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EFFECTS OF CONSUMER LOYALTY AND PRODUCT WEB COMPATIBILITY ON COOPERATIVE ADVERTISING AND PRICING POLICIES IN A DUAL-CHANNEL SUPPLY CHAIN

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Abstract. A common practice for brand manufacturers is to operate dual distribution channels in which they offer an online channel for direct sales to end consumers and an independently-managed retail channel for sales in physical stores. This structure enables the manufacturers to reach multiple segments of consumers with different online and offline shopping preferences, but it may create channel conflicts due to the manufacturers' competitive position in the end market. Cooperative advertising programs can be implemented in response to the emerging competitive dynamics between the manufacturers and the retailers. We investigate the impact of the consumers' sales channel preference (*i.e.*, "consumer loyalty") and the product compatibility with online shopping (*i.e.*, "product web compatibility or web fit") on the cooperative advertising and pricing decisions of a manufacturer and a retailer in a dual-channel supply chain. We use game-theoretical models and characterize the firms' equilibrium behaviors under different power structures in the channel. Our results indicate that the level of the retailer's advertising investment and the manufacturer's reimbursement in the cooperative advertising program depend critically on consumer loyalty, product web compatibility, and the power distribution among the channel members. For example, when the channel power is symmetrically distributed or held asymmetrically by the retailer, the retailer's local advertising level increases as the product web compatibility decreases or the proportion of store-loyal consumers increases; whereas this trend is reversed when the manufacturer is the channel leader. We examine how the introduction of the direct channel affects the profits, and we generate additional managerial insights from numerical experiments.

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1. Introduction

The Internet and related technologies play an important role in shaping consumers' shopping behavior. Today, an increasing number of consumers turn to web-based stores to shop. In 2019, consumers spent \$569.80 billion (seasonally adjusted) on online retail purchases in the United States, which accounted for a 12.4% increase in comparison with the purchases in 2018 [12]. During the COVID-19 pandemic, the annual growth in e-commerce

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retail sales has accelerated, as more consumers shopped online in response to physical store closures and online channel offerings by many companies.

In 2021, e-commerce retail sales reached \$960.15 billion in the United States, representing an 18.3% increase over 2020 and a 68.5% increase over 2019 [12]. Note, however, that online retail sales constitute a small portion of the total retail sales in the United States and worldwide, implying that in-store shopping remains the preferable option for many consumers [46]. For example, even with the strong growth due to the global pandemic, e-commerce retail sales represented only 14.6% of all retail sales in 2021 in the United States [12]. Customers' in-store shopping preference is especially prevalent in product categories where the purchase process benefits highly from personal assistance and customers' "touch and feel" of the items, which is best supported by brick-and-mortar stores.

These trends and developments have led firms to restructure their sales channels and given rise to dual distribution channels. Many suppliers and brand manufacturers have set up online channels to sell directly to end consumers, thereby complementing their traditional retail channels operated by independent brick-and-mortar retailers. Adopted by companies such as IBM, Nike, and Cisco, dual distribution channels help firms reach a large consumer base and have direct control of distribution and pricing of their products while also maintaining the benefits from the wide reach of their retailers' footprint without committing high capital expenditures. However, since partners in dual distribution channels become direct competitors in the end market, conflicts may emerge. By introducing a direct online channel, a manufacturer can attract consumers to purchase through this channel. Such a shift in consumers' purchasing behavior may hurt the retailer's profits and is more likely to occur when the product is highly compatible with online shopping and/or customers have strong preference or loyalty towards the manufacturer's brand. Similarly, the retailer can compete by offering prices to target customers who prefer in-store shopping to online shopping or by reducing its order quantities, both of which can reduce the manufacturer's sales.

One strategy that firms in dual-channel supply chains can employ to alleviate conflicts is to engage in cooperative programs. For example, in a practice called "cooperative advertising", the manufacturer shares the cost of the retailer's local advertising efforts in an attempt to stimulate demand and increase product sales. Companies such as Apple, Dell, and HP offer these programs [37]. Implementing a cooperative advertising scheme in a dual-channel environment may result in interesting dynamics because the firms in the channel compete (horizontally and vertically) and cooperate at the same time. Furthermore, customers' purchasing behavior when presented with online and offline channels can play an important role in the resulting outcomes. In this study, we investigate the cooperative advertising and pricing decisions of firms in dual-channel supply chains. We focus on consumer loyalty and product web compatibility (or product web-fit) as factors for consumers' channel preferences and the firms' advertising and pricing decisions.

There are many factors affecting consumers' purchase decisions on the online and offline channels. According to a survey, the main reasons driving the popularity of online shopping include the availability, convenience, the ability to compare prices, better prices, greater variety, and free shipping offers [14]. It is worth noting that the selling prices offered on the online and offline channels can influence consumers' purchase decisions. Product categories frequently bought online include consumer electronics, clothing, books, movies, games, small household appliances, and office supplies. Products that are conducive to online purchasing are those with few nondigital attributes. These attributes relate to fit and feel of items which are difficult to observe and assess without a physical inspection [4]. Consequently, they may negatively affect the demand volume of online channels. In other words, the extent to which a product is compatible with the web on the basis of its features is an important factor for online sales [17]. In comparison with online shopping, the main reasons driving in-store purchases include the ability to touch/feel/try-on items, personal service, avoiding delivery times and shipping costs, and shopping experience [14]. Home furnishing, jewelry, personal care items, and large appliances are some examples of products that customers prefer to purchase in physical stores. In a dual-channel environment where the manufacturer sells through an online channel and a brick-and-mortar retailer channel, product web-fit characteristics and the selling prices across the two distribution channels can affect company profits.

Consumer loyalty towards the manufacturer's brand or the retail store is another aspect that can affect the profits. Brand reputation and loyalty rewards are important factors for consumers' choice of shopping at branded online stores [43]. Therefore, manufacturers with established brands can induce brand-loyal consumers to purchase through the direct online channel for reasons such as convenience of online shopping and high shopping or transportation costs of store visits. Meanwhile, other customers might show loyalty towards the brick-and-mortar retailers because they may find it critical to physically examine the items before purchasing or may value other advantages of in-store shopping experience. In practice, such customer behavior may be encouraged by the retailers' rewards programs. Surveys reveal that, "69% of store card holders believe they are valuable and encourage them to shop at the same store" [34]. The preceding discussion indicates that firms in a dual-channel setting may face a market with "brand loyal" and "store loyal" consumer segments [9, 15, 30]. When the degree of product web-fit and the market size of brand-loyal customers increase, a positive demand effect can be expected for the online channel. However, the net effects of these factors on the firm profits depend also on the manufacturer's and retailer's selling prices and the specific terms of the cooperative advertising scheme in the dual-channel setting. To explore these effects, we develop a model by explicitly addressing the consumer loyalty and product web compatibility. Ours is the first work that unifies these two aspects in the context of cooperative advertising.

In cooperative advertising, the retailer implements advertising efforts locally and shares the associated expenditures with the manufacturer. As noted by Aust and Buscher [3], *vertical* cooperative advertising is the most common interpretation of this practice, and it entails the manufacturer paying a certain fraction of the advertising cost of its retailer, which is referred to as the manufacturer's participation rate. The primary motivation for manufacturers to offer these programs is to encourage the retailers to promote their brand locally and increase consumer demand and immediate sales of their products. Although specific policies may vary, Aust and Buscher [3] provide data that indicate an increasing trend of cooperative advertising applications in practice. In North America, $36 billion is allocated for these programs annually, and digital channels can present further opportunities for brand manufacturers and local retailers [7]. Given the strong growth of e-commerce and dual distribution channels, the significance of these programs can be expected to increase in the future. Moreover, it is beneficial to consider recent developments in channel power relationships to fully understand the mechanisms underlying the channel partners' decisions. Particularly, although brand manufacturers have traditionally maintained a dominant position in their channels, the emergence of large retailers has changed power relationships. Motivated by these observations, we consider scenarios that reflect different power structures in the dual-channel supply chain. Our main research questions are as follows:

(1) How do the consumer loyalty and product compatibility with the web affect the channel members' pricing and cooperative advertising strategies in a dual distribution channel?
(2) What are the profit implications of adding a direct online channel in the described model setting?
(3) What is the impact of alternative structures of channel dominance on the supply chain members' decisions?

We employ a game-theoretic approach to analyze our research problems. The model setting is a single-manufacturer, single-retailer supply chain where the manufacturer sells through the retail channel and a direct online channel. The retailer operates brick-and-mortar stores and purchases the product from the manufacturer at a wholesale price, which is determined by the manufacturer. The selling prices for in-store and online purchases are decided by the retailer and the manufacturer, respectively. The channel partners engage in a cooperative advertising scheme in which the manufacturer determines the participation rate and the retailer determines the level of local advertising. We develop a model that studies both vertical and horizontal channel competitions in which consumers show heterogeneous loyalty towards different channels and the product is not perfectly compatible with the web. We characterize the equilibrium decisions of the manufacturer and the retailer by analyzing scenarios of alternative channel power structures. We conduct a series of numerical analysis to compare the equilibrium strategies under different scenarios and test the sensitivity of the results to the model parameters.

Our results suggest that the manufacturer's profit improves with the introduction of the online channel, and it further increases as the consumer brand loyalty or the degree of product web-fit increases. The introduction of the online channel reduces the retailer's profit, but the cooperative advertising program alleviates the detrimental effects. Each channel member prefers to be in the leader position in both single and dual-channel supply chains; however, when the online channel is introduced in a single-channel supply chain under cooperative advertising, both members achieve a more advantageous outcome (*i.e.*, higher profit increase for the manufacturer and lower profit loss for the retailer) in the follower position. Moreover, the manufacturer charges a higher wholesale price when either more brand-loyal consumers switch to the direct channel, or the channel competition intensifies under the retailer's dominance or the symmetrically-distributed channel power. In cooperative advertising program, the manufacturer's participation rate increases as the product becomes more compatible with the online shopping or the consumers are less loyal to retail channel. The retailer, on the other hand, adopts different advertising policies with respect to the consumer loyalty and product web-fit parameters depending on the type of the power dominance, which underpins the salience of considering different power structures between the channel members.

Our research contributes to the literature in two major ways. Firstly, it enhances our understanding of dual-channel supply chains by assessing the simultaneous effects of consumer loyalty and product web compatibility. Secondly, it explores the impact of underlying power distribution among the channel members on the pricing and cooperative advertising strategies.

This paper is organized as follows. In Section 2, we position our work in the relevant literature. In Section 3, we describe our model and assumptions. Section 4 presents the game-theoretical analysis of the problems under different channel dominance scenarios. In Section 5, we provide a numerical analysis to generate further managerial insights. Section 6 concludes our paper with a discussion of research extensions. All proofs and technical details are provided in the Appendices A and B.

2. Literature review

Our work is related to two main streams of research in supply chains: dual-channel distribution systems and cooperative advertising.

