

A PROFIT MAXIMIZATION SINGLE ITEM INVENTORY PROBLEM CONSIDERING DETERIORATION DURING CARRYING FOR PRICE DEPENDENT DEMAND AND PRESERVATION TECHNOLOGY INVESTMENT

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Abstract. This paper addresses a single item two-level supply chain inventory model considering deterioration during carrying of deteriorating item from a supplier's warehouse to a retailer's warehouse as well as deterioration in the retailer's warehouse. The model assumes preservation technology in the retailer's warehouse to prevent the rate of deterioration. An upper limit for the preservation technology investment has been set as a constraint to the model. The model maximizes the retailer's profit per unit time, simultaneously calculated optimal order quantity. A price dependent demand and storage-time dependent holding cost is considered to develop the model. Some theorems are proven to get optimal values of the total cost. A numerical problem is worked out as per the developed algorithm and with the help of MATLAB software to study the applicability of our theoretical results.

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1. INTRODUCTION

Basic inventory models are classified into two types such as Economical Order Quantity (E.O.Q.) and Economical Production Quantity (E.P.Q.). In E.O.Q. model, we optimize order quantity to maximize the total profit of the retailer or minimize total cost. In this regard, first inventory model was introduced by Harris [15]. There are many developments on the basic E.O.Q model as of now considering different realistic assumptions which we will discuss in the literature review section.

We know that deterioration plays an important role in an inventory model as if the deterioration rate increases then the profit of a retailer decreases and if the deterioration rate decreases then profit of a retailer increases that is profit varies dis-proportionally with the deterioration rate. So, we can not ignore the deterioration rate in the present study of an inventory model. The deterioration is generally suitable for the items such as raw food items (fruit, vegetables, fishes, meat, eggs, etc.), processed food items, grocery items (salt, sugar, etc.), medical items (Bloods, medicine, vaccine, etc.), radioactive elements, alcohol etc.

This paper is developed considering different types of realistic assumptions. One such realistic assumption is deterioration during carrying. During carrying some quantity of the total order is spoiled because of many reasons. Some of the reasons may be long journey of the carrying vehicle (This type of reason is suitable for

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radioactive elements, blood, medicine, vaccine, vegetables, fruits, etc.), weather during carrying (This type of reason is suitable for salt, sugar, eggs, fruit, vegetable, meat, fish, etc.), carelessness during loading and unloading (It's the main reason for untrained labor force and this type of reason is suitable for any type of product), etc.

Preservation technology is very much useful to increase products lifetime. Also, during the carrying time, we can not apply preservation technology for more lifetime of the product but in the retailer's warehouse we can apply preservation technology to prevent deterioration. So, deterioration rate of the item during carrying is higher than the deterioration rate in the warehouse.

Another realistic assumption is retailer can sell the defective products in a certain price (which is less than the buying cost) to reduce the loss. This phenomena is used in our model. Retailer can use the equipment (for example – refrigeration equipment) to increase the self life of the item in retailer's warehouse after the item arrives. Thus, retailer has to invest some amount of money to operate the equipment for preservation; which is called preservation technology investment. If preservation technology investment increases then the deterioration rate decreases and if preservation technology investment decreases then deterioration rate increases that is preservation technology investment varies dis-proportionally with the deterioration rates. Also, retailer's can not invest most of the money in preservation technology and hence in this paper we consider an upper bound for the preservation technology investment.

In most cases, tendency of a customer is to buy a product in lesser price. So, if price of an item goes down then demand of the item increases. In this paper we have considered that the demand of an item is depending on the selling price and it varies dis-proportionally with the selling price of an item. Another realistic assumption is that the holding cost varies proportionally to time due to the rent of the warehouse that is if stock time increases then the holding cost increases and if stock time decreases then holding cost decreases. Now we will discuss on the various inventory models developed during the past years in literature review section.

