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A SINGLE-STAGE CLEANER PRODUCTION SYSTEM WITH WASTE MANAGEMENT, REWORKING, PRESERVATION TECHNOLOGY, AND PARTIAL BACKLOGGING UNDER INFLATION

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Abstract. Waste management and reworking are very crucial issues in the cleaner production system. The adaptation of preservation mechanism in inventory control is also a key aspect from an economic and environmental point of view. In the current study, an inventory model for a cleaner production system is modelled considering all these practical issues and inflation. Deterioration process takes place in the production system. In the model, market demand is viewed sales team efforts and selling price dependent. Here, rate of production along with the unit production cost are taken as variables. An investment in preservation technology is made with the goal to lower the percentage of defective products. Further, partial backordering is considered. In order to demonstrate the model, numerical example is provided. A Hessian matrix is used to establish the concavity of the objective function. A theoretical result is provided to obtain the concavity of the objective function. Sensitivity analysis along with managerial implications is also provided in the manuscript. Results indicate that by implementing high-efficiency preservation technology, the detrimental effects of deterioration on profit can be mitigated. Due to this, 1.6% rise in profit is observed. Thus, selection of right preservation technology is crucial for both financial and environmental sustainability. In addition to this, higher reworking rates and capital investment in quality improvement result in high profit for the system.

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1. Introduction

Making decisions about clean production systems is a difficult task for manufacturing systems as clean production system is directly clubbed with the environment. Nowadays, environmental sustainability is a major concern all over the world. Manufacturing activities contribute to various environmental issues along with global warming. It is an obvious fact that waste generated due to deterioration or imperfect production has an adverse impact on the environment as well as the economy. Manufacturing waste which has the properties such as toxicity, corrosiveness, or ignitability can be classified as hazardous waste. On focussing in this direction, Unilever aspires to have a waste-free manufacturing system by understanding the harmful effects of waste on

Keywords. Cleaner production system, reworking, partial backlogging, selling price-and-sales dependent demand, preservation technology.

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the environment and becoming an environmentally conscious firm[1]. Towards this direction, legendary company Microsoft moves a step forward and sets the goal of zero waste by 2030[2]. It is reported that by 2050, across the world, generation of municipal solid waste is expected to increase by 70% to 3.4 billion metric tons[3]. Thus, it is very important for the manufacturer to maintain the solid waste properly and keeping the clean environment as a primary concern while taking the decisions related to production system. Furthermore, reworking is a crucial technique for manufacturers to use in order to reduce solid waste.

Deterioration presents a significant obstacle in the direction of clean manufacturing processes. Products like electronic gadgets, bakery products, dairy products, etc., lose their utility with time. This happens due to oxidation, chemical reactions, and the presence of micro-organisms like mold, yeast, etc. This results in financial loss for the system and has a hazardous impact on the environment. Hence controlling measure is imperative as products have some economic value. As a result, due to its benefits in terms of reducing inventory loss, saving money, and raising service levels, the application of preservation technology has gained significance to attain the goal of cleaner manufacturing processes. Therefore, manufacturers are exploring the use of technology for preservation to reduce or avoid deterioration to make the production process cleaner. Several methods, such as cooling, freezing, smoking, pickling, vacuum packing, jugging, etc., are utilized by the manufacturers as preservation technology.

Environmental and economical sustainability are big challenges in front of the manufacturer. They look at several approaches to achieving these goals. Therefore, it's crucial to investigate how the manufacturers achieve the goal of environmental and economic sustainability as a result of incorporating reworking, waste management, and preservation technology while modelling their inventory problem.

Thus, current study extends the previously developed flexible manufacturing inventory model by considering waste management, reworking, preservation technology, and partial backordering. Effect of inflation and investment to reduce fraction of imperfect production process are also incorporated. Following sections are included in the current work: Section 1 contains introduction. Relevant literature review included in Section 2. Section 3 consist list of assumptions and notations. Section 4 have mathematical formulation for cleaner-production system. Solution methodology is presented in Section 5 to obtain the feasible as well as optimal solution. Numerical illustration consists in Section 6. Sensitivity analysis included in Section 7. In Section 8, managerial implications are presented. Finally, concluding remark with future extension is presented in Section 9. Flow of the study is presented in Figure 1.

2. Literature review

Traditionally, inventory researchers assumed that the manufacturing process was always flawless and also ignored quality concerns. This assumption was incorrect in practice. In this line, a single-stage production system was considered by Porteus [1] and considered capital investment in quality improvement. Cárdenas-Barrón [2] also looked into a production system having single-stage production process including reworking. Dem and Prasher [3] proposed a production system based on a flawed manufacturing process. They anticipated that defective items would be reworked or discarded. A production model was explored by Taleizadeh *et al.* [4] by considering faulty manufacturing process using a reworking mechanism to determine the optimal number of deliveries, price, and lot size. Stochastic production model was presented by Kim and Sarkar [5] considering imperfect production process, budget restriction, and lead time of a multi-stage cleaner production process. By including carbon emissions in the models, Daryanto and Wee [6] produced two ecologically friendly economic production quantity models. Rout *et al.* [7] developed a flawed production-inventory system that took into account deterioration, inspection errors, rework, and backordering. By considering reduction in setup cost and investment in process improvement, Tayyab *et al.* [8] designed a cleaner production system for textile industry.

[1] https://www.unilever.com/planet-and-society/waste-free-world/ visited on 30.01.2022

[2] https://blogs.microsoft.com/blog/2020/08/04/microsoft-direct-operations-products-and-packaging-to-be-zero-waste-by-2030/ visited on 02.02.2022

[3] https://www.statista.com/topics/4983/waste-generation-worldwide/ visited on 01.02.2022

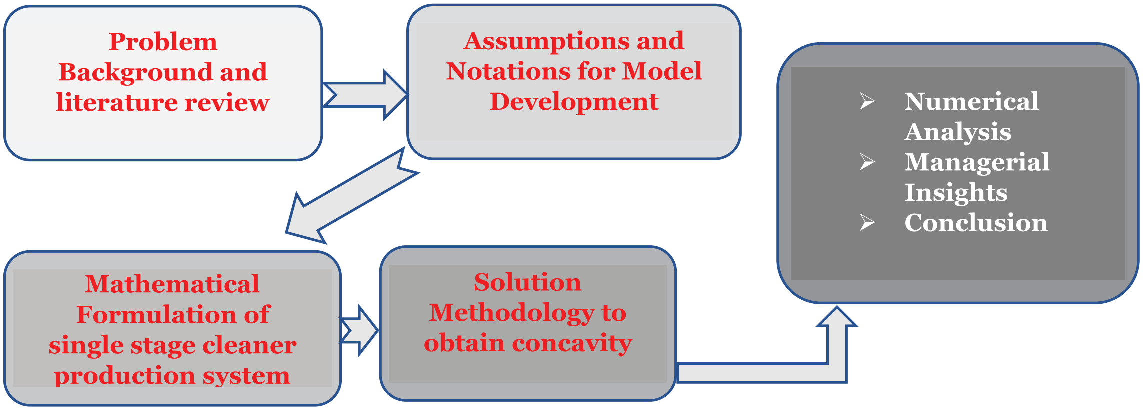


FIGURE 1. The flow of current study.

Rout *et al.* [9] examined several regulatory strategies for a supply chain with an imperfect production process to trade off carbon emissions and total cost. Reworking process for imperfect production inventory system was adopted by Manna *et al.* [10]. In inflationary environment, they carried out the study under the effect of learning. Sepehri *et al.* [11] examined a sustainable production-inventory system that took carbon reduction technology into account in order to lessen carbon emissions due to the manufacturing process. Moon *et al.* [12] presented EPQ model with the aim to increase the reliability of the production system. They applied geometric programming approach to obtain the optimal solution. Kumar *et al.* [13] examined the manufacturing/remanufacturing process for the supply chain, taking into account carbon emissions and advertisement-dependent demand, in order to clean up the environment. Sarkar and Bhunia [14] developed a flexible integrated model by considering manufacturing-remanufacturing system. They observed that through green investment profit of the integrated system increases. Sarkar *et al.* [15] presented an inventory model with the aim to nullify the food-waste which is responsible for greenhouse gases. Further in this direction, Sarkar *et al.* [16] presented three production models with different production strategies with the aim to reduce waste generation by adopting green innovative products. They observed that model's objective is achieved with the highly green innovative products rather than less innovative green products.