One line of research on dual supply chains has investigated the economic impact of the introduction of a direct sales channel. Moriarty and Moran [35] discuss potential channel conflicts due to hybrid sales channels and note some opportunities for enhancing firm profits. Rhee and Park [38] show that it is optimal for the manufacturer to offer a direct online store and sell through a brick-and-mortar retailer in a market with price or service sensitive customer segments. By considering a price-setting game between a manufacturer and its independent retailer, Chiang *et al.* [17] show that the manufacturer can strategically use the direct channel to improve the efficiency of the retailer channel and increase the profits of both parties. Tsay and Agrawal [42] provide a general model with logistics and marketing costs and find a similar win–win outcome for the manufacturer and the retailer. Cattani *et al.* [11] investigate the effects on profits and market share when the direct channel is introduced under specific pricing strategies. More recent research has investigated various topics, including, channel coordination and contracts [8, 9], pricing, service, and quality effort decisions [16], pricing decisions in the presence of multiple retailers [23], pricing and asymmetric reference effect [47], pricing and inventory decisions [21], pricing, quality, and stocking decisions [20], and the effect of information structure and risk aversion [33]. In the context of multi-channel distribution systems, Tahirov and Glock [41] carry out an exhaustive review of the manufacturer's drivers behind opening a direct channel and possible mechanisms to mitigate the channel conflicts. Similar to the studies in this line of research, we examine the economic impact of the direct sales channel but we focus on the cooperative advertising scheme and consider product web compatibility and consumer loyalty.

Among the aforementioned works, Chiang *et al.* [17] address product web compatibility and Cai *et al.* [9] address consumer loyalty. Consistent with the empirical findings suggesting that customers prefer the conventional retail channels and may not accept the web-based channels as perfect substitutes [26, 32], Chiang *et al.* [17] consider the effect of customer acceptance of a direct channel on supply chain design. The authors assume that a product is worth less to a buyer when purchased through the online channel rather than the conventional retail channel, where the reduced valuation is determined by a parameter between zero and one, which can be interpreted as the compatibility of the product with the online channel. Cai *et al.* [9] adopt a market structure consisting of store-loyal and brand-loyal customers and investigate price discount contracts under the scenarios of supplier Stackelberg, retailer Stackelberg, and Nash games. Such a market structure was first introduced by Kumar and Ruan [30] to examine the impact of market characteristics on the strategic role of the direct Internet channel. Consumer loyalty settings are also considered by Dan *et al.* [18] and Li *et al.* [31]. Their findings indicate that the degree of consumer loyalty to retail channel can be an important factor on the firms' pricing decisions. A similar result is found by Chen *et al.* [16] for the effect of direct-channel loyalty on the manufacturer's and the retailer's service and quality strategies. We contribute to this stream of literature by considering the simultaneous effects of product web compatibility and consumer loyalty in the dual channel.

The second relevant research area is cooperative advertising. Cooperative advertising has been extensively investigated in the supply chain literature from a multitude of perspectives, including the design and profit impact of the program [6, 22], competition [28, 29], joint pricing and advertising decisions [2], dynamic effects of national or local advertising [25], price and quality decisions in a three-echelon supply chain [40], and the impact of power relationships [13, 39]. Aust and Buscher [3] and Jørgensen and Zaccour [24] review the related literature. Consistent with most papers in this area, we model the cooperative advertising scheme such that the retailer decides the level of local advertising effort to exert and the manufacturer determines the percentage of the advertising costs to share. We also consider joint pricing and advertising decisions and study game structures reflecting alternative power relationships in the channel. Different from these papers, we focus on a dual-channel supply chain setting.

The literature that investigates cooperative advertising and pricing in a dual-channel supply chain environment contains the studies by Cao *et al.* [10], Chen [15], Pei *et al.* [37], Yan and Pei [44], and Yan *et al.* [45]. Cao *et al.* [10] focus on advertising decisions under centralized and decentralized channels in the presence of customer returns. Including product web compatibility as a model component, Yan and Pei [44] show that a cooperative advertising program can alleviate the dual-channel competition and improve the performance of the channel members and the entire channel. Similar to Yan and Pei [44], we incorporate the factor of product web-fit into demand; however, the demand model in our paper addresses different segments of customers in the presence of dual sales channels as well as the degree of consumer loyalty towards a particular channel. Pei *et al.* [37] investigate how the introduction of online channel impacts the manufacturer's advertising efforts for products that have lower, higher or equal compatibility with online shopping compared to the in-store shopping. However, they assume that the manufacturer is solely in charge of the advertising investment and do not account for consumer loyalty. Yan *et al.* [45] examine the effect of information sharing on the manufacturer's investment in cooperative advertising. We do not consider demand uncertainty or information sharing, but we explicitly model the retailer's advertising effort and the manufacturer's participation rate decisions; whereas Yan *et al.* [45] assume that the manufacturer fully supports the retailer's advertising expenditures, which is a special case of the manufacturer's participation rate in our model setting.

Chen [15] investigates the impact of pricing and cooperative advertising decisions on channel competition and considers a market structure with store-loyal and brand-loyal customer segments who can exhibit channel switching behavior. Chen [15] considers the manufacturer's global advertising in addition to cooperative advertising but assumes that the participation rate of the manufacturer in the cooperative advertising scheme is exogenous and does not address product web compatibility. In this study, we focus on cooperative advertising and treat the participation rate endogenously. We contribute to the literature in this stream by investigating the pricing and cooperative advertising policies in dual-channel supply chains with the consideration of consumer loyalty and product web-fit aspects. This unified framework leads to managerial insights different from those when either factor is considered in isolation. We also analyze the proposed model in game structures resulting from different power configurations in the channel (*i.e.*, manufacturer Stackelberg, retailer Stackelberg, Nash), whereas this stream of literature has focused only on the manufacturer Stackelberg game scenario.

3. The Model

We consider a two-echelon supply chain consisting of a manufacturer (he) and an independent retailer (she). The manufacturer sells his product through the retail channel and can also sell directly to end consumers through his online channel; that is, the manufacturer operates a dual-channel supply chain. The retailer has physical presence only and operates a brick-and-mortar store or a chain of stores. We assume that the base level of demand for the product, *i.e.*, market size, is a constant value (denoted with α), which is quite standard in the literature (see, *e.g.*, [18]). Moreover, consumers have developed distinct preferences or loyalty towards either the manufacturer's brand or the retail store. Particularly, consistent with Dan *et al.* [18] and Li *et al.* [31], we assume that $\lambda \in (0, 1)$ represents the degree of consumer loyalty to the store and $1 - \lambda$ represents the degree of consumer loyalty to the brand. Alternatively, one can think of a market with two consumer segments, namely, store-loyal and brand-loyal consumer segments, which constitute λ and $1 - \lambda$ proportion of the total market, respectively. The base market sizes of the store-loyal and brand-loyal consumers are then given by $\lambda\alpha$ and $(1 - \lambda)\alpha$.

Under the dual-channel structure, consumers' purchase options differ according to their loyalty type. Brand-loyal consumers purchase only the manufacturer brand but may choose to buy from the retailer or the manufacturer, whereas store-loyal consumers purchase only from the retailer and may choose to buy the manufacturer brand or substitute brands [9, 30]. We characterize the demand in the retail and online channels by using a model proposed by Cai *et al.* [9], which we discuss shortly. We incorporate the unique features of our setting into this model, namely, the product web-fit and supply chain members' cooperative advertising, both of which are assumed to affect the demand.

Product web-fit is a measure of compatibility with the web-based shopping. The increasing values of this parameter indicate an increasing demand in an online channel. We denote the degree of product web-fit with $g \in (0, 1]$, where customers accept the online channel as a perfect substitute to the physical store when $g = 1$, whereas the limiting value of 0 represents web incompatibility due to, for example, the need for customizing, tasting, and experimenting (*e.g.*, [17]).

We assume that the manufacturer and the retailer implement a promotional support program in the form of vertical cooperative advertising, which is widely offered in practice [3, 24]. Consistent with this literature, we assume that the retailer can undertake costly local advertising efforts to stimulate the sales of the manufacturer's brand in the store, and the manufacturer reimburses a fraction of these expenditures, which is referred to as the manufacturer's participation rate (see, *e.g.*, [5]). The retailer's advertising level and the manufacturer's participation rate are decision variables by the respective parties. The level of local advertising effort a makes a direct demand-increasing impact in the retail channel through the term ka, where $k \geq 0$ represents the effectiveness of the advertising program. In addition, we assume that the retailer's local advertising efforts influence the demand in the manufacturer's online channel. This assumption can be motivated from the observation that consumers' exposure to in-store promotional programs can help to build brand awareness and preference and is consistent with the empirical evidence on the physical stores' function as "living billboards" that benefit online sales [48]. We assume that this influence is independent of whether the customer is brand or store loyal but it is proportional to how compatible the product is with the direct channel. Correspondingly, we model the advertising effect on the online channel with a demand-increasing term kga. Overall, for every unit of advertising effort, the demand increases by k and kg units in the physical and online stores, respectively; and the higher the product web-fit is, the more efficient the advertising endeavors will be in stimulating the demand in the online channel. For an analogy, consider the demand model by Chiang *et al.* [17]. In the model, the consumer's consumption value of the product, ν is adjusted to the value $g\nu$ when the product is purchased in the direct channel, which captures the proportional impact of the product web-fit parameter. We use a linear additive form to formulate the effect of advertising on demand, which is commonly adopted in the literature (see, *e.g.*, [24, 45]).

The demands in both channels are also affected by the selling prices, where p_m is the manufacturer's direct channel selling price and p_r is the retail channel selling price. Then, by extending the model in [9], we let y

TABLE 1. List of notations.

	Parameters
α	Base market size ($\alpha > 0$)
$\lambda, (1-\lambda)$	Fraction of store-loyal and brand-loyal customers ($\lambda \in (0,1)$)
α_r, α_m	Base market sizes of store and brand-loyal consumers, *i.e.*, $\alpha_r = \lambda\alpha$ and $\alpha_m = (1-\lambda)\alpha$
g	Product web-fit ($g \in (0,1]$)
$y, (1-y)$	Percentage of brand-loyal customers buying products with $g = 1$ from the direct and retail channels offering equal prices ($y \in (0,1]$)
k	Effectiveness of local advertising efforts ($k \geq 0$)
θ, β	Cross-price sensitivity and own price sensitivity of demand ($0 < \theta < \beta$)
	Decision variables and dependent variables
w	Manufacturer's wholesale price
p_r, p_m	Selling price in the retail channel and in the direct channel
m	Retailer's profit margin, $m = p_r - w$
a	Retailer's local advertising effort
t	Manufacturer's participation rate ($0 \leq t \leq 1$)
Π_m, Π_r, Π_{sc}	Manufacturer's, retailer's, and the supply chain's profit

TABLE 2. Base demand across channels.