2. LITERATURE REVIEW

Mondal *et al.* [27] first introduced a model for an ameliorating item with demand of the product dependent on price. Then, Mukhopadhyay *et al.* [28] proposed an ordering policy on pricing inventory model for deteriorating items. Later, Mukhopadhyay *et al.* [29] modified their previous problem by introducing the deterioration rate as weibull distribution and price dependent demand. Roy and Chaudhari [33] formulated a model for a deteriorating item and demand depends on price with special sale of the product and later Roy [32] modified his previous problem by incorporating time dependent holding cost. Then Maiti *et al.* [24] formulated an inventory problem with price dependent demand and stochastic lead time in advance payment system. Next, Sridevi *et al.* [40] built a price dependent model for deteriorating items with weibull rate of replenishment. Then, Sana [35] formulated an inventory model for perishable items with price-sensitive demand. Maihaimi and Kamalabadi [23] formulated a model with time and price dependent demand for non-instantaneous deteriorating items on jointly time and price. Then, Avinadav *et al.* [4] introduced a supply chain problem for perishable items. Then, Bhunia and Shaikh [5] formulated an inventory problem for deteriorating items with selling price dependent demand and three-parameter weibull distribution. Next, Ghorieshi *et al.* [14] formulated an inventory model with price dependent demand and customer returns for non-instantaneous deteriorating items with partial backlogging. Then, Alfares and Ghaitan [2] formulated quantity discount offered to the customer with time dependent holding cost. Then, Jaggi *et al.* [18] formulated an inventory model for demand dependent on price with credit financing in two storage facilities and non-instantaneous deterioration. Shaikh *et al.* [36] developed an inventory model with variable demand dependent on price for three-parameter Weibull distributed deteriorating item. Next, Dey *et al.* [9] formulated an integrated inventory model with price-dependent demand and discrete setup cost reduction. Then, Khanna *et al.* [20] considered non-linear price-dependent demand for an inventory model with inspection error. In this direction, Gautam *et al.* [12] developed an inventory model with price-dependent demand for defective items.

An appreciable amount of research paper has been published on inventory control models for deteriorating products. Ghare and Schrader [13] first introduced the concept of deterioration. Then Philip [30] has extended

from constant deterioration to three parameters weibull deterioration. Thereafter a lot of researches have been modified by several authors with different types of deterioration. Regarding this context, to reduce deterioration effect Hsu *et al.* [17] first introduced the preservation technology in his research paper. They introduced an inventory model for deteriorating inventory with preservation technology investment. Next Dye [8] developed a model for non-instantaneous deteriorating item with preservation technology investment. Zhang *et al.* [43] formulated an inventory model with stock dependent demand and preservation technology investment for deteriorating inventory model. Mishra *et al.* [26] formulated an inventory model under price and stock dependent demand for deteriorating item with shortage and preservation technology investment. Then Shaikh *et al.* [39] investigated preservation technology for deteriorating item and time dependent demand with partial backlogging. Li *et al.* [22] considered preservation technology investment for non-instantaneous deteriorating item and replenishment. Aditi Khanna *et al.* [21] formulated an inventory model for deteriorating items with stock-dependent demand and time-dependent holding cost. Yongrui and Yu Cao [10] developed an inventory model with stochastic demand with reference price effect for the deteriorating product. Next, Mashud *et al.* [25] considered a joint pricing inventory model with price-dependent demand and time-dependent deterioration rate under a discount facility system. Also, Asim Paul *et al.* [3] formulated an inventory model for a deteriorating product with price-sensitive demand and discussed the effect of default risk on optimal credit period in the inventory model. Then, Bashair Ahmad and Lakdere Benkherouf [1] derived an optimal replenishment policy inventory model for the deteriorating product with stock-dependent demand and partial backlogging. Then, Rout *et al.* [31] formulated a production inventory model for deteriorating items with constant deterioration rate and backlog-dependent demand. After that, Ali Akbar Shaikh *et al.* [37] developed an inventory model for the deteriorating product with constant deterioration rate and price dependent demand also discussed the decision support system for customers under trade credit policy. Then, Choudhury and Mahata [7] developed an inventory model for growing deteriorating items with price-dependent demand.