The strategies used by a manufacturer to maintain optimal inventory control are heavily influenced by consumer demand. According to the rule of demand, a customer's demand for economic things is diminished if the price is high. As a result, there is a negative correlation between market demand and price. Manufacturers believe that by cutting the price, they will be able to entice customers away from competitors. The manufacturer's per-unit profits, however, will suffer as a result of low price. Thus, effective pricing methods must be implemented to make it easier for manufacturers to manage inventory systems economically. Furthermore, the sales team's efforts have been shown to have a positive impact on market demand. Customers' demand was assumed to be constant in traditional production models. However, demand for many products, such as cellphones, fans, televisions, and watches, is influenced by many factors such as selling price, the sales team's initiative, etc. Constant production rate was taken by Sarkar *et al.* [17] while considering production-inventory system where demand increases with respect to time. Mishra [18] looked at a production-inventory system with three different rates of production by looking at price- and advertisement-dependent demand. Singh [19] investigated the production-inventory model by taking into account demand that is influenced by selling prices and stock levels. By integrating selling price-dependent demand and environmental issues, an integrated model was developed by Dey *et al.* [20]. Manna *et al.* [21] looked at a manufacturing inventory system that took into account demand that was affected by warranty periods and selling price discounts, inspection errors, and time-dependent development costs. By taking into account selling price-dependent demand and reworking of faulty items, Ruidas *et al.* [22] examined production system that is not perfect. To get the optimal profit for the manufacturer, they used quantum behaved particle swarm optimization (QBPSO) technology. A non-perfect production-inventory model was investigated by Kamna *et al.* [23] taking price-dependent demand and production-related carbon emissions. By incorporating time-and price-dependent demand, a two-stage storage production-inventory system was investigated by Aarya *et al.* [24]. By taking rebate-value-based dependent demand into account, Mishra *et al.* [25] constructed a four-level production rate inventory model. They observed that customers are encouraged to acquire things when rebate marketing is used. According to Gautam *et al.* [26], manufacturers should invest more in advertisement in order to generate more profit and to increase demand. Hence, taking into account demand that is dependent on sales team efforts and selling price is more feasible in the case of production-inventory modelling.

Practically, it is observed that a manufacturer's economy is badly influenced by deterioration. It is important for the manufacturer to control the deterioration process by adopting different means or mechanisms, such as preservation technology. First of all, Hsu *et al.* [27] determined the optimal investment in preservation mechanism to manage the rate of deterioration for an inventory problem. Further, Dye [28] obtained the optimal investment in preservation mechanism in the case when deterioration is non-instantaneous. A production-inventory model was examined where demand depends on time by Hsieh and Dye [29] by considering investment in preservation technology. Zhang *et al.* [30] investigated an inventory system considering price-dependent demand to manage the deterioration process with the help of investment in preservation mechanism. A manufacturer-distribution model was designed by Tayal *et al.* [31] considering shortages, mechanism for preservation, and trade credit. For the inventory problem of seasonal products, optimal values of ordering frequency, investment in preservation mechanism, and selling price derived by Mishra *et al.* [32]. For non-instantaneous deteriorating goods, an inventory model was developed by Pal *et al.* [33] considering preservation technology. Shen *et al.* [34] looked at how preservation technology and carbon tax policies affected a supply chain production-inventory model for degrading goods. Ullah *et al.* [35] analyzed the effect of preservation methodology on the integrated inventory model and observed a 13% reduction in the total inventory cost. Iqbal and Sarkar [36] developed an integrated system focusing on environmental issues and controlling the deterioration rate by adopting preservation technology. A sustainable production-inventory model was modelled by Sepehri *et al.* [11] to manage deterioration rate by applying preservation technology and capital investment to diminish carbon emissions because of the production process. Mahapatra *et al.* [37] presented three EOQ models with time dependent deterioration using preservation technology. They observed that order quantity increases as the investment in preservation technology increases.

Most of the time, effect of inflation in a classical production-inventory system is avoided by inventory practitioners. Practically, ignorance of inflation gives a blurred picture of the economic position of the manufacturer. The rate of increase in the price of goods and services is termed the inflation rate. High inflation means a high selling price and high holding cost of the products. Thus, inflation plays a critical role in a production-inventory model. In this direction, Buzacott [38] was the first inventory practitioner who discussed the impact of inflation on inventory models. De and Goswami [39] presented an EOQ model with a fuzzy deterioration rate and fuzzy inflation rate. Research carried out by the researchers like Yavari *et al.* [40], Saha and Sen [41], Khanna *et al.* [42], Shaikh *et al.* [43], Hemapriya and Uthayakumar [44], Alamri *et al.* [45], Sarkar *et al.* [46] are noteworthy in this direction.

In the case of shortages, the reactions of different customers are different. This reaction depends on the availability of products in the market, their choice, and market conditions. Thus, partial backlog represents the practical situation of the market. Wee [47] presented an inventory model with pricing, quantity discount, and partial backordering. Zeng [48] analyzed the situation of partial backordering in an inventory model. Taleizadeh [49], Pal and Adhikari [50], Kung *et al.* [51], De and Mahata [52], Ahmed *et al.* [53], Öztürk [54], Khan *et al.* [55], Choi *et al.* [56], explored the inventory problems with partial backordering.

2.1. Research gap and objectives

From the literature carried out in review section, it is found that researchers explored different issues such waste management, reworking, preservation technology related to manufacturing systems considering environmental sustainability. Many research papers have been published in the recent years considering these issues

Table 1. Author's contribution.

Author	Model type	Investment in quality	Preservation technology	Reworking	Waste management	Shortages	Inflation	Demand
Hsieh and Dye [29]	EPQ	No	Yes	No	No	No	No	Time dependent
Kim and Sarkar [5]	EPQ with IP	Yes	No	No	No	BO	No	Constant
Manna *et al.* [57]	EPQ with IP	No	No	Yes	No	No	Yes	Depends on SP and AD
Kung *et al.* [51]	EPQ with IP	No	No	Yes	No	PB	No	Constant
Shen *et al.* [34]	EPQ	No	Yes	No	No	No	No	Constant
De and Mahata [52]	SC	No	No	No	No	PB	No	Constant
Hemapriya and Uthayakumar [44]	SC	No	No	No	No	No	Yes	Constant
Ahmed *et al.* [58]	SC with IP	No	No	Yes	No	PB	No	Constant
Manna *et al.* [10]	EPQ with IP	No	No	Yes	No	PB	Yes	Depends on SP and stock
Kamna *et al.* [23]	EPQ with IP	No	No	Yes	No	Yes	No	Depends on SP
Sepehri *et al.* [11]	EPQ with IP	Yes	Yes	Yes	No	No	No	Depends on SP
Kumar *et al.* [13]	EPQ with IP	No	No	Yes	Yes	No	No	Depends on time and AD
Gautam *et al.* [26]	EPQ with IP	No	No	Yes	No	No	No	Depends on SP and AD
This Paper	EPQ with IP	Yes	Yes	Yes	Yes	PB	Yes	Depends on SP and AD

Notes. EPQ: Economic order quantity; SC: Supply Chain; IP: Imperfect production; B: Backorder; PB: Partial backordering; BO: Backorder SP: Selling Price; AD: Advertisement.

independently. In context of production system, researchers such as Ahmad *et al.* [53] explored waste management, Öztürk [59] explored reworking, and Shen *et al.* [34] explored the impact of preservation mechanism. Yadav *et al.* [60] explored the integrated impact of preservation mechanism and waste management. Manna *et al.* [61] analyzed the impact of reworking and waste management, and Sepehri *et al.* [11] observed the impact of preservation technology and reworking. From literature and Table 1, it is evident that rarely any researcher integrated these three issues for faulty production process by considering partial backlogging, inflation, and investment to reduce fraction of imperfect production proportion. Thus, the objective of the current study is to outline how the decision-maker of manufacturing industries can modify their decision under the above-mentioned issues.

