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THE EQUITY THEORY: A QUANTITATIVE PERSPECTIVE USING DATA ENVELOPMENT ANALYSIS

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Abstract. Equity theory (ET) is an organizational theory investigating how fairly people feel they have been treated. The literature on ET does not address two essential questions: what is the magnitude of the equity that one may perceive compared to other members in an organization?, and how much should be the resources (outcomes) of an underpaid member reduced (increased) to feel equal? The group members may respond to these questions emotionally, and their answers could be biased based on their personalities. This paper proposes a novel method using data envelopment analysis (DEA) to quantify the ET and answer these questions more logically. DEA is a mathematical model that is conceptually similar to ET. We will show how DEA can estimate the degree of equity perceived by members of a group with different personalities, including optimistic, pessimistic, benevolent, and entitled characters.

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1. Introduction

People give inputs to the organizations and receive outcomes instead. Each person compares his/her inputs and outcomes with others and may feel dissatisfied. This comparison forms the foundation of the equity theory (ET) introduced by Adams [1, 2]. In other words, each person compares his/her outcome-input ($Y-X$) ratio with others; if this ratio is lower than others, he/she feels dissatisfied [2,3]. Feeling inequity motivates individuals to achieve equity or reduce inequity [2]. Some studies applied ET concepts to distribute outputs. For example, Wijck [4] presents a method to estimate utility functions based on ET to evaluate income distributions. ET is based on social exchange. However, it is applied in other areas, including commercial transactions [5], tax evasion behavior [6], tax reporting decisions [7, 8], software piracy behavior [9], development of practical solutions to the Emiratization challenge [10], online group buying intention [11], and pricing footprint of country-of-origin [12].

Keywords. Data envelopment analysis (DEA), equity theory (ET), personality, fairness.

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ET suffers from shortcomings that challenge its application in social issues, especially those related to quantifying this theory. ET does not quantify the magnitude of inputs, outcomes, and the degree of perceived inequity [13]. Measuring the perceived equity is essential because the more inequity people perceive, the more distress they feel, and therefore, the more efforts they make to reduce the inequity [3]. Adams [2] introduces different approaches that a person may use to reduce inequity, including decreasing inputs or increasing outcomes. However, these approaches are often undesirable and may damage the organization [14].

Procedural justice positively affects the evaluation of reciprocity [15]. However, ET overlooks procedural fairness [16, 17] and cannot determine the desirable values of inputs or outcomes. In other words, a question arises that the ET cannot answer: how much should an underpaid person's inputs (outcomes) be decreased (increased) to feel satisfied? Finding the exact answer to this question is critical because if the changes in inputs (outcomes) are insufficient, it will not lead to the feeling of equity by that person, although it may reduce the degree of dissatisfaction. On the other hand, too much decrease (increase) of inputs (outcomes) may cause the disadvantageous inequity shifts from the unsatisfied person to the satisfied one [13].

Another problem of ET is related to the aggregation process when there are multiple inputs and outcomes. This aggregation can be done by assigning the weights to inputs and outcomes according to their perceived importance and then forming a weighted $Y-X$ ratio [18]. Therefore, the problem can be rewritten as this question: what are the weights each person assigns to inputs and outcomes? The answer to this question depends on some factors; one of the most important is the magnitude of the inputs and outcomes of that person compared to others. On the other hand, choosing the people who play the role of comparison references for a person is a controversial topic because they significantly influence the degree of inequity perceived by that person. Nevertheless, ET does not elaborate on how or with whom a person chooses to compare his/her $Y-X$ ratio [18].

ET also cannot predict the magnitude of changes in satisfaction in advance when changing the values of inputs (outcomes) is necessary. Decreasing (increasing) the values of the inputs (outcomes) not only affects the satisfaction perceived by that person but also may affect the dissatisfaction perceived by the others. However, ET cannot estimate the magnitude of these changes, especially before changing the values of inputs (outcomes). This is worthy to note that these changes may also change the reference people for that person. Identifying the new reference people and determining the degree of inequity these changes have led to is complex and requires a lot of time and resources. This will be more complicated in a dynamic environment where such changes may occur rapidly and intermittently. It means managers need to analyze each complicated situation immediately to keep their companies competitive and retain their workforces. They should periodically identify underpaid persons and then determine the exact values of inputs reduction or outcome increase so that the underpaid persons become satisfied and the feeling of inequity does not shift to the others.

ET evaluates the inequity for each person from his/her point of view, generally a mental judgment rather than a logical procedure. In other words, ET is not about how justice should be but how people perceive equity. It is why persons with equal amounts of inputs and outcomes may perceive different feelings of inequity. People are also more likely to judge something fairly when they are in a good mood. Also, everyone may feel dissatisfied, even a person with the lowest inputs and most outcomes. In other words, the feeling of inequity perceived by each person is not necessarily based on a rational reason. These explanations imply that the inequity perceived by the persons could not justify the reduction (augmentation) of the inputs (outcomes).

If the organizational decisions are made based on the persons' mental judgments, it can be the origins of perceived inequities. It means the inequity perceived by people should not be the basis for organizational decisions such as reward allocation or promotion. These decisions must be made logically, not based on people's mental or emotional judgments. All of these indicate that ET lacks the competencies needed to form the basis for organizational decision-making. As a result, the magnitude of inequity that is supposed to be the basis of organizational decisions must be obtained from a consistent, neutral, and unbiased procedure [19].

This study aims to provide a procedure that estimates people's feelings of inequity based on logic, not personal judgment and emotions. For this purpose, we use the data envelopment analysis (DEA). DEA is a mathematical model that is conceptually similar to ET. DEA and ET aim to measure the desirability of each

unit's $Y-X$ ratio compared to the other units. Therefore, this study applies the DEA capabilities to cover the shortcomings of ET. In other words, DEA models are extended in this study to logically estimate the degree of inequity perceived by each person. DEA can also estimate the importance of inputs and outcomes for each person separately and identify the reference persons for a dissatisfied person. DEA estimates these values for each person based on his/her inputs and outputs compared to others. Another ability of DEA is to estimate the optimal rate of inputs reduction (outcomes increase) for an underpaid person so that both that person is satisfied and that dissatisfaction is not shifted to others. This predictability helps estimate the degree of equity people may perceive by changing their inputs and outcomes. These calculations are all based on mathematical logic.

The degree of equity obtained from a DEA model corresponded to a person is not necessarily the exact equity perceived by that person; nevertheless, it suffers from less bias and deserves to be the basis for organizational decisions. DEA has a clear, scientific, consistent, and unbiased procedure and follows the same procedure for everyone. Therefore, the use of DEA is expected to lead the more satisfaction when making organizational decisions, especially those related to the feeling of equity, including salary increases or reward allocation. As a result, the proposed approach in this study helps managers better understand the employees' feelings and develop equal working satisfaction. Although DEA is used to measure the degree of equity from a rational point of view, it can also be extended to estimate the degree of equity perceived by people with different personalities. For example, DEA models are employed in this study to estimate the degree of equity perceived by four different optimistic, pessimistic, benevolent, and entitled persons. We also show that DEA helps understand how people with different personalities assign the weights to inputs and outcomes differently. To the best of our knowledge, this is the first time organizational theories, such as ET, are criticized from a quantitative viewpoint.

This study is organized as follows: Section 2 presents the literature review related to ET and DEA; Section 3 extends DEA to measure the ET; Section 4 gives numerical examples to illustrate the proposed approaches; Section 5 gives a real example to show the application of the proposed approach; Section 6 compares DEA and ET and gives their similarities and differences; finally, Section 7 concludes and presents future directions.

