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SOLVING THE MULTI-MODAL TRANSPORTATION PROBLEM *VIA* THE ROUGH INTERVAL APPROACH

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Abstract. This research studies a transportation problem to minimize total transportation cost under the rough interval approximation by considering the multi-modal transport framework, referred to here as the rough Multi-Modal Transportation Problem (MMTP). The parameters of MMTP are rough intervals, because the problem is chosen from a real-life scenario. To solve MMTP under a rough environment, we employ rough chance-constrained programming and the expected value operator for the rough interval and then outline the main advantages of our suggested method over those existing methods. Next, we propose an algorithm to optimally solve the problem and present a numerical example to examine the proposed technique. Finally, the solution is analyzed by the proposed method with rough-chance constrained programming and expected value operator.

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1. Introduction

Transportation Problems (TPs) are widely studied within the operations research literature, with many real-life problems can be considered and modeled by them. Most studies in the TP literature consider to satisfy the availabilities of sources and demands for destinations and reduce transportation costs from sources to different destinations. In reality, a number of additional decision-making challenges appear such as, product benefits, benefits for purchasers, making decisions on various objective functions in real life, and so on. Multi-Model Transportation Problem (MMTP) is comparable to a TP involving the use of several modes of transportation (see Fig. 1). It is also called the combined TP that permits transporting goods under a solitary contract, but it operates under more than one mode of transportation; the bearer is liable (in the usual sense) for the whole movement, despite the fact that several/different modes of transportation are considered, such as road, sea, train, etc. The bearer does not need to utilize all modes for transport, and in normal practice this is not valid. The

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transportation is frequently executed by subcontracted bearers (normally one for each unique transportation mode).

The fundamental transportation model was first started by Kantorovich [11], who proposed an incomplete calculation for figuring the arrangement of the TP. Hitchcock [7] examined the issue of limiting the cost of giving an item from a few distribution centers to various buyers. Since the reality of TP has many facets, each one of them is usually studied separately. Murphy [25] developed a TP, described its advantages for the dual of TP and gave its implications for land-use and transport planning. Huang *et al.* [9] solved a stochastic dairy TP with considering collection and delivery phases. Maity *et al.* [17] analyzed an MMTP and its application to artificial intelligence. Kaur *et al.* [12] investigated a capacitated TP with two-stage and restricted flow for the aim of time minimization. Roy *et al.* [35] described a new approach for solving intuitionistic fuzzy multi-objective TP. James *et al.* [10] improved transportation benefit quality in view of a combination of data. Maity *et al.* [16] proposed a new approach for solving dual-hesitant fuzzy TP with restrictions. Mardanya and Roy [18] introduced the time-variant in the multi-objective linear fractional transportation problem. Later, Mardanya *et al.* [20] introduced the just-in-time concept in the multi-objective environment. Ebrahimnejad [3] addressed a problem including interval-valued trapezoidal fuzzy variables and provided a method for linear programming. Myung *et al.* [26] developed freight transportation network model with bundling option. Zhang *et al.* [40] derived a reactive tabu search algorithm for the multi-depot container truck transportation problem. Other studies related to transportation security arrangement have been presented by Ergun *et al.* [4], Luathep *et al.* [14], Zhi-Chun *et al.* [41], Abdel-Aty *et al.* [1] and Xu *et al.* [38]. Moreover the multi-objective transportation problem has been discussed by many authors, such as Dalman [2], Ghosh and Roy [5], Kumar *et al.* [13], Mahapatra *et al.* [15], Mardanya *et al.* [19], Midya and Roy [21], Midya *et al.* [22,23], Paraman *et al.* [28], Roy *et al.* [33,34,36], and others.

Vagueness appears in reality within TP. This is due to the lack of increasing awareness within a transportation policy and different types of unexpected elements such as a lack of evidence, etc. In reality, the main aim of the problem is to optimize the objective function under certain conditions.

In 1982, Pawlak [29,30] proposed the rough set theory. It is a technique that simultaneously considers vagueness and uncertainties. The rough set theory plays an important role in analyzing a vaguely described problem for finding a different course of actions in real-life decision-making problems [31]. The rough set theory has a wide scope of application in several fields, such as decision analysis, machine learning, and knowledge discovery from a database, civil engineering problems, and other areas. Recently, rough programming accommodates real-world vagueness and uncertainty, thus spurring the attention of researchers (*cf.* [6,27,32,39]).

Many researchers have focused on Rough Interval (RI) concepts in order to apply the various properties of RI on several optimization problems related to transporting systems.

However, there seems to be a gap in the application of RI concepts on MMTP. Since transportation cost in the transportation system depends on many factors, they have an inherent inclination for being complex, inexact, and/or vague. The interval approximation can imagine this inaccuracy or vagueness so that the Decision Maker (DM) can have an idea of the overall cost of transportation in interval form. Furthermore, in an multi-modal transportation system, DMs have no exact information about the data related to different transportation parameters in different modes, the parameters in the problem that are an approximate representation of the demands at the destinations, the capacity of supply at origins, selling price, unit cost, etc. Due to these unpredictable factors, the feasible region becomes uncertain. Considering the rough inclination approximation to an uncertain feasible region, DMs acquire better flexibility in the decision-making process. This practical necessity motivates us to study MMTP with RI approximation under the following objectives.

- To investigate MMTP with RI coefficients.
- To explain how the decision-making system smooth and relaxed with the help of RI in contrast with utilizing an exact crisp number.
- To derive the simplest form of the solution for the linear optimization problem, *i.e.*, an MMTP is solved by using existing methods, *viz.*, expected value operator and rough chance-constrained programming (RCCP).

The remaining paper is arranged as follows. Section 2 formulates the problem studied. Section 3 discusses the basic definitions and properties of RI. Section 4 proposes a new model for MMTP. Section 5 presents a solution procedure of the proposed method. To demonstrate the utilization of the proposed model of MMTP, Section 6 gives a numerical illustration. Section 7 shows the results and discussion of the proposed method. Finally, this paper ends with conclusions and future study in Section 8.

2. Problem background

To define MMTP, we need to know ground origins, final destinations, and transshipment origins.

Definition 2.1 (Ground Origin (GO))**.** In a TP, ground origins are the nodes with no capacity to gather the goods, but from where the next nodes receive goods.

Definition 2.2 (Final Destination (FD))**.** In a TP, final destinations are the nodes that receive goods with no capability to supply them.

Due to the capacity/multiple routes of transportation, goods cannot supply on-demand from the GO nodes to the FD nodes. In this case, certain destination nodes with the capacity to simultaneously supply and receive goods are needed. Such nodes are called *transshipment origins*.

Definition 2.3 (Transshipment Origin (TO))**.** In a TP, the destination nodes that can collect the goods and the capacity to deliver the goods are called transshipment origins.

In Figure 1, $G_1,\ G_2,\dots,G_{m1}$ are the nodes of GD; $T_{11},\ T_{12},\dots,T_{1m2}$ are the nodes of TO of level 1; and $D_1,\ D_2,\dots,D_{n1}$ are taken as the nodes of FD.

The TP under the consideration of at least one node of TO is depicted as MMTP. In order to accommodate the real-life TP, the usage of a single mode of transportation cannot always possibly fulfill customer demand at the destination points. There are sometimes certain restrictions on transporting the goods, and so the multi-modes of transportation from different nodes need to be addressed. At that point the transportation is not a simple TP, but turns into an MMTP.

3. Prelimiaries

We now provide some background and properties on rough set theory along with a rough interval TP.

Approximation space. Suppose $X \neq \phi$ is a finite set of objects. We define an equivalence relation R on X that partitions X into a family of pairwise disjoined subsets $E_1, E_2, \dots, E_n$, each of which is an equivalence class of R and called elementary sets. The pair (X, R) is called the approximation space and is denoted by $\mathrm{Appr}(R)$.

In the approximation space, $\mathrm{Appr}(R) = (X, R)$, and given an arbitrary set $B \subseteq X$, one may represent B by a pair of lower approximation (LA) and upper approximation (UA):

$$\underline{\mathrm{Appr}}(B) = \bigcup_{E_i \subseteq B} E_i = \{y \in X : [y]_R \subseteq B\},$$
$$\overline{\mathrm{Appr}}(B) = \bigcup_{E_i \cap B \neq \phi} E_i = \{y \in X : [y]_R \cap B \neq \phi\}.$$

Here, $[y]_R$ signifies the equivalence class containing y. The LA and UA of B can be equivalently described as:

$$\underline{\mathrm{Appr}}(B) = \{y \in X : \forall\ z \in X,\ yRz \Rightarrow z \in B\},$$
$$\overline{\mathrm{Appr}}(B) = \{y \in X : \exists\ z \in X \text{ such that } yRz \text{ and } z \in B\}.$$

The pair $(\underline{\mathrm{Appr}}(B), \overline{\mathrm{Appr}}(B))$ is called the rough set of B.

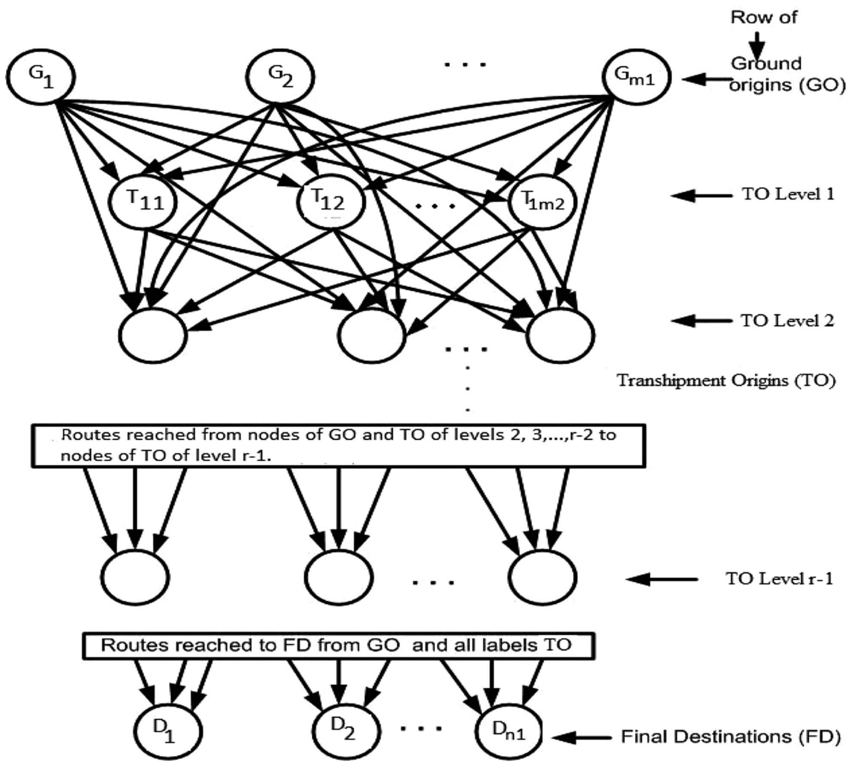

Figure 1. Graphical representation of an MMTP.

Rough intervals. The concept of RI [32] was proposed as a unique instance of a rough set that fulfills all of the rough set properties. In addition, fundamental ideas are defined including definitions of UA and LA. There are two components of RI: Lower Approximation Interval (LAI) and Upper Approximation Interval (UAI). In particular, RI may be expressed as a trademark an incentive from a suspicious idea defined on a variable $x \in R$.

The concept of RIs was introduced by Rebolledo [32] as a special case of a rough set. The distinctive point of RIs is that they can be used to treat parameters and variables that are partially unknown. RIs are applied in the literature to overcome the inherent limitations of the rough set property, as rough sets were originally used only to treat discrete object, and could not define continuous variables. However, RIs can use the concepts of the rough set as continuous variables in a model by applying them. This is a special case of a rough set that satisfies all the rough set properties and basic concepts including the UA and LA definitions. Some explanations are presented on RI as follows.

(i) If $y \in \underline{\text{Appr}}(B) \Rightarrow y \in B$ then y is surely taken by B.
(ii) If $y \in \overline{\text{Appr}}(B)$, then y is possibly taken by B.
(iii) If $y \notin \overline{\text{Appr}}(B) \Rightarrow y \notin B$ then y surely does not belong to B.

Remark. Let $S = \left(\left[\underline{s}^l, \underline{s}^u\right], \left[\overline{s}^l, \overline{s}^u\right]\right)$ be a RI. Presently, if $\underline{s}^l = \overline{s}^l$ and $\underline{s}^u = \overline{s}^u$, then there are no extraordinary cases in that RI, and at that point this RI degenerates into simply an interval. Along these lines, the RI is a sensible era of the widely known crisp interval.

Rough interval arithmetic. The RI arithmetic operations are similar to Moore's interval arithmetic [24]. In this consequence, some arithmetic operations of RIs are presented minutely. A detailed discussion of Rough

Interval Arithmetic (RIA) is in [32]. Presently, as indicated by Hamzehee *et al.* [6], $S = \left(\left[\underline{s}^l, \underline{s}^u\right], \left[\overline{s}^l, \overline{s}^u\right]\right)$ and $T = \left(\left[\underline{t}^l, \underline{t}^u\right], \left[\overline{t}^l, \overline{t}^u\right]\right)$ are two RIs. At that point, the RIA on these two RIs is given by the following.

(3.1) *Addition:* $S + T = \left(\left[\underline{s}^l + \underline{t}^l, \underline{s}^u + \underline{t}^u\right], \left[\overline{s}^l + \overline{t}^l, \overline{s}^u + \overline{t}^u\right]\right)$.

(3.2) *Subtraction:* $S - T = \left(\left[\underline{s}^l - \underline{t}^u, \underline{s}^u - \underline{t}^l\right], \left[\overline{s}^l - \overline{t}^u, \overline{s}^u - \overline{t}^l\right]\right)$.

(3.3) *Negation:* $-S = \left(\left[-\underline{s}^u, -\underline{s}^l\right], \left[-\overline{s}^u, -\overline{s}^l\right]\right)$.

(3.4) *Intersection:* $S \cap T = \left(\left[\max\{\underline{s}^l, \underline{t}^l\}, \min\{\underline{s}^u, \underline{t}^u\}\right], \left[\max\{\overline{s}^l, \overline{t}^l\}, \min\{\overline{s}^u, \overline{t}^u\}\right]\right)$.