3. Assumptions and notations

Following assumptions are utilized to derive the mathematical formulation for single stage manufacturing model:

3.1. Notations

Decision variables

P Production rate (unit/month)
s Selling Price (\$)
t_1 Shortage period without production (month)
t_3 Production period (month)

Parameters

μ	Fraction of imperfect production rate
μ_r	Fraction of non-reworkable item
θ	Deterioration rate
R	Rate of Reworking (units/month)
S_m	Manufacturer's setup cost ($/setup)
S_r	Re-working station's setup cost ($/setup)
i	The discount rate represents the time value of money
f	Inflation rate
r	The net discount rate of inflation *i.e.*, $r = i - f$
C_R	Re-working cost per unit item ($)
C_{wm}	Waste management cost per unit item ($)
C_{hs}	Holding cost at manufacturer end per unit item ($)
C_{hr}	Holding cost at re-working station per unit item ($)
C_s	Shortage cost per unit item ($)
C_L	Lost sale cost per unit item ($)
C_d	Deterioration cost per unit item ($)
k	Scale parameter of sales team efforts
$\aleph$	Shape parameter of sales team efforts

Following assumptions are utilized to develop the model for manufacturing system where manufacturing process carried out in single stage.

3.2. Assumptions

(1) Planning horizon is infinite.
(2) Single stage manufacturing system is considered for the production of single item. The manufacturer establishes the setup for the manufacturing and reworking process at the beginning of the planning horizon.
(3) Customer's demand is the function of sales team efforts (ρ) and selling price (s). It is of the form $D(s, \rho) = d_0 - d_1 s + d_2 \rho$ where d_0, d_1, $d_2 > 0$.
(4) Production process is not perfect. μ_0 is the proportion of defective items. It is assumed that proportion of defectiveness μ_0 can be mitigated to μ by the capital investment $\frac{\emptyset}{\alpha} \ln\left(\frac{\mu_0}{\mu}\right) T$ where $0 \leq \mu \leq \mu_0$, α is the percentage of the decrease in defectiveness, T is the cycle length, and $\emptyset$ is the opportunity cost.
(5) It is taken that μ_r fraction of imperfect items is scrap and $1-\mu_r$ fraction of imperfect items can be reworked.
(6) The effective management of scrap products and deteriorated units presents a significant problem for manufacturers when implementing a clean manufacturing system. The circular economy has advanced with this crucial stage. Thus, it is assumed that the manufacturer uses effective waste management strategies for scrap and deteriorated goods in light of this.
(7) Production rate is taken as variable. Further, unit production cost $\omega(P)$ is the function of P where $\omega(P) = C_0 + \frac{C_1}{P} + C_2 P$. C_0 is raw material cost, C_1 is labor charges, C_2 is tool/die cost.
(8) Selling price is linked with the unit production cost by the relation $s = m\omega(P)$ where $m(>1)$ is the markup price.
(9) Deterioration is unavoidable because of physical presence of items in the stock. So, to mitigate the rate of deterioration (θ), an investment β is made in preservation technology. Consider $\chi(\beta) = 1 - e^{-\gamma\beta}$ as the function of β and γ is its efficiency coefficient. Now, $1 - \chi(\beta)$ *i.e.*, $e^{-\gamma\beta}$ is the proportion of decreasing deterioration rate which is the function of preservation technology. Under the effect of preservation technology, revised deterioration rate is $\theta e^{-\gamma\beta}$.
(10) Shortages are allowed and further, during the stock-out period, only $\sigma(t)$ fraction of demand is satisfied and $1 - \sigma(t)$ fraction of demand are lost where $\sigma(t) = \sigma_0 e^{-\sigma_1 t}$, $\sigma_0 < 1, \sigma_1 \geq 0$, and t is the waiting period for the customers.

4. Mathematical formulation of single-stage cleaner production system

This section contains mathematical formulation of a single-stage cleaner production system in which it is assumed that production system is faulty. At $t = 0$, inventory level is zero and shortages build up upto $t = t_1$ and at $t = t_1$ cleaner production process starts which is utilized to meet the current demand and backlogged demand. At $t = t_2$, inventory level becomes zero. After that production process continues upto $t = t_3$. At that point, cleaner production process stops. During the production process, proportion of defective items μP transferred to the reworking station where reworking starts at the end of production process. During the period $[t_4, t_5]$ reworking process undergoes and it is assumed that rate of reworking is more than the demand rate. Certain proportion μ_r is found to remain non-reworkable. That items treated as scrap and go through the waste management process taking into account of environment. Remaining proportion $(1 - \mu_r)$ is considered as good items and transferred to service station from where the demand of customers is satisfied. Pictorial representation of the single-stage cleaner production system is represented in Figures 2a and 2b.

Inventory level at the service station can be depicted with the help of following differential equation:

$$\frac{\mathrm{d}I_1(t)}{\mathrm{d}t} = -D(s,\rho)\sigma_0 e^{-\sigma_1(t_1-t)}, \qquad 0 \le t \le t_1,\ I_1(0) = 0 \tag{1}$$

$$\frac{\mathrm{d}I_2(t)}{\mathrm{d}t} = (1-\mu)P - D(s,\rho), \qquad t_1 \le t \le t_2,\ I_2(t_2) = 0 \tag{2}$$

$$\frac{\mathrm{d}I_3(t)}{\mathrm{d}t} = (1-\mu)P - D(s,\rho) - \theta e^{-\gamma\beta} I_3(t), \qquad t_2 \le t \le t_3,\ I_3(t_2) = 0 \tag{3}$$

$$\frac{\mathrm{d}I_4(t)}{\mathrm{d}t} = (1-\mu_r)R - D(s,\rho) - \theta e^{-\gamma\beta} I_4(t), \qquad t_3 \le t \le t_4,\ I_4(t_3) = I_3(t_3) \tag{4}$$

$$\frac{\mathrm{d}I_5(t)}{\mathrm{d}t} = -D(s,\rho) - \theta e^{-\gamma\beta} I_5(t), \qquad t_4 \le t \le T,\ I_5(T) = 0. \tag{5}$$

Inventory level at the re-working station can be depicted with the help of following differential equation:

$$\frac{\mathrm{d}I_6(t)}{\mathrm{d}t} = \mu P - \theta e^{-\gamma\beta} I_6(t), \qquad t_1 \le t \le t_3,\ I_6(t_1) = 0 \tag{6}$$

$$\frac{\mathrm{d}I_7(t)}{\mathrm{d}t} = -R - \theta e^{-\gamma\beta} I_7(t), \qquad t_3 \le t \le t_4,\ I_7(t_4) = 0. \tag{7}$$

Following results are obtained on solving the above equations:

$$I_1(t) = \frac{D(s,\rho)\sigma_0}{\sigma_1}\left(e^{-\sigma_1 t_1} - e^{-\sigma_1(t_1-t)}\right), \qquad 0 \le t \le t_1 \tag{8}$$

$$I_2(t) = ((1-\mu)P - D(s,\rho))(t - t_2), \qquad t_1 \le t \le t_2 \tag{9}$$

$$I_3(t) = \frac{(1-\mu)P - D(s,\rho)}{\theta e^{-\gamma\beta}}\left(1 - e^{-\theta e^{-\gamma\beta}(t-t_2)}\right), \qquad t_2 \le t \le t_3, \tag{10}$$

$$I_4(t) = \frac{(1-\mu_r)R - D(s,\rho)}{\theta e^{-\gamma\beta}}\left(1 - e^{\theta e^{-\gamma\beta}(t_3-t)}\right) + \frac{(1-\mu)P - D(s,\rho)}{\theta e^{-\gamma\beta}}\left(e^{-\theta e^{-\gamma\beta}(t-t_3)} - e^{-\theta e^{-\gamma\beta}(t-t_2)}\right), \qquad t_3 \le t \le t_4, \tag{11}$$

$$I_5(t) = \frac{D(s,\rho)}{\theta e^{-\gamma\beta}}\left(e^{\theta e^{-\gamma\beta}(T-t)} - 1\right), \qquad t_4 \le t \le T, \tag{12}$$

$$I_6(t) = \frac{\mu P}{\theta e^{-\gamma\beta}}\left(1 - e^{\theta e^{-\gamma\beta}(t_1-t)}\right), \qquad t_1 \le t \le t_3, \tag{13}$$

$$I_7(t) = \frac{R}{\theta e^{-\gamma\beta}}\left(e^{\theta e^{-\gamma\beta}(t_4-t)} - 1\right), \qquad t_3 \le t \le t_4. \tag{14}$$

Taking the equation of continuity for inventory level, following relations are obtained:

$$t_2 = t_1 + \frac{D(s,\rho)\sigma_0}{\sigma_1((1-\mu)P - D(s,\rho))}\left(1 - e^{-\sigma_1 t_1}\right) \tag{15}$$

$$T = t_4 + \frac{1}{\theta e^{-\gamma\beta}} \log\Bigg(1 + \frac{(1-\mu_r)R - D(s,\rho)}{D(s,\rho)}\left(1 - e^{\theta e^{-\gamma\beta}(t_3 - t_4)}\right) + \frac{(1-\mu)P - D(s,\rho)}{D(s,\rho)}\left(e^{-\theta e^{-\gamma\beta}(t_4 - t_3)} - e^{-\theta e^{-\gamma\beta}(t_4 - t_2)}\right)\Bigg) \tag{16}$$

$$t_4 = t_3 + \frac{1}{\theta e^{-\gamma\beta}} \ln\left(1 + \frac{\mu}{R}\left(1 - e^{-\theta e^{-\gamma\beta}(t_3 - t_1)}\right)\right). \tag{17}$$

Now, different costs associated with single-stage cleaner production system are presented here under the effect of inflation.

