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CARBON-EMISSION AND WASTE REDUCTION OF A MANUFACTURING-REMANUFACTURING SYSTEM USING GREEN TECHNOLOGY AND AUTONOMATED INSPECTION

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Abstract. Environmental-friendly technology helps to reduce waste and carbon emissions of an imperfect production system. In general, the defective products generated during the "out-of-control" state are treated as waste. The single-stage manufacturing-remanufacturing system effectively depletes such defective spare parts within the same cycle but causes a tremendous amount of carbon. In such a circumstance, green technology to reduce carbon emissions is highly recommended. Also, the autonomated inspection makes defective detection more reliable and is ultimately helpful for waste reduction. Hence, in this study, we optimize the production plan along with the investments for applying green technology and autonomated inspection in an assembled product manufacturing-remanufacturing system. The numerical result shows that the appropriate green technology decreases carbon emissions up to 2.81% and autonomated inspection reduces the waste up to 2.37%, along with a reduction of entire production cycle cost up to 18.26%. In addition, the setup cost reduction is considered due to the characteristics of assembled product production.

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1. Introduction

In recent years, the issue of environmental pollution caused by manufacturing systems has become significant, directly affecting our daily lives and playing a major role in damaging the earth day by day. This causes global warming, air pollution, soil damage, water pollution, and climate change. Hence, controlling the emission of greenhouse gases (GHGs) has been a trending research topic for the last few years [37]. Carbon dioxide (CO_2), water vapor, "nitrous oxide (N_2O), ozone (O_3), methane (CH_4), and chlorofluorocarbons (CFC)" are the most powerful GHGs that result in severe damage to Earth. Of all the above gases, carbon dioxide results in the greatest effect.

In 1997, the "Kyoto Protocol" was established, and it was an indication by "developed countries" that they are directing approaches to moderate "carbon emission [23]". Significant emissions are caused by different production industries, and it is necessary to develop policies that can reduce the emission during the production process in

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order to achieve sustainable development. Most firms or industries are more concerned about reducing carbon emissions. To achieve this reduction, they invest in various technologies, *e.g.*, greening technology [21, 37, 43], carbon tax [18, 51], and carbon cap-and-trade policy [49].

However, waste reduction in production systems is one of the most trending and crucial topics. For an "imperfect production system (IPS)," the generation of defective products was random, and they have been considered waste. Intelligent production systems can remove those imperfect products from the production system. However, it is necessary to determine how to clean the production process from this defectiveness and manage the generated waste. This can be done in several ways, including discarding faulty products and using them for landfill purposes. However, this will incur some costs and is environmentally harmful. Those imperfect products can be used as the raw materials for different products. However, there will still be some costs incurred. The most effective and profitable way of managing those imperfect products is remanufacturing [3, 35, 45]. By investing a minimal amount in remanufacturing, imperfect products can be made as perfect and sold in the market at their original price. The first task of this remanufacturing is to identify the defective products, which were treated as waste, and for this identification, a precise inspection process is required. Most traditional production systems use a human-centric inspection process.

Nevertheless, with human inspection, it is challenging to achieve an error-free inspection Sarkar and Saren [34], and it is well known that inspection errors can significantly impact the industry, as a human inspection cannot fully clean the production process from defectiveness due to the tardiness and skill level of human labour. In such a situation, machine-based autonomated inspection may be the best alternative to control the waste and perform an error-free inspection. Based on Dey *et al.* [9], autonomated inspection confirms the presence of error-free inspection, and can directly help to maintain the waste of an imperfect manufacturing system. Hence, we apply such autonomated inspection to make the detection of defects more reliable, thus reducing the number of defects and helping to make a cleaner manufacturing system.

We also consider the investment required for setup cost reduction. A hybrid manufacturing-remanufacturing system for assembled products has to change the setup more frequently to deal with different product spare parts. Such an idea of setup cost reduction was considered by supply chain management [8, 42], but setup cost reduction for a hybrid manufacturing-remanufacturing system has not considered yet. The investment in setup costs eventually helps reduce the cost of the hybrid system and enhance economic sustainability.

Further, a large budget is required to reduce carbon emission, autonomated inspection, and setup costs. A large space is also necessary to build a hybrid manufacturing-remanufacturing manufacturing process and to store spare parts of assembled products. However, there is limited budget and space in the manufacturing industry Malik and Sarkar [19]. Therefore, budgetary and space constraints are incorporated to make our model more sustainable and cost-effective.

In summary, this study developed a hybrid manufacturing-remanufacturing system for assembled products. Hybrid production systems emit carbon, which is harmful to the environment, and which is controlled by green technology investment. Generated waste in the form of defective products is identified by machine-based autonomated inspection, which ensures proper waste management and enhances environmental sustainability. To reduce the cost of the entire system, an investment is incorporated into the setup cost. Owing to limitations concerning budget and space, two constraints (budget and space) are considered in this model to obtain a realistic result.

Several studies have reported the control of carbon emission and waste management [37, 45], but there have been no reported studies on the control of both carbon emission and waste for the imperfect production of assembled products using green technology and autonomated inspection. Achieving the cost reduction of the entire production system by reducing the setup cost is another finding of this study. No single-stage production models have been constructed for assembled products under the concept of setup cost reduction by some continuous investment.

Based on the above discussion, our research makes the following contribution:

(1) Several production models were developed with the concept of reduced "carbon emission" [21,37]. However, the realization of carbon emission reduction in manufacturing and remanufacturing for an "assembled product" remains a gap in the literature.
(2) In the traditional imperfect production model, it was considered that faulty products were identified through human inspection, which may be error-prone [34]. Advanced autonomated inspection can perform an error-free inspection and clean the production system [38]. However, there have been few studies of such an automated inspection in imperfect manufacturing systems for assembled products.
(3) Different models for assembled or multi-product production have been developed and reported in the literature [40,41], and "single-stage production models" for the single item were also studied in the literature [3,35]. However, there have been few studies on single-stage manufacturing models with manufacturing and remanufacturing for assembled products [36]. In contrast, the reduction of the setup cost in a hybrid manufacturing-remanufacturing system for assembled product production using continuous investment has also not been previously studied.
(4) Some manufacturing models were studied under budgetary and space constraints to optimize the profit of the manufacturing system for a single product [9, 19]. However, the application of a single-stage manufacturing system for an "assembled product" with different investments and a budgetary constraint along with a space constraint has still not been reported in the existing literature.

1.1. Objective and research contribution

Nowadays, everyone cares about the environment and become more careful about the substance, which mainly causes environmental damage. A large amount of carbon is emitted from the manufacturing process. Moreover, the long-run manufacturing tends to produce more defective items, and those produced defective items were treated as waste. The industry can remanufacture that defective product in the same cycle and sell them as perfect products in the market. In such a situation, one big issue is the inspection error. The current study is developed for an imperfect manufacturing system, which produces an assembled product with different spare parts and identifies the defectives by error-free autonomated inspection. The generated waste was managed through remanufacturing in the same production cycle. To protect the environment and reduce carbon emissions, selective investments in green technology along with setup cost reduction are also considered which enhance the environmental and economic sustainability. Finally, a production plan, *i.e.*, the optimized ordered quantity and backorder quantity, is determined along with the investments for a single-stage manufacturing and remanufacturing system.

The rest of the manuscript is structured as follows. Section 2 presents detailed explanations of the existing literature, and Section 3 describes the problems, formulation, and derivation of the model along with symbols and assumptions considered to develop this research. Owing to inequality constraints, the most popular Karush–Kuhn–Tucker(KKT) solution procedure is implied for the solution methodology and is described in Section 4. One example of assembled products with three spare parts used for practical implementation is discussed in Section 5. In this section, some exceptional cases and comparisons with existing literature are also discussed. In Section 6, the effect of critical parameters is illustrated. Finally, the conclusion explains managerial and theoretical contributions, research limitations, and suggestions for future research in Section 8.

2. Literature review

An extensive illustration of existing studies is discussed in this section, along with the gaps and questions that we address. The novelty of the study compared to existing studies is tabulated in Table 1.

2.1. Reduction of carbon emission using green technology investment

The main reason for the rapid increase in global warming is the emission of greenhouse gases (GHGs), and it is well known that CO_2 is the most effective greenhouse gas, causing daily damage globally. Owing to the crucial effect of this carbon emission, production industries and customers have exercised more care to save

Table 1. Contributions made by existing literature towards this research goal.

Author(s)	Carbon treatment	Inspection strategy	Waste management	Model type	Product type	Backorder type	Constraint	Investment
Sepehri *et al.* [37]	RC	NC	NC	EPQ	Single	NC	NC	GT & PreT
Mishra *et al.* [21]	RC	NC	NC	EPQ	Single	Partial	NC	GT
Cárdenas-Barrón [3]	NC	NC	NC	SS	Single	Partial	NC	NC
Sarkar *et al.* [35]	NC	MI	NC	EPQ	Single	Planned	NC	NC
Sarkar and Saren [34]	NC	MIT	NC	SS	Single	NC	NC	NC
Dey *et al.* [9]	NC	AIE	NC	SS	Single	Planned	BS	NC
Tiwari *et al.* [42]	NC	MIT	NC	SCM	Single	Partial	NC	SC
Tayyab *et al.* [41]	RC	NC	NC	MS	MT	Planned	NC	NC
Midya *et al.* [20]	NC	NC	NC	SCM	Single	NC	NC	GT
Mondal and Roy [22]	NC	NC	NC	SCM	Single	NC	NC	NC
Das and Roy [6]	RC	NC	NC	Transportation	Single	NC	Budget	NC
Paul *et al.* [25]	CT	NC	NC	EOQ	Single	NC	NC	GT
Sahebjamnia *et al.*[28]	NC	NC	NC	EPQ	AP	NC	NC	NC
Das Roy *et al.* [27]	NC	MI	NC	EOQ	Single	Partial	NC	NC
Yong *et al.* [50]	RC	NC	WR	EPQ	Single	NC	NC	NC
Khanmohammadi *et al.* [17]	RC	NC	WR	SSCM	Single	NC	NC	NC
Bai *et al.* [2]	RC	NC	NC	VMI	Single	Partial	NC	NC
Dey *et al.* [7]	NC	NC	NC	SCM	Single	Partial	NC	SC
Ghosh *et al.* [12]	RC	NC	WM	Transportation	Single	NC	Source	NC
Pervin *et al.* [26]	NC	NC	NC	EOQ	Single	Shortage	NC	NC
This study	RC	AIE	WR	SS	AP	Planned	BS	SC & GT

Notes. AP: Assembled product with different spare parts, SCM: Supply chain management, SS: Single-stage production system, MS: multi-stage production system, NC: Not considered, AIE: Autonomated error-free inspection, MI: Manual inspection, MIT-Manual inspection with two types of inspection error (TYPE-I and TYPE-II error), SSCM: Sustainable supply chain management, VMI: Vendor-managed inventory, MT: Multi-item textile product; WR: Reduction of waste; BS: Budget & space, CT: Carbon taxation, SC: Setup cost reduction, EPQ: Economic production quantity, EOQ: Economic order quantity, GT: Green technology, RC: Reduction in carbon emission, WM: Waste managed, PreT: Preservation technology

the environment. To keep the environment clean, manufacturing industries have attempted to use different technologies and invest in these technologies.

Carbon tax and cap-and-trade policies help to reduce carbon emissions [18, 49]. A production model that employs green and standard technology with a substitution policy was developed by Chen *et al.* [5]. They considered three models and proved that the "carbon emission trading" model is more beneficial than the "carbon emission allowance" model.

All those models discussed different carbon tax policies to reduce carbon emissions for a single item production system. In contrast, product type and production strategy are most important in those days to control carbon emissions. Reminding this a hybrid manufacturing/remanufacturing model under carbon emission constraints was developed by Zouadi *et al.* [52]. They considered that although carbon was emitted during manufacturing, remanufacturing, and transportation, in reality, a carbon emission boundary is not always possible. Therefore, a case study about energy constraints and green technology development to control carbon emission was performed by Wang *et al.* [47] from 2001 to 2013 in "Chinese industrial enterprises". All of those studies focused on the implementation of carbon tax policies to reduce carbon emissions for a single-item production system, as well as carbon emissions during production, and they considered the emitted carbon as a constraint. However, in reality, it is almost impossible to set the amount of carbon emissions. Thus, we have to focus on the technologies with which industry can reduce the emission of carbon.