(3.5) *Union:* $S \cup T = \left(\left[\min\{\underline{s}^l, \underline{t}^l\}, \max\{\underline{s}^u, \underline{t}^u\}\right], \left[\min\left\{\overline{s}^l, \overline{t}^l\right\}, \max\left\{\overline{s}^u, \overline{t}^u\right\}\right]\right)$.

The set and logic operations with RIs are basically analyzed by similar operations with a fuzzy set. Fuzzy sets must deal with continuous membership functions and cannot utilize Moore's interval calculus.

Order relation of a rough interval. Suppose S and T are any two RIs, and then the order relations "$\leq$" and "$<$" between S and T are characterized as $S \leq T \Leftrightarrow \frac{\underline{s}^l + \underline{s}^u}{2} \leq \frac{\underline{t}^l + \underline{t}^u}{2}$, $\frac{\overline{s}^l + \overline{s}^u}{2} \leq \frac{\overline{t}^l + \overline{t}^u}{2}$; and $S < T \Leftrightarrow S \leq T$ with $S \neq T$, respectively.

The order relation either "$\leq$" or "$<$" demonstrates the inclination of DMs for the distinctive decisions in view of the uppermost extreme midpoint in the ordinary case and in the exceptional case, by regarding a maximization problem. Expected value maximization and vulnerability minimization are the choices of reasoning. We note that the order relations "$\leq$" and "$<$" so-characterized are partially ordered relations.

Definition 3.1. Abdel-Aty *et al.* [1] presented the concept of a measure on the approximation space. If $X = (\underline{\mathrm{Appr}}(X), \overline{\mathrm{Appr}}(X))$ is a rough value on the approximation space, then the lower and upper trust measures of the rough event $\{X \leq r\}$ are respectively defined by:

$$\underline{\mathrm{Tr}}\{X \leq r\} = \frac{\mathrm{Card}\left(p \in \underline{\mathrm{Appr}}(X) | p \leq r\right)}{\mathrm{Card}\left(\underline{\mathrm{Appr}}(X)\right)}, \ \overline{\mathrm{Tr}}\{X \leq r\} = \frac{\mathrm{Card}\left(p \in \overline{\mathrm{Appr}}(X) | p \leq r\right)}{\mathrm{Card}\left(\overline{\mathrm{Appr}}(X)\right)}.$$

Here, Card() denotes the cardinality of a given set.

The trust measure of the rough event is given by:

$$\mathrm{Tr}\{X \leq r\} = \frac{1}{2}\left[\underline{\mathrm{Tr}}\{X \leq r\} + \overline{\mathrm{Tr}}\{X \leq r\}\right].$$

The trust measure may be defined as any convex combination of the lower and upper trusts.

Definition 3.2. Suppose $\gamma = \left([\underline{a}_l, \underline{a}_u], \left[\bar{a}^l, \bar{a}^u\right]\right)$, where $\bar{a}^l \leq \underline{a}_l \leq \underline{a}_u \leq \bar{a}^u$, is RI; then according to Definition 3.1, the trust measures of rough events $\{\gamma \leq r\}$ and $\{\gamma \geq r\}$ are denoted by $\mathrm{Tr}\{\gamma \leq r\}$, $\mathrm{Tr}\{\gamma \geq r\}$ and defined as follows:

$$\mathrm{Tr}\{\gamma \leq r\} = \begin{cases} 0, & \text{if } r \leq \bar{a}^l; \\ \frac{1}{2}\left(\frac{\bar{a}^l - r}{\bar{a}^l - \bar{a}^u}\right), & \text{if } \bar{a}^l \leq r \leq \underline{a}^l; \\ \frac{1}{2}\left(\frac{\bar{a}^l - r}{\bar{a}^l - \bar{a}^u} + \frac{\underline{a}^l - r}{\underline{a}^l - \underline{a}^u}\right), & \text{if } \underline{a}^l \leq r \leq \underline{a}^u; \\ \frac{1}{2}\left(\frac{\bar{a}^l - r}{\bar{a}^l - \bar{a}^u} + 1\right), & \text{if } \underline{a}^u \leq r \leq \bar{a}^u; \\ 1, & \text{if } r \geq \bar{a}^u. \end{cases} \qquad \mathrm{Tr}\{\gamma \geq r\} = \begin{cases} 0, & \text{if } r \geq \bar{a}^u; \\ \frac{1}{2}\left(\frac{\bar{a}^u - r}{\bar{a}^u - \bar{a}^l}\right), & \text{if } \underline{a}^u \leq r \leq \bar{a}^u; \\ \frac{1}{2}\left(\frac{\bar{a}^u - r}{\bar{a}^u - \bar{a}^l} + \frac{\underline{a}^u - r}{\underline{a}^u - \underline{a}^l}\right), & \text{if } \underline{a}^l \leq r \leq \underline{a}^u; \\ \frac{1}{2}\left(\frac{\bar{a}^u - r}{\bar{a}^u - \bar{a}^l} + 1\right), & \text{if } \underline{a}^l \leq \bar{a}^l; \\ 1, & \text{if } r \leq \bar{a}^u. \end{cases}$$

The α-optimistic value and the α-pessimistic value of the rough interval are given by equations (3.1) and (3.2) as:

$$\gamma_{\sup}(\alpha)=\begin{cases}(1-2\alpha)\bar{a}^u+2\alpha\bar{a}^l, & \text{if } \alpha\le\frac{\bar{a}^u-\underline{a}^u}{2(\bar{a}^u-\bar{a}^l)};\\ 2(1-\alpha)\bar{a}^u+(2\alpha-1)\bar{a}^l, & \text{if } \alpha\ge\frac{2\bar{a}^u-\underline{a}^l-\bar{a}^l}{2(\bar{a}^u-\bar{a}^l)};\\ \frac{\bar{a}^u(\underline{a}^u-\underline{a}^l)+\underline{a}^u(\underline{a}^u-\underline{a}^l)-2\alpha(\underline{a}^u-\underline{a}^l)(\bar{a}^u-\bar{a}^l)}{(\underline{a}^u-\underline{a}^l)+(\underline{a}^u-\bar{a}^l)}, & \text{otherwise.}\end{cases}\tag{3.1}$$

$$\gamma_{\inf}(\alpha)=\begin{cases}(1-2\alpha)\bar{a}^l+2\alpha\bar{a}^u, & \text{if } \alpha\le\frac{\underline{a}^l-\bar{a}^l}{2(\bar{a}^u-\bar{a}^l)};\\ 2(1-\alpha)\bar{a}^l+(2\alpha-1)\bar{a}^u, & \text{if } \alpha\ge\frac{\underline{a}^u+\bar{a}^u-2\underline{a}^l}{2(\underline{a}^u-\bar{a}^l)};\\ \frac{\bar{a}^l(\underline{a}^u-\underline{a}^l)+\underline{a}^l(\underline{a}^u-\underline{a}^l)-2\alpha(\underline{a}^u-\underline{a}^l)(\bar{a}^u-\bar{a}^l)}{(\underline{a}^u-\underline{a}^l)+(\bar{a}^u-\bar{a}^l)}, & \text{otherwise.}\end{cases}\tag{3.2}$$

Expected value of a rough interval. The expected value operator is used to reduce a RI to a crisp interval. As our discussion is confined into a TP in rough intervals, we have to define some important concepts on the expected value operator.

Definition 3.3. Suppose $X=\{z\in X:\gamma(z)\in B\}$, where $\gamma: X\to\mathbb{R}$ is a real function, $B\subset\mathbb{R}$; and X is approximated by $(\underline{X},\overline{X})$ according to the equivalence relation R. The lower expected, upper expected, and expected values of X are then defined as follows:

$$\underline{E}[X]=\int_0^{+\infty}\underline{\mathrm{Appr}}\{\gamma\ge x\}\mathrm{d}x-\int_{-\infty}^0\underline{\mathrm{Appr}}\{\gamma\le x\}\mathrm{d}x,$$
$$\overline{E}[X]=\int_0^{+\infty}\overline{\mathrm{Appr}}\{\gamma\ge x\}\mathrm{d}x-\int_{-\infty}^0\overline{\mathrm{Appr}}\{\gamma\le x\}\mathrm{d}x,$$
$$E[X]=\int_0^{+\infty}\mathrm{Appr}\{\gamma\ge x\}\mathrm{d}x-\int_{-\infty}^0\mathrm{Appr}\{\gamma\le x\}\mathrm{d}x.$$

The correlation among the expected value $E(X)$, the lower expected value $\underline{E}(X)$, and the upper expected value $\overline{E}(X)$ is placed into the following proposition.

Proposition. *Let $X=\{z\in X:\gamma(z)\in B\}$, where $\gamma:X\to\mathbb{R}$ is a real function, $B\subset\mathbb{R}$; and X is approximated by $\left(\underline{X},\overline{X}\right)$ according to the similarity relation R, and η is a given parameter predetermined by using the DM preference. The expected value of X is denoted by $E(X)$ and is defined as $E(X)=\eta\underline{E}(X)+(1-\eta)\overline{E}(X)$.*

Proof. For proof, one can see [37]. □

Theorem 3.4 ([37])**.** *Suppose $B=([a_1,b_1],[c_1,d_1])$ is a RI, where $c_1\le a_1\le b_1\le d_1$. The expected value of B is then $E(B)=\frac{1}{2}[\eta\cdot(a_1+b_1)+(1-\eta)\cdot(c_1+d_1)]$.*

Proof. For proof, one can see [37]. □

Remark. For $\eta=0.5$ the expected value of B is $\frac{1}{4}(a_1+b_1+c_1+d_1)$.

The mathematical model of the proposed MMTP is depicted in points of interest in the following section.

Theorem 3.5. *If $\gamma=\left([\underline{a}_l,\underline{a}_u],\left[\bar{a}^l,\bar{a}^u\right]\right)$, where $\bar{a}^l\le\underline{a}_l\le\underline{a}_u\le\bar{a}^u$ is a rough variable, then for a predetermined α, $0<\alpha\le1$, $\mathrm{Tr}\{\gamma\le r\}\ge\alpha$ is equivalent to*

(i) $(1-2\alpha)\bar{a}^u+2\alpha\bar{a}^l\le r$, *if* $\alpha\le\frac{\bar{a}^u-\underline{a}^u}{2(\bar{a}^u-\bar{a}^l)}$;

(ii) $2(1-\alpha)\bar{a}^u+(2\alpha-1)\bar{a}^l\le r$, *if* $\alpha\ge\frac{2\bar{a}^u-\underline{a}^l-\bar{a}^l}{2(\bar{a}^u-\bar{a}^l)}$;

(iii) $\frac{\bar{a}^u(\underline{a}^u-\underline{a}^l)+\underline{a}^u(\underline{a}^u-\underline{a}^l)-2\alpha(\underline{a}^u-\underline{a}^l)(\bar{a}^u-\bar{a}^l)}{(\underline{a}^u-\underline{a}^l)+(\underline{a}^u-\bar{a}^l)}\le r$, *otherwise.*

Proof. Given a value r and $\gamma = ([\underline{a}_l, \underline{a}_u], [\bar{a}^l, \bar{a}^u])$, given by equation (3.1), consider the following cases.

Case I. For $\bar{a}^l \le r \le \underline{a}_l$ and a predetermined α from equation (3.1) we have

$$\begin{aligned}\mathrm{Tr}\{\gamma \le r\} \ge \alpha \implies & \frac{r - \bar{a}^u}{2(\bar{a}^l - \bar{a}^u)} \ge \alpha \\ \implies & (1-2\alpha)\bar{a}^u + 2\alpha\bar{a}^l \le r.\end{aligned}$$

However, in this case, the maximum possible value of $\mathrm{Tr}\{\gamma \le r\} \ge \alpha$ can be $\frac{\bar{a}^u - \underline{a}^u}{2(\bar{a}^u - \bar{a}^l)}$ and the minimum possible value is 0 so the value of α must be less than or equal to $\frac{\bar{a}^u - \underline{a}^u}{2(\bar{a}^u - \bar{a}^l)}$.

Case II. For $\underline{a}_u \le r \le \bar{a}^u$ and a predetermined α,

$$\begin{aligned}\mathrm{Tr}\{\gamma \le r\} \ge \alpha \implies & \frac{1}{2}\left(\frac{r - \bar{a}^u}{(\bar{a}^l - \bar{a}^u)} + 1\right) \ge \alpha \\ \implies & 2(1-\alpha)\bar{a}^u + (2\alpha - 1)\bar{a}^l \le r.\end{aligned}$$

In this case, the maximum possible value of $\mathrm{Tr}\{\gamma \le r\} \ge \alpha$ can be 1 and the minimum possible value is $\frac{1}{2}\left(\frac{r-\bar{a}^u}{(\bar{a}^l - \bar{a}^u)} + 1\right)$ so the value of α must be greater than or equal to $\frac{1}{2}\left(\frac{r-\bar{a}^u}{(\bar{a}^l - \bar{a}^u)} + 1\right)$, which implies $\alpha \ge \frac{2\bar{a}^u - \underline{a}^l - \bar{a}^l}{2(\bar{a}^u - \bar{a}^l)}$.

Case III. For $\underline{a}_l \le r \le \underline{a}_u$ and a predetermined α,

$$\begin{aligned}\mathrm{Tr}\{\gamma \le r\} \ge \alpha \implies & \frac{1}{2}\left(\frac{r - \bar{a}^u}{\bar{a}^u - \bar{a}^l} + \frac{r - \underline{a}^u}{\underline{a}^u - \underline{a}^l}\right) \ge \alpha \\ \implies & \frac{\bar{a}^u(\underline{a}^u - \underline{a}^l) + \underline{a}^u(\underline{a}^u - \underline{a}^l) - 2\alpha(\underline{a}^u - \underline{a}^l)(\bar{a}^u - \bar{a}^l)}{(\underline{a}^u - \underline{a}^l) + (\underline{a}^u - \bar{a}^l)} \le r.\end{aligned}$$

In this case (*i.e.*, $\underline{a}_l \le r \le \underline{a}_u$), minimum possible value of $\mathrm{Tr}\{\gamma \le r\} \ge \alpha$ can be $\frac{\bar{a}^u - \underline{a}^u}{2(\bar{a}^u - \bar{a}^l)}$ and maximum possible value is $\frac{1}{2}\left(1 + \frac{\bar{a}^u - \underline{a}^u}{2(\bar{a}^u - \bar{a}^l)}\right) = \frac{2\bar{a}^u - \underline{a}^l - \bar{a}^l}{2(\bar{a}^u - \bar{a}^l)}$ and hence the proof is completed.

