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EFFECT OF CREDIT FINANCING ON THE SUPPLY CHAIN FOR IMPERFECT GROWING ITEMS

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Abstract. Several EOQ models have developed over time to guarantee that the appropriate quantity of inventory is ordered in every batch, so that a firm does not have to place orders too frequently or have an excess of inventory on hand. Inventory management, or the inspection of the ordering, storing, and utilization of a company's inventory, necessitates the use of EOQ models. This paper aims at developing an EOQ model when the supplier offers the trade credit policy to the buyer, for a particular class of items that is, growing items. However, it is not always necessary for all goods to be of perfect quality. There may also be some defective goods. Keeping this in mind, a wide-ranging scientific model has been intended, followed by a specific numerical model which is represented with the help of numerical examples. Sensitivity research is presented to assess the influence of the model's key factors taking into consideration it's decision variables and the objective function.

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1. Introduction

The groundwork for the inventory management was laid by Harris in the year 1913 [16] when he developed the first Economic Order Quantity (EOQ) model. Since then, several EOQ models have been developed to make sure that the appropriate quantity of inventory is ordered to avoid frequent placing of orders or excess of inventory. However, certain assumptions were made in the classic EOQ model which were relaxed by various researchers who extended the model. For example, EOQ models were developed considering consumable merchandises like food, vegetables [9, 12, 24, 26]. Several EOQ models were also developed for electronic products [28, 30, 32, 37]. There has been a significant research on items which are repairable, for example, military products [21, 31] and products that are reusable, for example, soft drink bottles [6, 20]. Perera *et al.* [27] analysed the literature on inventory ordering decisions using behavioural experiments. Glock and Grosse [14] presented a systematic review of inventory models with controllable production rates. Gautam *et al.* [13] presented an extensive literature review, based on the papers that considered the effect of imperfect quality items in inventory replenishment or production models. Mosca *et al.* [23] reviewed Integrated Transportation-Inventory (ITI) models developed over the last two decades for various supply chain configurations.

Keywords. Growing items, imperfect quality, poultry, credit financing.

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Considering growing items, which have the ability to grow with time, various EOQ models were developed [29]. The work was further extended by Zhang *et al.* [39] who integrated environmental sustainability to Rezaei's work in a carbon controlled condition. While calculating the progress of the growing items, Nobil *et al.* [25] extended the model by relaxing the assumption that shortages are not tolerated. Huang [17] developed a model considering both the vendor and the buyer taking into consideration items which were not of perfect quality. Chang [3] studied a model assuming the rate of demand and the percentage of poor quality items to be fuzzy variables. Jaber *et al.* [18] created a model for inventory subject to the effects of learning considering imperfect quality items. Chung *et al.* [10] created a model for inventory considering two-warehouses for imperfect quality items. Chang and Ho [4] built a model for items with imperfect quality where shortages were allowed without applying the concept of differential calculus. Chen and Kang [5] created a model considering both the vendor and buyer for imperfect quality items under allowable deferment in payments. Wang *et al.* [36] created a model for imperfect quality items considering partial backorders wherein the process of screening was constrained. There has been very less research considering such items which necessitates the development of more models. De-la-Cruz-Márquez *et al.* [11] proposed an inventory model for growing items with imperfect quality when the demand is price sensitive under carbon emissions and shortages. Yadav *et al.* [38] proposed a model with imperfect quality items when end demand is sensitive to price and marketing expenditure. Jaggi *et al.* [19] developed a model with credit financing for deteriorating imperfect-quality items under inflationary conditions.

Several EOQ models have been developed considering the trade credit policy [2, 7, 8, 15, 35]. A seller-buyer model with trade credit was developed by Abad and Jaggi [1] taking into consideration cooperative and non-cooperative relationships. Also, a vendor-buyer model for freight cost with trade credit policy was presented by Sheen and Tsao [34]. Mittal and Sharma [22] presented an EOQ model for growing items with trade credit policy taking the growth function to be logistic.

However, the researchers overlooked the impact of some aspects such as restricted time periods and poor quality of growing items. With this in mind, the inventory model has been developed with trade credit and an unlimited time period to control the optimal inventory cycle duration and optimum order quantity that needs to be ordered.

In this paper, a supplier-buyer model has been established in which the supplier's total profit is measured by taking into account growing goods, including items of poor quality, in the presence of trade-credit financing. The purpose of this study is to analyze the behavior of growing items with imperfect quality while they are being considered for sale in the market. Also, it will show that how defective goods and trade-credit affect the net profit of the manufacturer and the optimum quantity?

2. Assumptions and notations

2.1. Firstly, the notations for the anticipated model are incorporated

See Table 1.

2.2. Assumptions

The following asssumptions are made while the model is being derived.

- The annual demand for the item remains constant throughout time, and shortages are not allowed.
- The time period is considered to be infinite.
- The revenue generated from the sales is deposited in an account from which interest is received, till the time the account is unsettled. After the account is settled, the interest charges are paid on the items that are in stock [15].
- The ordered items are capable of growing, prior to being slaughtered. The cost of feeding the items is proportional to the weight gained by the items.
- Holding costs are incurred for the duration of the consumption period.
- A random fraction of the slaughtered items is of poorer quality.

TABLE 1. Notations used when deriving the mathematical model.

