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PRICE DECISIONS OF FREEBIE PROMOTION WITH RETAILER INFORMATION SHARING

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Abstract. This study investigates two-echelon pricing game models in which a manufacturer provides freebies during a promotional period and a retailer considers whether to share private demand information with the manufacturer. In particular, the manufacturer produces products (*e.g.*, high-end cosmetics) and sells them wholesale to the retailer during the regular selling period. During the promotional period, the manufacturer offers freebies as a means of gaining more profits. First, we investigate four pricing game models by considering the manufacturer's and retailer's different power structures under the condition that the manufacturer provides freebies. Then, numerical examples are used to comparatively analyze the equilibria in the different models. The results offer valuable managerial insights by performing a sensitivity analysis of three parameters, which are the relative length of the promotional period and a sensitivity coefficient and cost coefficient of the freebies.

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1. Introduction

On November 11, 2009, Taobao.com (a comprehensive shopping website) held its first online promotion named Double 11. Since then, the firm's number of participating merchants, promotional strength, and final transaction volume have increased every year. Gradually, owing to the rise in consumer willingness and consumption ability, the number of annual promotional activities, including 618 (June 18), Double 12 (December 12), and special purchases for the Spring Festival, have become popular. Therefore, as a type of promotion, freebie promotion is worth studying. Freebies are appealing and can attract a large number of consumers during promotional periods. For instance, Estee Lauder (the world's largest skincare, cosmetics, and fragrance company) produces a kind of high-end face cream and sells them in wholesale to retailers, and then the retailers sell them to consumers at a retail price (2500, prices for Mainland China) during the regular sales period. However, during the Double 11 promotional period in 2020, the retailer offered an additional freebie (*e.g.*, the same face cream) worth 1500 (prices for Mainland China) to consumers. The offering of freebies during a promotional period not only enhances the customers' expectation of purchasing the product by increasing the individual utility towards this product but also attracts more customers and expands the market without harming the manufacturer's and retailer's payoffs.

Keywords. Freebie promotion, pricing, supply chain, information sharing.

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To increase profits, the manufacturers and retailers in e-commerce platforms (*e.g.*, Taobao. com and JD.com) or offline physical retail stores have adopted different sales promotional strategies [2,7,10,13,16–18,20,21,27,29], including price discounts [1,2,12,13,21,22,25], rebate [16], and freebies (*i.e.*, offered freebies with a certain market value along with the product) [7,10,17,21]. The duration and promotional efforts of sales promotions vary. In China, sales promotions in June and November usually have the greatest intensity and last the longest, followed by promotions in March and September. With the increase in the number of promotions every year, consumers are given opportunities to visit online platforms or offline retailers during the promotional period. The marketers' choice of promotion type is influenced by product category, and firms are more likely to choose freebies than rebates or discounts. Freebies are beneficial to both consumers and promotion providers to a certain extent because, as opposed to consumers who buy a product during a regular sales period, consumers who buy the product during the promotional period not only obtain the product at the same price but also obtain a freebie with a certain value. In the long run, compared with price discounts, the freebie offerings would not affect the overall value of products [1].

In a vertical supply chain, a manufacturer sells at wholesale the products to a retailer, and then the retailer sells them to the end consumers. Both firms possess private information about the product, but they need to decide whether to share the information with the other firm. In practice, the retailer can privately access the precise demand information, whereas the manufacturer can hardly access it because the manufacturer usually does not directly serve the consumers. Furthermore, the retailer may not voluntarily share the observed demand information with the manufacturer because it may be used by the upstream manufacturer to improve its pricing and marketing strategies. In fact, some of the related studies [8,16,23] suggest that the retailer should maintain its own information advantage over the manufacturer.

This study considers a simple two-echelon supply chain with information asymmetry between the manufacturer and the retailer, in which the former provides freebies during the promotional period and the latter considers whether to share her demand information with the manufacturer. In particular, the manufacturer produces the products (*e.g.*, high-end cosmetics) and sells them wholesale to the retailer during the regular sales period. To gain more profits during the promotional period, a manufacturer may also offer freebies. Montaner *et al.* [18] show that high equity brand products with a fitted freebie can positively influence consumers' purchasing intention, from which large profit margins can be derived by these high-equity companies thereafter.

We take the cosmetics industry as an example to illustrate our model. Cosmetics can be roughly divided into two categories: low- and mid-end cosmetics and high-end cosmetics. Freebie promotion is common throughout the cosmetics industry. Cosmetics companies (*e.g.*, L'Oreal Group and Estee Lauder Group) have different pricing and promotional strategies between high-end and low- and mid-end products. First, the retail price of high-end cosmetics during the promotional period is the same as that during the regular sales period, except that retailers provide additional freebies to attract consumers during the promotional period. Furthermore, during the promotional period, retailers who sell low- and mid-end cosmetics not only offer freebies but also combine them with price reductions for promotion. Therefore, the freebies and the products with a retail price during the regular sales period both affect the consumers' purchase decisions, which makes the model more complicated. This study focuses on investigating the impact of freebies on the manufacturer's promotional strategies from the aspect of freebie promotion and sale of high-end cosmetics. In the future, researchers may focus on low- and mid-end cosmetics and freebie promotion and investigate the impact of price reduction promotions on a firm's profits.

This research makes several contributions. First, few researchers have attempted to investigate the topic on freebie promotion. On the basis of the existing sales promotional strategies, this research investigates the impact of the manufacturer's freebie promotion and the retailer's information sharing on their pricing decisions. Therefore, this research fills the gap in the literature with respect to freebie promotion and information sharing. Second, the related studies have generally focused on trade promotion (from a manufacturer to a retailer, *e.g.*, [2,7,27,29]), retailer promotion (from a retailer to consumers through the retail selling channel, *e.g.*, [13,20,27,29]), and manufacturer promotion (from a manufacturer to consumers through the direct selling channel, *e.g.*, [16,29]). In contrast to the previous studies, we consider a freebie promotion scenario in which a manufacturer provides freebie to consumers through the retail channel. Then, we focus on the impact of freebie promotion decisions of the manufacturer and information sharing decisions of the retailer, including the cost of freebies provided by the manufacturer and the strength of the promotion (*i.e.*, measured by the relative length of the promotional period and the sensitivity coefficient of freebies). Third, from the retailer's perspective, we consider the retailer's decision of sharing demand information on the premise that the manufacturer provides freebies. Our findings indicate that when the relative length of the promotional period is short, the retailer has no incentive to actively share her private information with the manufacturer. By contrast, when the relative length of the promotional period is sufficiently long, the retailer is willing to actively share demand information with the manufacturer. Finally, we consider different power structures among members to obtain some insights into the decision-making of firms by comparing the equilibrium results for scenarios in which the retailer does not share information and the manufacturer offers freebie promotion.

The rest of this paper is organized as follows. Section 2 describes the problem and provides the notations of the model setup. Section 3 discuss the proposed five pricing game models and the corresponding equilibrium results. Section 4 gives the comparative equilibrium solutions of the different Stackelberg and Bertrand game models and presents the sensitivity analysis of the main parameters used to compare different models. Section 5 presents the conclusions. The proof of the proposition and corollaries are given in the Appendix.

2. Literature

Our study covers many studies on promotions [2, 7, 10, 13, 16–18, 20, 21, 27], promotional period [29]. From the perspective of promotion methods, including rebate promotion [16], price discounts [1, 2, 12, 13, 21, 22, 25], gift card promotion [12, 13], and freebie promotion [7, 10, 17, 21], our work is mainly related to the last type of promotion. The studies related to promotion can be divided into two categories. The first category can be explored *via* experiments to investigate the following influence factors: promotion type and strategies [10, 21], consumers' response to gift promotions [8, 18], and consumers' preference for freebie promotional packages (buying the product and obtaining a different free item) [17].

The second category can be used to analyze the impact of promotion on the pricing strategies and profits of firms in the supply chain by establishing quantitative models. The organizer of the promotion can be the manufacturer (or the supplier) [7, 24] or the retailer [12, 13] or both the manufacturer and retailer [27, 29]. Some industries, such as electronic products [7], food, and cosmetics, currently adopt a combination of freebie promotion and advance selling. Sarkar *et al.* [22] have studied the promotional strategy of a supplier to a buyer *via* the price discount mechanism and showed that this policy can sufficiently coordinate the supply chain. Pal and Sarkar [20] have considered a dual channel in which the retailer implements strategic promotional effort to expand market demand, thereby increasing the product's recyclability and green innovation level. Regarding two-level of promotion, the manufacturer and retailer usually offer promotions at the same time. The difference is that the object of the manufacturer's promotion usually varies; that is, the object may be the retailer or the consumers. For instance, Yang *et al.* [29] have investigated the effects of product promotion in remanufacturing, in which the manufacturer and retailer offer promotions *via* the direct channel and retail channel, respectively. Meanwhile, Tsao *et al.* [27] have built two-level promotion models to compare three different promotion policies, in which the manufacturer provides trade promotion to a retailer, whereas the retailer offers the sales promotion to customers. In addition, the power structure between upstream and downstream firms can also affect the promotion decisions. For example, Zhang *et al.* [30] have examined the impact of relative channel status on the pricing decisions of firms under different power structures of a dual exclusive channel system (*i.e.*, two manufacturers distribute their respective products through an exclusive retailer). Our work is related to this category of literature in that during the promotional period, the manufacturer offers freebies to a retailer, whereas the retailer sells the product and offers a freebie to the end retail market. Furthermore, in consideration of the power structure between the manufacturer and retailer, we specify three different scenarios to examine the impact of the power structure on the pricing strategies of supply chain members under the condition that the retailer does not share information to the manufacturer.