2. Literature review

2.1. Equity theory

Justice and fairness are essential for any organization because they impact job satisfaction and effective job performance [20]. These concepts have been presented as organizational theories such as equity, justice, and allocation theories in several studies like [1,2,21–24]. Greenberg [25] categorized these theories into four distinct classes by combining a reactive-proactive dimension and a process-content dimension. The reactive theories focus on people's attempts to avoid perceived unfair states, while proactive theories are concerned about the behaviors designed to promote justice. On the other hand, the process and the content approaches focus on determining the various outcomes and the fairness of distribution of these outcomes, respectively.

The reactive-content theories focus on how people react to inequitable distributions of outcomes [25]. One of the well-known theories in this category is ET, presented by Adams [2]. According to this theory, a person compares his/her $Y-X$ ratio with others and may feel inequity. If this ratio is unequal, the exchange is inequitable, and that person feels dissatisfied [2,3]. A person whose $Y-X$ ratio is lower than others is an underpaid person and experiences disadvantageous inequity; this, in turn, produces feelings of anger [1,2,21,22]. Some studies like [1,26–30] confirm feelings of anger. On the other hand, a person with a higher $Y-X$ ratio than others is an overpaid person and experiences advantageous inequity, leading to feelings of guilt.

Different inputs and outcomes may be considered in an exchange relation. Inputs include all factors perceived by a person to be relevant for getting some return on personal investment [18]. Depending on the recognition and relevance, the inputs are different from each person's point of view. These inputs may be effort, education, experience, skill, training, intelligence, talent, seniority, ethnic background, social status, personal appearance or attractiveness, health, possession of specific tools, and responsibility [2,14,18]. In contrast, outcomes include all factors perceived by the person as returns to himself/herself, that is, the factors that have utility or value to

him/her [18]. Outcomes can be categorized in different ways, such as formal and informal or tangible and intangible outcomes. Examples of outcomes are salary, benefits, promotions, recognition, prestige, rewards intrinsic to the job, satisfying supervision, seniority benefits, job status, and full-time employee [2, 14, 18].

Feeling the inequity motivates a person to achieve equity or to reduce inequity [2]. Adams [2] reviewed the literature and found the approaches used to reduce inequity. These approaches are: to alter the inputs or outcomes, distort the inputs and outcomes cognitively, leave the field, act on others, and change the object of the comparison. Feeling angry or guilty affects how a person chooses to reduce inequity when applying these strategies. For example, altering the inputs for underpaid and overpaid persons means reducing and increasing their inputs, respectively; altering the outcomes for them means increasing and reducing their outcomes, respectively [1, 2, 26, 27, 31, 32].

The magnitude of perceived inequity varies from person to person, even for the same conditions, and depends on various factors, with one of the most important being the people's personality. This leads to different reactions to the same conditions for people with different personalities discussed in [3]. They presented three classes of people: equity sensitive, benevolent, and entitled people. Equity-sensitive people conform to the norm of equity, while benevolent (entitled) people violate the norm of equity because they give while expecting little (more) in return.

A benevolent person tends to sacrifice his/her interest for others. There are several sources for this preference mentioned in [3]. Some sources, such as personal philosophy or empathic arousal, have unselfish roots and emanate from the altruistic perspective. However, some other sources, such as a need for social approval or a desire to enhance one's self-image, are rooted in selfishness and do not necessarily arise from an altruistic perspective. Therefore, whether altruistic behavior arises from selfishness or unselfishness is not entirely clear. On the contrary, the entitled tend to view life as unfair to his/her benefit. He/she has high thresholds for feeling indebted and believes that any outcome that he/she receives in return for his/her input is not sufficiently compensated [33]. Unlike the benevolent who is the giver, the entitled is the getter [3].

ET has been criticized, revised, and extended by researchers. For example, Weick [34] discussed the ambiguities and limitations of early formulations of ET and suggested extensions of the theory. Walster *et al.* [22] revisited the ET by presenting a general theory of social behavior. Huseman *et al.* [3] proposed the equity-sensitive model following the traditional equity model. They argued that the type and the severity that each person shows to the same inequity depends on his/her preferences; and accordingly presented three classes of people: benevolent, equity sensitive, and entitled. Furthermore, many research findings supporting ET cannot be generalized to organizations because they have been conducted mostly in a laboratory setting with student subjects and consider only one or two inputs or outcomes [35]. Leventhal [23] criticized the unidimensional ET approach as this theory's basic problem. This approach employs a unidimensional concept of fairness that emphasizes only the fairness of distribution, ignoring the fairness of the procedure. ET employs a unidimensional rather than multidimensional conception of fairness. ET ignores the possible role of other standards of justice that influence the perception of distributive fairness. In contrast, some studies have recognized the need for a multidimensional concept of distributive fairness [36–41].

2.2. Data envelopment analysis

Data envelopment analysis (DEA) is a mathematical model proposed in [42] to evaluate the relative efficiency of homogeneous decision-making units (DMUs) that consume similar inputs to produce similar outputs (outcomes). Generically a DMU is an entity responsible for converting inputs into outcomes (outputs) and whose performance should be evaluated. In managerial applications, DMUs can be bank branches, department stores, supermarkets, hospitals, schools, public libraries, etc. DEA forms a ratio of the outcome to the input for each DMU and then measures this ratio compared to the other DMUs. If DMUs have multiple inputs and outcomes, DEA forms a ratio of weighted outcome to weighted inputs ($Y-X$ ratio) for each DMU. The weights that DEA assigns to the same factors may vary from DMU to DMU.

The classical DEA models evaluate the performance of each DMU optimistically [43] because they assign the weights to inputs and outcomes in a way that the efficiency of the DMU under assessment is maximized. In

other words, these models have an optimistic perspective and evaluate the performance of each DMU considering the best possible conditions. This perspective may lead to drawbacks investigated in several studies [44–49]. Therefore, some studies emphasized that efficiency should be measured from both optimistic and pessimistic perspectives. In contrast to the optimistic approach, the pessimistic approach assigns unfavorable weights to factors (inputs and outcomes) to evaluate the performance of under-assessment DMU as low as possible compared to other DMUs. Then, an approach should be used that considers both optimistic and pessimistic points of view [50].

Based on a combined approach, Entani *et al.* [43] developed DEA with an interval efficiency in which the upper and the lower efficiencies are obtained from the optimistic and pessimistic perspectives, respectively. Wang and Luo [51] measured the optimistic and pessimistic performance of the DMUs by introducing two ideal and anti-ideal virtual DMUs. Wang and Yang [52] also presented the interval DEA models to measure each DMU's optimistic and pessimistic performance. They provided an anti-ideal DMU consuming the maximum inputs to produce the minimum outcomes. Azizi [53] argued that both optimistic and pessimistic approaches should be considered simultaneously, and any approach that considers only one of them will be biased. Then, he proposed the bounded DEA models to generate both efficiencies as an interval reflecting various perspectives of decision-makers.