Parameters	
w_0	New-born item's weight (g)
w_t	Item's weight at time t (g)
D	Annual demand rate of item (g/year)
Q_t	Inventory's total weight at time t (g)
A	Asymptotic weight (g) (for exponential growth function)
b	Integration constant (for exponential growth function)
I_e	Interest earned
I_p	Interest paid
y	Fraction of poor-quality slaughtered items
λ	Exponential growth rate per unit time (for exponential growth function)
γ	Linear growth rate per weight unit per unit time (for linear growth function) (g/year)
δ_j	Linear growth rate in growth region j (for split linear growth function) (g/year)
w_1'	Supremum on weight in the first growth region (for split linear growth function) (g)
t_1'	Supremum on growth period duration in the first growth region (for split linear growth function) (years)
w_1''	Supremum on weight in the second growth region (for split linear growth function) (g)
t_1''	Supremum on growth period duration in the second growth region (for split linear growth function) (years)
Decision variables	
p	Number of new-born items ordered per cycle
Cost components	
P	Cost of purchasing a single unit item (ZAR/g)
S	Cost of selling a single unit good quality item (ZAR/g)
V	Cost of selling a single unit poor quality item (ZAR/g)
C	Cost of feeding a single unit item (ZAR/g/year)
H	Cost of holding a single unit item (ZAR/g/year)
K	Cost of setting up per growing cycle (ZAR/g/year)
Z	Cost of screening of each unit (ZAR/g)
r	Rate of screening (g/min)
Time periods	
T	Duration of a cycle (years)
t_1	Duration of the growing period (years)
t_2	Time of screening (years)
t_3	Time period of selling the items (years)
t_s	Time of setup (years)

- The screening process is considered to be 100 percent effective. All poorer quality items are sold in the market as a single batch [33].
- The selling price of good quality items is greater than that of the poorer quality items.
- There is no rework or replacement of poorer quality items.

3. Model definition

3.1. General model

A firm buys $\boldsymbol{p}$ new-born items at the beginning of the growth cycle, which can grow with time. Initially, each new born item weighs w_0 which gives the total weight of the inventory as $Q_0 = pw_0$. After feeding the items during the time period t_1, the items reach the target weight of w_1, which gives the total weight of the inventory as $Q_1 = pw_1$. A screening process is conducted to separate the good quality items from the defective quality items during the period t_2 where r is the rate of screening. The fraction of the items that are slaughtered and are of poor quality is taken to be y with distribution $g(y)$ and the expectation $\mathrm{E}[y]$. The poor-quality items are sold in the market at a discounted price during the time period t_3. The total time period of a cycle is taken as T that is, $T = t_1 + t_2 + t_3$. Now, the optimal quantity of the items to be ordered in the beginning and the ideal day of slaughter needs to be regulated by the firm. The overall profit is maximized after applying the trade credit policy considering the various cases on the basis of time period that the seller provides, so that the ideal values of the decision variables can be calculated.

The difference of the total revenue and total costs gives the total profit function. Various costs are included in the total cost like the cost of purchasing the new-born items, the cost of feeding the items, the cost of setting up the required equipments, the cost of screening the items and holding costs after the items are slaughtered. The total profit function is given as:

Hence, the expected value of the total profit will be equal to,

$$\begin{aligned}\text{Total Profit (TP)} &= \text{Total Revenue (TR)} - [\text{Purchasing costs (PC)} + \text{Feeding costs (FC)} + \text{Holding costs (HC)} \\ &\quad + \text{Screening costs (ZC)} + \text{Setup costs (SC)}] \\ \mathrm{E}[\mathrm{TP}] &= \mathrm{E}[\mathrm{TR}] - \mathrm{PC} - \mathrm{FC} - \mathrm{E}[\mathrm{HC}] - \mathrm{ZC} - \mathrm{SC}.\end{aligned}$$

Considering the notations for the anticipated model, the costs are calculated as:

$$\mathrm{E}[\mathrm{TR}] = Spw_1, \quad \mathrm{PC} = Ppw_0, \quad \mathrm{SC} = K, \quad \mathrm{ZC} = Zpw_1.$$

Taking the average weight of the items, that is $\left(\frac{pw_1}{2}\right)$,

$$\mathrm{E}[\mathrm{HC}] = H\left[\frac{p^2w_1^2(1-\mathrm{E}[y])^2}{2D} + \frac{p^2w_1^2\mathrm{E}[y]}{r}\right]$$

where t_2 is the holding time period of the items after they are slaughtered.

To calculate FC, the growth function w_t is considered which gives

$$\mathrm{FC} = Cp\int_0^{t_1} w_t\,\mathrm{d}t$$

where t_1 is the feeding time period of the items.

The necessary values are substituted to find the total profit function. That is,

$$\begin{aligned}\mathrm{E}[\mathrm{TP}] = Spw_1(1-\mathrm{E}[y]) + Vpw_1\mathrm{E}[y] - Ppw_0 - Cp\int_0^{t_1} w_t\,\mathrm{d}t - H\Bigg[\frac{p^2w_1^2(1-\mathrm{E}[y])^2}{2D} \\ + \frac{p^2w_1^2\mathrm{E}[y]}{r}\Bigg] - K - Zpw_1. \qquad (3.1)\end{aligned}$$

3.2. Investigating different growth functions

In order to solve equation (3.1), the feeding function is required for growing items which is different for different items. The quantity of feed stock that the items consume is assumed to be proportional to their weight to develop a comprehensive equation for E[TP]. To calculate the feeding cost, the function of growth is necessary for the items. There are three main growth functions that are taken into account. The logistic function of growth is the first. The linear function of growth is the second and split linear function of growth is the third function [33].