	Customer purchase channel	
Customer segment	Direct channel	Retail channel
Brand loyal	$\alpha_m gy$	$\alpha_m(1-gy)$
Store loyal	–	α_r

represent the initial portion of brand-loyal customers who prefer purchasing from the direct online channel if the prices are consistent and the product web-fit is perfect, *i.e.*, $p_m = p_r$ and $g = 1$. The parameter y can be thought of as a measure of the online shopping service, *e.g.*, convenient shipping, payment, and returns, and is independent of the compatibility of the product with online shopping. For values of $g < 1$, we expect that a lower number of brand-loyal customers will have the aforementioned preference as the benefits brought by online shopping service may not be fully exploited when the product has imperfect web compatibility. We address this situation by adjusting the parameter as yg. Consequently, $(1-yg)$ represents the portion of brand-loyal customers who prefer purchasing products with $g < 1$ from the retailer when $p_m = p_r$. When the prices are inconsistent (*i.e.*, $p_m \neq p_r$), horizontal price competition between the direct and retail channels results in the brand-loyal customers' switching from one channel to the other on the basis of the price difference and the customers' degree of price sensitivity, which is denoted with parameter $\theta > 0$. Customers are also sensitive to price changes within each channel; that is, the demand function for the percent market size is linearly decreasing in the price at a rate denoted with $\beta > 0$.

For ease of reference, we summarize our notation in Table 1. We provide Table 2 and Figure 1 to illustrate the market and channel structure that we have discussed thus far.

We express the demands in the retail channel and online channel in equations (3.1) and (3.2), respectively. The resulting demand functions have linear structures, which are commonly used in the literature for analytical tractability (*e.g.*, [17, 29]).

$$D_r = \lambda\alpha(1-\beta p_r) + (1-\lambda)\alpha((1-gy)(1-\beta p_r) + \theta(p_m - p_r)) + ka \tag{3.1}$$

$$D_m = (1-\lambda)\alpha(gy(1-\beta p_m) - \theta(p_m - p_r)) + kga. \tag{3.2}$$

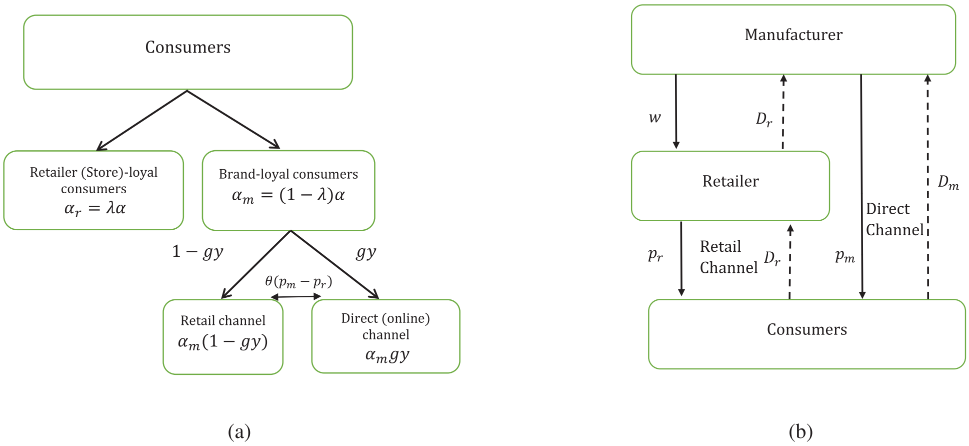


FIGURE 1. Market demand and channel structure in the dual-channel. (a) Market structure. (b) Channel structure.

The demand functions address the effects of consumer loyalty and product web-fit. Demand in the manufacturer's direct channel (D_m) increases when the brand-loyal market size increases and/or when the product web-fit or the preference of brand-loyal consumers towards the online channel increases (*i.e.*, $(1-\lambda)\uparrow, g\uparrow$, and/or $y\uparrow$). The model addresses the interplay of these parameters and price competition in affecting the retail channel demand. For example, increasing values of $(1-\lambda)$ can also increase the demand in the retail channel (D_r), if the manufacturer sets the direct channel price high, thereby diverting customers to the retail channel. The demand model can be reduced to that of Cai *et al.* [9] for the special case of $a = 0$ and $g = 1$.

We provide a practical motivation for the demand model. As discussed in Kumar and Ruan [30], store-loyal consumers represent the type of consumers that are loyal to the brick-and-mortar retailer and would never consider purchasing online. Therefore, the manufacturer can reach this segment only by selling through the retailer. By contrast, brand-loyal consumers have a strong preference for the manufacturer's product and would never consider purchasing a different brand. These consumers shop in-store in the absence of an online channel, however, they can shop online or in-store when an online channel becomes available. Suppose the manufacturer introduces a direct online channel and offers the same price as that in the retail channel. Then, a fraction of the brand-loyal consumers will switch from the retailer to the online channel due to the convenience and the product compatibility with online shopping, whereas the rest of these consumers will continue to buy in store. When the prices offered on the manufacturer's direct channel and the retail channel are different, the demand in each channel will be influenced by the sensitivity of the consumers to the price difference across the two channels. The retailer's advertising effort also affects the demand in both channels, with different intensities depending on the degree of the product web compatibility.

As an example, suppose a customer is interested in purchasing a personal computer. Store-loyal customers don't have any brand preference but are loyal to a retailer, such as Best Buy. These customers visit one of Best Buy's brick-and-mortar stores to make a purchase, and they are sensitive to the retailer's selling price of the product. Brand-loyal customers have a certain brand preference, such as HP, Dell, or Compaq, and some of these customers might buy in store, whereas others buy through the online channel. In doing so, the segment size of the brand-loyal consumers is adjusted by not only how convenient the online shopping is at the manufacturer's channel but also by how much a personal computer is convenient to be purchased directly online. If some of the brand-loyal consumers find it more appealing to test the performance of a personal computer before purchasing, then they might avoid the direct online channel and join the pool of Best Buy's store-loyal consumers. On the manufacturer's side, participating in the advertising campaign in the dual channel aims to attract more consumers from both the brand-loyal and store-loyal consumers. The effectiveness of the advertising campaign in convincing brand-loyal consumers to buy online is affected by how strongly these consumers feel it would be desirable to first inspect the personal computer in a Best Buy store before making any purchasing decision. In practice, companies that consider opening a direct online channel need to recognize the salient role of product web-fit in the success of an online channel in order to make more informed decisions based on the nuances of consumer behaviors such as brand and store loyalty and other factors that influence customers' channel preferences [41].

We next formulate the retailer's and manufacturer's profit functions. The retailer obtains the product from the manufacturer at the wholesale price w. Furthermore, the retailer incurs her share of local advertising cost which we assume to be of quadratic form, *i.e.*, $(1-t)a^2$. The quadratic form represents diminishing returns on sales effort expenditures (*e.g.*, returns on sales effort expenditures [42]). Correspondingly, the manufacturer incurs ta^2 through cooperative advertising and collects revenue from the sales on retail and direct channels. We assume that the production cost is zero and the manufacturer has ample capacity.

Then, the retailer's and the manufacturer's profit functions are:

$$\Pi_r = (p_r - w)D_r - (1-t)a^2, \tag{3.3}$$

$$\Pi_m = wD_r + p_m D_m - ta^2. \tag{3.4}$$

From equations (3.1) and (3.2), we equivalently have

$$\Pi_r = (p_r - w)(\alpha_r(1-\beta p_r) + \alpha_m((1-gy)(1-\beta p_r) + \theta(p_m - p_r)) + ka) - (1-t)a^2, \tag{3.5}$$

$$\begin{aligned}\Pi_m = {} & w(\alpha_r(1-\beta p_r) + \alpha_m((1-gy)(1-\beta p_r) + \theta(p_m - p_r)) + ka) \\ & + p_m(\alpha_m(gy(1-\beta p_m) - \theta(p_m - p_r)) + kga) - ta^2.\end{aligned} \tag{3.6}$$

The retailer and manufacturer make their decisions to maximize individual profits. The manufacturer determines w, t, and p_m, whereas the retailer determines p_r and a. For convenience of analysis, we make a change of variables and treat the retailer's profit margin, $m = p_r - w$, as the decision variable. Once the margin is established, the retail price can be obtained subsequently. We make some technical assumptions to facilitate the analysis. We assume $\theta < \beta$, which implies that the price effect of competing channel (*i.e.*, cross effect) does not exceed the effect of the channel's own price. This is a common assumption under price competition (*e.g.*, [27]). We ensure the conditions $1 - \beta p_r > 0$ and $1 - \beta p_m > 0$ are satisfied, which guarantee that positive demand exists in both channels when no cross-effects of prices occur and advertising level is zero.

4. Analysis

We analyze the decisions of the retailer and the manufacturer. The dual-channel structure results in competitive dynamics between the retailer and the manufacturer. Furthermore, the nature of competition can be influenced by possible power relationships in the channel. To capture such effects, we consider scenarios of alternative power structures. In line with studies such as Edirisinghe *et al.* [19] and SeyedEsfahani *et al.* [39], we model the supply chain power structure through the order of decisions of the supply chain members. The member who leads the interactions is regarded to be in a more powerful status in the channel and hence has channel dominance over the other member who takes a follower role in making decisions.

Consequently, three cases arise within a game-theoretical framework. In the first case, the retailer and the manufacturer have equal power, *i.e.*, Nash (simultaneous-move) game. In the second case, the retailer is the dominant member in the channel and takes the leadership role in the sequence of decisions, *i.e.*, retailer Stackelberg game, and in the third case, the manufacturer has channel power and leads the interactions, *i.e.*, manufacturer Stackelberg game. We characterize the equilibrium decisions of the channel members in each of these cases and denote them with the superscripts N, R, and M, respectively. In addition, we investigate the centralized system in which all channel power is exercised by a single decision maker, and we use the superscript C to denote the optimal decisions. Backward induction is used to derive the equilibrium decisions, where we impose certain

Table 3. Equilibrium decisions and profits in the single-channel supply chain.

