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# A META HEURISTIC APPROACH FOR RELIABLE CAPACITATED FACILITY JOINT INVENTORY-LOCATION PROBLEM WITH ROUND-TRIP TRANSPORTATION UNDER IMPERFECT INFORMATION OF DISRUPTION IN A FUZZY ENVIRONMENT

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**Abstract.** In today's systems and networks, disruption is inevitable. Designing a reliable system to overcome probable facility disruptions plays a crucial role in planning and management. This article proposes a reliable capacitated facility joint inventory-location problem where location-independent disruption may occur in facilities. The system tries to satisfy customer's demands and considers penalty costs for unmet customer demand. The article aims to minimize total costs such as establishing inventory, uncovered demand's penalty, and transportation costs. While many articles in this area only use exact methods to solve the problem, this article uses a metaheuristic algorithm, the red deer algorithm, and the exact methods. Various numerical examples have shown the outstanding performance of the red deer algorithm compared to exact methods. Sensitivity analyses show the impacts of various parameters on the objective function and the optimal facility layouts. Lastly, managerial insights will be proposed based on sensitivity analysis.

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## 1. Introduction

Facility location problems are one of the most widely used issues in scientific topics, and so far, many studies and research have been done in this field, and many models have been developed. Today, facility location problems are considered alongside inventory problems. Since inventory control is essential in providing better customer service, paying attention to such issues is important [7]. A typical location-inventory problem aims to determine the optimum number and facilities' location, allocate customers to facilities and optimize inventory level at facilities with considered different costs such as fixed (opening) cost, operation cost, inventory cost, and transportation cost. This problem combines both long-term and short-term decision-making together [16].

*Keywords.* Joint inventory-location problem, customers satisfaction, independent disruption, red deer algorithm, reliable capacitated facility location problem.

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Despite the significance of the standard location-inventory problem, the importance of planning to enhance reliability and resilience against disruptions hasn't been considered in the standard form. The occurrence of various disruptions caused by natural and unnatural (human) factors is inevitable. The event of disruptions leads to an increase in system's costs. Therefore, by predicting and managing system's disruptions, the effects of unpredictable disruptions can be reduced. As the probability of a facility failure increases, so does the cost of the total expected system and the number of facilities built [14]. Therefore, studying the joint inventory-location problem with disruptions occurrence in facilities is required [29]. In such problems, which are a combination of facility location and disruption problems, we seek to find suitable locations for the facilities to reduce system costs. The possibility of disruption in the facility is usually independent of each other. Customers go to different facilities to receive services. In the event of a facility disruption and failure to receive services, the customer will go to the next facility to receive the services they need.

In today's world, considering uncertainty is an essential part of planning. Over the past years, many studies and research have been conducted to apply uncertainty modeling techniques in planning problem (*e.g.*, [9,17]). One of the critical uncertainties in any supply chain is the amount of customer demand, in which various fluctuations can lead to extensive changes in the supply chain's performance and impose a cost on to supply chain [20]. For instance, in recent years, the coronavirus (COVID-19) outbreak has seriously affected the whole world and led to many changes in the demand for various products [30]. However, the probability distribution obtained from historical data recorded in the past cannot be easily accessible and reliable for use in various issues. Hence, fuzzy set theory [10] is an excellent tool to show uncertainty in different problems. For this reason, fuzzy numbers are used to express the uncertainty of customer demands.

Two types of assumptions have been used in the literature: perfect information problems and imperfect information problems. Most research has concentrated on perfect information problems in which customers are aware of the facility states and always choose the proper facility to visit. On the other hand, under imperfect information, customers do not know facilities' states. They have to visit a series of pre-assigned facilities until they find a functioning facility to get the service or return with no service [56]. It is usual for customers not to have enough information about the disruptions in facilities. Therefore, an imperfect information problem is considered in this study.

The goal of the reliable uncapacitated facility location problem (RUFLP) is to meet customers' demands by determining the facility's location while minimizing total system costs, including fixed costs and variable network costs, so that the number of facilities is uncertain. RUFLPs are also referred to as simple facility location problems. In these problems, it is assumed that the candidate locations for establishing the facilities are predetermined, and the customer demands are determined. An essential feature of this type of location problem is the possibility of deciding to determine the size of the facility without considering any constraints such as physical and budgetary constraints. While applying such restrictions can lead to the proximity of the problem to the real world. The UFLP that involves these constraints is called a reliable capacitated facility location problem (RCFLP) [49]. RUFLP and RCFLP are the main problems related to the location of facilities in the discrete state and the variety of articles and their variety in facility location problems indicates the importance of this area. This article focuses on the former problems, the capacitated facility location problems, unlike Asl-Najafi *et al.* [7], Chen *et al.* [14] and Liu *et al.* [29] researches.

This article discusses a fuzzy non-linear integer problem model for joint inventory-location facilities under facility disruptions. It tries to minimize the system's total costs, including the cost of creating facilities, transportation costs, customers' penalty costs, and inventory costs. In this article, we use a meta-heuristic method that has been proposed in recent years. Although exact methods are commonly used for small-scale problems, in solving the large-scale problem and NP-hard, heuristic and meta-heuristic methods are being used most often. The red deer algorithm is one of the recent meta-heuristic methods used to solve large-scale problems in this study.

Model contributions are as follows:

(1) Considering the inventory problem within the typical capacitated reliable facility location problem under imperfect disruption information with fuzzy programming.

(2) Using a minimum service level in the proposed model due to the high importance of customer satisfaction under uncertainty in customer demand.
(3) Solving the model for large-scale sizes using a recent metaheuristics method, the red deer algorithm for facility location problem.

The rest of this article is as follows: In Section 2, the literature review of the facility location problem, inventory problem, reliable transportation system, and disruption types is discussed. In Section 3, the main assumptions of the problem and first formulation of the model are presented, and then we use fuzzy programming methods to convert our fuzzy mixed integer nonlinear programming model to a crisp integer linear programming model. In Section 4, the solution method used in this article are introduced. In Section 5, we review some numerical examples and the results of solving them. Sensitivity analysis, discussion, and managerial insights will be covered in Section 6, and finally, the concluding remarks are given in Section 7.

## 2. Literature review

In this following section, some articles regarding the facility location problem have been reviewed, and their features are mentioned as follows. In some facility location problems, due to the need of the problem and its applicability in the real world, some other concepts such as inventory and routing are added to the research. For the first time in the literature, Shen *et al.* [43] mixed the location problem with the concept of inventory and presented them as an integrated problem. By combining these two concepts and determining the amount of safety stock, they tried to provide an acceptable level of service to their customers. Zhang *et al.* [57] formulated an integrated location and inventory problem with heterogeneous disruptions for the supply chain facilities. It finds an optimal number of facilities and locations of them with considerate inventory management. It presented two solution methods: (1) an exact approach like Special ordered sets of type two (SOS2) and (2) heuristic methods like Lagrangian relaxation. Jalali *et al.* [23] proposed a bi-objective reliable location-routing problem in a supply chain under different scenarios with the possibility of disruption in distribution centers. The first objective was minimizing the total cost, and the second objective was maximizing the fill rate. Hiassat *et al.* [21] developed an inventory-location-routing model for perishable products. They determined the number and location of warehouses, the level of inventory at each retailer, and the routes traveled by vehicles. Xie *et al.* [51] developed an integer linear programming for reliable location-routing design under probabilistic facility disruptions that minimize total cost, including fixed setup costs and routing costs. It is solved by a combination of the LR with a column generation (CG) algorithm. Rezaei *et al.* [38] proposed a bi-objective location-allocation-inventory model to design a dual-channel, multi-level supply chain network. Salari *et al.* [40] developed a bi-objective transportation–location–inventory–routing problem in three echelons with stochastic constraints.

The occurrence of a network disturbance is possible, and the exact time of its event cannot be determined. But considering it in problems can play an essential role in the efficiency of research. These disruptions can be dependent or independent depending on the definition of the problem. The source of disruption in the facility location can be endogenous or exogenous [46]. Various natural and human factors such as unfavorable weather, labor actions, breakdown of facilities and equipment lead to the disruption. These disruptions can have high costs for the service network [45]. Also, in most cases, customers are not aware of these disruptions in advance and do not have information about their occurrence.

Peng *et al.* [36] designed logistics networks with facility disruptions and it led to an increase in the reliability of the supply chain network. The model was mixed-integer programming and a single objective that minimizes costs when normal conditions and disruption costs when abnormal. Yun *et al.* [53] considered an integer programming model for reliable facility location design considering disruption for each facility, and the distribution of disruption is independent. This model strives to determine the facilities' optimal location and allocates it to customers to minimize overall costs and customers don't know exact information about facilities. Li and Ouyang [25] studied a reliable facility location model that disruptions are correlated, and when a disruption to one of the facilities occurs, the customer's need is used by another. Disruptions were location-dependent on where the facility was located. It uses the continuum approximation approach to solve the model. Shishebori *et al.* [44]

developed a reliability facility location-network design problem. In this model, if the facility fails, the closest facility to the customer will be replaced to serve it. Sabahi and Parast [39] discussed that innovative companies are more resilient to disruption because innovation can increase a company's ability, such as flexibility, agility, and share knowledge, and significantly positively affect its risk management ability. Snyder *et al.* [46] Focused on endogenous disruptions and have examined the behavior of managers in conditions of disruption.

Some authors considered disruption under uncertainty. Lu *et al.* [31] proposed robust mathematical programming for reliable facility location design under uncertain correlated disruptions. It minimizes the total expected costs under the worst-case distribution. Yun *et al.* [54] formulated a nonlinear integer programming for reliable facility location. Some binary variables in their research are the relocation of customers between facilities. Facilities have disruption, and it is related to their site. The information of customers is imperfect, and they don't know about real-time transportation. Abazari *et al.* [1], in their proposed model, have considered relief centers as facilities to provide services in disaster time. This model aims to minimize total costs such as locating and establishing relief centers' costs before a disaster, fixed transportation costs, and inventory costs. Lim *et al.* [27] presented a stylized continuous approximation model to determine optimal locations of facilities. It considered each facility has disruption with random distribution, and information of customers is imperfect. The results showed investing in preventing disruption leads to increased costs but increases the level of customer satisfaction. Tran *et al.* [48] proposed a mixed-integer nonlinear programming model for hub location planning with random facility disruption and minimize travel costs and satisfy demands. They considered a penalty if all hubs fail. Yun *et al.* [55] developed a continuous model for reliable facility location problems considering round-trip transportation. The model was suitable for a large scale, and it is proposed for situations where information is imperfect. The model is designed so that the customer returns to his starting point after receiving the service or lacking it. The objective function of this model was to minimize shipping and transportation costs.

Various researches on facility location problems have been conducted in this field over the past decades. Berman *et al.* [12] analyzed the effects of failure probabilities, availability of information, correlations, and problem objectives on finding the optimal location for facilities. They studied both complete and incomplete customer information. Alfandari [5] studied a general soft-capacitated facility location problem in which connection costs don't systematically satisfy the property of triangle inequality. Aboolian *et al.* [2] addressed a reliable facility location problem (RFLP) that more than one facility can assign to customers, and facilities have different failure rates. This model aims to minimize the total cost, including transportation costs and fixed costs. An *et al.* [6] studied a robust optimization for reliable p-median facility location problems. It was two-stage and considered two features for contribution: (1) demand changes due to disruptions and (2) facility capacities. Li *et al.* [26] formulated mixed-integer stochastic programming for reliable facility location design problems. The facility's failures are related, and the model presented a structure considered independent and identically distributed disruptions facility failure risks. Xie *et al.* [51] presented mathematical programming for a reliable location routing problem (RLRP) with the risk of disruptions. The model determined optimal facility locations and outbound delivery routing, and backup plans.