Total revenue generated by the manufacturer under inflation is

$$\int_{t_1}^{T} sD(s,\rho)e^{-rt}\mathrm{d}t + sI_1(t)e^{-rt_1} = \frac{sD(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) + \frac{sD(s,\rho)\sigma_0}{\sigma_1}\left(e^{-(r+\sigma_1)t_1} - e^{-rt_1}\right).$$

Inflation induced setup cost for the single-stage cleaner production system and re-working station is $S_m + S_r$.

Inflation induced production cost for the single-stage cleaner production system is

$$\int_{t_1}^{t_2} \omega(P)Pe^{-rt}\mathrm{d}t + \int_{t_2}^{t_3} \omega(P)Pe^{-rt}\mathrm{d}t = \frac{\omega(P)P}{r}\left[e^{-rt_1} - e^{-rt_3}\right].$$

Production process is not perfect. To make the system cleaner, re-working process is done at re-working station. Thus, inflation induced re-working cost is

$$\int_{t_3}^{t_4} C_R R e^{-rt}\mathrm{d}t = \frac{C_R R}{r}\left(e^{-rt_3} - e^{-rt_4}\right).$$

Here, deterioration takes when the inventory physically present at the manufacturer end as well as the re-working station. Thus, deterioration cost under the effect of inflation is

$$\begin{aligned}
&\int_{t_1}^{t_3} C_d P e^{-rt}\mathrm{d}t - \int_{0}^{t_1} C_d D(s,\rho)\sigma_0 e^{-\sigma_1(t_1 - t)}e^{-rt}\mathrm{d}t - \int_{t_1}^{T} C_d D(s,\rho)e^{-rt}\mathrm{d}t - \int_{t_3}^{t_4} C_d \mu_r R e^{-rt}\mathrm{d}t\\
&= \frac{C_d P}{r}\left(e^{-rt_1} - e^{-rt_3}\right) - \frac{C_d D(s,\rho)\sigma_0}{r - \sigma_1}\left(e^{-\sigma_1 t_1} - e^{-\sigma_1 t_1 - (r-\sigma_1)t_1}\right)\\
&\quad - \frac{C_d D(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) - \frac{C_d \mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right).
\end{aligned}$$

After performing the re-working process, some proportion of defective items cannot be used to serve the customer. This proportion of items is termed as scrap items and it has hazardous impact on the environment. Thus, proper waste management is very necessary for these scrap items as well as the deteriorated items. Inflation induced waste management cost for scrap items is

$$\int_{t_3}^{t_4} C_{wm}\mu_r R e^{-rt}\mathrm{d}t = \frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right),$$

and waste management cost for deteriorated items is

$$\frac{C_{wm}P}{r}\left(e^{-rt_1} - e^{-rt_3}\right) - \frac{C_{wm}D(s,\rho)\sigma_0}{r - \sigma_1}\left(e^{-\sigma_1 t_1} - e^{-\sigma_1 t_1 - (r-\sigma_1)t_1}\right)$$

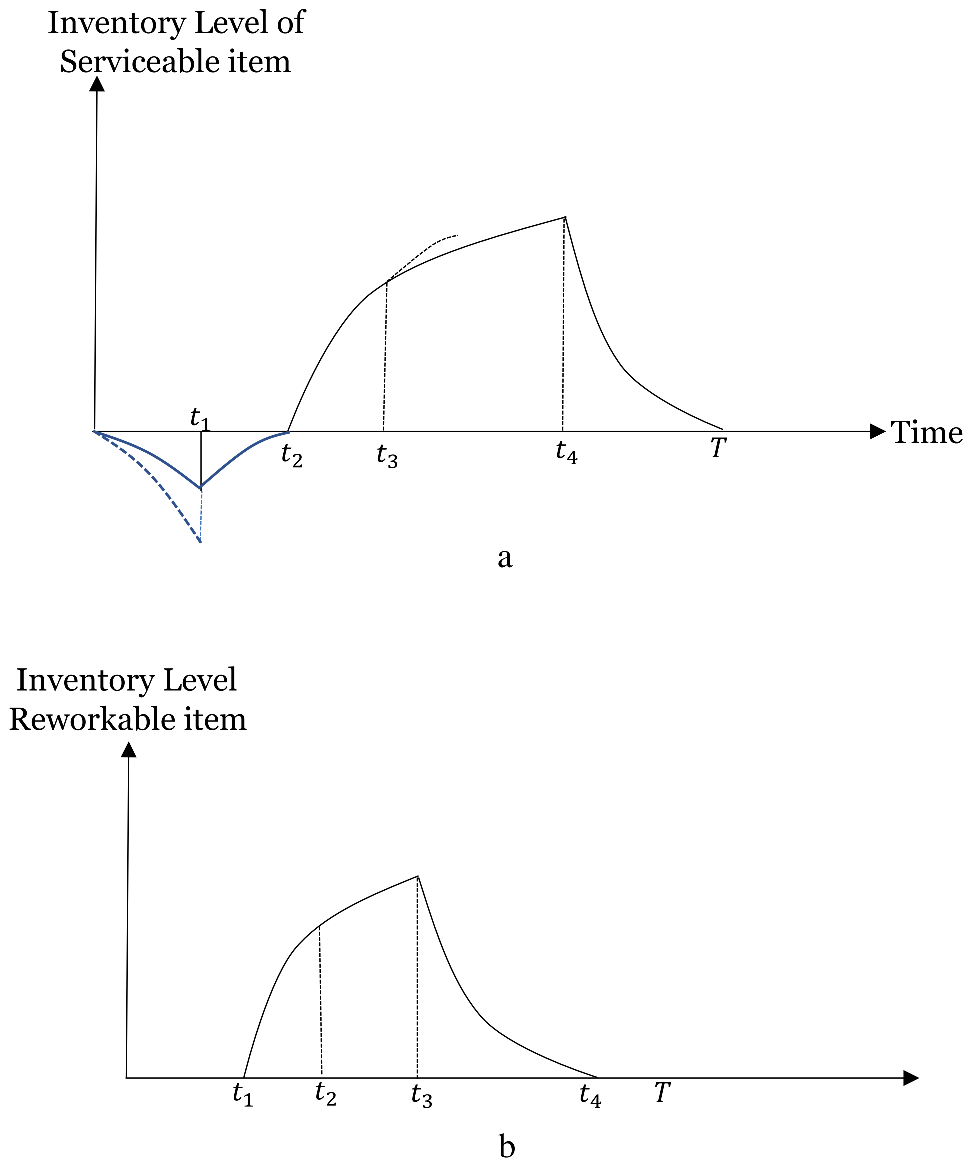


FIGURE 2. (a) Inventory level of serviceable item. (b) Inventory level of reworkable item.

$$- \frac{C_{wm} D(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) - \frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right).$$

Thus, total waste management cost is

$$\frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right) + \frac{C_{wm} P}{r}\left(e^{-rt_1} - e^{-rt_3}\right) - \frac{C_{wm} D(s,\rho)\sigma_0}{r-\sigma_1}\left(e^{-\sigma_1 t_1} - e^{-\sigma_1 t_1 - (r-\sigma_1)t_1}\right)$$
$$- \frac{C_{wm} D(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) - \frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right).$$