	Nash	Retailer Stackelberg	Manufacturer Stackelberg	Centralized
Wholesale price	$\frac{2\alpha}{(6\alpha\beta-k^2)}$	$\frac{2\alpha}{(8\alpha\beta-k^2)}$	$\frac{16\alpha\beta-3k^2}{\beta(32\alpha\beta-9k^2)}$	–
Retail price	$\frac{4\alpha}{(6\alpha\beta-k^2)}$	$\frac{6\alpha}{(8\alpha\beta-k^2)}$	$\frac{24\alpha\beta-3k^2}{\beta(32\alpha\beta-9k^2)}$	$\frac{2\alpha}{(4\alpha\beta-k^2)}$
Participation rate	0	0	$\frac{1}{3}$	–
Local advertising level	$\frac{k\alpha}{(6\alpha\beta-k^2)}$	$\frac{k\alpha}{(8\alpha\beta-k^2)}$	$\frac{6k\alpha}{(32\alpha\beta-9k^2)}$	$\frac{k\alpha}{(4\alpha\beta-k^2)}$
Manufacturer's profit	$\frac{4\alpha^3\beta}{(6\alpha\beta-k^2)^2}$	$\frac{4\alpha^3\beta}{(8\alpha\beta-k^2)^2}$	$\frac{4\alpha^2}{(32\alpha\beta-9k^2)}$	–
Retailer's profit	$\frac{\alpha^2(4\alpha\beta-k^2)}{(6\alpha\beta-k^2)^2}$	$\frac{\alpha^2}{(8\alpha\beta-k^2)}$	$\frac{8\alpha^2(8\alpha\beta-3k^2)}{(32\alpha\beta-9k^2)^2}$	–
Channel profit	$\frac{\alpha^2(8\alpha\beta-k^2)}{(6\alpha\beta-k^2)^2}$	$\frac{\alpha^2(12\alpha\beta-k^2)}{(8\alpha\beta-k^2)^2}$	$\frac{12\alpha^2(16\alpha\beta-5k^2)}{(32\alpha\beta-9k^2)^2}$	$\frac{\alpha^2}{(4\alpha\beta-k^2)}$

conditions on the model parameters to ensure that the objective functions are concave, and the equilibrium decisions, demands, and profits are nonnegative in all games. We provide these conditions in Appendix A. These parameter conditions can be justified in most practical settings that we focus on, which are single-channel and dual-channel supply chains with a sufficiently large market size, α. Under such settings, the manufacturer would more likely pursue a dual-channel strategy, which is the main model setting in our paper. To present the expressions and equilibrium results concisely, we define a set of parameters based on the model parameters given in Table 1 and denote them with H_i's, for $i = 1, \ldots, 12$ (see Appendix A). We provide all proofs in Appendix B.

As we investigate the manufacturer's introduction of a direct channel, the single-channel setting provides a benchmark case against which we can compare the equilibrium decisions of the dual-channel supply chain. We provide the results in Proposition 4.1.

Proposition 4.1. *When the manufacturer sells through the retailer in a single-channel setting, then* $D_m = 0$, $D_r = \alpha(1-\beta p_r)+ka$, *and the profit functions of the manufacturer, retailer, and the channel are given by* $\Pi_m = w(\alpha(1-\beta p_r)+ka)-ta^2$, $\Pi_r = (p_r-w)(\alpha(1-\beta p_r)+ka)-(1-t)a^2$, *and* $\Pi_{sc} = p_r(\alpha(1-\beta p_r)+ka)-a^2$. *Table 3 summarizes the equilibrium results in the single-channel supply chain. Furthermore, the following comparison results hold for the equilibrium decisions and profits:* $w^M > w^N > w^R$; $p_r^M > p_r^N > p_r^R > p_r^C$; $t^M > t^R = t^N$; $a^C > a^M > a^N > a^R$; $\Pi_m^M > \Pi_m^N > \Pi_m^R$; $\Pi_r^R > \Pi_r^N > \Pi_r^M$; *and* $\Pi_{sc}^C > \Pi_{sc}^N > \Pi_{sc}^M > \Pi_{sc}^R$.

The manufacturer shares the retailer's advertising cost only when he holds the channel leadership. In the decentralized channel, the wholesale price and local advertising level take their highest values when the manufacturer is the leader, generating the largest profit for him in comparison with the other scenarios.

In the dual-channel setting, we investigate the effects of introducing a direct channel and study how the equilibrium results are influenced by the product web-fit and consumer loyalty characteristics. We begin by analyzing the Nash game.

4.1. Nash game

The Nash game addresses the situation where the manufacturer and the retailer have equal power in the dual-channel supply chain. Consequently, they make their decisions simultaneously; that is, the manufacturer determines the wholesale price w, direct channel selling price p_m, and the participation rate t, whereas the retailer determines the local advertising level a and the retail margin m to maximize their profits in equations (3.5) and (3.6), respectively. In this game, we obtain a unique Nash equilibrium, which is stated in Proposition 4.2.

Proposition 4.2. *In the Nash game, the equilibrium decisions of the manufacturer (w^N, p_m^N, t^N) and the retailer (m^N and a^N) are as follows:*

$$
\begin{aligned}
w^N &= \frac{2\alpha H_1(2H_2+\theta(1-\lambda))+\theta g k^2(1-y(1+g)(1-\lambda))}{2\beta H_1(6\alpha H_2-k^2)},\\
p_m^N &= \frac{H_2(6\alpha(1-\lambda)H_1+gk^2(1-y(1+g)(1-\lambda)))}{2\beta(1-\lambda)H_1(6\alpha H_2-k^2)},\\
t^N &= 0,\\
m^N &= \frac{2\alpha(1-gy(1-\lambda))}{(6\alpha H_2-k^2)},\ (p_r^N = w^N+m^N),\\
a^N &= \frac{k\alpha(1-gy(1-\lambda))}{(6\alpha H_2-k^2)}.
\end{aligned}
$$

We provide the manufacturer's and the retailer's equilibrium profit expressions in equations (B.1) and (B.2) in Appendix B. Consistent with the previous findings in dual channels, the manufacturer does not support the retailer's local advertising when the channel members are balanced in their channel power (see, *e.g.*, [1,22]).

4.2. Retailer Stackelberg game

In the retailer Stackelberg game, the retailer acts as the leader in the channel, and the manufacturer makes his decisions in response to the retailer's decisions. Proposition 4.3 provides the equilibrium decisions that result from the backward induction procedure.

Proposition 4.3. *In the retailer Stackelberg game, the equilibrium decisions of the manufacturer (w^R, p_m^R, t^R) and the retailer (m^R and a^R) are as follows:*

$$
\begin{aligned}
w^R &= \frac{4\alpha H_1(H_2+\theta(1-\lambda))+\theta g k^2(1-y(1+g)(1-\lambda))}{2\beta H_1(8\alpha H_2-k^2)},\\
p_m^R &= \frac{H_2(8\alpha H_1(1-\lambda)+gk^2(1-y(1+g)(1-\lambda)))}{2\beta(1-\lambda)H_1(8\alpha H_2-k^2)},\\
t^R &= 0,\\
m^R &= \frac{4\alpha(1-gy(1-\lambda))}{8\alpha H_2-k^2},\ (p_r^R = w^R+m^R),\\
a^R &= \frac{k\alpha(1-gy(1-\lambda))}{8\alpha H_2-k^2}.
\end{aligned}
$$

The equilibrium profits of the manufacturer and the retailer are given in equations (B.6) and (B.7) in Appendix B. The manufacturer is not willing to participate in the retailer's advertising expenditures when the retailer is the leader. This observation is consistent with the prior literature on dual-channel supply chains (see, *e.g.*, [2,39]) and is generalized to the setting with product web-fit and consumer loyalty.

4.3. Manufacturer Stackelberg game

The manufacturer Stackelberg game considers the case where the interactions are led by the manufacturer who holds channel dominance. The analysis of this scenario reveals the equilibrium results in Proposition 4.4.

Proposition 4.4. *In the manufacturer Stackelberg game, the equilibrium decisions of the manufacturer (w^M, p_m^M, t^M) and the retailer (m^M and a^M) are as follows:*

$$
w^M = \frac{32\alpha(1-\lambda)H_1H_2^2-k^2H_5}{2H_2(32\alpha\beta(1-\lambda)H_1H_2-k^2H_3)},
$$

$$p_m^M = \frac{32\alpha(1-\lambda)H_1H_2 - 3k^2(H_1(1-\lambda) - 2gH_2(1-y(1+g)(1-\lambda)))}{2(32\alpha\beta(1-\lambda)H_1H_2 - k^2H_3)},$$
$$t^M = \frac{4\alpha(1-\lambda)H_1H_2(H_2(1+4g) + 3\theta(1-\lambda)) + k^2H_{12}}{4\alpha H_1H_2(1-\lambda)(\theta(1-\lambda) + (3+4g)H_2)},$$
$$m^M = \frac{8\alpha\beta(1-\lambda)(1-gy(1-\lambda))H_1H_2 - k^2H_4}{H_2(32\alpha\beta(1-\lambda)H_1H_2 - k^2H_3)},\ \left(p_r^M = m^M + w^M\right),$$
$$a^M = \frac{2\alpha k(1-\lambda)H_1(\theta(1-\lambda) + (3+4g))}{32\alpha\beta(1-\lambda)H_1H_2 - k^2H_3}.$$

See equations (B.10) and (B.11) in the Appendix B for the equilibrium profits of the manufacturer and the retailer. Manufacturer's channel dominating position results in a positive participation rate to be an equilibrium decision for the manufacturer.

4.4. Centralized dual-channel supply chain

It is well known that decentralized decision-making can lead to inefficiencies in supply chains as each member acts in his/her best interest, making decisions that may not be optimal from the system perspective. The centralized dual-channel scenario, which we study in this section, represents the situation where a single decision-maker considers the supply chain as one entity, *e.g.*, as a vertically integrated firm, and is concerned with maximizing the entire chain's profit. The profit function of the centralized supply chain is given by $\Pi_{sc} = p_rD_r + p_mD_m - a^2$, and the relevant decision variables are the selling price in the retail channel p_r, the online price p_m, and the advertising effort level a. From equations (3.5) and (3.6), we have

$$\begin{aligned}\Pi_{sc} &= p_r(\alpha_r(1-\beta p_r) + \alpha_m((1-gy)(1-\beta p_r) + \theta(p_m - p_r)) + ka)\\ &\quad + p_m(\alpha_m(gy(1-\beta p_m) - \theta(p_m - p_r)) + kga) - a^2.\end{aligned} \tag{4.1}$$

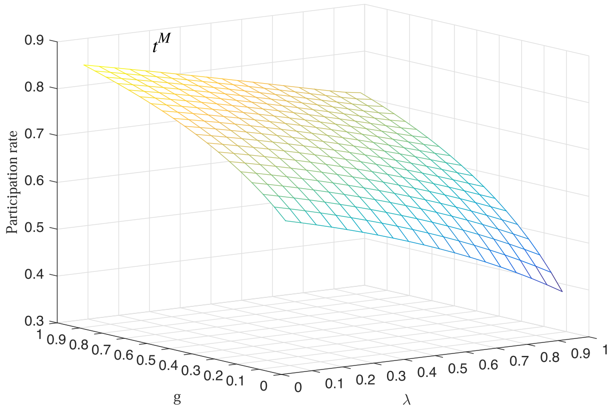
Proposition 4.5 presents the optimal decisions in the centralized supply chain.

