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OPTIMAL PRODUCT QUALITY AND PRICING STRATEGY FOR A TWO-PERIOD CLOSED-LOOP SUPPLY CHAIN UNDER RETURN POLICY

Tarun Maiti*

Abstract. This article considers a two-period closed-loop supply chain (CLSC) model, where a manufacturer and a retailer are trading one product. The retailer's demand rate for each period is dependent on the selling price, product quality, and refund price. The first period's product quality has an impact on the second period's demand rate. In the first period, returned products are remanufactured and sold through the retailer with the new products in the second period. The manufacturer is the leader of the Stackelberg game who declares wholesale price(s) and quality of the product to the retailer who follows the manufacturer's decision and sets his selling prices for two consecutive periods. The manufacturer implements two pricing policies: (I) sets the same wholesale price for both periods (II) sets different wholesale prices for two different periods. The present research's main aim is to find the optimal strategies for lower pricing and high-quality products. Under these circumstances, four different decision strategies between the manufacturer and the retailer are developed and compared these strategies analytically and numerically. The effects of different decision strategies on the optimal supply chain results are developed with a numerical example. An optimal solution for all four strategies is obtained using Mathematica 9. In addition, graphical analyses are developed to determine under what circumstances a particular decision strategy is dominant over others. Numerical analysis suggests that fast-acting strategies produce dominant results, but adopting strategies with advanced notice can produce higher quality products.

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1. Introduction

In the present economic era, the forward flow of products to customers is not enough for progressing in the business market. The reverse flow of products is now important in the supply chain model. Effective return management can have a great impact on your company's financial performance and build a stronger relationship with potential customers. Every return represents a failure in service encounter or not a satisfactory quality of the product. The main features of returns have two key aspects: the source of returns and the reason for returns. In a supply chain context, a manufacturer's products may be returned from intermediaries (distributors or retailers) or they may be consumer (or end-customer) returns, *i.e.*, from the actual users of products. Returns from end

Keywords. Two-echelon closed loop supply chain, two-period, decision strategy, price and quality dependent demand, dynamic pricing, return with refund price.

Department of Mathematics, Gurudas College, Kolkata 700054, West Bengal, India.
*Corresponding author: tarun.ju@gmail.com

customers are different from intermediaries. The manufacturer needs to identify the reason for returns to improve supply chain processes, decision-making, and product quality. Defective products will return after purchase, and some use or quality problems need to be rectified in manufacturing or refurbishing units. Sometimes, returned products are quickly resolved by inspecting, repairing, and then restoring them as good as new products.

As such, the above discussion shows how return policy has become an important issue in today's competitive business environment. It usually allows consumers to return products for any reason within a specific period after their purchases. The type of return policy practised in business varies across industries and stores. The policy may be defined in terms of the percentage of selling price returned or the quantity returned. The return policy can also be in the form of an unconditional money-back guarantee, or store credit only reduced by some "restocking fee" or no refund whatsoever. Restrictions are product return time, unused product, refund of original packaging, and specific instructions given on the label. Return issues have become even more significant over the last two decades, especially due to the Internet, which has given birth to e-commerce and e-marketplaces. Notable e-tail enterprises Amazon.com, Crutchfield.com, and Fipkart.com accept returns on most items within 7–30 days (varies from product to product) and pay for return shipping only for their own mistakes. In the online shopping environment, ease of return is one of the major motivations for consumers' buying decisions [36]. The return policy becomes more important in online selling because of the heterogeneity of consumers' demand and the diversity of quality in products and services. Su [43] addressed that more than 70% of online consumers consider return policy before making purchase decisions. Thus, return policies stimulate consumer demand and increase product sales.

Another reason behind commercial return is the quality problem. This is happening in the market not only due to defects in conformance quality, but also due to consumer dissatisfaction with product quality design, related specifications or product features that meet consumer needs and preferences. Low-quality products and reduced customer service lead to frequent returns. At the same time, high quality and service deserve high selling prices that lead to decreased customer demand, especially when customer demand is price sensitive. If the consumers are not worried about the price but the quality of the product, then the manufacturer adopts a "high price, high quality" strategy. Different consumer returns depend on the complexity of the process needed to put the product back on the shelf. In the apparel sector, for example, most of the returned products, after a quick visual inspection, can be put back on the shelf and sold again as new products. However, sectors such as consumer electronics and domestic appliances need more than an inspection process to decide whether the product can go back to the shelf. In this case, some rework may have to be done on the product to make it suitable for sale. Because of this return policy and reselling the product with improved quality, most companies are interested in a reverse supply chain process that is designed to minimize related costs.

To justify the above-said problems, a model is developed in this study with a manufacturer who leads the system and opts for one of the pricing strategies: (I) sets the same wholesale price for both periods. (II) sets different wholesale prices for two different periods. Under both pricing strategies, the manufacturer and the retailer adopt two different decision strategies [30] as follows:

(1) Announce their price information at the beginning of the selling horizon (Strategies (I,c) and (II,c)).
(2) Announce all decisions related to the individual periods before the starting of the respective period (Strategies (I,d) and (II,d)).

All these four strategies are developed under the manufacturer-led Stackelberg game. The quality problem is the primary reason for taking a return policy into account. To address the complex relationships between the return policy, product quality, and the selling prices in the reverse supply chain process, this paper aims to find the answers to the following questions:

(1) How do the manufacturer's optimal return and pricing policies affect the customer's demand and return decisions? Given that customers' return is sensitive to the refund price and the product quality [16, 24].
(2) What will be the best time to declare the wholesale price(s) so that customers will well know about the product's valuation and make the decision to purchase it?

(3) How does the manufacturer manage the reverse supply chain when the returned products during the first period are reproduced/repaired and ready to sell in the second period? Given that the returned products are sold in the second period with the newly produced product and with the same quality level.
(4) Which decision strategy gives the best optimal decision for the supply chain? Can one strategy prevail in all views or do different strategies prevail in different optimal views?

In short, the novelty (and contributions) of this paper are: What is the optimal decision of manufacturer and retailer based on different quality and price of product returns? A dynamic pricing modeling approach should be followed to allow product returns to be considered in the decision-making process. To be as economical as possible while emphasizing the impact of product returns on price and quality decisions, some consumers who purchased in the first period returned the product in the second period and maintain the model of buying new products launched in second period.

The rest of this manuscript is outlined as follows. In the next section, a brief review of the relevant works of literature is presented. Assumptions and notations adopted for developing the model are given in Section 3. Section 4 is devoted to the model formulation. Manufacturer-led decentralized models are developed in Section 5. Section 6 presents numerical results, sensitivity analyses of key model parameters, and a comparison of dominating areas. Some practical implications of the model are presented in Section 7. Finally, conclusions are drawn from the study with limitations and future research directions in Section 8.

2. Literature review

This study is mainly focused on customer return policy in a two-period CLSC model related to pricing strategy, and product quality. So, the literature review is focused on these matters which are discussed following subsections.

2.1. Return policy in reverse logistics

Return policies have been investigated in the manufacturer-retailer context. In this study, consumers' behaviour toward the return policy offered by the retailer or the manufacturer is reviewed with the pieces of related literature. In this context, Mukhopadhyay and Setoputro [35] analysed joint pricing and return policy decision which influences customers' purchase and returns decisions for a build-to-order product. Liu and Lei [27] were the first who proposed manufacturer's uniform pricing and return policies jointly within three channel structures (*i.e.*, Bricks-only, Clicks-only, and Bricks & Clicks). Su [43] demonstrated that consumer return policies may distort incentives under common supply contracts and proposed strategies to coordinate the supply chain in the presence of consumer returns. Xiao *et al.* [45] integrated consumer returns policy and manufacturer buyback policy within a two-stage supply chain under demand uncertainty.

Later, Shulman *et al.* [42] investigated the pricing and restocking fee decisions of two competing firms with horizontally differentiated products associated with returns and diminished return rates without an excessive loss in sales revenue. Chen and Bell [7] verified that perfect supply chain coordination can be achieved by a simple and easy-to-implement policy between a manufacturer and a retailer when customer returns are present. Chen and Bell [8] extended their study on market segmentation by using different returns policies and a dual-channel design by examining how these factors affect the firm's pricing and ordering decisions as well as the firm's profitability. Ai *et al.* [1] identified the conditions under which manufacturers and retailers prefer or do not prefer full refund policies in two competing supply chain settings for a substitutable product, with demand uncertainty. Li *et al.* [23] developed an optimal order (production) and pricing decision problem in two different supply chain channels of fashion products by taking B2B product returns concern. Chen and Grewal [9] examined the impact of customer returns, returns policies, and customer purchasing behaviour on pricing decisions in a supply chain that consists of two competing retailers and a manufacturer. They generated new insights into the consideration of customer return policies for a new entrant in a competitive market. Mishra *et al.* [32] discussed discretely changing the demand function dependent on the selling price and its rebate value. Some of the inventory models include pricing decisions with return policies under deteriorating products [2, 31, 38], greening and waste management [4, 15, 39].

Yoo *et al.* [47] investigated pricing and return policies under various supply contracts in a CLSC in which a supplier has more bargaining power than a retailer. Ruiz-Benitez and Muriel [40] analysed supply chain coordination in a vendor–buyer system facing stochastic demand and consumer returns and compared two decision policies – one that considers consumer returns as a decision variable and a second one that represents the current practice of ignoring consumer returns in the optimization model. Ruiz-Benitez *et al.* [41] showed a consumer return process where the operational decision of interest is the frequency in which returns are picked up from a collection point and then processed at a centralized location. Hsiao and Chen [19] also compared two return policies: – money back guarantee and hassle-free policies in the presence of consumer heterogeneity in their valuations as well as hassle costs. Liu *et al.* [28] investigated how buyback policy contracts influence the coordination of the supply chain, where the manufacturer buys back both unsold inventory and the returned products from customers. Khouja *et al.* [22] analysed the effect of return policy duration on the product return rate and price adjustment policies on a retailer's performance. Wang *et al.* [44] investigated the impact of customer returns and a bidirectional option contract on refund price and order decisions. Li *et al.* [25] developed the return strategy in a dual-channel supply chain model. Assarzadegan and Rasti-Barzoki [3] introduced a CLSC model with two retailers and one manufacturer playing a game-theoretic approach to pricing under a return policy and a money-back guarantee. Fan *et al.* [13] used return policies with store credit or gift cards and cash-only options to attract consumers. Jena and Meena [20] investigated a supply chain model with a retailer and two manufacturers who are Stackelberg leaders. They explored the retailer's operational decisions with or without a return policy. Cao and Yao [5] considered joint pricing and inventory control strategies with continuous review of the stochastic inventory model under customer demand and product return policies. Han *et al.* [17] established a single-stage CLSC with one manufacturer and one retailer. They investigated the effects of fixed and variable rebate mechanisms under reference price-sensitive demand. They also determined the optimal rebate mechanism and returning strategy that enterprises should adopt to recycle used products.

2.2. Return policy with product's quality

On the other hand, few previous studies have focused on the relationship between product quality and the return policy. Otake and Min [37] analysed inventory and investment in quality improvement policies under return on investment (ROI) maximization for a decision maker of an inventory system with a single finished product. Mukhopadhyay and Setoputro [36] considered the relationship between design quality and price of the product, and the firm's return policy, and showed how quality levels of the product influence the amount of return directly. Ketzenberg and Zuidwijk [21] addressed a firm that sells a product to consumers sensitive to both prices and returns policy. They developed a model for a single selling season but split into two periods where the boundary between periods is delineated by the opportunity to recover product returns and resell them. Li *et al.* [24] studied the impact of an online distributor's return policy, product quality, and pricing strategy on the customer's purchase and return decisions. They assumed that customer demand is sensitive to both the selling price and refund, and customer return is sensitive to refund and product quality. They considered only one player in the system (distributor) and a single-period situation. Yoo [46] studied a comprehensive model incorporating quality enhancement efforts and a return policy. They explored the prioritization between quality improvement and a generous return policy in the current business environment. De Giovanni and Zaccour [11] investigated a two-period model in which a manufacturer invests in quality improvements and sets the product prices over time under two return frameworks, namely, passive and active returns. Recently, Zhang *et al.* [48] integrated product quality and returns caused by quality problems under a dual-channel coordination mechanism in the CLSC.

TABLE 1. A comparison of the present study with related existing models.

Model	Demand dependency			Dynamic pricing	CLSC	Two-period
	Price	Quality	Refund			
Assarzadegan and Rasti-Barzoki [3]	✓		✓		✓	
Dong and Wu [12]	✓			✓		✓
Fan *et al.* [13]	✓		✓			
Li *et al.* [24]	✓		✓			
Li *et al.* [26]	✓		✓			✓
Li *et al.* [25]	✓		✓			
Maiti and Giri [29]	✓	✓			✓	
Maiti and Giri [30]	✓			✓		✓
Yoo [46]	✓		✓			
Yoo *et al.* [47]	✓		✓		✓	
Giri *et al.* [16]	✓	✓	✓		✓	✓
Zhang *et al.* [48]	✓	✓	✓		✓	
Present study	✓	✓	✓	✓	✓	✓

2.3. Return policy in two-period model

Very little literature is developed in a two-period environment under customers' return policies. Hematyar and Chaharsooghi [18] discussed a two-period newsboy problem under consumers' return, and coordinated the model using an insurance contract. Genc and Giovanni [14] optimized the return and rebate mechanism in a CLSC under a two-period model framework. Giri *et al.* [16] considered a two-period CLSC where they optimized product quality and pricing strategy. Dong and Wu [12] investigated the effects of strategic customer behaviour in a dynamic pricing policy in a selling season that is divided into two periods. Li *et al.* [26] examined returns policy, pricing and ordering decisions in a supply chain selling seasonal products over two periods where the manufacturer is the Stackelberg leader. Later, Mondal and Roy [33] developed a multi-objective, multi-product, and multi-period two-stage supply chain planning to maintain supply among production centres and various hospitals during the COVID-19 pandemic situation.

Overall, the above review of the literature suggests that no prior research has explored a two-period model under the influence of product quality and return policy (refund money). However, there are several studies on the return policy and product quality in CLSC models but not in individual models. Most return policy models are developed in a pricing strategy environment. The dynamic pricing situation where manufacturers can update their prices periodically and also upgrade product features timely is another aspect of this model. Also, another focus of this study is to visualize the different kinds of pricing decision strategies. The timing of announced decisions plays a vital role in product introduction in the commercial market. To fill this gap, a model with a return policy as part of a product quality-participating pricing strategy in the two-period CLSC will be discussed. A comparative study is shown in Table 1 to understand the contribution of the present study compared to previous literature.