Our study also contributes to the work on the impact of information sharing of the retailer [3, 4, 6, 8–10, 12, 19, 23, 28, 30]. In a traditional simple supply chain, upstream companies (*e.g.*, a manufacturer or a supplier) possess private quality information [26] or cost information [5, 11, 24, 31] about the products, whereas downstream buyers (or the retailer) possess private customer demand information, including demand forecast information, [6, 8–10, 12, 19, 23, 28, 30], disrupted demand information [3], and product information [4]. This situation implies that information asymmetry exists among members of the supply chain. Wang *et al.* [28] have examined how information asymmetry including that between a supplier and a buyer with private cost information, affects the supplier's expected profit, in which the level of information asymmetry is measured by the variance of the supplier's subjective distribution. An appropriate combination of information sharing and contrast strategy can achieve win-win outcomes for both the supply chain and its members [14]. Besides the private product information of third parties, they can also publish product-cost information, resulting in consumers increasingly acquiring such an information. Therefore, supply chain members need to decide whether to share with each other or disclose information about products to consumers. For example, a manufacturer can disclose certain product information to consumers [19,26,28], or a retailer can share product-related information with consumers [4]. Many studies have focused on the sharing of product demand information by downstream retailer to the upstream manufacturer (or the supplier) [8, 19, 23]. Furthermore, Sun and Tyagi [26] have proven that the manufacturers' decision to disclose information is affected by the degree of retail competition.

In addition to the sharing of vertical information in the supply chain, horizontal information sharing has also been investigated by scholars [12, 31]. Zhou *et al.* [31] have shown that the manufacturer's horizontal competition and incomplete information affect the efficiency of the supply chain. Johnsen *et al.* [12] have found that quantity discounts combined with the efficient sharing of information affect cooperation in a supply chain when a buyer holds private information pertaining to end-customer demand. A general assumption is a member of a supply chain should withhold its own private demand forecast information, such as a retailer who is afraid of information leakage [3]; alternatively, the retailer can maintain its own information advantage over the manufacturer, as upstream private information can improve channel efficiency and consumer surplus [5]. If a firm unconditionally shares information with another firm, then the other and the total supply chain would benefit from it [19]. Huang *et al.* [8] have obtained a different conclusion in which a retailer may prefer to voluntarily share information with the manufacturer when the retailer anticipates the supplier to be harmed. Consequently, a stream of literature has investigated the motivation of information sharing. For example, Shang *et al.* [23] have determined that the retailer's incentive to share information strongly depends on nonlinear production cost, the competition intensity between two manufacturers, and whether the retailer can provide a contract to charge for the information sharing, in which case the information contracting affects the retailer's decision to share information for free. Our study complements the studies about information sharing in a vertical supply chain; for instance, a downstream retailer holds demand forecast information and decides whether to share it with an upstream manufacturer. However, in contrast to the abovementioned study, we mainly focus on whether the retailer is willing to share their demand information when the upstream firm offers freebie promotion. We find that the retailer is willing to proactively share her private demand information when the relative length of the promotion is sufficiently high.

3. Problem description and notations

We consider a two-echelon supply chain composed of a manufacturer (he) and a retailer (she). The manufacturer simultaneously produces a product (*e.g.*, high-end cosmetics) and freebies and sells it at a wholesale price w to the retailer, who sells the product to the end consumers at a retail price p in a selling period. In our models, we divide the selling season into two parts: promotional period and regular selling period. Before the onset of the promotional period, the manufacturer sells at wholesale the product to the retailer with a corresponding freebie for each product unit, and then the retailer sells the product and gives the freebie to consumers during the promotional period simultaneously. During the regular sales period (the rest of the selling period), the manufacturer only sells at wholesale the product to the retailer. We assume that the

Table 1. Definition of notations.

Symbols	Explanation
w	Wholesale price for the product
p	Retail price
c	Unit cost (the sum of production and operation cost)
f	Value of freebies provided by the manufacturer
θ	Cost coefficient of freebies provided by the manufacturer
$a+\varepsilon$	Primary market base of the product
a	Base demand that is known to both firms
ε	Uncertain part of the demand, which is a random variable with a mean value of 0 and a variance of σ^2
Y	Unbiased estimate of ε
α	Self-price sensitivity of demands
λ	Sensitivity coefficient of freebies
β	Relative length of the promotional period
M	The manufacturer
R	The retailer
π^{NN}	Superscript indicates the scenario NN
π^{FN}	Superscript indicates the scenario FN
π^{FS}	Superscript indicates the scenario FS
π^{FN-N}	Superscript indicates the scenario FN-N
π^{FN-B}	Superscript indicates the scenario FN-B
$E[\pi_M^k]$	The expected profit of the manufacturer of model k
$E[\pi_R^k\|Y]$	The expected profit of the retailer of model k

market demand functions are linear, as this approach has been widely used in the literature [15, 16, 20, 23, 29]. On the one hand, during the promotional period (when the manufacturer offers the freebies to the retailer), the demand function is expressed by $d_0 = \beta(a+\varepsilon - \alpha p + \lambda f)$. On the other hand, during the regular selling period, the demand function is $d_1 = (1-\beta)(a+\varepsilon-\alpha p)$, where β represents the relative length of the promotional period when the manufacturer provides freebies. The total demand over the entire season is given by $D = d_0 + d_1 = \beta(a+\varepsilon-\alpha p+\lambda f) + (1-\beta)(a+\varepsilon-\alpha p)$. Table 1 lists all notations used in this study.

For the demand potential, $a+\varepsilon$ denotes the market potential, where a is a constant that can be observed by all members of the chain and ε represents the uncertain part for the market potential with a mean zero and a variance of σ^2. We further assume that the retailer can obtain private demand information about ε, which is denoted as Y, based on the data of previous sales or market research. Consistent with Li [15], the signal Y is an unbiased estimate of ε. Subsequently, $E[\varepsilon|Y] = Yt\sigma^2/(1+t\sigma^2)$ and $E[Y^2] = (\sigma^2 + 1/t)$, where $t = 1/[E(Var(Y|\varepsilon))]$.

Next, three main scenarios and two extending scenarios are discussed in this study. (i). NN: NN Stackelberg model, the manufacturer does not provide freebies and the retailer does not share her information with the manufacturer, in which the manufacturer is the Stackelberg leader in the vertical competition. (ii). FN: FN Stackelberg model, the manufacturer offers freebies on the premise that the retailer does not share the demand information with him, where the manufacturer is the Stackelberg leader. (iii). FS: FS Stackelberg model, the manufacturer offers freebies under the condition that the retailer shares her information with him, and the manufacturer is still the Stackelberg leader.

Aiming to examine the impact of power structure on the decisions of members when the manufacturer offers freebie promotion and the retailer does not share demand information, two extended scenarios compared with scenario FN: (i). FN-N: FN Nash game model, the manufacturer provides freebies under the premise that the

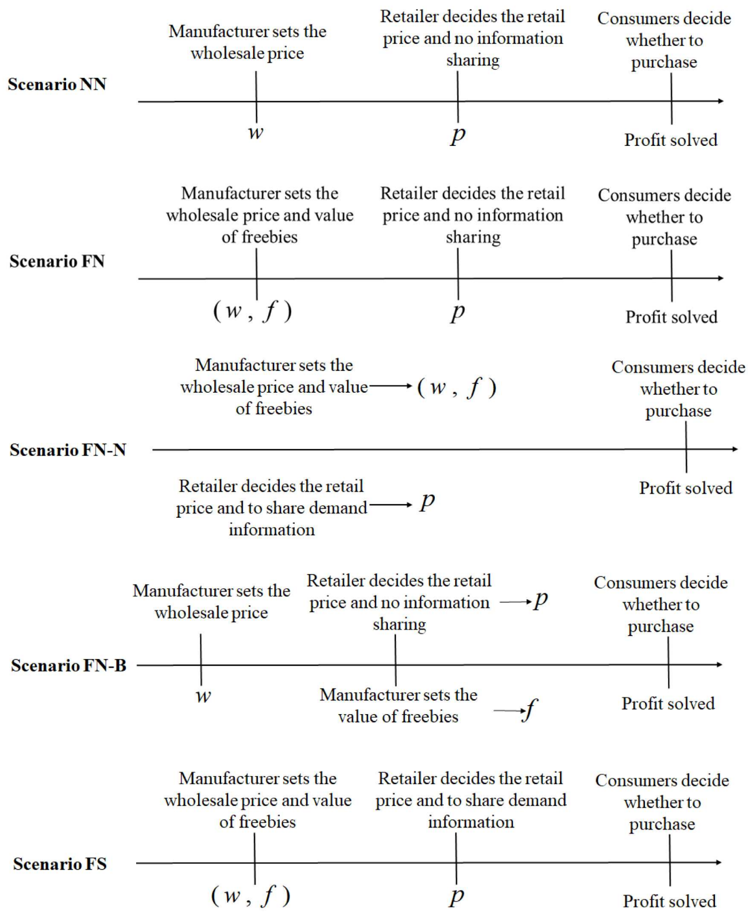

FIGURE 1. Sequence of events under different scenarios.

retailer does not share the demand information. Firms have the same market power, and they determine their pricing decisions independently and simultaneously. (ii). FN-B: FN Bertrand model, the manufacturer provides freebies under the premise that the retailer does not share the demand information. The manufacturer and retailer simultaneously determine the freebie's value and retail price. The sequence of events of our models is illustrated in Figure 1.

