and 800,000 women employed on protected munitions work." [Hazelhurst 1919–1922]

⁷ The leading figures in the formation of the AAES were Horace Darwin, founder of the Cambridge Scientific Instrument Company, and the electrical engineer Sir Alexander Kennedy. For further details, see [Barrow-Green 2014, 89–92].

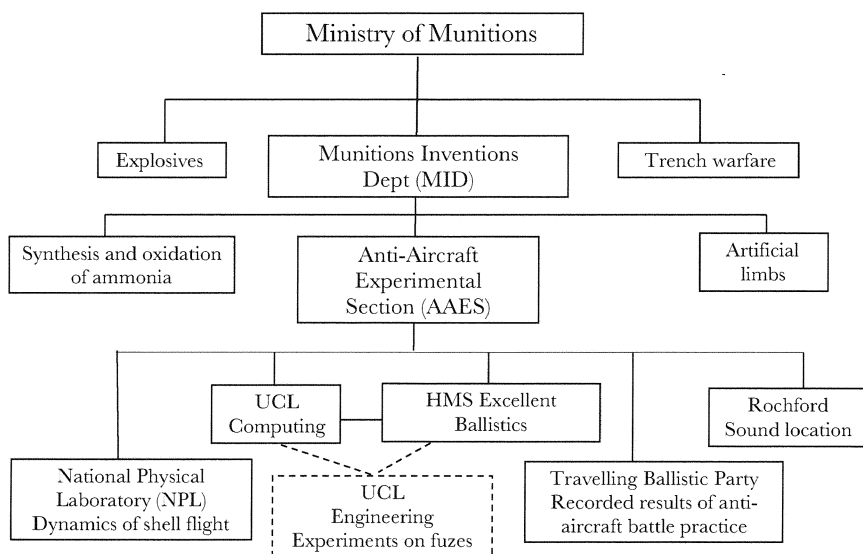


FIGURE 1. The structure of the Ministry of Munitions showing the divisions of the AAES.⁸ The personnel of the AAES were stationed at different locations according to the type of work, the main centre of activity being the naval proving ground at HMS *Excellent*.⁹

wrangler in the Cambridge Mathematical Tripos of 1907, and who was then serving as Brigade Musketry Instructor in the Infantry.¹⁰

In assembling his staff, Hill sought advice from G.H. Hardy, whom he knew from his time as a student at Cambridge. He also asked Hardy if he himself would be willing to join the group. Hardy was emphatic in his response, as Hill later recalled:

⁸ The structure of the Ministry of Munitions, and how the Munitions Inventions Department related to other Inventions Departments, is described in [Pattison, 1983, 529–531].

⁹ During the nineteenth century HMS *Excellent* was the name given to the succession of ships in Portsmouth Harbour which served as the Royal Navy's school of gunnery. In order to accommodate the increase in the range of guns and in the number of men requiring training, HMS *Excellent* moved ashore in 1891. For the early history of HMS *Excellent*, see [Lloyd 1955].

¹⁰ For an autobiographical account of Hill's involvement with the A.A.A.S., see [Hill 1970]. After the War, Hill went on to have a very distinguished career as a physiologist, winning the 1922 Nobel Prize for medicine for his discovery relating to the production of heat in the muscle. He continued to devote a large part of his life to public service, and in 1935 was part of the committee responsible for setting up radar in Britain.

He [Hardy] was always an odd fish and I remember him expressing great indignation and saying that although he was prepared to go off and have his body shot at he was not prepared to prostitute his brains for the purposes of war.¹¹

Nevertheless, Hardy did recommend R.H. Fowler, a research student in pure mathematics convalescing from wounds sustained at Gallipoli, and E.A. Milne, one of his best undergraduate students. They were joined by the algebraic geometer H.W. Richmond, a university lecturer of long-standing. After some false starts, the group eventually settled at the naval gunnery school at HMS *Excellent*, Whale Island, Portsmouth.¹² As the war progressed, the group increased in size but it was the three Cambridge men—Fowler, Milne, and Richmond—who took the lead in the research.¹³ Their investigations led in three directions: the general theory of anti-aircraft gunnery (including experimental work), some branches of ordinary gunnery (including the effects of winds and the dynamics of shell flight), and problems relating to the detection of aircraft by sound.¹⁴

While stationed at HMS *Excellent*, Milne wrote several letters to his younger brother Geoffrey telling him about life as a member of the AAES. These letters included vivid (and at times rather hair-raising) descriptions of "joyrides" in aeroplanes, as well as mathematical problems and commentaries on issues of public concern relating to anti-aircraft defences.¹⁵ In a letter of 30 October 1917, he gave a clear description of the general activities of his section of AAES:

¹¹ A.V. Hill to W.H. McCrea, 2 November 1950. [Milne Papers, 38] Hardy did volunteer for service under the Derby scheme but was rejected on medical grounds. [Hardy 1967, 39] The Derby Scheme—so-called because it was promoted by Lord Derby, an active proponent of conscription—was a personal canvass of every man on the Electoral Register. The Scheme began on 21 October 1915 and ended on 11 December 1915 with the realisation that over half a million men had refused to fill in the form.

¹² Later some members of the group were based at the National Physical Laboratory at Teddington, Middlesex. The group also had a 'travelling circus' which toured military positions in England and France. [Barrow-Green 2014, 91–92]

¹³ The group became known collectively as Hill's Brigands. [Weston-Smith 1990, 243] For information about other members of the group, which included, for example, William and Douglas Hartree, see [Barrow-Green 2014, 90–91]

¹⁴ For a detailed list of the functions of the AAES, as detailed in a report written by Hill in 1918, see [Barrow-Green 2014, 97–98].

¹⁵ The letters are preserved in the [Milne Papers]. After the war, Milne changed from pure to applied mathematics, making a career in astrophysics. He returned to ballistics in WW2 and found the subject again to his liking: "Hardy's view that ballistics is a poor subject was very much mistaken. I have derived intense aesthetic satisfaction from 3 or 4 of my recent ballistics papers—good, solid, satisfying stuff." E.A. Milne to Herbert Dingle, 29 November 1941. [Milne Papers, D560]

Our own section is not concerned with executive action, personnel, gun drill etc, we suggest the principles of designing new guns, new fuzes, new methods of fire, and point out the faults in the present ones. But we are a full 9 months ahead of what is in practice. The height finders we played about with at Teddington last summer have this summer been installed all over England. We tell the authorities that new anti-aircraft guns should have such and such muzzle velocities, project such and such weights of shell, have certain sighting systems and arrangements of mounting; the designing and manufacture of the first pair (experimental pair) of such new guns take four or five months.¹⁶

What is striking, although not altogether surprising, is the length of delay between recommendation and implementation. Nine months is a significant time-lag when considered in the overall context of the length of the War.

Further points made by Milne in the same letter concern the actual use of the guns. Every time a gun is fired it wears the barrel, hence after a certain number of rounds the gun will be worn out. Thus it makes sense only to fire when you believe you have a reasonable chance of success. Or does it? As a result of the increasing number of Zeppelin raids, it was decided to fire the guns continuously at night, not with the aim of actually hitting anything but rather to reassure the public that they were being adequately defended. While this policy may have fulfilled its purpose, it was not without its cost since replacing the worn-out guns was both difficult and expensive. Milne also mentioned to his brother that the AAES had demonstrated that a certain type of gun currently being used for anti-aircraft defence was in fact useless for that purpose, and that what was needed was a gun with a much lower muzzle velocity. Such a gun would last much longer with consequent savings to the War Office.

3. PEARSON AND THE AAES

How did Pearson get involved with the AAES? As mentioned above, Pearson began the War working for the Board of Trade but in the summer of 1915 he lost a number of members of his staff for other war-related work and the work was discontinued. He set about rebuilding his staff and in December 1915 offered their services to the Admiralty who handed the offer on to the Ministry of Munitions but, as he related in a Confidential Report, on this occasion his offer was not accepted:

¹⁶ E.A. Milne to G. Milne, 30 October 1917. [Milne Papers, A8] In the same letter Milne explains why the public outrage at the failure of the anti-aircraft defences to shoot down any of the 11 Zeppelins which had attacked London on 19 October, was completely misplaced. [Barrow-Green 2014, 92].

We consisted then [January 1915] of four men and six women, of whom one of the former and two of the latter were not on the regularly paid staff. My view then was and now is that the Laboratory can do far better work nationally as a whole than scattered, as it is trained to work together. However, I lost four members for special war work in the summer of 1915 ... I started therefore after July 1915 to rebuild my Laboratory staff and commenced training it afresh. By December 1915, I thought we were strong enough with three men and four women to offer ourselves for Government service again. I offered ourselves as a working whole [K.P. emphasis] first to the Admiralty who handed on the offer to the Ministry of Munitions. The Ministry of Munitions did not accept the offer.¹⁷

Striking in this account is the importance placed by Pearson on keeping his staff together. Pearson saw his staff as a single entity, not as a group of individuals. They could do "far better work nationally as a whole than scattered" because they were trained to work together. This emphasis on group working was to become a recurrent theme in Pearson's dealings with the Ministry.

As 1916 progressed with no end of hostilities in sight, Pearson began to despair of ever getting life in his laboratories back to normal. As he wrote in May that year:

... the war seems to have come as a cloud of poison and gas choking all one's usual activities and I don't feel that I shall ever have the energy to build up a new staff, fit up a big laboratory, and take up old treads again, should the war ever end.¹⁸

The following month he started work for the Royal Aircraft Factory and the Admiralty Air Department. And then in December 1916 he heard from his friend at Cambridge, the applied mathematician W.H. Macaulay, that Hill would be glad of his help with anti-aircraft gunnery work.¹⁹ He wrote to Macaulay offering his services, Macaulay forwarded the letter to Hill and Hill wrote to Pearson snapping up the offer.²⁰ Since the work Hill's group was doing on fuzes was new (for anti-aircraft fire, unlike flat fire, it is vital

¹⁷ K. Pearson *Confidential Report on the Work of the Laboratory Staff*, 23 February 1916. [Pearson Papers, 600]

¹⁸ K. Pearson to Mrs Weldon, 23 May 1916, quoted in [Magnello 2009, 22].

¹⁹ K. Pearson. *Journal of the Galton Laboratory*, 25 May 1918, p.105. [Pearson Papers, 246] Pearson also may have been encouraged to take up the work with Hill by Alexander Kennedy (see footnote 7). Pearson had met Kennedy in 1883 when he, Pearson, began lecturing at UCL and it was Kennedy who, in 1884, had encouraged him "to give up law and return to mathematics", although in truth he needed little encouragement. Pearson had also been very impressed by Kennedy's science teaching and when in 1903 he founded the Biometric Laboratory he modelled it on Kennedy's engineering laboratory. [G. Udny Yule, Louis N.G. Filon 1936, 74–75]

²⁰ A.V. Hill to K. Pearson, 15 December 1915. [Pearson Papers, 606] The content of the letter is reproduced in [Barrow-Green 2014, 93].

to know when a shell-burst is going to occur) and had been developed by his team at Whale Island, Hill suggested that he should send members of his staff up to London so that Pearson could hear first-hand what work they were doing. Hill was not only eager to liberate his own men from the grind of laborious computations, but also immediately saw the value in having a group of computers who were trained to work together. With Pearson's staff on board, Hill's men would be free to perform more experiments and to expand their research to include additional factors such as meteorological effects.