Zheng *et al.* [58] formulated a two-level mathematical programming model to find the optimal location for charging stations position that provides service to electric vehicles (EVs). Ma *et al.* [32] believe that one of the sustainability factors in urban transportation networks is bicycle-sharing, and they evaluate the quality of the bicycle-sharing system. Also, Zhou *et al.* [59] believe that since environmental issues and cost reduction are of great importance to people today, shared bikes can significantly impact these goals. Tellez *et al.* [47] formulated a model to reduce transportation costs and increase the quality of transportation in a healthy transportation network. This model seeks to optimize transportation strategies in a geographical area, including social and Medico-Social Institutions (MSI).

In totally, research on facility location problems can be divided into two general categories: (1) capacitated facility location problem (UFLP) and uncapacitated facility location problem (UFLP). Benedyk *et al.* [11] studied mixed-integer linear programming for facility locations with capacity limitations. The facility had uncertain demand. Therefore, they assigned an optimal strategic intermodal facility in their scenario-based study. Santiváñez and Carlo [41] proposed a linear mixed-integer model for reliable capacitated facility location (RCFL) problems that meet the expected minimum level of customer service. The model was presented in an uncertain environment, and it used the worst-case method for service-level in different disruption scenarios

Snyder and Daskin [45] presented various mathematical models based on the P-median problem (PMP) and the uncapacitated fixed-charge location problem (UFLP). They assumed that system costs would increase due to uncertainty factors such as equipment failures imposed on the customer. Cui *et al.* [15] proposed the reliable uncapacitated fixed charge location problem (RUFL), which reduces initial setup cost and transportation cost on their scenario-based study to determine the optimal facility locations and customer assignments. The model considered possible customer reassignment and unexpected failures with site-dependent probabilities. Shen *et al.* [42] presented two uncapacitated reliable facility location problems (URFLP): (1) a scenario-based stochastic programming (SP) model for random parameters and a nonlinear integer programming (NIP) model for high data numbers problems. This model is intended to replace another facility when a facility fails to service the customer again.

There are two general approaches to expressing uncertainty: (1) fuzzy mathematical programming and (2) Stochastic programming. Each of these two approaches can be used to describe the uncertainty following the problem. Inuiguchi and Ramik [22] have pointed out two main differences between stochastic and fuzzy mathematical programming approaches. First, If the random vector has a normal distribution, stochastic programming offers a better solution. But if the random vector has a general distribution, fuzzy mathematical programming has a better solution than stochastic programming. Second, If the uncertain variables are independent of each other, fuzzy programming is better than stochastic programming because only a few decision variables take on non-zero values. Given the reasons mentioned, using a fuzzy approach to express uncertainty in the amount of customer demand may be more appropriate.

Different methods have been used to solve facility location problems in the literature. Frequently facility location problem is an NP-hard problem, and if the dimensions of the problem increase, it requires the use of meta-heuristic methods to solve the problem until reducing the time to solve it to find the solution. In some research, exact methods have also been used. An *et al.* [6] used the column-and-constraint generation method for reliable p-median facility location problems. The result showed this method performs better than the Benders cutting plane method. Also, Marufuzzaman and Ekşioğlu [33] presented mixed-integer programming for designing a reliable transportation network for biofuel supply chains. They used an integrated Benders decomposition algorithm and a hybrid rolling horizon algorithm to solve this problem.

Kratica *et al.* [24] considered a simple plant location problem that they solved it with a genetic algorithm. Chauhan *et al.* [13] formulated an integer linear programming model that includes range and power consumption. The objective function was to maximize the area to cover more areas. It was NP-hard and used greedy and three-stage heuristics (3SH) to solve it. Mohammadi *et al.* [35] have designed a single problem-allocation hub location so that uncertainties in society, such as natural disasters, are taken into account in the model. The first objective function minimizes total transportation costs, and the second objective function reduces transportation time. The model was complex and NP-hard. It considered an efficient approximation method for defining a lower bound and used a hybrid meta-heuristic algorithm, including the genetic algorithm (GN) and VNS algorithm (SGV-II) to solve the problem. Peng *et al.* [37] proposed an approximation of discrete spatial data used for continuous facility location design and can be applied to discrete facility location problems. The model used a Voronoi diagram and discrete data to calculate the total cost and optimal facility location. Also, it presented a disk model for finding a near-optimum facility location. Liu and Wang [28] presented a location problem for charging stations that use charge battery electric vehicles (BEV). One of the major global issues is reducing greenhouse gas emissions, and this article tries to reduce travel costs and time by appropriately locating charging stations. The model is solved by an efficient surface response approximation model-based solution algorithm.

According to Table 1, the review of the above articles has shown that most of the existing articles in this field focus on a certain environment and less on the uncertainty in the parameters. So that in case of considering uncertainty, they have paid more attention to stochastic programming. In addition, location problems have less inventory concept in their model, while the shortage of inventory is one of the important reasons for the lack of service to customers and their dissatisfaction. Many articles in this area have focused on perfect information

Table 1. Summary of literature review.

| Row | Author's Name | Year | Terms of objective function: Transportation | Terms of objective function: Capacitated | Terms of objective function: Travel time | Terms of objective function: Inventory | Terms of objective function: Other Cases | Disruption/failure | Information: Imperfect | Information: Perfect | Uncertainty: Certainty | Uncertainty: Stochastic | Uncertainty: Fuzzy | Model Features: Linear | Model Features: Non-linear | Model Features: Single objective | Model Features: Multi objectives | Method & Solver |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Shishebori *et al.* | 2014 | * | | | | | | | * | * | | | | * | * | | |
| 2 | Yun *et al.* | 2015 | * | | | | * | * | * | | * | | | | * | * | | Lagrangian relaxation |
| 3 | Xie *et al.* | 2016 | * | | | | * | * | | * | | * | | * | | * | | Column generation algorithm |
| 4 | Jalali *et al.* | 2016 | * | | | | | * | | * | | * | | | * | | * | Multi-objective biogeography-based optimization |
| 5 | Zhang *et al.* | 2016 | * | | | * | | * | | * | * | | | | * | * | | Lagrangian relaxation |
| 6 | Xie *et al.* | 2016 | * | | | | * | * | | * | * | | | | * | * | | Lagrangian relaxation with embedded column generation and local search |
| 7 | Yun *et al.* | 2017 | * | | | | | * | * | | * | | | | * | * | | CPLEX |
| 8 | Marufuzzaman & Ekşioğlu | 2017 | * | | | | | * | | * | * | | | * | | * | | Integrated Benders decomposition algorithm and a hybrid rolling horizon algorithm |
| 9 | Tran *et al.* | 2017 | * | | | | * | * | | * | | * | | | * | | * | Tabu search algorithm |
| 10 | Zheng *et al.* | 2017 | * | | | | * | | | * | * | | | | * | | * | |
| 11 | Liu & Wang | 2017 | * | | * | | | | | * | * | | | * | | * | | Approximation model-based solution |
| 12 | Santiváñez & Carlo | 2018 | * | * | | | * | * | | * | | * | | * | | * | | Worst-case method |
| 13 | Xie *et al.* | 2019 | * | | | | | * | | * | * | | | | * | | * | Lagrangian relaxation |
| 14 | Yun *et al.* | 2019 | * | | | | | * | * | | * | | | * | | * | | |
| 15 | Mohammadi *et al.* | 2019 | * | | * | | | | | * | | * | | | * | | * | Genetic algorithm and VNS algorithm |
| 16 | Chauhan *et al.* | 2019 | * | | | | | * | | * | * | | | | * | * | | Greedy and three-stage heuristics (3SH) |
| 17 | This Research | 2021 | * | | | * | | * | * | | | | * | | * | * | | Red deer algorithm |

about the occurrence of disruptions. However, it is not possible to know enough about these disruptions in the real world. We proposed a novel mathematical model for the joint location-inventory problem in an uncertain environment to minimize the system's total costs, including construction costs of facilities, inventory costs, and transportation costs. We use fuzzy programming for present uncertainty, such as customer demand. Customer refers to the facility to receive the services they want. We considered site-independent disruption for facilities. In case of disruption in the facility, refer to the following facilities, and after receiving the service, return to her/his original location. We consider penalty cost if the customer demand doesn't meet, and the cost of transportation to the dummy facility is equal to the penalty cost imposed on the system. Each facility has the capacity and minimum level demand for the establishment, and if the demand for the facility is under minimum level demand, that facility doesn't establish. To meet customer demand, a maximum time has been set for the customer to present service level in the system so that the customer's presence in the system should not exceed that time. We also used a new meta-heuristic method, the red deer algorithm, to solve problems on a large scale to reduce solution time. Table 1 presented the characteristics of some of the Pervious researches in the literature review such as "Terms of objective function", "Disruption", "Type of information", "Uncertainty" and "Model Features".

## 3. Problem description and formulation

In this article, we try to find optimal locations for the facilities from among the candidate locations and optimally allocate the available facilities to the customers. Simultaneously, we want to minimize the total system costs: (1) facility construction costs, (2) transportation costs, (3) penalty costs, and (4) inventory costs.

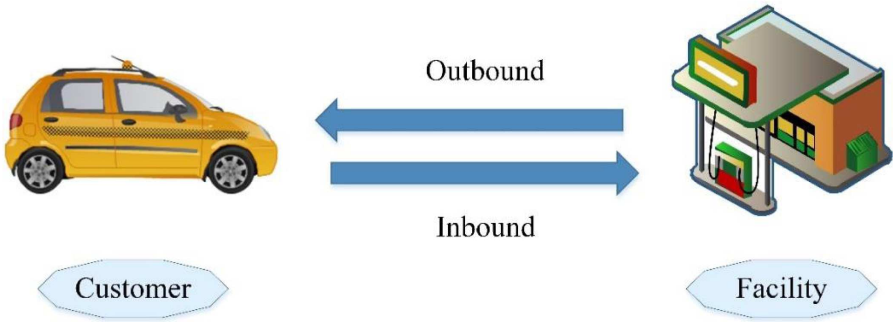


FIGURE 1. Transportation system.

In this model, the probability of site-dependent disruption is considered for facilities. This model assumes that one of the facilities will initially be allocated to the customer. In this case, if a disruption occurs in the assigned facility, another facility will be assigned to the customer. This work will continue until the customer receives their service. The facilities are assigned to the customer in advance, and the customer is not aware of the occurrence of these disruptions and the lack of service provided by the facilities. On the other hand, if the customer finally cannot receive the desired services, the penalty cost is charged.

Usually, people return to their first location after receiving services and meeting their demands or not receiving services. As a result, this model assumes that customers return to their first location. The transportation system in this article is round-trip transportation. In this type of transportation system, it is assumed that the customer will return to his first location whether they received services or not. Therefore, the proposed transportation system is divided into two categories: (1) outbound transportation (customer movement from its first location to the facility) and (2) inbound transportation (customer return to its first location). Figure 1 shows the transportation system.

The penalty cost is imposed on the system if the customer does not receive the service. This cost is applied when the last facility allocated to the customer fails to meet the customer's needs. Dummy facilities are provided to integrate transportation costs and penalty costs. Visiting a dummy facilitated by the customer does not mean meeting the customer's needs.