We assume that all members are risk-neutral and strive to maximize their payoffs, and the costs of the product and freebies produced by the manufacturer are c and θf, respectively. To ensuring that the manufacturer and retailer are profitable in our models, we set $p > w > c$ and $w > c + \theta f$. Furthermore, $a > c\alpha$ must be satisfied to guarantee a positive demand. $\lambda > \alpha\theta$ ensures an increase in the value of freebies after the retailer shares the information with the manufacturer.

4. Analysis

In this section, we investigate the equilibrium pricing decisions of firms in each scenario. We begin with a benchmark model in which the manufacturer does not provide freebies during the selling period and the retailer does not share her private demand information with him. The payoff of the manufacturer is only obtainable from the retailer. Therefore, the expected profit functions of the manufacturer and retailer based on the demand functions are

$$E[\pi_M^{NN}] = E[(w-c)(a+\varepsilon-\alpha p)], \tag{4.1}$$
$$E[\pi_R^{NN}|Y] = (p-w)(a+E[\varepsilon|Y]-\alpha p). \tag{4.2}$$

Second, under the condition that the retailer does not share her information, the manufacturer offers freebies during the promotional period and no freebies are given during the regular selling period. The expected profit functions of the manufacturer and retailer are defined as equations (4.3) and (4.4), respectively. In equation (4.3), the first term represents the manufacturer's expected profit from selling the products with freebies during the promotional period, and the second term represents the expected profit from selling the products without freebies during the regular selling period.

$$E[\pi_M^{FN}] = E[\beta(w-c-\theta f)(a+\varepsilon-\alpha p+\lambda f)+(1-\beta)(w-c)(a+\varepsilon-\alpha p)] \tag{4.3}$$
$$E[\pi_R|Y] = \beta(p-w)(a+E[\varepsilon|Y]-\alpha p+\lambda f)+(1-\beta)(p-w)(a+E[\varepsilon|Y]-\alpha p). \tag{4.4}$$

If the retailer shares her demand information with the manufacturer, then the expected profit function of the manufacturer is given by

$$E[\pi_M^{FS}|Y] = \beta(w-c-\theta f)(a+E[\varepsilon|Y]-\alpha p+\lambda f)+(1-\beta)(w-c)(a+E[\varepsilon|Y]-\alpha p). \tag{4.5}$$

4.1. Scenario NN

We begin with a benchmark case in which the manufacturer does not provide freebies and the retailer does not share information. In this case, the manufacturer is the Stackelberg leader, and the retailer is the follower in the vertical competition. In particular, the manufacturer determines the wholesale price, and then the retailer sets the retail price to maximize her expected profit. This scenario can be formulated as

$$\begin{cases} \max\limits_{(w)} E[\pi_M^{NN}(w, p^*(w))] \\ p^*(w) \text{ is derived from solving the problem} \\ \max\limits_{(p)} E[\pi_R^{NN}(p)|Y] \end{cases}, \tag{4.6}$$

where $E[\pi_M^{NN}]$ and $E[\pi_R^{NN}|Y]$ are defined according to equations (4.1) and (4.2), respectively. We adopt backward induction to solve this problem, and the optimal solutions of the manufacturer and retailer are formulated as follows.

In accordance with the standard approach, we obtain the following proposition to show the equilibrium decisions:

Proposition 4.1. *In scenario NN, the manufacturer's optimal wholesale price and the retailer's retail price are* $w^{NN} = \frac{a+c\alpha}{2\alpha}$ *and* $p^{NN} = \frac{3a+c\alpha}{4\alpha} + \frac{Yt\sigma^2}{2\alpha(1+t\sigma^2)}$*, respectively.*

On the basis of the firms' equilibrium results, we characterize the payoffs of the manufacturer and retailer in Tables 2 and 3, respectively. Proposition 4.1 shows that the self-price sensitivity α has a negative effect on the firms' equilibrium prices, and equilibrium profits. A decline in α represents an increase in consumer brand loyalty for the product. This situation means that the manufacturer's wholesale price and the retailer's retail price both increase as the consumer brand loyalty for the product increases. Therefore, the manufacturer and retailer can develop consumer brand loyalty by adopting more appropriate marketing strategies to gain more profit. The analysis presented above is consistent with that studied by Kurata *et al.* [14].

As the retailer does not share private demand information with the manufacturer, the manufacturer's equilibrium wholesale price is not affected by the value of Y. However, the retailer's retail price increases with the rise in Y because it indicates that the demand for the product will also increase. The retailer observes this signal and then increases her revenue by increasing the retail price [16].

Table 2. Manufacturer's payoff under different scenarios.

Equilibrium result	Payoffs of the manufacturer under different scenarios
$E[\pi_M^{NN}]$	$\frac{1}{8\alpha}(a-c\alpha)^2$
$E[\pi_M^{FN}]$	$\frac{(1-\beta)\theta\lambda(a-c\alpha)^2}{B}$
$E[\pi_M^{FS}\|Y]$	$\frac{(1-\beta)\theta\lambda(a-c\alpha)^2}{B}+\frac{(1-\beta)\theta\lambda t\sigma^4}{B(1+t\sigma^2)}$
$E[\pi_M^{FN-N}]$	$\frac{(1-\beta)(a-c\alpha)^2\theta\lambda(4\alpha\theta\lambda-2\alpha\beta\theta\lambda-\beta\lambda^2-\alpha^2\beta\theta^2)}{B1^2}$
$E[\pi_M^{FN-B}]$	$\frac{(1-\beta)\theta\lambda(a-c\alpha)^2}{B2}$

Table 3. Retailer's payoff under different scenarios.

Equilibrium result	Payoffs of the retailer under different scenarios
$E[\pi_R^{NN}\|Y]$	$\frac{1}{16\alpha}(a-c\alpha)^2+\frac{t\sigma^4}{4\alpha(1+t\sigma^2)}$
$E[\pi_R^{FN}\|Y]$	$\frac{4\alpha(a-c\alpha)^2\theta^2\lambda^2(1-\beta)^2}{B^2}+\frac{t\sigma^4}{4\alpha(1+t\sigma^2)}$
$E[\pi_R^{FS}\|Y]$	$\frac{4\alpha(a-c\alpha)^2\theta^2\lambda^2(1-\beta)^2}{B^2}+\frac{4\alpha\theta^2\lambda^2(1-\beta)^2t\sigma^4}{B^2(1+t\sigma^2)}$
$E[\pi_R^{FN-N}\|Y]$	$\frac{4\alpha(a-c\alpha)^2\theta^2\lambda^2(1-\beta)^2}{B1^2}+\frac{\theta\lambda(2\alpha\theta-\alpha\beta\theta-\beta\lambda)(4\lambda-\alpha\beta\theta-3\beta\lambda)t\sigma^4}{B1^2(1+t\sigma^2)}$
$E[\pi_R^{FN-B}\|Y]$	$\frac{(a-c\alpha)^2\lambda^2(1-\beta)^2(2\alpha\theta+\beta\lambda)^2}{\alpha B2^2}+\frac{2(2-\beta)t\sigma^4}{\alpha(4-\beta)^2(1+t\sigma^2)}$

4.2. Scenario FN

In this case, the manufacturer provides freebies during the promotional period on the premise that the retailer does not share her private demand information with him. The decision sequence can be briefly described as follows: the manufacturer initially sets the wholesale price w and the value of freebies f, and then the retailer makes a retail price decision. We can write this case as follows:

$$\begin{cases} \max\limits_{(w,f)} E[\pi_M^{FN}(w,f,p^*(w))] \\ p^*(w) \text{ is derived from solving the problem} \\ \max\limits_{(p)} E[\pi_R^{FN}(p)|Y] \end{cases}, \tag{4.7}$$

where $E[\pi_M^{FN}]$ and $E[\pi_R^{FN}(p)|Y]$ are formulated according to equations (4.3) and (4.4), respectively. We also use backward induction to solve this problem. The detailed equilibrium results of the manufacturer and retailer are summarized in Proposition 4.2.

Proposition 4.2. *In scenario FN, the equilibrium wholesale price and the value of freebies are* $f^{FN}=\frac{(a-c\alpha)(\lambda-\alpha\theta)}{B}$ *and* $w^{FN}=\frac{a\theta(4\lambda-\alpha\beta\theta-3\lambda\beta)+c\lambda(4\alpha\theta-3\alpha\beta\theta-\lambda\beta)}{B}$, *respectively. The retail price is* $p^{FN}=\frac{c\lambda(2\alpha\theta-\alpha\beta\theta-\lambda\beta)+a\theta(6\lambda-\alpha\beta\theta-5\lambda\beta)}{B}+\frac{Yt\sigma^2}{2\alpha(1+t\sigma^2)}$, *where* $B=8\alpha\theta\lambda-\alpha^2\beta\theta^2-6\alpha\beta\theta\lambda-\beta\lambda^2$.