Many inventory model developed assuming the holding cost per unit is not constant. Ferguson *et al.* [11] introduced an inventory model with holding cost as non-linear dependence on the storage time. Next San-Jose *et al.* [34] developed an E.O.Q. model with the holding cost function having two components: a fixed cost and a variable cost that increases with storage time and partial backlogging. Then Alfares and Ghaithan [2] formulated an inventory and pricing model with time varying holding cost, price dependent demand and quantity discount. Then, Ali-Akbar Shaikh [38] formulated an E.O.Q. model for time dependent holding cost and price discount facility with stock dependent demand.

3. RESEARCH GAPS AND OUR CONTRIBUTIONS

The main highlights of our contribution in this paper are

- Table 1 depicts that several researchers considered deterioration and preservation technology in the retailer's warehouse but no one considered deterioration during carrying of an item from supplier warehouse to

TABLE 1. The comparison between earlier published work and our present work.

Source	Deteriorating items	Demand	Holding cost	Preservation technology	Deterioration during carrying
Maiti <i>et al.</i> [24]	No	Price dependent	Constant	NO	NO.
He and Huang [16]	Yes	Price dependent	Constant	Yes	NO.
Zhang <i>et al.</i> [43]	Yes	Price dependent	Constant	Yes	NO.
Tayal <i>et al.</i> [42]	No	Price and exponential rate	Constant	NO	NO.
Taleizadeh <i>et al.</i> [41]	No	Price dependent	Constant	No	NO.
Alfares and Ghaithan [2]	No	Price dependent	Time dependent	No	NO.
Mishra <i>et al.</i> [26]	Yes	Price dependent	Constant	Yes	NO.
Al-Amin Khan <i>et al.</i> [19]	Yes	Price dependent	Time dependent	No	NO.
Ali Akbar Shaikh <i>et al.</i> [38]	Yes	Price and stock dependent	Time dependent	No	NO.
Aditi Khanna <i>et al.</i> [21]	Yes	Stock dependent	Time dependent	Yes	NO.
Present paper	Yes	Price dependent	Time dependent	Yes	Yes.

retailer's warehouse. This model fulfills this gap of research considering different deterioration rate for two intervals. This model can be consider as a generalization of the existing work on deterioration and preservation technology.

- Many authors considered that the preservation technology function ($\lambda(\alpha)$) dependent on initial deterioration rate. But during transportation of the items from the supplier warehouse to the retailer's the items has to survive various extreme condition (such as weather during transportation, road condition, etc.) and once the items are arrived at retailer's warehouse the items will be in under preservation technology system, which is a stable system. So, the deterioration rate during carrying and deterioration rate under preservation technology can not be always dependent. So, here we consider the deterioration rate under preservation technology ($\lambda(\alpha)$) is an independent function of initial deterioration rate (θ) and it is define by

$$\lambda(\alpha) = e^{-\eta\alpha}$$

where η is the sensitivity parameter of investment to the deterioration rate and α is the preservation technology investment per unit time and $\lambda(\alpha) \ll \theta$.

- To prove the optimality of the total cost function in this paper we derive two theorems and by using the results 3.2.1 & 3.2.10 from Combini and Martein [6] we prove those theorems analytically.
- Here to handle the numerical problem we modify an algorithm from Khan *et al.* [19] and the modified algorithm is suitable for any similar type of model, which is considered in this paper.

4. NOTATION AND ASSUMPTIONS

The following assumptions and notation have been used to formulate the mathematical model.

4.1. Notation

Tables 2 and 3 describes the notation for the developed model.

4.2. Assumptions

Our inventory model is established based on the following assumptions:

- (1) The model is developed for a single deteriorating item.
- (2) We consider the deterioration during carrying. We assume that during carrying the deterioration rate is θ ($0 < \theta \ll 1$) is constant.
- (3) We assume that the retailer will sell the defective item with a price P ; Which is less than the purchasing cost c per unit item.
- (4) Any replacement or repair for the deteriorated products is not considerable during the cycle length T .
- (5) We know that if the price of a product is low then demand appears highly in the market. From this observation, we assume that the price is dependent on demand same as Alfares and Ghaithan [2], which is expressed below

$$D(p) = \delta_1 - \delta_2 p$$

where δ_1 is the constant amount demand of the item when selling price $p = 0$ and δ_2 is a constant such that $\delta_1 - \delta_2 p > 0$.