Owing to maintenance, manufacturing, and remanufacturing processes, production systems emit a lot of carbon, which can be reduced through different tax policies, as stated by Hajej *et al.* [13]. In the same year, Mishra *et al.* [21] considered an economic order quantity model (EOQ) along with "green technology investments" to reduce "carbon emissions". Mishra *et al.* [21] was extended by Sepehri *et al.* [37] by considering preservation technology for deteriorating items. Sepehri *et al.* [37] proved that investments in green technology for carbon emission reduction and preservation as well as investments in technology to reduce the deterioration rate have been significant compared to what has been reported in the existing literature. A "closed-loop supply chain (CLSC)" model along with carbon tax and "carbon emission reduction" was formulated by Wang and Wu [46]. In a similar direction, Sana [31] developed a supply chain model for green and non-green products, where demand depends on the selling price and the green level of the product. He proved that the green level makes a more profitable supply chain. However, he only focused on the green level and ignored the concept of uncertainties. To fill the gap in an uncertain environment, a green supply chain model for multi-stage, multi-objective optimization under a fuzzy uncertain environment was proposed by Midya *et al.* [20]. In this study, they used the concept of intuitionistic fuzzy numbers for the fixed-charge solid transportation problem. However, they ignored the concept of single-stage manufacturing/remanufacturing to control waste. Midya *et al.* [20] model was again extended by Mondal and Roy [22] by considering open and closed-loop supply chains under mixed uncertainty. In this model, they focused on the open and closed-loop supply chains, whereas autonomated inspection was not studied. In a similar direction, Das and Roy [6] proposed a multi-objective transportation problem under the consideration of the effect of variable carbon emission. For this study, Das and Roy [6] considered a neutrosophic fuzzy uncertain environment. Under the consideration of carbon taxation, a green inventory model was analyzed by Paul *et al.* [25]. Paul *et al.* [25] ignored the concept of setup cost reduction or waste reduction through remanufacturing for their study.

Most of the models were developed under different policies that help reduce carbon emission for a single item (some were for multi-items). To the best of the authors' knowledge, an assembled item production model, where manufacturing and remanufacturing are performed in a single stage for various spare parts, that realizes carbon emission reduction by green technology investment has not yet been reported in the existing literature. Thus, in this research, an effort is made to address this research gap.

2.2. Reduction of waste in a single-stage imperfect production system using autonomated inspection

At present, waste is a major problem for most production industries. Although the traditional production model was developed considering perfect production [4]. Practically, it is not always possible for all production systems to produce an ideal quality item owing to different problems [24]. Sana [29,30] developed two inventory models for imperfect production under variable demand rates and stated that an imperfect production system had a huge impact on cost optimization.

As per Sana [29] model, a manufacturing system produced perfect and imperfect items, but the rate of production of imperfect items increased during an "out-of-control" state. Those imperfect items cannot be sold in the market for different reasons, and the imperfect items are at times useless. Thus, a massive amount of waste was generated in an imperfect production system, which needs to be managed to clean the production system and optimize the profit of the production industries. Remanufacturing is one of the best solutions that is employed to manage and reduce industrial waste.

In this current study, it was considered that different spare parts of an assembled product were generated in a single-stage manufacturing process, and remanufacturing was also performed within the same manufacturing cycle. Due to several spare parts, the amount of generation of defective items also increased [10]. In contrast, some manufacturing processes considered different stages for manufacturing and remanufacturing [40]. Tayyab and Sarkar [40] considered human-based inspection to identify defective products. Whereas, human inspection can lead to inspection error due to the skill level of the labor. Thus, for an imperfect production system preventive maintenance plays a very impotent role, and during this maintenance period measurement of buffer stock is too crucial to prevent the stock out situation [33].

Manufacturing and remanufacturing in a single stage are beneficial for an imperfect production system [15]. Jamal *et al.* [15] considered a fixed defective rate and neglected the inspection strategy. Cárdenas-Barrón [3] enhanced Jamal *et al.* [15] model with the backorder. Sarkar *et al.* [35] again extended the Cárdenas-Barrón [3] model with random defectiveness, where different distributions were adopted for the defectiveness, and it was established that the cost of the manufacturing system was optimized in a triangular distributed defective rate. However, they ignored the inspection of the imperfect products. In the same year, imperfect production management along with rework consideration was established by Tang *et al.* [39] and Jokić *et al.* [16]. They considered the effect of human inspection on the product without managing waste or carbon emission.

Different studies were conducted and reported in the literature for manufacturing and remanufacturing, but few studies focused on the identification of faulty products by autonomated inspection [9, 38]. Several models and technologies have been developed to reduce the waste for a production system [44,48]. Using the concept of remanufacturing returnable products for a closed-loop three-echelon sustainable supply chain management was performed by Ullah *et al.* [45], which helps to manage the waste. They used the concept of third-party logistic to collect the waste.

Most of the existing models were developed considering the use of human-based inspection to determine the product's defectiveness. The inspection error played a vital role in managing the waste properly, and it is nearly impossible to realize 100% error-free inspection through a human-based inspection strategy for inventory systems [42]. However, the defectiveness of the items can be identified through some advanced techniques, which can help to build a smart factory [38]. Sett *et al.* [38] stated that the defectiveness of the production system for a single product could be controlled by autonomated inspection. Lean manufacturing is one of the most trending topics over the last decade, and it was expected that industry 4.0 would introduce a cleaner manufacturing system Amjad *et al.* [1]. However, Amjad *et al.* [1] focused on a single product and ignored the effect of carbon emission for their study, whereas during lean manufacturing a huge amount of carbon can be emitted due to advanced technology. Similarly, autonomated inspection can detect the faulty product perfectly and performs an error-free inspection, which can directly help to reduce waste for a single type of item means a non-deteriorating single type of product, *e.g.*, home appliances [9]. Dey *et al.* [9] focused on manufacturing planning with the effect of autonomated inspection but ignored carbon emissions and the large cost associated with setup. Recently, Ghosh *et al.* [12] developed a multi-objective transportation problem under the consideration of waste reduction, where they used the "Pythagorean hesitant fuzzy uncertain environment". In this study, Ghosh *et al.* [12] also focused on the enhancement of sustainability through carbon mechanisms. However, they ignored the concept of advanced autonomated inspection for an assembled product.

All those models were constructed for a single product. Few models are considered multi-products. In the existing literature, there have been no reports on the assembly of product production models in a single stage with a random defective rate and advanced autonomated technology to reduce waste and green technology investment for carbon emission reduction.

2.3. Cost effective investment to reduce setup cost with budget and space constraints

As mentioned in previous sections, we consider the investment in green technology and autonomated inspection in the manufacturing-remanufacturing in a single stage. To develop a manufacturing system with all of that setup, an advanced and intelligent setup is required, leading to a huge cost for the entire manufacturing system. Thus, it is required to find some way, such that the setup cost of the system can be reduced. Investment in the initial setup cost can reduce the whole system's setup cost [14]. Huang *et al.* [14] developed an "integrated inventory model" where lead time demand follows a compound Poisson distribution and setup cost will be reduced through some investments. They considered a discrete investment for setup cost reduction. However, all of those setups are for supply chain or integrated inventory models. Letter different production, integrated inventory, and supply chain models were developed under the concept of setup cost reduction through discrete or continuous investment [7, 8, 11, 42]. However, there is little interest in a single-stage imperfect manufacturing system for "assembled product" production, in which distinctive spare parts are created in a single setup and can be sold in the market separately along with advanced technology and investment for setup cost reduction.

Different investments were considered in this model to reduce carbon emission, waste, and setup costs, which lead to huge manufacturing costs. Moreover, a large space is needed to set all those hybrid manufacturing-remanufacturing processes and to maintain the assembled products along with spare parts. However, it is well known that the budget or space of a production industry is not infinite [19]. To manage this situation, in this study, a budget and space constraints were incorporated to make a cost-effective hybrid manufacturing-remanufacturing process. Malik and Sarkar [19] described the effect of disruption in an imperfect manufacturing process for multi-items, where they proved that budget and space constraints were helpful to optimize the system cost. In the same direction, an advanced production model for imperfect manufacturing was studied by Dey *et al.* [9]. They established that budget and space constraints help to increase the profit of the manufacturing system. However, they ignored the effect of carbon emissions.

All of those models focus on single-product production and errors in the inspection. However, to the best of the author's knowledge, to identify the faulty or low-quality products and reduce waste through remanufacturing under an error-free autonomated policy, where two types of backorders occur, green technology is applied to control the "carbon emissions". Continuous investments were incorporated to reduce the setup cost, and the total production cost for an assembled product with budget and space constraints has not yet been reported in the literature. This current framework will fulfill these gaps and build a sustainable production model with reduced waste and carbon emissions.

3. Symbols, hypothesis, and model description

3.1. Symbol

The following notation is utilized to illustrate the current model.

Index	
x	Spare parts number of an item, $x = 1, 2, \ldots, n$
Decision	**Variables**
Υ_x	Ordered quantity of item x (units)
β_x	Number of backorder units of item x (units)
Γ_x	Amount of capital investment in green technology for item x (\$/cycle)
I_{A_x}	Amount of capital investment for autonomated inspection (\$/batch)
Y_x	Amount of capital investment in the reduction of setup cost (\$/setup)
Parameters	
S	Maximum allowable space (square meter)
$\epsilon_{P_{cs}}$	Emitted carbon due to the setup of the manufacturing process (kg/setup)
D_x	Demand rate of item x (units/time)
$\epsilon_{R_{cs}}$	Emitted carbon due to the setup of the remanufacturing process (kg/setup)
H_{C_x}	Cost to hold the item x per unit time (\$/unit/unit time)
ϵ_{P_x}	Emitted carbon owing to the production of item x (kg/unit)
F_{bc_x}	Per unit fixed cost related to backorder cost of item x (\$/unit)
ϵ_{r_x}	Emitted carbon owing to remanufacturing of item x (kg/unit)
$\epsilon_{c_{h_x}}$	Emitted carbon because of holding the item (kg/unit)
T	Cycle length (time units)
ξ_{c_x}	Carbon tax per cycle of item x (\$/kg)
ν_x	Rate of defectiveness for item x (random)
J_{ab_x}	Backorder average of item x (units)
ω	Green technology investment fraction to reduce carbon emission
C_{lb_x}	Cost for linear backorder of item x per unit (\$/unit)
C_{P_x}	Cost of fabrication of item x (\$/unit)
K_x	Budget of item x (\$)
$I_{\max}$	Level of maximum inventory (units)
A_{0_x}	Initial inspection cost of item x (\$/item)
U_x	Required space for a spare parts of an assembled product item x (square meter)
$\bar{I}_x$	Average value of inventory of item x (units)
C_{R_x}	Cost related to remanufacturing of defective item x (\$/item)

R_{P_x}	Rate of production of item x (units/time)
R_x	Parameter related to production rate
Y_{0_x}	Initial cost for setup of item x (\$/setup)
B_t	Maximum allowable budget (\$)
λ_m	Lagrangian constant, $m = 1, 2$
δ	Scaling parameter related to autonomated inspection investment
$E[\nu_x]$	Expected value of defectiveness in each cycle for item x
TC	Total cost of the system (\$)

3.2. Assumptions

Some assumptions are made to develop this single-stage cleaner production model.

(1) Carbon is emitted owing to the hybridization of the manufacturing process. To enhance the environmental sustainability by controlling the emission of carbon and saving the environment, some investment is incorporated and is characterized as $L(\Gamma_x) = \omega_x(1 - e^{-z_x\Gamma_x})$, where z_x is the efficiency of the carbon reduction technology and $z_x \in [0, 1]$. $L(\Gamma_x)$ converges to zero if $\Gamma_x = 0$, and if $\Gamma_x \to \infty$, then $L(\Gamma_x)$ converges to ω_x. In addition, $L(\Gamma_x)$ is a "persistently differentiable function" with respect to the carbon reduction investment Γ_x [37].
(2) Demand and production rates are constant and known over the planning horizon. The production rate is greater than the demand rate, hence there is no shortage. If the model faces some shortages initially those shortages are fully backordered [36].
(3) To control waste, all items are collected and inspected through a machine that executes an autonomated inspection approach for assembling and inspection within the manufacturing system, making the model more intelligent [9]. It is also accepted that machines are first conceptualized and a perfect item then created. Over time, it begins to create imperfect items owing to specific apparatus-related and other issues. Those defective items are treated as waste. The waste generation is random and a particular distribution follows. To perform different cases, this model considered five different distributions [36].
(4) Practically, "budget and space" are not limitless. Therefore, budget and space are treated as constraints along with infinite planning horizons. Moreover, it is considered that "manufacturing and remanufacturing" are performed within the same manufacturing cycle to manage waste and carbon emissions.
(5) To reduce waste and carbon emission, it was necessary to develop an advanced production system for which a large quantity was required during the setup. To minimize the setup cost and establish a cost-effective cleaner system with reduced waste and carbon emission, continuous investment is considered in this model [42].
(6) An imperfect manufacturing system is considered for assembled items such as automobile spare parts, electronic gadgets, etc., where different "spare parts of an assembled item" are created in a single arrangement with an arbitrary defective rate.