Proposition 4.5. *In the centralized dual-channel supply chain, the optimal decisions and the resulting system profit are as follows:*

$$p_r^C = \frac{4\alpha(1-\lambda)H_1 - g^2k^2(1-y(1-\lambda)(1+g))}{8\alpha\beta(1-\lambda)H_1 - 2k^2(\beta + (1+g)(2\theta(1-\lambda) - (1-g)H_2))},$$
$$p_m^C = \frac{4\alpha(1-\lambda)H_1 + gk^2(1-y(1-\lambda)(1+g))}{8\alpha\beta(1-\lambda)H_1 - 2k^2(\beta + (1+g)(2\theta(1-\lambda) - (1-g)H_2))},$$
$$a^C = \frac{2\alpha k(1+g)(1-\lambda)H_1}{8\alpha\beta(1-\lambda)H_1 - 2k^2(\beta + (1+g)(2\theta(1-\lambda) - (1-g)H_2))},$$
$$\Pi_{sc}^C = \frac{\alpha(4\alpha(1-\lambda)H_1 - g^2k^2(1-y(1+g)(1-\lambda))^2)}{2(8\alpha\beta(1-\lambda)H_1 - 2k^2(\beta + (1+g)(2\theta(1-\lambda) - (1-g)H_2)))}.$$

5. Numerical analysis

We conduct a numerical analysis using MATLAB in order to illustrate our analytical results and to provide managerial insights. We focus on the effects of the consumer loyalty and product web-fit parameters on the equilibrium decisions of the manufacturer and the retailer under different channel power structures. In the base case, we select the following values for the model parameters: $\alpha = 1000, \beta = 0.1, \theta = 0.05, k = 0.7$ and $y = 0.5$. The insights are qualitatively unaffected by variations in these values. In all numerical examples, we ensure that the conditions for nonnegative equilibrium outcomes hold.



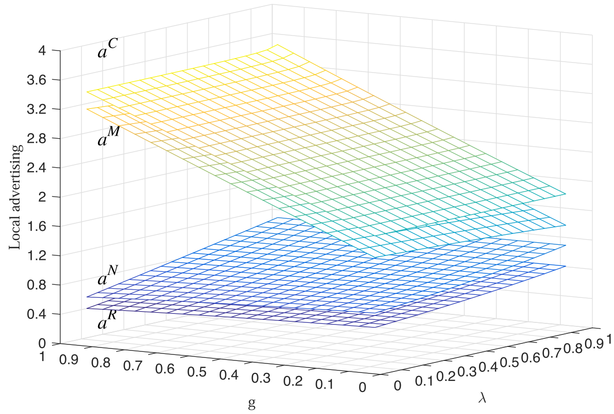


FIGURE 2. Cooperative advertising decisions under different channel power structures. (a) Manufacturer's participation rate. (b) Retailer's local advertising effort.

5.1. Cooperative advertising decisions

The key decisions in the cooperative advertising scheme are the manufacturer's participation rate and the retailer's local advertising level. Figures 2a and 2b present how these decisions are affected by the consumer loyalty to the retail store, λ, and the product web compatibility, g.

Recall that the manufacturer actively participates in the cooperative advertising program (*i.e.*, sets a positive participation rate) only if he has more channel power than the retailer. From Figure 2a, we can see that the manufacturer tends to reimburse a larger proportion of the retailer's local advertising expenditure when the product web-fit increases and consumer loyalty to the retail channel decreases. Under such settings, the retailer faces a stronger competition from the online channel. As the channel leader, the manufacturer finds it in his best interest to provide some compensation to induce the retailer to invest in local advertising, thereby reducing the channel conflict due to competition while also improving his own sales through the retailer's advertising. Figure 2b shows that local advertising level takes its highest value when the supply chain members operate as an integrated firm, where local advertising can be used effectively as a tool to boost system profits without any channel inefficiencies. In the decentralized channel, the retailer exerts the highest level of local advertising effort under the manufacturer's leadership. Furthermore, the amount of advertising becomes more significant as the product web-fit increases or the consumer loyalty to the retail channel decreases, which occurs as a result of the manufacturer's increasing support for the cooperative advertising program. The observation regarding the effect of product web-fit on local advertising is consistent with Pei *et al.* [37] and Yan and Pei [44].

When the retailer is the dominating member or the channel power is symmetrically distributed between the two firms, the retailer lacks the support from the manufacturer, and although she chooses to advertise locally in order to stimulate demand, the retailer is less aggressive in her advertising efforts. This implies that the channel conflict intensifies in Nash and retailer Stackelberg games as the retailer retaliates the manufacturer's non-involvement in the cooperative advertising program by investing less in local advertising. In contrast to the observation in the manufacturer Stackelberg game, the retailer decreases her local advertising when the product web-fit increases and increases her efforts when the consumer loyalty to the retail channel increases. This strategy enables the retailer to take advantage of the consumers' preference for offline shopping while restricting the advertising's impact on the manufacturer's online demand for products of high web-fit. Moreover, this observation underscores how different power structures can lead to different patterns in advertising decisions, thereby contributing to the related literature which assumes the manufacturer to be the dominant player.

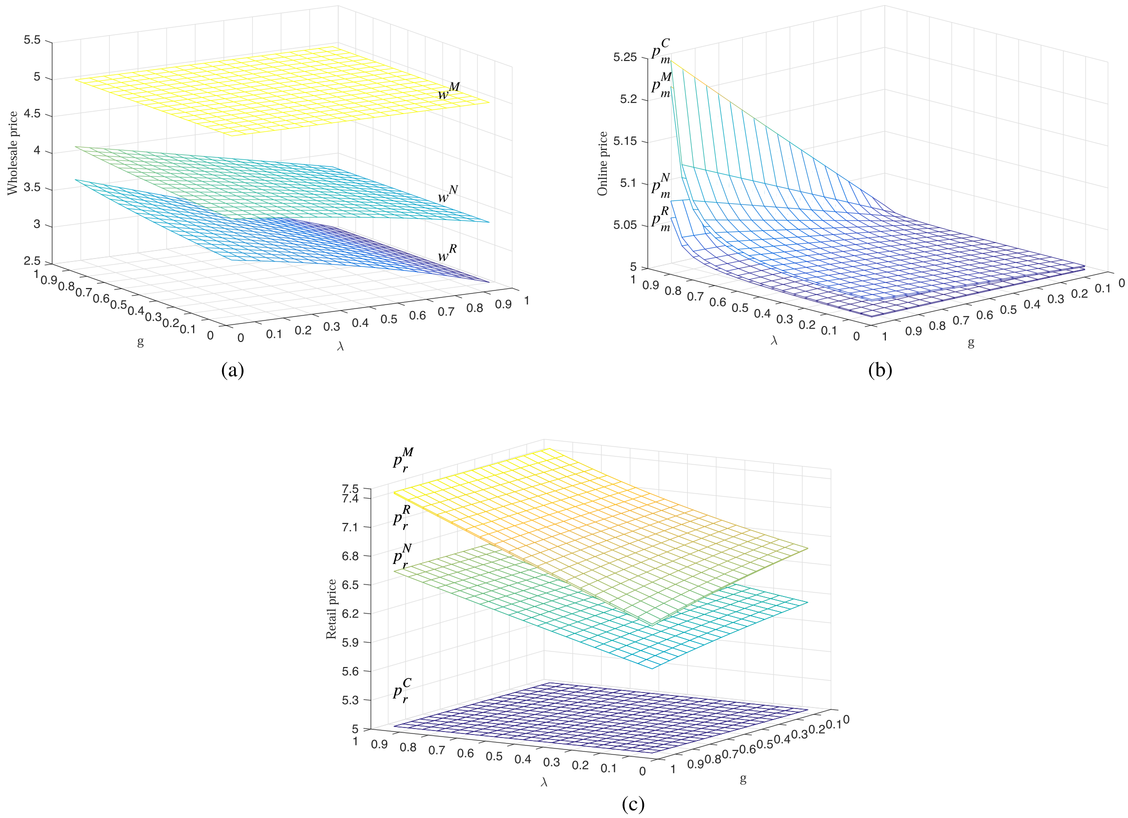


FIGURE 3. Pricing decisions under different channel power structures. (a) Manufacturer's wholesale price. (b) Manufacturer's online price. (c) Retailer's selling price.

5.2. Pricing decisions

This section discusses the effects of consumer loyalty and the product web-fit on the channel members' pricing decisions within three game structures. The results are illustrated in Figures 3a–3c.

We make several observations from these figures. The manufacturer charges the highest value for the wholesale price when he is the dominating member in the chain and the lowest value when the retailer is the more powerful party (Fig. 3a). Although the manufacturer shares the local advertising cost to reduce channel conflict and stimulate demand, he takes advantage of his channel power to charge the retailer a high wholesale price which helps him recover some advertising costs and generate profit gains in his sales to the retailer. Furthermore, as the product web-fit increases or the consumer loyalty to the retail channel decreases, the manufacturer tends to increase his wholesale price in the retailer Stackelberg and Nash games. However, the wholesale price is relatively stable in the manufacturer Stackelberg game as the manufacturer maximizes his equilibrium profit by adjusting the participation rate without significantly altering the wholesale price.

In the decentralized channel, high wholesale prices offered by the manufacturer when holding the channel power are transferred to high selling prices to the end customers. That is, both the retailer's selling price and the manufacturer's online price are the highest under the manufacturer's channel leadership (Figs. 3b and 3c). As the consumer loyalty to the retail channel increases, the retailer can secure higher profit margins, which is also facilitated by the relatively lower wholesale prices; meanwhile the manufacturer increases his online price to maintain the profitability of his direct channel under competition. In situations where the product is

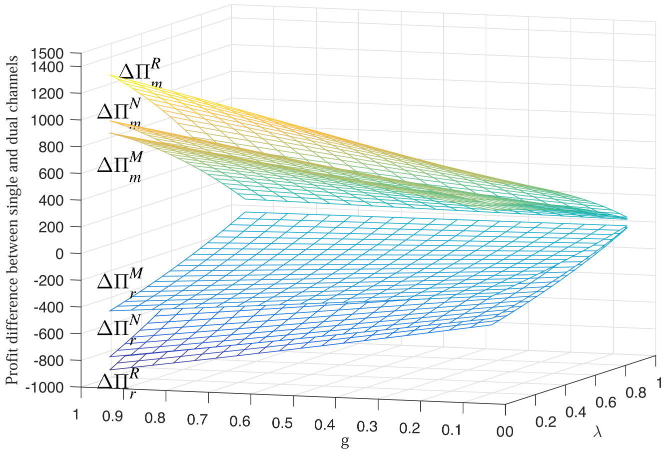


FIGURE 4. Profit differences between single and dual channels under cooperative advertising.

highly compatible with the online shopping, the manufacturer offers high direct prices to take advantage of the strong web-fit, whereas the retailer reduces the selling price to attract consumers to the retail channel. The retail price behavior is opposite to that identified in Pei *et al.* [37], and it arises due to differences in demand modeling. Our model accounts for a portion of brand-loyal consumers switching to the online channel. As the product web-fit gets stronger, the retailer's base demand decreases due to those consumers' switching behavior, while the manufacturer's base demand increases. The retailer resorts to reducing her selling price to prevent customer attrition, and the price reduction becomes more aggressive when the consumer loyalty to the retail channel decreases or the portion of brand-loyal consumers switching to the online channel increases. As the product web-fit gets weaker, the retailer's selling price tends to be higher, especially when the channel power is asymmetrically distributed in the channel; whereas the direct channel prices are relatively similar under all power configurations in the supply chain. Finally, as the supply chain members move to a centralized scenario, the selling prices on the two channels tend to converge, and the retail price decreases due to the elimination of the issue of double marginalization.