In this article, a model is developed for a sales season which is divided into two periods. In the first period, the manufacturer produces a product and sells it through a retailer and also offers a return policy which is limited in this period. The retailer's demand in the first period is dependent on the selling price, product quality and the refund price offered by the manufacturer. The returned quantity is sensitive to the product quality and refund price. After the end of the first period, the manufacturer reproduces/repairs the returned product and sells it through the retailer together with the newly produced products. He maintains the same quality level of both reproduced/repaired and newly produced products. The manufacturer is the Stackelberg leader and the retailer is the follower and follows the manufacturer's decisions. The optimal decisions in all strategies are analysed and compared. Numerical analysis reveals some important differences between the four strategies.

3. Notations and assumptions

The following notations are used throughout the paper:

D_1	First period's demand rate at the retailer.
D_2	Second period's demand rate at the retailer.
D_{r1}	First period's returned quantity.
D_{r2}	Second period's returned quantity.
ϕ_1	First period's basic return quantity.
ϕ_2	Second period's basic return quantity.
p_1	First period's selling price.
p_2	Second period's selling price.
q_1	Product quality of product at first period.
q_2	Product quality of product at second period.
r	Refund price offer per unit of returned product.
w	Common wholesale price for both the periods for strategies (I,c) and (I,d).
w_1	First period's wholesale price for strategies (II,c) and (II,d).
w_2	Second period's wholesale price for strategies (II,c) and (II,d).
c	Production cost per unit product.
c_m	Production cost per unit of reproduced/repaired product.
g_1	Goodwill lost cost for the totally impure product at first period.
g_2	Goodwill lost cost for the totally impure product at second period.
Π_{m1}	Profit of the manufacturer in the first period.
Π_{m2}	Profit of the manufacturer in the second period.
Π_m	Profit of the manufacturer in the whole season.
Π_{r1}	Profit of the retailer in the first period.
Π_{r2}	Profit of the retailer in the second period.
Π_r	Profit of the retailer in the whole season.
Π	Profit of the whole system in the whole season.

The following assumptions are made to develop the proposed model:

(1) The two-echelon supply chain is comprised of one manufacturer and one retailer. The manufacturer produces a single product and sells it through the retailer in a season that is divided into two periods.

(2) The demand rate at the retailer place depends on the selling price, refund money and product quality of the product for the first period. But in the second period, demand was additionally influenced by the first period's product quality. Therefore, the demand rate in the first period is $D_1 = d_1 - \alpha_1 p_1 + \beta_1 q_1 + \gamma_1 r$, where $\alpha_1, \beta_1, \gamma_1 > 0$, so that the demand is always positive. The return policy with high refund may lead to high market demand and a high selling price has a negative impact on the demand [10, 24, 29, 34, 49]. The return rate of the product during the first period is dependent on the refund price and the product quality. The demand rate in the second period is $D_2 = d_2 - \alpha_2 p_2 + \beta_2 q_2 - \delta q_1 + \gamma_2 r$, where $\alpha_2, \beta_2, \gamma_2, \delta > 0$. In the demand functions, δ indicates the product review from the first period to the second period made by the purchasing customers. The return rate at each period is taken as $D_{ri} = \phi_i + \eta_i r - \rho_i q_i$, where $i = 1, 2$ and η_i and ρ_i are positive constants such that the return rate is non-negative [24, 46]. Higher return compensation has a positive impact on the return quantity. It is quite natural that if the product quality is high, then the return rate would be low.

(3) The manufacturer is the Stackelberg leader and the retailer is the follower. First, the manufacturer declares wholesale price(s), product quality, and refund price and then the retailer sets his selling prices for two periods in different decision strategies.
(4) For the feasibility of the model $p_i > w_i(w) > c$, $i \in \{1, 2\}$ will be holds.
(5) The manufacturer remanufactures by error inspecting, and refurbishing the returned products at a cost c_m less than the production cost c and makes them "as good as new" products and sells in the next period.
(6) Lead time is negligible.

4. Model formulation

The manufacturer produces a single product and sells it through the retailer to satisfy the market demand. Two consecutive selling periods are considered in the total selling season. In both periods, the manufacturer offers a return policy that is dependent on the refund price and the product quality. The manufacturer decides to use two different strategies for setting the wholesale price(s) for the product. The manufacturer sets the same wholesale price w for both periods in the first strategy. In the second strategy, (s)he declares two wholesale prices w_1 and w_2 for two periods, respectively.

In practice, it is often observed for products such as cars, computers, smartphones, etc. the production of these products is highly technical. As a result, the production costs of these products are rapidly changing within a short period due to technological change or up-gradation. Several smartphone companies like Sony, Apple, Samsung, LG, Motorola, and also some PC companies, *e.g.*, Sony, Dell, and HP are all upgrading their models with an operating system, new software, and new technology and declared new prices for these products. In both the above strategies, the retailer sets two different selling prices p_1 and p_2 for two consecutive periods [30]. Firstly, the manufacturer declares his wholesale prices, product qualities q_1 and q_2 and the refund price r per unit product. Then the retailer sets his selling prices for the two periods. The manufacturer needs to maintain product quality by controlling the failure rate in the manufacturing stage as well as product delivery and service quality (speed and carelessness) in the sales stage. For this, a cost component of $\lambda_i q_i^2$, (where $\lambda_i > 0$ is a scaler parameter and $i = (1, 2)$) for each period is considered which is a continuous differentiable convex function [6,24]. A goodwill loss cost ($g_i, i = (1, 2)$) is incorporated if the quality label is below 100%. Also, let's ignore some cost factors such as set-up costs, ordering costs, and shipping costs that have no direct impact on optimal decisions. A schematic diagram of all strategies is shown in Figure 1.

Therefore, the profit functions of the manufacturer when the first strategy (symbolically use the superscript I) is adopted (wholesale price w) for the first and second period are respectively shown in (1) and (2) respectively

$$\Pi^{\mathrm{I}}_{m1}(w, r, q_1) = (w - c)D_1 - rD_{r1} - (1 - g_1)q_1 - \lambda_1 q_1^2 \tag{1}$$

$$\Pi^{\mathrm{I}}_{m2}(w, r, q_2) = (w - c)(D_2 - D_{r1}) + (w - c_m)D_{r1} - rD_{r2} - (1 - g_2)q_2 - \lambda_2 q_2^2, \tag{2}$$

and the retailer's first and second periods' profit functions are respectively shown in (3) and (4) for the first strategy.

$$\Pi_{r1}(p_1)^{\mathrm{I}} = (p_1 - w)D_1 \tag{3}$$

$$\Pi_{r2}(p_2)^{\mathrm{I}} = (p_2 - w)D_2. \tag{4}$$

Also, the profit functions of the manufacturer for the second strategy (different wholesale prices for both periods, symbolically use II) in the first and second periods are given in (5) and (6).

$$\Pi^{\mathrm{II}}_{m1}(w_1, r, q_1) = (w_1 - c)D_1 - rD_{r1} - (1 - g_1)q_1 - \lambda_1 q_1^2 \tag{5}$$

$$\Pi^{\mathrm{II}}_{m2}(w_2, r, q_2) = (w_2 - c)(D_2 - D_{r1}) + (w_2 - c_m)D_{r1} - rD_{r2} - (1 - g_2)q_2 - \lambda_2 q_2^2, \tag{6}$$

respectively. Similarly, the retailer's first and second periods' profit functions are respectively shown in (7) and (8) for the second strategy.

$$\Pi^{\mathrm{II}}_{r1}(p_1) = (p_1 - w_1)D_1 \tag{7}$$

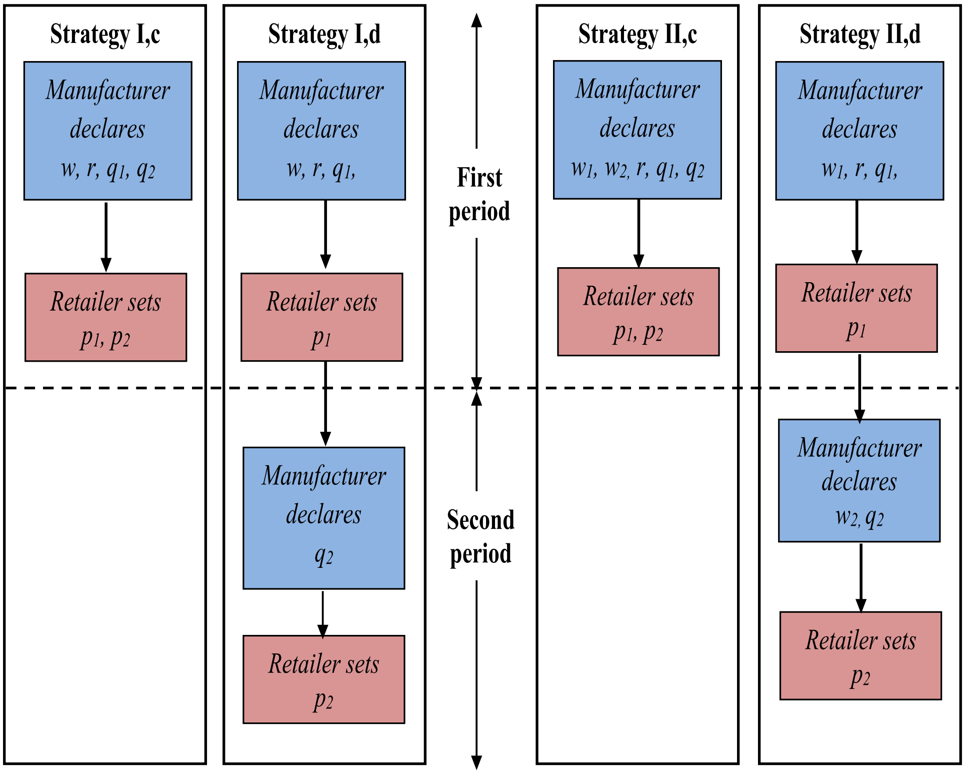

FIGURE 1. Sequences of pricing decisions under different strategies.

$$\Pi_{r2}^{\mathrm{II}}(p_2) = (p_2 - w_2)D_2. \tag{8}$$

Therefore, in the first and second strategies, the manufacturer's and the retailer's profit functions for the entire selling season are given in the (9)–(12).

$$\Pi_m^{\mathrm{I}}(w, r, q_1, q_2) = \Pi_{m1}^{\mathrm{I}}(w, r, q_1) + \Pi_{m2}^{\mathrm{I}}(w, r, q_2) \tag{9}$$

$$\Pi_r^{\mathrm{I}}(p_1, p_2) = \Pi_{r1}^{\mathrm{I}}(p_1) + \Pi_{r2}^{\mathrm{I}}(p_2) \tag{10}$$

$$\Pi_m^{\mathrm{II}}(w_1, w_2, r, q_1, q_2) = \Pi_{m1}^{\mathrm{II}}(w_1, r, q_1) + \Pi_{m2}^{\mathrm{II}}(w_2, r, q_2) \tag{11}$$

$$\Pi_r^{\mathrm{II}}(p_1, p_2) = \Pi_{r1}^{\mathrm{II}}(p_1) + \Pi_{r2}^{\mathrm{II}}(p_2). \tag{12}$$

For both strategies, the profit of the whole system is the same and is provided by

$$\begin{aligned}\Pi(p_1, p_2, r, q_1, q_2) = (p_1 - c)D_1 + (p_2 - c)D_2 + (c - c_m)D_{r1} - r(D_{r1} + D_{r2}) \\ - (1 - g_1)q_1 - (1 - g_2)q_2 - \lambda_1 q_1^2 - \lambda_2 q_2^2.\end{aligned} \tag{13}$$

5. MANUFACTURER-LED DECENTRALIZED MODELS

The manufacturer acts as the leader and the retailer as the follower in all strategies. The manufacturer declares wholesale price(s), product quality, and refund price first and then the retailer follows the manufacturer's decision and sets his selling prices for the two consecutive periods. Under two wholesale pricing strategies (*i.e.*, I and II), the retailer takes two different types of reactions to both wholesale pricing strategies of the manufacturer. The following four subsections describe four different strategies.

5.1. Decision strategy (I,c)

In this strategy, the decisions for both periods (centrally for the whole selling season) are made sequentially by each player at the beginning of the first period. At the beginning of the first period, the manufacturer declares the wholesale price (w), product qualities (q_1, q_2) and refund money (r) for both periods. Following the manufacturer's decisions, the retailer decides his two selling prices p_1 and p_2 for two consecutive periods, optimizing the total profit of the entire selling season. So, customers are well of this decision before purchasing a product. The decision sequence of this commitment contract is as follows:

– The manufacturer declares his common wholesale price w, refund money r and product qualities q_1 and q_2 for both periods.
– The retailer follows the manufacturer's and announces his selling prices p_1 and p_2 by maximizing the total profit of the two periods.

In practice, this type of pre-announced dynamic pricing is generally practised indirectly. For example, *via* promotional offers or coupons, firms may set a regular price and offer introductory price cuts. The cinema/theatre premieres charge introductory prices-premiums are also a real case of this strategy.

The following proposition gives the optimal results of this strategy.