Items are stored at serviceable centre and re-working station. So, inflation induced holding cost is

$$\int_{t_2}^{t_3} C_{hs} I_3(t) e^{-rt} \mathrm{d}t + \int_{t_3}^{t_4} C_{hs} I_4(t) e^{-rt} \mathrm{d}t + \int_{t_4}^{T} C_{hs} I_5(t) e^{-rt} \mathrm{d}t + \int_{t_1}^{t_3} C_{hr} I_6(t) e^{-rt} \mathrm{d}t + \int_{t_3}^{t_4} C_{hr} I_7(t) e^{-rt} \mathrm{d}t$$

$$= C_{hs}\left(\frac{(1-\mu)P - D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left\{\frac{1}{r}\left(e^{-rt_2} - e^{-rt_3}\right) + \frac{e^{\theta e^{-\gamma\beta} t_2}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_2}\right)\right\}$$

$$+ C_{hs}\left\{\left(\frac{(1-\mu_r)R - D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left(\frac{1}{r}\left(e^{-rt_3} - e^{-rt_4}\right) + \frac{e^{\theta e^{-\gamma\beta} t_3}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right)\right)\right.$$

$$+ \left(\frac{(1-\mu)P - D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left(\frac{e^{\theta e^{-\gamma\beta} t_3}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right)\right.$$

$$\left.\left. + \frac{e^{\theta e^{-\gamma\beta} t_2}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right)\right)\right\} + \frac{C_{hs} D(s,\rho)}{\theta e^{-\gamma\beta}}\left(\frac{e^{\theta e^{-\gamma\beta} T}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4} - e^{-\left(\theta e^{-\gamma\beta}+r\right)T}\right)\right.$$

$$\left. + \frac{1}{r}\left(e^{-rT} - e^{-rt_4}\right)\right) + \frac{C_{hr}\mu P}{\theta e^{-\gamma\beta}}\left(\frac{1}{r}\left(e^{-rt_1} - e^{-rt_3}\right) + \frac{e^{\theta e^{-\gamma\beta} t_1}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_1}\right)\right)$$

$$+ \frac{C_{hr} R}{\theta e^{-\gamma\beta}}\left(\frac{e^{\theta e^{-\gamma\beta} t_4}}{\theta e^{-\gamma\beta} + r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3} - e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right) + \frac{1}{r}\left(e^{-rt_4} - e^{-rt_3}\right)\right).$$

Shortage cost under the effect of inflation is

$$\int_0^{t_1} C_s\left(-I_1(t) e^{-rt}\right)\mathrm{d}t + \int_{t_1}^{t_2} C_s\left(-I_2(t) e^{-rt}\right)\mathrm{d}t = C_s\left[\frac{D(s,\rho)\sigma_0 e^{-\sigma_1 t_1}}{\sigma_1}\left(\frac{\left(e^{-rt_1} - 1\right)}{r} + \frac{\left(e^{(\sigma_1 - r)t_1} - 1\right)}{\sigma_1 - r}\right)\right.$$

$$\left. + ((1-\mu)P - D(s,\rho))\left\{\frac{(t_2 - t_1)e^{-rt_1}}{r} + \frac{e^{-rt_2} - e^{-rt_1}}{r^2}\right\}\right].$$

Lost sale cost under the effect of inflation is

$$\int_0^{t_1} C_L\left(1 - \sigma_0 e^{-\sigma_1(t_1 - t)}\right) D(s,\rho) e^{-rt}\mathrm{d}t = C_L D(s,\rho)\left[\frac{\sigma_0}{\sigma_1 - r}\left(e^{-\sigma_1 t_1} - e^{-rt_1}\right) + \frac{1}{r}\left(1 - e^{-rt_1}\right)\right].$$

Capital invested in preservation technology to diminish the deterioration rate under inflation is $\beta T e^{-rT}$.

Capital investment to reduce the fraction of imperfect production proportion is $\frac{\emptyset}{\alpha}\ln\left(\frac{\mu_0}{\mu}\right)T e^{-rT}$.

Investment in promotion activities made by the sales team under inflation is $k\rho^{\aleph} e^{-rT}$.

Inflation induced screening cost for the single-stage cleaner production system paid at the end of cycle is $P(t_3 - t_1)e^{-rT}$.

Finally, total profit of the manufacturer = revenue-setup cost – production cost-re-working cost – deterioration cost – waste management cost – holding cost – shortages cost – lost sale cost – preservation technology cost – capital investment to reduce the fraction of imperfect production proportion.

Hence, total profit of the manufacturer per unit time

$$(\mathrm{TP}) = \frac{1}{T}\left[\frac{sD(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) + \frac{sD(s,\rho)\sigma_0}{\sigma_1}\left(e^{-(r+\sigma_1)t_1} - e^{-rt_1}\right) - (S_m + S_r) - \frac{\omega(P)P}{r}\left(e^{-rt_1} - e^{-rt_3}\right)\right.$$

$$- \frac{C_R R}{r}\left(e^{-rt_3} - e^{-rt_4}\right) - \frac{C_d P}{r}\left(e^{-rt_1} - e^{-rt_3}\right) + \frac{C_d D(s,\rho)\sigma_0}{r - \sigma_1}\left(e^{-\sigma_1 t_1} - e^{-\sigma_1 t_1 - (r - \sigma_1)t_1}\right)$$

$$+ \frac{C_d D(s,\rho)}{r}\left(e^{-rt_1} - e^{-rT}\right) + \frac{C_d \mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right) - \frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3} - e^{-rt_4}\right)$$

$$
\begin{aligned}
&-\frac{C_{wm}P}{r}\left(e^{-rt_1}-e^{-rt_3}\right)+\frac{C_{wm}D(s,\rho)\sigma_0}{r-\sigma_1}\left(e^{-\sigma_1 t_1}-e^{-\sigma_1 t_1-(r-\sigma_1)t_1}\right)+\frac{C_{wm}D(s,\rho)}{r}\left(e^{-rt_1}-e^{-rT}\right)\\
&+\frac{C_{wm}\mu_r R}{r}\left(e^{-rt_3}-e^{-rt_4}\right)-C_{hs}\left(\frac{(1-\mu)P-D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left\{\frac{1}{r}\left(e^{-rt_2}-e^{-rt_3}\right)+\frac{e^{\theta e^{-\gamma\beta}t_2}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right.\right.\\
&\left.\left.-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_2}\right)\right\}-C_{hs}\left\{\left(\frac{(1-\mu_r)R-D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left(\frac{1}{r}\left(e^{-rt_3}-e^{-rt_4}\right)+\frac{e^{\theta e^{-\gamma\beta}t_3}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right.\right.\right.\\
&\left.\left.-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right)\right)+\left(\frac{(1-\mu)P-D(s,\rho)}{\theta e^{-\gamma\beta}}\right)\left(\frac{e^{\theta e^{-\gamma\beta}t_3}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right)\right.\\
&\left.\left.+\frac{e^{\theta e^{-\gamma\beta}t_2}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right)\right)\right\}-\frac{C_{hs}D(s,\rho)}{\theta e^{-\gamma\beta}}\left(\frac{e^{\theta e^{-\gamma\beta}T}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right.\right.\\
&\left.\left.-e^{-\left(\theta e^{-\gamma\beta}+r\right)T}\right)+\frac{1}{r}\left(e^{-rT}-e^{-rt_4}\right)\right)-\frac{C_{hr}\mu P}{\theta e^{-\gamma\beta}}\left(\frac{1}{r}\left(e^{-rt_1}-e^{-rt_3}\right)+\frac{e^{\theta e^{-\gamma\beta}t_1}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}\right.\right.\\
&\left.\left.-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_1}\right)\right)-\frac{C_{hr}R}{\theta e^{-\gamma\beta}}\left(\frac{e^{\theta e^{-\gamma\beta}t_4}}{\theta e^{-\gamma\beta}+r}\left(e^{-\left(\theta e^{-\gamma\beta}+r\right)t_3}-e^{-\left(\theta e^{-\gamma\beta}+r\right)t_4}\right)+\frac{1}{r}\left(e^{-rt_4}-e^{-rt_3}\right)\right)\\
&-C_s.\left(\frac{D(s,\rho)\sigma_0 e^{-\sigma_1 t_1}}{\sigma_1}\left(\frac{\left(e^{-rt_1}-1\right)}{r}+\frac{\left(e^{(\sigma_1-r)t_1}-1\right)}{\sigma_1-r}\right)+((1-\mu)P-D(s,\rho))\right.\\
&\left.\times\left\{\frac{(t_2-t_1)e^{-rt_1}}{r}+\frac{e^{-rt_2}-e^{-rt_1}}{r^2}\right\}\right)-C_L D(s,\rho)\left(\frac{\sigma_0}{\sigma_1-r}\left(e^{-\sigma_1 t_1}-e^{-rt_1}\right)+\frac{1}{r}\left(1-e^{-rt_1}\right)\right)\\
&\left.-\beta T e^{-rT}-\frac{\emptyset}{\alpha}\ln\left(\frac{\mu_0}{\mu}\right)Te^{-rT}-k\rho^{\aleph}e^{-rT}-P(t_3-t_1)e^{-rT}\right]. \qquad (18)
\end{aligned}
$$

The manufacturer's goal in the current problem is to maintain the clean production process as the primary focus while making the production system financially sustainable. In order to maximize profit, a manufacturer must choose the optimal values for production rate, selling price, shortage time, and manufacturing length. Thus, objective function of the current problem is as follows:

$$
\begin{aligned}
&\text{Max TP}(P,s,t_1,t_3)\\
&P>0, s>0, t_1>0, t_3>0. \qquad (19)
\end{aligned}
$$

5. Theoretical result and solution methodology

In this section, we theoretically explore the concavity of the objective function with respect to different decision variables. To derive the theoretical results, we apply some of the theorems of Cambini and Martein ([62], p. 245). According to them, if any function can be written as

$$f(x)=\frac{F(x)}{G(x)}, x\varepsilon R^n$$

$f(x)$ is (strictly) pseudo concave function, if $F(x)$ is concave and differentiable and $G(x)$ is positive and affine. Objective of the decision-maker is to optimize the objective function given in equation (19) where P, s, t_1, and t_3 are decision variables.