3. The quantitative approach to equity theory

Here, we aim to estimate the magnitude of equity perceived by individuals. For this purpose, we consider each person as a DMU and present an approach that gradually quantifies the ET. According to ET, the person p forms a ratio of his/her outcome to his/her input as y_P/x_P. When there are multiple inputs and outcomes, they are weighted according to their perceived importance [18], and consequently, the general form of the $Y-X$ ratio for person p is shown below:

$$\frac{y_{1p}\cdot u_1 + y_{2p}\cdot u_2 + \ldots + y_{sp}\cdot u_s}{x_{1p}\cdot v_1 + x_{2p}\cdot v_2 + \ldots + x_{mp}\cdot v_m} = \frac{\sum_{r=1}^{s} y_{rp}\cdot u_r}{\sum_{i=1}^{m} x_{ip}.v_i} \tag{1}$$

where, y_{rp} and x_{ip} are the values of rth outcome and ith input of person p, respectively; and u_r and v_i are their corresponding weights; s and m are the numbers of outcomes and inputs, respectively. The numerator and the denominator of equation (1) are the weighted sums of outcomes and inputs, respectively, which are perceived to be relevant to a particular exchange, assuming that persons probably do not weigh outcomes (inputs) equally [2]. By assigning the values to u_r and v_i, the final "value" of the $Y-X$ ratio is determined [18]. This ratio can be considered the degree of equity perceived by the person p. However, each person assigns the weights to these variables by comparing their values with those of others. Therefore, it is necessary to determine the $Y-X$ ratios of reference people. We assume that n people are going to be compared; therefore, for each person j, there are *n-1* reference persons whose $Y-X$ ratios are obtained as below:

$$\frac{\sum_{r=1}^{s} y_{rj}\cdot u_r}{\sum_{i=1}^{m} x_{ij}\cdot v_i}, \quad j = 1,\ldots,n;\ \ n \neq p. \tag{2}$$

Each person usually compares his/her $Y-X$ ratio, equation (1), with the ratios of others, equation (2). This comparison leads to a value indicating the degree of equity perceived by that person. In this study, we consider this value to be the satisfaction degree for that person because, according to [3], there should be a positive relationship between the person's perception of equity and job satisfaction.

Note that the weights assigned to the same inputs and outcomes may vary from person to person. It depends on different factors, including the values of inputs and outcomes for a person compared to others and the personality of that person. We present different approaches in the following sections to estimate the magnitude of perceived equity from the perspective of four characters, *i.e.*, optimist, pessimist, benevolent, and entitled.

3.1. The optimistic point of view

An entirely optimistic person usually allocates weights to inputs and outcomes to imagine his/her status in the best possible position compared to others. This person aims to show his/her $Y-X$ ratio as high as possible, as shown in the objective function of Model (3). On the other hand, the maximum value for the $Y-X$ ratio for this person and the reference people can be equal to 1. This leads to n constraints shown in Model (3). Finally, the weights assigned to inputs and outcomes should be non-negative, as given in this model.

$$\begin{aligned}
&\max \ \frac{\sum_{r=1}^{s} y_{rp} \cdot u_r}{\sum_{i=1}^{m} x_{ip} \cdot v_i} \\
&\text{s.t.} \\
&\quad \frac{\sum_{r=1}^{s} y_{rj} \cdot u_r}{\sum_{i=1}^{m} x_{ij} \cdot v_i} \leq 1, \qquad j = 1, \ldots, p, \ldots, n; \\
&\quad u_r \geq 0, \qquad r = 1, \ldots, s \\
&\quad v_i \geq 0, \qquad i = 1, \ldots, m
\end{aligned} \tag{3}$$

where p represents the optimistic person within a group of n members compared to each other. Model (3) can be considered a mathematical equivalent for an entirely optimistic mind. This model assigns significant weight to the input (outcome) whose relative value for person p is less (more) than others and, therefore, aims to maximize the objective function's value as much as possible. This is, in fact, the same philosophy that we believe an optimist follows when evaluating his/her feeling of equity. An optimist looks on the bright side of the equity and highlights his/her relatively low inputs and high outcomes compared to others. In this way, he/she achieves the highest possible degree of equity that can be perceived by him/her. It is worthy to say that Model (3) is the same CCR model proposed in [43]. Therefore, we can use the properties of the CCR model here to analyze the ET concepts. For example, the fractional Model (3) can be converted into the following linear programming Model (4).

$$\begin{aligned}
&\max \ \sum_{r=1}^{s} y_{rp} \cdot u_r \\
&\text{s.t.} \\
&\quad \sum_{i=1}^{m} x_{ip} \cdot v_i = 1 \\
&\quad \sum_{r=1}^{s} y_{rj} \cdot u_r - \sum_{i=1}^{m} x_{ij} \cdot v_i \leq 0, \qquad j = 1, \ldots, p, \ldots, n; \\
&\quad u_r \geq 0, \qquad r = 1, \ldots, s \\
&\quad v_i \geq 0, \qquad i = 1, \ldots, m.
\end{aligned} \tag{4}$$

Model (4) is the input-oriented CCR model presented in [43] and can be used to estimate the reduction in the inputs of an underpaid optimistic person to achieve the feeling of equity. The fractional Model (3) can also be converted into the linear programming Model (5), representing the outcome-oriented CCR model. By analyzing the results of Model (5), it can be estimated how much the outcomes of an underpaid optimist should be increased to achieve the feeling of equity.

$$\begin{aligned}
&\min \ \sum_{i=1}^{m} x_{ip} \cdot v_i \\
&\text{s.t.}
\end{aligned}$$

$$\begin{aligned}
&\sum_{r=1}^{s} y_{rp} \cdot u_r = 1 \\
&\sum_{r=1}^{s} y_{rj} \cdot u_r - \sum_{i=1}^{m} x_{ij} \cdot v_i \le 0, \qquad j = 1, \ldots, p, \ldots, n; \\
&u_r \ge 0, \qquad r = 1, \ldots, s \\
&v_i \ge 0, \qquad i = 1, \ldots, m.
\end{aligned} \tag{5}$$

Models (4) and (5) are, in fact, the quantitative equivalents of two strategies identified by Adams [2] that an underpaid person may use to achieve the feeling of equity. Models (4) and (5) estimate the values of inputs reduction and outcomes increase, respectively, so that mathematically, the underpaid optimist achieves the feeling of equity, and the feeling of inequity does not shift to the satisfied persons.

3.2. The benevolent point of view

Benevolent people show altruistic tendencies because they give while expecting little in return [3]. They are, generally, givers [3] and therefore donate a part of their outcomes to reduce their $Y-X$ ratios. Some benevolent people are inherently benevolent, and others pretend to be benevolent for reasons such as political motives. We believe that the inherent benevolent people have feelings such as maternal love and try to magnify their satisfaction. For this purpose, they may assign the weights to inputs and outcomes to make their $Y-X$ ratio greater than 1. In this way, they have a rational excuse to dedicate a part of their outcomes until their $Y-X$ ratio becomes equal to 1. The mathematical models for estimating the satisfaction degree of a benevolent are similar to the models presented for an optimist; the only difference is that the constraint corresponding to the benevolent is removed. Therefore, the $Y-X$ ratio for a benevolent can be more than 1. The fractional model to estimate the satisfaction of a benevolent can be formed as follow:

$$\begin{aligned}
&\max \frac{\sum_{r=1}^{s} y_{rp} \cdot u_r}{\sum_{i=1}^{m} x_{ip} \cdot v_i} \\
&\text{s.t.} \\
&\quad \frac{\sum_{r=1}^{s} y_{rj} \cdot u_r}{\sum_{i=1}^{m} x_{ij} \cdot v_i} \le 1, \qquad j = 1, \ldots, n,\ j \ne p, \\
&\quad u_r \ge 0, \qquad r = 1, \ldots, s \\
&\quad v_i \ge 0, \qquad i = 1, \ldots, m
\end{aligned} \tag{6}$$

where p represents a benevolent person. Model (6) is the same model proposed in [54]. Like Model (3), Model (6) can be converted into either input- or outcome-oriented linear programming.