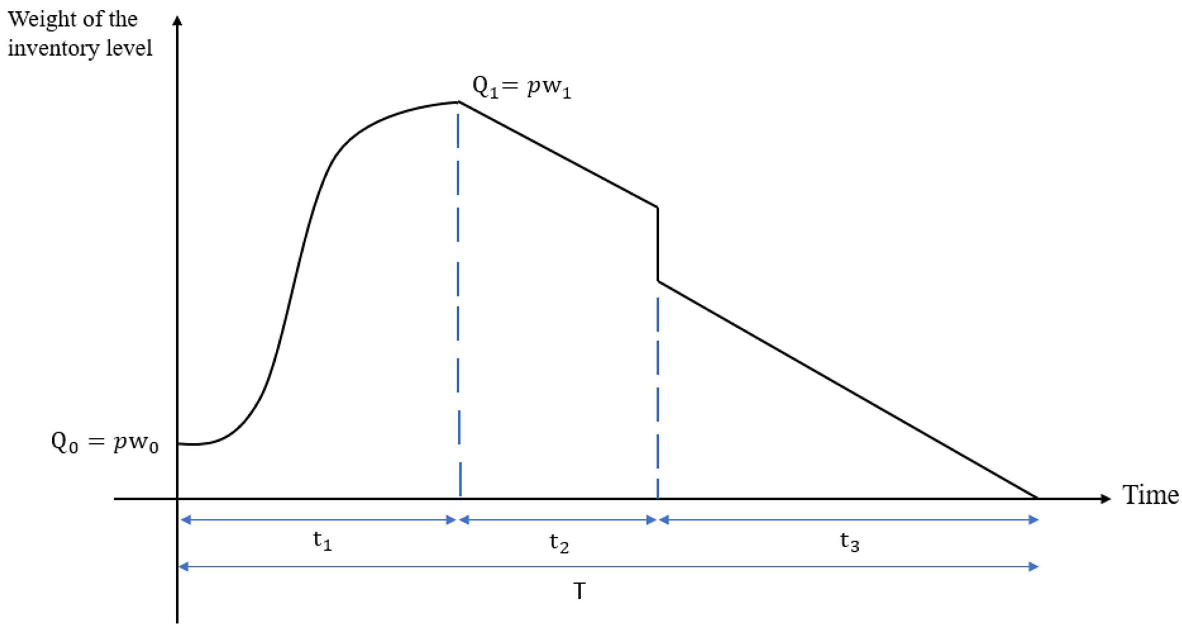

FIGURE 1. Inventory system behaviour with a logistic growth function.

3.2.1. Growth function I (Logistic)

The weight increases slowly at first and gradually increases over time, until the items reach maturity, after which the weight gain slows down.

Initially, the level of inventory is $Q_0(= pw_0)$ but as t_1 ends, it rises to $Q_1(= pw_1)$ since the inventory weight increases after they are fed. Then, the items are screened during the time period t_2.

The function of growth of the items [33] is,

$$w_t = \frac{A}{1 + be^{-\lambda t}}. \tag{3.2}$$

After solving it,

$$t_1 = -\log\left(\frac{1}{b}\left(\frac{A}{w_1} - 1\right)\right)/\lambda. \tag{3.3}$$

Hence, the cost of feeding will be (from Fig. 1)

$$\text{FC} = C\int_0^{t_1} pw_t\,\text{d}t = Cp\int_0^{t_1} \frac{A}{1 + be^{-\lambda t}}\,\text{d}t = Cp\left[At_1 + \frac{A}{\lambda}\left\{\log\left(1 + be^{-\lambda t_1}\right) - \log(1 + b)\right\}\right]. \tag{3.4}$$

3.2.2. Growth function II (Linear)

The weight of each item increases at a constant rate of γ weight units per unit time. Initially, each newborn item weighs w_0. Hence, the growth function w_t is a linear function with γ as the gradient and w_0 as the x-intercept.

The growth function [33] is,

$$w_t = w_0 + \gamma t. \tag{3.5}$$

Figure 2 illustrates the behaviour of inventory when the growth function is taken as linear.

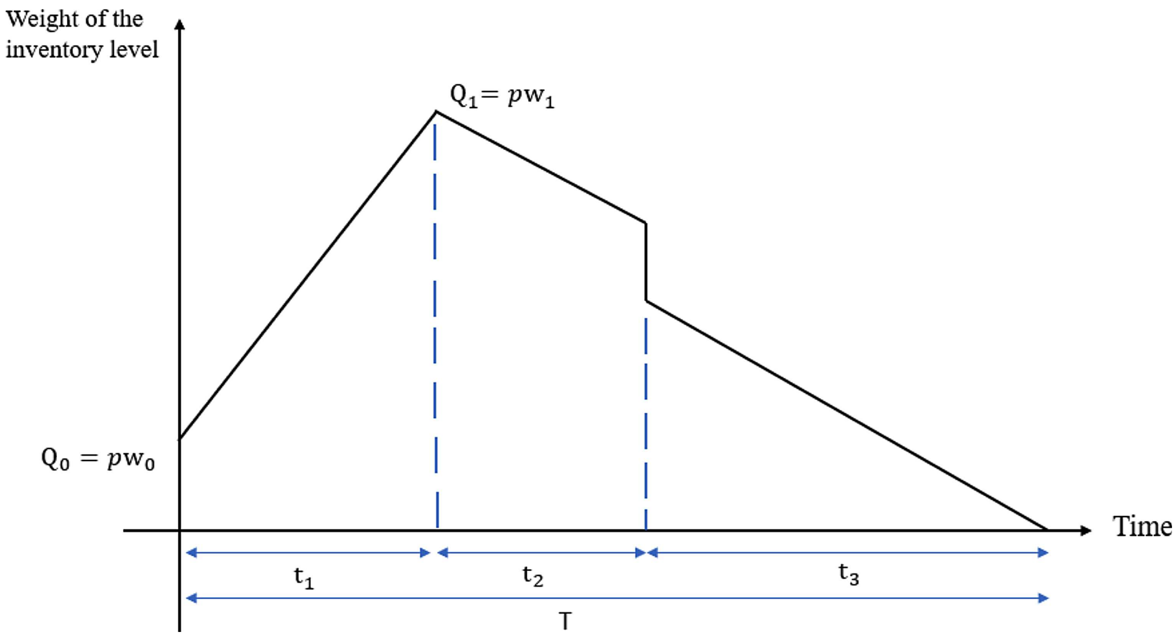

FIGURE 2. Inventory system behaviour with a linear growth function.

At time of slaughter, equation (3.5) becomes

$$w_1 = w_0 + \gamma t_1. \tag{3.6}$$

Hence, the slaughter age is

$$t_1 = \frac{w_1 - w_0}{\gamma}\cdot \tag{3.7}$$

The cost of feeding will be

$$\mathrm{FC} = C\left[\frac{t_1(yw_1 - yw_0)}{2}\right] = Cy\left[\frac{(w_1 - w_0)^2}{2\gamma}\right]\cdot \tag{3.8}$$

3.2.3. Growth function III (Split linear)

The growth function has been divided into 3 parts, slow growth (at the start of a growing cycle), quick growth (in the middle period), and slow growth (at the end of a growing cycle) (Fig. 3).