Theorem 1. *If* $\zeta_{11}<0, \zeta_{11}\zeta_{22}-\zeta_{12}\zeta_{21}>0, -\zeta_{13}\zeta_{22}\zeta_{31}+\zeta_{12}\zeta_{23}\zeta_{31}+\zeta_{13}\zeta_{21}\zeta_{32}-\zeta_{11}\zeta_{23}\zeta_{32}-\zeta_{12}\zeta_{21}\zeta_{33}+\zeta_{11}\zeta_{22}\zeta_{33}<0, \zeta_{14}\{\zeta_{14}(\zeta_{23})^2-\zeta_{14}\zeta_{22}\zeta_{33}-\zeta_{23}\zeta_{31}\zeta_{24}+\zeta_{12}\zeta_{33}\zeta_{24}+\zeta_{22}\zeta_{13}\zeta_{34}-\zeta_{12}\zeta_{23}\zeta_{34}\}+\zeta_{13}\{-\zeta_{24}\zeta_{23}\zeta_{14}+\zeta_{22}\zeta_{34}\zeta_{14}+\zeta_{12}\zeta_{23}\zeta_{44}+\zeta_{12}\zeta_{23}\zeta_{44}-\zeta_{12}\zeta_{34}\zeta_{24}-\zeta_{13}\zeta_{22}\zeta_{44}\}+\zeta_{12}\{\zeta_{33}\zeta_{14}\zeta_{24}-\zeta_{23}\zeta_{34}\zeta_{14}-\zeta_{24}\zeta_{13}\zeta_{34}+\zeta_{12}(\zeta_{34})^2+\zeta_{23}\zeta_{13}\zeta_{44}-\zeta_{12}\zeta_{33}\zeta_{44}\}+$

$\zeta_{11}\{-\zeta_{22}(\zeta_{34})^2-(\zeta_{23})^2\zeta_{44}+\zeta_{22}\zeta_{33}\zeta_{44}-(\zeta_{24})^2\zeta_{33}+\zeta_{23}\zeta_{34}\zeta_4 2+\zeta_{24}\zeta_{23}\zeta_{43}\}>0$, *where* $\zeta_{11}=\frac{\partial^2\mathrm{TP}}{\partial P^2};\zeta_{12}=\zeta_{21}=\frac{\partial^2\mathrm{TP}}{\partial P\partial s};\zeta_{13}=\zeta_{31}=\frac{\partial^2\mathrm{TP}}{\partial P\partial t_1};\zeta_{14}=\zeta_{41}=\frac{\partial^2\mathrm{TP}}{\partial P\partial t_3};\zeta_{22}=\frac{\partial^2\mathrm{TP}}{\partial s^2};\zeta_{23}=\zeta_{32}=\frac{\partial^2\mathrm{TP}}{\partial s\partial t_1};\zeta_{24}=\zeta_{42}=\frac{\partial^2\mathrm{TP}}{\partial s\partial t_3};\zeta_{33}=\frac{\partial^2\mathrm{TP}}{\partial t_1^2};\zeta_{34}=\zeta_{43}=\frac{\partial^2\mathrm{TP}}{\partial t_1\partial t_3};\zeta_{44}=\frac{\partial^2\mathrm{TP}}{\partial t_3^2}$ *then* $\mathrm{TP}(P,s,t_1,t_3)$ *will be maximum at the point* (P,s^*,t_1^*,t_3^*).

Proof. Consider the Hessian matrix

$$\begin{bmatrix} \frac{\partial^2\mathrm{TP}}{\partial P^2} & \frac{\partial^2\mathrm{TP}}{\partial P\partial s} & \frac{\partial^2\mathrm{TP}}{\partial P\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial P\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial s\partial P} & \frac{\partial^2\mathrm{TP}}{\partial s^2} & \frac{\partial^2\mathrm{TP}}{\partial s\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial s\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial t_1\partial P} & \frac{\partial^2\mathrm{TP}}{\partial t_1\partial s} & \frac{\partial^2\mathrm{TP}}{\partial t_1^2} & \frac{\partial^2\mathrm{TP}}{\partial t_1\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial t_3\partial P} & \frac{\partial^2\mathrm{TP}}{\partial t_3\partial s} & \frac{\partial^2\mathrm{TP}}{\partial t_3\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial t_3^2} \end{bmatrix}.$$

If the conditions given in statement of the theorem holds then

$$H_{11}<0;H_{22}>0;H_{33}<0;H_{44}>0.$$

Thus, Hessian matrix is negative semi-definite. Hence, obtained point on solving the following equations:

$$\frac{\partial\mathrm{TP}}{\partial P}=0;\frac{\partial\mathrm{TP}}{\partial s}=0;\frac{\partial\mathrm{TP}}{\partial t_1}=0;\frac{\partial\mathrm{TP}}{\partial t_3}=0$$

gives global optimal solution. □

5.1. Solution methodology

It is critical to tune up the manufacturing process for a cleaner production process so that the production system does not go out of control. Thus, getting an optimal production rate is very crucial for the production system. Furthermore, the selling price choice is critical since all of the manufacturer's inventory control strategies are based on the customer's demand, which is based on the selling price. Taking all of this into account, the manufacturer's profit is maximised by establishing the optimal values of t_1, t_3, P, and s. The objective function is a non-linear function of the decision variables in this case. As a result, traditional optimization approaches will not provide an optimal solution. The process for getting the optimum solution is described in this section. First, we suppose that t_1, t_3, P, and s are non-zero real numbers that maximise the manufacturer's profit and satisfy the first order optimality requirements as follows:

$$\frac{\partial\mathrm{TP}}{\partial P}=0 \tag{20}$$

$$\frac{\partial\mathrm{TP}}{\partial s}=0 \tag{21}$$

$$\frac{\partial\mathrm{TP}}{\partial t_1}=0 \tag{22}$$

$$\frac{\partial\mathrm{TP}}{\partial t_3}=0. \tag{23}$$

Further, TP is concave, if at P^*, s^*, t_1^*, and t_3^*, the Hessian matrix of TP is negative semidefinite. Hessian matrix is defined as follows:

$$\begin{bmatrix} \frac{\partial^2\mathrm{TP}}{\partial P^2} & \frac{\partial^2\mathrm{TP}}{\partial P\partial s} & \frac{\partial^2\mathrm{TP}}{\partial P\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial P\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial s\partial P} & \frac{\partial^2\mathrm{TP}}{\partial s^2} & \frac{\partial^2\mathrm{TP}}{\partial s\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial s\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial t_1\partial P} & \frac{\partial^2\mathrm{TP}}{\partial t_1\partial s} & \frac{\partial^2\mathrm{TP}}{\partial t_1^2} & \frac{\partial^2\mathrm{TP}}{\partial t_1\partial t_3} \\ \frac{\partial^2\mathrm{TP}}{\partial t_3\partial P} & \frac{\partial^2\mathrm{TP}}{\partial t_3\partial s} & \frac{\partial^2\mathrm{TP}}{\partial t_3\partial t_1} & \frac{\partial^2\mathrm{TP}}{\partial t_3^2} \end{bmatrix}.$$

TABLE 2. Optimal solution of problem.

t_1^* (month)	t_3^* (month)	P^* (units/ month)	s^* ($)	TP* ($)
2.78	11.81	18.85	1.58	2.94338×10^6

5.2. Solution algorithm

Step 1. Set $s = 0$, $t_3 = 0$ and $t_1 = 0$ and find P^* using equation (19).
Step 2. Evaluate the value of s^* using equation (20) by substituting the values of step 1.
Step 3. Evaluate the value of t_1^* using equation (21) by substituting the revised values of step 2.
Step 4. Evaluate the value of t_3^* using equation (22) by substituting the revised values of step 3.
Step 5. Again apply all of the above steps with the help of revised values of P^*, s^*, t_1^*, t_3^*. Continue this steps till there is no change in the values of P^*, s^*, t_1^*, t_3^*.
Step 6. Find TP(P^*, s^*, t_1^*, t_3^*) of the manufacturer.