3.3. The pessimistic point of view

An entirely pessimistic person generally allocates the weights to inputs and outcomes to imagine his/her status in the worst possible position compared to others. A pessimist aims to show his/her $Y-X$ ratio as low as possible, as the objective function of Model (7) shows. The maximum values for the $Y-X$ ratios for a pessimist and the reference persons are 1, leading to n constraints shown in Model (7) as

$$\begin{aligned}
&\max \frac{\sum_{i=1}^{m} x_{ip} \cdot v_i}{\sum_{r=1}^{s} y_{rp} \cdot u_r} \\
&\text{s.t.} \\
&\quad \frac{\sum_{i=1}^{m} x_{ij} \cdot v_i}{\sum_{r=1}^{s} y_{rj} \cdot u_r} \le 1, \qquad j = 1, \ldots, p, \ldots, n;
\end{aligned}$$

$$\begin{aligned} & u_r \geq 0, && r = 1, \ldots, s \\ & v_i \geq 0, && i = 1, \ldots, m \end{aligned} \tag{7}$$

where p represents the pessimist. This model, the same Inverted DEA (IDEA) model presented in [43], can be rewritten as below.

$$\begin{aligned} \min\ & \frac{\sum_{r=1}^{s} y_{rp} \cdot u_r}{\sum_{i=1}^{m} x_{ip} \cdot v_i} \\ \text{s.t.}\ & \\ & \frac{\sum_{r=1}^{s} y_{rj} \cdot u_r}{\sum_{i=1}^{m} x_{ij} \cdot v_i} \geq 1, && j = 1, \ldots, p, \ldots, n; \\ & u_r \geq 0, && r = 1, \ldots, s \\ & v_i \geq 0, && i = 1, \ldots, m. \end{aligned} \tag{8}$$

Model (8) can be considered a mathematical reflection of the way a pessimist thinks. In other words, this model assigns significant weight to the input (outcome) whose relative value is more (less) for person p compared to the others and thereby minimizes the value of the objective function, *i.e.*, the person's satisfaction. This is the philosophy that a pessimist may follow. A complete pessimist just looks on the dark side of the exchange comparison and usually highlights his/her relative inputs (outcomes) with high (low) values compared to others. The fractional Model (8) can, in turn, be rewritten to either an input or an outcome-oriented model (similar to Model (3)). Note that the optimal value obtained from the models presented for the pessimist does not necessarily indicate his/her relative satisfaction. In other words, due to the constraints of these models, the satisfaction of the reference persons may be greater than 1. Therefore, the optimal normal value of the objective function, which indicates the relative satisfaction of the pessimist, is obtained using the following relation:

$$e_p^* = \frac{z_p^*}{\max \frac{\sum_{r=1}^{s} u_{rp}^* y_{rj}}{\sum_{i=1}^{m} v_{ip}^* x_{ij}},\ j = 1, \ldots, n} \tag{9}$$

where z_p^* is the un-normal satisfaction degree of person p obtained from the optimization model and e_p^* is the normal one; u_{r0}^* and v_{i0}^* are the optimum values of rth outcome and ith input obtained by solving the optimization model for person p.

3.4. The entitled point of view

In contrast to benevolent, entitled people show egoism tendencies because they give while expecting more in return. Entitled people have much, but they want and expect more and consider everything as their right [3]. Entitled people tend to perceive life as unfair to them. They have high thresholds for feeling indebted and believe that any outcome that they receive in return for their input is not adequately compensated [33]. They are, generally, getters [3] and therefore try to get more outcomes in a mutual exchange. To rationalize their desires, the entitled assign the weights to inputs and outcomes that show their $Y-X$ ratio as low as possible. Therefore, they have an excuse to demand more outcomes until their ratio equals 1. The lower the $Y-X$ ratio can be determined, the more outcomes the entitled can demand. Therefore, we present mathematical models in this study for estimating an entitled's degree of satisfaction (DOS). These models are similar to those presented for a pessimist, with a difference, *i.e.*, removing the $Y-X$ ratio constraint related to the under assessment entitled. It means the feeling of equity for an entitled may be obtained less than 1, while those of all reference persons are more than 1. The fractional model to estimate the satisfaction of an entitled is presented in this study as below:

$$\min \frac{\sum_{r=1}^{s} y_{rp} \cdot u_r}{\sum_{i=1}^{m} x_{ip} \cdot v_i}$$

TABLE 1. Two different approaches to achieve equity for underpaid person B.

	Current situations		Proposed approaches for B	
Factors	Person A	Person B	Input-oriented	Outcome-oriented
Payment (\$)	1000	2000	2000	2500
Tables (number)	80	200	160	200
$Y-X$ ratio	12.5	10	12.5	12.5
DOS	$12.5 \div 12.5 = 1$	$10 \div 12.5 = 0.8$	$12.5 \div 12.5 = 1$	$12.5 \div 12.5 = 1$

$$\begin{aligned} &\text{s.t.} \\ &\frac{\sum_{r=1}^{s} y_{rj} \cdot u_r}{\sum_{i=1}^{m} x_{ij} \cdot v_i} \geq 1, \qquad j = 1, \ldots, n,\ j \neq p \\ &u_r \geq 0, \qquad r = 1, \ldots, s \\ &v_i \geq 0, \qquad i = 1, \ldots, m \end{aligned} \tag{10}$$

where p represents an entitled. In a similar approach to Model (3), this model can be converted to either input or outcome-oriented model. Like the explanations given for the pessimistic model, the optimal value obtained from the models presented for an entitled does not necessarily indicate his/her degree of relative satisfaction. Therefore, the relative satisfaction of an entitled is estimated using equation (9). This procedure results in the lowest feeling of equity perceived by an entitled mathematically.

4. Illustrative examples

This section gives two numerical examples to explain the proposed approaches. Example 1 estimates the satisfaction of two people compared to each other and considers only one input and one outcome. Example 2 considers two inputs and two outcomes and estimates the satisfaction degree of two people from the perspective of four different characters. This example indicates that the same inputs and outcomes may lead to different satisfaction degrees; it means the magnitude of perceived inequity varies from person to person due to their different attitudes.

Example 1. This example aims to estimate the satisfaction degrees of two persons, A and B, compared to each other. Person A makes 80 tables per month and earns 1000 \$, while person B makes 200 tables and earns 2000 \$. The $Y-X$ ratios indicate that persons A and B are paid 12.5 and 10 dollars for each table, respectively. It implies that person B is probably dissatisfied. Each person's DOS can be mathematically estimated by dividing his/her $Y-X$ ratio by the greatest ratio. Thus, the DOS for persons A and B are estimated equal to 1 and 0.8, respectively. Two different approaches can be used to increase person B's DOS to 1. In other words, person B should either reduce the produced tables or get more money. That is, he/she should produce 160 tables or be paid 2500 dollars. To calculate these values in both approaches, A plays the role of *referent person* for B. These approaches, which are the input-oriented (reducing inputs) and the outcome-oriented (increasing outcomes) approaches, respectively, are used to improve the DOS of person B. The related calculations are given in Table 1.

Table 1 gives the analyses which are computationally based on ET and are not necessarily in accordance with the real situations. In other words, there are some demographic and psychological variables such as age, experience, education, and expectation that affect the equity perceived by people and their satisfaction [3]. Therefore, both persons A and B may feel satisfied or dissatisfied in a real situation.

TABLE 2. The inputs and outcomes values for persons A and B.

Person	Education (input 1)	Experience (input 2)	Salary (outcome 1)	Job-status (outcome 2)
Person A	4	12 years	80 $	15
Person B	5	10 years	75 $	16

TABLE 3. The satisfaction degrees of persons A and B from an optimistic point of view.

Persons	Satisfaction degree	The weights of inputs		The weights of outcomes	
		Education	Experience	Salary	Job-status
A	1	0.25	0	0.0125	0
B	1	0	0.0625	0	0.1

TABLE 4. The satisfaction degrees of persons A and B from a benevolent point of view.

Persons	Satisfaction degree	The weights of inputs		The weights of outcomes	
		Education	Experience	Salary	Job-status
A	1.333	0.25	0	0.0166	0
B	1.28	0	0.1	0	0.08

Example 2. In this example, the satisfaction degrees of two persons, A and B, are estimated compared to each other. Here, education and experience are inputs, and the salary and job status are outcomes. Table 2 gives the values of these attributes for persons A and B.