Proposition 1. *Under decision strategy I,c; the manufacturer's optimal wholesale price, refund money and qualities, retailer's selling prices are given respectively by*

$$
\begin{aligned}
w^{\mathrm{I,c}} = \Big[& -8\delta\eta_1\lambda_2 + 8g_1\delta\eta_1\lambda_2 + 4c\delta^2\eta_1\lambda_2 - 8\delta\eta_2\lambda_2 + 8g_1\delta\eta_2\lambda_2 + 4c\delta^2\eta_2\lambda_2 + 4c\gamma_1^2\lambda_1\lambda_2 + 8c\gamma_1\gamma_2\lambda_1\lambda_2 \\
& + 4c\gamma_2^2\lambda_1\lambda_2 - 16d_1\eta_1\lambda_1\lambda_2 - 16d2\eta_1\lambda_1\lambda_2 - 16c\alpha_1\eta_1\lambda_1\lambda_2 - 16c\alpha_2\eta_1\lambda_1\lambda_2 - 8c\gamma_1\eta_1\lambda_1\lambda_2 \\
& + 8c_m\gamma_1\eta_1\lambda_1\lambda_2 - 8c\gamma_2\eta_1\lambda_1\lambda_2 + 8c_m\gamma_2\eta_1\lambda_1\lambda_2 - 16d_1\eta_2\lambda_1\lambda_2 - 16d2\eta_2\lambda_1\lambda_2 - 16c\alpha_1\eta_2\lambda_1\lambda_2 \\
& - 16c\alpha_2\eta_2\lambda_1\lambda_2 + 4\gamma_1\lambda_2\rho_1 - 4g_1\gamma_1\lambda_2\rho_1 + 4\gamma_2\lambda_2\rho_1 - 4g_1\gamma_2\lambda_2\rho_1 - 4c\gamma_1\delta\lambda_2\rho_1 - 4c\gamma_2\delta\lambda_2\rho_1 \\
& - 4c\delta\eta_1\lambda_2\rho_1 + 4c_m\delta\eta_1\lambda_2\rho_1 - 8c\delta\eta_2\lambda_2\rho_1 + 8c_m\delta\eta_2\lambda_2\rho_1 + 4d_1\lambda_2\rho_1^2 + 4d2\lambda_2\rho_1^2 + 4c\alpha_1\lambda_2\rho_1^2 \\
& + 4c\alpha_2\lambda_2\rho_1^2 + 4c\gamma_1\lambda_2\rho_1^2 - 4c_m\gamma_1\lambda_2\rho_1^2 + 4c\gamma_2\lambda_2\rho_1^2 - 4c_m\gamma_2\lambda_2\rho_1^2 + c\beta_2^2\big(4(\eta_1+\eta_2)\lambda_1 - \rho_1^2\big) \\
& + 4\gamma_1\lambda_1\rho_2 - 4g_2\gamma_1\lambda_1\rho_2 + 4\gamma_2\lambda_1\rho_2 - 4g_2\gamma_2\lambda_1\rho_2 - 2\delta\rho_1\rho_2 + 2g_2\delta\rho_1\rho_2 + 2\delta\rho_2^2 - 2g_1\delta\rho_2^2 \\
& - c\delta^2\rho_2^2 + 4d_1\lambda_1\rho_2^2 + 4d2\lambda_1\rho_2^2 + 4c\alpha_1\lambda_1\rho_2^2 + 4c\alpha_2\lambda_1\rho_2^2 + 2c\delta\rho_1\rho_2^2 - 2c_m\delta\rho_1\rho_2^2 \\
& + c\beta_1^2\big(4(\eta_1+\eta_2)\lambda_2 - \rho_2^2\big) + 8\gamma_1\lambda_1\lambda_2\phi_1 + 8\gamma_2\lambda_1\lambda_2\phi_1 - 4\delta\lambda_2\rho_1\phi_1 + 4\lambda_2(2(\gamma_1+\gamma_2)\lambda_1 - \delta\rho_1)\phi_2 \\
& + 2\beta_1(2c\gamma_1\lambda_2\rho_1 + 2c\gamma_2\lambda_2\rho_1 - 4\eta_2\lambda_2(-1+g_1+c\delta-c\rho_1+c_m\rho_1) \\
& - 2\eta_1\lambda_2(-2+2g_1+2c\delta-c\rho_1+c_m\rho_1) + \rho_1\rho_2 - g_2\rho_1\rho_2 - \rho_2^2 + g_1\rho_2^2 + c\delta\rho_2^2 - c\rho_1\rho_2^2 \\
& + c_m\rho_1\rho_2^2 + 2\lambda_2\rho_1\phi_1 + 2\lambda_2\rho_1\phi_2\big) + 2\beta_2(-4(-1+g_2)\eta_2\lambda_1 - \rho_1^2 + g_2\rho_1^2 + 2c(\gamma_1+\gamma_2)\lambda_1\rho_2 \\
& + \rho_1\rho_2 - g_1\rho_1\rho_2 + c\beta_1\rho_1\rho_2 - c\delta\rho_1\rho_2 + c\rho_1^2\rho_2 - c_m\rho_1^2\rho_2 + 2\eta_1\lambda_1(2-2g_2-c\rho_2+c_m\rho_2) \\
& + 2\lambda_1\rho_2\phi_1 + 2\lambda_1\rho_2\phi_2)\Big] \times \Big[\Big(\beta_2^2(4(\eta_1+\eta_2)\lambda_1 - \rho_1^2) + 4\lambda_2((\gamma_1+\gamma_2)^2\lambda_1 + (\eta_1+\eta_2)((\beta_1-\delta)^2 \\
& - 8(\alpha_1+\alpha_2)\lambda_1) + (\gamma_1+\gamma_2)(\beta_1-\delta)\rho_1 + 2(\alpha_1+\alpha_2)\rho_1^2) + 2\beta_2(2(\gamma_1+\gamma_2)\lambda_1 + (\beta_1-\delta)\rho_1)\rho_2 \\
& - \big((\beta_1-\delta)^2 - 8(\alpha_1+\alpha_2)\lambda_1\big)\rho_2^2\Big)\Big]^{-1}
\end{aligned}
$$

$$
\begin{aligned}
r^{\mathrm{I,c}} = \Big[& \big(w^{\mathrm{I,c}}\lambda_2(2(\gamma_1+\gamma_2)\lambda_1 + (\beta_1-\delta)\rho_1) - c\lambda_2(2(\gamma_1+\gamma_2-2\eta_1)\lambda_1 \\
& + \rho_1(\beta_1-\delta+2\rho_1)) - c\beta_2\lambda_1\rho_2 + w^{\mathrm{I,c}}\beta_2\lambda_1\rho_2 - 2(c_m\lambda_2(2\eta_1\lambda_1 - \rho_1^2) - (-1+g_2)\lambda_1\rho_2 \\
& + \lambda_2(\rho_1 - g_1\rho_1 + 2\lambda_1(\phi_1+\phi_2))))\Big] \times \Big[\big(8\eta_1\lambda_1\lambda_2 + 8\eta_2\lambda_1\lambda_2 - 2\big(\lambda_2\rho_1^2 + \lambda_1\rho_2^2\big)\big)\Big]^{-1}
\end{aligned}
$$

$$
q_1^{\mathrm{I,c}} = \frac{\big(-2+2g_1+w^{\mathrm{I,c}}\beta_1 - w^{\mathrm{I,c}}\delta + c(-\beta_1+\delta-2\rho_1) + 2\big(c_m + r^{\mathrm{I,c}}\big)\rho_1\big)}{4\lambda_1},
$$

$$q_2^{\mathrm{I,c}} = \frac{\left(-2 + 2g_2 - c\beta_2 + w^{\mathrm{I,c}}\beta_2 + 2r^{\mathrm{I,c}}\rho_2\right)}{4\lambda_2},$$

$$p_1^{\mathrm{I,c}} = \frac{\left(d_1 + w^{\mathrm{I,c}}\alpha_1 + q_1^{\mathrm{I,c}}\beta_1 + r^{\mathrm{I,c}}\gamma_1\right)}{2\alpha_1}, \quad p_2^{\mathrm{I,c}} = \frac{\left(d_2 + w^{\mathrm{I,c}}\alpha_2 + q_2^{\mathrm{I,c}}\beta_2 + r^{\mathrm{I,c}}\gamma_2 - q_1^{\mathrm{I,c}}\delta\right)}{2\alpha_2}.$$

Proof. See Appendix A. □

5.2. Decision strategy (I,d)

In this strategy, each player determines his optimal decisions for each period in a decentralized way, *i.e.*, they optimize the profit functions of each period individually. Here, the manufacturer decides his unique wholesale price, refund money, and product quality for the first period, optimizing the profit function of the first period only. Then the retailer maximizes his first-period profit function and sets his selling price. After that, the manufacturer sets the product quality for the second period, optimizing the second-period profit function and the retailer follows by setting its selling price for the second period by optimizing the second-period profit. The decision sequence is as follows:

- The manufacturer declares his common wholesale price w, refund r, and product quality q_1 at the beginning of the first period.
- The retailer decides his selling price p_1 by maximizing his first period's profit.
- The manufacturer sets product quality q_2 at the beginning of the second period.
- The retailer sets p_2 by maximizing his second period's profit.

This type of responsive pricing is commonly observed in online e-commerce. For example, `Amazon.com`, `Snapdeal.com`, `Flipkart.com`, etc. are known to employ complex dynamic-pricing algorithms.

The following proposition gives the optimal results of this strategy.

Proposition 2. *Under decision strategy I,d; the manufacturer's optimal wholesale prices and refund price, qualities, the retailer's optimal selling prices are given respectively by*

$$\begin{aligned}
w^{\mathrm{I,d}} &= \left(c\beta_1^2\eta_1 - 4d_1\eta_1\lambda_1 + \gamma_1\rho_1 - g_1\gamma_1\rho_1 + d_1\rho_1^2 + c_m\left(\gamma_1^2\lambda_1 - 4\alpha_1\eta_1\lambda_1 + \alpha_1\rho_1^2\right)\right.\\
&\quad \left. + 2\gamma_1\lambda_1\phi_1 + \beta_1(-2(-1+g_1)\eta_1 + \rho_1(c\gamma_1 + \phi_1))\right)/\left(\beta_1^2\eta_1 + \gamma_1^2\lambda_1 + \beta_1\gamma_1\rho_1 + 2\alpha_1\left(-4\eta_1\lambda_1 + \rho_1^2\right)\right),\\
q_1^{\mathrm{I,d}} &= \frac{(-1+g_1)\gamma_1^2 - 2d_1\beta_1\eta_1 + \gamma_1(-d_1\rho_1 + c\alpha_1\rho_1 + \beta_1\phi_1) + \alpha_1(2(4-4g_1+c\beta_1)\eta_1 + 4\rho_1\phi_1)}{2(\beta_1^2\eta_1 + \gamma_1^2\lambda_1 + \beta_1\gamma_1\rho_1 + 2\alpha_1(-4\eta_1\lambda_1 + \rho_1^2))},\\
r^{\mathrm{I,d}} &= -\frac{(-1+g_1)\beta_1\gamma_1 + 2d_1\gamma_1\lambda_1 + (d_1 - c\alpha_1)\beta_1\rho_1 + \beta_1^2\phi_1 - 2\alpha_1(c\gamma_1\lambda_1 + 2\rho_1 - 2g_1\rho_1 + 4\lambda_1\phi_1)}{2(\beta_1^2\eta_1 + \gamma_1^2\lambda_1 + \beta_1\gamma_1\rho_1 + 2\alpha_1(-4\eta_1\lambda_1 + \rho_1^2))}\\
p_1^{\mathrm{I,d}} &= \frac{d_1 + w^{\mathrm{I,d}}\alpha_1 + q_1^{\mathrm{I,d}}\beta_1 + r^{\mathrm{I,d}}\gamma_1}{2\alpha_1}\\
q_2^{\mathrm{I,d}} &= \frac{-2 + 2g_2 - c\beta_2 + w^{\mathrm{I,d}}\beta_2 + 2r^{\mathrm{I,d}}\rho_2}{4\lambda_2}\\
p_2^{\mathrm{I,d}} &= \frac{d_2 + w^{\mathrm{I,d}}\alpha_2 + q_2^{\mathrm{I,d}}\beta_2 + r^{\mathrm{I,d}}\gamma_2 - q_1^{\mathrm{I,d}}\delta}{2\alpha_2}
\end{aligned}$$

provided the condition $8\alpha_1\eta_1\lambda_1 > \beta_1^2\eta_1 + \gamma_1^2\lambda_1 + \beta_1\gamma_1\rho_1 + 2\alpha_1\rho_1^2$ *holds.*

Proof. See Appendix A. □

5.3. Decision strategy (II,c)

In this section, a strategy game is developed where manufacturers and retailers use a pre-announced pricing strategy. In the first period, the manufacturer and the retailer announce the full price path (w_1, w_2 and p_1, p_2, respectively), product qualities (q_1, q_2), refund price by maximizing their total profit for the whole selling season. As product prices, qualities, and refund money are pre-announced, customers make purchasing decisions for the first period. Also, they can wait and observe the product reviews of the first-period buyers, update their beliefs about product quality, and make their second-period purchasing decisions. In this decision strategy, the decision sequence is as follows:

– The manufacturer declares his wholesale prices w_1, w_2, product qualities q_1, q_2 and refund for the two periods at the beginning of the first period.
– The retailer sets his selling prices p_1 and p_2 by maximizing his total profit for the two periods at the beginning of the first period.

The optimal decisions can summarize in the following proposition:

Proposition 3. *Under decision strategy II,c, the manufacturer's optimal wholesale price, refund price and qualities, and the retailer's optimal selling prices are given respectively by*