On the basis of the equilibrium prices, as shown in Proposition 4.2, we can obtain the profits of the manufacturer and retailer, as shown in Tables 2 and 3, respectively. According to Proposition 4.2, the value of the freebies is positive if and only if $\lambda>\alpha\theta$ and $a>c\alpha$. Here, $a>c\alpha$ guarantees that the retail price is greater than the wholesale price, and $\lambda>\alpha\theta$ ensures an increase in the value of freebies after the retailer shares the information with the manufacturer. In the retailer's profit, the portion of the profit attributable to the private signal is unchanged. Following the optimal decisions of the manufacturer and retailer in scenarios NN and FN, we determine the impact of the value of freebies on their decisions in the next section.

4.3. Scenario FS

To analyze the impact of the retailer's information sharing with the manufacturer on the decisions of firms, we consider a scenario in which the retailer decides to share private demand information about market demand with the manufacturer under the condition that the manufacturer offers freebies during the promotional period. In this scenario, the manufacturer sets the wholesale price and the freebie's value based on the information shared by the retailer, and then the retailer announces the retail price. In particular, the manufacturer and retailer both make their pricing decisions based on the demand information observed by the retailer. This scenario can be formulated as

$$\begin{cases} \max\limits_{(w,f)} E[\pi_{_M}^{FS}(w,f,p^*(w))|Y] \\ p^*(w) \text{ is derived from solving the problem} \\ \max\limits_{(p)} E[\pi_R^{FS}(p)|Y] \end{cases}, \tag{4.8}$$

where $E[\pi_{_M}^{FS}|Y]$ and $E[\pi_R^{FS}|Y]$ are formulated as equations (4.5) and (4.4), respectively. The detailed equilibrium results of the manufacturer and retailer are presented in Proposition 4.3. In this circumstance, the members' equilibrium payoffs can be obtained, as shown in Tables 2 and 3.

Proposition 4.3. *Under scenario FS, the equilibrium results of the manufacturer are given by* $w^{FS} = \frac{a\theta(4\lambda-\alpha\beta\theta-3\lambda\beta)+c\lambda(4\alpha\theta-3\alpha\beta\theta-\lambda\beta)}{B} + \frac{\theta(4\lambda-\alpha\beta\theta-3\lambda\beta)Yt\sigma^2}{B(1+t\sigma^2)}$ *and* $f^{FS} = \frac{(a-c\alpha)(\lambda-\alpha\theta)}{B} + \frac{(\lambda-\alpha\theta)Yt\sigma^2}{B(1+t\sigma^2)}$*, and the optimal retail price is* $p^{FS} = \frac{a\theta(6\lambda-\alpha\beta\theta-5\lambda\beta)+c\lambda(2\alpha\theta-\alpha\beta\theta-\lambda\beta)}{B} + \frac{\theta(6\lambda-\alpha\beta\theta-5\lambda\beta)Yt\sigma^2}{B(1+t\sigma^2)}$.

As the manufacturer knows the additional uncertain part of the demand information, the manufacturer's equilibrium wholesale price and the value of freebies are related to the value of Y. Moreover, the value of the demand signal positively affects the manufacturer and the retailer's optimal prices. This scenario indicates that the manufacturer can respond to the changes in market demand by increasing the wholesale price and the value of freebies with the rise in Y. The specific analysis of the effect of the retailer's information sharing on the manufacturer and the retailer's optimal prices and maximal profits are discussed in Proposition 5.3.

4.4. Scenario FN-N

On the premise that the retailer does not share her private demand information with the manufacturer, we then consider the effect of different market power structures between the manufacturer and retailer on the decisions of firms. We begin with scenario FN-N, in which the retailer has the same market power as the manufacturer, and they make their decisions independently and simultaneously. In other words, the manufacturer announces the wholesale price and the value of freebies, and the retailer decides the retail price simultaneously. Scenario FN-N can be formulated as

$$\begin{cases} \max\limits_{(w,f)} E[\pi_{_M}^{FN-N}(w,f)] \\ \max\limits_{(p)} E[\pi_R^{FN-N}(p)|Y] \end{cases}, \tag{4.9}$$

where $E[\pi_{_M}^{FN-N}]$ and $E[\pi_R^{FN-N}(p)|Y]$ are defined according to equations (4.3) and (4.4), respectively. The following proposition gives the firms' equilibrium solutions and payoffs under scenario FN-N. The detailed equilibrium decisions of the manufacturer and retailer are shown in Table 4.

Correspondingly, the firms' profits can be derived, as shown in Tables 2 and 3. The value of the freebies is larger than zero if and only if $\lambda > \alpha\theta$ and $a > c\alpha$. Compared with scenario FN, in which the manufacturer's market power is reduced, he sets a relatively lower wholesale price and offers a higher value of freebies (*i.e.*, $w^{FN} > w^{FN-N}$ and $f^{FN-N} > f^{FN}$). This situation means that optimal pricing decisions correspond to the market power structure of the manufacturer and retailer. The impact of changes in the market power structure among supply chain members on their pricing decisions is discussed in Proposition 5.2.

TABLE 4. Members' equilibrium results in two scenarios.

Results	Equilibrium results in scenarios FN-N and FN-B
f^{FN-N}	$\frac{(\lambda-\alpha\theta)(a-c\alpha)}{B1}$
f^{FN-B}	$\frac{(a-c\alpha)(2\lambda-\alpha\theta-\beta\lambda)}{B2}$
w^{FN-N}	$\frac{a\theta(2\lambda-\beta\lambda-\alpha\beta\theta)+c\lambda(4\alpha\theta-3\alpha\beta\theta-\lambda\beta)}{B1}$
w^{FN-B}	$\frac{a\theta(4\lambda-\alpha\beta\theta-3\beta\lambda)+c\lambda(4\alpha\theta-3\alpha\beta\theta-\lambda\beta^2)}{B2}$
p^{FN-N}	$\frac{c\lambda(2\alpha\theta-\alpha\beta\theta-\lambda\beta)+a\theta(4\lambda-3\beta\lambda-\alpha\beta\theta)}{B1}+\frac{\lambda(2\alpha\theta-\beta\lambda-\alpha\beta\theta)Yt\sigma^2}{\alpha B1(1+t\sigma^2)}$
p^{FN-B}	$\frac{c\lambda(2\alpha\theta-\alpha\beta\theta-\lambda\beta)}{B2}+\frac{a(6\alpha\theta\lambda-\alpha^2\beta\theta^2-5\alpha\beta\theta\lambda+(1-\beta)\beta\lambda^2)}{\alpha B2}+\frac{2Yt\sigma^2}{\alpha(4-\beta)(1+t\sigma^2)}$

4.5. Scenario FN-B

From the manufacturer's perspective, his market power decreases gradually under scenarios FN, FN-B and FN-N, whereas the retailer's power increases gradually in scenarios FN, FN-B and FN-N. Under scenario FN-B, the manufacturer initially makes his wholesale price decision, and then the manufacturer and the retailer simultaneously decide the freebies' value and the retail price before the onset of the promotional period. This scenario can be formulated as

$$\begin{cases} \max\limits_{(w)} E[\pi_{_M}^{FN\text{-}B}(w, f^*(w), p^*(w))] \\ f^*(w), p^*(w) \text{ are derived from the problem} \\ \begin{cases} \max\limits_{(p)} E[\pi_R^{FN-B}(p)|Y] \\ \max\limits_{(f)} E[\pi_{_M}^{FN\text{-}B}(f)] \end{cases} \end{cases} \tag{4.10}$$

where $E[\pi_{_M}^{FN\text{-}B}]$ and $E[\pi_{_R}^{FN}|Y]$ are characterized according to equations (4.3) and (4.4), respectively. Using backward induction, we present the equilibrium results of firms in Table 4. According to the equilibrium decisions, we can characterize the payoffs of members, as shown in Tables 2 and 3. Compared with scenarios FN and FN-N, both the retail price and wholesale price are maximal in scenario FN-B. In addition, Proposition 5.2 shows that the manufacturer's power becomes more apparent, it increases the wholesale and retail prices and decreases the value of the freebies. The specific comparison is presented in a later section.

5. COMPARISON AND SENSITIVITY ANALYSIS

To gain more managerial insights, we compare the optimal pricing and profits of the manufacturer and retailer under different scenarios (*i.e.*, Subsects. 4.1–4.5) and conduct sensitivity analysis to investigate the impact of β, λ and θ on the equilibrium prices and profits. Furthermore, we provide numerical examples to illustrate the theoretical results and explore the differences among the three different scenarios (*i.e.*, FN, FN-N and FN-B). The parameters are set as $a = 10$, $c = 1$, $t = 0.1$, $\sigma = 1$, $Y = 0.2$, $\alpha = 1.2$, $\theta = 0.4$, and $\lambda = 0.6$. We obtain some managerial insights through numerical analysis, which are summarized in the following subsections.