The satisfaction degree of each person depends on two factors: his/her personality and the values of inputs and outcomes of this person compared to others. We estimate these persons' satisfaction degrees mathematically with four different characters. The approach used in all the following models is the input-oriented approach.

Optimistic characters. Assume that persons A and B are both optimistic. In this case, their satisfaction degrees are estimated using Model (3); the results are given in Table 3.

According to the weights assigned to factors, it is assumed that person A imagines that with less education, he/she receives a more relative salary than person B. Also, person B imagines that with less experience, he/she has a better job status than person A. These mental analyses of persons A and B come from the fact that they are both considered optimistic; therefore, they weigh the factors as they see the bright side of the exchange relation.

Benevolent characters. Assume that both persons A and B are benevolent. In this case, their satisfaction degrees are estimated using Model (6); the results are given in Table 4.

Table 4 shows that the satisfaction degrees of both A and B are estimated to be more than 1; therefore, both can dedicate part of their outcomes until their satisfaction degrees are equal to 1. Note that how these persons weigh the inputs and outcomes stems from their benevolent personalities.

Pessimistic characters. Assume that persons A and B are both pessimistic. In this case, their relative satisfaction degrees are estimated using Model (7); the results are shown in Table 5.

Note that the satisfaction degree for person A, obtained by Model (7), is equal to 1, which does not indicate his/her relative satisfaction. To calculate the relative satisfaction degree of person A, first, the satisfaction degree

TABLE 5. The satisfaction degrees of persons A and B from a pessimistic point of view.

Persons	Satisfaction degree		The weights of inputs		The weights of outcomes	
	Un-normal	Relative	Education	Experience	Salary	Job-status
A	1	0.781	0	0.083	0	0.067
B	1	1	0.131	0.034	0	0.063

TABLE 6. The satisfaction degrees of persons A and B from an entitled point of view.

Persons	Satisfaction degree		The weights of inputs		The weights of outcomes	
	Un-normal	Relative	Education	Experience	Salary	Job-status
A	0.781	0.781	0	0.083	0	0.052
B	0.75	0.75	0.2	0	0.01	0

of person B is obtained by applying the optimal weights of person A as

$$(75 \times 0 + 16 \times 0.067) \div (5 \times 0 + 10 \times 0.083) = 1.28.$$

Then, according to equation (9), the relative satisfaction degree of person A is calculated as

$$1 \div 1.28 = 0.781.$$

The satisfaction degree for person B obtained from Model (7) equals 1. Here, first, the satisfaction degree of person A is obtained by applying the optimal weights of person B as

$$(80 \times 0 + 15 \times 0.063) \div (4 \times 0.131 + 12 \times 0.034) = 1.$$

Therefore, according to equation (9), the relative satisfaction degree of person B is calculated as

$$1 \div 1 = 1.$$

Entitled characters. Now assume that persons A and B are both entitled. Therefore, their relative satisfaction degrees are estimated using Model (10) and equation (9); the results are given in Table 6.

Note that the satisfaction degrees of persons A and B obtained from Model (10) are equal to 0.781 and 0.75, respectively, which do not indicate their relative satisfaction. The satisfaction degree of person B by applying the optimal weights of person A, and *vice versa*, the satisfaction degree of person A by applying the optimal weights of person B, both are calculated equal to 1. Therefore, according to equation (9), the relative satisfaction degrees of persons A and B are estimated as 0.781 and 0.7, respectively.

According to the weights assigned to the factors, it can be assumed that person A imagines that with more experience, he/she has a lower job status compared to person B; person B also imagines that with more education than person A, he/she receives the lower salary. These people only highlight the benefits given to the reference person and ignore the benefits given to themselves. The mental analysis of persons A and B stems from the fact that they are both entitled. Therefore, they assign the weights to the factors so that the dark side of their exchange relation is highlighted, and they can demand more outcomes.

Comparison. Example 2 clearly shows how people with different personalities assign weights to the same inputs and outcomes differently. Table 7 summarizes the results of this example. This table shows that people with the same data but different personalities perceive different feelings of equity and therefore feel different degrees of satisfaction.

TABLE 7. The satisfactions degrees of persons A and B with different personalities.

Person	Optimist	Benevolent	Pessimist	Entitled
A	1	1.33	0.78	0.78
B	1	1.28	1	0.75

TABLE 8. The required data.

Professors	Outputs		Inputs	
	Financial compensation	Supporting facilities	Research core	Educational score
A	934	41	168	248
B	941	49	19	254
C	952	48	13	236
D	971	48	38	232
E	928	42	24	242
F	937	45	42	239
G	931	44	12	237
H	933	48	11	239
I	934	47	16	243

5. AN APPLICATION

This section shows the application of the approach presented in this study. For this purpose, a real example is used to estimate the equity perceived by nine professors working in an educational institution by considering their different personalities. This example considers two inputs and two outcomes. Inputs are educational activities' scores (derived from the quality of teaching, variety of courses, and student satisfaction) and research activities' scores (derived from book writing, publication of articles, the quantity and quality of articles). The scores of these inputs have been assigned to each professor by the educational institution based on different activities. The outcomes are financial compensation (including salary for teaching, reward incentives for articles) and supporting facilities (such as the proper office, computer, and managerial support). The values of these attributes were extracted for each professor. The inputs and outputs collected for each professor are related to the year 2021. These values are given in Table 8.

Here we estimate the degree of equity perceived by professors by considering four different personalities. Table 9 shows the weights of the inputs and outputs for each professor, considering the different characters for that professor. These weights are derived from different DEA models. This table shows how different personalities of people affect the weights they assign to the attributes. For example, if A is an entirely optimistic person, he/she may only assign weight to educational score and does not consider the research activities. In contrast, if A is benevolent, he/she may only assign weight to the research score because he/she has a remarkable performance in this attribute.

The magnitude of the degree of equity professors perceive from an optimistic (a benevolent) perspective can be estimated using Model (3) (Model (6)). From a pessimistic point of view, the equities perceived by the professors are estimated using Model (8) and relation (9). For this purpose, first, the weights of inputs and outcomes are obtained for each professor using Model (8). Then, the cross-efficiencies (the equity perceived by each professor taking into account the optimal weights of other professors) are calculated. Finally, the magnitude of the equity perceived by each professor is estimated from a pessimistic point of view using the relation (9). These calculations are given in Table 10.

TABLE 9. The weights of attributes for each professor under different personalities.

Professor		A	B	C	D	E	F	G	H	I
Optimistic	u_1	0.000963	0	0.00105	0.00103	0.001009	0.001	0.001063	0.001072	0
	u_2	0	0.019297	0	0	0	0	0	0	0.020199
	v_1	0	0.000622	0.015088	0	0.001406	0	0.015267	0.015395	0.000652
	v_2	0.004032	0.00389	0.003406	0.00431	0.003993	0.004184	0.003446	0.003476	0.004072
Benevolent	u_1	0.000963	0	0.001033	0.001069	0.001009	0.001	0.001063	0	0
	u_2	0	0.019297	0.000904	0	0	0	0	0.024621	0.020199
	v_1	0	0.000622	0.002517	0	0.001406	0	0.015267	0.090909	0.000652
	v_2	0.004032	0.00389	0.004099	0.00431	0.003993	0.004184	0.003446	0	0.004072
Pessimistic	u_1	0.000975	0.001063	0.001025	0.00104	0.001	0.001012	0.00102	0.001012	0.001
	u_2	0.002171	0	0.002281	0.00232	0.002225	0.002252	0.00227	0.002252	0.00222
	v_1	0	0	0	0	0	0	0	0	0
	v_2	0.004032	0.003937	0.004237	0.00431	0.004132	0.004184	0.00422	0.004184	0.00412
Entitled	u_1	0	0.001045	0.001025	0.00104	0.001	0.001012	0.00102	0.001012	0.001
	u_2	0.005556	0	0.002281	0.00232	0.002225	0.002252	0.00227	0.002252	0.00222
	v_1	0.005952	0	0	0	0	0	0	0	0
	v_2	0	0.003937	0.004237	0.00431	0.004132	0.004184	0.00422	0.004184	0.00412

TABLE 10. The calculations required to estimate perceived equity from a pessimistic view.