The slaughter weight [33] is

$$w_1 = w_1' + \delta_2(t_1 - t_1'). \tag{3.9}$$

Hence, the slaughter age will be

$$t_1 = t_1' + \left(\frac{w_1 - w_1'}{\delta_2}\right)\cdot \tag{3.10}$$

The feeding cost per cycle will be

$$\mathrm{FC} = Cy\left[\frac{(w_1' - w_0)^2}{2\delta_1} + \frac{(w_1 - w_1')^2}{2\delta_2} + \frac{(w_1 - w_1')(w_1' - w_0)}{\delta_2}\right]\cdot \tag{3.11}$$

The expected total profit function is formulated using the findings of the logistic growth function. In the numerical example section, the findings of the other two models will be used so that the effectiveness of various models can be compared.

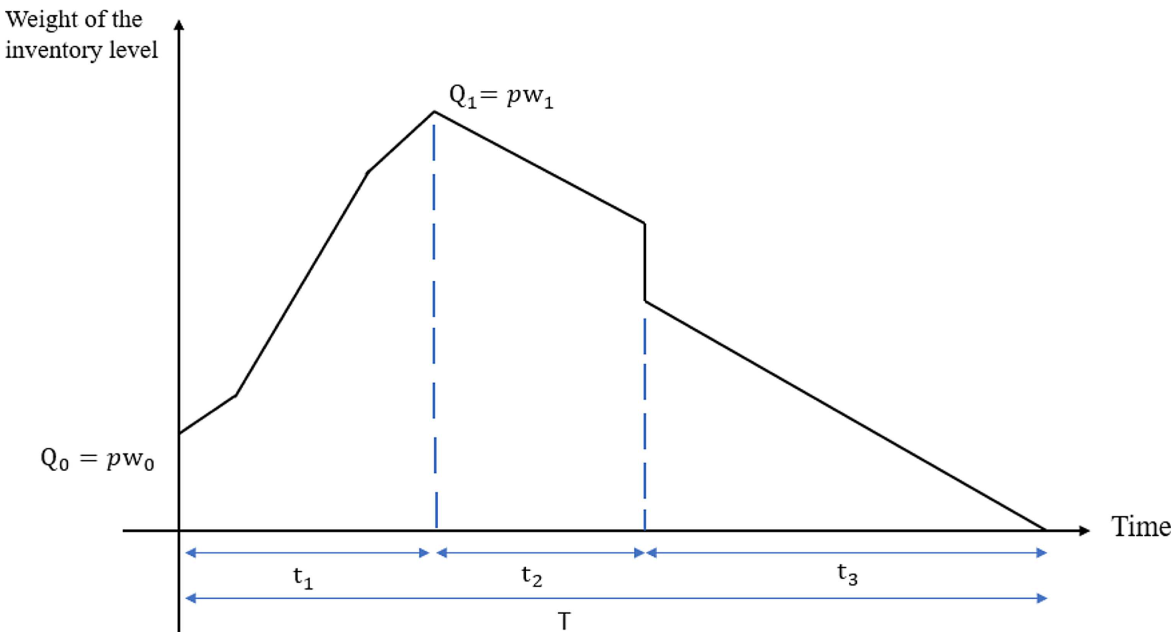

FIGURE 3. Inventory system behaviour with a split linear growth function.

3.3. Model with logistic growth function

3.3.1. Expected total profit function

Taking $\mathrm{E}[T] = \frac{pw_1(1-\mathrm{E}[y])}{D}$, the expected total profit function per unit time E[TPU] using equation (3.1) will be,

$$\mathrm{E[TPU]} = \frac{\mathrm{E[TP]}}{\mathrm{E}[T]} = SD + \frac{VD\mathrm{E}[y]}{1-\mathrm{E}[y]} - \frac{PDw_0}{w_1(1-\mathrm{E}[y])} - \frac{CD}{w_1(1-\mathrm{E}[y])}\left\{At_1 + \frac{A}{\lambda}\left\{\log\left(1+be^{-\lambda t_1}\right) - \log(1+b)\right\}\right\} - H\left[\frac{pw_1(1-\mathrm{E}[y])}{2} + \frac{Dpw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \frac{KD}{pw_1(1-\mathrm{E}[y])} - \frac{ZD}{(1-\mathrm{E}[y])}. \tag{3.12}$$

3.3.2. Model constraints [33]

To assure that the proposed inventory system is feasible, two fundamental constraints are required. The first constraint is to ensure that the products are available to consume at the specified time, while the second is to avoid shortages during the screening period.

Constraint 1. $t_1 + t_s \leq \mathrm{E}[T]$.

From equation (3.3),

$$\mathrm{E}[T] \geq \left\{-\frac{\log\left(\frac{1}{b}\left(\frac{A}{w_1}-1\right)\right)}{\lambda} + t_s = T_{\min}\right\}. \tag{3.13}$$