5.1. Comparison between scenario NN and scenario FN

Proposition 5.1. *A comparison of scenario NN with scenario FN obtains*

(i). $w^{FN} > w^{NN}$, $p^{FN} > p^{NN}$.

(ii). $E[\pi_M^{FN}] > E[\pi_M^{NN}], E[\pi_R^{FN}|Y] > E[\pi_R^{NN}|Y]$; $E[\pi_M^{FN}] - E[\pi_M^{NN}] \leq E[\pi_R^{FN}|Y] - E[\pi_R^{NN}|Y]$.

As shown in Proposition 5.1(i), if the manufacturer offers freebie promotion, then he and the retailer will both set a higher price (*i.e.*, the wholesale price and the retail price). This conclusion is intuitive because the manufacturer needs to bear a production cost when producing freebies. According to Proposition 5.1(ii), as oposed to the no-freebie promotion, the freebie promotion is beneficial to both the manufacturer and retailer, although the retailer benefits more from it than the manufacturer.

5.2. Comparison of scenarios FN, FN-N, and FN-B

Proposition 5.2. *In the equilibria of scenarios FN, FN-N, and FN-B, we obtain:*

(i). If $\frac{2\alpha\theta}{\theta+\lambda} \leq \beta < 1$, $f^{FN-N} > f^{FN-B} > f^{FN}$ *and* $p^{FN-N} > p^{FN} > p^{FN-B}$*; otherwise,* $f^{FN-B} > f^{FN-N} > f^{FN}$ *and* $p^{FN-B} > p^{FN} > p^{FN-N}$.

(ii). $w^{FN} > w^{FN-B}$, *if* $\beta \leq \frac{2\alpha\theta}{\alpha\theta+\lambda}$, $w^{FN-B} > w^{FN-N}$*; otherwise,* $w^{FN-B} < w^{FN-N}$.

(iii). $E[\pi_M^{FN}] > E[\pi_M^{FN-B}] > E[\pi_M^{FN-N}]$, $E[\pi_R^{FN-N}|Y] > E[\pi_R^{FN}|Y]$.

Combined with the analysis of the numerical example and Proposition 5.2, we analyze the effect of β on the retail price, the freebie's value, the wholesale price and the firms' profits. The following results can be obtained based on the trends of Figures 2–6:

(1). Figure 2 shows the impact of β on the freebie's value under different scenarios (*i.e.*, scenarios FN, FN-N, and FN-B). First, as β increases, the value of the freebies increases, that is, the longer the relative length of the promotional period, the manufacturer will likely prefer to offer a higher value of freebies to attract consumers, and the market capacity is expanded. Second, regardless of the value of β, the manufacturer will likely always provide a minimum value of freebies under scenario FN. This situation can be attributed to the manufacturer acting as the Stackelberg leader under scenario FN, and his advantage plays a major role. When β is relatively low (approximately less than 0.89), the freebie's value achieves a higher value under scenario FN-B, and the manufacturer's advantage is weakened. Once β is sufficiently high (*i.e.*, $\beta > \frac{2\alpha\theta}{\theta+\lambda}$), the manufacturer needs to offer a higher value of the freebie similar to that achieved in scenario FN-N; that is, he can only benefit from his advantage when the freebie's promotional period is relatively long.

(2). We then investigate the impact of β on the retail price (see Fig. 3) and the wholesale price (see Fig. 4). As β increases, the retail price and wholesale price increase under the three scenarios. In particular, under scenario FN, the impact of β on the wholesale and retail prices is identical. When β is sufficiently small (*e.g.*, $0 < \beta < 0.89$), the retailer will likely decide to set a higher retail price under scenario FN-B than those under the other two scenarios (*i.e.*, scenario FN and scenario FN-N). As presented in Proposition 5.2(1), when β is relatively high (approximately 0.89 in Fig. 3), the retailer will likely set a higher retail price under scenario FN-N than those under the other two scenarios (*i.e.*, scenario FN and scenario FN-B).

(3). We illustrate the impact of β on the profits of the manufacturer and retailer (see Figs. 5 and 6). Figure 5 shows the existence of a first-mover advantage in scenario FN. In this scenario, the manufacturer initially uses his advantage to announce his wholesale price and the freebie's value, resulting in him always obtaining a higher profit in scenario FN than in scenario FN-N and scenario FN-B. Under scenario FN-B, as β increases, the manufacturer's profit initially decreases and then increases. Under scenario FN-N, the manufacturer's advantage is shifted to the retailer, who achieves a higher payoff in this scenario than in the other scenarios (*i.e.*, scenario FN-N and scenario FN-B).

5.3. Comparison between scenario FN and scenario FS

We then examine the impact of the retailer's information sharing on members' optimal pricing decisions. Figure 7 illustrates the impact of the value of information sharing on the profits of the supply chain members, in which the parameter settings are the same as those mentioned above. Proposition 5.3. In the equilibria of scenario FN and scenario FS, we have

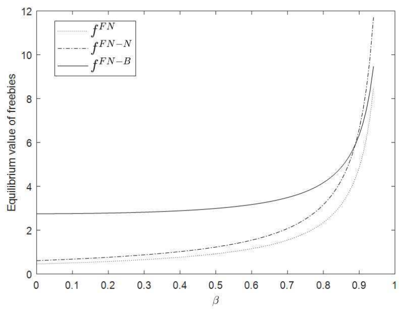

FIGURE 2. Impact of β on f.

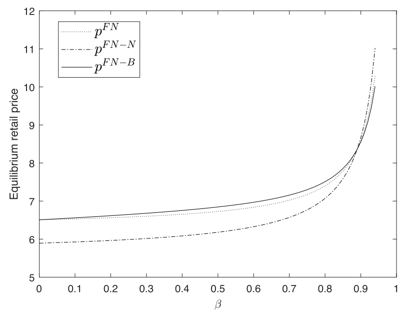

FIGURE 3. Impact of β on p.

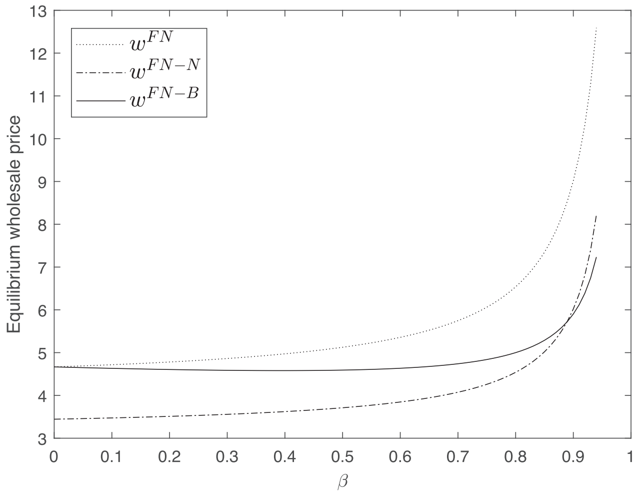

FIGURE 4. Impact of β on w.

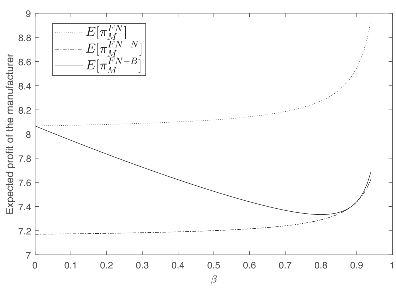

FIGURE 5. Impact of β on $E[\pi_M]$.

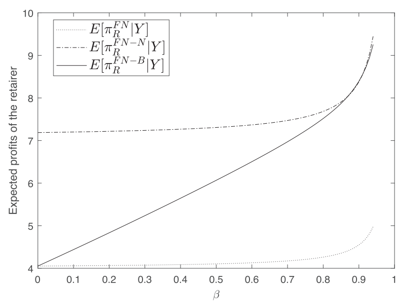

FIGURE 6. Impact of β on $E[\pi_R|Y]$.

Proposition 5.3. *In the equilibria of scenario FN and scenario FS, we have*

(i). $w^{FS} > w^{FN}$, $f^{FS} > f^{FN}$; *if* $0 < \beta \le \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2+2\alpha\theta\lambda}$, $p^{FS} \ge p^{FN}$; *if* $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2+2\alpha\theta\lambda} < \beta < 1$, $p^{FS} < p^{FN}$.

(ii). $E[\pi_M^{FS}] > E[\pi_M^{FN}]$.

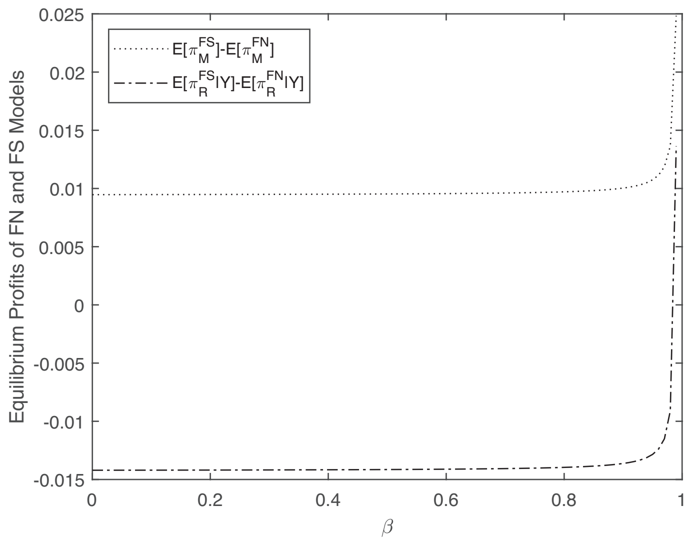

FIGURE 7. Comparison of the Value of Information Sharing.