Professors	A	B	C	D	E	F	G	H	I
A	1	1.017	1	1	1	0.911	1	1	1
B	1	1	1	1	1	0.896	1	1	1
C	1.085	1.089	1.085	1.086	1.086	0.976	1.086	1.085	1.085
D	1.123	1.130	1.124	1.124	1.124	1.012	1.124	1.124	1.123
E	1.021	1.035	1.021	1.021	1.022	0.928	1.021	1.021	1.021
F	1.049	1.059	1.050	1.050	1.050	0.948	1.050	1.050	1.049
G	1.050	1.061	1.050	1.051	1.051	0.950	1.051	1.050	1.050
H	1.052	1.054	1.053	1.053	1.053	0.944	1.053	1.052	1.052
I	1.034	1.038	1.034	1.034	1.034	0.930	1.034	1.034	1.033
Max	1.123	1.130	1.124	1.124	1.124	1.012	1.124	1.124	1.123
Perceived equity estimation	0.890	0.885	0.966	1	0.909	0.937	0.934	0.936	0.920

From an entitled point of view, the equities perceived by the professors are estimated using Model (10) and relation (9). Here, first, the weights of attributes are obtained using Model (10) for each professor; then, the cross-efficiencies are calculated; finally, the magnitude of the equity perceived by each professor is estimated from an entitled point of view using the relation (9). These calculations are given in Table 11.

Table 12 gives the estimations of equity perceived by professors from four different perspectives and ranks professors from the most satisfied to the most dissatisfied based on each perspective. This table also presents the average value of the estimated equities derived from four different perspectives. Note that three professors become fully satisfied based on the optimistic perspective; We rank these professors based on their benevolent scores. Table 12 also gives the real value of equity perceived by each professor extracted from the interview. The degrees of equity perceived by all professors were less than 1. Therefore, we normalized them. For this purpose, we divided these values by the largest value. It also makes it easy to compare the degree of real equity value perceived by each professor with those estimated based on different perspectives.

TABLE 11. The Calculations required to estimate perceived equity from an entitled view.

Professors	A	B	C	D	E	F	G	H	I
A	0.228	1	1	1	1	0.911	1	1	1
B	2.407	0.983	1	1	1	0.896	1	1	1
C	3.447	1.071	1.085	1.086	1.086	0.976	1.086	1.085	1.085
D	1.179	1.111	1.124	1.124	1.124	1.012	1.124	1.124	1.123
E	1.634	1.018	1.021	1.021	1.022	0.928	1.021	1.021	1.021
F	1	1.041	1.050	1.050	1.050	0.948	1.050	1.050	1.049
G	3.423	1.043	1.050	1.051	1.051	0.950	1.051	1.050	1.050
H	4.073	1.036	1.053	1.053	1.053	0.944	1.053	1.052	1.052
I	2.742	1.020	1.034	1.034	1.034	0.930	1.034	1.034	1.033
Max	4.073	1.111	1.124	1.124	1.124	1.012	1.124	1.124	1.123
Perceived equity estimation	0.056	0.885	0.966	1	0.909	0.937	0.934	0.936	0.920

TABLE 12. Estimating the perceived equity from different perspectives.

Professors	Benevolent		Optimist		Pessimist		Entitled		Mean		Real	
	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank	Score	Rank
A	0.899	9	0.899	9	0.890	8	0.056	9	0.686	9	0.385	9
B	0.946	7	0.946	7	0.885	9	0.885	8	0.916	8	0.816	8
C	1.027	3	1	3	0.966	2	0.966	2	0.990	3	0.928	2
D	1.038	2	1	2	1	1	1	1	1.010	2	1	1
E	0.963	5	0.963	5	0.909	7	0.909	7	0.936	6	0.843	6
F	0.937	8	0.937	8	0.937	3	0.937	3	0.937	5	0.854	5
G	0.989	4	0.989	4	0.934	5	0.934	5	0.962	4	0.886	4
H	1.182	1	1	1	0.936	4	0.936	4	1.014	1	0.907	3
I	0.949	6	0.949	6	0.920	6	0.920	6	0.935	7	0.831	6

Table 12 shows that the rankings of professors based on the real equity they perceive do not fully match the rankings estimated from each personality perspective. Its reason may come from the fact that professors do not necessarily have the same personality. This table also shows that the rankings of professors based on the average of equity degrees estimated from different perspectives are more consistent with the real perceived equality. This means that decision-makers can estimate professors' satisfaction from four different perspectives and consider their average as a measure of their satisfaction.

Considering an individual analysis, professor A has the highest potential for dissatisfaction according to his/her pessimistic scores (0.056) and the average score (0.686), which corresponds to the real degree of perceived equity (0.385). The decision-makers should provide facilities for this professor to reduce the severity of this negative potential. Provided that all professors look at the exchange with an optimistic perspective, H seems to be the most satisfied person. Conversely, if this perspective is pessimistic for all professors, D is probably the most satisfied person. Professor I rankings are equally based on all four different perspectives (the rank 6). However, the ranks of some professors have changed significantly based on different perspectives. For example, the ranks of F are 8 and 3 based on optimistic and pessimistic perspectives, respectively. In addition, H, who seems to be the most satisfied based on the benevolent perspective, is probably ranked fourth based on the entitled perspective.

6. Discussion

This section compares ET and DEA and gives their similarities and differences from different aspects. This section also discusses the advantages and disadvantages of quantifying the ET using DEA.

Conceptual framework. DEA is a mathematical model that evaluates the relative efficiency of a set of DMUs, while ET is a theoretical concept that evaluates the degree of inequity perceived by each person mentally. However, the definition of equity in ET and efficiency in DEA is similar, *i.e.*, the ratio of outcomes to inputs. Also, the approach used in both is the same, *i.e.*, the $Y-X$ ratios of subjects are compared to each other. Furthermore, DEA and ET both consider separate models (mathematically in DEA and mentally in ET) for each subject to obtain its private $Y-X$ ratio. DEA usually compares DMUs from a superior perspective, which means DEA investigates how much each DMU has generated considering the inputs it consumed. In other words, DEA investigates whether the $Y-X$ ratio for a DMU is acceptable compared to other DMUs. In contrast, ET is typically related to the perceived inequity from an inferior perspective. We mean ET investigates how much outcome a person receives in return for the inputs that he/she gives to the organization and whether his/her $Y-X$ ratio is fair compared to others.

Statistical methods. ET expresses that the more inequality the underpaid people feel, the more dissatisfied they are. However, it does not present any model to estimate the magnitude of this inequity. Some studies used statistical methods to estimate this inequity. For example, Finn and Lee [55] applied a multiple-regression model to predict equitable salaries. For this purpose, they provided a questionnaire to measure the demographics and work history information, perception of job inputs, salary treatment, and job-related attitudes. Their model predicted equitable salaries with a high correlation coefficient. Carrell and Dittrich [35] referred to some studies that support equity norm, including [56–59], in which the subject's perception of equity is the dependent variable.

The statistical methods, especially the multiple-regression model, come with some flaws. For example, people may experience various biases, intentionally or unintentionally, when completing the questionnaires. They may exaggerate their intangible inputs or underestimate their non-measurable outcomes. Another flaw is that the multiple-regression models bring inputs and outcomes together as the independent variables and consider only one dependent variable, *i.e.*, the inequity degree. Since the inputs are consumed to produce the outcomes, inputs and outcomes can be considered independent and dependent variables. In other words, the multiple-regression models do not consider the exchange relation between inputs and outcomes.