Constraint 2. Consider the difference between the weight of good quality slaughtered items and weight of poor-quality slaughtered items per cycle to be $N(pw_1, \mathrm{E}(y))$. Hence,

$$N(pw_1, \mathrm{E}(y)) = pw_1 - \mathrm{E}(y)pw_1 = (1-\mathrm{E}(y))pw_1.$$

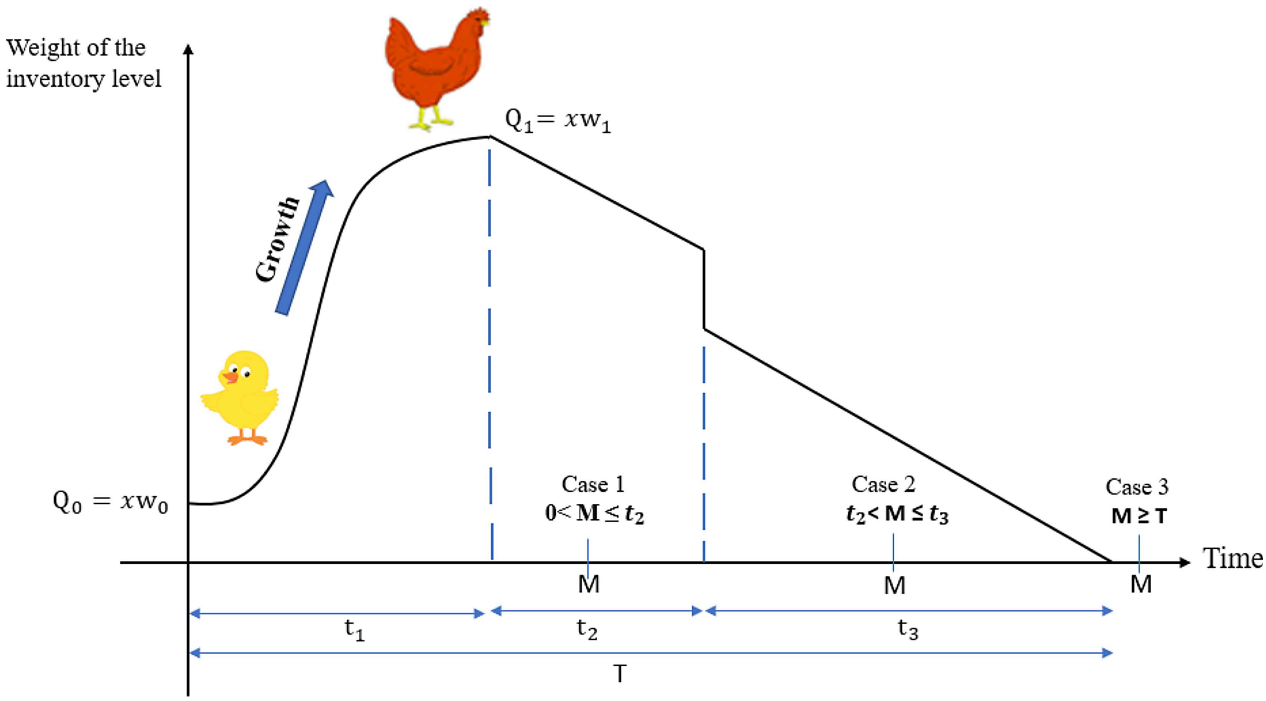

FIGURE 4. Inventory system with imperfect growing items under credit financing.

Since shortages are not allowed, hence the number of good quality items available throughout the screening period (t_2) must be at least equivalent to the demand. Thus,

$$N(pw_1, \mathrm{E}(y)) \geq Dt_2$$

which implies that

$$\mathrm{E}(p) \leq \left\{1 - \frac{D}{r} = x_{\mathrm{res}}\right\}. \tag{3.14}$$

In the next section, the effect of credit financing is shown on E[TPU] to calculate the overall profit of the supplier.

Effect of credit financing

Figure 4 shows how growing items including those with imperfect quality develop over time and the three cases under the existence of trade-credit are reflected. Hence,

$$\mathrm{E[TP]} = \mathrm{E[TR]} - \mathrm{PC} - \mathrm{FC} - \mathrm{E[HC]} - \mathrm{ZC} - \mathrm{SC} + \text{Interest Earned} - \text{Interest Paid}. \tag{3.15}$$

The supplier's overall profit is determined for the following three cases based on the credit period.

Case (i). $0 < M \leq t_2$.
Case (ii). $t_2 < M \leq t_3$.
Case (iii). $M \geq T$.

The earned interest and paid interest in all cases shall be calculated as:

Case (i): $0 < M \le t_2$

For the period 0 to M, the seller earns the interest on the revenue earned from the sales.

$$\text{Interest earned per cycle} = \frac{D(M-t_1)(M-t_1)I_eS}{2} \tag{3.16}$$

$$\text{Interest paid per cycle} = py(t_2-M-t_1)I_pP + \frac{D(T-M)(T-M)I_pP}{2}. \tag{3.17}$$

Hence,

$$\begin{aligned}\mathrm{E}[\mathrm{TP}] = {} & Spw_1(1-\mathrm{E}[y]) + Vpw_1\mathrm{E}[y] - Ppw_0 - Cp\int_0^{t_1} w_t\,\mathrm{d}t - H\left[\frac{p^2w_1^2(1-\mathrm{E}[y])^2}{2D} + \frac{p^2w_1^2\mathrm{E}[y]}{r}\right]\\ & - K - Zpw_1 + \frac{D(M-t_1)(M-t_1)I_eS}{2} - py(t_2-M-t_1)I_pP - \frac{D(T-M)(T-M)I_pP}{2}.\end{aligned}$$

From (3.12),

$$\begin{aligned}\mathrm{E}[\mathrm{TPU}] = {} & \mathrm{SD} + \frac{VD\mathrm{E}[y]}{1-\mathrm{E}[y]} - \frac{PDw_0}{w_1(1-\mathrm{E}[y])} - \frac{CD}{w_1(1-\mathrm{E}[y])}\left\{At_1 + \frac{A}{\lambda}\left\{\log\left(1+be^{-\lambda t_1}\right) - \log(1+b)\right\}\right\}\\ & - H\left[\frac{pw_1(1-\mathrm{E}[y])}{2} + \frac{Dpw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \left[\frac{KD}{pw_1(1-\mathrm{E}[y])} - \frac{ZD}{(1-\mathrm{E}[y])}\right] + \frac{D^2(M-t_1)^2I_eS}{2pw_1(1-\mathrm{E}[y])}\\ & - \frac{D^2(T-M)^2I_pP}{2pw_1(1-\mathrm{E}[y])} - \frac{yD(t_2-M-t_1)I_pP}{w_1(1-\mathrm{E}[y])}.\end{aligned} \tag{3.18}$$