For example, suppose a customer intends to receive fuel from a city gas station. After referring to the nearest gas station, the customer, if he cannot receive the desired fuel due to disruption and breakdown in that gas station, will refer to the next gas station. The customer repeats this operation until he gets the preferred fuel. Customers refer to different facilities and receive the services they need, as shown in Figure 2.

Another issue considered in the model is the inventory problem. Each facility can increase the level of its inventory if needed. There is also a cost for storing. On the other hand, some facilities are limited in providing services. One of these important constraints is capacity constraints, and each facility can ultimately meet the demand of a certain number of customers. Therefore, it is very important to consider the capacity constraint. Also, considering that the cost of establishing and creating some of these facilities is very high, it is important to consider that the condition for establishing these facilities is to have at least a certain amount of demand for services.

## 3.1. Assumptions

(1) Customers will return to their first location if they receive a service or give up their service Such as refueling centers such as gas stations.
(2) There is a specific capacity for each facility to provide services.
(3) The demand parameter is an uncertain and fuzzy number.

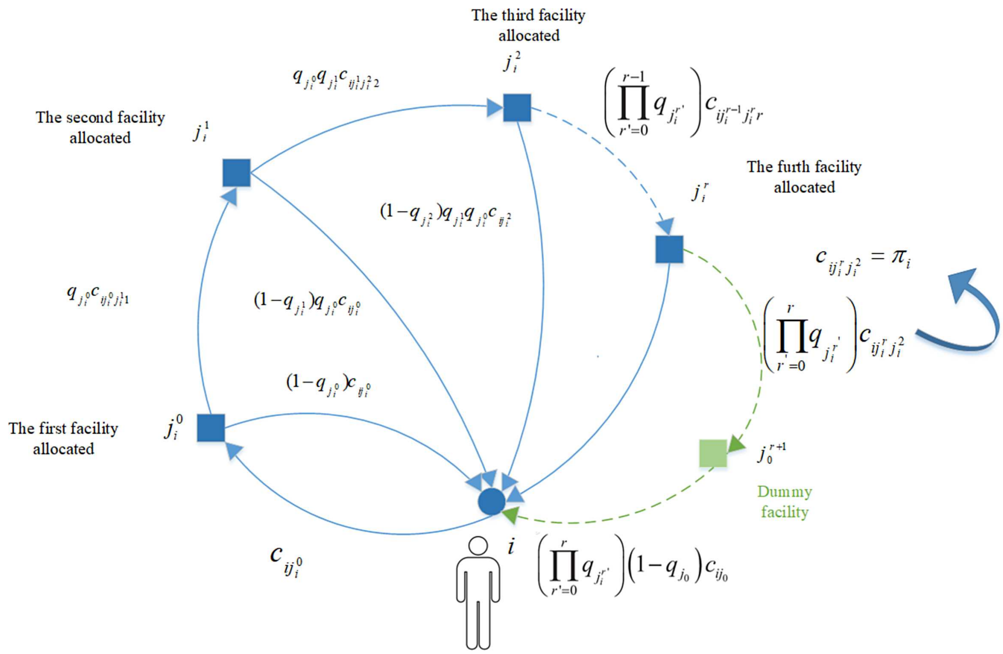


FIGURE 2. Network of the model.

(4) The proposed model is a single period.
(5) The possibility of disruption in the facilities is independent of each other.
(6) It is possible to order and maintain inventory for facilities.
(7) The duration in which customers spend on the system should not exceed the service level threshold

### 3.2. Notations

*Sets*

| | |
|---|---|
| $i \in I$ | Set of customer locations |
| $j \in J$ | Set of candidate facility locations |
| $k \in K$ | Set of locations $(I, J \subset K)$ |
| $K_j^+ \subset K$ | Set of candidate facility locations that can visited before visiting facility $j$ |
| $K_j^- \subset K$ | Set of candidate facility locations that can visited after visiting facility $j$ |
| $r \in R$ | Facility rank for a customer, the maximum facility rank is $R$ |

*Parameters*

| | |
|---|---|
| $\widetilde{d}_i$ | Demand level of customer $i$; $\widetilde{d}_i = (d_{i1}, d_{i2}, d_{i3})$ |
| $f_j$ | Fixed opening cost for facility $i$ |
| $\pi_i$ | Penalty cost when customer $i$ gives up on the service |
| $q_j$ | Disruption probability of facility $j$ |
| $c_{ij}$ | Unit-demand transportation cost from customer $i$ to facility $j$ |
| $t_{ij}$ | Unit-demand transportation time from customer $i$ to facility $j$ |
| $j_i^r$ | Facility for customer $i$ at rank $r$ |
| $c_{ijj'r}$ | Unit-demand transportation cost from facility $j$ to facility $j'$ for customer $i$ at rank $r$ |
| $cap_j$ | Maximum capacity of facility $j$ |
| $S_j$ | Minimum demand threshold from facility $j$ to build it |

| | |
|---|---|
| $b_j$ | Fixed ordering cost in facility $j$ |
| $v_j$ | Variable ordering cost per unit in facility $j$ |
| $h_j$ | Inventory (holding) cost in facility $j$ |
| $p_{ijj'r}$ | Probability that customer $i$ visits facility $j$ at rank $r$ after visiting facility $j'$ |
| $T$ | Maximum acceptable time threshold for service level |

The parameter $c_{ij}$ represents the cost of transportation from the first location of customer $i$ to the facility $j$. Similarly, $c_{ijj'r}$ is equal to the cost of transportation from the facility $j$ to the facility $j'$ at rank $r$. We consider $c_{ijj_0r}$ equal to $\pi_i$. Because the customer's referral to dummy facilities means that the previous facility does not receive service in advance, for this reason, the cost of a penalty is considered by referring the customer to the dummy facility.

*Decision variables*

| | |
|---|---|
| $z_j$ | Decision of facility location, if $z_j = 1$ denotes facility $j$ is open, $z_j = 0$ otherwise |
| $x_{ij}$ | Decision of customer assignment, if $x_{ij} = 1$ denotes customer $i$ is assigned to facility $j$ at rank 0, $x_{ij} = 0$ otherwise |
| $y_{ijj'r}$ | Decision of customer assignment, if $y_{ijj'r} = 1$ denotes customer $i$ is assigned to facility $j$ at rank $r-1$ and to facility $j'$ at rank $r$, $y_{ijj'r} = 1$ otherwise |
| $Q_j$ | Ordering quantity for facility $j$ |

As explained above, $p_{ijj'r}$ is the probability that customer $i$ visits facility $j$ at rank $r$ after visiting facility $j'$. When the $r = 1$ the $p_{ijj'r}$ would simply be $q_j$. However, for $r > 1$ the probability of visit facility $j$ at rank $r$ would rely on the probability of visit facility $j'$ at rank $r-1$ multiplied by the disruption probability of facility $j$. Therefore the value of $p_{ijj'r}$ would be as follows [55].

$$p_{ijj'r} = \begin{cases} q_j, & r = 1 \\ q_j \sum_{j' \in K_j^+} p_{ij'j(r-1)} y_{ij'j(r-1)}, & r > 1. \end{cases}$$

### 3.3. Model formulation

#### *3.3.1. Objective function*

The fixed cost of establishing the facility is as follows:

$$\mathrm{TFC} = \sum_{j \in J} f_j z_j. \tag{3.1}$$

Transportation costs are divided into two expressions. In the first term, it is assumed that the customer will receive the required services in the first facility that he/she refers to it. In the second term, we assume that the customer does not find her/her services in the first facility assigned to it and refers to other pre-assignments facilities.

The first expression is related to the time when the customer receives the desired services if she refers to the first facility and then returns to her original location.

$$\sum_{i \in I} \widetilde{d}_i \sum_{j \in K} (2 - q_j) c_{ij} x_{ij}. \tag{3.2}$$

The second expression refers to the time when the first facility allocated to the customer is disrupted and the customer refers to other services to receive services.

$$\sum_{i \in I} \widetilde{d}_i \sum_{j \in K} \left( \sum_{j' \in K_j^-} \sum_{r \in R} p_{ijj'r} (c_{ijj'r} + (1 - q_{j'}) c_{ij'}) y_{ijj'r} \right). \tag{3.3}$$

Therefore, total transportation cost for facility $j$ is:

$$\text{TTC} = \sum_{i\in I} \widetilde{d}_i \sum_{j\in K} \left( (2-q_j)c_{ij}x_{ij} + \sum_{j'\in K_j^-} \sum_{r\in R} p_{ijj'r}(c_{ijj'r} + (1-q_{j'})c_{ij'})y_{ijj'r} \right). \tag{3.4}$$

Inventory cost is divided into two types of costs: (1) holding cost and (2) ordering cost. For a facility at $j$, its demand is:

$$\sum_{i\in I} \widetilde{d}_i(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in K} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr}. \tag{3.5}$$

**Theorem 3.1.** *Let $B$ and $V$ be the fixed and variable ordering costs, then, the optimum order quantity would be $Q = \sqrt{\frac{2B}{h}}$.*

*Proof.* Similar to the theorem of Vujošević *et al.* [50] the total inventory costs would be:

$$\text{inventory cost} = \frac{1}{Q_j}B + V + \frac{h_jQ_j}{2} \tag{3.6}$$

by calculating the derivative of total inventory costs with respect to ordering quantity, the optimum quantity would be:

$$\frac{\mathrm{d}}{\mathrm{d}Q_j}\text{inventory cost} = -\frac{B}{Q_j^2} + \frac{h_j}{2} = 0 \to Q_j^* = \sqrt{\frac{2B}{h}}\cdot \tag{3.7}$$

Base on Theorem 3.1, inventory cost for facility $j$ is:

$$\frac{1}{Q_j}\left( \sum_{i\in I} \widetilde{d}_i b_j(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i b_j p_{ij'jr}(1-q_j)y_{ij'jr} \right) + \left( \sum_{i\in I} \widetilde{d}_i v_j(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i v_j p_{ij'jr}(1-q_j)y_{ij'jr} \right) + \frac{h_jQ_j}{2} \tag{3.8}$$

where the first and second part of it is associated with ordering cost and the third term is holding cost. The optimal order quantity would be as follows:

$$Q_j^* = \left[ \frac{2b_j}{h_j}\left( \sum_{i\in I} \widetilde{d}_i(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr} \right) \right]^{\frac{1}{2}}. \tag{3.9}$$

Therefore, total inventory costs are:

$$\text{TIC} = \sum_{j\in K} \left( \left( 2b_jh_j \left( \sum_{i\in I} \widetilde{d}_i(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr} \right) \right)^{\frac{1}{2}} + \left( \sum_{i\in I} \widetilde{d}_i v_j(1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i v_j p_{ij'jr}(1-q_j)y_{ij'jr} \right) \right). \tag{3.10}$$

Final objective function for this problem is:

$$\min_{X,Y,Z,Q} \sum_{j\in J} f_j z_j + \sum_{i\in I} \widetilde{d}_i \sum_{j\in K} \left( (2-q_j)c_{ij}x_{ij} + \sum_{j'\in K_j^-} \sum_{r\in R} p_{ijj'r}(c_{ijj'r} + (1-q_{j'})c_{ij'})y_{ijj'r} \right)$$
$$+ \sum_{j\in K} \left( \left( 2b_j h_j \left( \sum_{i\in I} \widetilde{d}_i (1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr} \right) \right)^{\frac{1}{2}} \right.$$
$$\left. + \left( \sum_{i\in I} \widetilde{d}_i v_j (1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i v_j p_{ij'jr}(1-q_j)y_{ij'jr} \right) \right). \tag{3.11}$$