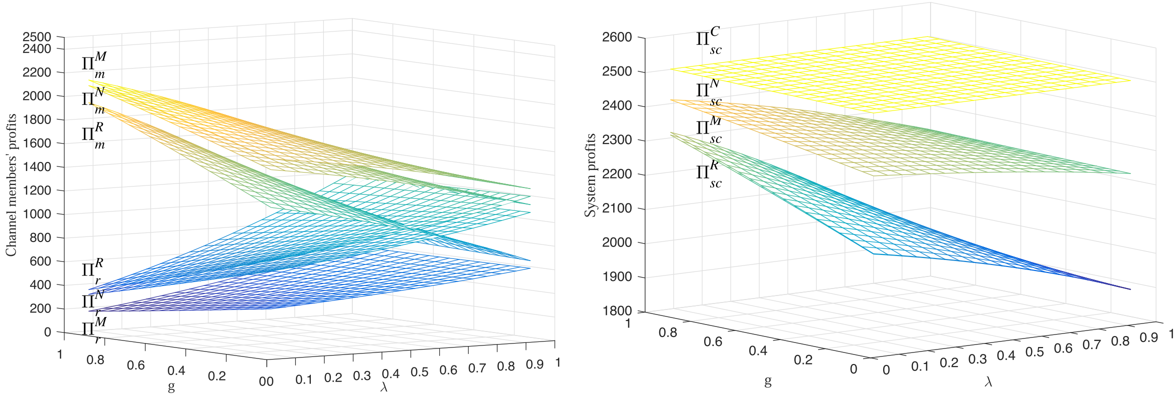
5.3. Channel profits

In this section, we first focus on the profit effects of the consumer loyalty and product web-fit parameters, and we next examine how the profits are affected by the introduction of the manufacturer's direct online channel.

Consider a single-channel setting in which the manufacturer exclusively sells his products through the retailer, and the two firms engage in cooperative advertising. Recall that the equilibrium decisions of the manufacturer and the retailer in this setting have been characterized for the different channel leadership scenarios (see Prop. 4.1). When the manufacturer introduces the online channel, the equilibrium decisions and profits change as a result of the competitive dynamics (see Props. 4.2–4.4). In Figure 4, we compare the profits of the supply chain members in the single-channel and dual-channel settings, where $\Delta\Pi_i^j$ represents the change in profit for the supply chain member $i = m, r$ in scenario $j = M, R, N$ after the introduction of the online channel.

The results indicate that the dual-channel structure benefits the manufacturer but negatively affects the retailer, and this observation holds under all three power configurations in the supply chain. The manufacturer increases his profits by introducing an additional sales channel and inducing competition in the end market, whereas the retailer loses profits as a result of this strategy.

The manufacturer obtains the highest profit gain when the retailer is the channel leader because he can better exploit the introduction of the online channel after shifting from a point where neither the channel power nor the online channel could be leveraged. Similarly, the retailer enjoys a more advantageous outcome (*i.e.*, lower



(a) (b)

Figure 5. Supply chain profits under different channel power structures. (a) Channel members' profits. (b) System profits.

profit loss) when the manufacturer is the channel leader because the manufacturer aims to mitigate the channel conflict by running a more aggressive cooperative advertising campaign in light of a higher level of advertising effort and participation rate. Furthermore, the manufacturer's profit improvement becomes more pronounced as the product web-fit increases and the consumer loyalty towards the retail chain decreases. By contrast, the retailer's profit loss becomes less severe as the product web-fit decreases and the consumer loyalty towards the retail chain increases, which is facilitated by increasing participation rate and local advertising effort. The preceding discussion implies that the manufacturer's introduction of the online channel is more harmful for a retailer who holds the channel leadership in the single-channel setting in comparison to a retailer who is the follower position in that scenario.

Figures 5a and 5b show the supply chain members' individual profits and the total channel profits under different power distributions in the dual-channel setting. Each member is better off when holding the more powerful status in the channel as opposed to holding the less powerful status, which can be attributed to the first-mover advantage in the decision sequence of the games. Furthermore, the profits increase when the product web-fit and the consumer loyalty parameter values change in a favorable manner for a particular channel member; that is, the manufacturer reaps higher profits when the product has higher compatibility with online shopping and the consumers show more loyalty towards the manufacturer's brand, whereas the retailer's profit increases when the product is less compatible with online shopping and consumers are more loyal to the conventional retail channel. Although having the channel leadership position guarantees a larger profit for the manufacturer in the channel, the same conclusion does not hold for the retailer. Particularly, when the consumer loyalty to the retail outlet is sufficiently large and the product web-fit is sufficiently low, the retailer's profit is higher than the manufacturer's profit when she is the dominating channel member. Otherwise, the manufacturer can secure a higher profit in the supply chain despite being in a less powerful position than the retailer.

Overall, the profit in the decentralized system is the highest when the channel power is balanced between the manufacturer and the retailer rather than held by one of the members. To a certain extent, symmetric power distribution helps the system move away from decisions that unilaterally benefit a particular member and thus improves profits. We also see that the decentralized system profits improve as the product web compatibility and the consumer brand loyalty increase. The situation of high web-fit and strong brand loyalty is favorable for online sales and hence enables the dual-channel structure to make a significant profit-improving effect on the system.

TABLE 4. Sensitivity of the equilibrium decisions and profits to model parameters.

	Π_m^j	Π_r^j	Π_{sc}^j	w^j	p_m^j	p_r^j	t^M	a^j
α	$\nearrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$
β	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$
θ	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\searrow$
k	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$	$\nearrow$
y	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$	$\searrow$	$\searrow$	$\nearrow$	$\nearrow$
y $(\lambda \to 1)$	$\nearrow$	$\searrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$	$\searrow$

Notes. $\nearrow$ ($\searrow$) increasing (decreasing) trend. $j = N, M, R$.

5.4. Sensitivity analysis

So far, we have focused on the parameters related with the store or brand loyalty of consumers and the product web compatibility. In this section, we investigate how the equilibrium outcomes are influenced by the changes in other model parameters. In particular, we investigate the sensitivity of the equilibrium decisions and profits to α, β, θ, k, and y. When we examine the impact of one parameter, we vary its values while keeping the remaining ones fixed. Table 4 summarizes the observations from our numerical experiments.

Increasing values of α point to expanding demand potential in the market and improve the profits of all supply chain members while reducing the wholesale price, selling prices to the end customers, and the intensity of the cooperative advertising program. As β increases, consumers become more sensitive to the price changes by the manufacturer and retailer in their respective channels, decreasing the demand and thus profit for both firms consistently. By contrast, the effect of θ follows a mixed pattern. Increasing values of θ indicate a higher sensitivity of consumers to the price differential between the two competing channels and enhance the manufacturer's profit but decrease the retailer's profit. As competition intensifies with growing θ, the manufacturer obtains profit gains by charging a higher wholesale price and by setting a lower direct price than the retail price, thereby shifting more consumers from the retail channel to the online channel. Furthermore, the manufacturer increases his participation in the retailer's local advertising efforts to mitigate the channel conflict due to intense competition.

The profit effects of the parameter k are intuitive as its increasing values imply higher effectiveness of the retailer's advertising in stimulating demand for both the manufacturer and the retailer, and therefore, the profits increase. The parameter y (*i.e.*, percentage of brand-loyal customers buying from the direct channel) makes a similar effect on the manufacturer's profit. As brand-loyal consumers increasingly prefer the online channel, this improves the manufacturer's sales and profits at the expense of the retailer. Interestingly, the impact of y on the equilibrium decisions is influenced by λ, *i.e.*, the fraction of store-loyal customers. When λ approaches its limiting value of 1, the potential size of the brand-loyal market is much smaller than that of the store-loyal market. Then, even though increasing values of y enable the manufacturer to capture some online sales, his main focus remains on the retail sales channel and he lowers the wholesale price to improve profits. In this setting, cooperative advertising is pursued less aggressively by the channel members. When λ is not too large, the brand-loyal and store-loyal markets are relatively closer in size. Increasing values of y further intensify the competition between the two members but strengthen the manufacturer's competitive position. Then, the manufacturer increases his wholesale price, and the channel members utilize the cooperative advertising program more aggressively.

6. Concluding remarks

With the growing popularity of the Internet and related technologies, consumers in many markets are increasingly demanding the option to shop online. This trend has been further strengthened by the COVID-19 pandemic and has motivated many firms to design their supply chains in a way to include an online sales channel in addition to the conventional brick-and-mortar retail channel. When the online and offline channels coexist, the sales volume in each channel is affected by various factors such as the difference in selling prices, product features relating to the compatibility with web-based sales, and consumer brand loyalty and preferences for online or offline shopping. In this study, we consider a dual-channel supply chain where a brand manufacturer sells through a direct online channel and an independently-owned brick-and-mortar retailer. We investigate the effects of consumer loyalty and product web compatibility on the pricing and cooperative advertising policies of the manufacturer and the retailer under different channel power structures.

Three scenarios arise depending on whether the manufacturer, retailer, or neither member is the dominating party in the channel. For each scenario, we characterize the equilibrium outcomes of the games in which the manufacturer determines the wholesale price, direct online price, and cooperative advertising participation rate, and the retailer determines the selling price and the local advertising effort. We also characterize the optimal decisions in a single-channel supply chain and an integrated dual-channel supply chain which serve as benchmark results.

Our analysis provides several managerial insights. Manufacturers with a more powerful status in the channel provide a strong advertising support to their retailer when the product has a high web-fit and the consumer loyalty towards the retail channel is weak. By following this strategy while holding the channel power, the manufacturer can obtain a high profit margin from the sales through the retailer as a result of high wholesale prices; furthermore, he can generate profit gains from increased sales in the online channel which occur as a result of a lower direct price than the retail price and the positive effect of the retailer's local advertising.

The retailer's advertising campaign behaves differently with respect to the consumer loyalty and product web-fit parameters across various game scenarios. Specifically, the retailer's advertising level increases with the product web-fit and decreases with the consumer store loyalty under the manufacturer's active advertising participation in the manufacturer Stackelberg game. However, this behavior is reversed when the manufacturer stops his advertising support in the Nash and the retailer Stackelberg games. The retailer advertises less aggressively as the product web-fit increases and more aggressively as the consumer store-loyalty increases, thereby promoting brick-and-mortar sales while impeding online sales.