□

Theorem 3.6. *If* $\gamma = \left([\underline{a}_l, \underline{a}_u], \left[\bar{a}^l, \bar{a}^u\right]\right)$, *where* $\bar{a}^l \le \underline{a}_l \le \underline{a}_u \le \bar{a}^u$ *is a rough variable, then for a predetermined* α, $0 < \alpha \le 1$, $\mathrm{Tr}\{\gamma \ge r\} \ge \alpha$ *is equivalent to*

(i) $(1-2\alpha)\bar{a}^l + 2\alpha\bar{a}^u \ge r$, *if* $\alpha \le \frac{\underline{a}^l - \bar{a}^l}{2(\bar{a}^u - \bar{a}^l)}$;

(ii) $2(1-\alpha)\bar{a}^l + (2\alpha - 1)\bar{a}^u \ge r$, *if* $\alpha \ge \frac{\underline{a}^u + \bar{a}^u - 2\underline{a}^l}{2(\underline{a}^u - \bar{a}^l)}$;

(iii) $\frac{\bar{a}^l(\underline{a}^u - \underline{a}^l) + \underline{a}^l(\underline{a}^u - \underline{a}^l) - 2\alpha(\underline{a}^u - \underline{a}^l)(\bar{a}^u - \bar{a}^l)}{(\underline{a}^u - \underline{a}^l) + (\bar{a}^u - \bar{a}^l)} \ge r$, *otherwise.*

Proof. Using the expression of $\mathrm{Tr}\{\gamma \ge r\} \ge \alpha$ as given in equation (3.2), the proof is similar to the proof of Theorem 3.4. □

4. Mathematical model

We use the following notations to design the mathematical formulation of the classical TP.

Notations

- p Origin
- q Destination
- s Total origin nodes
- t Total destination nodes
- c_{pq} Unit commodity transportation cost from pth node of origin to qth node of destination
- a_p Goods available at pth node of origin
- b_q Demand at qth node of destination
- x_{pq} The transported amount of goods from pth node of origin to qth node of destination

The mathematical model of a classical TP is defined as follows:

Model 1

$$\text{minimize} \quad z = \sum_{p=1}^{s}\sum_{q=1}^{t} c_{pq}x_{pq} \tag{4.1}$$

$$\text{subject to} \quad \sum_{q=1}^{t} x_{pq} \leq a_p \ (p = 1, 2, \ldots, s), \tag{4.2}$$

$$\sum_{p=1}^{s} x_{pq} \geq b_q \ (q = 1, 2, \ldots, t), \tag{4.3}$$

$$x_{pq} \geq 0 \ \forall \ p \text{ and } q. \tag{4.4}$$

The constraint (4.2) signifies that the amount of transported goods at the destinations should be less or equal to the availability at the origins. The constraints (4.3) represents that the amount of transported goods from the origins should be greater or equal to the demand at the destinations. The constraint (4.4) indicates that the amount of transported goods cannot be negative. The feasible condition for the optimal solution of Model 1 is $\sum_{p=1}^{s} a_p \geq \sum_{q=1}^{t} b_q$.

A TP with transshipment origins is referred to as MMTP [8]. We employ the following notations throughout the discussion to formulate the mathematical formulation of MMTP.

Notations

- s_1 Total GO nodes
- t_1 Total FD nodes
- s_m Total TO nodes at $(m-1)$th level, $m = 2, 3, \ldots, r$
- r Total levels for origins
- a_p^1 Goods availability at pth node of GO
- a_p^m Goods availability at pth node of mth level TO, $m = 2, 3, \ldots, r$
- b_q Demand at qth node of FD
- α_1^1 A single vehicle carrying capacity from GO nodes to FD nodes
- α_v^m A single vehicle carrying capacity from TO nodes of $(m-1)$th level, $v = 2, 3, \ldots, r$, to TO nodes of $(v-1)$th level, $v = r, r-1, \ldots, 2$
- c_{pq1}^1 Unit commodity transportation cost from pth node of GO to qth node of FD
- c_{pq1}^m Unit commodity transportation cost for transportation from pth node of TO of $(m-1)$th level to qth node of FD where $m = 2, 3, \ldots, r$
- c_{pqv}^1 Unit commodity transportation cost for transportation from pth node of GO to qth node of TO of $(v-1)$th level, $v = 2, 3, \ldots, r$,

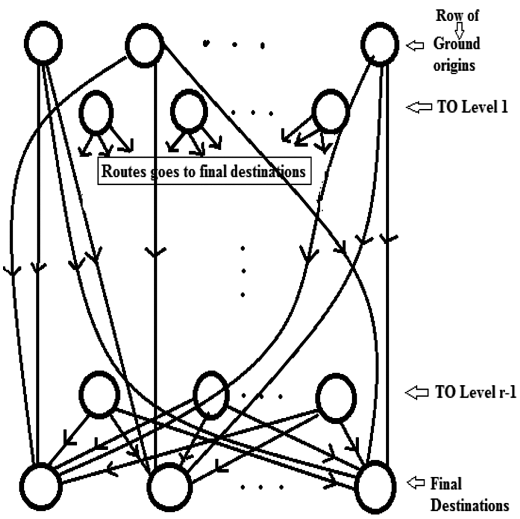

FIGURE 2. Graphical representation of transportation for z^1.

c^m_{pqv}	Unit commodity transportation cost for transportation from pth node of TO of $(m-1)$th level to qth node of TO of $(v-1)$th level, $m = 2, 3, \ldots, r-1$; $v = 2, 3, \ldots, r$ with $m < v$
x^1_{pq1}	Total vehicles required for transportation from pth node of FD to qth node of GO,
x^m_{pq1}	Total vehicles for transportation from pth node of TO of $(m-1)$th level to qth node of FD where $m = 2, 3, \ldots, r$
x^1_{pqv}	Total vehicles for transportation from pth node of GO to qth node of TO of $(v-1)$th level, $v = 2, 3, \ldots, r$
x^m_{pqv}	Total vehicles for transportation from pth node of TO of $(m-1)$th level to qth node of TO of $(v-1)$th level, $m = 2, 3, \ldots, r-1$; $v = 2, 3, \ldots, r$ with $m < v$
z^1	Objective function to minimize transportation cost from GO and all TO nodes to FD nodes
z^r	Objective function to minimize transportation cost from GO to nodes of 1st level TO,
z^i	Objective function to minimize transportation cost from GO and all TO nodes of $(r-i)$th level, $i = 2, 3, \ldots, r-1$ to $(r-i+1)$th level nodes.

To formulate the mathematical model of MMTP, we consider the following ways.

First, we take the objective function (z^1) for the transportation of goods from GO to FD and TO to FD in all levels.

We design the transportation network for the objective function z^1, which is shown in Figure 2, and the routes of transportation are summarized in Table 1.

$$z^1 = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1} \alpha_1^1\left(\left[\underline{c}^{1l}_{pq1}, \underline{c}^{1u}_{pq1}\right], \left[\bar{c}^{1l}_{pq1}, \bar{c}^{1u}_{pq1}\right]\right)x^1_{pq1} + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1} \alpha_1^2\left(\left[\underline{c}^{2l}_{pq1}, \underline{c}^{2u}_{pq1}\right], \left[\bar{c}^{2l}_{pq1}, \bar{c}^{2u}_{pq1}\right]\right)x^2_{pq1}$$
$$+ \ldots + \sum_{p=1}^{s_r}\sum_{q=1}^{t_1} \alpha_1^r\left(\left[\underline{c}^{rl}_{pq1}, \underline{c}^{ru}_{pq1}\right], \left[\bar{c}^{rl}_{pq1}, \bar{c}^{ru}_{pq1}\right]\right)x^r_{pq1}.$$

It is essential to satisfy the demands at the nodes of FD corresponding to the objective function z^1 (*cf.* Fig. 2). Therefore the following constraints must be satisfied.

$$\sum_{p=1}^{s_1} \alpha_1^1 x^1_{pq1} + \sum_{p=1}^{s_2} \alpha_1^2 x^2_{pq1} + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x^r_{pq1} \geq \left(\left[\underline{b}^l_q, \underline{b}^u_q\right], \left[\bar{b}^l_q, \bar{b}^u_q\right]\right) \; (q = 1, 2, \ldots, t_1).$$

TABLE 1. Routes of transportation corresponding to objective function z^1.

Route of transportation	Objective function
GO to FD	$\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_1^1\big([\underline{c}_{pq1}^{1l},\underline{c}_{pq1}^{1u}],[\bar{c}_{ij1}^{1l},\bar{c}_{pq1}^{1u}]\big)x_{pq1}^1$
TO of level 1 to FD	$\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_1^2\big([\underline{c}_{pq1}^{2l},\underline{c}_{pq1}^{2u}],[\bar{c}_{pq1}^{2l},\bar{c}_{pq1}^{2u}]\big)x_{pq1}^2$;
$\vdots$	$\vdots$
TO of level $r-1$ to FD	$\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_1^r\big([\underline{c}_{pq1}^{rl},\underline{c}_{ij1}^{ru}],[\bar{c}_{pq1}^{rl},\bar{c}_{pq1}^{ru}]\big)x_{pq1}^r$

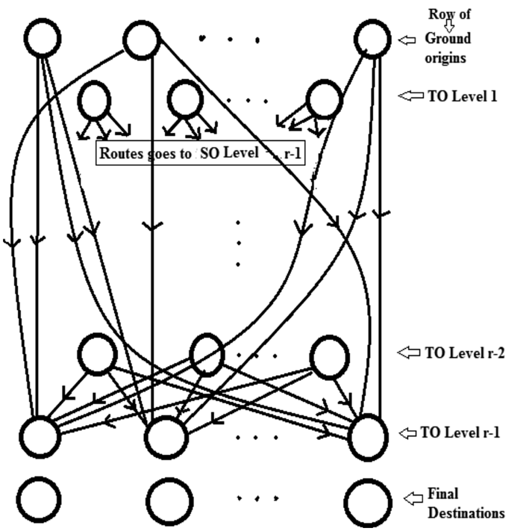

FIGURE 3. Graphical representation of transportation for z^2.

To construct the objective function (z^2) for transportation to TO of levels $r-1$ and $r-2$ from GO we design the network corresponding to the objective function z^2, which is shown in Figure 3, and routes of transportation are summarized in Table 2.

$$z^2=\sum_{p=1}^{s_1}\sum_{q=1}^{s_r}\alpha_r^1\big([\underline{c}_{pqr}^{1l},\underline{c}_{pqr}^{1u}],[\bar{c}_{pqr}^{1l},\bar{c}_{pqr}^{1u}]\big)x_{pqr}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{s_r}\alpha_r^2\big([\underline{c}_{pqr}^{2l},\underline{c}_{pqr}^{2u}],[\bar{c}_{pqr}^{2l},\bar{c}_{pqr}^{2u}]\big)x_{pqr}^2$$
$$+\ldots+\sum_{p=1}^{s_{r-1}}\sum_{q=1}^{s_r}\alpha_{r-1}^r\left(\left[\underline{c}_{pqr}^{(r-1)l},\underline{c}_{pqr}^{(r-1)u}\right],\left[\bar{c}_{pqr}^{(r-1)l},\bar{c}_{pqr}^{(r-1)u}\right]\right)x_{pqr}^{r-1}.$$

In the transportation corresponding to the objective function z^2 (*cf.* Fig. 3), the stored items in TO nodes of level $r-1$ must be larger than the amount of goods transported from TO nodes of level $r-1$ to FD nodes. Therefore, the following constraints need to be satisfied.

$$\sum_{q=1}^{t_1}\alpha_1^r x_{mq1}^r\leq\sum_{p=1}^{s_1}\alpha_r^1x_{pmr}^1+\sum_{p=1}^{s_2}\alpha_r^2x_{pmr}^2+\ldots+\sum_{p=1}^{s_{r-1}}\alpha_r^{r-1}x_{pmr}^{r-1}\ (m=1,2,\ldots,s_r).$$

TABLE 2. Routes of transportation corresponding to objective function z^2.

Route of transportation	Objective function
GO to TO of level $r-1$	$\sum_{p=1}^{s_1}\sum_{q=1}^{s_r} \alpha_r^1\big(\big[\underline{c}_{pqr}^{1l}, \underline{c}_{pqr}^{1u}\big], \big[\bar{c}_{pqr}^{1l}, \bar{c}_{pqr}^{1u}\big]\big)x_{pqr}^1$
TO of level 1 to TO of level $r-1$	$\sum_{p=1}^{s_2}\sum_{q=1}^{s_r} \alpha_r^2\big(\big[\underline{c}_{pqr}^{2l}, \underline{c}_{pqr}^{2u}\big], \big[\bar{c}_{pqr}^{2l}, \bar{c}_{pqr}^{2u}\big]\big)x_{pqr}^2$
$\vdots$	$\vdots$
TO of level $r-2$ to TO of level $r-1$	$\sum_{p=1}^{s_{r-1}}\sum_{q=1}^{s_r} \alpha_{r-1}^r\Big(\Big[\underline{c}_{pqr}^{(r-1)l}, \underline{c}_{pqr}^{(r-1)u}\Big], \Big[\bar{c}_{pqr}^{(r-1)l}, \bar{c}_{pqr}^{(r-1)u}\Big]\Big)x_{pqr}^{r-1}$

Again the goods stored at TO nodes of level $r-1$ must be less than its storing capacity.

$$\sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1} \leq a_r^m,\ (m = 1, 2, \ldots, s_r).$$

In level 1, for transporting the goods from GO to TO we construct z^p $(p = 2, 3, \ldots, r-1)$ in the same way as mentioned above.

In level 1, the construction of the objective function (z^r) for transportation from GO to TO is described in the following way.