Two weeks later Hill wrote to Pearson again to let him know that Milne would be coming to see him the following afternoon.²¹ He also mentioned that the Ordnance Committee at Woolwich were very keen to make use of him and his computers, asking him if he would be happy if J.E. Littlewood,²² as well as Milne, brought him some work. He went on:

In the meantime [Littlewood] and Douglas have been discussing the matter of what most needs doing, and so that we may not overlap he is going to make arrangements with Milne finally tomorrow as to what we are to do, and what they are.²³ Once you get started I expect you will tackle our work pretty quickly and will be able to do their long range naval guns as well. Though I think you will be busy until the end of the war now! Milne is a nice little modest person, very clever and rather shy: he tells me he knows your son very well. He will be very grateful indeed for your kind offer of hospitality. I have told him.

I hope I may come up some time this week and see how things are going on.

With regard to Goudie's letter, I think it will be best if I come up to the MID this week, see the Committee and tell them what Goudie can do.²⁴ It will be splendid if he can make these experiments at 30 000 revs.—I shall have to talk to Woolwich about it but I hope very much something may be done. It is astonishing that this speed can be reached, but I heard 3 days ago that they revolve the parts of a mechanical fuze about 500 times a second to "grind it in".

It will be a great help to us getting rid of a lot of the computation, and will give us more time for what is our proper function here—the collection of accurate data, and the investigation of the problems arising. From time to time also I expect you will help us with the problems as well as the arithmetic.²⁵

²¹ A.V. Hill to K. Pearson, 31 December 1915. [Pearson Papers, 720/3] The content of the first part of the letter is reproduced in [Barrow-Green 2014, 93].

²² Littlewood, who was working for the Ordnance Committee, was a Lieutenant in The Royal Garrison Artillery, the unit responsible for the large calibre guns positioned behind the front line. He was appointed Assistant Ballistic Officer to the Ordnance Committee.

²³ R.V. Douglas, Ordnance Committee Office, Woolwich.

²⁴ W.J. Goudie is discussed below.

²⁵ A.V. Hill to K. Pearson, 31 December 1916. [Pearson Papers, 720/3]

Since the AAES work was only just getting underway, the knowledge Milne could convey to Pearson was rather "narrow", as Pearson later recalled. But the visit marked the starting point of the collaboration, the main object of which was "to draw charts of the trajectories of the antiaircraft naval and military guns".²⁶

Hill and Littlewood, having been fellow undergraduates at Trinity College, were already well known to one another. Pearson was happy to work with Littlewood too, and although it is likely that as fellow mathematicians they were already at least acquainted, there was also a direct connection between them through Frances Cave-Brown-Cave, a lecturer at Girton College, who had worked for Pearson before the War and with whom Littlewood was already working on ballistics calculations. Although not a member of the AAES, Littlewood worked closely with members of the group and provided a link between them and the ballisticians at Woolwich.

W.J. Goudie was based in the Engineering Laboratory at University College and it was on Pearson's suggestion that he became funded by the MID to do experiments on fast-spinning fuzes for the AAES and the Ordnance Committee at Woolwich.²⁷ The problem of determining the pressure inside a fuze during flight was taxing both the AAES and Leonard Bairstow and his staff at the National Physical Laboratory,²⁸ and Pearson's intervention in this respect was particularly welcome.

The ballistics work Pearson and his staff undertook for the AAES took place at the Biometric Laboratory, the Laboratory sponsored by the city livery company, the Worshipful Company of Drapers. However, the Biometric Laboratory wasn't Pearson's only place of activity. Pearson was also running the Galton Eugenics Laboratory where his work on matters connected with studies in degeneration, which involved the construction of family pedigrees and the use of actuarial death rates, was being done. Although during this period the Biometric Laboratory acted as a training

²⁶ K. Pearson. *Journal of the Galton Laboratory*, 25 May 1918. [Pearson, 246]

²⁷ W.J. Goudie later became Professor of Theory and Practice of Heat Engines at Glasgow University.

²⁸ A.V. Hill to K. Pearson, 7 January 1917. [Pearson, 606]

ground for the Galton Laboratory²⁹ and members of staff from both Laboratories were engaged in war work, the two laboratories were funded independently and they occupied different physical spaces.³⁰ That all the ballistics work should take place in the Biometric Laboratory was quite natural since this was the Laboratory in which the more mathematical work was done and consequently where the calculating machines were located.

Pearson's staff employed on AAES work included several research students: Lindsey Ince and Adolph Zaiman from Cambridge,³¹ Andrew Young, a research student from E.T. Whittaker's mathematical laboratory at Edinburgh University,³² and Arthur Doodson, a graduate from Liverpool University and a gifted computer. There were also several female computers including Adelaide Davin, who had earlier helped Pearson with his work on calculating bomb trajectories, and Ethel Elderton who had joined UCL as Francis Galton's assistant in 1905 working on population statistics, and who had been working with Pearson since Galton's death in 1911.³³ As time marched on and the work became more intensive the Ministry of Munitions increased Pearson's staff, paying for new members, male and female, in both computing and draughtsmanship.

Doodson was unusual among Pearson's staff in that he was a conscientious objector against war service. When he joined Pearson's staff in the summer of 1916 he stated that he would refuse to do war work per se but that he was prepared to "acquiesce in war work incidental to [his] ordinary occupation", adding that he had recently refused a position created for him because it was "concerned purely and simply with military matters".³⁴ In December 1916 he agreed to do war work for Pearson because he had

²⁹ K. Pearson *Report to the Court of the Worshipful Company of Drapers on the Present Position and Past History of the Laboratories to which their Annual Grant has been made*, February 1918. [Pearson Papers, 233]

³⁰ As Eileen Magnello has shown, the fact that the two Laboratories were physically separated and independently funded was significant with regard to their individual modes of operation. [Magnello 1999].

³¹ See [Barrow-Green 2014, 96] for more information on Ince and Zaiman.

³² Young joined Pearson's staff in 1915 as an assistant in the Statistical Department at UCL. As a student in Whittaker's laboratory he was trained in practical computation (interpolation, numerical integration, practical harmonic analysis, etc.) and was described by Whittaker as 'a good lecturer' who 'can keep a class of Scotch students in order (not everybody can do this)'. E.T. Whittaker to K. Pearson, 21 June 1915. [Pearson Papers, 246]

³³ Ethel Elderton had a long career in the Galton laboratory, retiring in 1933 as an Assistant Professor. [Love, 1979]

³⁴ A. Doodson to K. Pearson, 6 June 1916. [Pearson Papers, 676/5]

undertaken to work for him and he was appreciative of the fact that Pearson had kept him from doing war work thus far.³⁵ While Doodson seems to have been alone in his views among the members of Pearson's laboratories, a similar stance was adopted by a number of mathematicians in Cambridge.³⁶

4. MATHEMATICAL BALLISTICS FOR HIGH-ANGLE FIRE³⁷

Prior to the First World War the study of external ballistics—the calculation of projectile trajectories—had been almost exclusively concerned with problems connected to firing flat at an effectively stationary target, the fundamental problem being to model the atmospheric drag on the moving projectile. But firing up at targets moving fast and through the air (moving in three dimensions) presented completely new and much more complicated problems. For flat fire only one point—the point of fall—is of paramount importance. For anti-aircraft gunnery every point of the arc of the trajectory should be known and the flight time is critical (since the time fuze has to be set). But that is not all. For anti-aircraft gunnery there are several additional factors to take into account. These include varying wind speed, varying air density, speed of aircraft, rapidity of manoeuvre of aircraft, height of aircraft, lack of visibility of aircraft, magnitude of unknown errors affecting gunfire at great heights, and the speed with which it is necessary to carry out the firing.³⁸

In more technical terms, external ballistics is defined as 'the determination of the motion of a heavy body in resisting medium under the action of gravity and the forces due to its motion in that medium'.³⁹ What adds substantially to the complications for anti-aircraft gunnery is that the density of air, since it varies with height, needs to be taken into account (for flat fire it can be considered as constant).

³⁵ A. Doodson to K. Pearson, 22 December 1916. [Pearson Papers, 676/5]

³⁶ For a discussion on pacifism among Cambridge mathematicians, see [Barrow-Green 2014, 69–78].

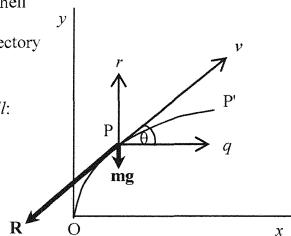
³⁷ The account in this section is based on [Richmond 1924–1925].

³⁸ For detailed information on the theoretical and experimental advances in anti-aircraft gunnery made by the AAES during the war, see the two volume work [Richmond 1924–1925]. These two volumes, the information in which was restricted to those 'holding an official position in His Majesty's Service', were edited (anonymously) by H.W. Richmond and contain a list of those engaged in the work described.

³⁹ Internal ballistics is concerned with the behaviour of the projectile from the moment of firing until the projectile leaves the gun barrel. Terminal ballistics is the study of the effect of the shell on hitting the target.

OPP' – trajectory of shell
 v – velocity of shell
 θ – inclination of trajectory

Forces acting on shell:
 R – air resistance
 mg – weight



$$R = kd^2 \rho(y) F(v)$$

m – mass of shell
 k – coefficient of reduction
 (effect of shape of shell)
 d – diameter of cylindrical portion of shell
 $\rho(y)$ – actual density function
 $F(v)$ – resistance-velocity function for given shell
 $f(y)$ – standard density function

$$\text{Ballistic coefficient } C = \frac{m}{kd^2}$$

Equations for velocity components used for calculation of trajectories (hodograph):

$$\frac{dq}{dt} = -\frac{R}{m} \cos \theta = -\frac{F(v)}{Cf(y)} \cos \theta$$

$$\frac{dr}{dt} = -\frac{R}{m} \sin \theta - g = -\frac{F(v)}{Cf(y)} \sin \theta - g$$

FIGURE 2. Primary Ballistic Problem⁴⁰.

External ballistics can be broken down into two parts:

1. The primary ballistic problem: The determination of the plane trajectory of a given shell in still air, assuming certain relations between resistance and velocity, and between density and height, and neglecting all effects of curvature and rotation of the earth.

2. The secondary ballistic problems:

(i) The errors caused by assumptions as to the shell and the types of air forces acting on it leads to theory and practice of 'twisted trajectory'.

(ii) Effects of air in motion with respect to the earth, and of variations of the resistance-velocity and density-height relations.

(iii) Effects of curvature and rotation of the earth.⁴¹

As shown in Figure 2, the motion of the shell for the primary ballistic problem can be described by a set of differential equations. A point to notice is that the ballistic coefficient, $C = m/kd^2$, the element of the equations which depends on the form of the shell, is a rather primitive notion. For a shell of a given size moving nose-on with constant velocity, the resistance, and therefore the variable k , depends principally on the shape of the head and of the base of the shell, and the ratio of the length of the cylindrical part of the body of the shell to the diameter of the shell, but the exact

⁴⁰ The diagram is taken from [Richmond 1924–1925, 111].