Case (ii): $t_2 < M \le t_3$

$$\text{Interest earned per cycle} = py(M-t_1-t_2)I_eV + \frac{D(M-t_1)(M-t_1)I_eS}{2} \tag{3.19}$$

$$\text{Interest paid per cycle} = \frac{D(T-M)(T-M)I_pP}{2}.$$

Hence,

$$\begin{aligned}\mathrm{E}[\mathrm{TP}] = {} & Spw_1(1-\mathrm{E}[y]) + Vpw_1\mathrm{E}[y] - Ppw_0 - Cp\int_0^{t_1} w_t\,\mathrm{d}t - H\left[\frac{p^2w_1^2(1-\mathrm{E}[y])^2}{2D} + \frac{p^2w_1^2\mathrm{E}[y]}{r}\right]\\ & - K - Zpw_1 + py(M-t_1-t_2)I_eV + \frac{D(M-t_1)(M-t_1)I_eS}{2} - \frac{D(T-M)(T-M)I_pP}{2}.\end{aligned}$$

From (3.12),

$$\begin{aligned}\mathrm{E}[\mathrm{TPU}] = {} & SD + \frac{VD\mathrm{E}[y]}{1-\mathrm{E}[y]} - \frac{PDw_0}{w_1(1-\mathrm{E}[y])} - \frac{CD}{w_1(1-\mathrm{E}[y])}\left\{At_1 + \frac{A}{\lambda}\left\{\log\left(1+be^{-\lambda t_1}\right) - \log(1+b)\right\}\right\}\\ & - H\left[\frac{pw_1(1-\mathrm{E}[y])}{2} + \frac{Dpw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \left[\frac{KD}{pw_1(1-\mathrm{E}[y])} - \frac{ZD}{(1-\mathrm{E}[y])}\right] + \frac{yD(M-t_1-t_2)I_eV}{w_1(1-\mathrm{E}[y])}\\ & + \frac{D^2(M-t_1)(M-t_1)I_eS}{2pw_1(1-\mathrm{E}[y])} - \frac{D^2(T-M)(T-M)I_pP}{2pw_1(1-\mathrm{E}[y])}.\end{aligned} \tag{3.20}$$

Case (iii): $M \geq T$

$$\text{Interest earned per cycle} = \frac{DTTSI_e}{2} + py(M - t_1 - t_2)I_eV + (M - T)DTI_eS \tag{3.21}$$
$$\text{Interest paid per cycle} = 0.$$

Hence,

$$\begin{aligned}\mathrm{E[TP]} = {} & Spw_1(1 - \mathrm{E}[y]) + Vpw_1\mathrm{E}[y] - Ppw_0 - Cp\int_0^{t_1} w_t \,\mathrm{d}t - H\left[\frac{p^2w_1^2(1-\mathrm{E}[y])^2}{2D} + \frac{p^2w_1^2\mathrm{E}[y]}{r}\right] \\ & - K - Zpw_1 + py(M - t_1 - t_2)I_eV + \frac{DTTSI_e}{2} + py(M - t_1 - t_2)I_eV + (M - T)DTI_eS.\end{aligned}$$

From (3.12),

$$\begin{aligned}\mathrm{E[TPU]} = {} & SD + \frac{VD\mathrm{E}[y]}{1-\mathrm{E}[y]} - \frac{PDw_0}{w_1(1-\mathrm{E}[y])} - \frac{CD}{w_1(1-\mathrm{E}[y])}\left\{At_1 + \frac{A}{\lambda}\left\{\log\left(1 + be^{-\lambda t_1}\right) - \log(1+b)\right\}\right\} \\ & - H\left[\frac{pw_1(1-\mathrm{E}[y])}{2} + \frac{Dpw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \left[\frac{KD}{pw_1(1-\mathrm{E}[y])} - \frac{ZD}{(1-\mathrm{E}[y])}\right] \\ & + \frac{D^2TTSI_e}{2pw_1(1-\mathrm{E}[y])} + \frac{yD(M - t_1 - t_2)I_eV}{w_1(1-\mathrm{E}[y])} + \frac{D^2(M-T)TSI_e}{pw_1(1-\mathrm{E}[y])}.\end{aligned} \tag{3.22}$$

E[TPU] is determined for all the cases. In the next section, E[TPU] is solved to achieve the optimum value of the number of items.

3.4. Solution procedure

The necessary condition for finding the optimum quantity that maximizes E[TPU] is $\frac{\mathrm{dE[TPU]}}{\mathrm{d}p} = 0$. As a result, find the first derivative of E[TPU] with respect to p to achieve the optimal number of items when accounting for the three scenarios described above.

Case (i). When $0 < M \leq t_2$.

$$\begin{aligned}\frac{\mathrm{dE[TPU]}}{\mathrm{d}p} = {} & \frac{KD}{p^2w_1(1-\mathrm{E}[y])} - \left[\frac{w_1H(1-\mathrm{E}[y])}{2} + \frac{DHw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \frac{D^2(M-t_1)^2I_eS}{2p^2w_1(1-\mathrm{E}[y])} - \frac{DyI_pP}{r(1-\mathrm{E}[y])} \\ & - \frac{I_pPw_1(1-\mathrm{E}[y])}{2} + \frac{D^2(M)^2I_pP}{2p^2w_1(1-\mathrm{E}[y])} = 0.\end{aligned}$$

On solving,

$$p = \sqrt{\frac{2KDr - DD(M-t_1)(M-t_1)I_eSr + DDMMI_pPr}{2DyI_pPw_1 + w_1w_1Hr(1-E1)(1-E1) + 2DHw_1w_1E1 + rI_pPw_1w_1(1-E1)(1-E1)}}. \tag{3.23}$$

Case (ii). When $t_2 < M \leq t_3$.