3.3. Mathematical model

Production starts with a backorder, and it then reaches the zero inventory state in time T_1. In the time interval $[0, T_2]$, the production system simultaneously manufactured perfect and imperfect products with a fixed production rate. Remanufacturing of the imperfect or waste item then started after time T_2 to perfect it and to reduce waste with the same production rate. After time T_3, only fixed demand takes effect up to time T_4, and it again reaches the zero inventory state at time T_4. The system again entered the backorder situation after time T_4, continuing up to time T_5. The description of the problem is graphically presented in Figure 1.

This study considers a single-stage and assembled product production system where waste can be generated owing to an imperfect production system. The behavior of the defective portion is random, and it follows a certain distribution. A machine-based autonomated inspection strategy detects the waste in terms of defective items, purging the production system from defects.

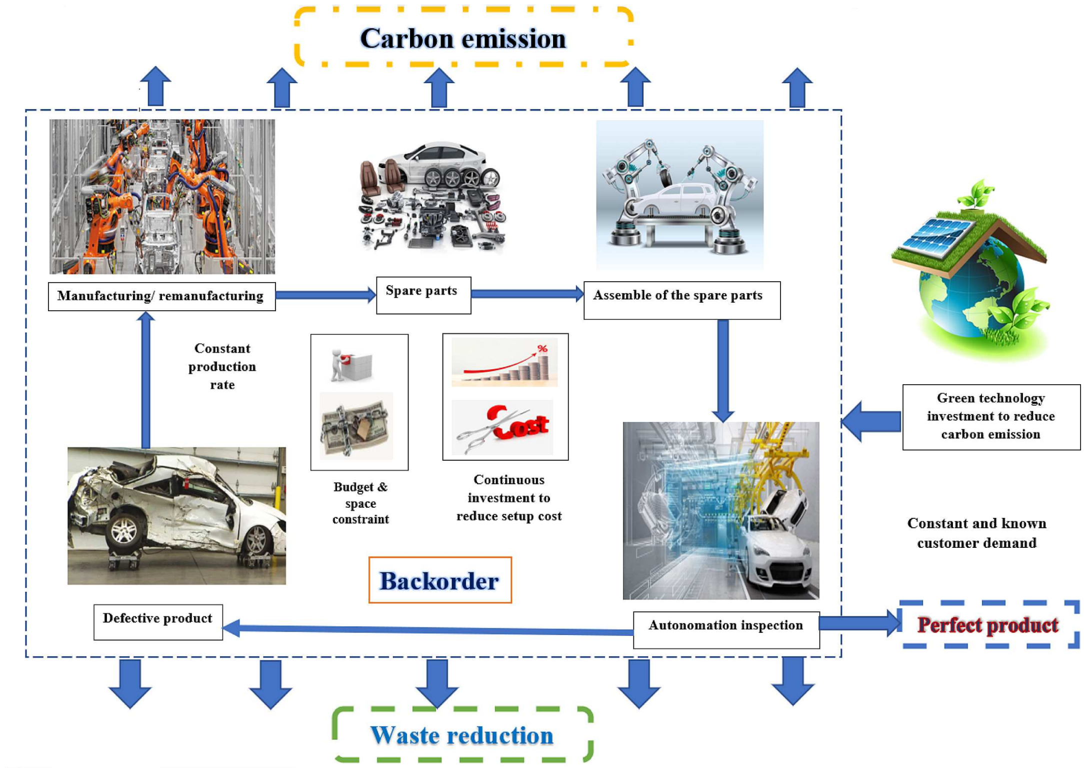


FIGURE 1. Graphical representation of the problem.

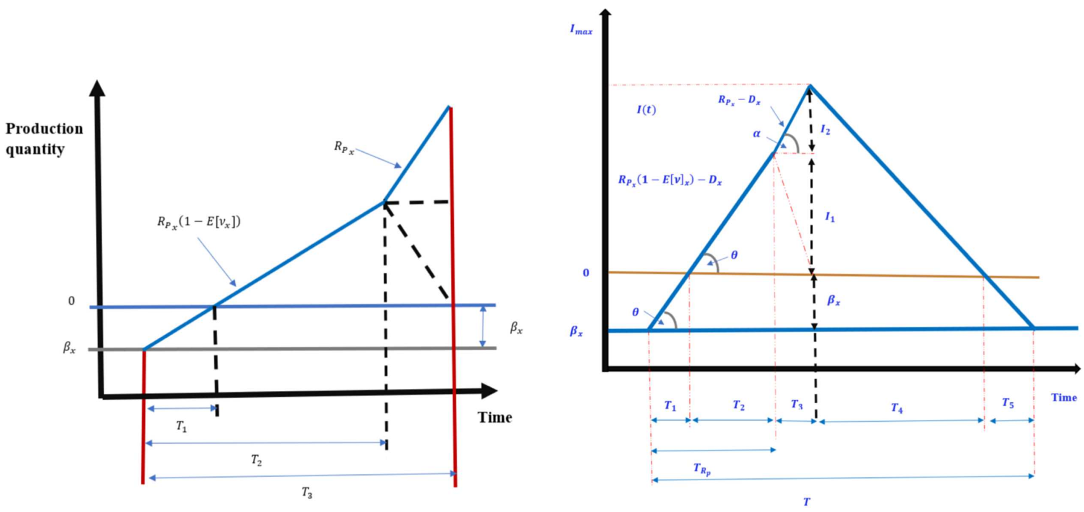
Figure 2 below illustrates the behavior of the inventory position, where production starts with a fixed rate of production where the $x-th$ item is $R_{P_x}(1-E[\nu_x])-D_x$, and this system continues up to time T_2, where within time T_1, the system fills the backorder. Thus, the production run time for the item Υ_x is $T_2 = \Upsilon_x/R_{P_x}$. After completing the backorder in the time interval $[T_1, T_2]$, the system completed the product at a fixed production rate of $R_{P_x}(1-E[\nu_x])-D_x$. After time T_2, remanufacturing began and continued up to time T_3 for the item $\Upsilon_x E[\nu_x]$ at a fixed production rate of $R_{P_x} - D_x$. Thus, remanufacturing is performed during the time interval $[T_2, T_3]$. After reaching the maximum inventory in time T_3, the inventory starts to diminish with a rate of $-D_x$, and reaches a zero-level inventory at time T_4. Thus, in the time interval $[T_3, T_4]$, only the demand is there, and it then again processes the backorder up to time T_5.

Now, the differential equations that govern the single-stage manufacturing system for the x-th item is provided as

$$\frac{\mathrm{d}\Upsilon_x}{\mathrm{d}t} = R_{P_x}(1-E[\nu]_x) - D_x; \quad \text{where} \quad \Upsilon_x(T_1) = 0, \quad T_1 \le t \le T_2 \tag{3.1}$$

$$\frac{\mathrm{d}\Upsilon_x}{\mathrm{d}t} = R_{P_x} - D_x; \quad T_2 \le t \le T_3 \tag{3.2}$$

$$\frac{\mathrm{d}\Upsilon_x}{\mathrm{d}t} = -D_x; \quad \text{where} \quad \Upsilon_x(T_4) = 0, \quad T_3 \le t \le T_4. \tag{3.3}$$



(a) (b)

FIGURE 2. Behaviour of the inventory: (a) Inventory level *versus* time graph of manufactured perfect and imperfect items with remanufacturing and backordered products. (b) Inventory *versus* time graph of entire production cycle.

After solving the above equation, one can obtain

$$\Upsilon_x(t) = \begin{cases} (D_x - R_{P_x}(1-E[\nu_x]))(T_1 - t); & T_1 \le t \le T_2 \\ R_{P_x}(T_1(E[\nu_x]-1) - T_2E[\nu_x] + t)D_x(T_1 - t); & T_2 \le t \le T_3 \\ T_1(D_x - R_{P_x}(1-E[\nu_x])) + R_{P_x}(T_3 - T_2E[\nu_x]) - D_xt; & T_3 \le t \le T_4. \end{cases} \tag{3.4}$$

Now, the original production time is given as (see Fig. 2).

$$T_1 + T_2 = \frac{(I_{1_x} + \beta_x)}{(R_{P_x}(1-E[\nu_x]) - D_x)} = \frac{\Upsilon_x}{R_{P_x}}$$

which gives,

$$I_{1_x} = \Upsilon_x\left((1-E[\nu_x]) - \frac{D_x}{R_{P_x}}\right) - \beta_x \tag{3.5}$$

where I_{1_x} denotes the inventory of manufactured items.

Now, the remanufactured time T_3 is calculated as

$$T_3 = \frac{I_{2_x}}{(R_{P_x} - D_x)}$$

$$\text{As} \quad T_3 = \frac{E[\nu_x]\Upsilon_x}{R_{P_x}},$$

$$\text{Thus,} \quad \frac{E[\nu_x]\Upsilon_x}{R_{P_x}} = \frac{I_{2_x}}{(R_{P_x} - D_x)}$$

$$\text{which gives,} \quad I_{2_x} = E[\nu_x]\Upsilon_x\left(1 - \frac{D_x}{R_{P_x}}\right). \tag{3.6}$$

Therefore, the inventory for the remanufactured items is represented by I_{2_x}.

Thus, the average inventory for the assembled production system is given as

$$I_{avg_x} = \frac{1}{2\Upsilon_x\left[(1-E[\nu_x]) - \frac{D_x}{R_{P_x}}\right]}\Bigg[\left(\Upsilon_x^2+\beta_x^2\right)(1-E[\nu_x]) + \frac{\Upsilon_x^2 D_x^2}{P_{R_m}^2}\left(1+E[\nu_x]+(E[\nu_x])^2\right) + \frac{\Upsilon_x^2 D_x}{R_{P_x}}\left((E[\nu_x])^3-2\right) + 2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}}+E[\nu_x]-1\right)\Bigg]. \tag{3.7}$$

For the values of T_1, T_2, T_3, T_4, and T_5, see Appendix A.

In time $[0, T_1]$, the process fills the backorder, and the time for the consumption of the backorder is again given by $T_5 - T_4 = \frac{\beta_x}{D_x}$ Now, the governing differential equations for the planned backorder of the single-stage manufacturing process are given as

$$\frac{\mathrm{d}\Upsilon_x(t)}{\mathrm{d}t} = R_{P_x}(1-E[\nu_x]) - D_x; \quad \text{with } \Upsilon_x(0)=0, \quad \Upsilon_x(T_1)=\beta_x,\ 0\le t\le T_1, \tag{3.8}$$

$$\frac{\mathrm{d}\Upsilon_x}{\mathrm{d}t} = -D_x, \quad \text{with } \Upsilon_x(T_4)=-\beta_x, \quad \Upsilon_x(T_5)=0,\ T_4\le t\le T_5. \tag{3.9}$$

The solution of the differential equation for the backorder is given by

$$\Upsilon_x(t) = \begin{cases} (R_{P_x}(1-E[\nu_x]) - D_x)(t-T_1)+\beta_x; & 0\le t\le T_1 \\ D_x(T_4-t)-\beta_x; & T_2\le t\le T_3. \end{cases} \tag{3.10}$$

Now, from the above solution, the average planned backorder for the production cycle is calculated as

$$I_{bavg_x} = \frac{\beta_x^2(1-E[\nu_x])}{2\Upsilon_x\left[(1-E[\nu_x]) - \frac{D_x}{R_{P_x}}\right]}. \tag{3.11}$$

To perform smooth manufacturing, some associated costs for the manufacturing system are required, and can be obtained as follows:

Setup cost with investment

To make an advanced production system with reduced carbon emissions and to manage waste, a significant setup is required, which can be reduced through some investment. Thus, the reduced setup cost with continuous logarithmic investment is provided as [42]

$$\frac{Y_x D_x}{\Upsilon_x} + I_{sr}\log\left(\frac{Y_{0_x}}{Y_x}\right).$$

Manufacturing cost

To produce perfect and imperfect spare parts of the assembled product, some manufacturing costs are necessary [36].