⁴¹ The secondary effects are so-called since their effect on the overall trajectory is relatively little compared to the primary.

nature of this dependence is complicated and not known exactly. In 1919 Forest Ray Moulton, who worked as a ballistician in the United States during the war, aptly summarised the issue when he remarked somewhat critically that the ballistic coefficient “had to carry heavy burdens” in its need to encompass all knowledge of the shell in a single number. [Gluchoff 2011, 516]

Since the differential equations (hodograph equations) cannot be solved exactly, because $F(v)$ is usually unknown, some approximate method of solution must be used for the calculation of any given trajectory. [Charbonnier 1928, 573]

Essentially there are three general methods of solution. One involves calculating each point on the trajectory independently of any other, by making use of the elements of the point at the origin and possibly some of those at the point required only. This method is suitable for constructing flat-fire range tables since in this case the point of fall is the only point on the trajectory for which the elements are required. It is not suitable for high-angle fire since there are too many points to calculate, although it has the advantage that an undetected error affects only one point. For flat fire, the theoretical work was done by Littlewood who refined the method of Francesco Siacci, [Siacci 1892] working out a series of approximate solutions for the elements of the trajectory at the point of fall (Littlewood-Siacci equations). [Richmond 1924–1925, 117] Littlewood wrote up his results in [Littlewood 1917, 1–7], later describing them in [Littlewood 1972] where he showed explicitly that the error in his range estimates were of $O(\phi^6)$ compared to that of Siacci of $O(\phi^4)$, where ϕ is the initial angle of inclination.

A second method involves integrating the equations numerically using a step-by-step (‘small arcs’) process, but the calculations at the end of one step affect those at the beginning of the next step. Thus the size of the interval must be sufficiently small for the accumulated error to be negligibly small for the accuracy required. And the third method involves calculating the trajectory by interpolation with previously calculated trajectories, although for best results the interpolations need to be performed on a quantity that varies slowly and slightly, rather than on one that varies rapidly or over a wide range. [Richmond 1924–1925, 125]

As noted in [Richmond 1924–1925, 287], the earliest methods of interpolation, which were exclusively graphical, were inadequate. The drawing process could not help but introduce errors, and the use in one drawing of data obtained from another drawing served to increase the inaccuracy. Furthermore, the final tables, and often the intermediary tables, had to

be smoothed, with an added increase in workload. However, by the time Pearson's computers faced the problem of calculating trajectories, graphical methods of interpolation had been superseded by numerical ones far superior in accuracy.

Initially Pearson and his staff worked with Littlewood's interpolation method.⁴² [Littlewood 1917, 8–16], [Richmond 1924–1925, 237–258], [Littlewood 1971]. This method has the advantage of requiring only two known trajectories, one of which being the simple case of vertical fire. But it can only be used up to a certain value of the time⁴³ after which the errors of the formulae increase to a point beyond which they cannot sensibly be ignored and the rest of the trajectory has then to be calculated by a small arcs method. Nevertheless, at the beginning of the trajectory Littlewood's formulae are easier to use than a small arcs method.

However, when it came to compiling high-angle range tables it was found that Littlewood's method was not sufficiently accurate. Pearson, bringing his mathematical skills into play, found a remedy for the situation.⁴⁴ He extended Littlewood's method by using five (rather than two) trajectories calculated initially and prepared formulae for the purpose of interpolation.⁴⁵ Although the calculation of the coefficients in Pearson's formula was laborious, it had the advantage that once obtained the coefficients could be used for a large number of calculations.

The interpolation involved several different processes, each one providing data for one or more of the others. First the ranges and mean heights (or trajectory data) for each quadrant elevation (initial angle of inclination of the gun—the only variable for a 'given gun') are calculated. In the next stage the time contour sheets—the sheets which give the calculations

⁴² [Richmond 1924–1925] gives details of various methods for calculating a plane trajectory with any given initial conditions, including both those with and those without time as the independent variable. As well as the methods used by the AAES, the former also included the methods of Moulton and Bennett (see [Gluchoff 2011]), while the latter included the methods of Mayevski, Bianchi, the *Méthode de Gavre* (see [Aubin 2014]) and the method of A.E.H. Love, the last two of which used angle as the independent variable. Love's method was the method generally used in England before the change to time as the independent variable.

⁴³ Approximately equivalent to the time of flight on the flat at 20° quadrant elevation. [Richmond 1924–1925, 237]

⁴⁴ Pearson was a strong mathematician. He graduated from Cambridge as third wrangler in 1879, and at UCL he held the Goldsmid Chair of Applied Mathematics and Mechanics from 1884 until 1911 when he was appointed the first Galton Professor of Eugenics and founded the department of applied statistics.

⁴⁵ For a detailed explanation of Pearson's method see [Richmond 1924–1925, 258–262].

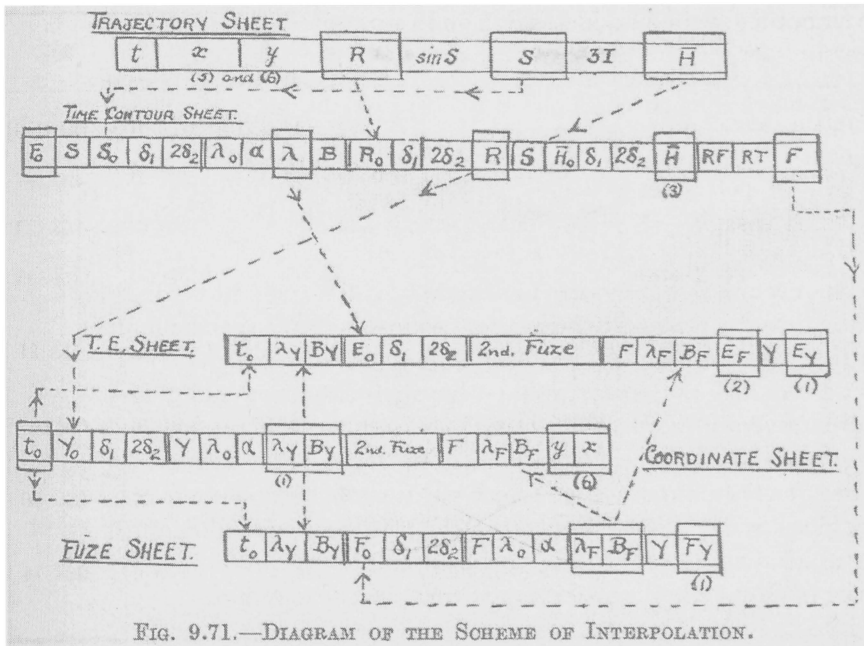


FIG. 9.71.—DIAGRAM OF THE SCHEME OF INTERPOLATION.

FIGURE 3. Diagram of the Scheme of Interpolation.

from quadrant elevations (in multiples of 5°) to clear angles of sight (in multiples of 5°)—are prepared. This information is then used to prepare the tangent elevation sheets, the fuze sheets (which contain the calculations for the fuze settings), and the coordinate sheets which contain both the calculations for the times corresponding to heights (in multiples of 500 feet) and the value of heights corresponding to particular fuzes.⁴⁶

Figure 3, which shows how the different sheets were connected, is a good illustration of the value of having a team of computers working together in one location.

The tables required were:

- (1) A height table, giving values of the tangent elevation, fuze setting and time of flight for clear heights (multiples of 500 feet) on clear angles of sight (multiples of 5°);
- (2) A table of tangent elevation for clear (or integral) fuze settings and to clear angles of sight;

⁴⁶ For further details, see [Richmond 1924–1925, 297–313]. Figure 3 is taken from p. 313.

- (3) A table of mean heights for clear angles of sight and for integral seconds of time;
- (4) A horizontal range table, giving quadrant elevation, time of flight and fuze settings for clear ranges (multiples of 200 yards) on the flat;
- (5) A table of coordinates of trajectories at integral time for clear quadrant elevations (multiples of 5°);
- (6) A graphic range table showing trajectories at every 5° interval of quadrant elevation, and also showing integral fuze setting contours. [Richmond 1924–1925, 282–283]

However, before the range tables can be calculated, the settings for the time fuze—the device intended to explode a shell at a specific moment after firing—have to be considered. Although there is no significant chance of a direct hit—the goal is to burst the shell close to the target—it is important to know (accurately) the behaviour of the fuze in order to be able to calculate the settings for the shell to burst at a given point in the plane of fire. If a shell bursts $\frac{1}{2}$ second early, that probably equates to 300 or 400 feet in distance.⁴⁷ The main factors which affect the burning of the (powder train) fuze are pressure, spin and temperature, and these were investigated by Goudie in his experiments at University College.

5. WORK AT THE BIOMETRIC LABORATORY

The *Fortnightly Reports of the AAES* prepared by Hill give a good idea of how hard and quickly Pearson and his team worked in preparing the tables. In these *Reports* Hill describes the activities of the various departments of the AAES, listing the personnel in each department, what they have achieved, and what he hopes they will achieve. In the *Report* for 5 March 1917 he detailed the activities of the “Computing Branch, University College”, and was unequivocal about the need for human computers to complete the work of his men at Whale Island:

The following diagrams and tables have been calculated and prepared under the direction of Professor Karl Pearson FRS at the Galton Laboratory for the 3 inch Q.F. [quick-firing] 20cwt gun No 80 fuze. They have been most carefully done, and represent the highest accuracy ever attained in the ballistics of High Angle Gunnery, both from the accuracy of the experimental data, the exactness of the drawing, and the care expended in the calculation. ...

If Professor Pearson is able to continue this work, and to develop it, not only will High Angle fuze scales and trajectory diagrams be constructed for all Anti-aircraft guns, but it is hoped that his experience and the resources of his Laboratory in computing work, may be employed in the investigation of the effects of wind and other meteorological conditions on High Angle or Long Range fire.

⁴⁷ E.A. Milne to G. Milne, 30 October 1917. [Milne Papers, A8]

There is a large amount of computing attached to Experimental Work in any form of High Angle or Long Range Gunnery, and the Experimental (Ballistic) Branch is incapable of carrying this all out unaided. In the further development of the new methods, ideas and facts which have arisen, partly from the experience of the war and partly as the result of the work of this Section, it is believed that the co-operation of the Galton Laboratory will be invaluable.

3 inch Q.F. 20 cwt Gun No. 80 Fuze

Figure G.L. 1. Trajectory, time and fuze curves in space

Figure G.L. 2. Tangent elevation⁴⁸ against angle of sight for constant fuze setting

Figure G.L. 3. Tangent elevation against angle of sight for constant height and for constant fuze setting

Table G.L. I. Tangent elevations for given heights and quadrant elevations

Table G.L. II. Co-ordinates of trajectories at given times

Table G.L. III. Tangent elevations for given fuze-settings and angles of sight

Table G.L. IV. Tangent elevations and fuze settings for given angles of sight and heights

The list of diagrams and tables is for only one type of gun using only one type of fuze but each gun needed a similar number of range tables for each type of fuze. By 22 May 1917, 74 tables and 22 diagrams, including some for flat fire as well as high-angle fire, had been completed.⁴⁹ Hill had originally attached Pearson's initials to the tables but Pearson, ever mindful of the fact that the work was a team effort, had asked him to use G.L. (for Galton Laboratory) instead so that the numbering reflected the work of all his staff.⁵⁰ For the same reason Pearson also insisted on the Laboratory's name (rather than his own) being used in Hill's *Fortnightly Reports of the AAES*.⁵¹

The lower left hand portion of a typical graphic range table representing the vertical plane of fire, such as those produced by Pearson's laboratory, is shown in Figure 4.⁵² This table is for a 4 inch Q.F. Mark V gun with a muzzle velocity 2,350 feet/sec and a ballistic coefficient of 2.5, and a fuze no. 80, using shells of 6-calibre radius. The curves for constant quadrant elevation for constant fuze, and for constant time are plotted with horizontal range (0-15,000 yards) and height (0-30,000 feet) as abscissa and ordinate

⁴⁸ The tangent elevation is the quadrant elevation minus the angle of sight.