Objective function equation (3.11) aims to minimize total costs, such as (1) fixed construction cost and (2) transportation costs. □

### 3.3.2. Constraints

$$x_{ij} = \sum_{j'\in K_j^-} y_{ijj'1}, \quad \forall i \in I, j \in K \tag{3.12}$$

$$\sum_{j'\in K_j^+} y_{ij'j(r-1)} = \sum_{j'\in K_j^-} y_{ijj'r}, \quad \forall i \in I, j \in K, r \in R \tag{3.13}$$

$$\sum_{j\in K} y_{ijj_0(R+1)} = 1, \quad \forall i \in I \tag{3.14}$$

$$S_j z_j \le \sum_{i\in I} \widetilde{d}_i (1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr}, \quad j \in K \tag{3.15}$$

$$\sum_{j\in K} x_{ij} = 1, \quad \forall i \in I \tag{3.16}$$

$$x_{ij} + \sum_{r\in R} \sum_{j'\in K_i^+} y_{ij'j(r-1)} \le z_j, \quad \forall i \in I, j \in J \tag{3.17}$$

$$\sum_{i\in I} \widetilde{d}_i (1-q_j)x_{ij} + \sum_{j'\in K_j^+} \sum_{i\in I} \sum_{r\in R} \widetilde{d}_i p_{ij'jr}(1-q_j)y_{ij'jr} \le cap_j, \quad j \in K \tag{3.18}$$

$$p_{ijj'1} = q_j, \quad \forall i \in I, j \in K, j' \in K_j^- \tag{3.19}$$

$$p_{ijj'r} = q_j \sum_{j'\in K_j^+} p_{ij'j(r-1)} y_{ij'j(r-1)}, \quad \forall i \in I, j \in K, j' \in K_j^-, r \in R \tag{3.20}$$

$$\sum_{j\in K} \left( t_{ij}x_{ij} + \sum_{j'\in K_i^-} \sum_{r\in R} p_{ijj'r} t_{ijj'r} y_{ijj'r} \right) \le T, \quad \forall i \in I \tag{3.21}$$

$$x_{ij} \in \{0,1\}, \quad \forall i \in I, j \in K \tag{3.22}$$

$$y_{ijj'r} \in \{0,1\}, \quad \forall i \in I, j \in K, j' \in K_j^-, r \in R \tag{3.23}$$

$$z_j \in \{0,1\}, \quad \forall j \in J \tag{3.24}$$

$$Q_j \ge 0, \quad \forall j \in J. \tag{3.25}$$

Constraints (3.12)–(3.14) ensure that in the event of a facility $j$ assigned to customer $i$, the subsequent facilities will be assigned consecutively until the dummy facility is assigned to their ranks. Constraint (3.15) ensures that facility $j$ is created when there is a minimum amount of demand to receive service by the customer in facilities. Constraint (3.16) indicates that a facility must be assigned to the customer. Constraint (3.17) ensures that the facility is built if the customer visits the facility. Constraint (3.18) guarantees that the facilities' demand should not exceed their capacity (capacity constraint). Constraints (3.19) and (3.20) indicate the probability of disruption in the facilities. Constraint (3.21) ensures that every customer's transportation time will not exceed the defined service level threshold. Constraints (3.22)–(3.24) are related to binary decision variables. Constraint (3.25) is related to nonnegative variable.

## 3.4. Fuzzy programming

As discussed in the earlier stages, the uncertainty of demand is taken care of using fuzzy numbers. To solve the proposed mathematical model, we have to convert the fuzzy representation to the crisp representation. For this purpose, we used the hybrid expected value and chance constraint programming (CCP) approach. Therefore, the expected value of the objective function should be used in the crisp model instead, which would be like this:

$$\begin{aligned}\min_{X,Y,Z,Q} \mathrm{EV}[\mathrm{OF}] = &\sum_{j\in J} f_j z_j + \sum_{i\in I} \mathrm{EV}\left(\widetilde{d}_i\right) \sum_{j\in K}\left((2-q_j)c_{ij}x_{ij} + \sum_{j'\in K_j^-}\sum_{r\in R} w_{ijj'r}(c_{ijj'r} + (1-q_{j'})c_{ij'})\right)\\ &+\sum_{j\in K}\left(\left(2b_jh_j\left(\sum_{i\in I}\mathrm{EV}\left(\widetilde{d}_i\right)(1-q_j)\;x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\mathrm{EV}\left(\widetilde{d}_i\right)w_{ijj'r}(1-q_j)\right)\right)^{\frac{1}{2}}\right.\\ &+\left.\left(\sum_{i\in I}\mathrm{EV}\left(\widetilde{d}_i\right)v_j(1-q_j)x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\mathrm{EV}\left(\widetilde{d}_i\right)v_j\mathrm{w}_{ijj'r}(1-q_j)\right)\right). \qquad (3.26)\end{aligned}$$

Constraints (3.15) and (3.18) have still fuzzy numbers within them. Suppose that $\widetilde{d}_i$ is a trapezoidal fuzzy number $(d_{i1}, d_{i2}, d_{i3})$ and we want the credibility of these constraints to be more than parameters $\vartheta_1$, $\vartheta_2$.

$$Cr\left(S_jz_j \le \sum_{i\in I}\widetilde{d}_i(1-q_j)x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\widetilde{d}_iw_{ijj'r}(1-q_j)\right) \ge \vartheta_1, \quad j\in K \qquad (3.27)$$

$$Cr\left(\sum_{i\in I}\widetilde{d}_i(1-q_j)x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\widetilde{d}_iw_{ijj'r}(1-q_j) \le cap_j\right) \ge \vartheta_2, \quad j\in K. \qquad (3.28)$$

Consequently, the equivalent crisp parametric constraint can be written as follows:

$$\begin{cases} S_jz_j \le \sum_{i\in I}\left[(2-2\vartheta_1)d_{i2} + (2\vartheta_1-1)d_{i1}\right]\,(1-q_j)\;x_{ij} & \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\;\left[(2-2\vartheta_1)d_{i2} + (2\vartheta_1-1)d_{i1}\right]\,w_{ijj'r}(1-q_j) & \text{if }\ \vartheta_1 \ge 0.5\\ S_jz_j \le \sum_{i\in I}\left[(1-2\vartheta_1)d_{i3} + 2\vartheta_1d_{i2}\right]\,(1-q_j)\;x_{ij} & \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\;\left[(1-2\vartheta_1)d_{i3} + 2\vartheta_1d_{i2}\right]\,w_{ijj'r}(1-q_j) & \text{if }\ \vartheta_1 \le 0.5\end{cases}, \quad j\in K \qquad (3.29)$$

$$\begin{cases} cap_j \ge \sum_{i\in I}\left[(2\vartheta_2-1)d_{i3} + (2-2\vartheta_2)d_{i2}\right](1-q_j)x_{ij} & \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[(2\vartheta_2-1)d_{i3} + (2-2\vartheta_2)d_{i2}\right]w_{ijj'r}(1-q_j) & \text{if }\ \vartheta_2 \ge 0.5\\ cap_j \ge \sum_{i\in I}\left[2\vartheta_2d_{i2} + (1-2\vartheta_2)d_{i1}\right](1-q_j)x_{ij} & \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[2\vartheta_2d_{i2} + (1-2\vartheta_2)d_{i1}\right]w_{ijj'r}(1-q_j) & \text{if }\ \vartheta_2 \ge 0.5\end{cases}, \quad j\in K. \qquad (3.30)$$

### 3.5. Final model after linearization

As a result, the final linear model is as follows based on the content about linearization presented in the appendix:

$$\min_{X,Y,Z,Q} \mathrm{EV}[OF] = \sum_{j\in J} f_j z_j + \sum_{i\in I} \mathrm{EV}\left(\tilde{d}_i\right) \sum_{j\in K}\left((2-q_j)c_{ij}x_{ij} + \sum_{j'\in K_j^-}\sum_{r\in R} w_{ijj'r}(c_{ijj'r} + (1-q_{j'})c_{ij'})\right)$$
$$+ \sum_{j\in K}\left(\left(2b_j h_j\left(\sum_{i\in I}\mathrm{EV}\left(\tilde{d}_i\right)(1-q_j)x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\mathrm{EV}\left(\tilde{d}_i\right)w_{ijj'r}(1-q_j)\right)\right)^{\frac{1}{2}}\right.$$
$$\left. + \left(\sum_{i\in I}\mathrm{EV}\left(\tilde{d}_i\right)v_j(1-q_j)x_{ij} + \sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\mathrm{EV}\left(\tilde{d}_i\right)v_j w_{ijj'r}(1-q_j)\right)\right). \quad (3.31)$$

Subject to:
Constraints (3.12)–(3.14), (3.16), (3.17), (3.19), (3.22)–(3.25),

$$S_j z_j \le \begin{cases} \sum_{i\in I}\left[(2-2\vartheta_1)d_{i2} + (2\vartheta_1 - 1)d_{i1}\right](1-q_j)\,x_{ij} \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[(2-2\vartheta_1)d_{i2} + (2\vartheta_1-1)d_{i1}\right] w_{ijj'r}(1-q_j) & \text{if } \vartheta_1 \ge 0.5 \\ \sum_{i\in I}\left[(1-2\vartheta_1)d_{i3} + 2\vartheta_1 d_{i2}\right](1-q_j)\,x_{ij} \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[(1-2\vartheta_1)d_{i3} + 2\vartheta_1 d_{i2}\right] w_{ijj'r}(1-q_j) & \text{if } \vartheta_1 \le 0.5 \end{cases}, \quad j\in K \quad (3.32)$$

$$cap_j \ge \begin{cases} \sum_{i\in I}\left[(2\vartheta_2 - 1)d_{i3} + (2-2\vartheta_2)d_{i2}\right](1-q_j)x_{ij} \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[(2\vartheta_2-1)d_{i3} + (2-2\vartheta_2)d_{i2}\right]w_{ijj'r}(1-q_j) & \text{if } \vartheta_2 \ge 0.5 \\ \sum_{i\in I}\left[2\vartheta_2 d_{i2} + (1-2\vartheta_2)d_{i1}\right](1-q_j)x_{ij} \\ +\sum_{j'\in K_j^+}\sum_{i\in I}\sum_{r\in R}\left[2\vartheta_2 d_{i2} + (1-2\vartheta_2)d_{i1}\right]w_{ijj'r}(1-q_j) & \text{if } \vartheta_2 \ge 0.5 \end{cases}, \quad j\in K \quad (3.33)$$

$$w_{ijj'r} \le p_{ijj'r} \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R \quad (3.34)$$
$$w_{ijj'r} \le y_{ijj'r} \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R \quad (3.35)$$
$$w_{ijj'r} \ge 0 \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R \quad (3.36)$$
$$w_{ijj'r} \ge p_{ijj'r} + y_{ijj'r} - 1 \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R \quad (3.37)$$
$$p_{ijj'r} = q_j \sum_{j'\in K_j^+} w_{ijj'(r-1)} \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R \quad (3.38)$$
$$\sum_{j\in K}\left(t_{ij}x_{ij} + \sum_{j'\in K_j^+}\sum_{r\in R} t_{ijj'r}w_{ijj'r}\right) \le T, \qquad \forall i\in I \quad (3.39)$$
$$w_{ijj'r} \in \{0,1\} \qquad \forall i\in I, j\in K, j'\in K_j^-, r\in R. \quad (3.40)$$

Constraints (3.34)–(3.37) have been added to the model for linearization. Constraints (3.40) are related to binary decision variable.