(iii). If $0 < \beta < \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$, $E[\pi_R^{FS}|Y] < E[\pi_R^{FN}|Y]$; *if* $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2} \leq \beta < 1$, $E[\pi_R^{FS}|Y] \geq E[\pi_R^{FN}|Y]$.

(iv). If $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2} \leq \beta < 1$, $E[\pi_R^{FS}|Y]+E[\pi_M^{FS}] \geq E[\pi_R^{FN}|Y]+E[\pi_M^{FN}]$.

According to Proposition 5.3(3), on the premise that the manufacturer provides freebies for the retailer, the manufacturer always benefits from the retailer's demand information sharing, but the retailer does not necessarily benefit from it. When β is sufficiently high ($i.e.$, $\beta \geq \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), she can benefit from her information sharing. As illustrated in Figure 7, the manufacturer's value of information sharing is positive, and it is less affected by β. With a sufficiently high β, the manufacturer's and retailer's values of information sharing are both positive, indicating that information sharing is beneficial to the manufacturers, the retailer and the total supply chain. When β is relatively small, the retailer's value of information sharing is negative, indicating that her information sharing is detrimental. When β is sufficiently high ($i.e.$, $\beta \geq \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), the retailer is more affected by the relative length of the promotional period.

According to Proposition 5.3(1), the freebie's value and the wholesale price both increase after the retailer shared the information with the manufacturer. When β is relatively small ($i.e.$, $\beta \leq \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2+2\alpha\theta\lambda}$), the retailer is likely to set a higher retail price than under no information sharing ($i.e.$, $p^{FS} \geq p^{FN}$). When β is sufficiently high ($i.e.$, $\beta > \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2+2\alpha\theta\lambda}$), the retailer will decrease her retail price. An increase in β denotes an increase in promotional strength, and a lower retail price and more freebies can attract more consumers. Thus, the retailer should not immediately increase her retail price after the manufacturer increases his freebie's value. During the regular sales period, a higher retail price will reduce the demand.

According to Proposition 5.3(3), there exists a threshold ($i.e.$, $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$). When β is relatively low ($i.e.$, $\beta < \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), the manufacturer benefits from information sharing, whereas the retailer does not gain from it. When β is sufficiently high ($i.e.$, $\beta \geq \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), information sharing is beneficial to all firms in the supply chain, and the retailer may voluntarily share her demand information with the manufacturer. Therefore, if the retailer is willing to share her information, then the manufacturer can compensate for the retailer's loss of information sharing by increasing the freebie's value, offering promotion (increasing the relative length of the promotional period) or subsidy.

5.4. Sensitivity analysis

In this subsection, Corollary 5.4 summarizes the impacts of the relative length of the promotional period β, the sensitivity coefficient of freebies λ and the cost coefficient of freebies θ on the equilibrium results and profits of the manufacturer and retailer under scenario FN. Corollary 5.5 shows a sensitivity analysis of information sharing for the channel members under scenario FS.

Corollary 5.4. *Under scenario FN, we have:*

(i). $\frac{\partial f^{FN}}{\partial \beta} > 0$, $\frac{\partial f^{FN}}{\partial \lambda} > 0$, $\frac{\partial f^{FN}}{\partial \theta} < 0$; $\frac{\partial p^{FN}}{\partial \beta} > 0$, $\frac{\partial p^{FN}}{\partial \theta} > 0$, $\frac{\partial p^{FN}}{\partial \lambda} < 0$.

(ii). $\frac{\partial E[\pi_M^{FN}]}{\partial \beta} > 0$, $\frac{\partial E[\pi_R^{FN}|Y]}{\partial \beta} < 0$; $\frac{\partial E[\pi_M^{FN}]}{\partial \lambda} > 0$, $\frac{\partial E[\pi_R^{FN}|Y]}{\partial \lambda} > 0$; $\frac{\partial E[\pi_M^{FN}]}{\partial \theta} < 0$, $\frac{\partial E[\pi_R^{FN}|Y]}{\partial \theta} > 0$.

Corollary 5.4(2) reveals that the relative length of the promotional period β and the cost coefficient of freebies θ have opposite effects on the profits of the manufacturer and retailer in scenario FN, whereas the sensitivity coefficient of freebies λ have the same positive effect. Some of the implications of Corollary 5.4 can be described as follows.

As illustrated in Corollary 5.4(2), λ positively affects the profits of the manufacturer and retailer, indicating that the number of consumers who prefer freebies will likely increase as λ increases, and the profits attributable to the freebie promotion will increase. A higher λ further increases the freebie's value, denoting that as λ increases, the manufacturer will likely add the freebie's value. By contrast, the impact of λ on the retail price is negative, indicating that an increase in the consumers' sensitivity to freebies will encourage the retailer to reduce the retail price. Therefore, the manufacturer and retailer should take certain marketing actions (*e.g.*, advertising and promotional efforts) to increase the consumers' sensitivity to freebies.

Corollary 5.4 shows that as the cost coefficient of freebies (θ) increases, although the manufacturer's profit decreases, the retailer's profit increases under scenario FN. A higher θ can increase the cost of the manufacturer rather than that of the retailer. Thus, as θ increases, the manufacturer will likely decrease the freebie's value to reduce the cost of provided freebies, whereas the retailer will likely increase the retail price to gain more revenue. A higher value of freebies can stabilize the number of existing consumers who prefer freebies, and new consumers can be attracted simultaneously.

Interestingly, β has an opposite effect on the revenue of the manufacturer and retailer. The manufacturer's profit increases, but the retailer's profit decreases when β increases. The freebie's value and the retail price are both higher when β rises. During the promotional period, a higher freebie's value brings more revenue to the manufacturer and attracts more consumers who prefer freebies. Correspondingly, the retailer also needs to raise the retail price to ensure profit gain in according with the higher wholesale price given by the upstream manufacturer. Therefore, as β increases, the retailer needs to adopt more promotions to attract more consumers who are interested in freebies, thus ensuring profits.

In real life, a manufacturer provides more freebies to attract consumers into the market as the relative length of the promotional period increases. As the sensitivity coefficient of freebies positively affects the value of freebies, the manufacturer and retailer can separately or cooperatively take certain marketing actions to increase the consumers' brand loyalty to the product, such as implementing advertising or promotional efforts. The manufacturer can develop consumer loyalty to the brand as a means of increasing the optimal wholesale price of the product, while the retailer can increase the retail price of the product by developing consumer loyalty to the brand.

Corollary 5.5. *Under scenario FN and scenario FS, we have*

(i). $\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial \lambda} > 0$, $\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial \theta} < 0$, $\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial \beta} > 0$.

(ii). $\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial \lambda} < 0$, $\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial \theta} > 0$, $\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial \beta} < 0$.

According to Corollary 5.5, on the premise that the retailer shares her information with the manufacturer, the retailer's profit decreases, and the manufacturer's profit increases when the sensitivity coefficient of freebies λ increases. Therefore, as the manufacturer is likely to offer freebies, we can summarize some implications on whether the retailer would share her information with the manufacturer. The parameters β, θ and λ have opposite effects on the profits of the manufacturer and retailer before and after information sharing. The value of the retailer's information sharing increases as λ or β decreases, but θ has a positive effect. Correspondingly, β and λ positively affects the change in profit before and after the manufacturer obtains the private demand information shared by the retailer, but θ has a negative effect.

First, as λ (sensitivity coefficient of freebies) increases, the retailer's value of information sharing decreases, but the greater change in the manufacturer's profits is affected by the retailer's information sharing. Therefore, the manufacturer may increase the value of freebies or decrease the cost coefficient of freebies, whereas the retailer may reduce the retail price or take promotional actions to attract consumers.

Second, when the relative length of the freebie promotional period β is relatively small, an increase in β harms the retailer's value of information sharing. As illustrated in Figure 6, when β is sufficiently high (*i.e.*, $\beta \geq \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), the value of information sharing increases, and the manufacturer can benefit more from it. At this moment, the retailer voluntarily shares her demand information with the manufacturer, leading to a win-win situation.

Finally, a higher θ increases the retailer's value of information sharing, and the manufacturer will likely hesitate to actively increase the cost coefficient of the freebie. Therefore, when β is sufficiently high (*i.e.*, $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2} \leq \beta < 1$), the retailer will likely volunteer to share her demand information with the manufacturer. When β is relatively low (*i.e.*, $0 < \beta < \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$), the manufacturer can appropriately increase the cost coefficient of providing freebies to encourage the retailer to share demand information.