⁴⁹ R.H. Fowler to K. Pearson (enclosing a list of all the diagrams and tables), 23 May 1917. [Pearson Papers, 606]

⁵⁰ A.V. Hill to K. Pearson, 16 February 1917. [Pearson Papers, 606]

⁵¹ A.V. Hill to K. Pearson, 24 March 1917. [Pearson Papers, 606]

⁵² G.R.T. XXIV. [Richmond 1924-1925, facing p.534] The original table was drawn to a scale of one inch to 500 yards. The initials G.R.T. stand for Graphic Range Table, the earlier tables constructed in Pearson's laboratories bear the initials G.L. for Galton Laboratory.

respectively. The trajectories are shown for every 5° of quadrant elevation and the fuze curves are shown for every setting (set full = 22), while the time curves are shown only where they intersect the trajectories.⁵³ Figure 5 shows the top-right hand corner of the same table where the angles of sight can be seen marked at 5° intervals around the edge of the chart. Once the height and horizontal range of a target is known, then the angle of sight, the quadrant elevation and the fuze setting required to hit it can be read off from the table. For example, if the height of the target is 11,000 feet, and the horizontal range is 5,500 yards, then, reading from the table, the angle of sight is 33.8° , the quadrant elevation is 40° , and the fuze setting is 13.9.⁵⁴

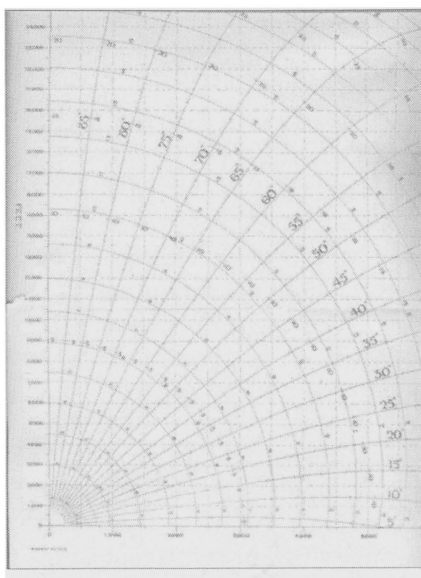


FIGURE 4. G.R.T. XXIV bottom left hand corner.

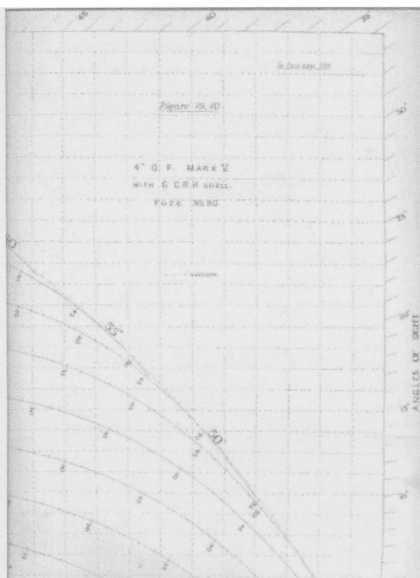


FIGURE 5. G.R.T. XXIV top right hand corner.

For range tables in tabular form, two tables were produced. Figure 6, which shows a page from one of them, is for a 12 pounder Q.F. 12 cwt

⁵³ The fuze time scale is arbitrary (i.e. not in seconds). Usually, as in the range table considered, the maximum setting was 22, which represented something between 40 and 60 seconds, depending on the type of fuze.

⁵⁴ The example is given in [Richmond 1924–1925, 534].

| | | Angles | | | | | | | | | | of sight. | | | | | | | | | | | |
|---------|-----------------|---------|---------|--------|--------|--------|--------|--------|--------|--------|--------|-----------|--------|--------|--------|--------|--------|--------|-------|-------|------|---------|--|
| Height. | | | | | | | | | | | | | | | | | | | | | | Height. | |
| | | 5° | 10° | 15° | 20° | 25° | 30° | 35° | 40° | 45° | 50° | 55° | 60° | 65° | 70° | 75° | 80° | 85° | V.F. | | | | |
| 500 | T. E. ... | 1° 55' | 0° 38' | 0° 24' | 0° 10' | 0° 12' | 0° 10' | 0° 9' | 0° 7' | 0° 6' | 0° 5' | 0° 4' | 0° 3' | 0° 3' | 0° 2' | 0° 1' | 0° 1' | 0° 0' | 0° 0' | 0° 0' | 500 | | |
| | P. (No. 80) ... | 4-97 | 2-30 | 1-09 | 1-20 | 1-01 | 0-88 | 0-80 | 0-74 | 0-69 | 0-65 | 0-63 | 0-61 | 0-59 | 0-58 | 0-57 | 0-56 | 0-55 | 0-55 | 0-55 | | | |
| | H. (yds.) ... | 1912 | 950 | 644 | 487 | 394 | 335 | 291 | 259 | 236 | 218 | 203 | 192 | 184 | 177 | 173 | 169 | 167 | 167 | 167 | | | |
| | T. (secs.) ... | 8-48 | 1-50 | 0-96 | 0-78 | 0-58 | 0-49 | 0-42 | 0-37 | 0-33 | 0-31 | 0-29 | 0-27 | 0-26 | 0-25 | 0-24 | 0-24 | 0-23 | 0-23 | 0-23 | | | |
| 1000 | T. E. ... | 4° 44' | 1° 34' | 0° 55' | 0° 32' | 0° 38' | 0° 22' | 0° 18' | 0° 15' | 0° 13' | 0° 11' | 0° 9' | 0° 7' | 0° 6' | 0° 4' | 0° 3' | 0° 2' | 0° 1' | 0° 0' | 0° 0' | 1000 | | |
| | P. (No. 80) ... | 10-28 | 4-77 | 3-11 | 2-34 | 1-89 | 1-62 | 1-41 | 1-28 | 1-18 | 1-10 | 1-04 | 0-99 | 0-97 | 0-92 | 0-91 | 0-90 | 0-88 | 0-88 | 0-88 | | | |
| | H. (yds.) ... | 3895 | 1920 | 1288 | 975 | 789 | 667 | 581 | 519 | 471 | 435 | 407 | 386 | 368 | 355 | 345 | 338 | 335 | 333 | 333 | | | |
| | T. (secs.) ... | 8-77 | 3-45 | 2-10 | 1-38 | 1-21 | 1-00 | 0-80 | 0-77 | 0-68 | 0-63 | 0-59 | 0-55 | 0-53 | 0-51 | 0-49 | 0-48 | 0-46 | 0-46 | 0-46 | | | |
| 1500 | T. E. ... | 8° 49' | 3° 1' | 1° 34' | 1° 2' | 0° 46' | 0° 35' | 0° 29' | 0° 23' | 0° 20' | 0° 16' | 0° 13' | 0° 11' | 0° 9' | 0° 7' | 0° 6' | 0° 5' | 0° 4' | 0° 3' | 0° 2' | 1500 | | |
| | P. (No. 80) ... | 16-16 | 7-30 | 4-74 | 3-51 | 2-82 | 2-34 | 2-08 | 1-85 | 1-69 | 1-57 | 1-47 | 1-40 | 1-36 | 1-30 | 1-25 | 1-22 | 1-22 | 1-22 | 1-22 | | | |
| | H. (yds.) ... | 6787 | 3279 | 1931 | 1462 | 1189 | 1005 | 872 | 778 | 707 | 658 | 610 | 577 | 549 | 522 | 518 | 508 | 502 | 500 | 500 | | | |
| | T. (secs.) ... | 15-64 | 6-94 | 4-48 | 3-46 | 2-46 | 1-91 | 1-57 | 1-34 | 1-26 | 1-16 | 1-08 | 1-00 | 0-95 | 0-82 | 0-79 | 0-76 | 0-75 | 0-74 | 0-73 | | | |
| 2000 | T. E. ... | 12° 40' | 5° 25' | 1° 52' | 1° 6' | 0° 51' | 0° 40' | 0° 33' | 0° 28' | 0° 27' | 0° 22' | 0° 18' | 0° 15' | 0° 13' | 0° 9' | 0° 7' | 0° 6' | 0° 5' | 0° 4' | 0° 3' | 2000 | | |
| | P. (No. 80) ... | 20-72 | 10-72 | 6-47 | 4-78 | 3-77 | 3-10 | 2-72 | 2-44 | 2-23 | 2-05 | 1-91 | 1-82 | 1-76 | 1-69 | 1-63 | 1-60 | 1-50 | 1-47 | 1-47 | | | |
| | H. (yds.) ... | 9880 | 5576 | 3481 | 2577 | 1981 | 1577 | 1305 | 1182 | 1087 | 1003 | 930 | 870 | 814 | 770 | 736 | 709 | 690 | 677 | 669 | | | |
| | T. (secs.) ... | 20-72 | 8-84 | 5-08 | 3-62 | 2-70 | 2-20 | 1-87 | 1-64 | 1-45 | 1-31 | 1-25 | 1-17 | 1-12 | 1-08 | 1-05 | 1-02 | 1-01 | 1-01 | 1-00 | | | |
| 2500 | T. E. ... | 17° 6' | 8° 29' | 3° 8' | 1° 51' | 1° 2' | 0° 54' | 0° 43' | 0° 36' | 0° 30' | 0° 23' | 0° 19' | 0° 15' | 0° 13' | 0° 9' | 0° 7' | 0° 6' | 0° 5' | 0° 4' | 0° 3' | 2500 | | |
| | P. (No. 80) ... | 27-50 | 13-16 | 8-26 | 6-00 | 4-76 | 3-67 | 3-08 | 2-68 | 2-42 | 2-08 | 1-84 | 1-75 | 1-69 | 1-63 | 1-57 | 1-50 | 1-40 | 1-38 | 1-38 | | | |
| | H. (yds.) ... | 14799 | 8220 | 5457 | 3972 | 3067 | 2467 | 2086 | 1826 | 1679 | 1588 | 1517 | 1462 | 1419 | 1387 | 1361 | 1345 | 1327 | 1315 | 1309 | | | |
| | T. (secs.) ... | 27-50 | 12-12 | 6-22 | 4-54 | 3-48 | 2-80 | 2-46 | 2-18 | 1-90 | 1-75 | 1-60 | 1-50 | 1-44 | 1-38 | 1-34 | 1-31 | 1-29 | 1-28 | 1-28 | | | |
| 3000 | T. E. ... | 21° 54' | 10° 44' | 4° 44' | 2° 52' | 1° 59' | 1° 22' | 1° 3' | 0° 55' | 0° 45' | 0° 37' | 0° 30' | 0° 24' | 0° 19' | 0° 15' | 0° 11' | 0° 9' | 0° 8' | 0° 7' | 0° 6' | 3000 | | |
| | P. (No. 80) ... | 35-34 | 18-34 | 10-11 | 7-51 | 6-78 | 4-80 | 4-13 | 3-66 | 3-31 | 3-03 | 2-83 | 2-67 | 2-56 | 2-46 | 2-39 | 2-35 | 2-31 | 2-30 | 2-30 | | | |
| | H. (yds.) ... | 22953 | 12894 | 8294 | 5294 | 3896 | 2996 | 2496 | 2196 | 2000 | 1805 | 1655 | 1555 | 1485 | 1405 | 1325 | 1245 | 1165 | 1095 | 1090 | | | |
| | T. (secs.) ... | 35-34 | 15-38 | 8-34 | 6-07 | 4-56 | 3-66 | 3-07 | 2-60 | 2-36 | 2-14 | 1-98 | 1-86 | 1-77 | 1-70 | 1-65 | 1-61 | 1-59 | 1-58 | 1-58 | | | |
| 3500 | T. E. ... | 25° 59' | 13° 29' | 6° 14' | 3° 48' | 2° 32' | 1° 52' | 1° 28' | 1° 3' | 0° 55' | 0° 45' | 0° 37' | 0° 30' | 0° 24' | 0° 19' | 0° 15' | 0° 11' | 0° 9' | 0° 8' | 0° 7' | 3500 | | |
| | P. (No. 80) ... | 40-03 | 20-03 | 12-03 | 8-03 | 6-81 | 5-04 | 4-33 | 4-20 | 3-85 | 3-54 | 3-30 | 3-10 | 2-87 | 2-85 | 2-77 | 2-72 | 2-68 | 2-67 | 2-67 | | | |
| | H. (yds.) ... | 26719 | 14608 | 9411 | 5761 | 4111 | 2761 | 2331 | 2031 | 1816 | 1658 | 1534 | 1447 | 1387 | 1342 | 1298 | 1265 | 1242 | 1221 | 1217 | | | |
| | T. (secs.) ... | 40-03 | 20-24 | 11-12 | 7-22 | 5-62 | 4-48 | 3-74 | 3-28 | 2-87 | 2-59 | 2-39 | 2-23 | 2-13 | 2-04 | 1-97 | 1-92 | 1-90 | 1-88 | 1-88 | | | |
| 4000 | T. E. ... | 29° 56' | 15° 41' | 8° 8' | 4° 18' | 2° 48' | 1° 58' | 1° 45' | 1° 22' | 1° 6' | 0° 54' | 0° 44' | 0° 35' | 0° 28' | 0° 22' | 0° 18' | 0° 14' | 0° 10' | 0° 8' | 0° 7' | 4000 | | |
| | P. (No. 80) ... | 44-02 | 22-02 | 13-02 | 8-54 | 7-54 | 6-50 | 5-67 | 4-92 | 4-42 | 4-05 | 3-77 | 3-55 | 3-39 | 3-26 | 3-16 | 3-10 | 3-06 | 3-05 | 3-05 | | | |
| | H. (yds.) ... | 31537 | 18088 | 11588 | 7188 | 5188 | 3688 | 2888 | 2488 | 2288 | 2148 | 2008 | 1868 | 1748 | 1658 | 1588 | 1534 | 1488 | 1458 | 1453 | | | |
| | T. (secs.) ... | 44-02 | 22-24 | 13-12 | 9-02 | 7-27 | 5-87 | 4-86 | 4-28 | 3-80 | 3-47 | 3-23 | 3-03 | 2-80 | 2-61 | 2-41 | 2-32 | 2-26 | 2-22 | 2-21 | | | |