$$
\begin{aligned}
q_1^{\mathrm{II,c}} = \Big[\Big(&\beta_2\big(-(1-g_2)(\beta_1\gamma_1\gamma_2+\gamma_1^2\delta+4\alpha_1\gamma_2\rho_1)+\beta_1(d_2\gamma_1-c\alpha_2\gamma_1-2d_1\gamma_2+2c\alpha_1\gamma_2)\rho_2\\
&-(d_1-c\alpha_1)\gamma_1\delta\rho_2+4\alpha_1\big((d_2-c\alpha_2)\rho_1\rho_2-2\gamma_2(-1+g_1-c\rho_1+c_m\rho_1)\rho_2\\
&+\delta\big(2(1-g_2)\eta_2+\eta_1(2-2g_2-c\rho_2+c_m\rho_2)+\rho_2(\phi_1+\phi_2)\big)\big)\big)\\
&+\beta_2^2\big(8\alpha_1\eta_1+\gamma_1^2(-1+g_1-c\rho_1+c_m\rho_1)+\gamma_1\big(-d_1\rho_1+c(-\beta_1\eta_1+\alpha_1\rho_1)\\
&+\beta_1(c_m\eta_1+\phi_1+\phi_2)\big)+2\big(-4g_1\alpha_1(\eta_1+\eta_2)-d_1\beta_1(\eta_1+\eta_2)+\alpha_1\big(4\eta_2+c\beta_1(\eta_1+\eta_2)\\
&+2c(\eta_1+2\eta_2)\rho_1+2\rho_1(-c_m(\eta_1+2\eta_2)+\phi_1+\phi_2)\big)\big)\big)-2\big(-d_2\lambda_2(\beta_1\gamma_1\gamma_2+\gamma_1^2\delta\\
&-8\alpha_1\delta(\eta_1+\eta_2)+4\alpha_1\gamma_2\rho_1)-2d_2\alpha_1\delta\rho_2^2+\alpha_2\big(\gamma_1^2\lambda_2(-4+4g_1+c(\delta-4\rho_1)+4c_m\rho_1)\\
&+2\big(d_1\beta_1(-4(\eta_1+\eta_2)\lambda_2+\rho_2^2)+\alpha_1\big(2c\gamma_2\lambda_2\rho_1+4\eta_1\lambda_2(4-4g_1-2c_m\rho_1+c(\beta_1-\delta+2\rho_1))\\
&+4\eta_2\lambda_2(4-4g_1-4c_m\rho_1+c(\beta_1-\delta+4\rho_1))+4\rho_1\rho_2-4g_2\rho_1\rho_2-4\rho_2^2+4g_1\rho_2^2-c\beta_1\rho_2^2\\
&+c\delta\rho_2^2-4c\rho_1\rho_2^2+4c_m\rho_1\rho_2^2+8\lambda_2\rho_1\phi_1+8\lambda_2\rho_1\phi_2\big)\big)+\gamma_1\big(4c_m\beta_1\eta_1\lambda_2+c\lambda_2\big(\beta_1(\gamma_2-4\eta_1)\\
&+4\alpha_1\rho_1\big)-2(-1+g_2)\beta_1\rho_2+4\lambda_2(-d_1\rho_1+\beta_1(\phi_1+\phi_2))\big)\big)+\gamma_2\big(d_1(\beta_1\gamma_2+\gamma_1\delta)\lambda_2\\
&+\alpha_1\big(\gamma_2\lambda_2(-4+4g_1+4c_m\rho_1-c(\beta_1+4\rho_1))-\delta\big(c(\gamma_1-4\eta_1)\lambda_2+2\big(\rho_2-g_2\rho_2\\
&+2\lambda_2(c_m\eta_1+\phi_1+\phi_2)\big)\big)\big)\big)\big)\big)\Big)\Big]\\
\times\Big[&\big(2\big(\beta_2^2\big(\gamma_1^2\lambda_1+2\alpha_1(-4(\eta_1+\eta_2)\lambda_1+\rho_1^2)\big)+\lambda_2\big(\gamma_1^2(\delta^2-8\alpha_2\lambda_1)-8\alpha_1\big(\gamma_2^2\lambda_1\\
&+(\eta_1+\eta_2)(\delta^2-8\alpha_2\lambda_1)-\gamma_2\delta\rho_1+2\alpha_2\rho_1^2\big)\big)+4\alpha_1\beta_2(-2\gamma_2\lambda_1+\delta\rho_1)\rho_2+2\alpha_1(\delta^2-8\alpha_2\lambda_1)\rho_2^2\\
&+\beta_1\gamma_1(2\gamma_2\delta\lambda_2+\beta_2^2\rho_1-8\alpha_2\lambda_2\rho_1+\beta_2\delta\rho_2)+\beta_1^2\big(\gamma_2^2\lambda_2+(\eta_1+\eta_2)(\beta_2^2-8\alpha_2\lambda_2)+\beta_2\gamma_2\rho_2\\
&+2\alpha_2\rho_2^2\big)\big)\big)\Big]^{-1},\\
q_2^{\mathrm{II,c}} = \Big[\big(&-\gamma_1^2\delta^2+g_2\gamma_1^2\delta^2+8\alpha_1\delta^2\eta_1-8g_2\alpha_1\delta^2\eta_1+8\alpha_1\delta^2\eta_2-8g_2\alpha_1\delta^2\eta_2+8\alpha_2\gamma_1^2\lambda_1-8g_2\alpha_2\gamma_1^2\lambda_1\\
&+8\alpha_1\gamma_2^2\lambda_1-8g_2\alpha_1\gamma_2^2\lambda_1-64\alpha_1\alpha_2\eta_1\lambda_1+64g_2\alpha_1\alpha_2\eta_1\lambda_1-64\alpha_1\alpha_2\eta_2\lambda_1+64g_2\alpha_1\alpha_2\eta_2\lambda_1\\
&-8\alpha_1\gamma_2\delta\rho_1+8g_2\alpha_1\gamma_2\delta\rho_1+16\alpha_1\alpha_2\rho_1^2-16g_2\alpha_1\alpha_2\rho_1^2+4\alpha_1\gamma_2\delta\rho_2-4g_1\alpha_1\gamma_2\delta\rho_2-d_1\gamma_1\delta^2\rho_2\\
&+c\alpha_1\gamma_1\delta^2\rho_2-4c\alpha_1\delta^2\eta_1\rho_2+4c_m\alpha_1\delta^2\eta_1\rho_2+8d_1\alpha_2\gamma_1\lambda_1\rho_2-8c\alpha_1\alpha_2\gamma_1\lambda_1\rho_2+8d_2\alpha_1\gamma_2\lambda_1\rho_2\\
&-8c\alpha_1\alpha_2\gamma_2\lambda_1\rho_2+32c\alpha_1\alpha_2\eta_1\lambda_1\rho_2-32c_m\alpha_1\alpha_2\eta_1\lambda_1\rho_2-16\alpha_1\alpha_2\rho_1\rho_2+16g_1\alpha_1\alpha_2\rho_1\rho_2\\
&-4d_2\alpha_1\delta\rho_1\rho_2+4c\alpha_1\alpha_2\delta\rho_1\rho_2+4c\alpha_1\gamma_2\delta\rho_1\rho_2-4c_m\alpha_1\gamma_2\delta\rho_1\rho_2-16c\alpha_1\alpha_2\rho_1^2\rho_2
\end{aligned}
$$

$$
\begin{aligned}
&+16c_m\alpha_1\alpha_2\rho_1^2\rho_2+4\alpha_1\delta^2\rho_2\phi_1-32\alpha_1\alpha_2\lambda_1\rho_2\phi_1+4\alpha_1\big(\delta^2-8\alpha_2\lambda_1\big)\rho_2\phi_2\\
&+\beta_2\big(\big(d_1-c\alpha_1\big)\gamma_1\big(2\gamma_2\lambda_1-\delta\rho_1\big)+\gamma_1^2\big(-2\big(d2-c\alpha_2\big)\lambda_1+\delta\big(-1+g_1-c\rho_1+c_m\rho_1\big)\big)\\
&-4\alpha_1\big(4c\alpha_2\eta_1\lambda_1-2c\gamma_2\eta_1\lambda_1+2c_m\gamma_2\eta_1\lambda_1+4c\alpha_2\eta_2\lambda_1+\gamma_2\rho_1-g_1\gamma_2\rho_1-c\alpha_2\rho_1^2+c\gamma_2\rho_1^2\\
&-c_m\gamma_2\rho_1^2+d_2\big(-4\big(\eta_1+\eta_2\big)\lambda_1+\rho_1^2\big)+2\gamma_2\lambda_1\phi_1+2\gamma_2\lambda_1\phi_2+\delta\big(2\eta_2\big(-1+g_1-c\rho_1+c_m\rho_1\big)\\
&+\eta_1\big(-2+2g_1-c\rho_1+c_m\rho_1\big)-\rho_1\big(\phi_1+\phi_2\big)\big)\big)\big)+\beta_1\big(2\big(-1+g_2\big)\gamma_1\big(\gamma_2\delta-4\alpha_2\rho_1\big)\\
&+\big(-\big(d2\gamma_1+\big(d_1-c\alpha_1\big)\gamma_2\big)\delta+\alpha_2\gamma_1\big(-4+4g_1+c\delta\big)+4\alpha_2\big(d_1+c_m\gamma_1-c\big(\alpha_1+\gamma_1\big)\big)\rho_1\big)\rho_2\\
&+\beta_2\big(-\big(d_1-c\alpha_1\big)\big(2\delta\big(\eta_1+\eta_2\big)-\gamma_2\rho_1\big)+\gamma_1\big(-2d2\rho_1+\gamma_2\big(-1+g_1-c\rho_1+c_m\rho_1\big)\\
&+c\big(-\delta\eta_1+2\alpha_2\rho_1\big)+\delta\big(c_m\eta_1+\phi_1+\phi_2\big)\big)\big)\big)+\beta_1^2\big(\big(-1+g_2\big)\gamma_2^2+8\alpha_2\eta_1\\
&+\gamma_2\big(-d2\rho_2+c\big(-\beta_2\eta_1+\alpha_2\rho_2\big)+\beta_2\big(c_m\eta_1+\phi_1+\phi_2\big)\big)+2\big(-4g_2\alpha_2\big(\eta_1+\eta_2\big)\\
&-d_2\beta_2\big(\eta_1+\eta_2\big)+\alpha_2\big(4\eta_2+c\beta_2\big(\eta_1+\eta_2\big)-2c\eta_1\rho_2+2\rho_2\big(c_m\eta_1+\phi_1+\phi_2\big)\big)\big)\big)\big)\Big]\\
&\times\Big[\big(2\big(\beta_2^2\big(\gamma_1^2\lambda_1+2\alpha_1\big(-4\big(\eta_1+\eta_2\big)\lambda_1+\rho_1^2\big)\big)+\lambda_2\big(\gamma_1^2\big(\delta^2-8\alpha_2\lambda_1\big)-8\alpha_1\big(\gamma_2^2\lambda_1\\
&+\big(\eta_1+\eta_2\big)\big(\delta^2-8\alpha_2\lambda_1\big)-\gamma_2\delta\rho_1+2\alpha_2\rho_1^2\big)\big)+4\alpha_1\beta_2\big(-2\gamma_2\lambda_1+\delta\rho_1\big)\rho_2+2\alpha_1\big(\delta^2-8\alpha_2\lambda_1\big)\rho_2^2\\
&+\beta_1\gamma_1\big(2\gamma_2\delta\lambda_2+\beta_2^2\rho_1-8\alpha_2\lambda_2\rho_1+\beta_2\delta\rho_2\big)+\beta_1^2\big(\gamma_2^2\lambda_2+\big(\eta_1+\eta_2\big)\big(\beta_2^2-8\alpha_2\lambda_2\big)+\beta_2\gamma_2\rho_2\\
&+2\alpha_2\rho_2^2\big)\big)\big)\Big]^{-1}
\end{aligned}
$$

$$
\begin{aligned}
w_1^{\mathrm{II,c}}=&\Big[\Big(-d_2\gamma_1\gamma_2-q_2^{\mathrm{II,c}}\beta_2\gamma_1\gamma_2+d_1\gamma_2^2+q_1^{\mathrm{II,c}}\beta_1\gamma_2^2+q_1^{\mathrm{II,c}}\gamma_1\gamma_2\delta-8d_1\alpha_2\eta_1-8q_1^{\mathrm{II,c}}\alpha_2\beta_1\eta_1\\
&+4c_m\alpha_2\gamma_1\eta_1-8d_1\alpha_2\eta_2-8q_1^{\mathrm{II,c}}\alpha_2\beta_1\eta_2+c\big(\alpha_1\gamma_2^2+\alpha_2\big(\gamma_1\big(2\gamma_1+\gamma_2-4\eta_1\big)-8\alpha_1\big(\eta_1+\eta_2\big)\big)\big)\\
&-4q_1^{\mathrm{II,c}}\alpha_2\gamma_1\rho_1-4q_2^{\mathrm{II,c}}\alpha_2\gamma_1\rho_2+4\alpha_2\gamma_1\phi_1+4\alpha_2\gamma_1\phi_2\Big)\Big]\times\Big[\big(2\big(\alpha_1\gamma_2^2+\alpha_2\big(\gamma_1^2-8\alpha_1\big(\eta_1+\eta_2\big)\big)\big)\big)\Big]^{-1}\\
w_2^{\mathrm{II,c}}=&\Big[\Big(q_2^{\mathrm{II,c}}\beta_2\gamma_1^2-d_1\gamma_1\gamma_2-q_1^{\mathrm{II,c}}\beta_1\gamma_1\gamma_2-q_1^{\mathrm{II,c}}\gamma_1^2\delta-8q_2^{\mathrm{II,c}}\alpha_1\beta_2\eta_1+4c_m\alpha_1\gamma_2\eta_1+8q_1^{\mathrm{II,c}}\alpha_1\delta\eta_1\\
&-8q_2^{\mathrm{II,c}}\alpha_1\beta_2\eta_2+8q_1^{\mathrm{II,c}}\alpha_1\delta\eta_2+d_2\big(\gamma_1^2-8\alpha_1\big(\eta_1+\eta_2\big)\big)+c\big(\alpha_1\gamma_2\big(\gamma_1+2\gamma_2-4\eta_1\big)\\
&+\alpha_2\big(\gamma_1^2-8\alpha_1\big(\eta_1+\eta_2\big)\big)\big)-4q_1^{\mathrm{II,c}}\alpha_1\gamma_2\rho_1-4q_2^{\mathrm{II,c}}\alpha_1\gamma_2\rho_2+4\alpha_1\gamma_2\phi_1+4\alpha_1\gamma_2\phi_2\Big)\Big]\\
&\times\Big[\big(2\big(\alpha_1\gamma_2^2+\alpha_2\big(\gamma_1^2-8\alpha_1\big(\eta_1+\eta_2\big)\big)\big)\big)\Big]^{-1}\\
r^{\mathrm{II,c}}=&\frac{w_1^{\mathrm{II,c}}\gamma_1+w_2^{\mathrm{II,c}}\gamma_2-c\big(\gamma_1+\gamma_2-2\eta_1\big)-2\big(c_m\eta_1-q_1^{\mathrm{II,c}}\rho_1-q_2^{\mathrm{II,c}}\rho_2+\phi_1+\phi_2\big)}{4\big(\eta_1+\eta_2\big)}\\
p_1^{\mathrm{II,c}}=&\frac{d_1+w_1^{\mathrm{II,c}}\alpha_1+q_1^{\mathrm{II,c}}\beta_1+r^{\mathrm{II,c}}\gamma_1}{2\alpha_1},\qquad p_2^{\mathrm{II,c}}=\frac{d_2+w_2^{\mathrm{II,c}}\alpha_2+q_2^{\mathrm{II,c}}\beta_2+r^{\mathrm{II,c}}\gamma_2-q_1^{\mathrm{II,c}}\delta}{2\alpha_2}.
\end{aligned}
$$

Proof. See Appendix A. □

5.4. Decision strategy (II,d)

This strategy is one type of responsive pricing strategy. A company can encourage the demand for its production by setting prices particular demand conditions, and the available capacity. Instances of such products comprise advanced consumers of electronics (smart-phone, tablets, computers), media products (movies, books), and digital products (computer software, smart-phone apps). In this decision strategy, the decision sequence is as follows:

– The manufacturer declares only the wholesale price w_1, refund price r, and product quality q_1 for the first period.
– The retailer sets his selling price p_1 based on the profit of the first period.

– At the end of the first period, the manufacturer declares wholesale price w_2 and product quality q_2 for the second period.
– The retailer sets his selling price p_2 for the second period.

Each period can be viewed as one selling season or a span over one product generation. Thus, each period in this model can span over one quarter, 6 months, or 2 years, depending on the nature of the product being considered.

The optimal decisions are given in the following proposition.