6. Conclusion

The continuous development of the e-commerce industry has led to the penetration of foreign high-end cosmetics into the Chinese market. Supply chain members, including manufacturers and retailers have adopted promotional strategies, such as freebie promotion, price discounts, and rebate to expand the market demand for such products. A retailer obtains private demand information about the products prior to selling them to consumers and considers the case in which private information can be shared with a manufacturer. This research mainly studies four scenarios in which the manufacturer provides freebies to the retailer under different market power structures. The impact on the profits of supply chain members when the retailer shares her own demand information with the manufacturer is also analyzed. The relevant findings of this study are as follows.

First, on the premise that the retailer does not share private demand information with the manufacturer, we find that freebie promotion can increase the profits of both the manufacturer and retailer, but the retailer benefits more than the manufacturer. When the manufacturer offers freebie promotion, he can always benefit from the retailer's information sharing. When the relative length of the promotional period is relatively low, the retailer has no incentive to actively share her demand information. When the relative length of the promotional period is sufficiently high, the retailer is likely willing to actively share information with the manufacturer, and information sharing is beneficial to the total supply chain at this time.

Second, on the premise that the manufacturer has a leadership advantage over the retailer, this power structure can benefit the manufacturer but hurt the retailer when the relative length of the promotional period is relatively small. When the manufacturer's market power is relatively low, the manufacturer's revenue is decreased, but the retailer's revenue is increased. Meanwhile, when the manufacturer's and the retailer's power relationship is equal, it indicates that the manufacturer does not have a leadership advantage; his profit be likely to decrease, and the retailer can benefit from his situation.

Finally, the value of demand information sharing decreases as the relative length of the promotional period and the sensitivity coefficient of freebies both increases, and the value increases as the cost coefficient of the freebies decreases. Additionally, the manufacturer can adopt strategies, such as increasing the relative length of the promotional period or adopting marketing strategies, to increase the consumers' sensitivity to freebie promotion. From the manufacturer's perspective, he can add the freebie's value to encourage the retailer to share private demand information or offset the loss as a result of her information sharing.

Our study has certain limitations that illuminate future research directions. First, in consideration of the differences in the promotional strategies of high-end cosmetics and low-end cosmetics, we focus on the impact of freebies on the manufacturer's promotional strategies. Therefore, in the future, one could consider the characteristics of low and mid-end cosmetics combined with freebies and price promotions and examine the effect of the freebies on the manufacturer's promotional strategies. Second, the unit cost of the product and freebies have been set as constants in our models. Further study may explore the impact of variable costs when producing freebies under uncertain environments. Third, for the hypothesis pertaining to demand in this research, a manufacturer is assumed to have only certain information about the product, and he cannot acquire information about the uncertain part of the demand. Future research may consider both the manufacturer and retailer having their own demand signals as they engage in the game, and the manufacturer can obtain his own private demand information through a third-party organization [31].

Appendix A.

Proof of Proposition 4.1. After analyzing the retailer's expected profit function equation (4.2), $E(\pi_R^{NN}|Y)$ is strictly concave with respect to p because $\frac{\mathrm{d}^2E(\pi_R^{NN}|Y)}{\mathrm{d}p^2}=-2\alpha<0$. Thereafter, the retailer's response function is $p=\frac{a+\alpha w+E(\varepsilon|Y)}{2\alpha}$; substituting this expression into equation (4.1), $E[\pi_M^{NN}]$. The expected profit function $E[\pi_M^{NN}]$ is strictly concave in terms of w because $\frac{\mathrm{d}^2E[\pi_M^{NN}]}{\mathrm{d}w^2}=-\alpha<0$. w^{NN} can be obtained by solving the first-order optimality condition. When w^{NN} is plugged into p, the expected profits of the manufacturer and retailer hold. □

Proof of Proposition 4.2. Given the manufacturer's wholesale price and the value of freebie, the retailer's expected profit function equation (4.4) $E(\pi_R^{FN}|Y)$ is strictly concave in terms of p because $\frac{\mathrm{d}^2E(\pi_R^{FN}|Y)}{\mathrm{d}p^2}=-2\alpha<0$. The retailer's response function is $p=\frac{a+\alpha w+\beta\lambda f+E(\varepsilon|Y)}{2\alpha}$. Substituting the retailer's response function into the manufacturer's expected profit function equation (4.3) to maximize his expected profit. The Hessian matrix H of $E[\pi_M^{FN}]$ in terms of w and f is $H=\begin{vmatrix}\partial^2\pi_M^{FN}/\partial w^2 & \partial^2\pi_M^{FN}/\partial w\partial f\\ \partial^2\pi_M^{FN}/\partial f\partial w & \partial^2\pi_M^{FN}/\partial f^2\end{vmatrix}=\begin{vmatrix}-\alpha & \beta(\lambda+\alpha\theta)/2\\ \beta(\lambda+\alpha\theta)/2 & \beta\theta\lambda(\beta-2)\end{vmatrix}$, where $\partial^2\pi_M^{FN}/\partial w^2<0$. Accordingly, we have $|H|=\frac{\beta}{4}(8\alpha\theta\lambda-\alpha^2\beta\theta^2-6\alpha\beta\theta\lambda-\beta\lambda^2)$. By $|H|>0$, the manufacturer's expected optimal profit function is a joint concave function with respect to w and f. Then, when $|H|>0$, we have $0<\beta<\frac{8\alpha\theta\lambda}{\alpha^2\theta^2+6\alpha\theta\lambda+\lambda^2}$. Furthermore, the manufacturer's and retailer's optimal prices and value of freebies are obtained as p^{FN}, w^{FN} and f^{FN}, respectively. The corresponding expected profits of the manufacturer and retailer are $E[\pi_M^{FN}]$ and $E[\pi_R^{FN}|Y]$, where $B=8\alpha\theta\lambda-\alpha^2\beta\theta^2-6\alpha\beta\theta\lambda-\beta\lambda^2>0$. □

Proof of 4.3. In the FS Stackelberg model, the retailer shares the information with the manufacturer, and the profit of the manufacturer changes. As the proof is similar to Proposition 4.2, we omit this part in this research. □

Proof of Proposition 5.1. $w^{FN}-w^{NN}=\frac{\beta(a-c\alpha)(\lambda^2-\alpha^2\theta^2)}{2\alpha B}$, because $a>c\alpha, \lambda>\alpha\theta$, and we obtain $w^{FN}>w^{NN}$. $p^{FN}-p^{NN}=\frac{(a-c\alpha)\beta(\lambda-\alpha\theta)(3\lambda+\alpha\theta)}{4\alpha B}$, because $\lambda>\alpha\theta$, and we obtain $p^{FN}>p^{NN}$. $E[\pi_M^{FN}]-E[\pi_M^{NN}]=\frac{(a-c\alpha)^2\beta(\lambda-\alpha\theta)^2}{8\alpha B}>0$. $E[\pi_R^{FN}|Y]-E[\pi_R^{NN}|Y]=\frac{(a-c\alpha)^2(8\alpha\theta\lambda(1-\beta)+B)(8\alpha\theta\lambda(1-\beta)-B)}{B^2 16\alpha}>0$. As $\frac{8(1-\beta)\alpha\theta\lambda+B}{2B}-1>0$, $E[\pi_M^{FN}]-E[\pi_M^{NN}]\leq E[\pi_R^{FN}|Y]-E[\pi_R^{NN}|Y]$. □

Proof of Proposition 5.2. (1). *Proof of the value of freebies*

$f^{FN-B}-f^{FN}=\frac{(a-c\alpha)\lambda(1-\beta)(8\alpha\theta\lambda-\alpha^2\beta\theta^2-5\alpha\beta\theta\lambda-2\beta\lambda^2)}{BB2}>0.$

As $B > B1$, we have $f^{FN-N} > f^{FN}$. $f^{FN-N} - f^{FN-B} = \frac{(a-c\alpha)\lambda(1-\beta)(\beta\theta+\beta\lambda-2\alpha\theta)}{B1B2}$. $f^{FN-N} > f^{FN-B} > f^{FN}$, if $\beta\theta + \beta\lambda - 2\alpha\theta > 0$; $f^{FN-B} > f^{FN-N} > f^{FN}$, if $\beta\theta + \beta\lambda - 2\alpha\theta \leq 0$.

(2). *Proof of the retail prices*

$p^{FN-B} - p^{FN} = \frac{\beta t\sigma^2 Y}{2\alpha(4-\beta)(1+t\sigma^2)} + \frac{(a-c\alpha)(1-\beta)\beta\lambda^3(2\alpha\theta-\alpha\beta\theta-\beta\lambda)}{\alpha BB2}$. $p^{FN-B} > p^{FN}$ if and only if $2\alpha\theta - \alpha\beta\theta - \beta\lambda \geq 0$, namely, $0 < \beta \leq \frac{2\alpha\theta}{(\alpha\theta+\lambda)}$.

By comparing p^{FN} with p^{FN-N}, we obtain $p^{FN} - p^{FN-N} = \frac{2(a-c\alpha)(1-\beta)\theta\lambda^2(2\alpha\theta-\alpha\beta\theta-\beta\lambda)}{BB1} + \frac{Yt\sigma^2}{\alpha(1+t\sigma^2)}(2\lambda - 2\beta\lambda - \alpha\beta\theta) > 0$; thus, $p^{FN} > p^{FN-N}$.