Therefore, the required cost for production is given by

$$\sum_{x=1}^{n} C_{P_x} D_x (1+E[\nu]_x).$$

Autonomated inspection cost

To reduce waste, it is vital to distinguish waste legitimately. As is the custom, human inspection was utilized to distinguish inadequate items. Owing to diverse parameters such as stress, tardy human inspection cannot guarantee a 100% error-free inspection. To perform machine-based autonomated inspection, an investment is proposed. In addition, the assembly of spare parts is also done by autonomation [9]. Hence, the cost related to autonomated inspection is given by:

$$I_{I_{A_x}} = \sum_{x=1}^{n} \frac{I_{A_x} D_x}{\Upsilon_x} + \delta \log\left(\frac{A_{0_x}}{I_{A_x}}\right).$$

Cost for unit remanufacturing

Remanufacturing the defective item is one of the best options to manage the waste of the industry. Thus, to manage the waste through remanufacturing, some costs are required, which were calculated as

$$\mathrm{RC} = \sum_{x=1}^{n} C_{R_x} R_{P_x} E[\nu]_x \frac{D_x}{Q_m}.$$

Cost to hold

To hold the spare parts and the amassed items in conjunction with waste, a little space is required. This space may be in a stockroom or leased distribution centre, and little cost is required for this, and is known as the holding cost.

To hold the product, the holding cost is given by

$$= \frac{H_{C_x}}{2\Upsilon_x\left[(1-E[\nu_x]) - \frac{D_x}{R_{P_x}}\right]}\left[(\Upsilon_x^2+\beta_x^2)(1-E[\nu_x]) + \frac{\Upsilon_x^2 D_x^2}{P_{R_m}^2}\left(1+E[\nu_x]+(E[\nu_x])^2\right) + \frac{\Upsilon_x^2 D_x}{R_{P_x}}\left((E[\nu_x])^3-2\right)\right.$$
$$\left. + 2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}} + E[\nu_x] - 1\right)\right].$$

Cost of backorder

Owing to large demand, there may be shortages, which can be controlled through the backorder. This backorder may be planned or partial. In this model, two types of backorder (linear and fixed) are calculated to obtain the best result [35].

Therefore, the total cost related to the backorder is given as

$$\mathrm{BC} = \frac{F_{bc_x}\beta_x D_x}{\Upsilon_x} + C_{lb_x} I_{bavg_x}.$$

Cost related to carbon emission

– The setup for production and remanufacturing emits carbon, and some costs are associated with this, namely:

$$\frac{D_x}{\Upsilon_x}\xi_{c_x}(\epsilon_{P_{cs}}).$$

– Owing to the production process, some carbon is emitted during the time interval $[T_1, T_2]$, and the related cost is obtained as

$$\frac{\xi_{c_x}\epsilon_{P_x} R_{P_x}(T_1+T_2)}{T} = \xi_{c_x}\epsilon_{P_x} D_x.$$

– Carbon emission cost for the remanufacturing is given by

$$\frac{\xi_{c_x}\epsilon_{r_x}(R_{P_x}-D_x)T_3}{T} = \delta_c\epsilon_{r_x}R_{P_x}E[\nu_x]\frac{E[\nu_x]B_{S_m}}{R_{P_x}}\frac{D_x}{\Upsilon_x} = \xi_{c_x}\epsilon_{r_x}R_{P_x}(E[\nu_x])^2D_x.$$

– Emission related to holding the items is given by

$$E_h = \xi_{c_x}\epsilon_{ch}I_{avg_x} = \delta_c\epsilon_{ch}\frac{1}{2\Upsilon_x\left[(1-E[\nu_x])-\frac{D_x}{R_{P_x}}\right]}\left[(\Upsilon_x^2+\beta_x^2)(1-E[\nu_x])\right.$$
$$\left.+\frac{\Upsilon_x^2D_x^2}{P_{R_m}^2}\big(1+E[\nu_x]+(E[\nu_x])^2\big)+\frac{\Upsilon_x^2D_x}{R_{P_x}}\big((E[\nu_x])^3-2\big)+2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}}+E[\nu_x]-1\right)\right].$$

Investment for green technology

Carbon is emitted owing to the hybridization of the manufacturing process. To control the emission of carbon and reduce the environmental impact, some investment is incorporated, which is characterized as $L(\Gamma_x) = \omega_x(1-e^{-z_x\Gamma_x})$, where z_x is the efficiency of the carbon reduction technology. $L(\Gamma_x)$ converges to zero if $\Gamma_x = 0$, and if $\Gamma_x \to \infty$, then $L(\Gamma_x)$ converges to ω_x. In addition, $L(\Gamma_x)$ is a "persistently differentiable function" with respect to the carbon reduction investment Γ_x [37]. Thus, the total green investment is given by

$$= \omega_x(1-e^{-z_x\Gamma_x}) + \Gamma_x.$$

Therefore, one can obtain the cost for this hybrid production by adding all of the above-mentioned related costs as follows

$$\begin{aligned}
\mathrm{TC}(\Upsilon_x,\beta_x,I_{A_x},Y_x,\Gamma_x) = \sum_{x=1}^{n}&\left[\frac{Y_xD_x}{\Upsilon_x}+I_{sr}\log\left(\frac{Y_{0_x}}{Y_x}\right)+H_{C_x}\left[\frac{1}{2\Upsilon_x\left[(1-E[\nu_x])-\frac{D_x}{R_{P_x}}\right]}\right.\right.\\
&\times\left[(\Upsilon_x^2+\beta_x^2)(1-E[\nu_x])+\frac{\Upsilon_x^2D_x^2}{P_{R_m}^2}\left(1+E[\nu_x]+(E[\nu_x])^2\right)+\frac{\Upsilon_x^2D_x}{R_{P_x}}\big((E[\nu_x])^3-2\big)\right.\\
&\left.\left.+2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}}+E[\nu_x]-1\right)\right]\right]+\frac{F_{bc_x}\beta_xD_x}{\Upsilon_x}+C_{lb_x}\left[\frac{\beta_x^2(1-E[\nu_x])}{2\Upsilon_x\left[(1-E[\nu_x])-\frac{D_x}{R_{P_x}}\right]}\right]\\
&+C_{P_x}D_x(1+E[\nu_x])+R_{P_x}E[\nu_x]C_{R_x}\frac{D_x}{\Upsilon_x}+\frac{I_{A_x}D_x}{\Upsilon_x}+\delta\log\left(\frac{A_{0_x}}{I_{A_x}}\right)\\
&+\left[\frac{D_x}{\Upsilon_x}\xi_{c_x}\epsilon_{P_{cs}}+\xi_{c_x}\epsilon_{P_x}D_x+\xi_{c_x}\epsilon_{r_x}R_{P_x}(E[\nu_x])^2D_x+\frac{\xi_{c_x}\epsilon_{ch}}{2\Upsilon_x\left[(1-E[\nu_x])-\frac{D_x}{R_{P_x}}\right]}\right.\\
&\times\left[(\Upsilon_x^2+\beta_x^2)(1-E[\nu_x])+\frac{\Upsilon_x^2D_x^2}{P_{R_m}^2}\big(1+E[\nu_x]+(E[\nu_x])^2\big)+\frac{\Upsilon_x^2D_x}{R_{P_x}}\big((E[\nu_x])^3-2\big)\right.\\
&\left.\left.\left.+2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}}+E[\nu_x]-1\right)\right]\right]\left(1-\omega_x(1-e^{-z_x\Gamma_x})\right)+\Gamma_x\right].
\end{aligned}\tag{3.12}$$

Owing to the realistic assumption of the limitation with respect to the total storage and total system budget, the constraints related to space and budget are provided as:

$$\sum_{x=1}^{n}\Upsilon_xK_x \le B_t \tag{3.13}$$

$$\sum_{x=1}^{n} U_x \Upsilon_x \leq S. \tag{3.14}$$

The entire cost of the system along with the constraints are presented in equations (3.12)–(3.14), which need to be optimized based on the decision variable Υ_x, β_x, Y_x, Γ_x, and I_{A_x}.

4. Solution methodology

It is clear that equations (3.12)–(3.14) construct a non-linear equation with some constraints. Thus, to determine the optimum cost for this "hybrid production system" and the values of the "decision variables," one can utilize the KKT optimization technique.

By equating $\frac{\partial L}{\partial \beta_x}$, $\frac{\partial L}{\partial \Upsilon_x}$, $\frac{\partial L}{\partial I_{A_x}}$, $\frac{\partial L}{\partial Y_x}$, and $\frac{\partial L}{\partial \Gamma_x}$ to zero, one can obtain

$$\beta_x = \frac{\Upsilon_x \left[\Omega_1 + H_{C_x} - \frac{F_{bc_x} D_x}{\Upsilon_x}\right]\left[1 + E[\nu_x] - \frac{D_x}{R_{P_x}}\right]}{(1 - E[\nu_x])\Omega_4}, \tag{4.1}$$

$$\Upsilon_x = \left[\frac{\Omega_4\Omega_5 + D_x\Omega_3}{\Omega_6 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.2}$$

$$I_{A_x} = \frac{\delta}{D_x}\left[\frac{\Omega_4\Omega_5 + D_x\Omega_3}{\Omega_6 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.3}$$

$$Y_x = \frac{I_{sr}}{D_x}\left[\frac{\Omega_4\Omega_5 + D_x\Omega_3}{\Omega_6 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.4}$$

$$\begin{aligned}\Gamma_x = \frac{1}{z_x}\log\Bigg(z_x\omega_x\Bigg[&\frac{D_x}{\Upsilon_x}\xi_{c_x}(\epsilon_{P_{cs}}) + \xi_{c_x}\epsilon_{P_x} D_x + \xi_{c_x}\epsilon_{r_x} R_{P_x}(E[\nu_x])^2 D_x\\ &+ \frac{\xi_{c_x}\epsilon_{ch_x}}{2\Upsilon_x\left[(1 - E[\nu_x]) - \frac{D_x}{R_{P_x}}\right]}\Bigg[(\Upsilon_x^2 + \beta_x^2)(1 - E[\nu_x]) + \frac{\Upsilon_x^2 D_x^2}{P_{R_m}^2}\left(1 + E[\nu_x] + (E[\nu_x])^2\right)\\ &+ \frac{\Upsilon_x^2 D_x}{R_{P_x}}((E[\nu_x])^3 - 2) + 2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}} + E[\nu_x] - 1\right)\Bigg]\Bigg]\Bigg).\end{aligned} \tag{4.5}$$

[See Appendix A for detailed calculations.]

Now, one can find the Lagrangian constraint as follows:

$$\Rightarrow\ \lambda_1 = \sum_{x=1}^{n}\frac{K_x}{B_t^2}(\Omega_4\Omega_5 + D_x\Omega_3) - \sum_{x=1}^{n}\frac{(\Omega_6 + \lambda_2 U_x)}{K_x}\cdot \tag{4.6}$$

[See Appendices A and B for calculations.]

And

$$\Rightarrow\ \lambda_2 = \sum_{x=1}^{n}\frac{U_x}{S^2}(\Omega_4\Omega_5 + D_x\Omega_3) - \sum_{x=1}^{n}\frac{(\Omega_6 + \lambda_1 K_x)}{U_x}\cdot \tag{4.7}$$

[See Appendices A and C for simplification.]

4.1. 0 investment ($\Gamma_x = 0$)

The important concern related to developing this framework is achieving a reduction of carbon emissions through green technology investment. Now, we find the analytic values of the decision variables when green technological investment cannot take place. Therefore, the optimum values for the decision variables are provided as:

$$\beta_x = \frac{\Upsilon_x\left[\xi_{c_x}\epsilon_{ch} + H_{C_x} - \frac{F_{bc_x}D_x}{\Upsilon_x}\right]\left[1 + E[\nu_x] - \frac{D_x}{R_{P_x}}\right]}{(1 - E[\nu_x])\Omega_4}, \tag{4.8}$$

$$\Upsilon_x = \left[\frac{\Omega_8\Omega_5 + D_x\Omega_3}{\Omega_9 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.9}$$

$$I_{A_x} = \frac{\delta}{D_x}\left[\frac{\Omega_8\Omega_5 + D_x\Omega_7}{\Omega_9 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.10}$$

$$Y_x = \frac{I_{sr}}{D_x}\left[\frac{\Omega_8\Omega_5 + D_x\Omega_7}{\Omega_9 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.11}$$

$$\lambda_1 = \sum_{x=1}^{n}\frac{K_x}{B_t^2}(\Omega_8\Omega_5 + D_x\Omega_7) - \sum_{x=1}^{n}\frac{(\Omega_9 + \lambda_2 U_x)}{K_x} \tag{4.12}$$

$$\lambda_2 = \sum_{x=1}^{n}\frac{U_x}{S^2}(\Omega_8\Omega_5 + D_x\Omega_7) - \sum_{x=1}^{n}\frac{(\Omega_9 + \lambda_1 K_x)}{U_x}\cdot \tag{4.13}$$