$$\begin{aligned}\frac{\mathrm{dE[TPU]}}{\mathrm{d}p} = {} & \frac{KD}{p^2w_1(1-\mathrm{E}[y])} - \left[\frac{w_1H(1-\mathrm{E}[y])}{2} + \frac{DHw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right] - \frac{DyI_eV}{r(1-\mathrm{E}[y])} - \frac{w_1PI_p(1-\mathrm{E}[y])}{2} \\ & + \frac{PI_pD^2M^2}{2(1-\mathrm{E}[y])p^2w_1} - \frac{SI_eD^2(M-t_1)^2}{2(1-\mathrm{E}[y])p^2w_1} = 0.\end{aligned}$$

On solving,

$$p=\sqrt{\frac{2KDr-DD(M-t_1)(M-t_1)I_eSr+DDMMI_pPr}{2DyI_eVw_1+w_1w_1Hr(1-E1)(1-E1)+2DHw_1w_1E1+rI_pPw_1w_1(1-E1)(1-E1)}}. \quad (3.24)$$

Case (iii). When, $M\geq T$

$$\frac{\mathrm{dE[TPU]}}{\mathrm{d}p}=\frac{KD}{p^2w_1(1-\mathrm{E}[y])}-\left[\frac{w_1H(1-\mathrm{E}[y])}{2}+\frac{DHw_1\mathrm{E}[y]}{r(1-\mathrm{E}[y])}\right]-\frac{DyI_eV}{r(1-\mathrm{E}[y])}-\frac{w_1SI_e(1-\mathrm{E}[y])}{2}=0.$$

On solving the above equation,

$$p=\sqrt{\frac{2KDr}{2DyI_eVw_1+w_1w_1Hr(1-E1)(1-E1)+2DHw_1w_1E1+rI_eSw_1w_1(1-E1)(1-E1)}}. \quad (3.25)$$

The maxima minima test is used to verify if the equation meets the sufficient concavity constraint, that is, $\frac{\mathrm{d}^2\mathrm{E[TPU]}}{\mathrm{d}p^2}<0$.

Case (i). $\frac{\mathrm{d}^2\mathrm{E[TPU]}}{\mathrm{d}p^2}=\frac{-2KD}{p^3w_1(1-\mathrm{E}[y])}-\frac{PI_pD^2M^2}{(1-\mathrm{E}[y])p^3w_1}+\frac{SI_eD^2(M-t_1)^2}{(1-\mathrm{E}[y])p^3w_1}<0.$
Case (ii). $\frac{\mathrm{d}^2\mathrm{E[TPU]}}{\mathrm{d}p^2}=\frac{-2KD}{p^3w_1(1-\mathrm{E}[y])}-\frac{PI_pD^2M^2}{(1-\mathrm{E}[y])p^3w_1}+\frac{SI_eD^2(M-t_1)^2}{(1-\mathrm{E}[y])p^3w_1}<0.$
Case (iii). $\frac{\mathrm{d}^2\mathrm{E[TPU]}}{\mathrm{d}p^2}=-\frac{4KDr}{p^3}<0.$

The concavity is thus shown in all situations, *i.e.*, p is the maximum point that maximizes E[TPU]. In addition, an algorithm is intended to give the best-case scenario for determining the optimal value of the number of items.

Algorithm

Step 1. Calculate the values of t_1 and $T_{\min}$ using equations (3.3) and (3.13) respectively.
Step 2. Examine if $T_{\min}\geq 0$, the problem is feasible.
Step 3. Calculate the value of x_{res} using equation (3.14). If $\mathrm{E}(x)\leq x_{\mathrm{res}}$, then problem is feasible and proceed to step (4).
Step 4. Substitute the value of p from equation (3.23) into the equation (3.12) to obtain the value of $\mathrm{E}[T]$. If $0<M\leq t_2$, put all the necessary values in equation (3.18) so that the optimal value of E[TPU] is obtained.
Step 5. Substitute the value of p from equation (3.24) into the equation (3.12) to obtain the value of $\mathrm{E}[T]$. If $t_2<M\leq t_3$, put all the necessary values in equation (3.20) so that the optimal value of E[TPU] is obtained.
Step 6. Substitute the value of p from equation (3.25) into the equation (3.12) to obtain the value of $\mathrm{E}[T]$. If $M\geq T$, put all the necessary values in equation (3.22) so that the optimal value of E[TPU] is obtained.
Step 7. Compare the values of E[TPU] in all the cases and choose the optimal value of p that maximizes E[TPU].

4. Numerical example

The proposed model is developed with the help of a numerical example considering a firm that buys some new born items, feeds them after which they are slaughtered and sold in the market. The following parameters have been used [33]:

$D=1\,000\,000$ g/year; $K=1000$ ZAR/cycle; $H=0.04$ ZAR/g/year; $C=0.2$ ZAR/g/year; $w_1=1500$ g; $P=0.025$ ZAR/g; $S=0.05$ ZAR/g; $A=6870$ g; $b=120$; $\lambda=40$/year; $r=10$g/min, $V=0.02$ ZAR/g; $Z=0.00025$ ZAR/g; $w_0=57$ g; $\gamma=15\,330$ g/year; $t_s=0.01$ year; $\delta_1=10\,220$ g/year; $w_1'=550$ g; $t_1'=0.0521$ year; $\delta_2=27\,375$ g/year; $w_1''=5350$ g; $t_1''=0.2274$ year; $\delta_3=10\,220$ g/year; $I_e=0.05$; $I_p=0.08$; $E[y]=0.02$; $r=5\,256\,000$; $M=0.3$.

TABLE 2. Summary of results obtained from the considered numerical example.