## 4. Solution method

The proposed model in this study is a mixed-integer nonlinear mathematical model with uncertain parameters. While we took care of nonlinearity and uncertainty in the previous section and turned the MINLP model to a MILP model, solving the transformed model with exact methods is still a computationally expensive task. Because the remained problem is an NP-hard problem, using heuristic and metaheuristic algorithms seems to be a reasonable alternative for large instances. Therefore, after defining five numerical examples (one small size, one medium size, and three large-size instances), a new metaheuristic algorithm named red deer optimizer will be solved. In order to validate our answers, we will also solve the small and medium-size problems using exact methods with GAMS software.

### 4.1. Computation of the reliability of the system

A system's reliability can be defined as the probability of operating successfully over a specific period. Any system comprises many units, subsystems, and components arranged and connected in series, parallel or meshed structure. A set of $n$ components is said to be in series (or non-redundant) if the system's success depends on the success of all the components. If we assume that the unit failures are independent within the system, then the reliability and unreliability of a series system would be as follows [8]:

$$R = R_1 R_2 \dots R_n = \prod_{i=1}^{n} R_i$$

$$Q = 1 - R = 1 - \prod_{i=1}^{n} R_i = 1 - \prod_{i=1}^{n} (1 - Q_i)$$

where $R$ and $Q$ are reliability and unreliability, respectively. On the other hand, a set of components is said to be parallel (or completely redundant system) if the system can succeed when at least one component succeeds. The reliability and unreliability of a parallel system with individual units are as follows [19]:

$$R = 1 - (1 - R_1)(1 - R_2) \dots (1 - R_n) = 1 - \prod_{i=1}^{n} Q_i$$

$$Q = \prod_{i=1}^{n} Q_i = \prod_{i=1}^{n} (1 - R_i).$$

The system in the proposed model follows the parallel system design.

### 4.2. Red deer algorithm

In this study, due to the high dimensions of location problems, the use of integer and binary variables, and the problem is NP-hard, the use of exact solution methods leads to an increase in solution time and consequently causes inefficiency of the proposed model. For this reason, the use of the metaheuristic method can be a reasonable alternative for large instances and lead to reduced solution time [34]. The RDA method is one of the new methods that has been introduced in recent years. Based on the results of previous studies, the proposed RDA can explore areas of promising search and, in most cases, find global solutions. In general, setting the RDA was simple and could be ordered for various real-world problems. By changing or adjusting the RDA parameters, the interaction between the in intensification and diversification phases is applied according to the type and dimensions of the problem [18].

This section will first briefly introduce the red deer algorithm and discuss the equations in this method. Then, we examine a few numerical examples and the results obtained in the next section and finally analyze the problem parameters' sensitivity.

The proposed mathematical model will be solved by a novel metaheuristic algorithm named red deer algorithms. Red Deer algorithm is one the newest and most efficient Darwinian evolutionary algorithms reported so far [18]. Therefore, we will use this algorithm to solve the joint location-inventory-routing problem for the real-size problems. The red deer algorithm's procedure is as follows.

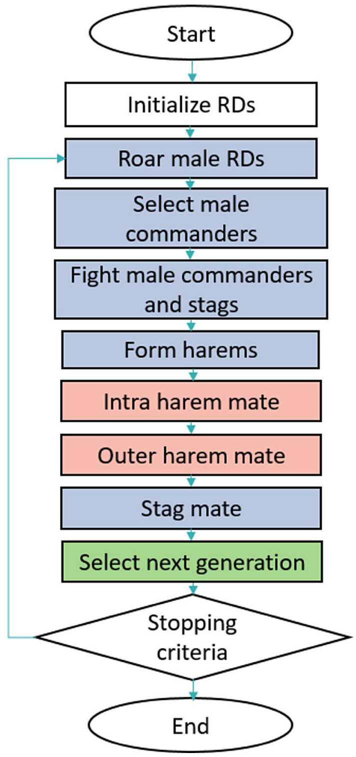


FIGURE 3. Flowchart of Red Deer Algorithm.

At first, a set of red deers would generate the initial solutions. The initial red deers have a set of male deers (stag) and female deers (hind). All red deers will be initialized randomly and better solutions will be tagged as stags and the others will be hinds. Afterward, there is a roaring step for stags to evaluate their neighbors and try to move their location to a better position. Then, a portion of the male deers would be selected as commanders, and after that, each commander would fight with stags randomly to increase its fitness. Like crossover in the genetic algorithm, the fighting of commander and stag generates two new locations. The commander moves to the best location among all four locations in the fight (which are the commander's position itself, the fighting stag, and two new positions). After that, the Harems should be formed based on the objective function of each commander. The better objective function commander has, the more hinds would be on his harem. Commander would mate with a portion of hinds in his harem to generate new solutions. Additionally, he will randomly choose one other harem and mate, which hinders new solutions. Non-commander deers would also mate with the nearest hind no matter what harem they are in, in order to generate new solutions. Finally, in selecting the algorithm to generate new populations, commanders and stages simply go to the next generations and for the reminder population, which consists of both hinds and new generated solutions, one of the fitness tournaments or roulette wheel mechanisms can be used. The flowchart of the algorithm can be seen in Figure 3.

There are three forms of operators in this algorithm which are being marked with various colors. Blue procedures are intensification operators, red procedures are diversification operators, and the green procedure is for escaping local optimum. We have used the following matrix that shows in Figure 4, representation to use a solution as a red deer in the algorithm. For $I$ number of customers and $J$ number of facilities, we have a $((I+1), J)$ size matrix.

The first row, which only contains binary values, represent whether the $j$th facility is open or not $(z_j)$. Other rows, which are a permutation of a number of facilities, show the order in which the customer visits without

| | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ | $j = 5$ | | $j = J$ |
|---|---|---|---|---|---|---|---|
| facilities | 1 | 0 | 1 | 0 | 0 | … | 0 |
| $i = 1$ | 1 | 3 | 4 | 2 | 5 | ⋮ | 6 |
| $i = 2$ | 5 | 7 | 4 | 1 | 3 | | 2 |
| | | | | | | | ⋮ |
| | | | … | | | | |
| $i = I$ | 4 | 2 | 1 | 3 | 5 | | 7 |

Figure 4. Solution representation in the Red deer algorithm.

| | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ | $j = 5$ |
|---|---|---|---|---|---|
| facilities | 1 | 0 | 1 | 1 | 0 |
| $i = 1$ | 1 | 3 | 4 | 2 | 5 |
| $i = 2$ | 5 | 2 | 4 | 1 | 3 |

Figure 5. Solution representation sample for the Red deer algorithm.

considering that the facility is open or close. For example, for a 2 customer and 5 facility problem, we will have the representation in Figure 5.

The order in which the customer $i = 1$ will visit facilities is (1−4−2−3−5). But as the facilities 2 and 5 are not open the order visit of customer $i = 1$ will be (1−4−3). The same thing will be applied for customer $i = 2$ with the facility order of (4−3−1).

Having this representation will help the algorithm to handle a variety of the problem constraints. However, some of the constraints cannot be covered using this representation. Constraint (3.35) for minimum demand threshold, constraint (3.38) for facilities' capacity, and constraint (3.39) for required service level needs to be taken care of differently in the algorithm. Therefore, we have defined a penalty function that will get value when one of these three containers is not satisfied. The relation between penalty function and the deviation from acceptable values of this constraints is exponential. Finally, the algorithm's fitness function will be the summation of both the objective function and the penalty function.

As mentioned above, there are three Phases in the red deer algorithm: roaring, fighting, and mating. In the roaring phase, we generate neighbors using 2-opt algorithm. In this phase, for each male red deer, we set the facility row as constant and by changing the values of customers' order two by two, generate new neighbors and select the best-found neighbor for the new location of the red deer. We generate two new solutions using RRC crossover (which is being used for binary chromosomes in the genetic algorithm). The best solution found between two existing answers and two new answers will be the commander's new location. In the mating phase, we use RRC crossover for facility row and PMX crossover (which is being used for order-based chromosomes in the genetic algorithm) for customer rows to generate new offspring. After generating new offspring, the new generation will be formed using tournament selection.

## 5. Numerical examples

In this section, we look at five different numerical examples, two of which are discussed in more detail. In these cases, we used the information published by Iran's Transport and Fuel Management Organization. This information, which indicates the amount of fuel demand, the cost of setting up a gas station, etc., have been used to bring the examples produced closer to the real world.

TABLE 2. Test problems' sizes.

| Test problem number | Test problem size | Number of customers ($I$) | Number of facilities ($J$) |
|---|---|---|---|
| 1 | Small | 2 | 8 |
| 2 | Medium | 3 | 12 |
| 3 | Large | 3 | 18 |
| 4 | Large | 4 | 24 |
| 5 | Large | 5 | 28 |

TABLE 3. Source of randomly generated parameters.

| Parameter | Ranges (unit) | Parameter | Ranges (unit) |
|---|---|---|---|
| $f_j$ | $\sim U(1000{-}2900)$ | $h_j$ | $\sim U(1{-}4)$ |
| $q_j$ | $\sim U(0{-}1)$ | $T$ | $\sim U(350{-}700)$ |
| $cap_j$ | $\sim U(200{-}550)$ | $\pi_i$ | $\sim U(200{-}300)$ |
| $S_j$ | $\sim U(40{-}150)$ | $c_{ij}$ | $\sim U(5{-}40)$ |
| $b_j$ | $\sim U(100{-}160)$ | $c_{ijj'r}$ | $\sim U(10{-}400)$ |
| $v_j$ | $\sim U(1{-}2)$ | $\vartheta_1, \vartheta_2$ | $\sim U(0.45{-}0.85)$ |
| $t_{ij}$ | $\sim U(10{-}300)$ | $t_{ijj'r}$ | $\sim U(4{-}30)$ |
| $\widetilde{d}_i$ | $\sim U((150, 200, 250){-}(300, 350, 400))$ | | |

Before turning into the metaheuristic algorithm for solving the problem, we evaluate our proposed model by calculating the gap between the RDA method results and the exact method with CPLEX solver only for small and medium-size instances. To understand the accuracy of the RDA method, we use equation (5.1). The gap shows the difference between the best possible objective and the best-found objective.

$$Gap = \frac{\text{best found objective} - \text{best possible objective}}{\text{best possible objective}} \times 100\%. \tag{5.1}$$

After ensuring the model's validation, we run the proposed algorithm for all instances, including large-sized problems. The Table 2 shows the size and characteristics of each test problem. The algorithm was executed on a laptop, with an Intel(R) Core (TM) i7-8550U CPU @1.80 GHz 1.99 GHz processor and 8 GB RAM of memory.

Table 3 depicts the values of defined parameters in test problems generated randomly, based on uniform distribution.

In following, the first two instances will be discussed in detail. The first example has two customers and eight facilities. Initially, each customer must be assigned to a facility. If the customer's needs are not met, the customer can refer to the pre-determined facilities to receive the services they want. Finally, in case of disruption in all facilities, a penalty cost will be added to the systems cost regarding not meeting the demand. The Tables 4–6 show the values of the various parameters such as facility parameters, customer parameters and transportation parameters in this example 1.