Proposition 4. *Under decision strategy II,d, the manufacturer's optimal wholesale prices, refund price and product qualities, the retailer's optimal selling prices are given respectively by*

$$\begin{aligned}
w_1^{\mathrm{II,d}} &= \Big[\big(c\beta_1^2\eta_1 - 4d_1\eta_1\lambda_1 + \gamma_1\rho_1 - g_1\gamma_1\rho_1 + d_1\rho_1^2 + c\big(\gamma_1^2\lambda_1 - 4\alpha_1\eta_1\lambda_1 + \alpha_1\rho_1^2\big) + 2\gamma_1\lambda_1\phi_1 \\
&\quad + \beta_1(-2(-1+g_1)\eta_1 + \rho_1(c\gamma_1 + \phi_1))\big)\Big] \times \Big[\big(\beta_1^2\eta_1 + \gamma_1^2\lambda_1 + \beta_1\gamma_1\rho_1 + 2\alpha_1\big(-4\eta_1\lambda_1 + \rho_1^2\big)\big)\Big]^{-1} \\
r^{\mathrm{II,d}} &= \frac{2(-1+g_1)\rho_1 - \Big(c - w_1^{\mathrm{II,d}}\Big)(2\gamma_1\lambda_1 + \beta_1\rho_1) - 4\lambda_1\phi_1}{8\eta_1\lambda_1 - 2\rho_1^2} \\
q_1^{\mathrm{II,d}} &= \frac{-2 + 2g_1 - c\beta_1 + w_1^{\mathrm{II,d}}\beta_1 + 2r\rho_1}{4\lambda_1} \\
p_1^{\mathrm{II,d}} &= \frac{d_1 + w_1^{\mathrm{II,d}}\alpha_1 + q_1^{\mathrm{II,d}}\beta_1 + r^{\mathrm{II,d}}\gamma_1}{2\alpha_1} \\
q_2^{\mathrm{II,d}} &= \frac{-\beta_2\Big(d_2 + r^{\mathrm{II,d}}\gamma_2 - q_1^{\mathrm{II,d}}\delta\Big) + \alpha_2\big(4 - 4g_2 + c\beta_2 - 4r^{\mathrm{II,d}}\rho_2\big)}{\beta_2^2 - 8\alpha_2\lambda_2} \\
w_2^{\mathrm{II,d}} &= \frac{d_2 + c\alpha_2 + q_2^{\mathrm{II,d}}\beta_2 + r^{\mathrm{II,d}}\gamma_2 - q_1^{\mathrm{II,d}}\delta}{2\alpha_2} \\
p_2^{\mathrm{II,d}} &= \frac{d_2 + w_2^{\mathrm{II,d}}\alpha_2 + q_2^{\mathrm{II,d}}\beta_2 + r^{\mathrm{II,d}}\gamma_2 - q_1^{\mathrm{II,d}}\delta}{2\alpha_2}
\end{aligned}$$

provided $4\eta_1\lambda_1 > \rho_1^2$ *and* $4\lambda_1(\beta_1^2\eta_1 + 2\alpha_1\rho_1^2) > 2\lambda_1(\gamma_1^2 + 16\alpha_1\eta_1\lambda_1) + 2\beta_1\gamma_1\rho_1 + (4\alpha_1 + \beta_1^2)\rho_1^2$ *hold.*

Proof. See Appendix A. □

6. Numerical results

In this section, the optimal results are established by considering a numerical example of four different strategies. Then, ratios of the two periods' optimal results are developed and compared amongst the four strategies. Lastly, a graphical illustration is shown for some regions where one decision strategy dominates the other three.

6.1. Numerical example

The following numerical example illustrates and validates the proposed model. Because it is difficult to collect real industry data, a realistic example [16] is created and considered a hypothetical data set related to that example as given below. The model applies to quality improvement and pricing strategies of manufacturers who may choose to have new release prices at the same level as their predecessors (*e.g.*, Apple iPhones). Alternatively, update this price according to your investments in quality (*e.g.*, Renault cars).

The parameter values are as follows: $d_1 = 150$, $d_2 = 180$, $\alpha_1 = 0.34$, $\alpha_2 = 0.30$, $\beta_1 = 0.8$, $\beta_2 = 0.75$, $\gamma_1 = 0.35$, $\gamma_2 = 0.3$, $\phi_1 = 10$, $\phi_2 = 15$, $\eta_1 = 0.3$, $\eta_2 = 0.25$, $\rho_1 = 0.55$, $\rho_2 = 0.6$, $c = 50$, $c_m = 15$, $g_1 = 25$, $g_2 = 20$, $\lambda_1 = 80$,

Table 2. Optimal results under different strategies.

Optimal Decisions	Strategy (I,c)	Strategy (I,d)	Strategy (II,c)	Strategy (II,d)
w or w_1	317.81	**271.735**	280.829	**271.735**
w_2	–	–	359.484	**350.448**
p_1	414.956	**382.602**	396.243	**382.602**
p_2	493.374	**461.11**	514.226	500.672
r	66.8708	**48.8054**	66.6676	**48.8054**
q_1	**0.887235**	0.872105	0.787573	0.872105
q_2	0.886395	0.730187	**0.972539**	0.894173
Π_m	19 827.5	19 241.8	**20 319.0**	20 168.8
Π_r	12 455.6	**14 938.0**	11 712.5	10 949.3
Π	32 283.1	**34 179.9**	32 031.5	31 118.0

$\lambda_2 = 90$, $\delta = 0.05$. Table 2 represents the optimal results in different strategies, taking the parameters into the account. An optimal solution for all four strategies is found with the help of Mathematica 9 software. Optimal results mean maximum profit with lower prices (wholesale, selling, and refund) and higher product quality (shown in bold font). In this assessment, Table 2 shows that the strategies (I,d) and (II,d) offer lower prices (except only (w_2) in (II,d) and p_2 in (I,d) is lower value). But, higher product quality is provided by strategy (I,c) in period 1, and by strategy (II,c) in period 2. Most profit is earned by the manufacturer in strategy (II,c), the retailer, and the whole system in strategy (I,d). Table 2 gives an important outcome that lower prices give lower quality of the product (in strategy (I,d)) and higher prices provide higher product quality (for period 1 strategy (I,c) and period 2 strategy (II,c)). Because of the higher wholesale price and product quality, the manufacturer gains more profit in (II,c) whereas the retailer and whole system profits are maximum in (I,d) for lower prices.

These optimal results also indicate quick responsive decisions (*i.e.*, (I,d) and (II,d)) give dominating outcomes but the good quality product can be shown in pre-announced strategies (*i.e.*, (I,c) and (II,c)). In real life, some online retailers (or manufacturers) change prices from time to time, meaning they also have to adjust their product quality. Also, some reputed companies keep their product prices the same or sometimes higher with time to keep product quality higher. As these results only depend on one set of parameter values, now a variation on some parameter values is done to understand the sensitivities of some key parameters.

6.2. Sensitivity analysis

Sensitivity analyses with respect to the parameters α_1, γ_1, η_1, λ_1, ϕ_1 and c is shown in the Figures 2–7. Run those parameters‘ values over the ratios of period 1 and period 2's optimal results are verified *via* way of means of retaining different parameters' values the same. So, if the ratio is greater than one (*i.e.*, >1) period one has higher values and less than one (<1) then the second period has higher values. With these figures, the following observations can be made.

(i) α_1 is the negative sensitivity of selling price on the first period's demand rate. Therefore, if it has a lower value, then the selling price in the first period becomes higher than that of the second period. As a result, increasing value provides lower selling price in the first period, so the value of $\Pi_{m1}/\Pi_{m2}, \Pi_{r1}/\Pi_{r2}, D_1/D_2, D_{r1}/D_{r2}$ and p_1/p_2 (see Figs. 2a–2e) become low. But, in the case of the unique wholesale price (*i.e.*, strategies (I,c) and (I,d) the product quality increases in period 1 when α_1 increases (Fig. 2f).

(ii) γ_1, refund price sensitivity on first-period demand rate, refund money increases when it becomes higher. This had a positive impact on the first period's profits of both players, demand rate, return rate (more rapidly) and also on selling price (Figs. 3a–3e). As refund money increases that means product quality in the first period becomes lower in strategies (I,c) and (I,d) (Fig. 3f).

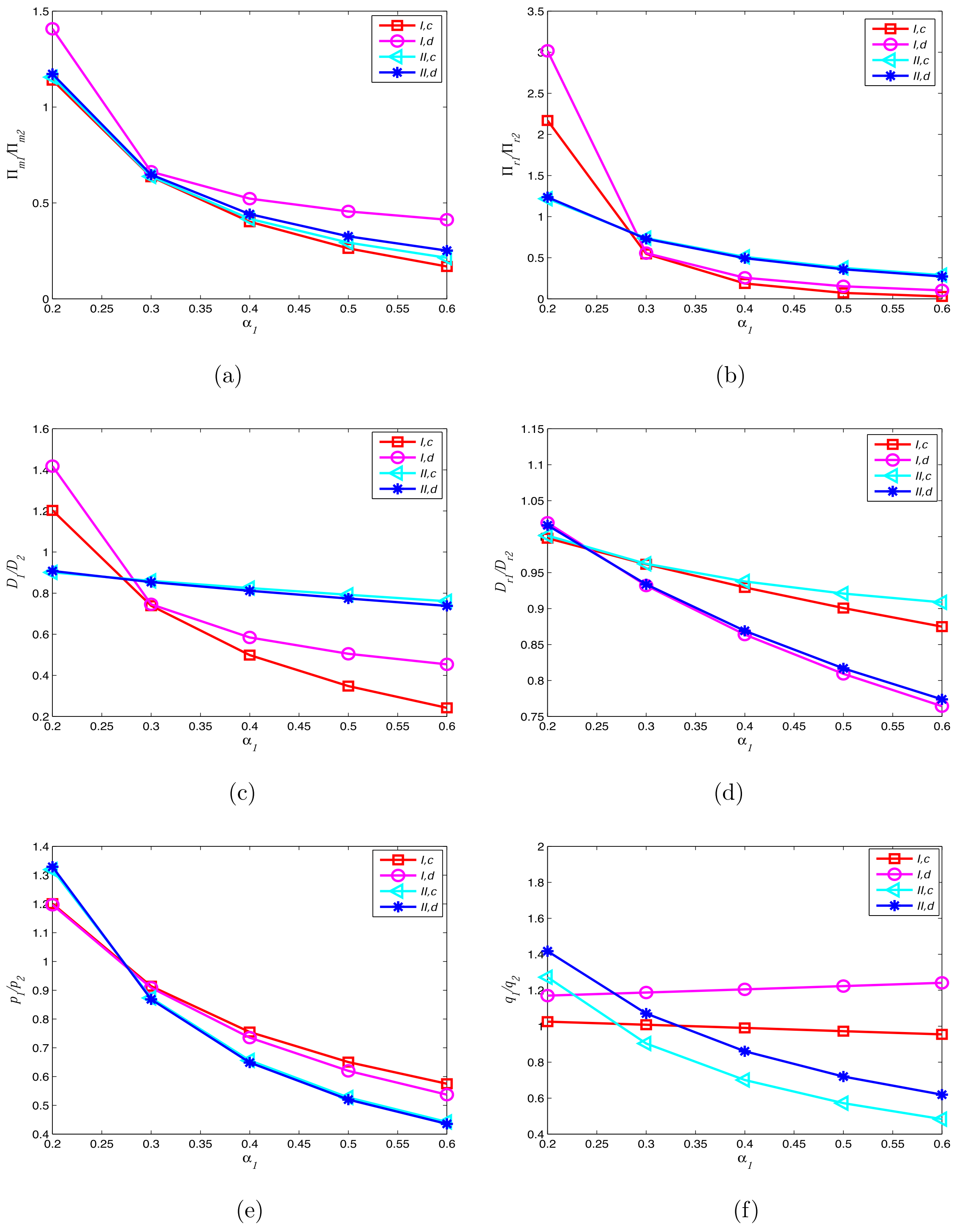

FIGURE 2. Sensitivity of the parameter α_1 on the ratios of two periods' optimal results. (a) α_1 *vs.* Π_{m1}/Π_{m2}. (b) α_1 *vs.* Π_{r1}/Π_{r2}. (c) α_1 *vs.* D_1/D_2. (d) α_1 *vs.* D_{r1}/D_{r2}. (e) α_1 *vs.* p_1/p_2. (f) α_1 *vs.* q_1/q_2.

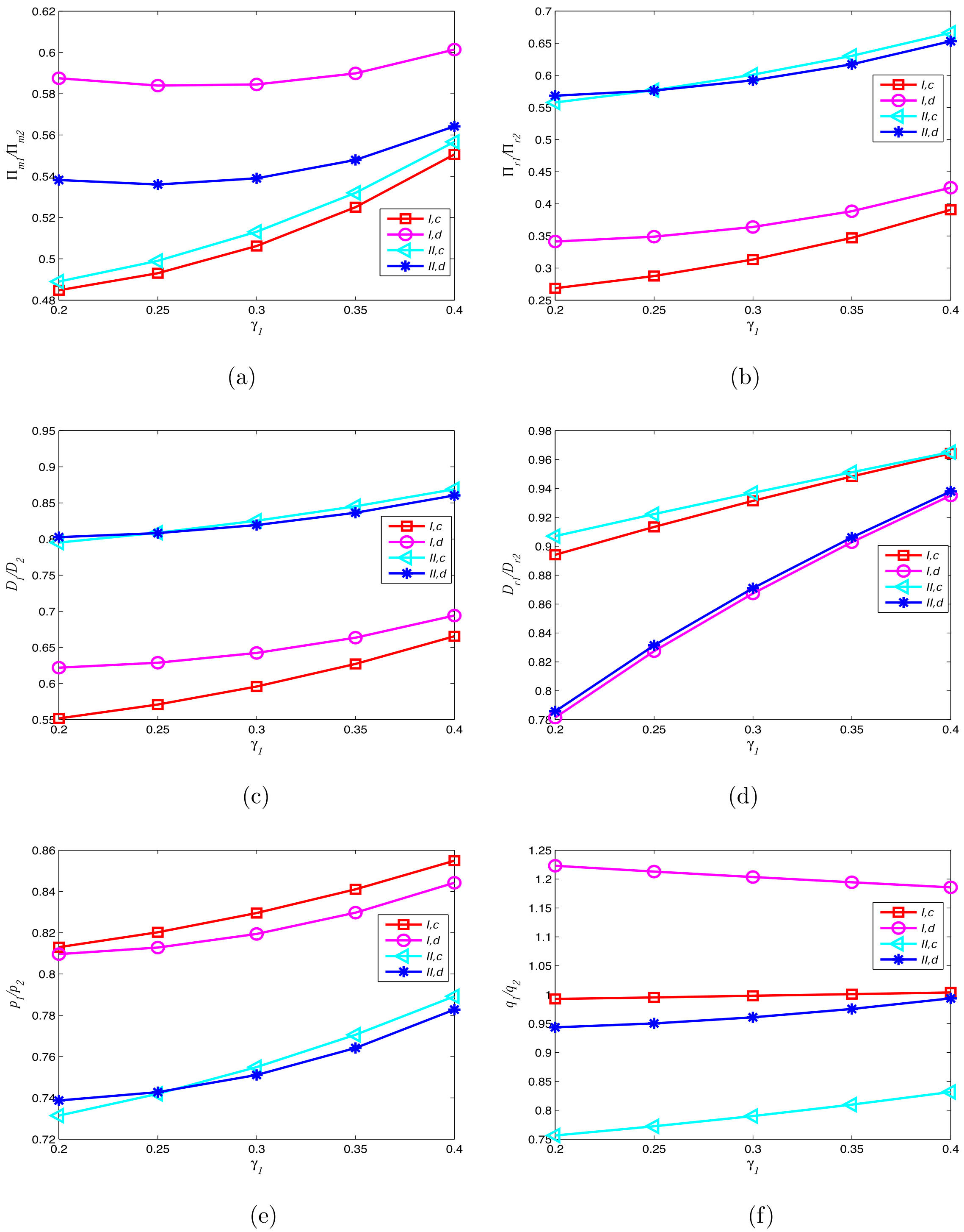

FIGURE 3. Sensitivity of the parameter γ_1 on the ratios of two periods' optimal results. (a) γ_1 *vs.* Π_{m1}/Π_{m2}. (b) γ_1 *vs.* Π_{r1}/Π_{r2}. (c) γ_1 *vs.* D_1/D_2. (d) γ_1 *vs.* D_{r1}/D_{r2}. (e) γ_1 *vs.* p_1/p_2. (f) γ_1 *vs.* q_1/q_2.