Variable	Units	Quantity		
		Logistic growth function	Linear growth function	Split linear growth function
t_1	In years	0.0878	0.0941	0.0868
t_2	In years	0.0423	0.0423	0.0423
E[T]	In years	0.218	0.218	0.218
p	No of items	148	148	148
E[TPU]	ZAR/year	35 114.8	31 439.5	34 491.3

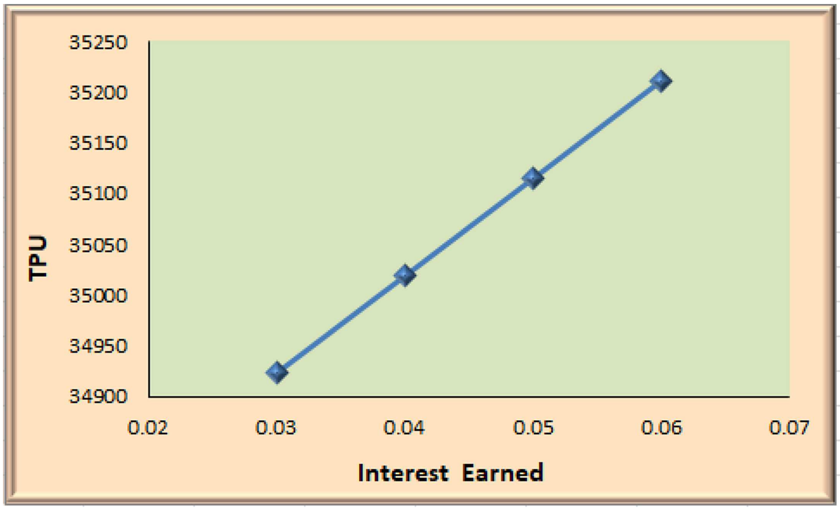

FIGURE 5. Effect of interest earned on E[TPU].

The above algorithm has been applied to the numerical example to calculate the value of E[TPU] in all the three cases. Table 2 presents an overview of the results from the numerical example.

Regardless of whether the growth function was logistic, linear or split linear, it was observed that the EOQ, cycle time, and screening time were the same while, on the other hand, there was a variation in the values of expected total profits and slaughter ages. The logistic model's annual expected profit is 35 114.8 ZAR, linear model's annual expected profit is 31 439.5 ZAR, whereas the split linear model's is 34 491.3 ZAR. Less deviation is observed in the profit of the split linear model from the logistic growth model.

5. Sensitivity analysis

Sensitivity analysis shows the effectiveness in assessing how the objective function and decision variables are significantly impacted by adjusting the model constraints. The effect of M, IE and IP on the net income of the company is seen in this model through a sensitivity study.

Following graphs summarize the results

Observation and managerial implication

As E[TPU] is maximum in the third case compared to the other two cases, the effect of the changes in three parameters, *i.e.*, purchase costs, feeding costs and holding costs on E[TPU] is shown for the third case. The

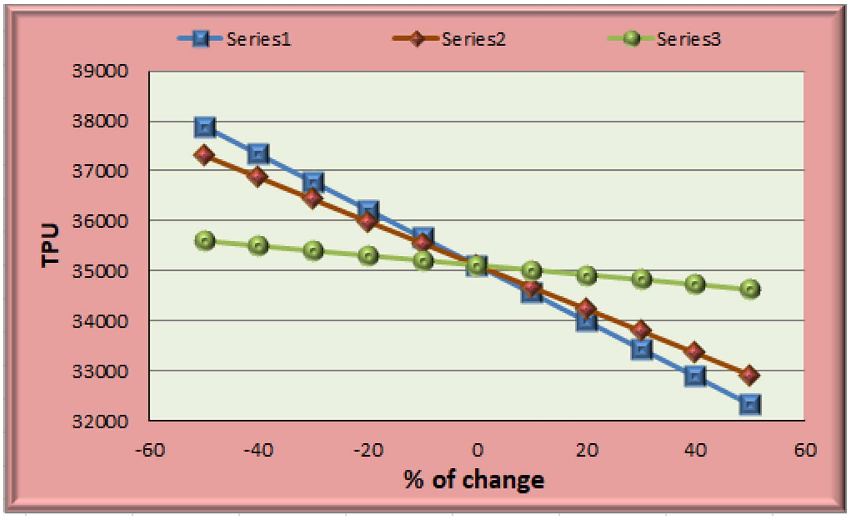

FIGURE 6. Effect of change in purchasing cost, feeding costs and holding costs on E[TPU].

retailer must be vigilant while ordering as the defective items has an impact on the order quantity and expected total profit. The observations are given below:

- Figure 5 indicates that with the increase in interest earned, E[TPU] tends to increase which suggests that as the supplier provides extended time period to the retailer, the interest gained is high which implies that the anticipated overall cost is relatively high.
- It is seen from Figure 6, changes are examined due to purchasing cost, holding cost and feeding cost that affect E[TPU]. The effect of changes in feeding costs on E[TPU] is shown by series 1. The effect of changes in holding costs on E[TPU] is shown by series 2. The effect of changes in purchasing costs on E[TPU] is shown by series 3. It is observed that all the three costs have a negative impact on E[TPU]. E[TPU] decreases while all the costs increase. Changes in purchasing costs doesn't affect E[TPU] that much as compared to the other two costs.

6. Conclusion and future scope

A mathematical model has been analysed in the context of growing items, such as chickens in which the inventory is optimized, that is further used to maximize the supplier's overall profit in the existence of allowable deferment in payments. Earlier, the researchers have developed the model ignoring the existence of imperfect quality of growing items. However, in this model, a condition is considered under which a firm acquires new-born chicks and feeds them to a certain weight following the process of screening resulting in poor quality chickens which are then sold in the market. In order to maximize the gross profit in both situations, the organization has to consider the optimal order quantity of new-born chicks. Sensitive research is carried out to assess the influence of vital parameters, *i.e.*, trade credit, interest earned, and interest paid on the inventory level and gross income of the supplier. The findings show that gross revenue is the maximum in the third case relative to the other two cases. Sensitivity analysis also reveals the impact of holding costs, feeding costs, and purchasing costs on E[TPU]. It is shown that all three components inversely affect the total profit per unit E[TPU].

However, the following model is only applicable with certain conditions and constraints that is, where shortages are not allowed.

Future scope

In this paper, both cases are considered where all chickens are good and flawed when they are slaughtered and sold by the company. However, the situation can be enlarged to defective products in addition of carbon emissions and several other natural events where the cost of screening will also be incorporated, resulting in changes in the total profit per unit of the supplier.

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