As shown in Figure 6, the second facility is initially assigned to the first customer. However, due to the occurrence of a disruption in the second facility, refer to the fourth facility, and after the event of a disorder in the fourth facility, refer to dummy facility (facility 8) then return to its initial location. The customer reference to the eighth facility shows that the customer demand is not met, and a penalty cost is imposed on the system.

Additionally, the third facility is assigned to the second customer. After having a disruption in the third facility, the customer refers to the sixth facility, and after receiving services in the sixth facility, returns to its

TABLE 4. Facility parameters for test problem 1.

| | Facilities | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Parameters | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $f_j$ | 1280 | 1900 | 1430 | 2290 | 1020 | 1310 | 1420 | 1000 |
| $q_j$ | 1 | 0.5 | 0.5 | 1 | 1 | 0 | 1 | 0 |
| $cap_j$ | 200 | 500 | 420 | 500 | 340 | 410 | 250 | 200 |
| $S_j$ | 70 | 80 | 90 | 140 | 110 | 120 | 70 | 40 |
| $b_j$ | 110 | 100 | 120 | 150 | 160 | 150 | 130 | 120 |
| $v_j$ | 1 | 2 | 1.5 | 2 | 1 | 1.5 | 2 | 1 |
| $h_j$ | 2 | 4 | 3 | 4 | 2 | 3 | 2 | 1 |

TABLE 5. Customers parameters for test problem 1.

| | Customers | |
|---|---|---|
| Parameters | A | B |
| $\widetilde{d}_i$ | $(203, 210, 218)$ | $(266, 280, 293)$ |
| $\pi_i$ | 200 | 300 |

TABLE 6. Transportation parameters for test problem 1.

| $c_{ij}$ | Facility 1 | Facility 2 | Facility 3 | Facility 4 | Facility 5 | Facility 6 | Facility 7 | Facility 8 |
|---|---|---|---|---|---|---|---|---|
| Customer A | 20 | 15 | 19 | 20 | 19 | 18 | 12 | 200 |
| Customer B | 18 | 14 | 17 | 20 | 19 | 18 | 13 | 300 |

initial location. Failure to apply the customer to the eighth facility indicates that the customer has received the services they want, and there is no need for a penalty cost.

Figure 7 shows the different costs in the objective function, including the cost of establishing the facility, the cost of transportation, and the cost of inventory separately.

In the next instance, we look at an example with larger dimensions. The assumptions of the second example are the same as the first example. In this example, three customers and 12 facilities (along with a dummy facility) are considered. The results of the numerical example are shown in Figure 8. As shown, Facility 5 is assigned to the first customer, Facility 3 to the second customer, and Facility 4 to the third customer. The customer first goes to facility 8 and then to facility 10 due to disruption in facility 5, and returns to the initial location after receiving his services. The second customer, after the disruption of facility 3, refers to facility 6 to receive services. Finally, after not receiving services in the facilities allocated to it, customer 3 refers to facility 12 (dummy facility), which imposes a penalty cost on the system.

After verifying and validating our model using the exact method, we have used Red Deer Algorithm to solve the bigger size problems. To evaluate the accuracy and efficiency of the RDA algorithm we have compared its results with exact method's results. As shown in the following Table 7, the gap between the metaheuristic method and the exact method is promising, which lets us use it for the large size instances.

The results from solving all the defined test problems from Table 2 can be seen in Table 8.

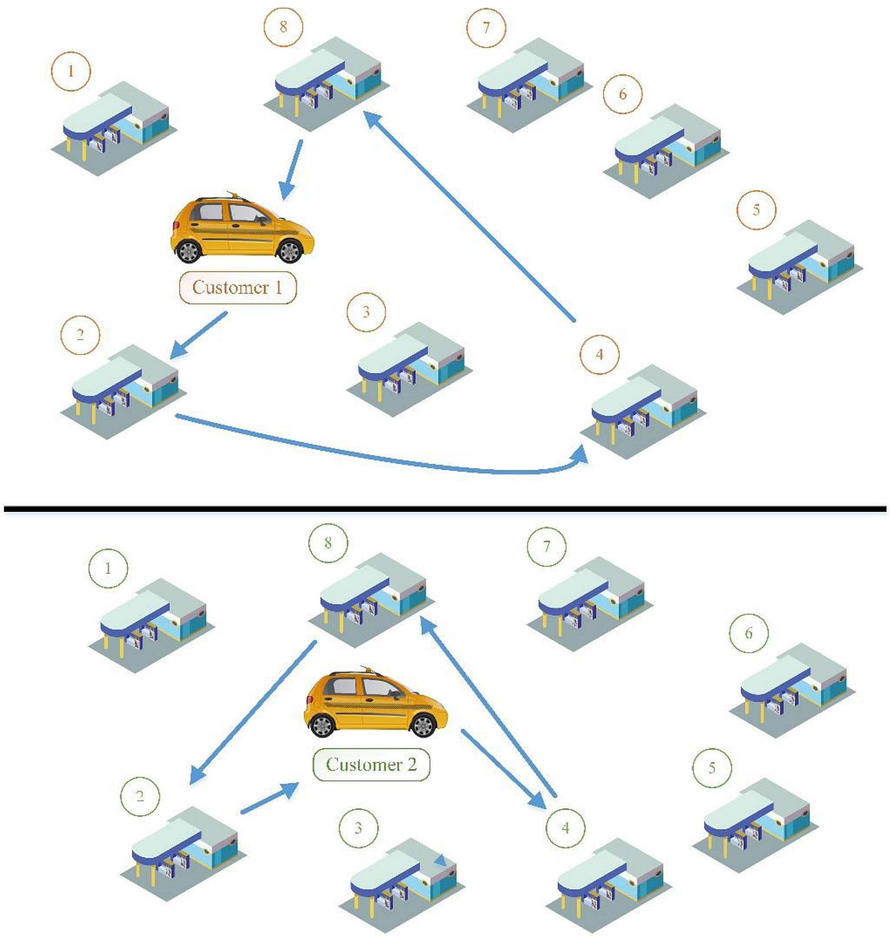


FIGURE 6. Results of example 1.

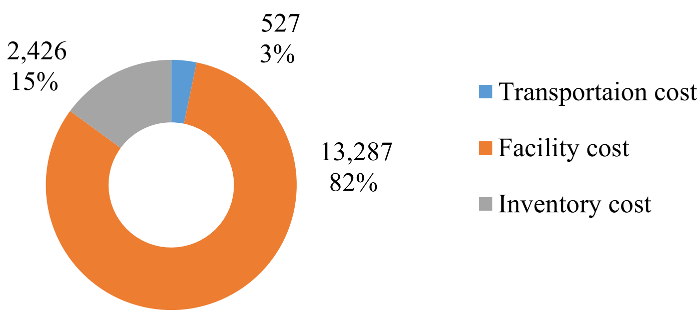


FIGURE 7. Values of different cost expressions for example 1.

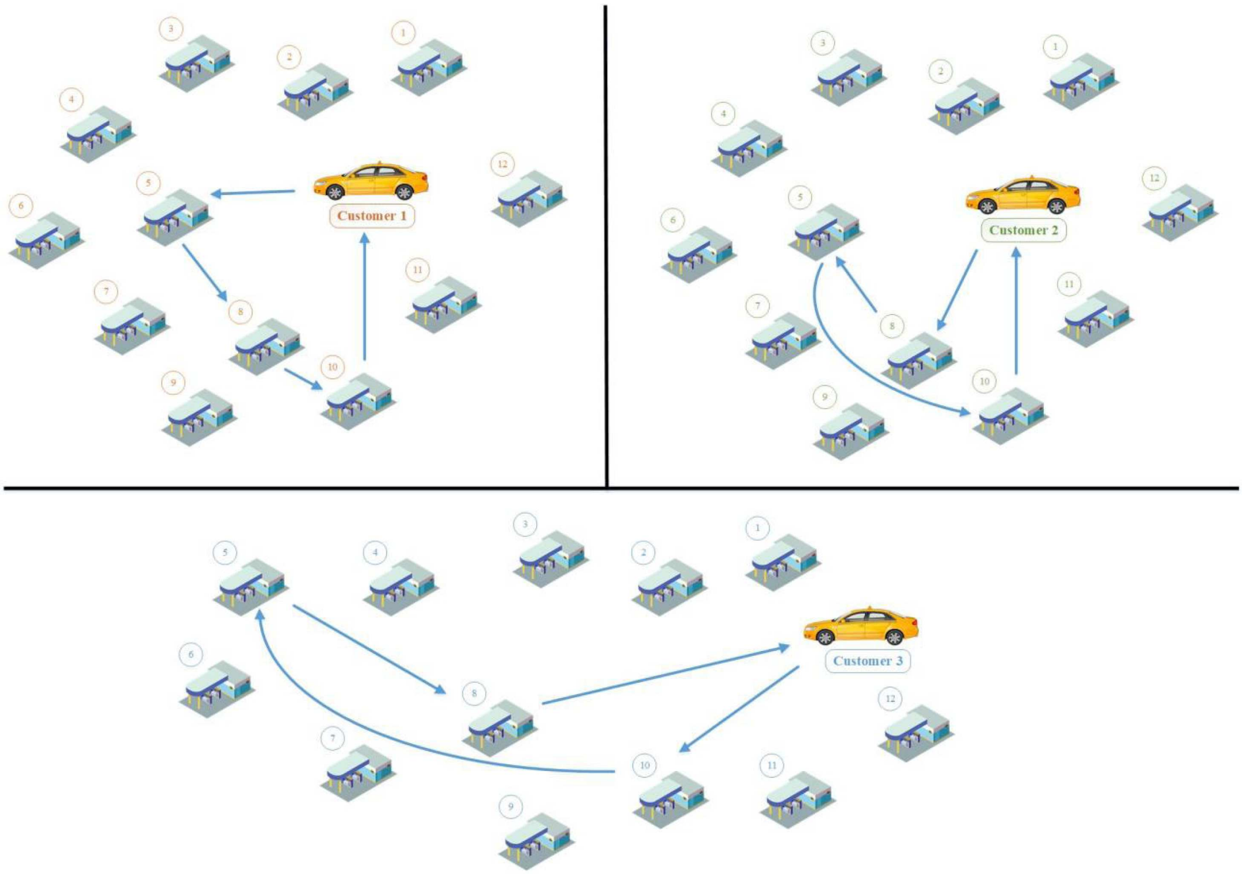


Figure 8. Results of example 2.

## 6. Sensitivity analysis and managerial insights

### 6.1. Sensitivity analysis

To perform the sensitivity analysis, we select five parameters such as the customer demand, capacity of facilities, disruption probability of facilities, maximum acceptable time threshold for service level, and unit-demand transportation cost and evaluate their impact on order quantity and objective function that including the total costs (TTL) such as total fixed cost of establishing the facilities (TFC), total inventory cost (TIC), total transportation cost (TTC), and total penalty cost (TPC).

The first parameter that is analyzed is the parameter related to the amount of customer demand. This parameter is essential in all models and issues because all systems worldwide are trying to meet demand. This section increases the expected value of the demand parameter in each step to examine the effect of changes related to this parameter on the objective function and various cost types. The results of changes in the demand parameter are shown in Table 9.