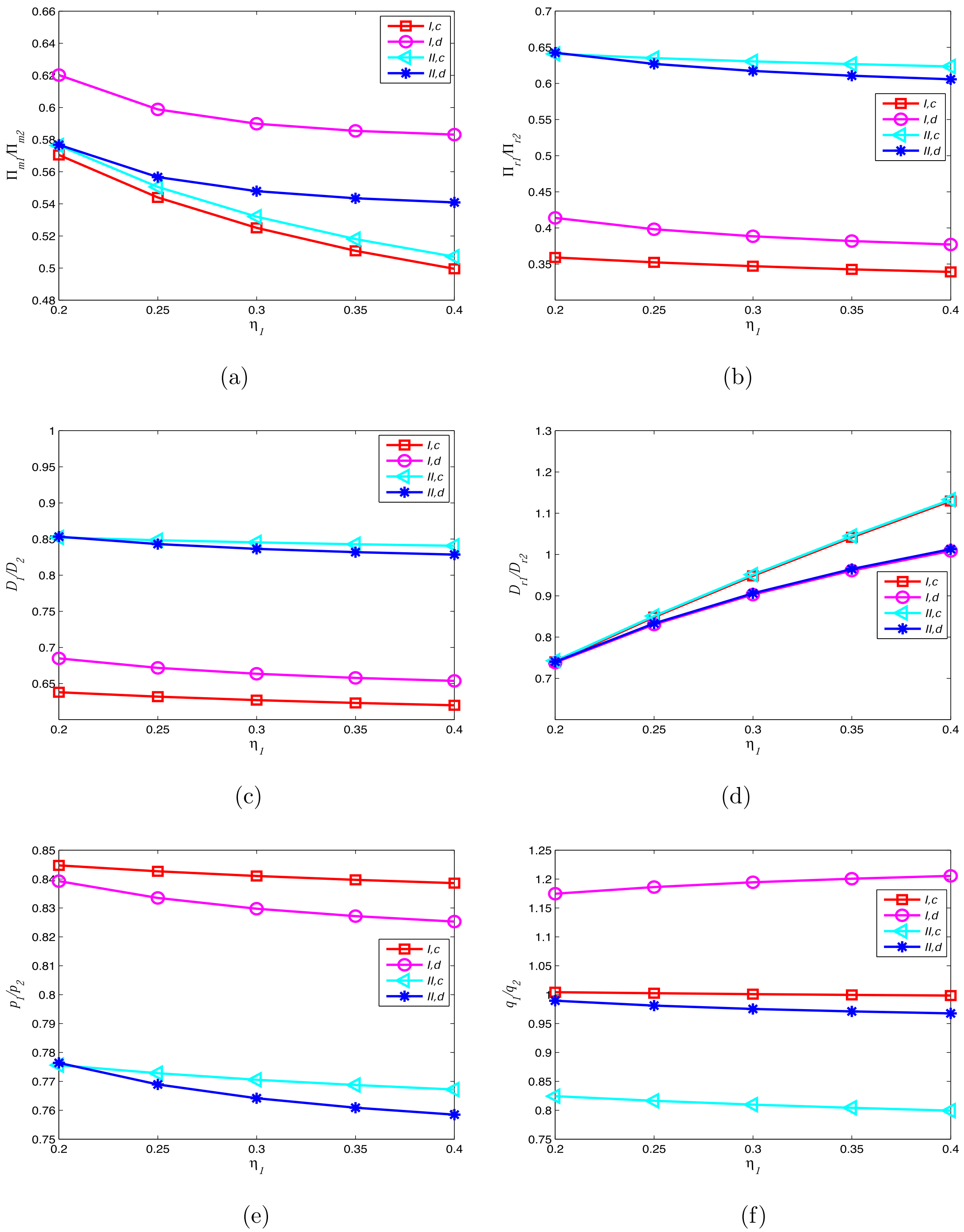

FIGURE 4. Sensitivity of the parameter η_1 on the ratios of two periods' optimal results. (a) η_1 *vs.* Π_{m1}/Π_{m2}. (b) η_1 *vs.* Π_{r1}/Π_{r2}. (c) η_1 *vs.* D_1/D_2. (d) η_1 *vs.* D_{r1}/D_{r2}. (e) η_1 *vs.* p_1/p_2. (f) η_1 *vs.* q_1/q_2.

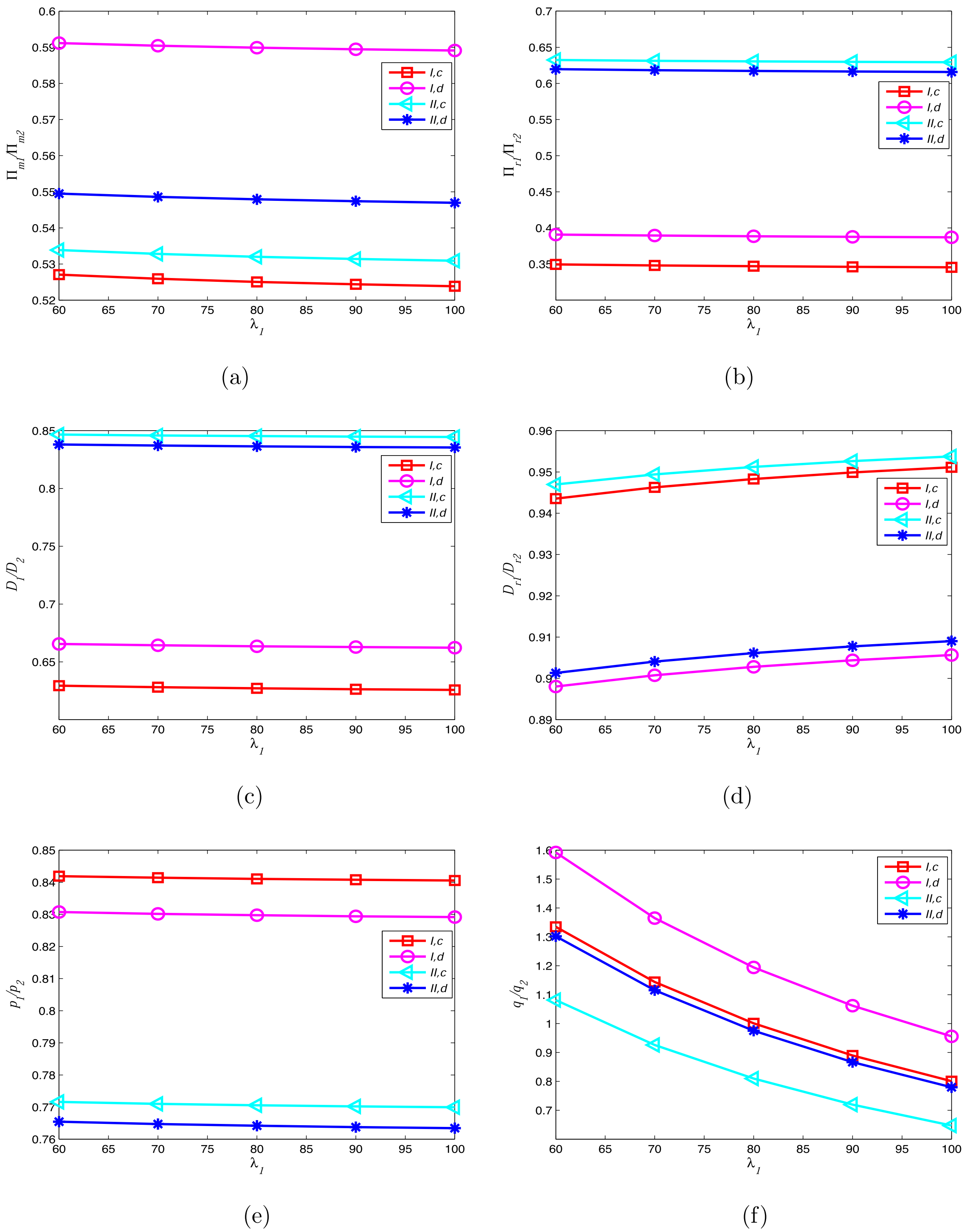

FIGURE 5. Sensitivity of the parameter λ_1 on the ratios of two periods' optimal results. (a) λ_1 *vs.* Π_{m1}/Π_{m2}. (b) λ_1 *vs.* Π_{r1}/Π_{r2}. (c) λ_1 *vs.* D_1/D_2. (d) λ_1 *vs.* D_{r1}/D_{r2}. (e) λ_1 *vs.* p_1/p_2. (f) λ_1 *vs.* q_1/q_2.

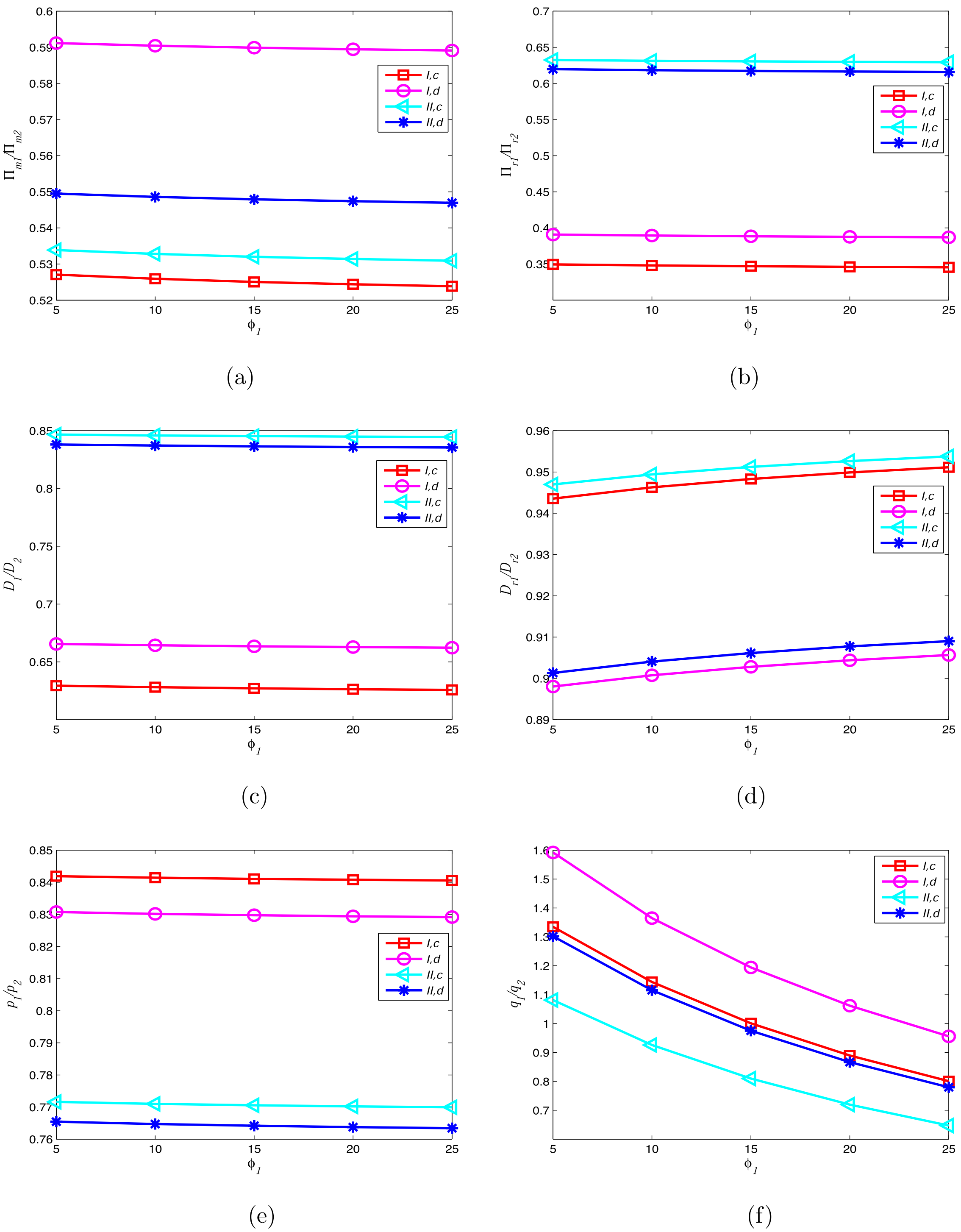

FIGURE 6. Sensitivity of the parameter ϕ_1 on the ratios of two periods' optimal results. (a) ϕ_1 *vs.* Π_{m1}/Π_{m2}. (b) ϕ_1 *vs.* Π_{r1}/Π_{r2}. (c) ϕ_1 *vs.* D_1/D_2. (d) ϕ_1 *vs.* D_{r1}/D_{r2}. (e) ϕ_1 *vs.* p_1/p_2. (f) ϕ_1 *vs.* q_1/q_2.