FIGURE 6. Range Table for 12 pounder QF, 12 cwt gun Mark 1, fuze 80.

gun, Mark 1, fuze 80.⁵⁵ This table tabulates tangent elevations, fuze settings, ranges and times of flight for clear heights and angles of sight. The second table gave the tangent elevation for each fuze setting and clear angles of sight.

These tables can then be used to construct, either by interpolation or by plotting curves, all other tables required, e.g. a table providing times of flight for integral fuze settings, and clear heights, which is the table required to construct a time rheostat dial. [Richmond 1924–1925, 536]

As noted above, accuracy of the tables was important. In April Fowler, following a suggestion from the Anti-Aircraft Equipment Committee, asked Pearson for “times and fuzes accurate to 1/100 sec or setting”.⁵⁶ However, given the uncertainty of the experimental data, this degree of accuracy was well beyond what could possibly have been required from a practical point of view, and Pearson queried Fowler’s request. Hill explained that such accuracy was needed because various inexperienced

⁵⁵ Published by the War Office, 2 August 1917. [Pearson Papers, 606]

⁵⁶ A.V. Hill to K. Pearson, 16 April 1917. [Pearson Papers, 606] A quotation from this letter is included in [Barrow-Green 2014, 98].

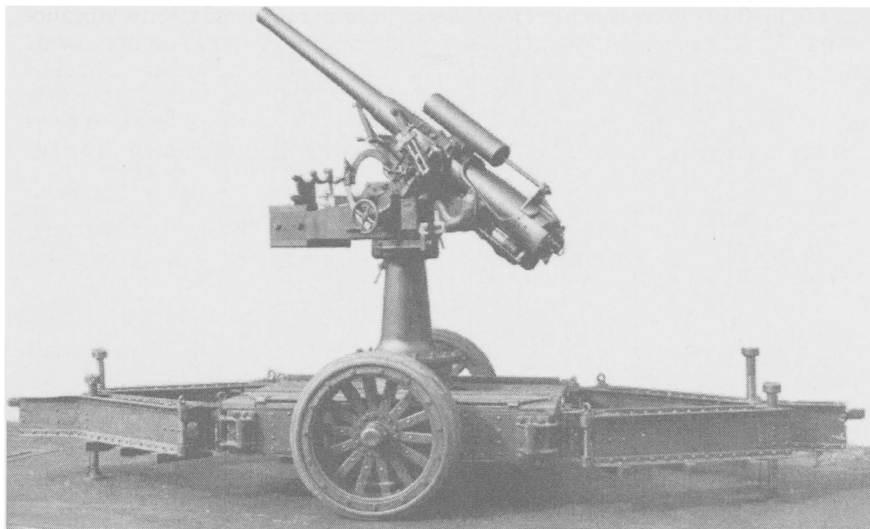


FIGURE 7. QF 12 pounder 12 cwt gun Mark 1* on high-angle anti-aircraft mount on travelling platform.

people wanted to plot the results and that in order to ensure consistency between them, the data needed to be smooth. Later Hill recalled that Pearson “complained sometimes when the gunners missed the Zeppelins by thousands of yards, after he had worked out his range tables to a hundredth of a foot.” [Hill 1960, 92] Thus for Pearson to question Fowler’s request was entirely reasonable. However, it was out of character. Pearson habitually produced results to an impractical degree of accuracy and, indeed, was criticised by colleagues for doing so.⁵⁷ Presumably the exigencies of wartime had brought about a (temporary) change of heart, conferring on him a more realistic attitude typical of an applied mathematician.

In general, communication between members of the AAES and members of the military was not easy. Many of those in the military had little or no understanding of the theory behind the work of the AAES and were incapable of appreciating the value of new ideas and new instruments, and even had difficulty in comprehending the significance of being able to measure height (rather than range). [Barrow-Green 2014, 99]. As Milne described to his brother:⁵⁸

⁵⁷ For an example, see [Sheynin 1994, 255].

⁵⁸ E.A. Milne to G. Milne, 30 October 1917. [Milne Papers, A8]

... the English officer is neither a mathematician nor a scientist, has an instinctive dislike for any method which will improve his shooting only on the case of being scientific, and prefers to trust to luck. Bit by bit we are taking control of the course of action away from the officer and putting it on instruments which work automatically. The responsibility then remaining on the officer is to keep his men thoroughly trained and to keep his instruments in adjustment. The latter he will wriggle out of if he possibly can.

But while the military may have had problems with the theory, they were pleased with the results. In February Hill reported to Pearson on the success of the diagrams:⁵⁹

I took your diagrams to the Ordnance Committee at Woolwich today and showed them to the Committee and to General Bland. They like them very much, and are very grateful to you, and are most anxious to have more them for other guns. If I may I will send you at once details of C and m.v. for the 4 inch Mk V, and fuze scales for 84 and 65A fuzes. In the meantime Bolton is bringing up to the Committee on Wed. next the question of increasing your staff, and I have no doubt whatever they will agree to it. I am dining with Kennedy tonight and shall talk it over with him, and he will certainly get the Comptroller to agree to it.

I went to MI5 at the War Office this morning, and they have agreed to make 200 copies within a week in 2 colours, of both diagrams. I will send you copies directly they are completed. The Ordnance Committee thought that they had better be reduced in size, and after printing mounted on linen, for use in the field. We have arranged therefore to have them reduced to what is almost exactly half size, viz 1 inch = 500 yards. I am sorry for the inches, but most of our gunners carry this relic of barbarism with them, and have no other scale. And being smaller it will not get so torn and crumpled in service.

And a few months later, Douglas wrote to Pearson expressing his satisfaction with the accuracy of the tables:

I think the accuracy is quite sufficient for all practical purposes. I tested the ranges and elevations of the horizontal range table by Littlewood's latest "X" formula and got differences of -7 and +5 yards at 10,200 and 11,200 yards respectively. This is very satisfactory as I imagine your figures were obtained from small arc calculations.⁶⁰

However, as far as actually using the tables and diagrams within the field of war is concerned, little information has so far come to light, although there is concrete evidence that the diagrams made their way across the English Channel. In May 1917, Hill, who at that time was with the British Expeditionary Force, wrote a glowing report of the reception of the diagrams in

⁵⁹ A.V. Hill to K. Pearson, 12 February [1917] [Pearson Papers, 720/3]

⁶⁰ R.V. Douglas to K. Pearson, 8 May 1917. [Pearson Papers, 606]

France, saying that they fulfilled “a long felt want”.⁶¹ While at the same time Fowler told Pearson that he had had “several letters from France which show that AA group commanders are very pleased with them and would like them for all guns”.⁶²

Such was the volume and urgency of the work that several of Pearson’s staff not only worked overtime, including Saturdays, but also gave up their leave. But their hard work did not go unnoticed and in July 1917, Captain Moore, Assistant Comptroller of the MID, wrote to the Provost of UCL to tell him that the Ministry proposed to supplement their pay with an additional three months salary to compensate for working during the summer vacation.⁶³ Included in his letter was a list of Pearson’s staff ‘whose assiduity has been continuous and most valuable’:

| | Salary |
|------------------------|--------|
| Miss Ethel M Elderton | £150 |
| Mr Andrew W Young | £150 |
| Mr Arthur Doodson | £150 |
| Mr Lindsey Ince | £150 |
| Miss Adelaide Davin | £120 |
| Miss Winifred Husbands | £100 |

Since all of the men named above were effectively in the position of post-graduate students, it might appear that their salaries were not unreasonable, particularly those being paid £150 per annum.⁶⁴ But the work was grinding and the salaries did not compare favourably to those being paid in other Government departments. So it was not long before dissatisfaction set in, at least for some of the staff, and once they had considered their positions from a commercial point of view their commitment to the work diminished and they began to seek employment in civilian war service elsewhere. That, combined with the fact that the recruiting sergeants were constantly in pursuit of anyone whom they believed could do a job in the Army, however menial, meant that Pearson was permanently struggling to

⁶¹ A.V. Hill to K. Pearson, 22 May 1917 (KP: UCL, 606). The main content of the letter is reproduced in [Barrow-Green 2014, 99]

⁶² R.H. Fowler to K. Pearson, 23 May 1917. [Pearson Papers, 606]

⁶³ A.E. Moore to G. Foster, 6 July 1917. [Pearson Papers, 606]

⁶⁴ It is notoriously difficult to compute the relative value of money. The website <http://www.measuringworth.com/ukcompare/> provides comparisons using five different indicators. Taking the year 2011, the relative value of £150 in 1917 is £6900 using the retail price index, and £35,800 using ‘average earnings’. Website accessed 20 November 2014.

find and keep personnel.⁶⁵ Typical was the case of Doodson, of whom Hill wrote to Pearson:

Yes, Doodson and his braves are doing most excellent work. The idiotic Mr Kellaway wanted to take him away to sweep floors in the Army, but we put a stopper on that. It really would be too ridiculous, as the work would collapse without him.⁶⁶

When Pearson began working for the AAES he had a staff of eight assistants, [Yule & Filon 1936, 78] but as the work increased so did their number until eventually he had twenty members of staff, computers and draughtsmen, each one differently trained and of different ability, and he had to keep them all fully employed. [Pearson 1938, 91] Added to which only some of them were directly under his control, others were "more or less bound by the Ministry".⁶⁷ No wonder he found running the Laboratory a time-consuming task. All told it left him with little time for anything else. As he wrote in June 1917, "I get no spare moments now; it is anti-aircraft guns all day long, and for me most Sundays also". [Pearson 1938, 92] Moreover, much of the ballistics work was dull and monotonous, and working on it was not made easier for him by the fact that he had to take instruction from men, such as Milne, much younger than himself.