(3). *Poof of wholesale prices*

$w^{FN} - w^{FN-B} = \frac{(1-\beta)(a-c\alpha)\beta\theta\lambda^2(4\lambda-\alpha\beta\theta-3\beta\lambda)}{BB2}$, given that $4\lambda - \alpha\beta\theta - 3\beta\lambda > 0$, namely, $\beta < \frac{4\lambda}{\alpha\theta+3\lambda}$, then $w^{FN} > w^{FN-B}$.

$w^{FN-N} - w^{FN-B} = \frac{(1-\beta)(a-c\alpha)(4-\beta)\theta\lambda^2(\beta\lambda+\alpha\beta\theta-3\alpha\theta)}{B1B2}$. $w^{FN-B} > w^{FN-N}$, if $\beta \leq \frac{2\alpha\theta}{\alpha\theta+\lambda}$; otherwise, $w^{FN-B} < w^{FN-N}$.

(4). *Proof of the manufacturer's profits*

According to the manufacturer's expected profit under the FN M-R Stackelberg model and Nash game model, we derive $E[\pi_M^{FN}] > E[\pi_M^{FN-B}]$. Given that $B < B2$ holds. We can derive that $E[\pi_M^{FN-B}] - E[\pi_M^{FN-N}] = \frac{(a-c\alpha)^2(1-\beta)^2\theta\lambda^3(\alpha\theta(\beta-2)+\beta\lambda)^2}{B2B1^2} > 0$ also holds. Thus, $E[\pi_M^{FN}] > E[\pi_M^{FN-B}] > E[\pi_M^{FN-N}]$. □

Proof of Proposition 5.3. The relationships of the optimal wholesale price, retail price, and the value of freebies between the FN and FS Stackelberg models are obvious.

$p^{FS} - p^{FN} = \frac{4\alpha\theta\lambda-\beta((\alpha\theta+\lambda)^2+2\alpha\theta\lambda)}{2\alpha B}\frac{Yt\sigma^2}{(1+t\sigma^2)}$, $p^{FS} < p^{FN}$ is established if $\frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2+2\alpha\theta\lambda} < \beta < 1$; otherwise, $p^{FS} \geq p^{FN}$.

$E[\pi_M^{FS}] - E[\pi_M^{FN}] = \frac{(1-\beta)\theta\lambda}{B}\frac{t\sigma^4}{(1+t\sigma^2)} > 0$,

$E[\pi_R^{FS}|Y] - E[\pi_R^{FN}|Y] = \frac{-(\alpha^2\beta\theta^2-4\alpha\theta\lambda+2\alpha\beta\theta\lambda+\beta\lambda^2)(\alpha^2\beta\theta^2-12\alpha\theta\lambda+10\alpha\beta\theta\lambda+\beta\lambda^2)t\sigma^4}{4\alpha B^2(1+t\sigma^2)}$. Given that $B > 0$, we obtain $\beta < \frac{8\alpha\theta\lambda}{\alpha^2\theta^2+6\alpha\theta\lambda+\lambda^2}$. Furthermore, $\alpha^2\beta\theta^2 - 12\alpha\theta\lambda + 10\alpha\beta\theta\lambda + \beta\lambda^2 = \frac{-4\alpha\theta\lambda(\lambda-\alpha\theta)^2}{\alpha^2\theta^2+6\alpha\theta\lambda+\lambda^2} < 0$. $E[\pi_R^{FS}|Y] > E[\pi_R^{FN}|Y]$, if $\alpha^2\beta\theta^2 - 4\alpha\theta\lambda + 2\alpha\beta\theta\lambda + \beta\lambda^2 > 0$, namely, $\beta > \frac{4\alpha\theta\lambda}{(\alpha\theta+\lambda)^2}$; otherwise, $E[\pi_R^{FS}|Y] \leq E[\pi_R^{FN}|Y]$. □

Proof of Corollary 5.4. (1). $\frac{\partial f^{FN}}{\partial\beta} = \frac{(a-c\alpha)(\lambda-\theta\alpha)(\alpha^2\theta^2+6\alpha\theta\lambda+\lambda^2)}{B^2} > 0$. $\frac{\partial f^{FN}}{\partial\lambda} = \frac{-(a-c\alpha)(\alpha^2(7\beta-8)\theta^2+2\alpha\beta\theta\lambda-\beta\lambda^2)}{B^2}$, because $\alpha^2(7\beta-8)\theta^2 + 2\alpha\beta\theta\lambda - \beta\lambda^2 = -8(1-\beta)\alpha^2\theta^2 - \beta(\lambda-\alpha\theta)^2 < 0$, and we obtain $\frac{\partial f^{FN}}{\partial\lambda} > 0$.

(2). $\frac{\partial p^{FN}}{\partial\beta} = \frac{-2\lambda\theta(a-c\alpha)(\alpha\theta+3\lambda)(\alpha\theta-\lambda)}{B^2} > 0$. $\frac{\partial p^{FN}}{\partial\lambda} = \frac{\beta\theta(c\alpha-a)(-B-(\alpha\theta-\lambda)(\alpha\theta-3\lambda+2\beta\lambda))}{B^2}$. As previously mentioned, $2\alpha\theta - \alpha\beta\theta - \beta\lambda > 0$, namely, $\beta < \frac{2\alpha\theta}{\lambda+\alpha\theta}$, $\alpha\theta - 3\lambda + 2\beta\lambda < \alpha\theta - 3\lambda + 2\lambda\frac{2\alpha\theta}{\lambda+\alpha\theta} = \frac{(\alpha\theta+3\lambda)(\alpha\theta-\lambda)}{\lambda+\alpha\theta} < 0$. Then $\alpha^2(\beta-2)\theta^2 + 2\alpha\beta\theta\lambda + (5\beta-6)\lambda^2 < 0$. Therefore, $\frac{\partial p^{FN}}{\partial\lambda} > 0$.

$\frac{\partial p^{FN}}{\partial\theta} = \frac{\beta\lambda(a-c\alpha)(\alpha^2(\beta-2)\theta^2+2\alpha\beta\theta\lambda+(5\beta-6)\lambda^2)}{B^2} < 0$.

(3). $\frac{\partial E[\pi_M^{FN}]}{\partial\beta} = \frac{\theta\lambda(\lambda-\alpha\theta)^2}{B^2} > 0$, $\frac{\partial E[\pi_M^{FN}]}{\partial\lambda} = \frac{(\beta-1)\beta\theta(\alpha^2\theta^2-\lambda^2)}{B^2} > 0$, $\frac{\partial E[\pi_M^{FN}]}{\partial\theta} = \frac{(\beta-1)\beta\lambda(\lambda^2-\alpha^2\theta^2)}{B^2} < 0$.

(4). $\frac{\partial E[\pi_R^{FN}|Y]}{\partial\beta} = \frac{8\alpha(\beta-1)(\lambda-\alpha\theta)^2}{B^3} < 0$, because $3\alpha\beta\theta + \beta\lambda - 4\alpha\theta < (3\alpha\theta+\lambda)\frac{2\alpha\theta}{\lambda+\alpha\theta} - 4\alpha\theta = \frac{2\alpha\theta(\alpha\theta-\lambda)}{\lambda+\alpha\theta} < 0$, $\frac{\partial E[\pi_R^{FN}|Y]}{\partial\lambda} > 0$ holds.

$\frac{\partial E[\pi_R^{FN}|Y]}{\partial\theta} = \frac{-16\alpha^2(\beta-1)^2(\alpha\beta\theta+3\beta\lambda-4\lambda)}{B^3}$. Similarly, $\alpha\beta\theta + 3\beta\lambda - 4\lambda < (\alpha\theta+3\lambda)\frac{2\alpha\theta}{\lambda+\alpha\theta} - 4\lambda = \frac{2(2\lambda+\alpha\theta)(\alpha\theta-\lambda)}{\lambda+\alpha\theta} < 0$, and $\frac{\partial E[\pi_R^{FN}|Y]}{\partial\theta} > 0$ holds. □

Proof of Corollary 5.5. $\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial\lambda} = \frac{(\beta-1)\beta\theta(\alpha^2\theta^2-\lambda^2)}{B^2} > 0$,

$\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial\theta} = \frac{(\beta-1)\beta\lambda(\lambda^2-\alpha^2\theta^2)}{B^2} < 0$,

$\frac{\partial(E[\pi_M^{FS}]-E[\pi_M^{FN}])}{\partial\beta} = \frac{\theta\lambda(\lambda-\alpha\theta)^2}{B^3} > 0$,

$$\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial\lambda} = \frac{8\alpha(1-\beta)^2\beta\theta^2\lambda(\alpha^2\theta^2-\lambda^2)}{B^3} < 0,$$
$$\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial\theta} = \frac{-8\alpha(1-\beta)^2\beta\theta\lambda^2(\alpha^2\theta^2-\lambda^2)}{B^3} > 0,$$
$$\frac{\partial(E[\pi_R^{FS}|Y]-E[\pi_R^{FN}|Y])}{\partial\beta} = \frac{8\alpha(\beta-1)\theta^2\lambda^2(\alpha\theta-\lambda^2)}{B^3} < 0. \qquad \square$$

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