The demand sensitivity analysis of different cost types in the objective function is shown in Figure 9. As shown, if demand increases, the amount of transportation and inventory will increase. As a result, the costs associated with them also increase. Also, if demand increases, the need to establish new facilities to meet demand will increase. Of course, this happens if the facility's total vacant capacity is less than the increment in demand. As a result, the cost of establishing a facility is more sensitive to changes in demand than other costs.

TABLE 7. Results' coparison between exact and metaheuristic's method.

| Test problem number | Test problem size | Opened facilities (Exact) | Opened facilities (RDA) | Customers' routing (Exact) | Customers' routing (RDA) | Total cost (Exact) | Total cost (RDA) | Gap |
|---|---|---|---|---|---|---|---|---|
| 1 | Small | 1–3–7 | 0–1 | [1 3 7]<br>[3 7 1] | [1 0]<br>[0 1] | 16 240.03 | 16 457.65 | 1.34% |
| 2 | Medium | 4–7–9 | 0–1–9 | [4 7 9]<br>[7 9 4]<br>[9 4 7] | [1 9 0]<br>[9 1 0]<br>[0 1 9] | 19 833.11 | 20 257.54 | 2.14% |

TABLE 8. Results' coparison between exact and metaheuristic's method.

| Test problem number | Test problem size | Number of customers ($I$) | Number of facilities ($J$) | Opened facilities | Customers' routing | Total cost | Facility construction costs | Transportation and penalty costs | Holding, ordering and inventory costs |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Small | 2 | 8 | 0–1 | [1 0]<br>[0 1] | 16 457.64 | 3455 | 12 288.01 | 714.64 |
| 2 | Medium | 3 | 12 | 0–1–9 | [1 9 0]<br>[9 1 0]<br>[0 1 9] | 20 257.51 | 5932 | 13 152.11 | 1173.42 |
| 3 | Large | 3 | 18 | 9–12–14–17 | [12 9 14 17]<br>[9 12 17 14]<br>[14 17 9 12] | 29 076.923 | 7174 | 20 395.10 | 1507.81 |
| 4 | Large | 4 | 24 | 1–10–11–20 | [20 1 10 11]<br>[1 10 20 11]<br>[20 11 1 10]<br>[10 1 11 20] | 29 986.810 | 8659 | 19 715.15 | 1612.65 |
| 5 | Large | 5 | 28 | 8–11–15–20–21–24 | [20 11 21 24 8 15]<br>[21 24 20 15 8 11]<br>[20 24 21 8 11 15]<br>[15 11 8 20 24 21]<br>[21 8 11 20 24 15] | 50 743.140 | 11 577 | 36 704.34 | 2461.79 |

The second parameter that is analyzed to examine the effect of its changes on the objective function is the parameter related to the capacity of the facility. In this step, we increase the value of the parameter in steps. The results are shown in Table 10.

Different values for different types of costs in exchange for variable variation in facility capacity are shown in Figure 10. The results show that by increasing the established facilities' capacity, the system's total cost decreases. The cost associated with establishing the facility plays a vital role in reducing total costs. By increasing the facility's capacity, a facility can meet the demand of more customers, and the need to establish a new facility is reduced. Also, as facility capacity increases, inventory, transportation and penalty costs will almost remain the same.

In the following, we analyze the sensitivity of the disruption probability parameter for facility $j$ and its effect on the objective function and the ordering quantity. The results are shown in Table 11.

The results obtained for the objective function and the order value due to the exchange in the probability of disruption in facility $j$ are shown in Figure 11. As the results show, when the probability of disruption of a facility increases, transportation costs increase due to customers turning to more facilities to meet their demand. Also, due to the rise in the probability of disruption, the cost of establishing facilities will increase. On the other hand, inventory cost will decrease, and penalty cost will increase because customer demand will not be met.

TABLE 9. Results of customer demand parameter sensitivity analysis.

| Test number | Rang of fuzzy demand parameter | Expected value demand | Objective function |
|---|---|---|---|
| 1 | 210–280 | 245 | 5257 |
| 2 | 260–330 | 295 | 5583 |
| 3 | 310–380 | 345 | 5904 |
| 4 | 360–430 | 395 | 6251 |

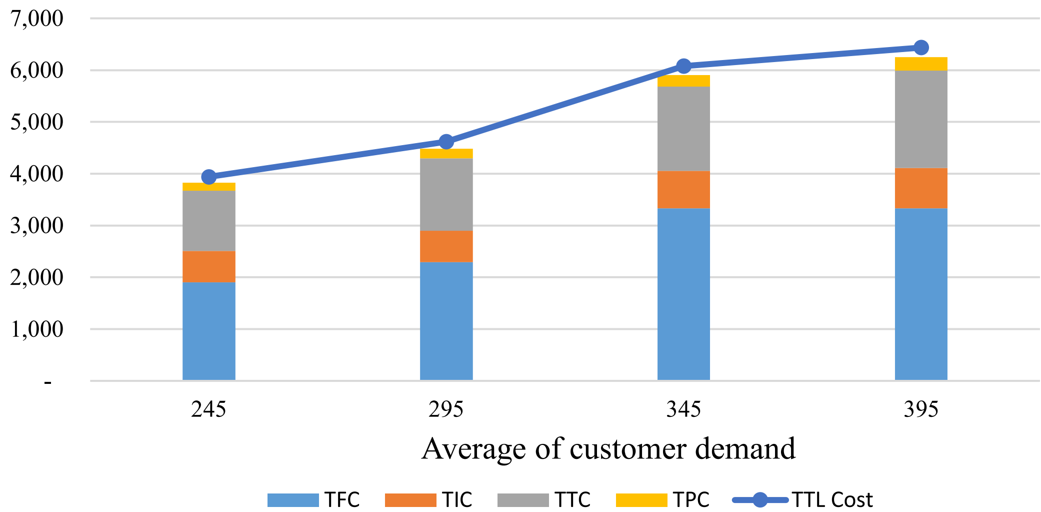


FIGURE 9. Customer demand parameter sensitivity analysis.

TABLE 10. Results of capacity of facility parameter sensitivity analysis.

| Test number | Rang of capacity | Average of capacity | Objective function |
|---|---|---|---|
| 1 | 200–500 | 350 | 5257 |
| 2 | 250–550 | 400 | 4334 |
| 3 | 275–575 | 425 | 3977 |
| 4 | 300–600 | 450 | 3448 |

TABLE 11. Results of disruption probability parameter sensitivity analysis.

| Test number | Average of probability | Objective function |
|---|---|---|
| 1 | 0.1 | 3041 |
| 2 | 0.3 | 4148 |
| 3 | 0.5 | 6158 |
| 4 | 0.7 | 8354 |

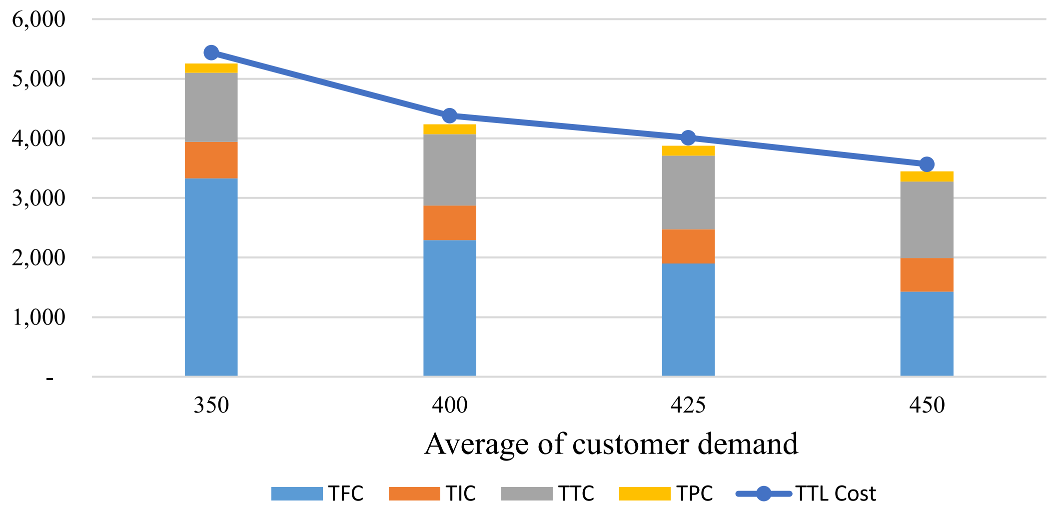


FIGURE 10. Capacity of facility parameter sensitivity analysis.

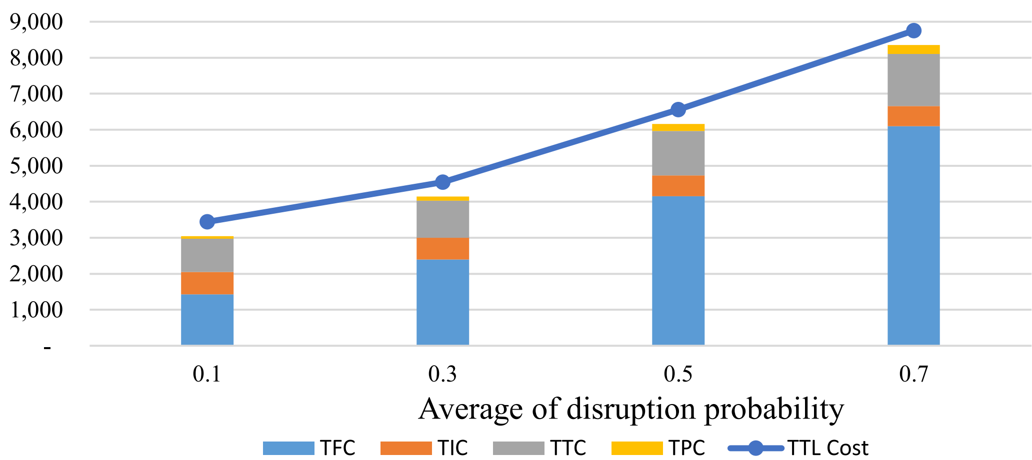


FIGURE 11. Disruption probability parameter sensitivity analysis.

As shown in Figure 12, as the probability of disruption in each facility increases, the order quantity in each facility decreases because it meets fewer customer demands. With an increase in the facility's disruption probability, the ordering quantity will decrease because higher disruptions in a facility make the model rely more on reliable facilities with lower disruptions. In that case, the model will cover demand mostly with reliable facilities, which causes the demand coverage of unreliable facilities to decrease and, consequently, lower the ordering quantity value.

The maximum acceptable time threshold for service level is the fourth parameter that is analyzed the effect of its changes on the objective function. In this step, we increase the value of the parameter in steps. The results are shown in Table 12.