A manufacturer in a single-channel supply chain can improve his profit if he introduces a direct online channel and continues to engage in the cooperative program. The profit gains are significant at high values of product web-fit and consumer brand loyalty. When the competition between the two channels is intense due to high cross-price sensitivity of consumers, both the online and retail prices tend to decrease, while the manufacturer's wholesale price increases, improving the manufacturer's profit but reducing the retailer's. As the proportion of brand-loyal consumers who switch to the online channel increases, the manufacturer tends to charge high wholesale prices. Interestingly, an opposite behavior is observed when the base demand from brand-loyal consumers is too low, as the manufacturer attempts to improve sales in the retail channel by offering low wholesale prices. We observe that both the manufacturer and the retailer prefer to be the channel leader in either single or dual-channel supply chains. However, when they switch from a single channel to a dual channel, both gain a more advantageous outcome in the follower position. Meanwhile, the highest system profit is obtained when the channel power is symmetrically distributed, and it increases as the product web-fit and consumer store loyalty parameter values increase. Finally, on the basis of the selling prices in the market, consumers are worse off in a supply chain under the manufacturer's leadership in comparison to the other power configurations.

Our study can be extended in several directions. In our model, we have assumed that demand is deterministic and the interactions between the channel partners occur in a single period under complete information. Model settings that consider demand uncertainty, multi-stage games with repeated bargaining/negotiating, and asymmetric information present opportunities for future research. Even though the demand model we have used can capture practical scenarios of multiple consumer segments as well as consumer loyalty and channel preference, it does not address settings in which the proportion of brand-loyal consumers switching to the online channel may be correlated with the product web compatibility. Consideration of such issues require developing more advanced demand models which can also be useful to explore scenarios where the brand-loyal consumers' channel preferences and purchasing choice can be endogenous.

Appendix A.

For the convenience of presenting the results and proofs, we define the following parameters:

$$\begin{aligned}
H_1 &= \beta gy(1-gy(1-\lambda))+\theta,\\
H_2 &= \beta(1-gy(1-\lambda))+\theta(1-\lambda),\\
H_3 &= 8(1+g)H_2(2\theta(1-\lambda)-(1-g)H_2)+\beta(H_1(1-\lambda)+8H_2),\\
H_4 &= H_2((2+g)H_1(1-\lambda)+2gH_2(g-y(1-\lambda)(1+g)^2))-\theta(1-\lambda)(2H_2+H_1(1-\lambda)),\\
H_5 &= \theta(1-\lambda)(H_1(1-\lambda)-2H_2)+2H_2((4+3g)H_1(1-\lambda)+gH_2(4g-y(1-\lambda)(1+g)(1+4g))),\\
H_6 &= 2\theta(1-\lambda)+2gH_2(-g+y(1-\lambda)(1+g)^2)-5H_1(1+g)(1-\lambda),\\
H_7 &= (\theta(1-\lambda)(2H_2+H_1(1-\lambda))+H_2(2gH_2((1+g)^2(1-\lambda)y-g)-(2+g)H_1(1-\lambda))),\\
H_8 &= 6(1+g)^2H_2^2(H_2-\theta(1-\lambda))+((3+4g)H_2+\theta(1-\lambda))(\theta(1-\lambda)+gH_2)^2)\\
&\quad -3\beta(H_2(H_2(2+4g)-(2+g)H_1(1-\lambda))+\theta(1-\lambda)(2H_2+H_1(1-\lambda))),\\
H_9 &= (1-y(1-\lambda)(1+g^3y(1-\lambda)-2g^2(1-y(1-\lambda)))-g(1-y(1-\lambda)(1-y(1-\lambda)))),\\
H_{10} &= (\theta(1-\lambda)+gH_2)(1-y(1+g)(1-\lambda)),\\
H_{11} &= ((1+g)H_2(2\theta(1-\lambda)-(1-g)H_2)+\beta(H_2-H_1(1-\lambda))),\\
H_{12} &= ((\theta(1-\lambda)+gH_2)(2gH_2(1-y(1+g)(1-\lambda))-H_1(1-\lambda))).
\end{aligned}$$

To ensure the concavity of the objective functions and nonnegativity of the equilibrium decisions, demands, and profits in all games, we assume that all the following conditions are satisfied by the model parameters.

$$\frac{4}{3}\alpha\beta-k^2>0 \tag{A.1}$$

$$2\alpha H_2-k^2>0 \tag{A.2}$$

$$2\alpha H_1(2H_2+\theta(1-\lambda))-k^2(\theta-(3+g)H_1+gy(1+g)H_2)>0 \tag{A.3}$$

$$8\alpha H_1H_2(1-\lambda)-k^2(2H_1(1-\lambda)+gH_2(1-y(1-\lambda)(1+g)))>0 \tag{A.4}$$

$$\begin{aligned}4\alpha\beta H_1(1-\lambda)(H_2-H_1(1-\lambda))-k^2(\beta(H_2-H_1(1-\lambda))+(1+g)(\theta(1-\lambda)(2H_2-3H_1(1-\lambda))\\ -H_2(H_2(1-g)+3H_1(1-\lambda)))))>0\end{aligned} \tag{A.5}$$

$$\frac{2\beta(8\alpha H_1H_2(1-\lambda)(H_2-\theta(1-\lambda))+k^2H_7)^2}{H_1(1-\lambda)((3+4g)H_2+\theta(1-\lambda))^2}-\frac{4\alpha\beta H_1H_2(1-\lambda)k^2-k^4H_8}{(3+4g)H_2+\theta(1-\lambda)}>0 \tag{A.6}$$

$$32\alpha\beta H_1H_2(1-\lambda)-k^2H_3>0 \tag{A.7}$$

$$\begin{aligned}2\alpha H_1H_2^2(1-\lambda)-k^2((\theta(1-\lambda)(H_1(1-\lambda)-2H_2)+2H_2(gH_2(4g-y(1+g)(1+4g)(1-\lambda))\\ -(4+3g)(1-\lambda)H_1)))>0\end{aligned} \tag{A.8}$$

$$16\alpha H_1H_2(1-\lambda)(H_2+H_1(1-\lambda))-k^2(H_1(1-\lambda)-2gH_2(1-gy(1-\lambda)(1+g)))^2>0 \tag{A.9}$$

$$\begin{aligned}16\alpha\beta H_1H_2(1-\lambda)(H_2+\theta(1-\lambda))-k^2(2(1+g)H_2^2((19+6g)\theta(1-\lambda)-(11-2g)H_2)\\ +\beta(2H_2((11+9g)-(5+4g)H_1(1-\lambda))(6H_2+H_1(1-\lambda))\theta(1-\lambda)))>0\end{aligned} \tag{A.10}$$

$$32\alpha\beta H_1H_2(1-\lambda)-k^2(\beta(10H_2-H_1(1-\lambda))-2(1+g)H_2((5-8g)H_2-13\theta(1-\lambda)))>0 \tag{A.11}$$

$$\begin{aligned}4\alpha H_1H_2(1-\lambda)(H_2(1+4g)+3\theta(1-\lambda))-k^2(\theta(1-\lambda)(2H_2+H_1(1-\lambda))\\ -H_2((2+g)(1-\lambda)H_1+2gH_2(g-y(1+g)^2(1-\lambda)))))>0\end{aligned} \tag{A.12}$$

$$8\alpha(1-\lambda)H_1(H_2-\theta(1-\lambda))-k^2H_6>0. \tag{A.13}$$

Appendix B.

Proof of Proposition 4.1. We first analyze the Nash game in the single-channel case.

Equilibrium in Nash game: Π_m is decreasing in t, so the manufacturer selects $t = 0$. Then, given $p_r = w+m$, the optimization problems of the manufacturer and the retailer are reduced to: $\max_{w} \Pi_m(w) = w(\alpha(1-\beta(w+m))+ka)$ and $\max_{m,a} \Pi_r(m,a) = m(\alpha(1-\beta(w+m))+ka)-a^2$. Note that $\partial^2\Pi_m/\partial w^2 = -2\alpha\beta < 0$; furthermore, $\partial^2\Pi_r/\partial m^2 = -2\alpha\beta < 0$ and $(\partial^2\Pi_r/\partial m^2)(\partial^2\Pi_r/\partial a^2) - (\partial^2\Pi_r/\partial m\partial a)^2 = 4\alpha\beta - k^2$. Due to assumption (A.1), the Nash equilibrium decisions can be obtained by solving the system of equations derived from the First-Order Conditions (FOCs): $\frac{\partial \Pi_m}{\partial w} = 0$, $\frac{\partial \Pi_r}{\partial m} = 0$ and $\frac{\partial \Pi_r}{\partial a} = 0$.
Equilibrium in retailer Stackelberg game: from the discussion in Nash game, the manufacturer's best response decisions are $w = \frac{\alpha(1-\beta m)+ak}{2\alpha\beta}$ and $t = 0$. By inserting these decisions into the retailer's objective function, we obtain $\frac{1}{2}m(\alpha - m\alpha\beta + ka) - a^2$. We can verify concavity under assumption (A.1) and obtain the retailer's equilibrium decisions from the FOCs. The manufacturer's equilibrium decisions are next found by substitution.
Equilibrium in manufacturer Stackelberg game: given w and t, we have $\partial^2\Pi_r/\partial m^2 = -2\alpha\beta < 0$ and $(\partial^2\Pi_r/\partial m^2)(\partial^2\Pi_r/\partial a^2) - (\partial^2\Pi_r/\partial m\partial a)^2 = 4\alpha\beta(1-t) - k^2$. Assuming $t < 1 - \frac{k^2}{4\alpha\beta}$, the retailer's best response is: $m = \frac{2\alpha(1-\beta w)(1-t)}{4\alpha\beta(1-t)-k^2}$ and $a = \frac{\alpha k(1-\beta w)}{4\alpha\beta(1-t)-k^2}$. By inserting these decisions into the manufacturer's objective function, we obtain the equilibrium w and t from the FOCs and the retailer's equilibrium m and a by substitution. Assumption (A.1) ensures that equilibrium decisions satisfy the Second-Order Conditions (SOCs) for the manufacturer's and the retailer's objective functions. Finally, we verify that the equilibrium demand is positive (*i.e.*, $1 - \beta p_r > 0$), under assumption (A.1).
Centralized system: profit function is $p(\alpha(1-\beta p) + ka) - a^2$. The proof is straightforward.