In Figure 4, the transportation network is shown corresponding to the objective function z^r. In level 1 the objective function for the transportation from GO to TO is:

$$z^r = \sum_{p=1}^{s_1}\sum_{q=1}^{s_2} \alpha_1^2\big(\big[\underline{c}_{pq2}^{1l}, \underline{c}_{pq2}^{1u}\big], \big[\bar{c}_{pq2}^{1l}, \bar{c}_{pq2}^{1u}\big]\big)x_{pq2}^1.$$

In the transportation corresponding to the objective function z^r (*cf.* Fig. 4), the stored items in TO nodes of level 1 must be larger than the amount of transported goods to TO nodes of levels t, $t = 2, 3, \ldots, (r-1)$ and FD node from there. Therefore, we consider the following constraints as:

$$\sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1\ (m = 1, 2, \ldots, s_2).$$

Again, the stored capacity at TOs of level 1 must be less than the storing capacity.

$$\sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1 \leq a_m^2\ (m = 1, 2, \ldots, s_2).$$

The complete MMTP model (see Fig. 2) is the network aggregated by using the objective functions z^i, $i = 1, 2, \ldots, r$ together with the constraints needed to construct the objective functions z^i, $i = 1, 2, \ldots, r$. We explain the mathematical model on MMTP as follows:

Model 2

$$\text{minimize} \quad z = z^1 + z^2 + \ldots + z^r,$$

$$z^1 = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1} \alpha_1^1\big(\big[\underline{c}_{pq1}^{1l}, \underline{c}_{pq1}^{1u}\big], \big[\bar{c}_{pq1}^{1l}, \bar{c}_{pq1}^{1u}\big]\big)x_{pq1}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1} \alpha_1^2\big(\big[\underline{c}_{pq1}^{2l}, \underline{c}_{pq1}^{2u}\big], \big[\bar{c}_{pq1}^{2l}, \bar{c}_{pq1}^{2u}\big]\big)x_{pq1}^2$$

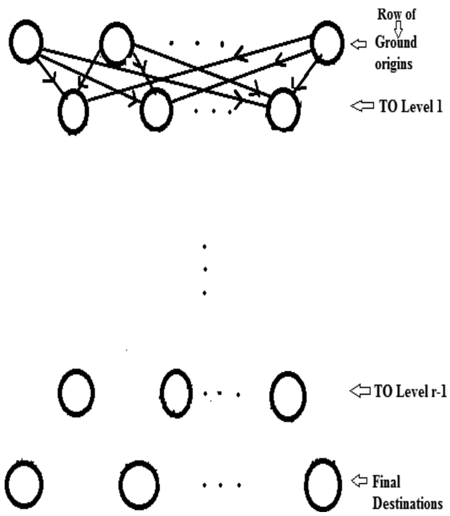

FIGURE 4. Graphical representation of transportation for z^3.

$$
+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_1^r\big(\big[\underline{c}_{pq1}^{rl},\underline{c}_{pq1}^{ru}\big],\big[\bar{c}_{pq1}^{rl},\bar{c}_{pq1}^{ru}\big]\big)x_{pq1}^r,
$$

$$
z^2=\sum_{p=1}^{s_1}\sum_{q=1}^{s_r}\alpha_r^1\big(\big[\underline{c}_{pqr}^{1l},\underline{c}_{pqr}^{1u}\big],\big[\bar{c}_{pqr}^{1l},\bar{c}_{pqr}^{1u}\big]\big)x_{pqr}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{s_r}\alpha_r^2\big(\big[\underline{c}_{pqr}^{2l},\underline{c}_{pqr}^{2u}\big],\big[\bar{c}_{pqr}^{2l},\bar{c}_{pqr}^{2u}\big]\big)x_{pqr}^2
$$

$$
+\ldots+\sum_{p=1}^{s_{r-1}}\sum_{q=1}^{s_r}\alpha_r^{r-1}\Big(\Big[\underline{c}_{pqr}^{(r-1)l},\underline{c}_{pqr}^{(r-1)u}\Big],\Big[\bar{c}_{pqr}^{(r-1)l},\bar{c}_{pqr}^{(r-1)u}\Big]\Big)x_{pqr}^{r-1},
$$

$$
z^3=\sum_{p=1}^{s_1}\sum_{q=1}^{s_{r-1}}\alpha_{r-1}^1\Big(\Big[\underline{c}_{pq(r-1)}^{1l},\underline{c}_{pq(r-1)}^{1u}\Big],\Big[\bar{c}_{pq(r-1)}^{1l},\bar{c}_{pq(r-1)}^{1u}\Big]\Big)x_{pq(r-1)}^1
$$

$$
+\sum_{p=1}^{s_2}\sum_{q=1}^{s_{r-1}}\alpha_r^2\Big(\Big[\underline{c}_{pq(r-1)}^{2l},\underline{c}_{pq(r-1)}^{2u}\Big],\Big[\bar{c}_{pq(r-1)}^{2l},\bar{c}_{pq(r-1)}^{2u}\Big]\Big)x_{pq(r-1)}^2
$$

$$
+\ldots+\sum_{p=1}^{s_{r-2}}\sum_{q=1}^{s_{r-1}}\alpha_{r-2}^{r-1}\Big(\Big[\underline{c}_{pq(r-1)}^{(r-2)l},\underline{c}_{pq(r-1)}^{(r-2)u}\Big],\Big[\bar{c}_{pq(r-1)}^{(r-2)l},\bar{c}_{pq(r-1)}^{(r-2)u}\Big]\Big)x_{pq(r-1)}^{r-2},
$$

$$
\vdots
$$

$$
z^r=\sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_2^1\big(\big[\underline{c}_{pq2}^{1l},\underline{c}_{pq2}^{1u}\big],\big[\bar{c}_{pq2}^{1l},\bar{c}_{pq2}^{1u}\big]\big)x_{pq2}^1,
$$

subject to the constraints regarding availability at GO and TO of all levels

$$
\sum_{q=1}^{t_1}\alpha_1^1x_{pq1}^1+\sum_{q=1}^{s_r}\alpha_r^1x_{pqr}^1+\ldots+\sum_{q=1}^{s_2}\alpha_2^1x_{pq2}^1
$$

$$
\leq\big(\big[\underline{a}_p^{1l},\underline{a}_p^{1u}\big],\big[\bar{a}_i^{1l},\bar{a}_p^{1u}\big]\big)\ (p=1,2,\ldots,s_1), \tag{4.5}
$$

$$
\sum_{p=1}^{t_1}\alpha_1^2x_{pq1}^1+\sum_{p=1}^{s_r}\alpha_r^2x_{pqr}^2+\ldots+\sum_{p=1}^{s_3}\alpha_3^2x_{pq3}^2
$$

$$\leq \left(\left[\underline{a}_p^{2l}, \underline{a}_p^{2u}\right], \left[\bar{a}_p^{2l}, \bar{a}_p^{2u}\right]\right) \; (p = 1, 2, \ldots, s_2), \tag{4.6}$$

$$\sum_{q=1}^{t_1} \alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{m_4} \alpha_4^3 x_{pq4}^3$$

$$\leq \left(\left[\underline{a}_p^{3l}, \underline{a}_p^{3u}\right], \left[\bar{a}_p^{3l}, \bar{a}_p^{3u}\right]\right) \; (p = 1, 2, \ldots, s_3), \tag{4.7}$$

$$\vdots$$

$$\sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq \left(\left[\underline{a}_p^{rl}, \underline{a}_p^{ru}\right], \left[\bar{a}_p^{rl}, \bar{a}_p^{ru}\right]\right), \tag{4.8}$$

the constraints regarding least demands at the FD

$$\sum_{p=1}^{s_1} \alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2} \alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x_{pq1}^r$$

$$\geq \left(\left[\underline{b}_q^{1l}, \underline{b}_q^{1u}\right], \left[\bar{b}_q^{1l}, \bar{b}_q^{1u}\right]\right), \tag{4.9}$$

the constraints regarding storing and distributing

of goods at nodes of TO of all levels

$$\sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1, \tag{4.10}$$

$$\sum_{q=1}^{t_1} \alpha_1^3 x_{mq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{mqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{mq4}^3$$

$$\leq \sum_{p=1}^{s_1} \alpha_3^1 x_{pm3}^1 + \sum_{p=1}^{s_2} \alpha_3^2 x_{pm3}^2, \tag{4.11}$$

$$\vdots$$

$$\sum_{q=1}^{t_1} \alpha_1^r x_{mq1}^r \leq \sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1}$$

$$\leq \left(\left[\underline{a}_r^{ml}, \underline{a}_r^{mu}\right], \left[\bar{a}_r^{ml}, \bar{a}_r^{mu}\right]\right), \tag{4.12}$$

$$x_{pqk}^{(v)} \geq 0 \; \forall \; p, \; q, \; v \text{ and } k. \tag{4.13}$$

Model 2 has a feasible solution only when $\sum_{p=1}^{s_1}\left(\left[\underline{a}_p^l, \underline{a}_p^u\right], \left[\bar{a}_p^l, \bar{a}_p^u\right]\right) \geq \sum_{q=1}^{t_1}\left(\left[\underline{b}_q^l, \underline{b}_q^u\right], \left[\bar{b}_q^l, \bar{b}_q^u\right]\right)$.

In Model 2, the maximum number of decision variables is $(s_1 \times s_2 \times \cdots \times s_r \times t_1)$. The feasible region of the proposed model is composed of considering the accompanying discussions.

- There is s_1 number of availability constraints that are present in equation (4.5) for GO nodes.
- For FD nodes, the number of demand constraints considered in equation (4.6) is t_1.
- Since there are storage capacity constraints in the TO nodes, we assume $(s_2 + s_3 + \ldots + s_r)$ number of inequalities from equations (4.7) and (4.8).
- Again, the conveyed measure of merchandise from the TOs does not exceed the supplied amount of merchandise to the separate TOs. To do this, we present $(s_2 + s_3 + \ldots + s_r)$ number of in-equations from equations (4.9) to (4.13).

In this manner, the detailed mathematical model comprises $(s_1 \times s_2 \times \dots \times s_r \times t_1)$ number of factors and $[2(s_2 + s_3 + \ldots + s_r) + s_1 + t_1]$ limitations along with the non-negative conditions.

4.1. Equivalent deterministic model

Due to the presence of RI in MMTP, we cannot directly solve MMTP. Thus, we convert MMTP into the deterministic model by accompanying the following properties.

- $\left[\underline{c}^{kl}_{pq1}, \underline{c}^{ku}_{pq1}\right] \subseteq \left[\bar{c}^{kl}_{pq1}, \bar{c}^{ku}_{pq1}\right] \Rightarrow \bar{c}^{kl}_{pq1} \leq \underline{c}^{kl}_{pq1} \leq \underline{c}^{ku}_{pq1} \leq \bar{c}^{ku}_{pq1}$ $(k = 1, 2, \ldots, r)$.
- $\left[\underline{c}^{vl}_{pq2}, \underline{c}^{vu}_{pq2}\right] \subseteq \left[\bar{c}^{vl}_{pq2}, \bar{c}^{vu}_{pq2}\right] \Rightarrow \bar{c}^{vl}_{pq1} \leq \underline{c}^{vl}_{pq2} \leq \underline{c}^{vu}_{pq2} \leq \bar{c}^{vu}_{pq2}$ $(v = 1, 2, \ldots, r-1)$.
- $\left[\underline{c}^{1l}_{pqr}, \underline{c}^{1u}_{pqr}\right] \subseteq \left[\bar{c}^{1l}_{pqr}, \bar{c}^{1u}_{pqr}\right] \Rightarrow \bar{c}^{1l}_{pqr} \leq \underline{c}^{1l}_{pqr} \leq \underline{c}^{1u}_{pqr} \leq \bar{c}^{1u}_{pqr}$.
- $\left[\underline{a}^{l}_{p}, \underline{a}^{u}_{p}\right] \subseteq \left[\bar{a}^{l}_{p}, \bar{a}^{u}_{p}\right] \Rightarrow \bar{a}^{l}_{p} \leq \underline{a}^{l}_{p} \leq \underline{a}^{u}_{p} \leq \bar{a}^{u}_{p}$.
- $\left[\underline{b}^{l}_{q}, \underline{b}^{u}_{q}\right] \subseteq \left[\bar{b}^{l}_{q}, \bar{b}^{u}_{q}\right] \Rightarrow \bar{b}^{l}_{q} \leq \underline{b}^{l}_{q} \leq \underline{b}^{u}_{q} \leq \bar{b}^{u}_{q}$.