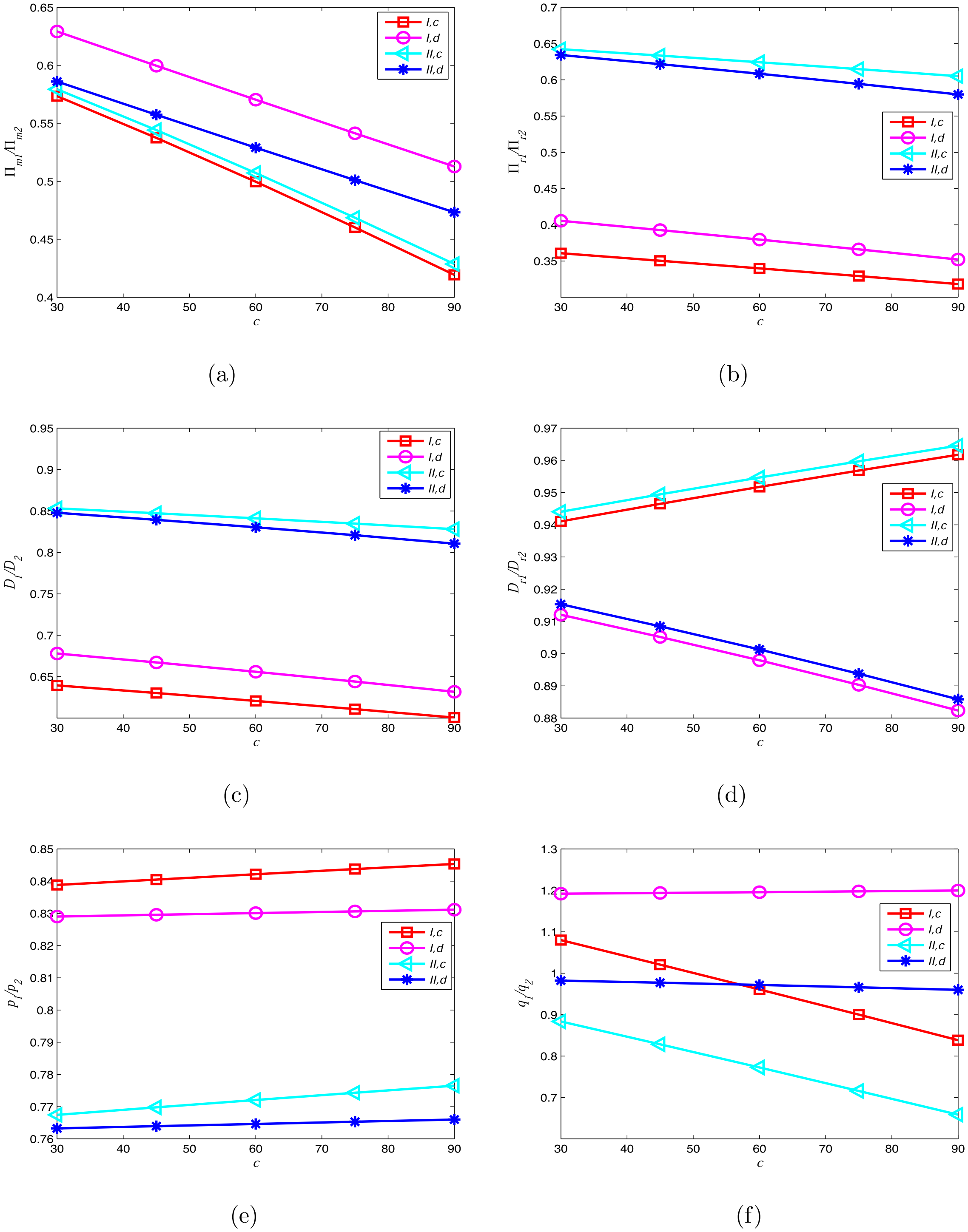

FIGURE 7. Sensitivity of the parameter c on the ratios of two periods' optimal results. (a) c *vs.* Π_{m1}/Π_{m2}. (b) c *vs.* Π_{r1}/Π_{r2}. (c) c *vs.* D_1/D_2. (d) c *vs.* D_{r1}/D_{r2}. (e) c *vs.* p_1/p_2. (f) c *vs.* q_1/q_2.

(iii) If η_1 increases, then refund money rapidly increases, which gives high returned products in period 1 (Fig. 4d). Due to the huge return in period 1, profits and selling prices went down as well as the product quality (Figs. 4a–4c, 4e, 4f). But in strategy (I,d) period 2's quality is more affected. As they provide the same wholesale price, the quality label decreases in the second period, he can compensate for his losses.
(iv) As λ_1 is the quality improvement parameter in the first period, so higher cost means a drop in product quality in period 1 (Fig. 5f). A drop in product quality implies a return rate increase in period 1 (Fig. 5c). Other optimal values are almost unaffected by this parameter (Figs. 5a, 5b, 5d, 5e). Similar outcomes are found when ϕ_1 changed (Figs. 6a–6f).
(v) Changes in the cost of production (c) lead to a lower first-period profit and lower demand rate (Fig. 7a–7c). Higher production cost implies higher wholesale price as well as selling price (Fig. 7e). Return of product increases in strategies (I,c) and (II,c) but decreases in (I,d) and (II,d) (Fig. 7d). Product quality in (I,c) and (II,c) decrease rapidly because of rapid changes in wholesale price in pre-announced decisions. But, this effect is balanced by quick responsive strategies *i.e.*, (I,d) and (II,d) (Fig. 7f).

6.3. Comparison of dominating areas

This subsection is devoted to showing the dominating strategies sequences in (d_1, d_2) region graphically. Comparisons of the manufacturer's profit, retailer's profit and selling prices, product quality and refund money sequences under four different strategies are developed. The outcomes of the figure are given as follows:

(i) The region is created in variations of d_1 and d_2 where $d_1 \in (150, 250)$ and $d_2 \in (150, 250)$. Figure 8a shows (II,c) is dominating strategy in the manufacturer's profit perspective, which is a similar result to Table 2.
(ii) The retailer's profit is dominated by strategy (I,d) similar to Table 2. But, when d_1 is high and d_2 is low, then the retailer gains more profit by (I,c) strategy, which is second in place in Table 2 results.
(iii) Similar to Table 2 in all region p_1 is same in (I,d) and (II,d) strategies gives lower value when d_2 is higher. But, when d_2 is lower and d_1 is higher, strategy (I,c) gives the lowest value (Figure 8(c)).
(iv) Figure 8d shows that p_2 is lowest when d_2 is high and d_1 is low. Then the situation shifts to strategies (II,d) and (II,c) when d_1 becomes higher and d_2 gradually decreases.
(v) Figure 8e shows q_1 is same in (I,d) and (II,d) strategies (as in Tab. 2) and provide higher quality when d_1 is higher. But, when d_2 is higher than d_1 then (I,c) strategy gives a better quality product as reflected in Table 2.
(vi) q_2 is high when d_2 is high in (II,c) as in Table 2, but shifts to strategy (I,c) and then (I,d) when d_1 is higher than that of d_2 (Fig. 8f).
(vii) As in Table 2, in all region r is same in (I,d) and (II,d) strategies gives lower value when d_2 is higher. But when d_1 is more higher than d_1 then strategy (I,c) gives a lower refund money.

7. Managerial implications

In practice, when new products such as fashion goods, mobiles, laptops, and other electronics goods are launched in the market, the manufacturers often offer a certain return policy to attract customers for some specific periods. Later, manufacturers cut down the return policy when products are very familiar to the customers by their brand value or quality. However, they may offer lower selling prices to keep intact their market demand. A product's high quality or service can indeed reduce the product's return. Key findings of this study are outlined as managerial insights in the following.

- This study shows that the pre-announcement strategy ((I,c), (II,c)) has a higher selling price in the second period.
- If the retailer jointly optimizes the overall profit for the whole season (strategies (I,c) and (II,c)), in the second period, the retailer is expected to meet market demand. As a result, he demands higher selling prices for these strategies than strategies (I,d) and (II,d).

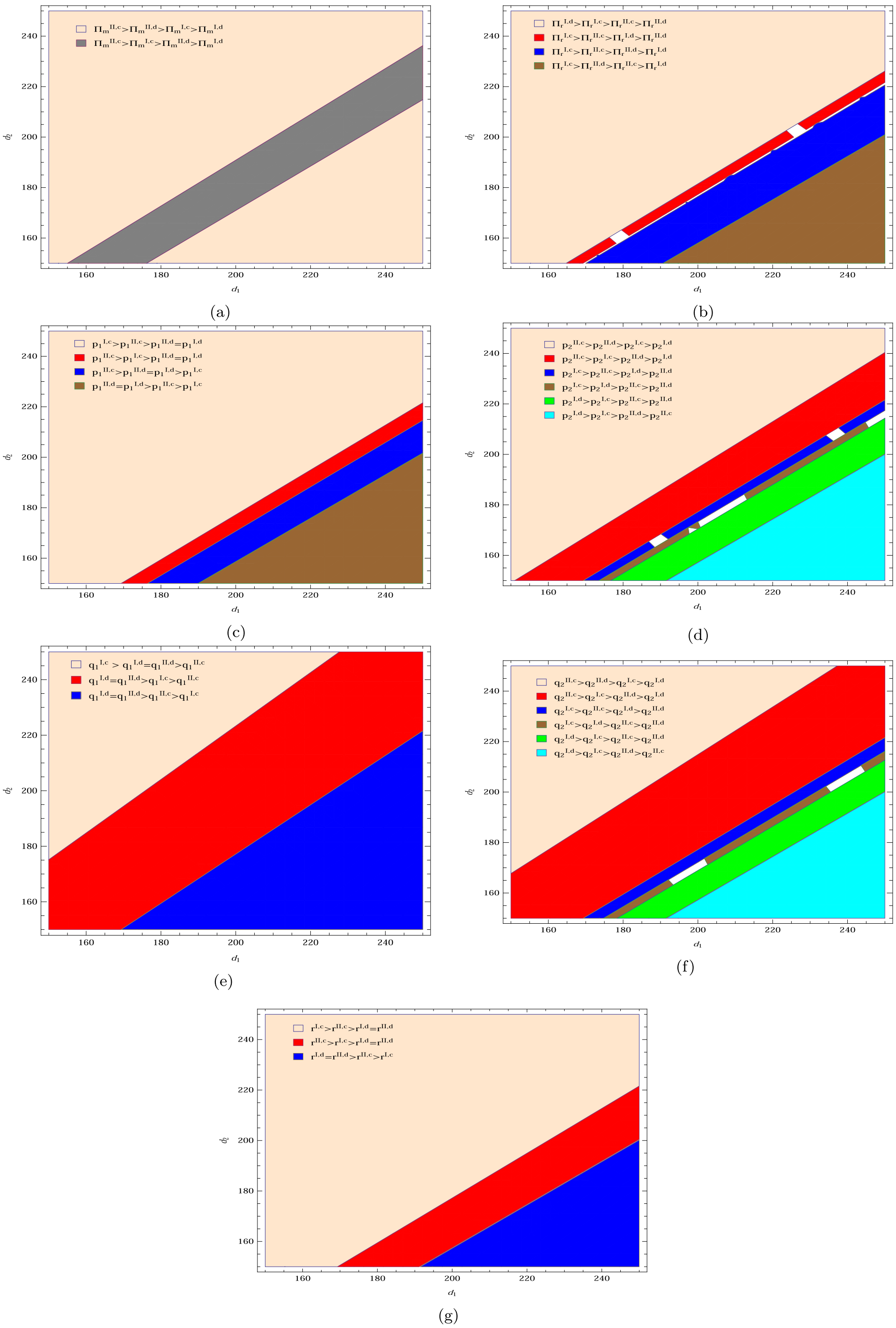

FIGURE 8. Dominance of different strategies' optimal results over (d_1, d_2) region. (a) Manufacturer's profit. (b) Retailer's profit. (c) Selling price (p_1). (d) Selling price (p_2). (e) Product quality (q_1). (f) Product quality (q_2). (g) Refund money (r).

- Optimal results are similar to the penetration pricing strategy under real market conditions. Some new mobile operators, food and beverage companies, and Internet and cable providers are introducing new products at low prices (sometimes free) for customers to try.
- If the product is known and trusted in the market, the price of the product can be high. This research shows that management must adopt a pre-announced strategy with higher production values if they want to deliver higher quality products.
- Companies that want to update their products frequently need to lower their product prices and make faster decisions. Prices for products can be high if the product is well known and trusted in the market.

8. Conclusions and future research directions

Consider a two-period closed-loop supply chain model with one manufacturer and one retailer to develop a model where selling price, product quality, and refund amount affect the demand rate. The manufacturer leads the chain and the retailer follows his decisions. A two-period model with pre-announced decisions and corresponding decisions illustrates the difference between these policies. In the pre-registration model, well-known companies follow the policy and offer high quality at a high price. However, new companies are introducing responsive pricing models that offer lower prices for lower-quality products. They launch their products in a competitive market using low prices or some counter offers to gain trust in their products. It also learns from feedback of previous periods to improve decision-making for the next period.

This study has some limitations. Firstly, all of the decision-making strategies are based on the manufacturer-driven Stackelberg model. A different outcome can occur if the retailer is at the forefront of his chain of supply. In the future, this research may include dual channels in both directions (as developed by Zhang *et al.* [48]). One can analyse the proposed model by considering the effect of various coordinating contracts in a closed-loop supply chain [47]. A risk-aversion supplier or manufacturer inclusion in this study is a good future direction for this study [46]. Finally, this study is limited to a two-period sales season. It can be extended to a multi-objective and multi-period model [33] with some other limitations.

Appendix A.

Proof of Proposition 1. Analytically, first the retailer's whole season profit function is optimized $\Pi_r^{\mathrm{I}}(p_1, p_2)$ and find the reaction $p_1 = \frac{d_1+w\alpha_1+q_1\beta_1+r\gamma_1}{2\alpha_1}$, $p_2 = \frac{d_2+w\alpha_2+q_2\beta_2+r\gamma_2-q_1\delta}{2\alpha_2}$. The reaction is unique as the determinant of 2×2 Hessian matrix $4\alpha_1\alpha_2 > 0$ and all pivot terms are negative, so it is negative definite.