The computational logic. According to Adams [2], people may respond to injustice in various ways. The problem is that his theory is not very explicit about predicting when people will respond one way rather than another [16, 17]. Besides, even if ET suggests a way for this purpose (for example, increasing outcomes or decreasing inputs), it cannot determine the exact values of these changes as both the dissatisfied person feels equal and the dissatisfaction feeling does not shift to the satisfied persons. Furthermore, it cannot answer this question: Should all outcomes (inputs) increase (decrease) in the same proportion? ET cannot estimate the rational magnitude of the equity perceived by a person to alter the values of outcomes (inputs) accordingly. ET's answers are generally based on people's emotions that may be biased and inconsistent.

Like ET, DEA cannot determine when and which approach will be used by an inefficient DMU to achieve efficiency. However, it can estimate the magnitude of the perceived equity by a person. It can also rationally estimate the exact values of the outcomes (inputs) that should be increased (decreased) so that both an underpaid person will feel equal and the satisfied persons will remain satisfied. This rationality arises from the mathematical logic of DEA. Therefore, DEA can answer some questions that ET is unable. These answers are based on mathematical logic and are not emotional or biased.

Referents. ET also expresses that each person compares his/her $Y-X$ ratio with others consciously or unconsciously [18]. The problem is identifying peers chosen as the comparison references by that person in an exchange relationship [2]. Also, even if the reference individuals for an underpaid person are identified, EA cannot determine their contribution to the inequity perceived by that person. Furthermore, a person does not necessarily

choose homogenous persons with him/her as the comparison references. Contrary to ET, the reference DMUs are determined in DEA, and their contributions are obtained.

DEA determines the referents from a mathematical perspective based on a specific set of homogeneous DMUs with similar inputs and outcomes. Also, the reference DMUs for under-assessment DMU in DEA are usually selected among the efficient ones (DMUs with the $Y-X$ ratio equals 1). Based on these explanations, DEA can be used to respond to the shortcomings of ET mentioned here. However, DEA can be used for this purpose by ignoring some facts. In other words, unlike DEA, ET states that the persons in a comparison exchange relation do not necessarily consider the similar inputs and outcomes attributes. Besides, an underpaid person does not necessarily compare himself/herself with the satisfied persons and even may compare with the dissatisfied persons (the persons whose $Y-X$ ratio are less than 1).

In DEA, comparisons are made only between homogeneous DMUs in a specific set. Therefore, DEA's use to quantify the organizational theories, including ET, can be a controversial issue because, according to different researchers, this may or may not be confirmed. For example, according to Adams [2], the reference persons may include someone in another job or social role. In other words, each person does not necessarily compare himself/herself with homogeneous persons, and this comparison may also be with heterogeneous persons. A broad class of internal or external individuals and standards could be perceived by a person as relevant for comparison when determining equity [3,18].

On the other hand, the theory of distributive justice presented by Jasso ignores the outside comparisons. It defines justice by comparing persons' actual share of goods and their beliefs about a just share [25]. As a result, the researchers' point of view on comparison references can be categorized into two classes defined by Zelditch *et al.* [60]: the local comparison in which a person compares himself/herself with particular persons and the referential comparison in which the comparisons are made to the generalized other persons. Considering its specifications, the use of DEA seems to be more acceptable for the local comparisons.

Predictability. ET is a reactive theory [25] investigating the perceived equities in an exchange relation. In other words, it does not have the ability of prediction and cannot be applied until the exchange relation is finished. The difficulties in predicting the inequity perceived by a person come from the variables that must be considered; they are summarized in [18]. DEA, like ET, has a retrospective approach and is a reactive model. Nevertheless, DEA can be considered a futuristic model. It can estimate how the change in each person's inputs and outcomes will affect his/her satisfaction and the satisfaction of others. In other words, changing a person's inputs and outcomes will change the DOS of other persons, which DEA can predict. As a result, although DEA is inherently a reactive model, it can also be considered a proactive model to predict the satisfaction degree of individuals in advance by considering different values for their inputs and outcomes.

The ambiguous nature of attributes. Some studies like [3,18,61,62] emphasize that a specific job element, for example, doing challenging work, may be perceived by some persons as an input, while others view it as an outcome. ET does not explain how these variables should be considered when measuring people's satisfaction. DEA can help ET to deal with this problem. Some studies like [63–66] extended DEA models to deal with a measure playing an input role for some DMUs and an output role for others. These models can measure the perceived equity by different people when a measure is considered input for some people and output for others.

Disagreement in selecting attributes. People can select many attributes as inputs or outcomes, some of them are tangible, and others are intangible [14]. Examples of inputs include education, intelligence, experience, training, skill, seniority, talent, effort, superior to age, experience, responsibility, versatility, ethnic background, and social status. Examples of outcomes include salary, better-paid job, a higher status, benefits, promotions, prerequisites, recognition, prestige, full-time employee, rewards intrinsic to the job, satisfying supervision, seniority benefits, job status, health, and personal appearance or attractiveness. Adams [2] suggests two conceptually distinct characteristics of attributes, recognition and relevance. The problem is that there is disagreement when choosing the final attributes. Different persons may select different sets of relevant attributes [13]. The inputs and outcomes selected by a person are not necessarily the same perceived by others [2]. Likewise, there may

be different points of view to selecting attributes at different levels of an organization. In other words, the attributes perceived by the managers are not necessarily the same as those perceived by the employers [14].

Note that considering the logic of each person separately about the expectations and variables affecting the perceived inequity may lead to encountering a problem in terms of procedural justice. For example, consider different people giving the same inputs to an organization; however, they receive different salaries based on the variables, including their expectations, age, and gender. Payment of the salaries based on these variables may lead to procedural injustice. Contrary to ET, the inputs and outcomes should be the same for all persons in an exchange relation when using DEA to estimate their satisfaction degree. DEA compares the performance of DMUs that consume similar inputs and produce similar outcomes. Therefore, it should be handled in some ways, such as selecting essential attributes based on the majority of persons' opinions or consensus.

Inability to identify all attributes. Some attributes affect the perceived equity but are often ignored. Some of these attributes positively affect the perceived equity, such as rewards intrinsic to the job, affection, love, formal courtesies, satisfying supervision, expressions of friendship, and a variety of formally and informally sanctioned perquisites. In contrast, the other negatively affects the perceived equity, including insult, rudeness, and rejection. These attributes may subconsciously affect the magnitude of the equity perceived by a person. Also, contingent factors such as the labor market and unemployment rate affect the perceived equity; these factors are not considered in this study. Identifying all practical attributes and their corresponding weights for each person is difficult. Consider that all attributes make the problem complex and may distort the magnitude of the perceived equity, both theoretically and computationally. Also, if the number of attributes is high, the DEA cannot distinguish satisfied people from dissatisfied ones.

The norm of equity. The ET's distress prediction is based upon the assumption that persons are equally sensitive to equity; that is, the general preference is that the $Y-X$ ratios are equal to those of the comparison of others. This premise is termed the "norm of equity" [3]. However, some variables generally arise from individual differences and affect people's fair behavior [67]. These variables violating the norm of equity are classified into demographic variables and personality traits [3]. They include commitment [68], responsibility [29], sex [69,70], expecting equity or inequity [59], ego-oriented *vs.* task-oriented conditions [71], intrinsic job satisfaction [72], intelligence level, social and religious values, and quantitative aptitude [73], current pay level and earnings potentials like education [74], different initial conditions, and inequities originated from chance advantageous or intentional ones [13]. These variables can also distort the results obtained from DEA.