Concerning Model 2, we develop the methodology and theoretical background by defining some sets that lead us to solve MMTP with RI approximation.

$$
\bar{U}^l = x \in \mathbb{R}^n : \begin{cases}
\sum_{q=1}^{t_1} \alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r} \alpha_r^1 x_{pqr}^1 + \ldots + \sum_{q=1}^{s_2} \alpha_2^1 x_{pq2}^1 \leq \bar{a}_p^{1l} \\
\sum_{p=1}^{t_1} \alpha_1^2 x_{pq1}^1 + \sum_{p=1}^{s_r} \alpha_r^2 x_{pqr}^2 + \ldots + \sum_{p=1}^{s_3} \alpha_3^2 x_{pq3}^2 \leq \bar{a}_p^{2l} \\
\sum_{q=1}^{t_1} \alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{pq4}^3 \leq \bar{a}_p^{3l} \\
\vdots \\
\sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq \bar{a}_p^{rl} \\
\sum_{p=1}^{s_1} \alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2} \alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x_{pq1}^r \geq \bar{b}_q^{1l} \\
\sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1 \\
\sum_{q=1}^{t_1} \alpha_1^3 x_{mq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{mqr}^3 + \ldots + \sum_{q=1}^{m_4} \alpha_4^3 x_{mq4}^3 \\
\leq \sum_{p=1}^{s_1} \alpha_3^1 x_{pm3}^1 + \sum_{q=1}^{s_2} \alpha_3^2 x_{pm3}^2 \\
\vdots \\
\sum_{q=1}^{t_1} \alpha_1^r x_{mq1}^r \leq \sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1} \\
\leq \bar{a}_r^{ml} \\
x_{pqk}^{(v)} \geq 0 \ \forall\ p,\ q,\ v \text{ and } k.
\end{cases}
$$

$$\underline{U}^l = x \in \mathbb{R}^n : \begin{cases} \sum_{q=1}^{t_1} \alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r} \alpha_r^1 x_{pqr}^1 + \ldots + \sum_{q=1}^{s_2} \alpha_2^1 x_{pq2}^1 \leq \underline{a}_p^{1l} \\ \sum_{p=1}^{t_1} \alpha_1^2 x_{pq1}^1 + \sum_{p=1}^{s_r} \alpha_r^2 x_{pqr}^2 + \ldots + \sum_{p=1}^{s_3} \alpha_3^2 x_{pq3}^2 \leq \underline{a}_p^{2l} \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{pq4}^3 \leq \underline{a}_p^{3l} \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq \underline{a}_p^{rl} \\ \sum_{p=1}^{s_1} \alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2} \alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x_{pq1}^r \geq \underline{b}_q^{1l} \\ \sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1 \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{mq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{mqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{mq4}^3 \\ \leq \sum_{p=1}^{s_1} \alpha_3^1 x_{pm3}^1 + \sum_{p=1}^{s_2} \alpha_3^2 x_{pm3}^2 \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{mq1}^r \leq \sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1} \\ \leq \underline{a}_r^{ml} \\ x_{pqk}^{(v)} \geq 0 \ \forall \ p, \ q, \ v \text{ and } k. \end{cases}$$

$$\underline{U}^u = x \in \mathbb{R}^n : \begin{cases} \sum_{q=1}^{t_1} \alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r} \alpha_r^1 x_{pqr}^1 + \ldots + \sum_{q=1}^{s_2} \alpha_2^1 x_{pq2}^1 \leq \underline{a}_p^{1u} \\ \sum_{p=1}^{t_1} \alpha_1^2 x_{pq1}^1 + \sum_{p=1}^{s_r} \alpha_r^2 x_{pqr}^2 + \ldots + \sum_{p=1}^{s_3} \alpha_3^2 x_{pq3}^2 \leq \underline{a}_p^{2u} \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{pq4}^3 \leq \underline{a}_p^{3u} \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq \underline{a}_p^{ru} \\ \sum_{p=1}^{s_1} \alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2} \alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x_{pq1}^r \geq \underline{b}_q^{1u} \\ \sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1 \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{mq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{mqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{mq4}^3 \\ \leq \sum_{p=1}^{s_1} \alpha_3^1 x_{pm3}^1 + \sum_{p=1}^{s_2} \alpha_3^2 x_{pm3}^2 \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{mq1}^r \leq \sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1} \\ \leq \underline{a}_r^{mu} \\ x_{pqk}^{(v)} \geq 0 \ \forall \ p, \ q, \ v \text{ and } k. \end{cases}$$

$$\bar{U}^u = x \in \mathbb{R}^n : \begin{cases} \sum_{q=1}^{t_1} \alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r} \alpha_r^1 x_{pqr}^1 + \ldots + \sum_{q=1}^{s_2} \alpha_2^1 x_{pq2}^1 \leq \bar{a}_p^{1u} \\ \sum_{p=1}^{t_1} \alpha_1^2 x_{pq1}^1 + \sum_{p=1}^{s_r} \alpha_r^2 x_{pqr}^2 + \ldots + \sum_{p=1}^{s_3} \alpha_3^2 x_{pq3}^2 \leq \bar{a}_p^{2u} \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{pq4}^3 \leq \bar{a}_p^{3u} \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq \bar{a}_p^{ru} \\ \sum_{p=1}^{s_1} \alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2} \alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r} \alpha_1^r x_{pq1}^r \geq \bar{b}_q^{1u} \\ \sum_{q=1}^{t_1} \alpha_1^2 x_{mq1}^2 + \sum_{q=1}^{s_r} \alpha_r^2 x_{mqr}^2 + \ldots + \sum_{q=1}^{s_3} \alpha_3^2 x_{mq3}^2 \leq \sum_{p=1}^{s_1} \alpha_2^1 x_{pm2}^1 \\ \sum_{q=1}^{t_1} \alpha_1^3 x_{mq1}^3 + \sum_{q=1}^{s_r} \alpha_r^3 x_{mqr}^3 + \ldots + \sum_{q=1}^{s_4} \alpha_4^3 x_{mq4}^3 \\ \leq \sum_{p=1}^{s_1} \alpha_3^1 x_{pm3}^1 + \sum_{p=1}^{s_2} \alpha_3^2 x_{pm3}^2 \\ \vdots \\ \sum_{q=1}^{t_1} \alpha_1^r x_{mq1}^r \leq \sum_{p=1}^{s_1} \alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2} \alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}} \alpha_r^{r-1} x_{pmr}^{r-1} \\ \leq \bar{a}_r^{mu} \\ x_{pqk}^{(v)} \geq 0 \;\forall\; p,\; q,\; v \text{ and } k. \end{cases}$$

To proof the equivalence between the deterministic model and the MMTP *i.e.*, Model 2, we derive the following proposition.

Proposition 4.1. *The relation between the sets* $\bar{U}^l$, $\underline{U}^l$, $\underline{U}^u$, *and* $\bar{U}^u$ *is* $\bar{U}^l \leq \underline{U}^l \leq \underline{U}^u \leq \bar{U}^u$.

Proof. From the properties of rough interval we have,

$$\left[\underline{a}_p^{il}, \underline{a}_p^{iu}\right] \subseteq \left[\bar{a}_p^{il}, \bar{a}_p^{iu}\right] \implies \bar{a}_p^{il} \leq \underline{a}_p^{il} \leq \underline{a}_p^{iu} \leq \bar{a}_p^{iu},\; i = 1, 2, \ldots, r.$$
$$\left[\underline{b}_p^{1l}, \underline{b}_p^{1u}\right] \subseteq \left[\bar{b}_p^{1l}, \bar{b}_p^{1u}\right] \implies \bar{b}_p^{1l} \leq \underline{b}_p^{1l} \leq \underline{b}_p^{1u} \leq \bar{b}_p^{1u}.$$
$$\left[\underline{a}_r^{ml}, \underline{a}_r^{mu}\right] \subseteq \left[\bar{a}_r^{ml}, \bar{a}_r^{mu}\right] \implies \bar{a}_r^{ml} \leq \underline{a}_r^{ml} \leq \underline{a}_r^{mu} \leq \bar{\alpha}_r^{mu}.$$

From the above relations, we have for any $x \in \underline{U}^l$, $x \in \bar{U}^l$, $x \in \underline{U}^u$ and $x \in \bar{U}^u$. □

Next, we formulate two problems from Model 2 using interval costs, and these are referred to as TP-1 and TP-2. Furthermore, TP-1 and TP-2 are reduced to two crisp valued TPs, referred to as TP-1.1 and TP-1.2 from TP-1 and TP-2.1 and TP-2.2 from TP-2.

TP-1

$$\begin{aligned} \text{minimize} \quad & \underline{z} = \underline{z}^{1l} + \underline{z}^{2l} + \ldots + \underline{z}^{rl}, \\ \underline{z}^{1l} = & \sum_{p=1}^{s_1} \sum_{q=1}^{t_1} \alpha_1^1 \left[\underline{c}_{pq1}^{1l}, \underline{c}_{pq1}^{1u}\right] x_{pq1}^1 + \sum_{p=1}^{s_2} \sum_{q=1}^{t_1} \alpha_1^2 \left[\underline{c}_{pq1}^{2l}, \underline{c}_{pq1}^{2u}\right] x_{pq1}^2 + \ldots \\ & + \sum_{p=1}^{s_r} \sum_{q=1}^{t_1} \alpha_1^r \left[\underline{c}_{pq1}^{rl}, \underline{c}_{pq1}^{ru}\right] x_{pq1}^r, \end{aligned}$$

$$\underline{z}^{2l} = \sum_{p=1}^{s_1}\sum_{q=1}^{s_r}\alpha_r^1\left[\underline{c}_{pqr}^{1l}, \underline{c}_{pqr}^{1u}\right]x_{pqr}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{s_r}\alpha_r^2\left[\underline{c}_{pqr}^{2l}, \underline{c}_{pqr}^{2u}\right]x_{pqr}^2$$

$$+ \ldots + \sum_{p=1}^{s_{r-1}}\sum_{q=1}^{s_r}\alpha_{r-1}^r\left[\underline{c}_{pqr}^{(r-1)l}, \underline{c}_{pqr}^{(r-1)u}\right]x_{pqr}^r,$$

$$\vdots$$

$$\underline{z}^{rl} = \sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_2^1\left[\underline{c}_{pq2}^{1l}, \underline{c}_{pq2}^{1u}\right]x_{pq2}^1,$$

$$\text{subject to}\quad \sum_{q=1}^{t_1}\alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r}\alpha_r^1 x_{pqr}^1 + \ldots + \sum_{q=1}^{s_2}\alpha_2^1 x_{pq2}^1 \le \left[\underline{a}_p^{1l}, \underline{a}_p^{1u}\right], \tag{4.14}$$

$$\sum_{p=1}^{t_1}\alpha_1^2 x_{pq1}^1 + \sum_{p=1}^{s_r}\alpha_r^2 x_{pqr}^2 + \ldots + \sum_{p=1}^{s_3}\alpha_3^2 x_{pq3}^2 \le \left[\underline{a}_p^{2l}, \underline{a}_p^{2u}\right], \tag{4.15}$$

$$\sum_{q=1}^{t_1}\alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r}\alpha_r^3 x_{pqr}^3 + \ldots + \sum_{q=1}^{s_4}\alpha_4^3 x_{pq(4)}^3 \le \left[\underline{a}_p^{3l}, \underline{a}_p^{3u}\right], \tag{4.16}$$

$$\vdots$$

$$\sum_{q=1}^{t_1}\alpha_1^r x_{pq1}^r \le \left[\underline{a}_p^{rl}, \underline{a}_p^{ru}\right], \tag{4.17}$$

$$\sum_{p=1}^{s_1}\alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2}\alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r}\alpha_1^r x_{pq1}^r \ge \left[\underline{b}_q^{1l}, \underline{b}_q^{1u}\right], \tag{4.18}$$

$$\sum_{q=1}^{t_1}\alpha_1^r x_{mq1}^r \le \sum_{p=1}^{s_1}\alpha_r^1 x_{pmr}^1 + \sum_{p=1}^{s_2}\alpha_r^2 x_{pmr}^2 + \ldots + \sum_{p=1}^{s_{r-1}}\alpha_r^{r-1} x_{pmr}^{r-1}$$

$$\le \left[\underline{a}_r^{ml}, \underline{a}_r^{mu}\right], \tag{4.19}$$

the constraints (4.10)–(4.13).

Replacing $\underline{c}_{pq1}^{kl}$ by $\bar{c}_{pq1}^{kl}$ $(k = 1, 2, \ldots, r)$ $\underline{c}_{pq1}^{ku}$ by $\bar{c}_{pq1}^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{c}_{pqr}^{vl}$ by $\bar{c}_{pqr}^{vl}$ $(v = 1, 2, \ldots, r-1)$ $\underline{c}_{pqr}^{vu}$ by $\bar{c}_{pqr}^{vu}$ $(v = 1, 2, \ldots, r-1)$, $\underline{c}_{pq2}^{1l}$ by $\bar{c}_{pq2}^{1l}$, $\underline{c}_{pq2}^{1u}$ by $\bar{c}_{pq2}^{1u}$, $\underline{a}_p^{kl}$ by $\bar{a}_p^{kl}$ $(k = 1, 2, \ldots, r)$, $\underline{a}_p^{ku}$ by $\bar{a}_p^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{a}_r^{ml}$ by $\bar{a}_r^{ml}$, $\underline{a}_r^{mu}$ by $\bar{a}_r^{mu}$, we consider TP-1 as TP-2.

TP-1.1

$$\text{minimize}\quad \underline{Z} = \underline{Z}^{1l} + \underline{Z}^{2l} + \ldots + \underline{Z}^{rl},$$

$$\underline{Z}^{1l} = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_1^1\underline{c}_{pq1}^{1l}x_{pq1}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_1^2\underline{c}_{pq1}^{2l}x_{pq1}^2 + \ldots +$$

$$\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_1^r\underline{c}_{pq1}^{rl}x_{pq1}^r,$$

$$\underline{Z}^{2l} = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_2^1\underline{c}_{pq2}^{1l}x_{pq2}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_2^2\underline{c}_{pq2}^{2l}x_{pq2}^2 + \ldots +$$

$$\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_2^r \underline{c}_{pq2}^{(r-1)l} x_{pq2}^r,$$

$$\vdots$$

$$\underline{Z}^{rl} = \sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_r^1 \underline{c}_{pqr}^{1l} x_{pqr}^1,$$

$$\text{subject to} \quad \sum_{q=1}^{t_1}\alpha_1^1 x_{pq1}^1 + \sum_{q=1}^{s_r}\alpha_2^1 x_{pq2}^1 + \ldots + \sum_{q=1}^{s_2}\alpha_r^1 x_{pqr}^1 \leq \underline{a}_p^l, \tag{4.20}$$

$$\sum_{p=1}^{s_1}\alpha_1^1 x_{pq1}^1 + \sum_{p=1}^{s_2}\alpha_1^2 x_{pq1}^2 + \ldots + \sum_{p=1}^{s_{r-1}}\alpha_1^r x_{pq1}^r \geq \underline{b}_q^l, \tag{4.21}$$

$$\sum_{q=1}^{t_1}\alpha_1^2 x_{pq1}^2 + \sum_{q=1}^{s_r}\alpha_2^2 x_{pq2}^2 + \ldots + \sum_{q=1}^{s_3}\alpha_{r-1}^2 x_{pq(r-1)}^2 \leq \underline{a}_p^{1l}, \tag{4.22}$$

$$\sum_{q=1}^{t_1}\alpha_1^3 x_{pq1}^3 + \sum_{q=1}^{s_r}\alpha_2^3 x_{pq2}^3 + \ldots + \sum_{q=1}^{s_4}\alpha_{r-2}^3 x_{pq(r-2)}^3 \leq \underline{a}_p^{2l}, \tag{4.23}$$

$$\vdots$$

$$\sum_{q=1}^{t_1}\alpha_1^r x_{pq1}^r \leq \underline{a}_p^{rl} \ (p = 1, 2, \ldots, s_r), \tag{4.24}$$

the constraints (4.10)–(4.13).