6. Numerical example

To illustrate the developed model, following data has been taken from the literature in appropriate units: $d_0 = 114$ (units/month); $d_1 = 0.21$; $d_2 = 5.0$; $\rho = 1.5$ ($); $\mu_0 = 0.3$; $\mu = 0.2$; $\emptyset = 263$ ($); $\alpha = 2$; $\gamma = 0.0005$; $\beta = 160$ ($); $\mu_r = 0.2$; $\theta = 0.04$ (unit); $R = 30$ (units/month); $S_m = 300$ ($); $S_r = 100$ ($); $C_1 = 5$ ($); $C_{wm} = 4$ ($); $C_{hs} = 1$ ($); $C_{hr} = 0.5$ ($); $C_s = 2.5$ ($); $C_l = 3$ ($); $k = 10$; $\aleph = 2$; $C_0 = 3$ ($); $C_1 = 2$ ($); $C_2 = 0.05$ ($); $\sigma_0 = 0.9$; $\sigma_1 = 0.2$; $r = 0.1$; $cd = 0.5$ ($).

On applying the solution methodology mentioned in Section 5, solution of the equation (19)–(22) is $(18.85, 1.58, 2.78, 11.81)$. Now, check the nature of Hessian matrix. It is observed that

$$H_{11} = -2.03 \times 10^9, H_{22} = 3.84 \times 10^{14}, H_{33} = -6.28 \times 10^{28}, H_{44} = 8.01 \times 10^{30}.$$

Thus, Hessian matrix is negative semi-definite. Therefore, at $(18.85, 1.58, 2.78, 11.81)$ objective function is maximum. Hence, optimal solution of the current problem is as follows (see Tab. 2).

Remark 1. It is clear from Table 1 and the literature evaluation that no researcher has presented the cleaner production system focussing on environment sustainability, social sustainability, and economic sustainability simultaneously along with quality production. These are the focussed areas of current study. Practically, it is not impossible to compare the results of current study numerically with the published research. However, it has been observed that after relaxing the certain assumptions, base model of the current study is similar to the published paper in this direction.

– If we relax the assumptions of deterioration, preservation technology, capital investment in quality, waste management and considered stock and selling price dependent demand and constant production rate then the base model of current study reduces to Manna *et al.* [10].
– If we relax the assumptions of deterioration, preservation technology, capital investment in quality, inflation, waste management and considered constant demand and constant production cost then the base model of current study reduces to Öztürk [54].
– If we relax the assumptions of capital investment in quality, waste management, reworking, inflation, partial backlogging and considered constant demand and constant production cost then the base model of current study reduces to Shen *et al.* [34].
– If we relax the assumptions of deterioration, preservation technology, capital investment in quality, inflation, partial backlogging, reworking and considered constant demand and constant production cost then the base model of current study reduces to Ahmad *et al.* [53].

7. Sensitive analysis

In order to observe the change in profit due to variation in different key parameters associated with inventory system. Effect of different parameters graphically summarized in Figures 3a–3i and Table 3.

(i) Figure 3a shows that as the reworking rate rises, the system's total profit goes up as well. Due to an increase in the reworking rate, more imperfect items can be sold with the price of perfect ones.
(ii) Figure 3b shows that an increase in different parameters associated with partial backlogging rate results in a decline in profit for the system. This is because, for higher customer satisfaction, the decision-maker has to compromise on the profit.
(iii) Large amount of capital is associated in inventory in the form of holding cost. Figure 3c shows the impact of holding cost on the profit. Results pointed out that as the holding cost increases total profit decreases.
(iv) Increase in manufacturer's setup cost and re-working station's setup cost results in a decline in the profit (Fig. 3d). The negative impact of these costs can be mitigated by implementing setup cost-cutting technologies.
(v) Effect of efficiency coefficient of preservation technology and deterioration are presented in Figure 3e. Results indicate that as the deterioration rate increases, profit decline. Further, profit increases due to increase in the efficiency coefficient of preservation mechanism. Thus, as the efficiency coefficient of preservation mechanism increases, the rate of deterioration reduces, which leads to accumulating more revenue by selling them.
(vi) Figure 3f shows the effect of α, and $\emptyset$ on the profit. It is observed that increase in opportunity cost results decline in the total profit while profit inclined due to the increase in the percentage of the decreasing defectiveness.
(vii) Figure 3g shows the impact of inflation on the system and shows the current scenario of the financial position of the inventory. Results indicate that profit decline as the inflation rate increases.
(viii) Through Figure 3h effect of deterioration cost is visualized on the profit. It is observed that profit decline due to increase in deterioration cost. All of these increase the importance of preservation technology for the system.
(ix) As the value of μ_0 and μ_r increase, the profit of the system decreases. More reworking is required for higher value of μ_0 and more scrapping occurs due to the high value of μ_r. All of this leads to lower profit for the system (See Figure 3i).

8. Managerial implications

The goal of this paper is to give decision-makers a basic understanding of how to deal with inventory when the manufacturing process isn't perfect. This section contains a few recommendations to the decision-maker. These are as follows:

(a) Decision-makers must be more cautious while adopting the mechanism of preservation technology for the system where deterioration takes place. This is because the efficiency coefficient of preservation technology has a positive effect on the profit. Preservation technology helps to increase the profit of the system and also reduces the waste generation in the production system. In addition to this, adoption of proper preservation technology helps to create a clean and green atmosphere, which is the primary goal of every government. Without preservation, because of deterioration, production system faces shortage.
(b) This analysis also recommends preservation technologies to the decision-maker based on the expense of deterioration. The decision-maker should use preservation technology when the expense of deterioration or negative impact of waste on the environment is substantial. Selection of right preservation technology is very important. Because for this, managers have to pay attention on different aspects such as availability of fund, and compatibility of the mechanism with the production system.
(c) Generally, setup cost of any flexible manufacturing system is very high. Current study suggest that decision-makers have to adopt the mechanism that reduce setup cost for the flexible manufacturing process.

Table 3. Sensitivity analysis with respect to different parameters.

Parameter	% Change in parameter	Profit of the system	Parameter	% Change in Parameter	Profit of the System
R	−25	2 413 571.60	C_{hs}	−25	3 037 568.16
	−10	2 707 909.60		−10	2 981 643.94
	10	3 178 850.40		10	2 890 399.16
	25	3 473 188.40		25	2 855 078.60
σ_0	−25	2 949 266.76	C_{hr}	−25	3 055 228.44
	−10	2 946 323.38		−10	3 031 681.40
	10	2 943 115.10		10	2 943 377.06
	25	2 942 202.65		25	2 913 651.86
σ_1	−25	2 952 210.14	S_m	−25	2 969 870.42
	−10	2 947 795.07		−10	2 949 266.76
	10	2 943 377.06		10	2 943 085.66
	25	2 942 791.32		25	2 932 195.15
r	−25	3 031 681.40	S_r	−25	2 946 029.04
	−10	2 978 700.56		−10	2 943 380.29
	10	2 884 512.40		10	2 937 552.10
	25	2 855 078.60		25	2 881 833.92
γ	−25	2 919 832.96	α	−25	2 943 321.13
	−10	2 925 719.72		−10	2 943 350.57
	10	2 955 153.52		10	2 943 409.43
	25	2 966 927.04		25	2 943 438.87
θ	−25	2 946 323.38	$\emptyset$	−25	2 943 438.87
	−10	2 944 851.69		−10	2 943 403.55
	10	2 943 232.83		10	2 943 350.57
	25	2 937 493.24		25	2 943 262.26
C_d	−25	2 955 742.19	μ_r	−25	2 945 146.03
	−10	2 950 149.77		−10	2 944 263.01
	10	2 932 783.83		10	2 943 377.06
	25	2 928 368.76		25	2 943 294.64
μ_0	−25	2 944 263.01			
	−10	2 943 674.34			
	10	2 943 374.11			
	25	2 943 229.89			

(d) Current study suggests that capital investment in quality improvement, which mitigates the fraction of defective production, increases the profit of the system as well as reduces the waste from the system. Thus, it is beneficial for the production system to explore technologies that can mitigate the rate of imperfect production. This helps to achieve the target of cleaner production.