In early July 1917, Pearson heard rumours that the Germans were flying higher than he had previously been led to believe was possible. This was a matter of some concern since an increase in the maximum height would necessitate extending the tables. But Hill believed Pearson's fears to be unfounded. As he explained to Pearson, once again the problem lay with the army:

⁶⁵ The Military Service Act which brought in the first wave of Conscription became law on the 24 January 1916. By April 1918, after four revisions to the Act, Conscription applied to men between the ages of 17 and 51.

Much of the correspondence between Pearson and Hill (and others connected with the AAES) during 1917 concerns the suitability or otherwise of possible recruits to Pearson's staff. One mathematician who rejected the idea of working as a computer in Pearson's laboratory was the analyst G.N. Watson. Watson, who had been an exact contemporary of Hill's at Cambridge and senior wrangler in their year (1907), evidently considered the work beneath him, as Hill makes clear in a letter to Pearson of the 20 February. [Pearson Papers, 606] The letter is reproduced in [Barrow-Green 2014, 97]. Pearson was also in contact with members of the University of Cambridge Appointments Board from whom he received lists of Cambridge Scholars who were available for war work.

⁶⁶ A.V. Hill to K. Pearson, 19 July 1917. [Pearson Papers, 720/3]

⁶⁷ A.E. Moore to K. Pearson, 25 March 1918. [Pearson Papers, 606]

With reference to your queries of the Boche flying above 21000 feet. Only freak machines have ever been to heights of 24 and 25000 and there is absolutely no reliable evidence that his machines are flying at these heights. In the recent raids they have been flying at 14000. Various incompetent people have turned long base height finders on to the whole enemy squadron and probably got the opposite ends of the base on different targets. That is the explanation of these so-called heights of 24000. They might equally (and probably do) get negative heights by the same means but they cannot bring themselves to believe the latter so we do not hear of them. 18000 is the greatest recorded height of any German military machine. As regards oxygen apparatus, it is really necessary for prolonged flights at 16000. I think it would be necessary for the engines at 22000. Of course the difficulties are not insuperable, and the Boche could do it and we could do it equally well if we really tried. The Boche is not really a whit cleverer or more energetic than we are, once we rouse ourselves from our peace time lethargy. We are far ahead of him in many ways.⁶⁸

Hill did, however, seek further advice on this question from R.V. Douglas of the Ordnance Committee Office at Woolwich, who was working with Littlewood and for whom Pearson was also producing tables. Douglas thought that the Germans would soon be flying higher than 21,000 feet and hence recommended that the tables should be extended to 24,000 or 25,000 feet.

Douglas, who had his own team of computers at Woolwich, was interested in the methods used by Pearson, and in particular was curious about the effect of calculating machines on the rate of work. He told Pearson that his "best small arc computer" could calculate "about 46 in seven hours or at the rate of 1 complete small arc in 10 minutes".⁶⁹ Pearson had nine calculating machines and although there is no record of their effect on the rate of work they were in constant use. In fact so constantly were they in use that they were beginning to wear out and, because they mostly originated from the Continent (e.g., the German Brunsviga) it was virtually impossible to get them quickly and properly repaired. Consequently, any new ones available were on the market at rather inflated prices. In an effort to make up the shortfall, Pearson made an appeal through the columns of the journal *Nature* for machines in "reasonable condition" for which he was prepared to pay a "reasonable price". [Pearson 1917] To make his request seem more attractive he suggested that the high prices currently being asked would not be sustained after the war since "When the war is over machines will return to their normal price—indeed, will probably be at reduced prices, for the war has taught many persons their value, and the market will be

⁶⁸ A.V. Hill to K. Pearson, 6 July 1917. [Pearson Papers, 606]. Boche was one of the pejorative words used for "German."

⁶⁹ R.V. Douglas to K. Pearson 7 July 1917. [Pearson Papers, 606]

wider than it has hitherto been, so that foreign monopolies are certain to be broken down."

His appeal met with at least one offer. This was from Arthur Schuster, the erstwhile Professor of Physics at Manchester, who was retired but keeping himself active as Secretary for the Royal Society. Schuster had been born in Germany of German parents but had lived in Britain since 1870 and been naturalised in 1875. He was fervently loyal to Britain during the war although not without encountering difficulties.⁷⁰ It was a characteristically patriotic gesture on his part to offer his machines to Pearson:

I have read your letter in *Nature* and should be glad to help you. I have a "Millionaire" calculating machine (6 figures) which is in perfect condition and I could lend it to you till the end of the war, on an undertaking that it would be carefully used. The difficulty is to get it to London as it is rather heavy. I think I could manage to get it as far as Paddington Station, if you could get somebody to meet me at the train. ... I also have an American machine, which can be used for multiplication only, and wants of a little practice, but is still quite reliable when carefully used.⁷¹

The Millionaire, which was made by the Hans Egli Company of Zurich from 1899-1935 and was the first commercially successful 'direct multiplication' calculating machine, was advertised as 'The Fastest Multiplying or Dividing Machine in the World'. Schuster was certainly not exaggerating when he said the Millionaire was rather heavy. The standard model weighed 72 pounds and they could weigh up to 120 pounds!

6. THE RICHMOND INCIDENT⁷²

As the months wore on, the German bombing raids worsened and from May 1917 Gotha bi-plane bombers as well as Zeppelins were used to carry out the raids.⁷³ The already inadequate anti-aircraft defences were brought under increasing pressure, and by the end of September it seems that an exhausted Pearson was no longer convinced that what he was doing was worthwhile. Fowler wrote to encourage him:

⁷⁰ In July 1915 Schuster won a libel action against *Pearson's Weekly* (no connection to K. Pearson) for 'an imputation of disloyalty'. 'Libel on Professor Schuster' *The Times* 31 July 1915.

⁷¹ A. Schuster to K. Pearson, 14 July 1917. [Pearson Papers, 606]

⁷² This section is a development of the summary given in [Barrow-Green 2014, 100].

⁷³ The Gotha was the first German heavier-than-air aircraft to be used in bombing raids over England.

I don't think you need feel that any energy properly applied to anti-aircraft gunnery is wasted. All forms of defence are useless against aeroplanes at night, guns, searchlights, and aeroplanes themselves. Personally I always feel very strongly that it is no good pretending that they are any use at all. Possibly they have some moral effect on the pilot! In day time, however, it is a very different story. I am sure it is just as wrong to think that anti-aircraft gunnery is useless in daytime as it is to think that it is any use at night (except against the out of date Zepp). After all the bag this season to gunfire in England is 5 Gothas, which is by no means bad, considering the state of development of scientific methods of shooting. Why they fire at night at all, except to encourage us all by the noise, I have no idea. ... Night raids on London is a small part of the AA problem.⁷⁴

But on the 4 October Pearson received a letter from Richmond outlining what he believed to be a mistake in one of the tables produced by Pearson's staff:

I have been trying (for the sake of some work on wind effects) to find the velocity and inclination of the shell at each point of certain trajectories. It has always been easy to do this from the 1st and 3rd differences of coordinates, until this afternoon I used your table of coordinates for the 4" BL MkVII (2800:2.40). I was considering the 50° trajectory and found that after 37 secs, the second differences become impossibly irregular. Now I find the 45° trajectory is similarly bad, and the irregularities begin almost at the start. Can your computers have made a mistake in the sign of some of the small correcting terms in interpolations?

...

I have not tested other parts of the sheet; it is as well to call your attention to the mistake at once.⁷⁵

Richmond could hardly have anticipated the strength of Pearson's reaction which, judging by the next letter written by Hill to Pearson, must have been extreme. It appears that Pearson did not take at all kindly to what he considered as an accusation of error in the methods used by his staff, and on the strength of the accusation, and at a meeting with Hill, declared that he wished to break off his association with the AAES. Hill, the consummate diplomat, tried to smooth things over, although he was not going to let Pearson ride roughshod over him and was not prepared to accept Pearson's unjust accusations:

The work may seem to you trivial and unscientific and difficult: but it isn't our fault that the data and methods are not more adequate, and that difficulties and annoyances continually arise in it. I daresay you feel I was rather rude yesterday. If you feel that I am sorry. ... If you want to break with us because of my ill manners I shall be very sorry. If you want to break with us because of a private letter from an individual I think you are unjust to me and to other people

⁷⁴ R.H. Fowler to K. Pearson, 26 September 1917. [Pearson Papers, 606]

⁷⁵ H.W. Richmond to K. Pearson, 4 October 1917. [Pearson Papers, 606]

who until 2 days ago thought we were your friends. If you believe we depreciate and should publicly speak unjustly of your work I can only say you are mistaken; and if you do not believe me when I say so, I can only regret that I have inspired you with so little trust in my straightforwardness. The matter rests entirely with you. I have no "charges" to bring & it would give me no pleasure to bring them. I am always ready to apologise for my bad manners, but to apologise for bringing charges against you and your staff, or for allowing charges to be brought, I cannot honestly do because I have no idea or intention of bringing any such charges. It seems to me we shall only perpetuate our differences by going into legal arguments about Richmond's letters, and what their meaning is. My own object is to get on with the war and to get it over and to help my friends who are fighting much more real battles in France. On this I do count on your co-operation—if you deny me this I shall just have to get on without it. But it seems a real pity that the beautiful work you have done for and with us should terminate in this trivial way.⁷⁶

In an effort to resolve the actual point at issue, Richmond wrote again at length to Pearson going into even greater detail about the problematic calculations. But by now Richmond's blood was up. He pointed out to Pearson that it was not only his computing staff that had had to work under difficult conditions: "conditions of being told half of all manner of useful facts, of having no useful books, and no central authority to consult, of having to glean computer knowledge of essential things in any way that can be contrived, and of course being compelled to develop methods independently." He also drew attention to the inadequate communication between the "two departments", a situation which was hardly surprising in wartime conditions:

... when Milne went to you, it was plainly understood by us how much work we were asking of you in wishing you to accept dogmatic rules without explanations or reasons. We could only tell you what we knew; when you asked why, we could only answer that Littlewood says it is, or some such inadequate reason. It was a question either of telling you all we could, knowing that it was obscure in all sorts of ways, where you could with justice expect enlightenment, or of doing nothing. Secondly, I think it would have been better if we had developed our methods with more exchange of ideas between the two departments. Our needs are different and the forms of working must be distinct; but I regret that we have scarcely even compared ideas until we each had a finished scheme in working order.⁷⁷

But worse, he called Pearson's statement that he, Richmond, had "brought a general charge against the laboratory by implication in his letters" both "preposterous and baseless". Clearly such language was not

⁷⁶ A.V. Hill to K. Pearson, [?] October 1917 [Pearson Papers, 606]

⁷⁷ H.W. Richmond to K. Pearson, 7 October 1917. [Pearson Papers, 606]

going to endear him to Pearson. Hill intervened again and this time capitulated a little with respect to Richmond's letters, but he was not going to let Pearson get away with misinterpreting his own:

I agree that Richmond's letters were rude and tactless—why he wrote them I don't know, unless it was simply carelessness—but as for a general charge against your people it is quite impossible for me to suppose it was intended and I cannot understand why you should not believe my statement that I at any rate have neither heard nor seen nor made any statement other than one of respect for your work [Hill emphasis].⁷⁸

However, it would clearly have been a terrible blow for the MID if Hill and the work of the AAES were to be compromised by the loss of the use of Pearson and his staff, and so when Pearson wrote to Captain Moore, the assistant Comptroller of the MID, Moore immediately informed Colonel Goold Adams, the Comptroller, who hastily arranged a meeting with Pearson in order to try to resolve the matter.⁷⁹

After his meeting with Pearson, Goold Adams put structures in place to prevent anything like “the ‘Richmond’ incident” ever happening again, telling Pearson “that it is fully recognised that any errors which may appear in computation are due to the method employed by us and not in any way to lack of care on the part of the staff serving under you”.⁸⁰ It did the trick and Pearson withdrew his resignation.