Replacing $\underline{c}_{pq1}^{kl}$ by $\underline{c}_{pq1}^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{c}_{pqr}^{vl}$ by $\underline{c}_{pqr}^{vu}$ $(v = 1, 2, \ldots, r-1)$, $\underline{c}_{pq2}^{1l}$ by $\underline{c}_{pq2}^{1u}$, $\underline{a}_p^{kl}$ by $\underline{a}_p^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{a}_r^{ml}$ by $\underline{a}_r^{mu}$, we choose TP-1.1 as TP-1.2.

Similarly, substituting $\underline{c}_{pq1}^{kl}$ by $\bar{c}_{pq1}^{kl}$ $k = 1, 2, \ldots, r)$, $\underline{c}_{pqr}^{vl}$ by $\bar{c}_{pqr}^{vl}$ $(v = 1, 2, \ldots, r-1)$, $\underline{c}_{pq2}^{1l}$ by $\bar{c}_{pq2}^{1l}$, $\underline{a}_p^{kl}$ by $\bar{a}_p^{kl}$ $(k = 1, 2, \ldots, r)$, $\underline{a}_r^{ml}$ by $\bar{a}_r^{ml}$, we take TP-1.1 as TP-2.1, and replacing $\underline{c}_{pq1}^{kl}$ by $\bar{c}_{pq1}^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{c}_{pqr}^{vl}$ by $\bar{c}_{pqr}^{vu}$ $(v = 1, 2, \ldots, r-1)$, $\underline{c}_{pq2}^{1l}$ by $\bar{c}_{pq2}^{1u}$, $\underline{a}_p^{kl}$ by $\bar{a}_p^{ku}$ $(k = 1, 2, \ldots, r)$, $\underline{a}_r^{ml}$ by $\bar{a}_r^{mu}$, we denote TP-2.1 as TP-2.2.

5. Solution procedure

Model 2 (MMTP) contains the transportation parameters in the form of rough intervals. Thus, an algorithm is adopted for producing four crisp MMTP models as follows:

Algorithm

The procedure to extract the solution of Model 2 is prescribed in the following steps.

Step 1. From Model 2, we develop two TPs involving interval cost, given by TP-1 and TP-2 in Section 4.

Step 2. We find the surely optimal range $[\underline{z}^{vl}, \underline{z}^{vu}]$ by solving TP-1 by breaking it into two classical TPs that are given by TP-1.1 and TP-1.2 in Section 4.

Step 3. In a similar way as described in Step 2, we extract the possible optimal range $[\bar{z}^{kl}, \bar{z}^{ku}]$ by solving TP-2.

Step 4. We have three possible outcomes according to the set of decision variables as follows.

Step 4.1. The main problem, *i.e.*, transportation problem rough interval cost (TPRIC), has a rough range when TP-1 and TP-2 have their optimal ranges. The rough range of TPRIC is $([\underline{z}^{vl}, \underline{z}^{vu}], [\bar{z}^{kl}, \bar{z}^{ku}])$.

Step 4.2. TPRIC has an unbounded range when TP-1 and TP-2 have unbounded range.

Step 4.3. TPRIC has no feasible solution once TP-1 and TP-2 have no feasible solution.

The corresponding flowchart of the algorithm is presented in Figure 5.

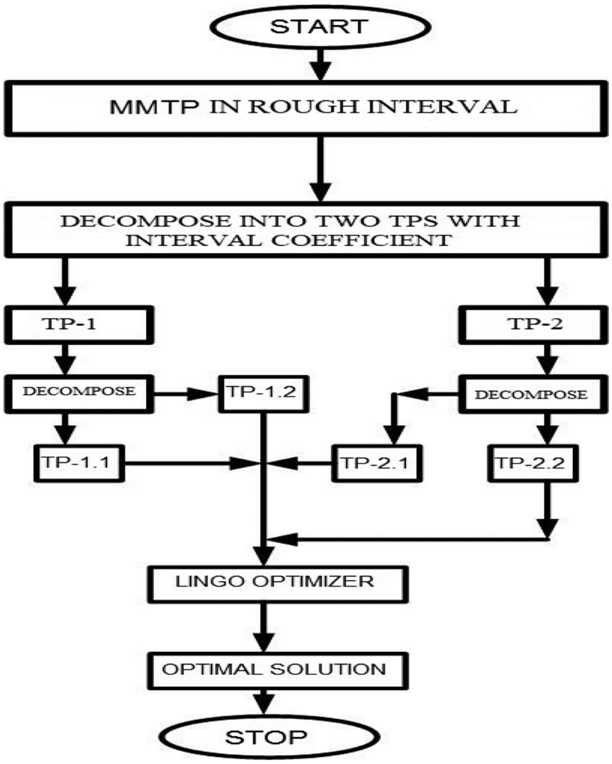

FIGURE 5. Flowchart for solving MMTP under RI.

5.1. Rough chance constrained programming

Before utilizing rough chance constrained programming (RCCP) in the proposed model, we first construct the following objective functions and use these in the RCCP model.

$$\begin{aligned}
\text{Consider}\quad & Z_1 = z_{11} + z_{21} + \ldots + z_{r1},\\
& Z_2 = z_{12} + z_{22} + \ldots + z_{r2},\\
& Z_3 = z_{13} + z_{23} + \ldots + z_{r3},\\
& Z_4 = z_{14} + z_{24} + \ldots + z_{r4}.\\
\text{where}\quad & z_{11} = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1} \alpha_1^1 \underline{c}_{pq1}^{1l} x_{pq1}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1} \alpha_1^2 \underline{c}_{pq1}^{2l} x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r}\sum_{q=1}^{t_1} \alpha_1^r \underline{c}_{pq1}^{rl} x_{pq1}^r,\\
& z_{21} = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1} \alpha_2^1 \underline{c}_{pq2}^{1l} x_{pq2}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1} \alpha_2^2 \underline{c}_{pq2}^{2l} x_{pq2}^2 + \ldots + \sum_{p=1}^{s_r}\sum_{q=1}^{t_1} \alpha_2^r \underline{c}_{pq2}^{(r-1)l} x_{pq2}^r,\\
& \vdots\\
& z_{r1} = \sum_{p=1}^{s_1}\sum_{q=1}^{s_2} \alpha_r^1 \underline{c}_{pqr}^{1l} x_{pqr}^1,\\
& z_{12} = \sum_{p=1}^{s_1}\sum_{q=1}^{t_1} \alpha_1^1 \bar{c}_{pq1}^{1l} x_{pq1}^1 + \sum_{p=1}^{s_2}\sum_{q=1}^{t_1} \alpha_1^2 \bar{c}_{pq1}^{2l} x_{pq1}^2 + \ldots + \sum_{p=1}^{s_r}\sum_{q=1}^{t_1} \alpha_1^r \bar{c}_{pq1}^{rl} x_{pq1}^r,
\end{aligned}$$

$$z_{22}=\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_2^1\bar{c}_{pq2}^{1l}x_{pq2}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_2^2\bar{c}_{pq2}^{2l}x_{pq2}^2+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_2^r\bar{c}_{pq2}^{(r-1)l}x_{pq2}^r,$$

$$\vdots$$

$$z_{r2}=\sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_r^1\bar{c}_{pqr}^{1l}x_{pqr}^1,$$

$$z_{13}=\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_1^1\underline{c}_{pq1}^{1u}x_{pq1}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_1^2\underline{c}_{pq1}^{2u}x_{pq1}^2+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_1^r\underline{c}_{pq1}^{ru}x_{pq1}^r,$$

$$z_{23}=\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_2^1\underline{c}_{pq2}^{1u}x_{pq2}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_2^2\underline{c}_{pq2}^{2u}x_{pq2}^2+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_2^r\underline{c}_{pq2}^{(r-1)u}x_{pq2}^r,$$

$$\vdots$$

$$z_{r3}=\sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_r^1\underline{c}_{pqr}^{1u}x_{pqr}^1,$$

$$z_{14}=\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_1^1\bar{c}_{pq1}^{1u}x_{pq1}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_1^2\bar{c}_{pq1}^{2u}x_{pq1}^2+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_1^r\left[\bar{c}_{pq1}^{rl},\bar{c}_{pq1}^{ru}\right]x_{pq1}^r,$$

$$z_{24}=\sum_{p=1}^{s_1}\sum_{q=1}^{t_1}\alpha_2^1\bar{c}_{pq2}^{1u}x_{pq2}^1+\sum_{p=1}^{s_2}\sum_{q=1}^{t_1}\alpha_2^2\bar{c}_{pq2}^{2u}x_{pq2}^2+\ldots+\sum_{p=1}^{s_r}\sum_{q=1}^{t_1}\alpha_2^r\bar{c}_{pq2}^{(r-1)u}x_{pq2}^r,$$

$$\vdots$$

$$z_{r4}=\sum_{p=1}^{s_1}\sum_{q=1}^{s_2}\alpha_r^1\bar{c}_{pqr}^{1u}x_{pqr}^1.$$

We need to minimize the least objective function $\bar{z}$, because the main problem is a minimization problem and all of its parameters are RIs that satisfy $\mathrm{Tr}\{z\geq\bar{z}\}\geq\alpha$, and $\alpha\in(0,1]$ is the degree of trust or confidence level, which implies maximizing the α-optimistic value $z_{\inf(\alpha)}$ of z, which explicitly means that the optimum objective value will be less than $\bar{z}$ with the trust level α. We also consider the α-pessimistic value of the source and conveyance constraints with confidence level α and α-optimistic value of demand constraints. Thus, RCCP is:

$$\min(\min\bar{z})=Z_1+Z_2+Z_3+Z_4,$$

$$\text{subject to}\quad \sum_{q=1}^{t_1}\alpha_1^1x_{pq1}^1+\sum_{q=1}^{s_r}\alpha_2^1x_{pq2}^1+\ldots+\sum_{q=1}^{s_2}\alpha_r^1x_{pqr}^1\leq a_{p\inf}(\alpha), \tag{5.1}$$

$$\sum_{p=1}^{s_1}\alpha_1^1x_{pq1}^1+\sum_{p=1}^{s_2}\alpha_1^2x_{pq1}^2+\ldots+\sum_{p=1}^{s_{r-1}}\alpha_1^rx_{pq1}^r\geq b_{q\sup}(\alpha), \tag{5.2}$$

$$\sum_{q=1}^{t_1}\alpha_1^2x_{pq1}^2+\sum_{q=1}^{s_r}\alpha_2^2x_{pq2}^2+\ldots+\sum_{q=1}^{s_3}\alpha_{r-1}^2x_{pq(r-1)}^2$$
$$\leq a_{p\inf}^1(\alpha), \tag{5.3}$$

$$\sum_{q=1}^{t_1}\alpha_1^3x_{pq1}^3+\sum_{q=1}^{s_r}\alpha_2^3x_{pq2}^3+\ldots+\sum_{q=1}^{s_4}\alpha_{r-2}^3x_{pq(r-2)}^3$$

$$\leq a^2_{p\,\inf}(\alpha), \tag{5.4}$$

$$\vdots$$

$$\sum_{q=1}^{t_1} \alpha_1^r x_{pq1}^r \leq a^r_{p\,\inf}(\alpha), \tag{5.5}$$

the constraints (4.10)–(4.13).

Here, $a_{p\,\inf}(\alpha)$, $b_{q\,\sup}(\alpha)$, $a^1_{p\,\inf}(\alpha)$, $a^2_{p\,\inf}(\alpha)$, and $a^r_{p\,\inf}(\alpha)$ are to be evaluated using the definitions of α-optimistic and α-pessimistic, for all p, q. Using the definition of the α-optimistic value, we have the objective function as well as the RCCP model equivalent to the following model.

Model 3

$$\begin{aligned} \text{minimize} \quad & z' = Z_1 + Z_2 + Z_3 + Z_4, \\ \text{subject to} \quad & \text{the constraints (5.1)–(5.5)} \\ & \text{the constraints (4.10)–(4.13).} \end{aligned}$$

where

$$z' = \begin{cases} (1-2\alpha)Z_4 + 2\alpha Z_1, & \text{if } \alpha \leq \frac{Z_1 - Z_2}{2(Z_4 - Z_2)}; \\ 2(1-\alpha)Z_4 + (2\alpha - 1)Z_1, & \text{if } \alpha \geq \frac{Z_3 + Z_4 - 2Z_1}{2(Z_3 - Z_2)}; \\ \frac{Z_2(Z_3 - Z_1) + Z_1(Z_3 - Z_1) - 2\alpha(Z_3 - Z_1)(Z_4 - Z_2)}{(Z_3 - Z_1) + (Z_4 - Z_2)}, & \text{otherwise.} \end{cases} \tag{5.6}$$

We now formulate another RCCP for the proposed model to minimize the maximum objective function $\bar{z}$, satisfying $\mathrm{Tr}\{z \leq \bar{z}\} \geq \alpha$, where $\alpha \in (0, 1]$ is the specified trust or confidence level, which implies that we maximize the α-optimistic value $z_{\sup(\alpha)}$ of z, which directly indicates that the optimum objective value will be less than $\bar{z}$ with the trust level α. In addition, we find the α-pessimistic value of the source and conveyance constraints with confidence level α and α-optimistic value of demand constraints. Thus, RCCP becomes:

$$\begin{aligned} \min(\max \bar{z}) &= \bar{z}^1 + \bar{z}^2 + \ldots + \bar{z}^r, \\ \text{subject to} \quad & \text{the constraints (5.1)–(5.5)} \\ & \text{the constraints (4.10)–(4.13).} \end{aligned}$$

Using the definition of α-pessimistic to the objective function, we have an equivalent form of the above RCCP model as follows.