(e) Waste produced during the production process has a hazardous impact on the environment and on future generations. This study provides direction to the decision-maker on how they can bear his/her responsibility towards society by adopting the mechanism to enhance the quality of production system.

(f) This study gives direction to the decision-maker on how they can take the decision regarding the selling price in developing countries like India, Indonesia, Iran, Iraq, etc., where the economy is price sensitive.

(g) According to numerical analysis, the decision-maker should pay special attention to the cost of deterioration, carrying the products, and re-working rate. profit is highly affected by the fluctuation in these costs.

(h) Finally, this study recommends that production managers use deterioration preventive strategies to control the financial and environmental effects on their production system. The amount of deteriorated or decayed

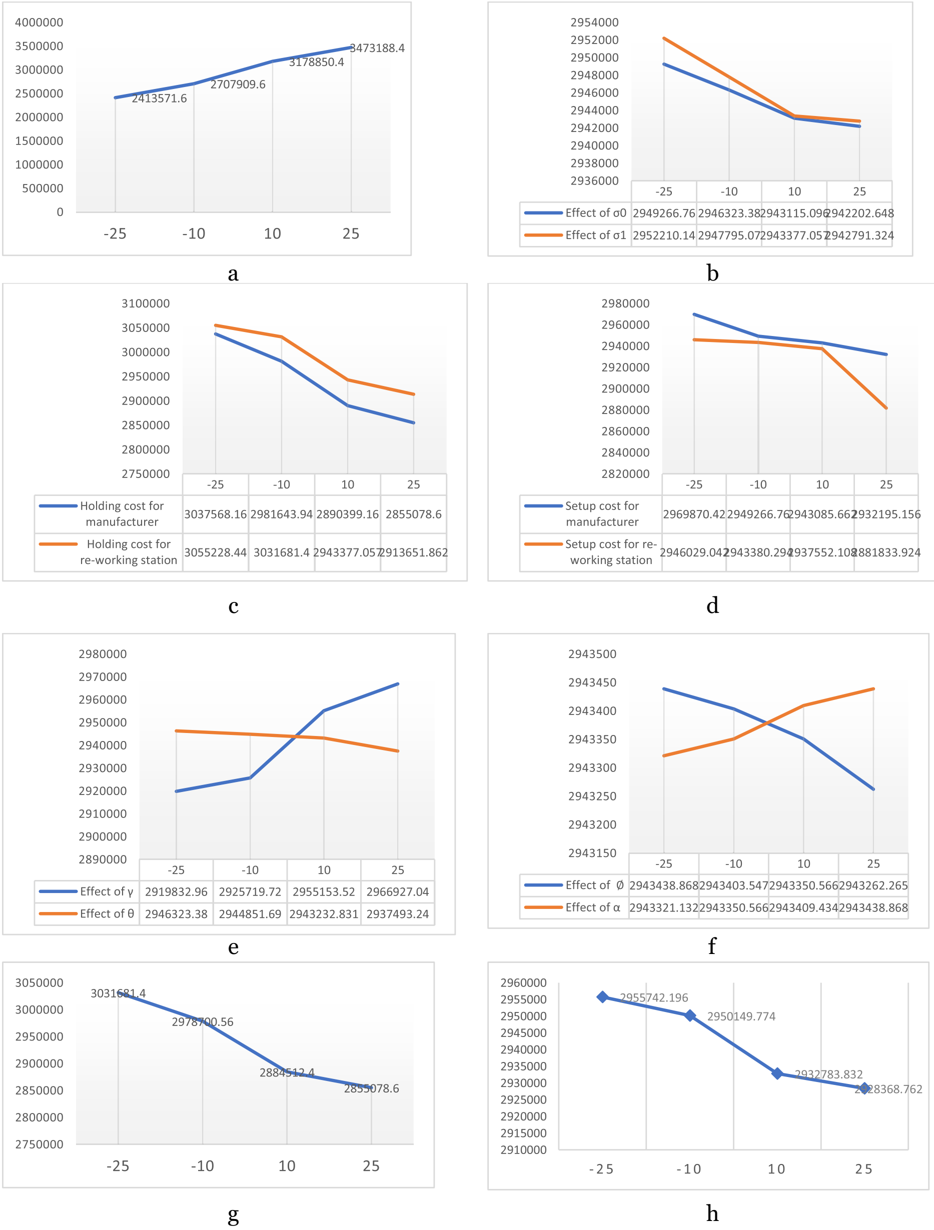
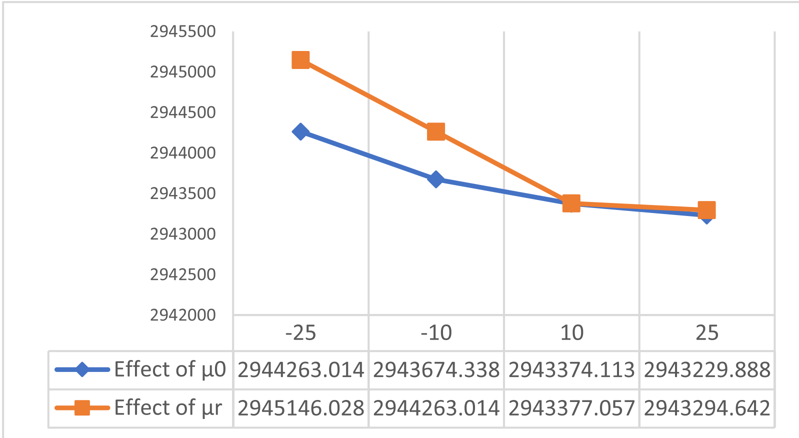


FIGURE 3. (a) Effect of reworking rate on profit. (b) Effect of σ_0 and σ_1 on profit. (c) Effect of holding cost of manufacturer and re-working station on profit. (d) Effect of setup cost of manufacturer and re-working station on profit. (e) Effect of γ and θ on the profit. (f) Effect of α and $\emptyset$ on the profit. (g) Effect of inflation on the profit. (h) Effect of deterioration cost on the profit. (i) Effect of μ_0 and μ_r on the profit.

items decreases, and the consumption of resources decreases as well, making preservation technology a definite sustainable strategy for perishable goods.



i

Figure 3. continued.

9. Concluding remarks and future extension

Management of scrap items due to the deterioration and imperfect manufacturing process is a challenging task for the decision-maker because nowadays everyone is more concerned about the environment. Thus, to make a cleaner production process, mechanism to control the deterioration rate and to mitigate the fraction of imperfect products has been discussed in the current study. Theoretical result is provided along with solution methodology to derive the optimal solution of the current problem. Numerically, concavity of the profit function was elaborated by using Hessian matrix.

From the quantitative analysis, it is found that the task of social responsibility has been achieved by controlling the rate of deterioration through preservation technology. Results indicate that adopting the preservation technology of high efficiency makes the inventory system financially more sustainable. Investment in quality improvement and reworking cuts down on the amount of scrap and makes the production process a cleaner one. Through this, the task of social responsibility is achieved by the decision-maker. Further, the incorporation of inflation into the model painted the actual financial picture of the inventory system in front of the decision-maker. From sensitive analysis, it is observed that a higher reworking rate results in a high profit for the inventory system by decreasing the deteriorated units at the re-working station. Analysis indicates that higher customer service results in less profit for the system. Thus, decision-makers have to trade-off between customer service and profit. The producer must explore various options for product storage and setup preparation, as higher carrying costs and setup costs result in less profit for the system. Increase in the value of α, decline the number of waste items generated from the system and hence decreases the waste management cost. Due to all this, the profit of the system increases.

There are a lot of promising areas in which the current study can be extended. One of the promising areas is consideration of carbon emissions due to different operation activities combined with an inventory system [60]. This will make the current model environment sustainable. The rates of imperfect production and deterioration can be considered stochastic in future extension [63]. Further, a time-varying deterioration rate and demand rate can be considered [64]. Analysing the model by considering trade credit could be a fruitful future work [65].

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