The comparison results among the equilibrium decisions and profits in the four scenarios can be shown under the assumption in (A.1). □

Proof of Proposition 4.2. Π_m is decreasing in t, so the manufacturer selects $t = 0$. Then, the objective functions of the manufacturer and the retailer are given in equations (3.5) and (3.6), respectively, by setting $t = 0$ and $p_r = w + m$. In the Hessian matrix of the manufacturer's objective function, the first principal minor is $-2\alpha H_2 < 0$, and second principal minor is $4\alpha^2\beta(1-\lambda)H_1 > 0$, where we define $H_1 = \beta gy(1-gy(1-\lambda))+\theta > 0$. The Hessian matrix is negative definite, and the SOCs are satisfied for the manufacturer's objective function. In the Hessian matrix of the retailer's objective function, the first principal minor is $-2\alpha H_2 < 0$, where we define $H_2 = \beta(1 - gy(1-\lambda)) + \theta(1-\lambda) > 0$, and the second principal minor is $4\alpha H_2 - k^2$. By assumption (A.2), the Hessian matrix is negative definite. We solve the system of equations derived from FOCs and obtain the equilibrium decisions expressed in the proposition. Then, we verify that the equilibrium demand $1 - \beta p_r^N$ is positive under the assumptions (A.2) and (A.3). Assumption (A.2) also ensures that the equilibrium demand $1 - \beta p_m^N$ is positive and the SOCs are satisfied for the retailer's objective function.

The equilibrium profits of the manufacturer and the retailer are as follows:

$$\Pi_m^N = \frac{\alpha H_2(4\alpha^2(1-\lambda)H_1(4H_2 + 5(1-\lambda)H_1) + k^2 g(1-y(1+g)(1-\lambda))(12\alpha(1-\lambda)H_1 + k^2 g(1-y(1+g)(1-\lambda)))}{4\beta(1-\lambda)H_1(6\alpha H_2 - k^2)^2}, \tag{B.1}$$

$$\Pi_r^N = \frac{\alpha^2(1-gy(1-\lambda))^2(4\alpha H_2 - k^2)}{(6\alpha H_2 - k^2)^2}. \tag{B.2}$$

□

Proof of Proposition 4.3. From the proof of Proposition 4.2, we can obtain the manufacturer's best response decisions as follows:

$$w = \frac{\alpha H_1(1-\beta m) + ak(\beta gy + \theta g + \theta)}{2\alpha\beta H_1} \tag{B.3}$$

$$p_m = \frac{\alpha(1-\lambda)H_1 + ak(\theta(1+g)(1-\lambda) + \beta g(1-gy(1-\lambda))}{2\alpha\beta(1-\lambda)H_1} \tag{B.4}$$

$$t = 0. \tag{B.5}$$

Then, the retailer's decision problem is expressed as:

$$\begin{aligned} \underset{m,\,a}{\text{maximize}} \quad & \Pi_r = m(\lambda\alpha(1-\beta(w+m)) + (1-\lambda)\alpha((1-gy)(1-\beta(w+m)) \\ & \qquad + \theta(p_m-(w+m))) + ka) - (1-t)a^2 \\ \text{subject to} \quad & \text{(B.3)}-\text{(B.5)}. \end{aligned}$$

We can verify that the Hessian matrix is negative definite under assumption (A.2). From the FOCs, we obtain the equilibrium decisions stated in the proposition. Under the assumptions (A.2) and (A.3), we can verify that the equilibrium demand $1-\beta p_r^R > 0$. Assumption (A.4) ensures that the equilibrium demand $1-\beta p_m^R > 0$.

The equilibrium profits of the manufacturer and the retailer are as follows:

$$\Pi_m^R = \frac{\alpha H_2(16\alpha^2(1-\lambda)H_1(H_2+3(1-\lambda)H_1) + k^2 g(1-y(1+g)(1-\lambda))(16\alpha(1-\lambda)H_1 + k^2 g(1-y(1+g)(1-\lambda)))}{4\beta(1-\lambda)H_1(8\alpha H_2-k^2)^2}, \tag{B.6}$$

$$\Pi_r^R = \frac{\alpha^2(1-gy(1-\lambda))^2}{(8\alpha H_2-k^2)^2}. \tag{B.7}$$

□

Proof of Proposition 4.4. The objective function of the retailer is given in equation (3.6), where $p_r = w+m$. The Hessian matrix is negative definite if $t < 1-\frac{k^2}{4\alpha H_2}$. Assuming this holds, the retailer's best response decisions from the FOCs are as follows:

$$m = \frac{2\alpha(1-t)(\theta(1-\lambda)(p_m-w) + (1-\beta w)(1-gy(1-\lambda)))}{(4\alpha(1-t)H_2-k^2)} \tag{B.8}$$

$$a = \frac{\alpha k(\theta(1-\lambda)(p_m-w) + (1-\beta w)(1-gy(1-\lambda)))}{(4\alpha(1-t)H_2-k^2)}. \tag{B.9}$$

Then, the manufacturer's decision problem is expressed as:

$$\begin{aligned} \underset{w,\,p_m,\,t}{\text{maximize}} \quad & \Pi_m = w(\lambda\alpha(1-\beta(w+m)) + (1-\lambda)\alpha((1-gy)(1-\beta(w+m)) + \theta(p_m-(w+m))) + ka) \\ & \qquad + p_m((1-\lambda)\alpha(gy(1-\beta p_m) - \theta(p_m-(w+m))) + kga) - ta^2 \\ \text{subject to} \quad & \text{(B.8) and (B.9)}. \end{aligned}$$

We solve the manufacturer's problem in two stages. In the first stage, we derive the optimal prices (*i.e.*, w and p_m) for any given participation rate t. In the second stage, we incorporate the first-stage results into the optimization problem and determine the optimal t. In the first stage, the FOCs yield:

$$w = \frac{16\alpha^2 H_1 H_2^2(1-t)^2 - gk^4 H_{10} - 2\alpha k^2(1-\lambda)\big(\theta(1-\lambda)H_1 + \theta(1-t)H_2 + (1+g(1-t)-3t)H_1H_2 + gy(1+g)(1-t)H_2^2\big)}{32\alpha^2\beta H_1 H_2^2(1-\lambda)(1-t)^2 - 4\alpha\beta H_1 H_2 k^2(1-\lambda)(2-3t) - k^4 H_{11}}$$

$$p_m = \frac{2\alpha H_2(8\alpha H_1 H_2(1-\lambda)(1-t)^2 - k^2((1-\lambda)(1-2t)H_1 + g(1-t)H_2(y(1+g)(1-\lambda)-1)))}{32\alpha^2\beta H_1 H_2^2(1-\lambda)(1-t)^2 - 4\alpha\beta H_1 H_2 k^2(1-\lambda)(2-3t) - k^4 H_{11}}.$$

Note that the parameters H_i, $i = 1,\ldots,12$ are given in Appendix A. The first principal minor of the Hessian matrix of the manufacturer's profit function at w and p_m is $\frac{-2\alpha H_2^2(8\alpha(1-t)H_2-k^2(2-3t))}{(4\alpha(1-t)H_2-k^2)^2}$, which is negative under the assumed condition $t < 1-\frac{k^2}{4\alpha H_2}$. The second principal minor is positive if the following condition holds: $\frac{\alpha^2(32\alpha^2\beta(1-\lambda)(1-t)^2 H_2^2 H_1 - 4\alpha\beta k^2 H_1 H_2(1-\lambda)(2-3t) - k^4(\theta(1-\lambda)+gH_2)^2)}{(4\alpha(1-t)H_2-k^2)^2} > 0$. We assume this condition holds and later verify it at the optimal t value. There is no other positive candidate solution, and the corner points result in nonpositive profits for the manufacturer. Therefore, the solution derived from the FOCs is a global maximum in this stage.

In the second stage, we embed the w and p_m values into the manufacturer's objective function and obtain a function of one variable (*i.e.*, a function of the variable t):

$$\max_t \Pi_m = \frac{\alpha^2 H_1(1-\lambda)(4\alpha H_2(H_2+H_1(1-\lambda))(1-t)^2 + k^2(2gH_2(1-t)(1-y(1+g)(1-\lambda)) + tH_1(1-\lambda)))}{32\alpha^2\beta H_1 H_2^2(1-\lambda)(1-t)^2 - 4\alpha\beta H_1 H_2 k^2(1-\lambda)(2-3t) - k^4 H_{11}}.$$

From the FOCs, we obtain the expression of t^M stated in the proposition. Then, the rest of the equilibrium decisions (*i.e.*, w^M, p_m^M, m^M, a^M) can be found by substitution. We impose assumption (A.5) to ensure that the second derivative of the manufacturer's profit function is negative at t^M. There is no other positive candidate solution, thus t^M is the unique global optimizer in the second stage. The second principal minor in the first-stage problem is positive at $t = t^M$ under assumption (A.6). Additionally, assumptions (A.7)–(A.12) guarantee that equilibrium demands, profits, and decisions are positive.

Finally, assumption (A.13) ensures that the SOCs are satisfied for Π_r at equilibrium.

The equilibrium profits of the manufacturer and retailer are as follows:

$$\Pi_m^M = \frac{\alpha\big(16\alpha(1-\lambda)H_1H_2((1-\lambda)H_1+H_2) - k^2(2gH_2(1-y(1+g)(1-\lambda)) - (1-\lambda)H_1)^2\big)}{4H_2(32\alpha\beta(1-\lambda)H_1H_2 - k^2H_3)} \tag{B.10}$$

$$\Pi_r^M = \frac{\alpha\big(8\alpha\beta(1-\lambda)(1-gy(1-\lambda))H_1 + k^2H_6\big)(8\alpha\beta(1-\lambda)(1-gy(1-\lambda))H_1H_2 - k^2H_4)}{(32\alpha\beta(1-\lambda)H_1H_2 - k^2H_3)^2}. \tag{B.11}$$

□

Proof of Proposition 4.5. The Hessian matrix of the centralized system's profit function, which is given in equation (4.1), is as follows:

$$\begin{bmatrix} -2\alpha H_2 & 2\alpha\theta(1-\lambda) & k \\ 2\alpha\theta(1-\lambda) & -2\alpha(1-\lambda)(\beta gy+\theta) & gk \\ k & gk & -2 \end{bmatrix}.$$

The first principal minor is $-2\alpha H_2 < 0$, and second principal minor is $4\alpha^2\beta(1-\lambda)H_1 > 0$. The following condition should be verified for the third minor: $8\alpha\beta(1-\lambda)H_1 - k^2(2(\theta(1+g)^2(1-\lambda) + \beta g(g(1-gy(1-\lambda)) + y(1-\lambda)))) > 0$, which we assume holds. Then, applying the FOCs, we can get the optimal solutions for the centralized case which are stated in the proposition. Furthermore, under the assumption (A.3) and the following assumptions, we can verify that the equilibrium prices and demands (*i.e.*, $1-\beta p_r^C$ and $1-\beta p_m^C$) are positive: $4\alpha(1-\lambda)H_1 - k^2g(y(1-\lambda)(1+g)-1) > 0$; $4\alpha\beta(1-\lambda)H_1 - k^2(\beta + (1+g)(3\theta(1-\lambda) - (1-2g)H_2)) > 0$; and $4\alpha\beta(1-\lambda)H_1 - k^2(\beta(2+g) + (1+g)((4+g)\theta(1-\lambda) - (2-g)H_2)) > 0$. □

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