Model 4

$$\begin{aligned} \text{minimize} \quad & z'' = Z_1 + Z_2 + Z_3 + Z_4, \\ \text{subject to} \quad & \text{the constraints (5.1)–(5.5)} \\ & \text{the constraints (4.10)–(4.13).} \end{aligned}$$

where

$$z'' = \begin{cases} (1-2\alpha)Z_2 + 2\alpha Z_4, & \text{if } \alpha \leq \frac{Z_4 - Z_3}{2(Z_4 - Z_2)}; \\ 2(1-\alpha)Z_2 + (2\alpha - 1)Z_4, & \text{if } \alpha \geq \frac{2Z_4 - Z_1 - Z_2}{2(Z_4 - Z_2)}; \\ \frac{Z_4(Z_3 - Z_1) + Z_3(Z_3 - Z_1) - 2\alpha(Z_3 - Z_1)(Z_4 - Z_2)}{(Z_3 - Z_1) + (Z_4 - Z_2)}, & \text{otherwise.} \end{cases} \tag{5.7}$$

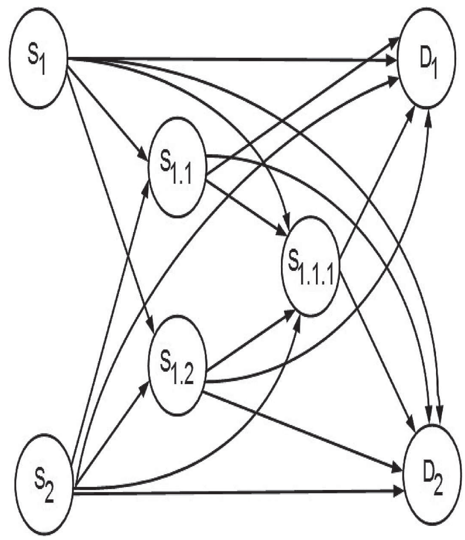

FIGURE 6. Network corresponding to numerical example.

6. Numerical example

In this section, we introduce a numerical example for the proposed model to clarify the utility of MMTP. Consider two factories of daily used goods for a company are located in cities S_1 and S_2. The company has three warehouses ($S_{1.1}$, $S_{1.2}$, and $S_{1.1.1}$) in which goods are stored and then sold into markets D_1 and D_2 (see Fig. 6). It is considered that any number of goods can be transported from S_1 and S_2 to $S_{1.1}$ and $S_{1.2}$, respectively; $S_{1.1}$ and $S_{1.2}$ to $S_{1.1.1}$; $S_{1.1.1}$ to D_1 and D_2. In any other routes, items are transported in multiple forms due to vehicle restrictions. The limit of the vehicle to transport products is 500 items from S_1 and S_2 to D_1 and D_2. Thus, there is an issue to deliver the products when the requests at the destinations D_1 and D_2 are not a multiple of 500. Once more, the vehicles are carrying the products from TOs $S_{1.1}$ and $S_{1.2}$ to destinations D_1 and D_2 with the limit of 50 items. With this consideration, again there is an issue to transfer products when the measures of goods are not in a multiple of 50. In addition to that, there is a destination $S_{1.1.1}$ that takes the products from S_1, S_2, $S_{1.1}$, and $S_{1.2}$ and supplies these to the destinations D_1 and D_2. The transportation from the inside $S_{1.1.1}$ to the destinations D_1 and D_2 has no such vehicle limit; *i.e.*, any measure of products can be transported between the nodes. The customary approach of the TP cannot provide any such numerical model to tackle the proposed problem. To take care of the problem, we design a numerical model known as MMTP, based on the above issues.

The accompanying documentations and presumptions are considered to define the numerical model of MMTP.

- The choice factors for transporting the items are taken into account as follows:
  Moving from S_1 and S_2 to D_1 and D_2 is as x^1_{ij1}, by utilizing the shipping path with vehicle limit $\alpha^1_1 = 500$. Moving from $S_{1.1}$ and $S_{1.2}$ to D_1 and D_2 is taken as x^1_{ij2}, by choosing the rail-route with vehicle limit $\alpha^2_1 = 50$. Moving from $S_{1.1.1}$ to D_1 and D_2 is assumed as x^1_{ij3}, **by** using the street path with no vehicle limitation; *i.e.*, $\alpha^3_1 = 1$.
  Moving from S_1 and S_2 to $S_{1.1.1}$ is taken as x^2_{ij1} with vehicle limitation $\alpha^1_2 = 250$.
  Moving from $S_{1.1}$ and $S_{1.2}$ to $S_{1.11}$ is assumed as x^2_{ij2} with no vehicle confinement.
  Moving from S_1 and S_2 to $S_{1.1}$ and $S_{1.2}$ is considered as x^3_{ij1} and there is not any vehicle limitation.
- The possibility of the numerical illustration comprises the accompanying number of constraints.
  The supply limits at the factories S_1 and S_2 are presented by two imperatives. The demands at the last destinations D_1 and D_2 are considered by two limitations. Keeping the limits at the warehouses $S_{1.1}$, $S_{1.2}$,

TABLE 3. Transportation cost from S_1 and S_2 to D_1 and D_2 (in \$).

	D_1	D_2
S_1	$([14, 17], [13, 18])$	$([12, 16], [10, 17])$
S_2	$([14, 16], [13, 18])$	$([17, 21], [16, 22])$

TABLE 4. Transportation cost from $S_{1.1}$ and $S_{1.2}$ to D_1 and D_2 (in \$).

	D_1	D_2
$S_{1.1}$	$([7, 10], [6, 11])$	$([9, 12], [8, 14])$
$S_{1.2}$	$([8, 11], [6, 12])$	$([6, 9], [4, 11])$

TABLE 5. Transportation cost from $S_{1.1.1}$ to D_1 and D_2 (in \$).

	D_1	D_2
$S_{1.1.1}$	$([5, 8], [3, 9])$	$([4, 8], [3, 9])$

TABLE 6. Transportation cost from S_1 and S_2 to $S_{1.1.1}$ (in \$).

	$S_{1.1.1}$
S_1	$([10, 13], [9, 14])$
S_2	$([11, 13], [10, 16])$

TABLE 7. Transportation cost from $S_{1.1}$ and $S_{1.2}$ to $S_{1.1.1}$ (in \$).

	$S_{1.1.1}$
S_1	$([7, 10], [6, 12])$
S_2	$([8, 10], [7, 14])$

TABLE 8. Transportation cost from S_1 and S_2 to $S_{1.1}$ and $S_{1.2}$ (in \$).

	$S_{1.1}$	$S_{1.2}$
S_1	$([4, 7], [3, 8])$	$([3, 5], [2, 8])$
S_2	$([5, 7], [8, 9])$	$([4, 6], [3, 9])$

and $S_{1.1.1}$, give three limitations. The amounts of products circulating from the warehouses $S_{1.1}$, $S_{1.2}$, and $S_{1.1.1}$ do not surpass the measure of putting away things that produce three requirements. Thus, the quantity of limitations in MMTP of the numerical problem is 10.

The transportation costs in the various routes are presented in Tables 3–8.

The accessibility of products at each factory S_1 and S_2 is $([1500, 1700], [1400, 1800])$ units in the RI shape. The most extreme limits of putting away at the warehouses $S_{1.1}$, $S_{1.2}$, and $S_{1.1.1}$ are $([1100, 1400], [1000, 1500])$ units,

$([1200, 1500], [1100, 1600])$ units, and $([900, 1200], [800, 1300])$ units separately in the RI shape. The mathematical model is planned for relating to the accessible information portrayed in Tables 3–8.

Model 5

$$
\begin{aligned}
\text{minimize} \quad & z \\
z &= z_1 + z_2 + z_3, \\
z_1 &= 500\big(([14,17],[13,18])x^1_{111} + ([12,16],[10,17])x^1_{121} + ([14,16],[13,18])x^1_{211} \\
&\quad + ([17,21],[16,22])x^1_{221}\big) + 50\big(([7,10],[6,11])x^2_{111} + ([9,12],[8,14])x^2_{121} \\
&\quad + ([8,11],[6,12])x^2_{211} + ([6,9],[4,11])x^2_{221}\big) + ([5,8],[3,9])x^3_{111} \\
&\quad + ([4,8],[3,9])x^3_{121}, \\
z_2 &= ([4,7],[3,8])x^1_{112} + ([3,5],[2,8])x^1_{122} + ([5,7],[8,9])x^1_{212} + ([4,6],[3,9])x^1_{222}, \\
z_3 &= 250\big(([10,13],[9,14])x^1_{113} + ([11,13],[10,16])x^1_{213}\big) + ([7,10],[6,12])x^2_{113} \\
&\quad + ([8,10],[7,14])x^2_{213}, \\
&\quad 500\big(x^1_{111} + x^1_{121}\big) + 250x^1_{113} + x^1_{112} + x^1_{122} \le ([1500,1700],[1400,1800]), \\
&\quad 500\big(x^1_{211} + x^1_{221}\big) + 250x^1_{213} + x^1_{212} + x^1_{222} \le ([1500,1700],[1400,1800]), \\
&\quad 500\big(x^1_{111} + x^1_{211}\big) + 50\big(x^2_{111} + x^2_{211}\big) + x^3_{111} \ge ([1355,1655],[1255,1755]), \\
&\quad 500\big(x^1_{121} + x^1_{221}\big) + 50\big(x^2_{121} + x^2_{221}\big) + x^3_{121} \ge ([1375,1675],[1275,1775]), \\
&\quad x^1_{112} + x^1_{212} \le ([1100,1400],[1000,1500]), \\
&\quad x^1_{122} + x^1_{222} \le ([1200,1500],[1100,1600]), \\
&\quad 250\big(x^1_{113} + x^1_{213}\big) + x^2_{113} + x^2_{213} \le ([900,1200],[800,1300]), \\
&\quad x^1_{112} + x^1_{212} \ge 50\big(x^2_{111} + x^2_{121}\big) + x^2_{113}, \\
&\quad x^1_{122} + x^1_{222} \ge 50\big(x^2_{211} + x^2_{221}\big) + x^2_{213}, \\
&\quad 250\big(x^1_{113} + x^1_{213}\big) + x^2_{113} + x^2_{213} \ge x^3_{111} + x^3_{121}, \\
&\quad x^k_{pqr} \ge 0, \ \forall\ p,\ q,\ r,\ k.
\end{aligned}
$$

Model 5 reduces to four deterministic TPs namely: TP-1.1, TP-1.2, TP-2.1, and TP-2.2. We then solve TPs by using our proposed algorithm. The minimum values of objective functions for TP-1.1, TP-1.2, TP-2.1, and TP-2.2 are \$30 185, \$51 280, \$21 980, and \$63 100, respectively. The optimal solutions of TP-1.1, TP-1.2, TP-2.1, and TP-2.2 are presented in Table 9. In Table 10, the total amounts of transported goods reaching the final destinations D_1 and D_2 are calculated. Similarly, in Tables 11 and 12, the total amounts of goods reached to the TOs are shown. Table 13 lists the amount of transported goods stored at the TOs and final destinations.

We now solve Model 5 by two different techniques such as RCCP and using the expected value operator.

TABLE 9. Optimal solutions by the proposed method.

Problem	Optimal value	Optimal solution
TP-1.1	30 185	$x^1_{121}=1$, $x^2_{111}=20$, $x^2_{211}=7$, $x^2_{221}=17$, $x^3_{111}=5$, $x^3_{121}=25$, $x^2_{113}=30$, $x^1_{122}=1000$, $x^1_{212}=1030$, $x^1_{222}=200$, and other variables are zero.
TP-1.2	51 280	$x^1_{211}=3$, $x^2_{111}=3$, $x^2_{121}=3$, $x^2_{221}=30$, $x^3_{111}=5$, $x^3_{121}=25$, $x^2_{113}=30$, $x^1_{112}=200$, $x^1_{122}=1500$, $x^1_{212}=130$, and other variables are zero.
TP-2.1	21 980	$x^1_{121}=1$, $x^2_{111}=18$, $x^2_{211}=7$, $x^2_{221}=15$, $x^3_{111}=5$, $x^3_{121}=25$, $x^2_{113}=30$, $x^1_{112}=900$, $x^1_{212}=30$, $x^1_{222}=1100$, and other variables are zero.
TP-2.2	63 100	$x^1_{121}=3$, $x^1_{211}=3$, $x^2_{111}=5$, $x^2_{221}=5$, $x^3_{111}=5$, $x^3_{121}=25$, $x^2_{113}=30$, $x^1_{112}=255$, $x^1_{122}=45$, $x^1_{212}=25$, $x^1_{222}=205$, and other variables are zero.

TABLE 10. The amounts of transported goods to final destinations D_1 and D_2.

TP	Variable	x^1_{111}	x^1_{121}	x^1_{211}	x^1_{221}	x^2_{111}	x^2_{121}	x^2_{211}	x^2_{221}	x^3_{111}	x^3_{121}
TP-1.1 (z^{TP1})	Value	0	500	0	0	1000	0	350	850	5	25
TP-1.2 (z^{TP2})	Value	0	0	1500	0	150	150	0	1500	5	25
TP-2.1 (z^{TP3})	Value	0	500	0	0	900	0	350	750	5	25
TP-2.2 (z^{TP4})	Value	0	1500	1500	0	250	0	0	250	5	25

TABLE 11. The amounts of transported goods to transhipment origins $S_{1.1}$ and $S_{1.2}$.

TP	Variable	x^1_{112}	x^1_{122}	x^1_{212}	x^1_{222}
TP-1.1 (z^{TP1})	Value	0	1000	1030	200
TP-1.2 (z^{TP2})	Value	200	1500	130	0
TP-2.1 (z^{TP3})	Value	900	0	30	1100
TP-2.2 (z^{TP4})	Value	255	45	25	205

TABLE 12. The amounts of transported goods to transhipment origin $S_{1.1.1}$.

TP	Variable	x^1_{113}	x^1_{213}	x^2_{113}	x^2_{213}
TP-1.1 (z^{TP1})	Value	0	0	30	0
TP-1.2 (z^{TP2})	Value	0	0	30	0
TP-2.1 (z^{TP3})	Value	0	0	30	0
TP-2.2 (z^{TP4})	Value	0	0	30	0

Table 13. The amounts of transported goods stored at all TOs and FDs.

TP	Node	$S_{1.1}$	$S_{1.2}$	$S_{1.1.1}$	D_1	D_2
TP-1.1 (z^{TP1})	Value	1030	1200	30	1355	1375
TP-1.2 (z^{TP2})	Value	330	1500	30	1655	1675
TP-2.1 (z^{TP3})	Value	930	1100	30	1255	1275
TP-2.2 (z^{TP4})	Value	280	250	30	1755	1775

Table 14. Optimal solutions by RCCP.