The manufacturer has four decision variables *i.e.*, $\Pi_m^{\mathrm{I}}(w, r, q_1, q_2)$. Corresponding Hessian matrix is

$$H = \begin{pmatrix} \frac{\partial^2\Pi_m}{\partial q_1^2} & \frac{\partial^2\Pi_m}{\partial q_1\partial q_2} & \frac{\partial^2\Pi_m}{\partial q_1\partial r} & \frac{\partial^2\Pi_m}{\partial q_1\partial w} \\ \frac{\partial^2\Pi_m}{\partial q_2\partial q_1} & \frac{\partial^2\Pi_m}{\partial q_2^2}. & \frac{\partial^2\Pi_m}{\partial q_2\partial r} & \frac{\partial^2\Pi_m}{\partial q_2\partial w} \\ \frac{\partial^2\Pi_m}{\partial r\partial q_1} & \frac{\partial^2\Pi_m}{\partial r\partial q_2} & \frac{\partial^2\Pi_m}{\partial r^2} & \frac{\partial^2\Pi_m}{\partial r\partial w} \\ \frac{\partial^2\Pi_m}{\partial w\partial q_1} & \frac{\partial^2\Pi_m}{\partial w\partial q_2} & \frac{\partial^2\Pi_m}{\partial w\partial r} & \frac{\partial^2\Pi_m}{\partial w^2} \end{pmatrix} = \begin{pmatrix} -2\lambda_1 & 0 & \rho_1 & \frac{\beta_1-\delta}{2} \\ 0 & -2\lambda_2 & \rho_2 & \frac{\beta_2}{2} \\ \rho_1 & \rho_2 & -2(\eta_1+\eta_2) & \frac{\gamma_1+\gamma_2}{2} \\ \frac{\beta_1-\delta}{2} & \frac{\beta_2}{2} & \frac{\gamma_1+\gamma_2}{2} & -(\alpha_1+\alpha_2) \end{pmatrix}.$$

From this get, $|H_2| = 4\lambda_1\lambda_2 > 0$, $|H_3| = -2\lambda_2(4\eta_1\lambda_1 - \rho_1^2) - 2\lambda_1(4\eta_2\lambda_2 - \rho_2^2) < 0$ if $\eta_i\lambda_i > \rho_i^2$ where $i = 1, 2$. Also the

$$\begin{aligned} |H_4| = 1/4\Big[&\Big[\Big(8(\alpha_1+\alpha_2)\lambda_1 - (\beta_1-\delta)^2\Big)\Big(4(\eta_1+\eta_2)\lambda_2 - \rho_2^2\Big)\Big] - 4(\gamma_1+\gamma_2)^2\lambda_1\lambda_2 \\ &- 4\lambda_2\rho_1[(\gamma_1+\gamma_2)(\beta_1-\delta) + 2(\alpha_1+\alpha_2)\rho_1] - \beta_2^2\big[4(\eta_1+\eta_2)\lambda_1 - \rho_1^2\big] \\ &- 2\beta_2[2(\gamma_1+\gamma_2)\lambda_1 + (\beta_1-\delta)\rho_1]\rho_2\Big] > 0, \end{aligned}$$

if $[(8(\alpha_1+\alpha_2)\lambda_1-(\beta_1-\delta)^2)(4(\eta_1+\eta_2)\lambda_2-\rho_2^2)] > [4(\gamma_1+\gamma_2)^2\lambda_1\lambda_2+4\lambda_2\rho_1[(\gamma_1+\gamma_2)(\beta_1-\delta)+2(\alpha_1+\alpha_2)\rho_1]+\beta_2^2[4(\eta_1+\eta_2)\lambda_1-\rho_1^2]+2\beta_2[2(\gamma_1+\gamma_2)\lambda_1+(\beta_1-\delta)\rho_1]\rho_2]$ holds.

Then, the manufacturer declares his unique product qualities, refund money and wholesale price by optimizing $\Pi_m^{\mathrm{I}}(w,r,q_1,q_2)$. Then optimal results cab substitute in previous results and get the optimal decisions which are shown in Proposition 1. □

Proof of Proposition 2. To determine the optimal reaction, At first the retailer's profit function during the second period *i.e.*, $\Pi_{r2}^{\mathrm{I}}(p_2)$ is optimized and find out the value of p_2. From the first order condition gives the value of $p_2=\frac{d_2+w\alpha_2+q_2\beta_2+r\gamma_2-q_1\delta}{2\alpha_2}$. Then the second order condition is also checked which gives $\frac{\mathrm{d}^2\Pi_{r2}^{\mathrm{I}}}{\mathrm{d}p_2^2}=-2\alpha_2<0$ *i.e.*, the reaction is unique. Then, the manufacturer optimizes the profit function of the second period *i.e.*, $\Pi_{m2}^{\mathrm{I}}(q_2)$ and determines $q_2=\frac{-2+2g_2-c\beta_2+w\beta_2+2r\rho_2}{4\lambda_2}$. Here also the second order optimality *i.e.*, $\frac{\mathrm{d}^2\Pi_{m2}^{\mathrm{I}}}{\mathrm{d}q_2^2}=-2\lambda_2<0$, implies the optimal quality label. Now, the first period's profit part is calculated in a similar way. The retailer maximizes the profit function for the first period *i.e.*, $\Pi_{r1}^{\mathrm{I}}(p_1)$ and determines the value of $p_1=\frac{d_1+w\alpha_1+q_1\beta_1+r\gamma_1}{2\alpha_1}$. Again the second order derivative $\frac{\mathrm{d}^2\Pi_{r1}^{\mathrm{I}}}{\mathrm{d}p_1^2}=-2\alpha_1<0$ which suggests that the reaction is unique. With these reactions, the manufacturer maximizes his profit function for the first period *i.e.*, $\Pi_{m1}^{\mathrm{I}}(q_1,r,w)$ and sets the wholesale price w and refund price r for the two periods and quality q_1 for the first period. Now, the Hessian is given as follows:

$$H=\begin{pmatrix}-2\lambda_1 & \rho_1 & \frac{\beta_1}{2}\\ \rho_1 & -2\eta_1 & \frac{\gamma_1}{2}\\ \frac{\beta_1}{2} & \frac{\gamma_1}{2} & -\alpha_1\end{pmatrix}.$$

All pivot terms are negative, and $|H_2|=4\eta_1\lambda_1-\rho_1^2>0$ if $4\eta_1\lambda_1>\rho_1^2$ and $|H_3|=\frac{1}{2}[\beta_1^2\eta_1+\gamma_1^2\lambda_1+\beta_1\gamma_1\rho_1+2\alpha_1\rho_1^2-8\alpha_1\eta_1\lambda_1]<0$ if $8\alpha_1\eta_1\lambda_1>\beta_1^2\eta_1+\gamma_1^2\lambda_1+\beta_1\gamma_1\rho_1+2\alpha_1\rho_1^2$. So all the condition indicates that if $8\alpha_1\eta_1\lambda_1>\beta_1^2\eta_1+\gamma_1^2\lambda_1+\beta_1\gamma_1\rho_1+2\alpha_1\rho_1^2$ then the Hessian is negative definite and gives a optimized result. □

Proof of Proposition 3. Here, the retailer maximizes the total profit $\Pi_r^{\mathrm{II}}(p_1,p_2)$. The Hessian matrix $\begin{pmatrix}-2\alpha_1 & 0\\ 0 & -2\alpha_2\end{pmatrix}$ clearly indicates the uniqueness. Therefore, $\frac{\partial\Pi_r^{\mathrm{II}}}{\partial p_1}=0=\frac{\partial\Pi_r^{\mathrm{II}}}{\partial p_2}$ give the unique selling prices of the retailer as

$$p_1=\frac{d_1+w_1\alpha_1+q_1\beta_1+r\gamma_1}{2\alpha_1},\quad p_2=\frac{d_2+w_2\alpha_2+q_2\beta_2+r\gamma_2-q_1\delta}{2\alpha_2}.$$

Knowing the reaction of the retailer, the manufacturer maximizes his total profit $\Pi_m^{\mathrm{II}}(q_1,q_2,r,w_1,w_2)$. Here, the Hessian matrix is

$$H=\begin{pmatrix}-2\lambda_1 & 0 & \rho_1 & \frac{\beta_1}{2} & -\frac{\delta}{2}\\ 0 & -2\lambda_2 & \rho_2 & 0 & \frac{\beta_2}{2}\\ \rho_1 & \rho_2 & -2(\eta_1+\eta_2) & \frac{\gamma_1}{2} & \frac{\gamma_2}{2}\\ \frac{\beta_1}{2} & 0 & \frac{\gamma_1}{2} & -\alpha_1 & 0\\ -\frac{\delta}{2} & \frac{\beta_2}{2} & \frac{\gamma_2}{2} & 0 & -\alpha_2\end{pmatrix}.$$

Here all pivot elements are negative.

Now $|H_2|=4\lambda_1\lambda_2>0$, $|H_3|=-2\lambda_2(4\eta_1\lambda_1-\rho_1^2)-2\lambda_1(4\eta_2\lambda_2-\rho_2^2)<0$ if $4\eta_i\lambda_i>\rho_i^2$, where $i=1,2$. Again, $|H_4|=(-\gamma_1^2\lambda_1\lambda_2+(\eta_1+\eta_2)(8\alpha_1\lambda_1-\beta_1^2)\lambda_2-\beta_1\gamma_1\rho_1\lambda_2-2\alpha_1\rho_1^2\lambda_2)-1/4(8\alpha_1\lambda_1-\beta_1^2)\rho_2^2>0$ if $(8\alpha_1\lambda_1-\beta_1^2)>\frac{\lambda_2}{\lambda_2(\eta_1+\eta_2)-\rho_2^2/4}(\gamma_1^2\lambda_1+\beta_1\gamma_1\rho_1+2\alpha_1\rho_1^2)$. and $|H_5|=1/8(-\beta_2^2(\gamma_1^2\lambda_1-8\alpha_1(\eta_1+\eta_2)\lambda_1+2\alpha_1\rho_1^2)+\gamma_1^2\lambda_2(8\alpha_2\lambda_1-\delta^2)+8\alpha_1\lambda_2(\gamma_2^2\lambda_1+(\eta_1+\eta_2)(\delta^2-8\alpha_2\lambda_1)-\gamma_2\delta\rho_1+2\alpha_2\rho_1^2)+4\alpha_1\beta_2(2\gamma_2\lambda_1-\delta\rho_1)\rho_2-2\alpha_1(\delta^2-8\alpha_2\lambda_1)\rho_2^2-\beta_1\gamma_1(2\gamma_2\delta\lambda_2-\rho_1(8\alpha_2\lambda_2-\beta_2^2)\rho_1+\beta_2\delta\rho_2)-\beta_1^2(\gamma_2^2\lambda_2-(\eta_1+\eta_2)(8\alpha_2\lambda_2-\beta_2^2)+\beta_2\gamma_2\rho_2+2\alpha_2\rho_2^2))<0$ if $(\beta_1^2\beta_2^2+64\alpha_1\alpha_2\lambda_1\lambda_2)(\eta_1+\eta_2)+\beta_2^2\gamma_1^2\lambda_1+\beta_1^2\gamma_2^2\lambda_2+2\beta_1\gamma_1\gamma_2\delta\lambda_2+\gamma_1^2\delta^2\lambda_2+\beta_1\beta_2^2\gamma_1\rho_1+8\alpha_1\gamma_2\delta\lambda_2\rho_1+2\alpha_1\beta_2^2\rho_1^2+\beta_1^2\beta_2\gamma_2\rho_2+$

$\beta_1\beta_2\gamma_1\delta\rho_2+4\alpha_1\beta_2\delta\rho_1\rho_2+2(\alpha_2\beta_1^2+\alpha_1\delta^2)\rho_2^2 > (8\alpha_1\beta_2^2\lambda_1+8\alpha_2\beta_1^2\lambda_2+8\alpha_1\delta^2\lambda_2)(\eta_1+\eta_2)+8\lambda_1\lambda_2(\alpha_2\gamma_1^2+\alpha_1\gamma_2^2)+8\alpha_2\beta_1\gamma_1\lambda_2\rho_1+8\alpha_1\beta_2\gamma_2\lambda_1\rho_2+16\alpha_1\alpha_2(\lambda_2\rho_1^2+\lambda_1\rho_2^2)$ holds.

So the Hessian is negative definite. This shows that there exist unique wholesale prices, qualities and refund price of the manufacturer, which are given in the Proposition 3. □

Proof of Proposition 4. The retailer's reaction for the first period can be obtained by optimizing the profit function $\Pi^{\text{II}}_{r2}(p_2)$ with respect to p_2 and get $p_2=\frac{d_2+w_2\alpha_2+q_2\beta_2+r\gamma_2-q_1\delta}{2\alpha_2}$ which is unique as second derivative is $-2\alpha_2<0$.

Then, the manufacturer maximizes the profit function $\Pi^{\text{II}}_{m2}(w_2,q_2)$ for the second period and determines the optimal wholesale price $w_2=\frac{d_2+c\alpha_2+q_2\beta_2+r\gamma_2-q_1\delta}{2\alpha_2}$
and product quality $q_2=\frac{-\beta_2(d_2+r\gamma_2-q_1\delta)+\alpha_2(4-4g_2+c\beta_2-4r\rho_2)}{\beta_2^2-8\alpha_2\lambda_2}$.
Here also Hessian $=\begin{pmatrix}-2\lambda_2 & \beta_2/2\\ \beta_2/2 & -\alpha_2\end{pmatrix}$ shows it will be negative definite if $8\alpha_2\lambda_2>\beta_2^2$.

The same process is applied to derive the selling and wholesale prices, refund money, and product quality of the retailer and the manufacturer for the second-period profit functions $\Pi^{\text{II}}_{r1}(p_1)$ and $\Pi^{\text{II}}_{m1}(q_1,r,w_1)$ respectively. The retailer's optimal selling price is obtained as $p_1=\frac{(d_1+w_1\alpha_1+q_1\beta_1+r\gamma_1)}{2\alpha_1}$.

Then knowing the reaction of the retailer, the manufacturer maximizes his total profit $\Pi^{\text{II}}_{m1}(q_1,r,w_1)$. Here, the Hessian matrix is

$$H=\begin{pmatrix}-2\lambda_1 & \rho_1 & \beta_1/2\\ \rho_1 & -2\eta_1 & \gamma_1/2\\ \beta_1/2 & \gamma_1/2 & -\alpha_1\end{pmatrix}.$$

Here all pivot elements are negative, now $|H_2|=4\eta_1\lambda_1-\rho_1^2>$ if $4\eta_1\lambda_1>\rho_1^2$ and $|H_3|=\frac{1}{4}(2\lambda_1(\gamma_1^2-2\beta_1^2\eta_1+16\alpha_1\eta_1\lambda_1)+2\beta_1\gamma_1\rho_1+(4\alpha_1+\beta_1^2-8\alpha_1\lambda_1)\rho_1^2)<0$ if $4\lambda_1(\beta_1^2\eta_1+2\alpha_1\rho_1^2)>2\lambda_1(\gamma_1^2+16\alpha_1\eta_1\lambda_1)+2\beta_1\gamma_1\rho_1+(4\alpha_1+\beta_1^2)\rho_1^2$.

Hence prove the proposition. □

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