The weights of attributes. It is reasonable to assume the weights of different inputs, like seniority, skill, effort, and sex, should not be considered equal [2]. This is also true about the different types of outcomes. Equal weighting of the attributes may represent an inadequate assumption of how respondents summed their attributes [75]. Outcomes and inputs are weighted according to their perceived importance in determining each person's final value of the $Y-X$ ratio [18]. This implicitly implies that the importance of a specific attribute varies from person to person. Nevertheless, EA does not explain on what logic each person determines the importance of an attribute. It is also unable to determine the weights that each person probably assigns to attributes. DEA, like ET, forms a unique model for each person and assigns different weights to the same attributes for each person. However, contrary to ET, DEA not only explains the reasons for different weights for the same attributes assigned by different persons but also estimates these weights mathematically.

Aggregating the attributes. Adams [2] states that the aggregation of attributes is not clear, and psychometric research is needed to determine how persons aggregate their outcomes and inputs. He asks a question: Is the assumptive model that net outcomes are the algebraic sum of elemental outcomes weighted by their importance a valid one? DEA can be this assumptive model. It considers the relations between the attributes to be linear and uses the additive functions to aggregate the weighted inputs and outcomes. Note that DEA determines these relations based on a mathematical perspective.

Desirable and undesirable attributes. Outcomes are inherently positive. However, some outcomes may have a negative nature equivalent to costs [21,22]. ET brings together positive and negative outcomes as the negative outcomes nullify the positive ones [13]. However, DEA applies different approaches and deals with them

as two separate categories called desirable and undesirable data. These approaches reviewed in [76], along with the examples for each approach, are: ignoring undesirable outcomes [77, 78], treating them as inputs [79, 80], treating them using directional distance functions [81–83], and transforming undesirable outputs into desirable attributes [84–87].

Interdependency of attributes. It is difficult to form the $Y-X$ ratio for a person with two inter-correlated inputs [1]. For example, assume that age and job experience may be highly correlated; however, they may be considered independent inputs by a person [18]. Thus, if a person assesses the sum of his/her inputs, he/she might well score these attributes separately [2]. This causes the score of weighted inputs to be obtained highly virtually when applying ET. In contrast to ET, in classical DEA models, the greater the dependence is between the two inputs, the less importance is obtained for one of them if the dependence of two inputs is 100%, the weight of one of them will be zero. This comes back to the characteristics of linear programming.

Problems with the $Y-X$ model. Carrell and Dittrich [35] identified three problems that may arise when applying the $Y-X$ model. Here, we review these problems and discuss how DEA deals with them. Firstly, the variables and referent persons specified in an exchange relation may have either great or little relevance to a person in the equities of his/her particular situation. DEA emphasizes the proper selection of attributes and the comparison of homogeneous persons. Nevertheless, DEA may suffer from this problem.

Secondly, a person may feel equal compared to the internal or external referent persons; however, he/she may see the rewards system as unfair or may feel inequality based upon earlier experience. DEA cannot solve the unfairness feeling of the reward system. However, it can solve the problem of earlier experience by considering the window data. In this case, persons are not only compared to each other but also to themselves in the past.

Thirdly, easy questions with limited variables are designed to measure the equity (using only pay as an outcome, for example). According to [35], a multiple variable model would be equally plausible and perhaps more realistic and more powerful than a limited variable model; however, it can be difficult to operationalize using the typical $Y-X$ comparison process. Here, we propose DEA as a powerful technique considering multiple inputs and outcomes. It can also estimate the DOS for each person and does not require knowing his/her perception.

7. Conclusion and future directions

ET is a valuable theory for recognizing the inequity perceived by each person when comparing his/her $Y-X$ ratio with others, considering that this feeling of inequity, in turn, leads to dissatisfaction. However, this theory suffers from a lack of quantification. It only states that if people feel unequal, they are dissatisfied and try to achieve equity by using the approaches, including reducing inputs or increasing outcomes. However, it cannot estimate the degree of dissatisfaction and the value of inputs reduction or outcomes increases to achieve the feeling of equity. On the other hand, considering the personal logic of each person separately about the expectations and the variables affecting his/her satisfaction and accordingly making managerial decisions may violate procedural justice.

This study aimed to cope with ET's shortcomings; therefore, it used DEA to quantify ET and its related concepts. The advantage of this approach is the quantitative responses to the questions that EA is unable to respond. DEA can also be adapted for different persons with different characteristics and personalities to estimate their feelings of equity. For example, conventional DEA approaches were used in this study to estimate the DOS of optimistic, benevolent, pessimistic, and entitled persons. However, applying DEA to quantify ET suffers from some shortcomings; the most important is conceptual. In other words, DEA mathematically measures a person's DOS, while that person does this mentally and judiciously. Nevertheless, DEA gives ET a quantitative aspect and a predictive ability.

DEA's mathematical logic helps estimate how much each person feels equal compared to others and how much an underpaid person's inputs (outcomes) should be reduced (increased) to achieve the feeling of equity. It is important to say that changing the inputs and/or outcomes values for a person changes not only the feeling of equity perceived by that person but also the feeling of equity perceived by others. The value of these changes can be estimated in advance using DEA. This helps predict how much changes in inputs and/or outcomes for

all persons cause the least possible dissatisfaction. Despite its shortcomings, DEA can create a special place for itself in organizational theories because it is based on an accepted logic and not on personal judgment. Therefore, the management decisions that arise from DEA are defensible and may be more acceptable.

Applying DEA in the long term may even gradually change the persons' mindset as they evaluate their performances compared to others based on a more rational pattern instead of their emotional and mental judgments. This implies that it is possible to change people's mental patterns when perceiving equity through the DEA over time and bring their logic closer to a defensible logic of mathematics. This can help reduce feelings of unreasonable inequity arising from emotional and mental judgments. We believe that making some organizational decisions, such as reward allocating and changing the inputs and outcomes, based on well-known logical methods like DEA leads to less organizational conflicts and less bias inequity perceived by persons. In other words, making these decisions based on personal judgments leads to more dissatisfaction. DEA can be developed continuously during the time to adapt to the real world gradually. Using DEA in a completely subjective issue, *i.e.*, feeling of equity, can open a new window for applying the mathematical techniques in some human and social theories to avoid their shortcomings.

This study presented an approach to mathematically quantifying Adams' ET as a reactive-content theory to answer some of its shortcomings. It is suggested that other organizational theories be quantified as well, such as justice theory or reward allocation, to answer some of their shortcomings from a new perspective and try to adapt these quantitative models to the real world as much as possible. Greenberg [25] classified these theories into Reactive Content, Proactive Content, Reactive Process, and Proactive Process. We also propose to extract the common concepts of theories in each category and develop a general model for that category. Despite its advantages, using DEA to quantify the organizational theory sometimes comes with difficulties. For example, DEA usually considers a finite number of inputs and outcomes and ignores others, simplifying the real world by limiting the number of attributes. DEA cannot distinguish well between satisfied and dissatisfied people if the number of inputs and outcomes increases. It is suggested to use other methods in these situations, particularly meta-heuristic algorithms and machine learning methods, which generally do not require simplifying the real world.

DEA can estimate how much an underpaid person's inputs (outcomes) should be decreased (increased) to feel satisfied. However, in a real case, not all inputs (outcomes) can be decreased (increased). For example, unlike working hours that can be reduced, some inputs like education and working experience cannot be reduced. Classical DEA models cannot handle these situations. Hence, it is suggested that future studies develop DEA models that reduce some inputs, not all of them. These models, for example, should reduce some inputs like working hours and leave others like education to feel equity. This study used DEA as a mathematical model to answer the questions that ET cannot answer. It can open a window for future studies to use mathematical models to overcome the weaknesses of the other organizational theories, such as Herzberg's two-factor theory, Victor Vroom's expectancy theory, and Maslow's hierarchy of needs. It can be another suggestion for future studies.

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