Problem	Optimal value	Optimal solution
min z'	17 071.20	$x^1_{111} = 2$, $x^2_{111} = 6$, $x^2_{121} = 23$, $x^2_{221} = 3$, $x^1_{122} = 215$, $x^1_{212} = 1490$, $x^1_{222} = 15$, and other variables are zero.
min z''	52 709	$x^1_{111} = 2$, $x^2_{111} = 5$, $x^2_{111} = 5$, $x^2_{111} = 5$, $x^2_{221} = 2$, $x^3_{111} = 15$, $x^3_{121} = 35$, $x^2_{113} = 50$, $x^1_{112} = 28$, $x^1_{122} = 100$, $x^1_{212} = 1422$, and other variables are zero.

Table 15. Optimal solutions by using the expected value operator.

Value of η	Optimal value	Optimal solution
0.5	33 385	$x^1_{121} = 1$, $x^2_{111} = 25$, $x^2_{211} = 5$, $x^2_{221} = 20$, $x^3_{111} = 5$, $x^3_{121} = 25$, $x^2_{213} = 30$, $x^1_{112} = 35$, $x^1_{122} = 1065$, $x^1_{212} = 1215$, $x^1_{222} = 215$, and other variables are zero.
1	46 500	$x^1_{211} = 3$, $x^2_{121} = 3$, $x^2_{221} = 27$, $x^3_{111} = 5$, $x^3_{121} = 25$, $x^2_{113} = 30$, $x^1_{112} = 180$, $x^1_{122} = 1350$, and other variables are zero.
1.5	26 180	$x^1_{121} = 1$, $x^2_{111} = 23$, $x^2_{211} = 7$, $x^2_{221} = 20$, $x^3_{111} = 5$, $x^3_{121} = 25$, $x^2_{113} = 30$, $x^1_{112} = 1100$, $x^1_{212} = 80$, $x^1_{222} = 1350$, and other variables are zero.
2	53 370	$x^1_{121} = 3$, $x^1_{211} = 3$, $x^3_{111} = 5$, $x^3_{121} = 25$, $x^2_{113} = 30$, $x^1_{112} = 30$, and other variables are zero.

Solution by RCCP. We reduce Model 5 into the form of Models 3 and 4, and the solutions after solving Models 3 and 4 are presented in Table 14.

On account of RCCP, we solve the problem using similar data by developing two RCCP models with a trust level $\alpha = 0.99$. Utilizing the α-optimistic and α-pessimistic definitions, we determine the numerical calculations associated with the source, demand, and conveyance capacities, individually and subsequently. Thus, we derive two RCCP results given in Table 14. Here, observing the outcome in Table 14, we conclude that the objective value lies in the interval $[17\,071.20, 527\,09]$ for a trust level of $\alpha = 0.99$; additionally for $0.93 < \alpha \leq 1$, $z_{\sup(\alpha)} \leq z_{\inf(\alpha)}$. We also observe that through the outcome, $z' \leq z''$ justifies the truth values of the outcome. In addition, we have the optimum expected objective value as 41 636.25 derived from $E(\text{least transportation cost}) = \frac{1}{4}(z^{\text{TP1}} + z^{\text{TP2}} + z^{\text{TP3}} + z^{\text{TP4}})$, which is inside the interval. It definitely validates the result.

Solution by using expected value operator. We solve the proposed model using the expected value operator by taking four different values of η. The solution by using the expected value operator is shown in Table 15.

TABLE 16. Optimal solutions by the proposed method.

Problem	Optimal value	Optimal solution
TP-1.1	35 470	$x^1_{111} = 0.25,\ x^1_{121} = 2.75,\ x^1_{211} = 2.46,\ x^1_{221} = 0$
TP-1.2	53 280	$x^1_{111} = 0,\ x^1_{121} = 3.35,\ x^1_{211} = 3.31,\ x^1_{221} = 0$
TP-2.1	29 065	$x^1_{111} = 0.25,\ x^1_{121} = 2.55,\ x^1_{211} = 2.26,\ x^1_{221} = 0$
TP-2.2	61 765	$x^1_{111} = 0.05,\ x^1_{121} = 3.55,\ x^1_{211} = 3.46,\ x^1_{221} = 0$

TABLE 17. The quantities of goods transported to final destinations D_1 and D_2.

TP	Variable	x^1_{111}	x^1_{121}	x^1_{211}	x^1_{221}
TP-1.1 (z^{TP1})	Value	125	1375	1230	0
TP-1.2 (z^{TP2})	Value	0	1675	1655	0
TP-2.1 (z^{TP3})	Value	125	1275	1130	0
TP-2.2 (z^{TP4})	Value	25	1775	1730	0

Special case of the numerical example

To establish the efficiency of MMTP, let us remove the TOs from the transportation system. Then the problem presented in the numerical example becomes a simple TP with origins $S_1,\ S_2$ and destinations $D_1,\ D_2$. The mathematical model is formulated as follows:

Model 6

We solve Model 6 using proposed method and the solution is listed in Tables 16 and 17 shows the quantities of goods that transported to final destinations D_1 and D_2.

$$
\begin{aligned}
\text{minimize}\quad & z_1\\
z_1 = {} & 500\big(([14,17],[13,18])x^1_{111} + ([12,16],[10,17])x^1_{121} + ([14,16],[13,18])x^1_{211} + ([17,21],[16,22])x^1_{221}\big),\\
& 500\big(x^1_{111} + x^1_{121}\big) \le ([1500,1700],[1400,1800]),\\
& 500\big(x^1_{211} + x^1_{221}\big) \le ([1500,1700],[1400,1800]),\\
& 500\big(x^1_{111} + x^1_{211}\big) \ge ([1355,1655],[1255,1755]),\\
& 500\big(x^1_{121} + x^1_{221}\big) \ge ([1375,1675],[1275,1775]),\\
& x^k_{pqr} \ge 0,\ \forall\ p,\ q,\ r,\ k.
\end{aligned}
$$

On account of RCCP, we solve the problem using similar data by developing two RCCP models with a trust level $\alpha = 0.99$. Utilizing the α-optimistic and α-pessimistic definitions, we determine the numerical calculations associated with the source, demand, and conveyance capacities, individually and subsequently. Thus, we derive two RCCP results as given in Table 18. Here, observing the outcome in Table 18, we conclude that the objective value lies in the interval $[13\,284.70, 47\,075.64]$ for a trust level of $\alpha = 0.99$; additionally for $0.93 < \alpha \le 1$, $z_{\sup(\alpha)} \le z_{\inf(\alpha)}$. We also observe that in our outcome, $z' \le z''$ justifies the truth values of the outcome. In addition, we have the optimum expected objective value as \$44 895 derived from $E(\text{least transportation cost}) = \frac{1}{4}(z^{\mathrm{TP1}} + z^{\mathrm{TP2}} + z^{\mathrm{TP3}} + z^{\mathrm{TP4}})$, which is inside the interval. It definitely validates the result.

In real situations, if there is any empty space in conveyance then the purchaser needs to pay transportation cost of a full trip. In this situation, the transportation cost to be paid and the number of conveyance can be obtained by Model 6 (see Tab. 19).

TABLE 18. Optimal solutions by RCCP.

Problem	Optimal value	Optimal solution
min z'	13 284.70	$x^1_{111} = 1.53$, $x^1_{121} = 1.57$, $x^1_{211} = 0$, $x^1_{221} = 0$
min z''	47 075.64	$x^1_{111} = 1.834$, $x^1_{121} = 2.55$, $x^1_{211} = 0.676$, $x^1_{221} = 0$

TABLE 19. Optimal cost and number of conveyance in optimal routes by RCCP.

Problem	Optimal value	Optimal solution
min z'	17 180	$x^1_{111} = 2$, $x^1_{121} = 2$, $x^1_{211} = 0$, $x^1_{221} = 0$
min z''	55 930	$x^1_{111} = 2$, $x^1_{121} = 3$, $x^1_{211} = 1$, $x^1_{221} = 0$

Based on the obtained solutions, we conclude that, in the absence of TOs the range of transportation cost $[17\,180, 55\,930]$ using simple TP is larger than the range of transportation cost $[17\,071.20, 52\,709]$ obtained by MMTP. Consequently, we establish that the absence of TOs increases the total transportation cost.

7. Results and discussion

We now compare the solution of Model 5, obtained by the proposed method, with the solutions obtained by two different techniques, RCCP and using the expected value operator.

Effectiveness of MMTP. To test the effectiveness of the proposed mathematical model of MMTP, we describe the diverse possibilities associated with the numerical example as follows.

- Suppose that the routes from the supply points, S_1 and S_2 to destination points D_1 and D_2 are by sea. Thus, delivering the goods is done by a ship. Clearly, a sufficient amount of goods is transported by the ship, and the amount is 500 units. In that situation, if there are no other nodes available like $S_{1.1}$, $S_{1.2}$, and $S_{1.1.1}$, at that point the defined TP is classical TP. In this case, we see that there exists a feasible solution to the proposed problem that does not minimize the transportation cost as in each destination the least requirements are $([1355, 1655], [1255, 1755])$ units and $([1375, 1675], [1275, 1775])$ units of goods that are not necessarily a multiple of 500 in the optimal solution. Furthermore, if the purchaser would like to purchased amount of goods which is not a multiple to 500, there will be empty place in conveyance but he should have to pay for the fully loaded conveyance cost. Due to this reason, D_1 and D_2 will have to pay transportation cost of a lesser amount of goods by giving transportation cost a larger amount of goods of fully loaded vehicles. Thus, transportation cost increases as shown in **"Special case of the numerical example"**. The traditional TP is hence not sufficient to provide a clear conclusion without considering the TOs as discussed this study.
- We again assume that there is a connection through the railway between $S_{1.1}$ and $S_{1.2}$ to D_1 and D_2. At that point, the capacity for transports through the railway is high for which we consider that a single transport needs a 50 units. In that circumstance, the problem is solved without considering the TO $S_{1.1.1}$ (*i.e.*, utilizing the value of the variables connecting the node $S_{1.1.1}$ is "0".), and the total transportation cost for TP-1.1, TP-1.2, TP-2.1, and TP-2.2 are separately \$30 800, \$43 400, \$24 300, and \$50 500. We note that the transportation cost obtained by the proposed method for TP-1.1 and TP-2.1 is less than the transportation cost in the case when we have not chosen the TO $S_{1.1.1}$. The transportation costs for TP-1.2 and TP-2.2 obtained by the proposed method are more when we have not taken the TO $S_{1.1.1}$. In the case when the node $S_{1.1.1}$ is not used, then the amount of transported goods must be a multiple of 50 at the final destinations D_1 and D_2. Therefore, the amount of transported goods is larger than the minimum

requirement at the final destinations. Due to this reason, the transportation cost increases when the node $S_{1.1.1}$ is not considered in the model. Choosing the TO, $S_{1.1.1}$ decreases the transportation cost, which is what we discuss herein the study.

– In a similar way, if we formulate the mathematical model without considering the transshipment origins $S_{1.1}$ and $S_{1.1.1}$ or $S_{1.2}$ and $S_{1.1.1}$, then the transportation cost will vary depending on the lower and upper values of the rough intervals, and the supply will be more or less.

According to the aforementioned discussions, we analyze that the introduction of a multi-modal system in TP is very much essential to reduce the transportation cost for delivering the goods. However in the classical TP, this is not so.

The above discussion also allows us to introduce the importance of RI and its usefulness in TP. The utility of rough programming is mentioned as follows.

Utility of rough programming. In this paper we solve MMTP by considering a numerical example using different techniques. We make a comparison with the traditional outcome and optimal solution presented by the proposed approach. In most of the solution procedures of an uncertain TP, the obtained solutions are defined as crisp values. Here, we present a rough solution for MMTP with RIs. The solutions obtained by the proposed method for TP-1.1, TP-1.2, TP-2.1, and TP-2.2 are \$30 185, \$51 280, \$21 980 and \$63 100, respectively. Therefore, the rough solution space obtained by the proposed method is $([30\,185, 51\,280], [21\,980, 63\,100])$, and the solution space obtained by RCCP is $([17\,071.20, 52\,709])$. The solutions obtained by using the expected value operator based on different values of η are \$33 385, \$46 500, \$26 180, and \$53 370.

By using RI, we make the solution space of the problem more flexible, but here we observe that the solution space obtained by RCCP does not contain all expected solutions. However, the solution space obtained by classical TP contains all the expected solutions. Thus, observing the results of the three methods, we derive a more flexible solution space when we use the proposed method. Government budgets are generally made yearly. Budget management requires the prediction of government income from several sources for the upcoming year. In this regard, for most cases the government income is considered by an interval number during the budget. Thus, the budget is made under uncertainty. Furthermore, the amounts assigned for different purposes also become uncertain numbers. In this situation if the uncertain parameters are considered as rough data, then the prediction of income and expenditure in several purposes must be included in a surely occurrence region of a rough approximated feasible region. In this context, the approach discussed inthis paper is more fruitful in a prediction-based decision making process. This study is also useful for decision making in several corporate sectors under uncertainty. Furthermore, existing studies of TP including interval parameters may be solved by RCCP to produce better optimal solutions.

8. Conclusion

It is difficult to formulate a mathematical model and to find the least-cost route of transportation when multiple modes are involved in a TP. Considering multiple modes of transportation, this paper has established a new model MMTP, and its solution suggests the selection of a mode of transportation as well as an optimal solution to the problem. In most real-life cases, the data are not crisp. To accommodate these situations, here we have considered the rough intervals in transportation parameters. Furthermore, we present an MMTP in which all parameters are taken as RIs. A new algorithm has been presented to solve MMTP with them. We have demonstrated a solution procedure to solve MMTP using RCCP. The usefulness of the multi-modal criterion has been illustrated along with a brief discussion on the utility of RI.

An MMTP brings new insight into organizations, such as systems, stations, data envelopment analysis models, portfolio selection, financial model, inventory model, and so on. Moreover the proposed study can be performed under various questionable conditions to accommodate more real-life circumstances for choosing an optimum mode of transportation.

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