4.2. Special Case II: Without autonomated inspection investment ($I_{A_x} = 0$)

Another major problem of an imperfect production system is waste, which can be identified and eliminated using the advanced autonomated inspection technique. Now, if $I_{A_x} = 0$, *i.e.*, without investment for autonomated inspection, then optimal values for decision variables are as follows:

$$\beta_x = \frac{\Upsilon_x\left[\Omega_1 + H_{C_x} - \frac{F_{bc_x}D_x}{\Upsilon_x}\right]\left[1 + E[\nu_x] - \frac{D_x}{R_{P_x}}\right]}{(1 - E[\nu_x])\Omega_4}, \tag{4.14}$$

$$\Upsilon_x = \left[\frac{\Omega_4\Omega_5 + D_x(\Omega_3 - I_{A_x})}{\Omega_6 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.15}$$

$$Y_x = \frac{I_{sr}}{D_x}\left[\frac{\Omega_4\Omega_5 + D_x(\Omega_3 - I_{A_x})}{\Omega_6 + \lambda_1 K_x + \lambda_2 U_x}\right]^{\frac{1}{2}} \tag{4.16}$$

$$\begin{aligned}\Gamma_x = \frac{1}{z_x}\log\Bigg(z_x\omega_x\Bigg[&\frac{D_x}{\Upsilon_x}\xi_{c_x}(\epsilon_{P_{cs}}) + \xi_{c_x}\epsilon_{P_x}D_x + \xi_{c_x}\epsilon_{r_x}R_{P_x}(E[\nu_x])^2 D_x \\ &+ \frac{\xi_{c_x}\epsilon_{ch}}{2\Upsilon_x\left[(1 - E[\nu_x]) - \frac{D_x}{R_{P_x}}\right]}\Bigg[\left(\Upsilon_x^2 + \beta_x^2\right)(1 - E[\nu_x]) + \frac{\Upsilon_x^2 D_x^2}{P_{R_m}^2}\left(1 + E[\nu_x] + (E[\nu_x])^2\right) \\ &+ \frac{\Upsilon_x^2 D_x}{R_{P_x}}((E[\nu_x])^3 - 2) + 2\beta_x\Upsilon_x\left(\frac{D_x}{R_{P_x}} + E[\nu_x] - 1\right)\Bigg]\Bigg]\Bigg)\end{aligned} \tag{4.17}$$

$$\lambda_1 = \sum_{x=1}^{n}\frac{K_x}{B_t^2}(\Omega_4\Omega_5 + D_x(\Omega_3 - I_{A_x})) - \sum_{x=1}^{n}\frac{(\Omega_6 + \lambda_2 U_x)}{K_x} \tag{4.18}$$

$$\lambda_2 = \sum_{x=1}^{n}\frac{U_x}{S^2}(\Omega_4\Omega_5 + D_x(\Omega_3 - I_{A_x})) - \sum_{x=1}^{n}\frac{(\Omega_6 + \lambda_1 K_x)}{U_x} \tag{4.19}$$

where Ω_3 equals $\Omega_2 + Y_x + F_{bc_x}\beta_x + I_{A_x} + R_{P_x}E[\nu_x]C_{R_x}$.

4.3. Special Case III: Without budget and space constraints ($\lambda_m = 0$)

When the constraint does not affect the total cost, that is, $\lambda_m = 0$, then by equating $\lambda_m = 0$, one can easily obtain the backorder quantity (β_x) and order quantity (Υ_x) as follows:

$$\beta_x = \frac{(\Upsilon_x \Omega_1 + \Upsilon_x H_{C_x} - F_{bc_x} D_x)\Big((1 - E[\nu_x]) - \frac{D_x}{R_{P_x}}\Big)}{(1 - E[\nu_x])\Omega_4}. \tag{4.20}$$

Using the above formula, the following optimum values are obtained:

$$\Upsilon_x = \sqrt{\frac{\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) F_{bc_x}^2 D_x^2 - 2D_x(1 - E[\nu_x])\Omega_3\Omega_4}{H_{C_x}\Big(H_{C_x}\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) - (1 - E[\nu_x])\Omega_4\Big[1 - (1 + E[\nu_x] + (E[\nu_x])^2)\frac{D_x}{R_{P_x}}\Big]\Big)}} \tag{4.21}$$

$$I_{A_x} = \sqrt{\frac{\delta^2\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) F_{bc_x}^2 D_x^2 - 2D_x(1 - E[\nu_x])\Omega_3\Omega_4}{D_x^2 H_{C_x}\Big(H_{C_x}\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) - (1 - E[\nu_x])\Omega_4\Big[1 - (1 + E[\nu_x] + (E[\nu_x])^2)\frac{D_x}{R_{P_x}}\Big]\Big)}} \tag{4.22}$$

$$Y_x = \sqrt{\frac{I_{sr}^2\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) F_{bc_x}^2 D_x^2 - 2D_x(1 - E[\nu_x])\Omega_3\Omega_4}{D_x^2 H_{C_x}\Big(H_{C_x}\Big(1 - E[\nu_x] - \frac{D_x}{R_{P_x}}\Big) - (1 - E[\nu_x])\Omega_4\Big[1 - (1 + E[\nu_x] + (E[\nu_x])^2)\frac{D_x}{R_{P_x}}\Big]\Big)}}. \tag{4.23}$$

Sufficient condition. By neglecting the constraint and calculating the values of the principal minors of the Hessian matrix, which are based on the decision variable Υ_x, Y_x, I_{A_x}, Γ_x and β_x, one can prove the sufficient condition of the optimization. The convexity of the cost function based on decision variables was guaranteed by the positivity of the principal minors of the 5×5 Hessian matrix.

Proof. Please see Appendix D. □

5. Numerical experiment and discussion

This segment provides numerical results for three spare parts assembled products when the defectiveness rate satisfied five different types of probabilistic random distributions. In this section, the effect of autonomated inspection in waste reduction and green technology in the reduction of carbon emission was also illustrated along with numerical values.

5.1. Parameter setup and optimum result

The numerical values of different parameters are taken from Sarkar *et al.* [36] model and Sepehri *et al.* [37] model. It is obvious that the setup for the Sarkar *et al.* [36] model or Sepehri *et al.* [37] model is different from the current study, and that is the reason for using the parametric value at their best fit for this current study. We used the software Mathematica 11.0 to obtain the result numerically. The values for the different parameters are as follows:

Assembled products with three spare parts, *i.e.*, $x = 3$. An "assembled product produced with three spare parts," *i.e.*, the number of spare and parts $x = 3$, and the remaining parametric values are considered as: spare parts demand $D_x = (250, 350, 400)$ (units/time); initial inspection cost $700, 550, 550$ ($/lot); fixed rate of production of spare and parts $R_{P_x} = (500, 550, 600)$ (units/time); carbon emission for manufacturing $(5, 6, 4)$ (kg/cycle); annual cost for linear backorder $C_{lb_x} = (10, 11, 10)$ ($/unit/year); carbon tax per cycle $(1, 1, 1)$ $/kg; annual holding cost $H_{C_x} = (50, 55, 55)$ ($/unit/year); cost for remanufacturing of each item $C_{R_x} = (5, 7, 6)$ ($/unit); cost for fixed backorder $F_{bc_x} = (1, 2, 1)$ ($/unit); carbon emission for setup $(20, 25, 25)$ kg/setup; initial cost to setup the process $Y_x = (350, 300, 250)$ ($/lot size); manufacturing cost for different spare and parts

TABLE 2. Optimal result for an assembled product with three-spare parts ($x = 3$).

DR	CD	RD	UD	TD	DTD
Y_x (\$/setup)	124.20, 179.34, 121.57	151.86, 211.97, 135.00	157.95, 221.79, 143.01	146.47, 185.47, 167.55	147.74, 184.97, 168.17
Υ_x (Units)	62.10, 74.73, 81.05	75.93, 88.32, 90.00	78.97, 92.41, 95.34	73.23, 77.28, 83.77	73.87, 77.07, 84.08
β_x (Units)	18.50, 37.91, 18.87	21.38, 42.88, 20.76	22.29, 44.98, 21.86	24.29, 32.32, 23.44	24.19, 32.06, 23.23
I_{A_x} (\$/batch)	207.00, 298.90, 202.62	253.11, 353.29, 225.01	263.25, 369.65, 238.34	244.11, 309.12, 279.25	246.24, 308.28, 280.28
Γ_x (\$/item)	98.26, 127.12, 184.92	101.72, 132.31, 195.31	102.86, 134.01, 198.71	109.90, 144.57, 219.83	113.09, 149.35, 229.40
Constraints	0.12, 0.17	0.13, 0.18	0.11,0.14	0.12, 0.17	0.14, 0.16
TC (\$/cycle)	**46 803.40**	47 446.90	49 456.70	53 216.10	54 765.50

Notes. DR: Defective rate follows; CD: χ^2- distribution; RD: Reciprocal distribution; UD: Uniform distribution; TD: Triangular distribution; DTD: Double triangular distribution.

TABLE 3. Optimal result of an assembled product with three-spare parts without green technology investment.

DR	CD	RD	UD	TD	DTD
Y_x (\$/setup)	125.00, 180.25, 122.22	152.50, 212.66, 135.62	158.56, 222.44, 143.56	147.02, 186.16, 168.14	148.27, 185.65, 168.74
Υ_x (Units)	62.49, 75.11, 81.48	76.25, 88.61, 90.42	79.28, 92.68, 95.71	73.51, 77.57, 84.07	74.13, 77.35, 84.37
β_x (Units)	18.65, 38.15, 19.00	21.51, 43.09, 20.90	22.42, 45.18, 22.00	24.41, 32.47, 23.56	24.31, 32.21, 23.34
I_{A_x} (\$/batch)	208.32, 300.42, 203.69	254.17, 354.43, 226.04	264.26, 370.74, 239.27	245.03, 310.27, 280.23	247.11, 309.42, 281.23
Constraints	0.12, 0.17	0.13, 0.18	0.11,0.14	0.12, 0.17	0.14, 0.16
TC (\$/cycle)	**48 116.10**	48 949.30	51 026.00	55 261.30	57 064.50

$C_{P_x} = (7, 8, 7)$ (\$/unit); carbon emission for holding spare parts $(1, 1, 1)$ (kg/cycle); budget for each spare and parts $K_x = (55, 65, 60)$ (\$/unit); efficiency of the carbon reduction technology $(0.03, 0.02, 0.01)$; space required for each spare and parts $U_x = (3, 4, 2)$ (sq.m./unit); δ =1000; carbon emission for remanufacturing (2, 2, 2); fraction of carbon emission reduction $(0.1, 0.1, 0.1)$. The rate of defectiveness, which follows certain distributions were calculated by the help of following numerical values: $(0.03, 0.03, 0.04)$, $(0.03, 0.06; 0.04, 0.07; 0.04, 0.06)$, $(0.03, 0.07; 0.03, 0.07; 0.04, 0.08)$, $(0.03, 0.04, 0.07; 0.03, 0.04, 0.07; 0.04, 0.04, 0.07)$, and $(0.03, 0.04, 0.07; 0.03, 0.04, 0.07; 0.04, 0.04, 0.07)$ for Chi-square distribution, Reciprocal distribution, Uniform distribution, Triangular distribution, and Double-triangular distribution respectively.

5.2. Analysis and discussion of the result

The optimum values for different cases are illustrated in Tables 2–6. From the above tables, it is clear that optimum results for the entire production were obtained when the defectiveness rate follows a "chi-square distribution" compared to other distributions. Moreover, the convexity of the total cost is based on different variables graphically represented in Figure 3.

TABLE 4. Optimal result of an assembled product with three-spare parts without investment for autonomated inspection.