Richmond was not the first member of the AAES to have a disagreement with Pearson; nor, indeed, were members of the AAES the only people to have arguments with Pearson: his career was studded with controversies.⁸¹ Earlier Pearson had had a “contretemps” with Milne over some discrepancies that had arisen from using two types of formulae for plotting trajectories.⁸² Milne patiently explained that the discrepancies were unlikely to produce great differences in the trajectories; and that inconsistencies, due to such things as wind effects and muzzle-velocity variations, rendered worthless any great refinement in the accuracy of the theoretical trajectories. Fortunately, Milne had recognised that the way to deal with Pearson was to adopt an air of humility. As a result, the situation was quickly and amicably resolved.

⁷⁸ A.V. Hill to K. Pearson 14 October 1917. [Pearson Papers, 606]

⁷⁹ A.E. Moore to K. Pearson, 16 October 1917. [Pearson Papers, 606]

⁸⁰ Goold Adams to K. Pearson, 18 October 1917. [Pearson Papers, 606]

⁸¹ See, for example, Pearson's long-standing controversies with, among others, R.A. Fisher and G.U. Yule. [Porter 1970–1990, 52–56] According to Fisher “Pearson's zest for polemics was unlimited.” [Edwards 1994, 102].

⁸² E.A. Milne to G. Milne, 21 October 1917. [Milne Papers, A8]

Doodson was another who clashed with Pearson. In one instance, he had come across a little-known paper originating from 1877 by John Couch Adams on calculating the trajectories of shot, an adaptation of which he tried to introduce into his work for Pearson.⁸³ [Adams 1877, 286–287] As Joseph Proudman, Doodson's colleague from Liverpool, who had joined the AAES in 1918, later described, Pearson took more than a little convincing:

The new work consisted of calculations of high-angle trajectories, a new subject in those days. The methods which had been evolved were rather poor and not sufficiently accurate for the purposes in view. The trajectories were calculated by 'small arcs' and a number of other processes, involving graphs, followed, the final tables requiring much 'smoothing'. From that time onwards Doodson had an incurable antipathy to graphical methods. On reading the Scientific papers of J. C. Adams, he found a paper on trajectories which appeared to be quite unknown to others working on the subject.

Doodson adapted Adams's methods and extended them, but Pearson rejected this procedure. There was a long struggle to get the 'small arc' methods amended, as Karl Pearson was not an easy man to argue with, and there was a convulsion among the staff when an attempt was made to abandon the graphical processes. Pearson then placed Doodson in charge of all the work on trajectories with instructions to apply a numerical method which Pearson had devised. This method proved to be a failure and Doodson was then able to demonstrate to Pearson that the procedure which he had rejected was able to give very good results. [Proudman 1968, 192]

Pearson conceded and thereafter left Doodson to work in the way he saw fit.

7. THE FINAL MONTHS

In November Pearson was asked to work on machine gun sights for low flying planes,⁸⁴ but Hill counselled against it:

I should have no hesitation in refusing to do the work on machine gun sights for low flying aeroplanes. I had heard nothing about it: the accuracy of your method would be absolutely wasted on the problem [Hill emphasis]. We have made a number of calculations on machine gun trajectories at Portsmouth—in particular DR Hartree has—and can easily do them there if the MID wants them done.

⁸³ Adams was the Lowndean Professor of Astronomy and Geometry at Cambridge. Today he is renowned for his role in the discovery of the planet Neptune.

⁸⁴ A.E. Moore to KP, 21 November 1917. [Pearson Papers, 606]

So if it will cause the least delay in your other work I should say do not on any account worry about machine gun sights. The merest crude “fudging” is sufficiently good for a machine gun sight for a low-flying aeroplane: accurate methods are quite impossible.⁸⁵

However, as is evident from his Journal, Pearson left Hill’s advice unheeded and took on the extra work:

We were also asked just before Xmas 1917 to take as urgent work the calculation of the sights for Vickers, Lewis and Hotchkiss machine guns for use against low flying airplanes. These were making the place unhealthy for our men at the front, and we had a very strenuous six weeks work on getting these tables off to France on Xmas Eve, 1917.⁸⁶

While Pearson was doing this “urgent work” he was still suffering from an acute shortage of appropriate calculating machines, a state of affairs which can only have added to the stress he was under. Having told Captain Moore of his difficulties, he was informed in January 1918 that he could expect the delivery of two Muldivo 18-figure machines although with the expectation that he would return a smaller Muldivo machine that he had been loaned in the interim.⁸⁷ It was a hardly satisfactory resolution to the problem.

During the same period, Pearson was also developing methods for dealing with the variations of trajectories due to wind effects. Since winds can have considerable and unexpected effects on the course of a trajectory—for example a following wind increases not only the horizontal range for a given time of flight, but also the height throughout all the early part of the trajectory, and commonly through its entire extent—work on them with respect to anti-aircraft fire was urgently needed. It was difficult work but in February 1918 Pearson sent Hill a report on methods of dealing with a constant wind.⁸⁸ In addition, he devised a formula for the lateral displacement in an oblique wind which was extremely well adapted for finding the total effects of cross-wind upon observed shell bursts. [Richmond 1924–1925, 370]

⁸⁵ A.V. Hill to K. Pearson, 19 November 1917. [Pearson Papers, 606]

⁸⁶ K. Pearson *Journal of the Galton Laboratory*, 25 May 1918. [Pearson Papers, 246]

⁸⁷ A.E. Moore to K. Pearson, 9 January 1918. [Pearson Papers, 606]

⁸⁸ A.V. Hill to K. Pearson, 17 February 1918. *Report on Methods of Dealing with the Influence of a constant horizontal Wind on the Position and Speed of a Shell* (handwritten). [Pearson Papers, 606]

Richmond and Milne wrote a memoir entitled ‘The equivalent constant wind’ [Milne & White 1948, 226] which appears to be unpublished but which may be AVHL 1/28, an anonymous memoir with the same title dated May 1918.

Pearson and his staff had worked unremittingly on ballistics problems for the whole of 1917 almost without a break. Apart from one week at Christmas the Biometric Laboratory was never closed from the 1 January 1917 to the 1 January 1918. Given the nature of the work and the long hours he spent on it, it was perhaps not surprising that Pearson somewhat over-reacted to Richmond's remarks. But his efforts and those of his staff were certainly valued both by those who were using their results as well as by the officials in charge of the administration of the enterprise. In February 1918 Pearson received a letter of appreciation from Vice-Admiral Bacon, the new Comptroller of the MID who had only been in post since the preceding December:

At a time of great pressure the Ministry found itself in need of a trained staff of computers provided with necessary machines to undertake gunnery work, and was indeed fortunate to find such a staff and machinery already in existence at the Drapers' Company Biometric Laboratory and the allied Galton Laboratory. The work done in connexion, inter alia, with the preparation of practically all the charts and high angle range tables for the a.a. [anti-aircraft] guns of both Services has proved of inestimable value; and it was no small advantage to find at a time of national stress that a school had been trained in times of peace for such computing work as became, on the outbreak of the war, a matter of such vital importance.⁸⁹

Bacon was also rather keen that Pearson should bring his letter to the notice of the Drapers' Company so that they would know that their money in sponsoring the Laboratory was being well spent. Pearson would undoubtedly have been gratified by Bacon's recognition of the importance of having "a school" of computers.

But as the New Year wore on, Pearson became increasingly ground down by the work, and the lack of time he had for doing other things, such as his biometric and eugenics work. It is no coincidence that the longest period between two consecutive issues of *Biometrika*, the journal Pearson had helped to found and of which he was sole editor from 1906 until his death in 1936, was between May 1917 and November 1918.⁹⁰

That is not to say that Pearson himself did not recognise the contribution he and his team were making to the war effort and indeed how essential it was. In February 1918 he wrote a Report for the Drapers Company on the "Present Position and Past History of the Laboratories to which their Annual Grant has been made" and which included details of the war work of the Laboratory:

⁸⁹ R.H.S. Bacon to K. Pearson, 14 February 1918. [Pearson Papers, 606]

⁹⁰ For further information on how *Biometrika* fared during the War years, see [Pearson 1938, 87–89].

We have had in charge the preparation of the whole of the charts and high-angled range tables for the anti-aircraft guns in the Army and Navy, and the preparation of data for the sights of these guns. All the organisation and control of the work, all the finer draughtsmanship of the charts, was undertaken by the trained members of my staff. ... The work has been so urgent and of such value that the Ministry of Munitions has placed eight to ten computers and draughtsmen at my disposal ...

Voluntary enthusiasm has been the mainspring of the whole enterprise. I venture to think that we may claim it as good evidence of the value of the training given in the Laboratory that our members could thus take upon themselves an entirely new field of work. It must be remembered that high-angled range tables for anti-aircraft guns were unknown before the war had developed the airplane as a new instrument of war, that in the majority of cases we had to develop new methods from the very rough processes originally suggested to us...⁹¹

While one may scent a whiff of self-congratulation in Pearson's account, it must be remembered that Pearson was writing for the sponsors of his Laboratory and he needed them to continue with their financial support. He gave a concise description of the work completed during the last twelve months, taking care to mention its effectiveness. And once again he referred to the importance of keeping his team of computers together. The idea of collective and/or collaborative working was absolutely fundamental to him, irrespective of whether the country was at war:

In the course of the work no fewer than 85 new tables have been computed for various guns, giving ranges, fuzes or sights, and, further, several of the old ballistic tables have been re-calculated or developed to higher degrees of accuracy. We have heard from the Front due appreciation of our charts and tables, and whereas before their existence practically no airplanes were shot down, we now hear of upwards of sixty in six months by direct anti-aircraft fire. ...