The results obtained from the change value of the maximum acceptable time threshold for service level on the objective function are shown in Figure 13. If we want to increase customer satisfaction with the system's services, we must reduce the time of service and customer expectations in the network. By reducing the maximum

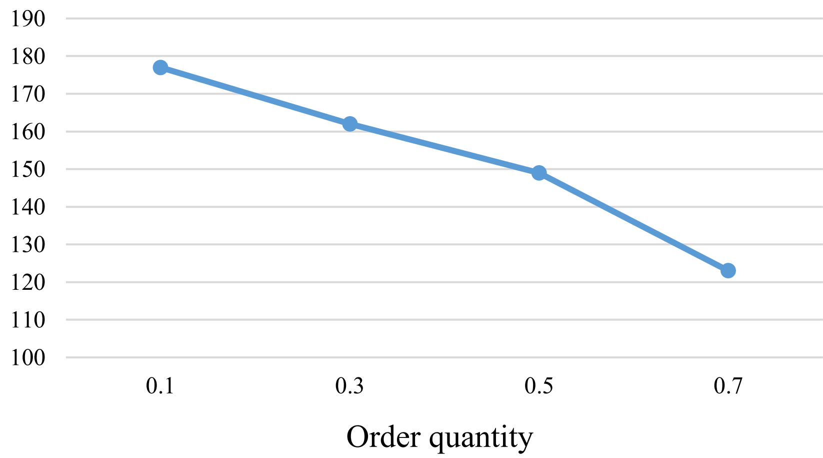


FIGURE 12. Impact of disruption probability parameter on order quantity.

TABLE 12. Results of maximum acceptable time threshold parameter sensitivity analysis.

| Test number | Rang of maximum acceptable time threshold | Average of maximum acceptable time threshold | Objective function |
|---|---|---|---|
| 1 | 600–700 | 650 | 6496 |
| 2 | 500–600 | 550 | 5454 |
| 3 | 400–500 | 450 | 4330 |
| 4 | 300–400 | 350 | 3440 |

acceptable time for service, the cost of transportation in the system is reduced, and establishing the facility's cost is increased so that customers can receive their service in the shortest possible time. Penalty cost is also reduced.

One of the interesting points in Figure 14 is the correlations between transportation cost and ordering quantity. More unit-demand transportation costs cause the model to decrease total transportation costs by reducing transportation units, lowering the number of facilities and even having higher unsatisfied demand and more shortage cost. Therefore, the ordering quantity will drop due to the effect of unit-demand transportation on-demand coverage.

## 6.2. Discussion and managerial insights

Most studies have only focused on the location problem of reliable systems, while this study considers the inventory problem within the typical reliable facility location problem and proposes a facility joint inventory-location problem. A minimum service level is considered because of the high importance of customer satisfaction. Proposing facilities' capacity and minimum economic demand limit for each facility in the mathematical model are also some of this research's contributions. Although most studies in this area have used deterministic parameters in the proposed model, this parameter is considered a fuzzy number due to demand's imperfect information and its uncertain nature. Additionally, few studies used metaheuristics algorithms to solve real-world

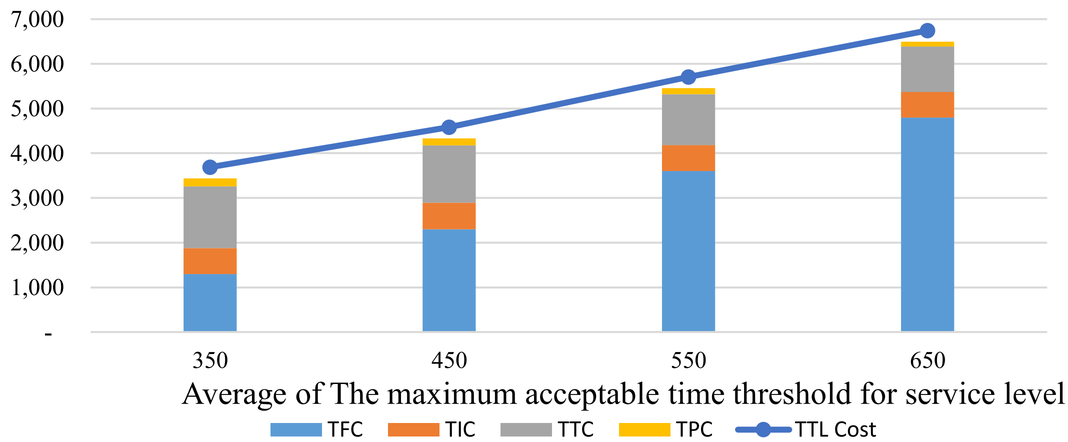


FIGURE 13. Maximum acceptable time threshold parameter sensitivity analysis.

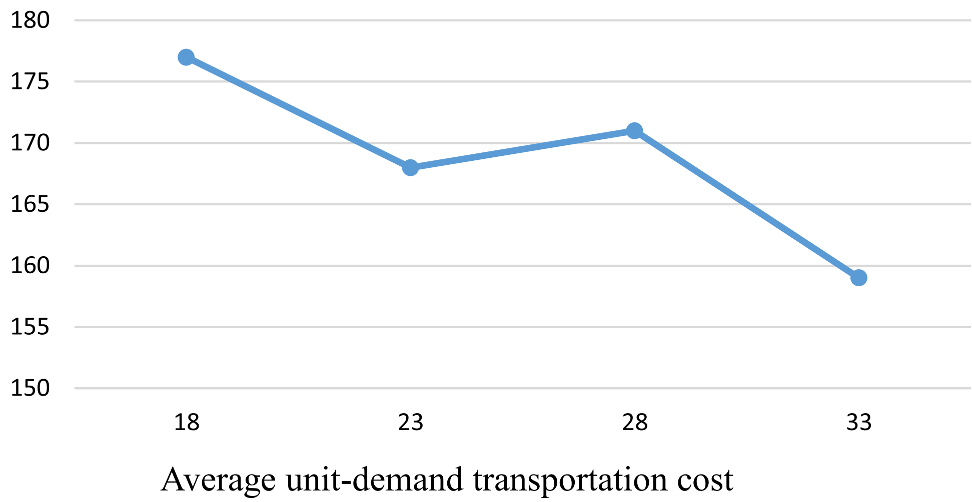


FIGURE 14. Impact of unit-demand parameter on order quantity.

size problems. We have taken care of large-size problems by solving the model using a recent metaheuristics method, the red deer algorithm.

Regarding the results from the first sensitivity analysis (demand parameter), the growth rate of construction (establishing) cost would be more than the growth rate of penalty costs when the demand increases. This happens because the model's defined penalty cost is high, and the model prefers to establish an additional facility instead of not meeting customers' demands. A high value for penalty cost was being used in the model. Due to the new era's competitive economy and business, all companies are customer-oriented. Therefore, unsatisfying a customer's need will cause a huge amount of costs in the long run. Therefore, each business should consider its future demand and growth over time before constructing its facilities and starting its business. When the demand exceeds the business's capacity, they can add more facilities or increase their existing facilities' capacity. The same thing can be seen in the fourth sensitivity analysis. As the customers are the most critical stakeholder for each business, satisfying customers means providing higher and better service levels which causes more construction costs. If managers aim to acquire more customers, they need to improve service level, which means they have to invest more and increase their costs. Therefore, it is rational to calculate the opportunity cost of this decision first and evaluate whether the added value of acquiring more customers can outweigh the added cost or not before taking any action.

As suggested by the second sensitivity analysis (capacity of facilities), if we use the second approach, it takes care of customers' demands. It decreases the construction of facilities costs, which results in a reduction in total expenses. As the model suggests, if businesses do not consider increasing their capacity while their demand is rising, they will face a lot more costs due to unsatisfied customers. Based on the third sensitivity analysis, we witnessed that having lower disruption rates leads to huge money saving and huge cost reductions. This analysis definitely will show the importance of preventive maintenance. Although preventive maintenance may sound like an additional cost for the company, it will reduce costs.

Additionally, this sensitivity analysis suggests that to optimize the costs of the system, managers and decision-makers should rely more on reliable facilities. If there is an unreliable facility, it is better to reduce its disruptions before increasing orders cover a high range of customers' demand. Also, due to the high sensitivity of total costs to disruption rate changes, it is probably better for management to assign a separate budget to deal with the disruptions.

The most crucial managerial insight from all these analyses was that managers and leaders should concentrate more on financial funding to establish new facilities costs than other costs when facing demand uplift.

## 7. Conclusions

Customers are the most critical stakeholder in each company, and the main goal of all systems and supply chains is to meet customers' demand. One of the factors in not meeting these demands is the occurrence of disruption. This article presented a reliable joint inventory-location problem with round-trip transportation and imperfect information of disruption consideration. This model's objective function was to minimize the system's total costs, including the cost of establishing the facility, transportation, inventory (holding and ordering costs), and penalty cost of unmet demand.

In this model, facilities are allocated to customers in advance. As mentioned before, disruption is inevitable, so the possibility of disruption of the facility is considered beforehand, and these disruptions are site-dependent. As customers do not have information about the facility's disruption, in case of disruption in the facility allocated to the customer, the customers refer to the next facility assigned to them. If any of the facilities do not meet the customer's request, the cost will be imposed on the system as a penalty cost. Customers, in the end, whether they have met their demand or not, will return to their initial location. In the real world, facilities have a limited and different capacity, which is included in this model. A minimum demand threshold has been set for each facility establishment, and a minimum service level has been set to take care of customers' satisfaction. Demand is also considered as a fuzzy number, and fuzzy programming and linearization methods are being used to convert the fuzzy MINLP to a crisp ILP.

The model presented in this article is NP-hard. One of the meta-heuristic methods called red deer algorithm (RDA), has been used to solve the model. In this article, several different numerical examples in different dimensions are examined. This algorithm is suitable for medium and large-scale. Finally, we investigated the effect of parameters on the results and sensitivity analysis. One of the sensitivity analysis results was that customers' demands should be forecasted in advance before taking any action or constructing any facilities. If the predicted customers' demand is underestimated or invalid or the company provides fewer facilities than the actual demand, the unmet demand's penalty cost will be higher than other costs. It will negatively affect the business in the long run. The next important thing which is discussed in the managerial insight part is the importance of preventive maintenance.

The lack of information on demand also limits the study. Data mining and deep learning algorithms were being used more often to predict and estimate customers' needs and demand more accurately in the last decade.

We suggest some directions for future research:

(1) Develop the model by considering other types of disruption, such as transportation disruption that customer uses it.
(2) Combining the model presented in this article with other supply chain problems such as pricing, routing, maintenance.
(3) Considering a queueing-inventory model in the facilities to serve the arrival customers [3,4].
(4) To tackle the data's uncertainty issue, introduce the model to other uncertain environments like robust or probabilistic methods.
(5) Use machine learning and time-series approaches to estimate customers' demand.

# Appendix A.

**Linearization**

The model is a mixed non-linear integer programming and it is difficult to solve it with commercial solvers, so after presenting the model, the method for linearization of the model is performed so that it can be easily solved with the help of commercial solvers like GAMS and LINGO.

According to the model presented in the previous Section 3.4, the expression $p_{ijj'r} \times y_{ijj'r}$ in objective function and the constraints has led to the non-linearization of the model of MINLP. Therefore, we linearize the model with the method.

We assume $Y$ is a binary variable and $X$ is a non-negative continuous variable between 0 to 1 and we define binary variable $W$ as follows:

$$W = X \times Y.$$

Then we define these constraints:

(1) $W \leq X$.
(2) $W \leq Y$.
(3) $W \geq X + Y - 1$.
(4) $W \geq 0$.

As a result, with the help of the structure presented above, the expression $p_{ijj'r} \times y_{ijj'r}$ can be replaced by the following expressions:

$$w_{ijj'r} = p_{ijj'r} \times y_{ijj'r}.$$

(1) $w_{ijj'r} \leq p_{ijj'r}$.
(2) $w_{ijj'r} \leq y_{ijj'r}$.
(3) $w_{ijj'r} \geq p_{ijj'r} + y_{ijj'r} - 1$.
(4) $w_{ijj'r} \geq 0$.

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