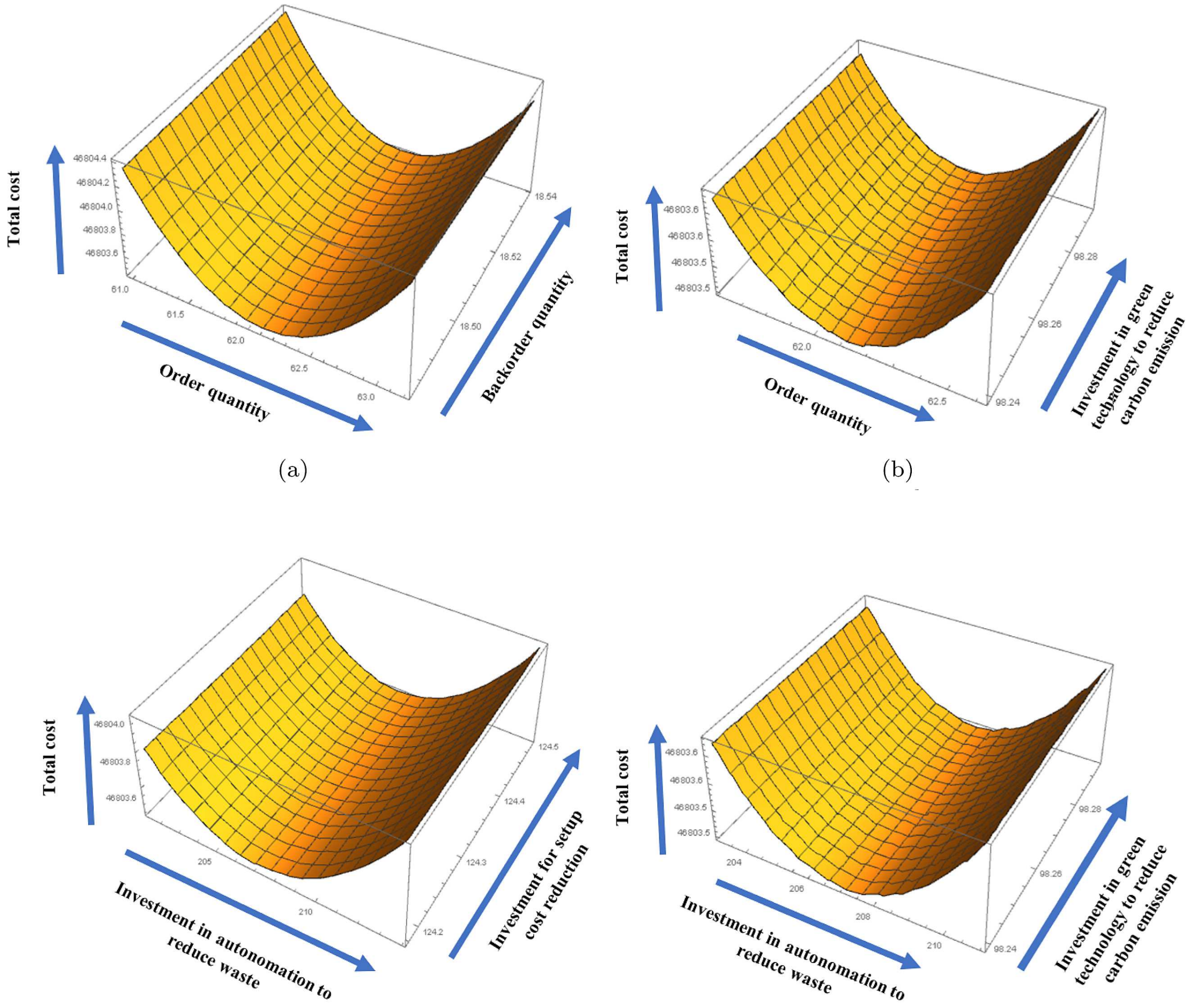
DR	CD	RD	UD	TD	DTD
Y_x (\$/setup)	162.93, 224.91, 156.39	186.34, 249.20, 169.78	191.52, 257.70, 176.25	175.86, 223.81, 197.16	176.03, 222.46, 196.74
Υ_x (Units)	81.47, 93.71, 104.26	93.17, 103.83, 113.18	95.76, 107.38, 117.50	87.93, 93.25, 98.58	88.01, 92.69, 98.37
β_x (Units)	24.80, 48.13, 24.83	26.72, 50.89, 26.72	27.49, 52.70, 27.50	29.50, 39.36, 27.88	29.15, 38.91, 27.46
Γ_x (\$/item)	98.05, 126.80, 184.29	101.62, 132.15, 194.98	102.77, 133.88, 198.45	109.82, 144.45, 219.60	113.01, 149.25, 229.18
Constraints	0.12, 0.17	0.13, 0.18	0.11,0.14	0.12, 0.17	0.14, 0.16
TC (\$/cycle)	**47 910.10**	48 189.10	50 099.60	53 920.20	55 470.30

TABLE 5. Optimal result of an assembled product with three-spare parts without setup cost reduction investment.

DR	CD	RD	UD	TD	DTD
Υ_x (Units)	72.03, 82.51, 87.34	84.04, 87.56, 95.57	86.69, 91.00 100.17	80.08, 78.10, 85.86	80.40, 77.90, 86.06
β_x (Units)	21.73, 42.10, 20.48	23.89, 42.49, 22.19	24.68, 44.25, 23.09	26.71, 32.68, 24.07	26.48, 32.42, 23.81
I_{A_x} (\$/batch)	240.10, 330.04, 218.35	280.13, 350.24, 238.93	288.96, 363.99, 250.43	266.92, 312.41, 286.21	268.00, 311.59, 286.86
Γ_x (\$/item)	98.16, 126.96, 184.61	101.69, 132.26, 195.21	102.83, 133.97, 198.63	109.88, 144.54, 219.77	113.07, 149.33, 229.34
Constraints	0.12, 0.17	0.13, 0.18	0.11,0.14	0.12, 0.17	0.14, 0.16
TC (\$/cycle)	**47 338.20**	47 561.00	49 549.10	53 310.50	54 857.50

TABLE 6. Optimal result of an assembled product with three-spare parts without budget and space constraints.

DR	CD	RD	UD	TD	DTD
Y_x (\$/setup)	148.00,230.03, 143.73	194.45, 286.26, 168.78	195.60, 288.16, 173.14	171.74, 227.03, 201.03	176.82, 232.47, 206.70
Υ_x (Units)	74.00, 95.84, 95.82	97.22, 119.28, 112.52	97.80, 120.07, 115.42	85.87, 94.60, 100.51	88.41, 96.86, 103.35
β_x (Units)	22.37, 49.28, 22.66	27.98, 58.86, 26.55	28.12, 59.24, 26.97	28.77, 39.95, 28.46	29.29, 40.74, 28.94
I_{A_x} (\$/batch)	246.67, 383.38, 239.56	324.08, 477.11, 281.31	326.00, 480.26, 288.56	286.23, 378.39, 335.04	294.70, 387.44, 344.49
Γ_x (\$/item)	98.09, 126.86, 184.41	101.60, 132.13, 194.94	102.76, 133.87, 198.43	109.82, 144.45, 219.59	113.01, 149.24, 229.16
TC (\$/cycle)	**54 047.60**	54 895.40	55 697.10	60 343.20	63 055.00



(c) (d)

Figure 3. Surface graph of convexity of total cost respect to different decision variables when rate of defectiveness follows chi-square distribution. (a) Convexity of total cost respect to order quantity and backorder quantity. (b) Convexity of total cost respect to order quantity and investment in green technology. (c) Convexity of total cost respect to investment in autonomated inspection and investment in setup cost. (d) Convexity of total cost respect to investment in autonomated inspection and investment in green technology.

Table 2, provided the optimum values for three spare-parts assembled products. The optimum or minimum value for three spare-parts assembled product systems is \$46 803.40, and green technology investment reduces the carbon emission up to 4.20%. The optimized cost without green technology is provided in Table 3, from which one can find that the optimum cost for the manufacturing cycle is \$48 116.10.

The effect of autonomated inspection technology to manage the waste is numerically explained in Table 4 for the assembled products, which consist of three spare parts. The minimum coat is attainted when the rate of defectiveness follows a chi-square distribution. However, Table 4 proves that the system cost without

TABLE 7. Effect of green technology investment on total cost (assembled product with three-spare parts).

Without	CD ($/cycle)	RD ($/cycle)	UD ($/cycle)	TD ($/cycle)	DTD ($/cycle)
Autonomated inspection investment	49 212.10	49 685.20	51 663.90	55 261.30	57 763.50
Setup cost reduction investment	48 645.70	49 061.50	51 117.00	55 354.20	57 155.00
Budget and space constraints	55 351.50	56 390.70	59 320.60	62 382.50	65 347.30

autonomated investment is $47 910.10, which means the autonomated inspection reduced the waste by about 2.37% in a production cycle. Moreover, from Table 4, one can estimate the effect of green technology investment and autonomated inspection investment. Without green technology investment and autonomated inspection, the system cost is 49 212.10 for three spare-parts assembled product manufacturing systems. Thus, autonomated inspection technology and green technology investment reduce costs by up to 5.47% for three spare parts.

The investment to reduce the setup cost of the hybrid manufacturing process is very effective in optimizing the cost of the manufacturing system. The effect of investment related to setup cost reduction is presented in Table 5. From Table 5, it is clear that investment in setup cost reduced the cost of the production cycle up to $534.80 per cycle, which is impressive. From Table 5, it is also proved that without investment in green technology and setup cost, increase the system cost up to $529.60 per cycle.

The effect of constraints is provided in Table 6. From Table 6, it is clear that budget and space constraints have a great impact on the production industry. The production cost is optimum under the chi-square distributed defective rate, as in the previous cases. The optimum value for three spare parts products is $54 047.60. Thus, "budget and space constraints" reduce the production cycle cost by up to 18.40% for three spare parts production systems. Table 7 shows the effects of green technology investment under different scenarios. When autonomated inspection and green technology are not used, the cost of the manufacturing cycle increases by $2408.7 (see Tab. 2). Similarly, setup cost investment reduces total manufacturing costs by up to $1842 under green technology (see Tab. 2). Budget and space constraints reduce the manufacturing cycle costs by up to 18.26% under green technology investments.

From the above explanation, it is clear that green technology investment reduces the carbon emission up to 1312.70 kg per production cycle (see Tabs. 2 and 3) due to the use of green technology and reduces the waste up to 1106.70 kg per cycle (see Tabs. 2 and 4) due to the use of autonomated inspection. Thus, one can conclude that, by the present research strategy, the production industry can reduce its carbon emission and manage the waste in terms of the defective item properly and build up a sustainable production industry with minimized costs.

5.3. Comparison with previous studies

As per the authors' knowledge, no study was constructed in the literature for a single-stage manufacturing system with reduced carbon emission and waste under controllable setup cost and a "budget and space" constraint. Thus, it is impossible to compare our study directly with the previous literature. However, this model converges with previous studies by ignoring some assumptions of this study and some different setups.

(i) If one ignored assembled product, green technology investment, autonomated inspection, constraints, and setup cost reduction, and considered the constant rate of defectiveness, then the base of this model converges to Cárdenas-Barrón [3]. Consequently, if one considered the random rate of defectiveness, then this study converges to the Sarkar *et al.* [35] model.

TABLE 8. Sensitivity for key parameters when rate of defectiveness follows chi-square distribution.

Parameters	Changes (%)	Changes in Υ^* (%)	Changes in β^*(%)	Changes in I_A^*(%)	Changes Γ^*(%)	Changes in Y^*(%)	Changes TC(%)
D	−50	−37.69	−3.68	+24.61	−25.83	+24.61	−41.26
	−25	−22.29	−1.48	+3.61	−10.71	+3.61	−19.21
	+25	+37.34	+1.23	–	+8.26	–	+16.84
	+50	–	–	–	–	–	–
R_P	−50	–	–	–	–	–	–
	−25	+50.07	−1.64	+50.07	−2.35	+50.07	−9.17
	+25	−14.27	+3.15	−14.27	+2.00	−14.27	+5.83
	+50	−20.43	+6.48	−20.43	+3.82	−20.43	+10.41
H_C	−50	+30.74	+11.05	+30.74	−0.13	+30.74	−5.79
	−25	+12.87	+6.50	+12.87	−0.08	+12.87	−2.65
	+25	−9.85	−6.47	−9.85	+0.08	−9.85	+2.33
	+50	−17.69	−12.39	−17.69	+0.17	−17.69	+4.42
C_P	−50	–	–	–	–	–	−19.77
	−25	–	–	–	–	–	−9.89
	+25	–	–	–	–	–	+9.89
	+50	–	–	–	–	–	+19.77
ξ_c	−50	−1.20	−1.63	−1.20	−26.93	−1.20	−16.76
	−25	−0.59	−0.81	−0.59	−11.17	−0.59	−8.36
	+25	+0.58	+0.78	+0.58	+8.66	+0.58	+8.33
	+50	+1.14	+1.55	+0.68	+15.74	+1.14	+16.65
ω	−50	+0.13	+0.17	+0.13	−26.94	+0.13	−1.59
	−25	+0.07	+0.08	+0.06	−11.18	+0.07	−0.82
	+25	−0.06	−0.09	−0.07	+8.67	−0.06	+0.84
	+50	−0.13	−0.18	−0.13	+15.69	0.05	+1.69
z	−50	+0.026	+0.030	+0.023	+46.12	+0.027	+0.68
	−25	+0.010	+0.005	+0.007	+18.42	+0.011	+0.25
	+25	−0.004	−0.010	−0.007	−13.07	−0.004	−0.17
	+50	−0.007	−0.020	−0.009	−22.84	−0.008	−0.29
B_t	−50	–	–	–	–	−24.58	+7.62
	−25	–	–	–	–	−10.20	+3.81
	+25	–	–	–	–	+7.91	−3.81
	+50	–	–	–	–	+14.38	−7.62

Notes. "–" denotes no effect on sensitivity.