My object throughout has been to maintain a body of trained computers together who would have the force of character and knowledge to meet new problems and remain as a nucleus for the Laboratory research work when peace returns.⁹²

But the strain of working so hard and trying to maintain the enthusiasm of his staff—some of whom, feeling underpaid, had begun to slacken—began to tell on Pearson. He found himself “more often scolding and irritated by inertia here and there”,⁹³ so that in March 1918 he decided to

⁹¹ K. Pearson *Report to the Court of the Worshipful Drapers on the Present Position and Past History of the Laboratories to which their Annual Grant has been made*, February 1918. [Pearson Papers, 233], [Pearson 1938, 165]

⁹² K. Pearson *Report to the Court of the Worshipful Drapers on the Present Position and Past History of the Laboratories to which their Annual Grant has been made*, February 1918. [Pearson Papers, 233], [Pearson 1938, 165–166]

⁹³ K. Pearson. *Journal of the Galton Laboratory*, 25 May 1918. [Pearson Papers, 246]

give up the ballistics work and told Hill of his plans. Hill expressed his appreciation for all that Pearson had done:

[I am] very sorry indeed that you should feel it advisable to give up your personal superintendence of the work. ...] I hope you will realise the gratitude we feel—and that I personally feel—for your work on our behalf and will believe the regret with which I personally shall cease my official relations with you. I very sincerely hope that the end of our official relationship will not mean the end of our personal and scientific relationship, and that you will allow me to come to you occasionally to discuss our problems and ask your advice exactly as hitherto. If it had not been for your help a large part of the work done by the AAES could never have been done, and the present stage of development could never have been reached.

"Archie" [anti-aircraft gun] has not done much up to date, but the production of your diagrams has done more I think to educate the gunners up to the developments which will now really occur within a few months than any other one link in the chain of "Archie's" evolution as a scientific weapon.⁹⁴

On 28 March Pearson wrote an official letter to Vice-Admiral Bacon to tell him that he was no longer able to continue with the work.⁹⁵ He recommended Doodson as his successor. Hill was happy with his choice, writing to Pearson:

I have seen something of Doodson lately—I am now quite sure you are right in advising us to put him in charge of the work. I think he will do it admirably. He is a very human person once one has got to know him & I think he will do it very well indeed, & get the best out of his people.⁹⁶

Pearson went gracefully, making pains to ensure that the handover to Doodson went smoothly and that all the arrangements were in place for the transfer.⁹⁷ Since he was no longer going to be involved in the work, it made little sense for it to be done at UCL and he made arrangements for the team to be transferred to Woolwich. Although the team would be the poorer without Pearson, they would now have the advantage of being in close proximity to ballisticians, among whom, in addition to Littlewood, were the mathematicians Henry Hassé and Joseph Proudman. [Barrow-Green 2014, 101]

⁹⁴ A.V. Hill to K. Pearson, 23 March 1918. [Pearson Papers, 606]

⁹⁵ Vice-Admiral Bacon acknowledged Pearson's letter of resignation in his reply. R.H.S. Bacon to K. Pearson, 2 April 1918. [Pearson Papers, 606]

⁹⁶ A.V. Hill to K. Pearson, 26 April 1918. [Pearson Papers, 606]

⁹⁷ K. Pearson *Journal of the Galton Laboratory*, 25 May 1918 (KP: UCL, 246). Doodson remained in charge of the Computing Section of the AAES until March 1919 after which Pearson offered him a job which he declined. He subsequently had a distinguished career at the Tidal Institute in Liverpool. A. Doodson to K. Pearson, 24 June 1919. [Pearson Papers, 676/5], [Proudman 1968, 193]

Not all of Pearson's staff went to Woolwich. Two of the computers, one of whom was Young, went to join the experimental staff at HMS *Excellent*, while Ince had already left to work at the Ministry for Food Control where he was earning double the salary he had been receiving under Pearson. The only one he considered "a serious loss", was Adelaide Davin, whom he considered "invaluable for the whole of the war work".⁹⁸

The transfer took place on 1 April, and later that month Moore wrote to Pearson to thank him for his work:

I have received your kind and generous letter this morning. Strangely enough, I had made a memorandum to write to you today as I wished to express my personal thanks for the unfailing kindness with which you have always put up with my shortcomings. I fear I have often failed to keep my promises owing to circumstances which have been too strong for me to overcome.

I realise, perhaps more fully than others, the self-sacrifice and devotion you have given so freely to the National cause. It must be some satisfaction to you to know that the results of your work are being applied with the utmost advantage to our anti-aircraft gunnery, and thus you and your Staff have contributed in a large measure to the success which has attended us in endeavouring to defeat the enemy's plan to lower the morale of our people at home by constant air raids.

I sincerely hope that the relief from this work will enable you to regain your health and energy. I was very glad indeed to be able to make temporary arrangements for Mr Doodson and his staff and I am confident that they can carry on quite comfortably until their permanent quarters are ready.⁹⁹

Pearson's long-standing assistant, Ethel Elderton, was the only one who remained at the laboratory with Pearson, and the two of them spent the summer completing "some special gunnery problems". [Pearson 1938, 92] Pearson had offered to continue giving Hill assistance and advice, and since by this time Hill had been transferred to London their communication undoubtedly continued. Pearson also kept in touch with some of his computers. In July 1918 he heard from Adolph Zaiman, who had been conscripted into the Meteorological Section at Shoeburyness, who wrote to him describing his new job. Zaiman, much to his own delight, had had to use the very range tables that he had helped to compile:

The work is very interesting and consists in determining the air pressures and temperatures at different heights, the nature and strength of the wind, etc., whilst gunnery practices are in progress—and then applying the corrections for these atmospheric conditions. So really this place combines the work they do at HMS *Excellent* with meteorological work. For anti-aircraft guns, the GL tables are the basic things. I have been using the GL2B this week—the dear old 3" 20cwt!

⁹⁸ K. Pearson *Journal of the Galton Laboratory*, 25 May 1918 (KP: UCL, 246).

⁹⁹ A.E. Moore to K. Pearson, 25 April 1918. [Pearson Papers, 768/3]

There is a certain amount of ballistics done here too, computing work, and in fact the work is very similar to what I did at Univ. College, except that we do experimental stunts too—pilot-balloon observations, shell-burst spotting in mirrors, and so on—I suppose the fact that I am wearing khaki makes the work more useful than when I was in the MID! I sometimes yearn for a Brunsviga!¹⁰⁰

By August, Pearson was able to take a much needed holiday and return to his biometric work, no longer dreaming of guns. [Pearson 1938, 92]. Rather auspiciously, on 11 November, the day the Armistice was signed, he was in the laboratory initiating a wounded L.J. Comrie, into the workings of a Brunsviga calculating machine.¹⁰¹ [Massey 1952, 98]

In May the following year, Moore's gratitude to Pearson for his war work was echoed by a member of the Ministry of Munitions with whom Pearson had been in contact in his effort to find replacement calculating machines which he badly needed for training "students returned from the Front":

For more than two years during the war Professor Pearson devoted his whole time voluntarily, with the assistance of his highly trained Computing Staff, to important ballistic work for this Department in connexion with anti-aircraft gunnery, contributing in a large measure to British supremacy in the air on the various battle fronts and to the defeat of the enemy's plans to lower the morale of our people at home by constant air raids. As a result of the strenuous work carried out during the war the calculating machines used by Professor Pearson's Staff sustained much wear and tear and he has now to undertake difficult and intricate work with insufficient and inefficient machines.

In view of Professor Pearson's endeavours and unselfish devotion in the National interest, it would be much appreciated if you could assist him in obtaining at reasonable cost the calculating machines which he so urgently needs.¹⁰²

Although Pearson had been able to start instructing students immediately at the end of the war, the post-war rise in costs meant it took time to get his two laboratories properly staffed and equipped. But by the end of 1919—after a vigorous fund-raising campaign and with the Drapers' Company continuing their support—ten members of staff were in place and Pearson could resume his work in something like its pre-war form. [Pearson 1938, 93–94]

¹⁰⁰ A. Zaiman to K. Pearson, 13 July 1918. [Pearson Papers, 906/1]

¹⁰¹ Comrie, a New Zealander, had lost a leg in action in France in February 1918. He was a pioneer in mechanical computation, founding in 1937 the first private company for scientific computing.

¹⁰² From a letter written by Mr. Merrifield to the Ministry of Supply, and copied to K. Pearson, 23 May 1919. [Pearson Papers, 768/3]

8. CONCLUDING REMARKS

How did Pearson view his experiences of working for the Ministry of Munitions? We know the answer because on the 25 May 1918 he wrote in his *Journal*:

I think it was right to do this gunnery work because I believe we were the only trained group of computers able at once to take it up and work in cooperation. I at first thought that it would keep my staff together and ready to start work again when the war should end. But Young and Doodson, to say nothing of Ince, were not satisfied with the financial prospects and would have left on the first chance—they had no special sympathy with eugenic or biometric work, and though both able men [they] would not have consented to stay on at our work, except for the sake of salaries such as I could not offer.¹⁰³

Thus, while there is no doubt that Pearson offered the services of his Laboratories to the war effort for patriotic reasons there was also an understandable element of self-interest attached to the offer. By working for the Ministry of Munitions he could help his country and at the same time hope to maintain a functioning work force ready to embark on research immediately peace was brokered. The ballistics work enabled him to keep the Biometric Laboratory running at full stretch and no doubt was one of the reasons that he kept going with the work for as long as he did. Indeed he admitted as much in his February 1918 *Report* to the Drapers' Company where he acknowledged that at the beginning of the war he had considered "it essential for the future to retain if possible a highly trained staff" but recognised that it was going to be very difficult to do so due to the fact that he had insufficient funds at his disposal "to compete with the high salaries offered to competent statisticians", and because he did not consider it "justifiable to keep members of the staff who were urgently needed for national work of importance".¹⁰⁴

What did Hill really think of Pearson? It is clear that Pearson was not always an easy person to deal with. He was a perfectionist, certainly, and tolerance was not always his strong suit. As evident in the case of the Richmond incident, he could exhibit a short temper combined with a lack of understanding of human nature. But that said, he worked extremely hard, was unwaveringly patriotic, garnered the unswerving loyalty of almost all

¹⁰³ K. Pearson *Journal of the Galton Laboratory*, 25 May 1918. [Pearson Papers, 246]

¹⁰⁴ K. Pearson *Report to the Court of the Worshipful Drapers on the Present Position and Past History of the Laboratories to which their Annual Grant has been made*, February 1918. [Pearson Papers, 233]

of his staff, and he made an important contribution to the war effort. Although Hill certainly encountered difficulties when working with Pearson, it is clear from Hill's *Memoirs*, that he retained affection for him:

Pearson was not easy to instruct (although his chief instructor was EA Milne) in which he regarded as rough and ready methods and he often upbraided me for the imperfections of the data we supplied. It was not easy to convince him that these, obtained by expensive firing trials and prolonged calculations, were all that we had or could have. I have about a hundred long and closely written letters of that time, nearly all in his own hand. They show, at an hour of trial, his dominating and pugnacious spirit, controlled by a passion of loyalty and desire to help, in what must have been, for him, the very distasteful role of accepting advice and instruction from people much younger and less experienced than himself. That he did so for those years, and that—as I know—he spoke in later times with affection of his taskmasters; that—except for a severe strain caused once by an error in his own calculations—we remained friends, not only in 1917–1918 but thereafter; all this is a sign of the essential generosity of a human nature which must have given even greater trouble to its owner than to his neighbours: they could laugh about it and he could not. [Hill 1970, 43]

Acknowledgements

I am particularly grateful to Eileen Magnello who prompted me to write about this aspect of Pearson's work and for sharing her deep knowledge of Pearson with me over several years. I thank the members of staff at University College London Library Special Collections who were helpful in assisting me find my way through the voluminous Pearson archive. I also thank the anonymous referees and the editor for their helpful comments and suggestions that greatly improved the manuscript.

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