(ii) Instead of the assembled item, if one considered a single item and ignored investment for green technology and setup cost reduction, then the base of our study merges with Dey *et al.* [9].

(iii) Instead of the assembled product, if one considered a single deteriorating item means a single type of product that is deteriorating over time, *e.g.*, food product with green technology investment and preservation technology without autonomated inspection and constraints, then our model is quite similar to Sepehri *et al.* [37] and Yadav *et al.* [48].

6. SENSITIVITY ANALYSIS

The effect of change in the different parameters as −50%, −25%, +25%, and +50%, on total optimal cost, are shown in the Sensitivity analysis in Table 8 and graphically presented in Figure 4. From the sensitivity analysis Table 8, one can conclude as follows:

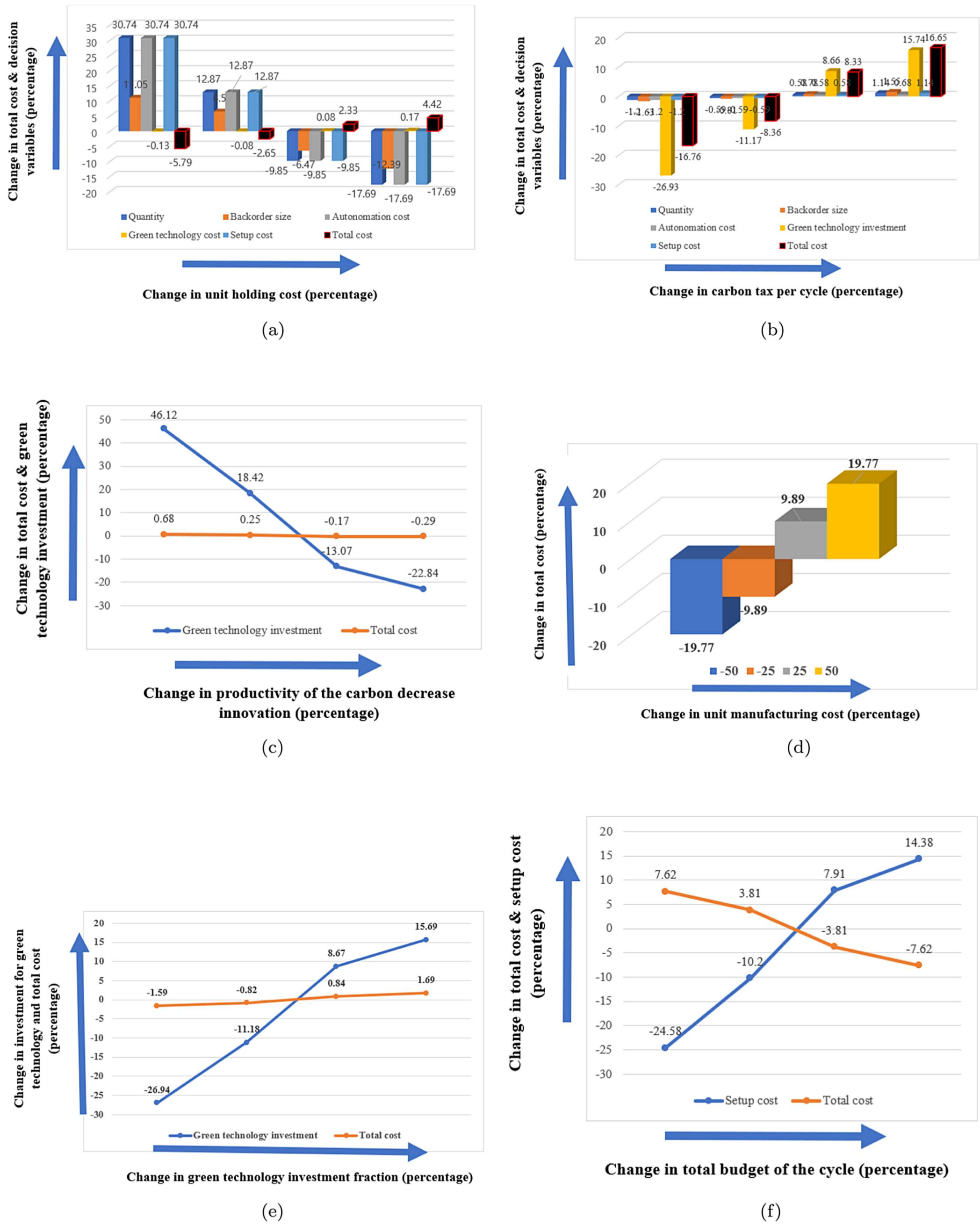


FIGURE 4. Effect of different parameters on decision variables and total cost. (a) Effect of holding cost on decision variable and total cost. (b) Effect carbon tax per cycle on decision variable and total cost. (c) Effect of productivity of the carbon decrease innovation on green technology investment and total cost. (d) Effect of unit manufacturing cost on total cost. (e) Effect of green technology investment fraction on green technology investment and total cost. (f) Effect of budget of the production industry on setup cost and total cost.

(i) The demand for the product always plays the most critical role in any industry. The cost of the production cycle is always directly proportional to the demand. The quantity and number of backordered items and green technology investment are also directly proportional to the demand. However, investment for autonomated inspection and setup cost reduction is inversely proportional to the demand, *i.e.*, autonomated inspection and investment for setup cost reduction in a small manufacturing house may be harmful.

(ii) The production rate of a production system is also directly proportional to the total system cost. Green technology investments and the number of backordered items are also directly proportional to the production rate. In contrast, ordered quantity, investment for autonomated inspection, and setup cost reduction are inversely proportional to the production rate. Since, in this study demand and production rate is considered fixed, whereas order quantity and investments are variable. Thus, the increase in production rate may increase the amount for the investments or order quantity. However, as all other parameters are fixed during the change of production rate, that's why production rate is inversely proportional to investments and order quantity. Otherwise, unit holding costs will increase which leads to higher system costs. Thus, order quantity and investment have to decrease along with the increased rate of production.

(iii) Unit manufacturing cost is directly proportional with the cost of the production cycle. The cost of the system varies within the range ± 19.77 for change in the parametric value of the manufacturing cost within the range $\pm 50\%$. However, unit manufacturing cost does not affect the decision variables.

(iv) The cost for holding the assembled product's spare parts is also slightly sensitive to the total production cycle cost. It is directly proportional to the total cost. It is quite obvious that an increment in unit holding cost is harmful to the total system cost.

(v) Carbon emission due to hybrid manufacturing and remanufacturing is too vital for any advanced production system to control carbon emissions. The total cost of the production cycle is directly proportional to the carbon emission due to manufacturing and remanufacturing. The total cost varies within the range of ± 12.83 for the increased carbon emission rate due to manufacturing and within the range ± 3.21 for remanufacturing. Moreover, carbon emission due to manufacturing and remanufacturing has a significant impact on green technology investment. If carbon emission is increased, it is apparent that investment in green technology will also be increased gradually.

7. Managerial insights

The manager of the industry can be made several essential decisions through this current study. Carbon which was emitted from the production industry is one of the tremendous problems nowadays. Those emitted carbon increase the global warming of the earth day by day. Thus, the reduction of carbon emissions is one of the essential issues for the industry. By the findings of this study, managers of the industry can decide how much green technology investment is required to control carbon emissions. Simultaneously, waste management in terms of defective items is another headache for the industry's manager. Before managing the waste, it is necessary to identify the waste correctly. An autonomated inspection policy is utilized in this study to identify defective items entirely without error. Thus, managers of the industry can decide on how much investment is appropriate for autonomated inspection technology. Moreover, an advanced setup is required to build an advanced production system that can reduce carbon emissions and manage waste smartly. The current study can decide how much investment can reduce the setup cost and build a sustainable production system. How many spare parts will be produced, and what amount of backorder items can provide the best result? managers can make that result. Finally, it can be shown by the numerical result of this study that green technology investment, autonomated inspection, investment for setup cost reduction along with budget and space constraints helps to make a sustainable single-stage manufacturing system for assembled products like automobile spare parts, and electronic gadgets.

8. Conclusion

Carbon emission and waste are two significant problems for production industries. The reduction of carbon emissions for production industries gained significant attention from the researcher in those days. An SSP system for assembled products is formulated under green technology investment to control carbon emissions. Moreover, managing waste in the production industry is one crucial task for industry managers. Due to imperfection in the process, faulty products were produced randomly, which resulted in a huge amount of industrial waste in terms of the defective product. The task for the managers was to identify those faulty products accurately and properly manage them. Remanufacturing those faulty products is the best solution for managing those waste, which is the reason to consider manufacturing and remanufacturing in the same production cycle. Machine-based automated inspection by the computerized numerical control (CNC) machines, which is autonomated inspection technology, is used to clean the system from defectiveness and manage waste properly.

Additionally, a tremendous amount of setup cost is required to build up a "hybrid manufacturing and remanufacturing" system to produce various parts of an assembled product along with controllable carbon emission and waste management. This study utilized an investment to control or reduce setup-related costs, which makes a cost-effective production system. Simultaneously, the application of space and budget constraints makes the model more realistic.

Provided numerical experiments prove that green technology investment significantly impacts controlling carbon emission and autonomated inspection helps manage the production system's waste accurately. From numerical results, one can claim that the cost of the entire production cycle was optimized under chi-square distributed defective rates.

Fixed demand and the production rate for assembled product production are two limitations of this model. In the near future, one can consider variable demand and variable production to extend this study [26]. Instead of single-stage production, one can utilize the multi-stage production system and investment to improve process quality. Using the concept of the supply chain, one can extend the current study to multiple players [32].

Appendix A. Value of T_1–T_5, and Ω_1–Ω_9

The values of T_1, T_2, T_3, T_4, and T_5 are as follows

$$T_1 = \frac{\beta_x}{(R_m D_x(1 - E[\zeta_i]) - D_x)};\quad T_2 = \frac{\Upsilon_x}{R_m D_x} - \frac{\beta_x}{(R_m D_x(1 - E[\nu_x]) - D_x)}$$

$$T_3 = \frac{Bs_m}{R_m D_x} E[\nu_x];\quad T_4 = \frac{\Upsilon_x \left[1 - (1 + E[\nu_x]) \frac{D_x}{R_m D_x}\right] - \beta_x}{D_x};\quad T_5 = \frac{\beta_x}{D_x}$$

$$\Omega_1 = \xi_{c_x} \epsilon_{ch} \left(1 - \omega_x \left(1 - e^{-z_x \Gamma_x}\right)\right); \Omega_2 = \xi_{c_x} \epsilon P_{cs} \left(1 - \omega_x \left(1 - e^{-z_x \Gamma_x}\right)\right)$$

$$\Omega_3 = (\Omega_2 + Y_x + F_{bc_x} \beta_x + I_{A_x} + R_{P_x} E[\nu_x] C_{R_x}); \Omega_4 = \Omega_1 + H_{C_x} + C_{lb_x};$$

$$\Omega_5 = \frac{\beta_x^2 (1 - E[\nu_x])}{2\left[1 + E[\nu_x] - \frac{D_x}{R_{P_x}}\right]}; \Omega_6 = \frac{(\Omega_1 + H_{C_x})\left[1 - \left(1 + E[\nu_x] + (E[\nu_x])^2\right) \frac{D_x}{R_{P_x}}\right]}{2};$$

$$\Omega_7 = (\xi_{c_x} \epsilon P_{cs} + Y_x + F_{bc_x} \beta_x + I_{A_x} + R_{P_x} E[\nu_x] C_{R_x}); \Omega_8 = \xi_{c_x} \epsilon_{ch} + H_{C_x} + C_{lb_x}$$

$$\Omega_9 = \frac{(\xi_{c_x} \epsilon_{ch} + H_{C_x})\left[1 - \left(1 + E[\nu_x] + (E[\nu_x])^2\right) \frac{D_x}{R_{P_x}}\right]}{2}.$$

Appendix B. Value of constraint λ_1

The calculation of constraint λ_1 is as follows:

$$\lambda_1\left(\sum_{x=1}^{n} K_x\left[\frac{\Omega_4\Omega_5+D_x\Omega_3}{\Omega_6+\lambda_1 K_x+\lambda_2 U_x}\right]^{\frac{1}{2}}\right)=\lambda_1 B_t$$

$$\Rightarrow \sum_{x=1}^{n} K_x\left[\frac{\Omega_4\Omega_5+D_x\Omega_3}{\Omega_6+\lambda_1 K_x+\lambda_2 U_x}\right]^{\frac{1}{2}}=B_t$$

$$\Rightarrow \sum_{x=1}^{n}\left[\frac{\Omega_4\Omega_5+D_x\Omega_3}{\Omega_6+\lambda_1 K_x+\lambda_2 U_x}\right]=\sum_{x=1}^{n}\frac{B_t^2}{{K_x}^2}$$

$$\Rightarrow \sum_{x=1}^{n}(\Omega_6+\lambda_1 K_x+\lambda_2 U_x)=\sum_{x=1}^{n}\frac{{K_x}^2}{B_t^2}(\Omega_4\Omega_5+D_x\Omega_3)$$

$$\Rightarrow \sum_{x=1}^{n}\lambda_1 K_x=\sum_{x=1}^{n}\frac{{K_x}^2}{B^2}(\Omega_4\Omega_5+D_x\Omega_3)-\sum_{x=1}^{n}(\Omega_6+\lambda_2 U_x)$$

$$\Rightarrow \lambda_1=\sum_{x=1}^{n}\frac{K_x}{B_t^2}(\Omega_4\Omega_5+D_x\Omega_3)-\sum_{x=1}^{n}\frac{(\Omega_6+\lambda_2 U_x)}{K_x}\cdot$$

Appendix C. Value of constraint λ_2

The calculation of constraint λ_2 is as follows:

$$\lambda_2\left(\sum_{x=1}^{n} U_x\left[\frac{\Omega_4\Omega_5+D_x\Omega_3}{\Omega_6+\lambda_1 K_x+\lambda_2 U_x}\right]^{\frac{1}{2}}-S\right)=0;\quad \Rightarrow \sum_{x=1}^{n}\frac{\Omega_4\Omega_5+D_x\Omega_3}{\Omega_6+\lambda_1 K_x+\lambda_2 U_x}=\sum_{x=1}^{n}\frac{U_x^2}{S^2}$$

$$\Rightarrow \sum_{x=1}^{n}\left[\frac{S^2}{U_x^2}(\Omega_6+\lambda_1 K_x+\lambda_2 U_x)\right]=\sum_{x=1}^{n}(\Omega_4\Omega_5+D_x\Omega_3)$$

$$\Rightarrow \lambda_2=\sum_{x=1}^{n}\frac{U_x}{S^2}(\Omega_4\Omega_5+D_x\Omega_3)-\sum_{x=1}^{n}\frac{(\Omega_6+\lambda_1 K_x)}{U_x}\cdot$$

Appendix D. Proof of convexity of the total cost function

To prove the sufficient condition of convexity of the cost function, one has to find the value of the principal minors of the following Hessian matrix

$$\det(H_{55})=\det\begin{pmatrix}
\frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\beta_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\\
\frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\beta_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}\\
\frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\beta_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\beta_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\beta_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial\beta_x}\\
\frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial Y_x}\\
\frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x^2}
\end{pmatrix}$$

$$\frac{\partial^2\mathrm{TC}}{\partial{\Upsilon_x}^2}=\frac{2}{{\Upsilon_x}^3}\left[\Omega_3 D_x+\frac{(C_{lb_x}+H_{C_x}){\beta_x}^2(1-E[\nu_x])}{2\left[1-E[\nu_x]-\frac{D_x}{R_{P_x}}\right]}+2\Omega_1\Omega_5\right]>0$$

$$\frac{\partial^2 \mathrm{TC}}{\partial {\beta_x}^2} = \frac{\Omega_4(1-E[\nu_x])}{\Upsilon_x\left[1-E[\nu_x]-\frac{D_x}{R_{P_x}}\right]} > 0$$

$$\frac{\partial^2 \mathrm{TC}}{\partial {\Gamma_x}^2} = e^{-\Gamma_x z_x}\omega_x z_x^2\left(\frac{\Omega_5}{\Upsilon_x}+\frac{\Upsilon_x\xi_{c_x}\epsilon_{ch}}{2}\left(1-\frac{\left(1+E[\nu_x]+(E[\nu_x])^2\right)D_x}{R_{P_x}}\right)-\beta_x^2\xi_{c_x}\epsilon_{ch}\right.$$
$$\left.+\frac{D_x}{\Upsilon_x}\xi_{c_x}(\epsilon_{P_{cs}})+\xi_{c_x}\epsilon_{P_x}D_x+\xi_{c_x}\epsilon_{r_x}R_{P_x}(E[\nu_x])^2D_x\right) > 0$$

$$\frac{\partial^2 \mathrm{TC}}{\partial {I_{A_x}}^2} = \frac{\delta}{I_{A_x}^2} > 0; \quad \frac{\partial^2 \mathrm{TC}}{\partial {Y_x}^2} = \frac{I_{sr}}{Y_x^2} > 0$$

$$\frac{\partial^2 \mathrm{TC}}{\partial I_{A_x}\partial\Upsilon_x} = \frac{\partial^2 \mathrm{TC}}{\partial\Upsilon_x\partial I_{A_x}} = \frac{\partial^2 \mathrm{TC}}{\partial Y_x\partial\Upsilon_x} = \frac{\partial^2 \mathrm{TC}}{\partial\Upsilon_x\partial Y_x} = -\frac{D_x}{\Upsilon_x^2}$$

$$\frac{\partial^2 \mathrm{TC}}{\partial I_{A_x}\partial\beta_x} = \frac{\partial^2 \mathrm{TC}}{\partial J_m\partial\beta_x} = \frac{\partial^2 \mathrm{TC}}{\partial I_{A_x}\partial J_m} = \frac{\partial^2 \mathrm{TC}}{\partial I_{A_x}\partial\Gamma_x} = \frac{\partial^2 \mathrm{TC}}{\partial Y_x\partial\Gamma_x} = 0$$

$$\frac{\partial^2 \mathrm{TC}}{\partial\Upsilon_x\partial\Gamma_x} = -e^{-z_x\Gamma_x}z_x\omega_x\left(\frac{\Omega_5}{\Upsilon_x^2}+\frac{\xi_{c_x}\epsilon_{ch}}{2}\left(1-\frac{\left(1+E[\nu_x]+(E[\nu_x])^2\right)D_x}{R_{P_x}}\right)-\frac{D_x}{\Upsilon_x^2}\xi_{c_x}(\epsilon_{P_{cs}})\right)$$

$$\frac{\partial^2 \mathrm{TC}}{\partial\beta_x\partial\Gamma_x} = -e^{-z_x\Gamma_x}z_x\omega_x\xi_{c_x}\epsilon_{ch}\left(\frac{2\Omega_5}{\Upsilon_x\beta_x}-1\right)$$

$$\frac{\partial^2 \mathrm{TC}}{\partial\Upsilon_x\beta_x} = -\left[\frac{F_{bc_x}D_x}{{\Upsilon_x}^2}+\frac{\Omega_4\beta_x(1-E[\nu_x])}{\Upsilon_x^2\left[1-E[\nu_x]-\frac{D_x}{R_{P_x}}\right]}\right].$$

Now, using the optimum value of I_{A_x}, Y_x, Γ_x, Υ_x and β_x, one can easily find that the first order principal minor of the hessian matrix that is $\frac{\partial^2\mathrm{TC}}{\partial I_{A_x}^2}$, $\frac{\partial^2\mathrm{TC}}{\partial {Y_x}^2}$, $\frac{\partial^2\mathrm{TC}}{\partial G_m^2}$, $\frac{\partial^2\mathrm{TC}}{\partial {\Upsilon_x}^2}$, or $\frac{\partial^2\mathrm{TC}}{\partial {\beta_x}^2}$ are greater than zero.

One can calculate the "second-order principal minor of the Hessian matrix" as

$$\det(H_{22}) = \det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2}\end{pmatrix} = \frac{\partial^2\mathrm{TC}}{\partial I_{A_x}^2}\frac{\partial^2\mathrm{TC}}{\partial {Y_x}^2}-\left(\frac{\partial^2\mathrm{TC}}{\partial Y_x I_{A_x}}\right)^2$$
$$= \frac{I_{sr}\delta}{Y_x^2 I_{A_x}^2}-\frac{D_x^2}{B_{sm}^4}$$

which is greater than zero if $I_{sr}\delta > D_x^2$ and $Y_x^2 I_{A_x}^2 < B_{sm}^4$.

Using the value of I_{A_x}, Υ_x and β_x, one can be established that provided "second-order principal minor" is greater than zero.

Similarly, to satisfy the sufficient condition of convexity, it is necessary to prove that the principal minor of third order is larger than zero, *i.e.*,

$$\det(H_{33}) = \det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}\end{pmatrix} = \frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}\end{pmatrix}$$
$$-\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}\end{pmatrix}+\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\end{pmatrix}.$$

We already prove that principal minors of orders one and two are positive and $\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x}=0$, $\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}=0$, $\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}=0$. Therefore,

$$\det(H_{33})=\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}\end{pmatrix}=\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}\left[\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2}\frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}-\left(\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x}\right)^2\right]$$

$$=\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}\times\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2}\times\frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2}>0.$$

The fourth-order principal minor is

$$\det(H_{44})=\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2}\end{pmatrix}$$

$$=\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2}\end{pmatrix}-\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2}\end{pmatrix}$$

$$+\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2}\end{pmatrix}-\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial G_m}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x^2}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\end{pmatrix}.$$

We already proved that all principal minor of order three is greater than zero and $\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2}>0$, $\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x}=0$, $\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x}=0$, $\frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x}=-\frac{D_x}{\Upsilon_x^2}<0$. Then the fourth order principal minor can be rewritten as

$$\det(H_{44})=\frac{\delta}{I_{A_x}^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial G_m\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x^2}\end{pmatrix}+\frac{D_x}{\Upsilon_x^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial G_m}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x^2}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x}\end{pmatrix}>0.$$

As all principal minors of order three are greater than zero, from the above calculation, one can conclude that the principal minors of order four of the Hessian matrix are greater than zero.

Now, to prove the sufficient condition of convexity, it needs to be proved that the principal minor of order five of the Hessian matrix is greater than zero.

Now, the principal minor of order five is given by

$$\det(H_{55})=\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial A_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial G_m} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial I_{A_x}}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial B^2 s_m} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial\Upsilon_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x^2}\end{pmatrix}$$

$$= \frac{\partial^2 \mathrm{TC}(.)}{\partial A_m^2} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial B^2 s_m} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x^2} \end{pmatrix}$$

$$- \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial Y_x} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial B^2 s_m} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x^2} \end{pmatrix}$$

$$+ \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial B^2 s_m} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x^2} \end{pmatrix}$$

$$- \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Upsilon_x} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x^2} \end{pmatrix}$$

$$+ \frac{\partial \mathrm{TC}(.)}{\partial I_{A_x} \partial \beta_x} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \Upsilon_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial B^2 s_m} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial I_{A_x}} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Upsilon_x} \end{pmatrix}.$$

From the discussion of fourth order principal minor, it is clear that all principal minor of order four is greater than zero, and it is already provided that $\frac{\partial^2 \mathrm{TC}(.)}{\partial A_m^2} = \frac{\delta}{I_{A_x}^2} > 0$, $\frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial Y_x} = 0$, $\frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Gamma_x} = 0$, $\frac{\partial^2 \mathrm{TC}(.)}{\partial I_{A_x} \partial \Upsilon_x} = -\frac{D_x}{\Upsilon_x^2} < 0$, $\frac{\partial \mathrm{TC}(.)}{\partial I_{A_x} \partial \beta_x} = 0$. Then the fifth order principal minor can be rewritten as

$$\det(H_{55}) = \frac{\delta}{I_{A_x}^2} \times \det \begin{pmatrix} \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial Y_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x^2} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Gamma_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial B^2 s_m} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \Upsilon_x \partial \beta_x} \\ \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial Y_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Gamma_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x \partial \Upsilon_x} & \frac{\partial^2 \mathrm{TC}(.)}{\partial \beta_x^2} \end{pmatrix}$$

$$+\frac{D_x}{\Upsilon_x^2}\times\det\begin{pmatrix}\frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_m^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial I_{A_x}\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial Y_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x^2} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Gamma_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\Upsilon_x\partial\beta_x}\\ \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial I_{A_x}} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial Y_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x\partial\Gamma_x} & \frac{\partial^2\mathrm{TC}(.)}{\partial\beta_x^2}\end{pmatrix}>0.$$

The above condition is always true because of the optimal value of the decision variables I_{A_x}, Y_x, Γ_x, Υ_x and β_x. Therefore, principal minors of order five are positive.

From the above discussion, it is clear that the value of the principal minors of the Hessian matrix are all positive, which indicates that the total cost function is positive, convex, and provides a global optimal result.

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