- (6) In the retailer's warehouse inventory level is gradually decreases because of the demand D and reduced deterioration rate due to the investment on preservation technology is

$$\lambda(\alpha) = e^{-\eta\alpha}$$

where η is the sensitivity parameter of investment to the deterioration rate and α is the preservation technology investment per unit time and $\lambda(\alpha) \ll \theta$. The relationship between deterioration rate and preservation technology investment parameter are

$$\frac{\partial \lambda(\alpha)}{\partial \alpha} < 0$$

TABLE 2. Notation.

Notation	Units	Description
δ_1	Constant	Constant part of demand rate ($\delta_1 > 0$).
δ_2	Constant	Co-efficient of the price in the demand rate ($\delta_2 > 0$).
p	\$/Unit	Selling price per unit item.
c	\$/Unit	Purchasing cost per unit item.
C	\$/Unit	Ordering cost per unit item.
θ	Constant	Deterioration rate at during carrying.
g	\$/Unit	Constant part of holding cost.
h	\$/Unit	Co-efficient of linearly time dependent holding cost.
P	\$/Unit	Selling price of the defective item.
$\lambda(\alpha)$	Constant	the proportion of reduced deterioration rate, $0 \leq \lambda(\alpha) \leq 1$.
$q(t)$	Units	Inventory level at a time t .
t_1	Week	Time at which the stock arrived at the retailer's warehouse.
T	Week	Length of each replenishment cycle.
Q	Units	The number of order placed per cycle.
T	Week	Length of each replenishment cycle.
α	\$/Unit time	the preservation technology investment per unit time.
M	Units	Inventory level at the time $t = t_1$.
S	Units	Defective product quantity due to carrying.
T^*	Week	Optimal length of the cycle.
p^*	\$/Unit	Optimal selling price.
Q^*	Unit	Optimal Order quantity.
S^*	Unit	Optimal quantity of defective items due to carrying.
M^*	Unit	Optimal quantity arrived at retailer's warehouse.
η	Constant	the sensitivity parameter of investment to the deterioration rate.
$\bar{\alpha}$	\$/Unit time	the maximum investment cost in preservation technology.

TABLE 3. Decision variables.

Notation	Units	Description
p	\$/Unit	Selling price per unit item.
T	Week	Length of each replenishment cycle.

and

$$\frac{\partial^2 \lambda(\alpha)}{\partial \alpha^2} < 0.$$

(7) In our model preservation technology investment per unit time is α and it satisfies the condition

$$0 \leq \alpha \leq \bar{\alpha}$$

where $\bar{\alpha}$ is the maximum investment on preservation technology.

(8) Similar to Alfares and Ghaithan [2], we consider storage time-dependent holding cost. The holding cost of the products increases linearly concerning the storage time of each unit and it's proportional to the purchase cost c per unit item. Also, the holding cost contained two constant parts, one is the g and another one is h . So, the holding cost function can be expressed as

$$H(t) = c(g + ht).$$

5. MATHEMATICAL MODEL

The retailer ordered Q units of deteriorating item. Therefore, the inventory level at $t = 0$ is Q . During carrying from the supplier's warehouse to retailers warehouse, the stock level gradually declines due to the deterioration rate θ and drops at M at the time $t = t_1$. In the retailer's warehouse inventory level gradually decreases because of the demand (D) and the reduced deterioration rate due to the investment on preservation technology ($\lambda(\alpha)$). To prevent deterioration in the retailer's warehouse we applied preservation technology (Fig. 1).

The demand function is $D(p) = \delta_1 - \delta_2 p$. Based on above description, the inventory at any time $t \in [0, T]$ is given by the differential equation:

$$\frac{dq_1}{dt} + \theta q_1 = -(\delta_1 - \delta_2 p), \quad 0 \leq t \leq t_1 \quad (5.1)$$

$$\frac{dq_2}{dt} + \lambda(\alpha) q_2 = -(\delta_1 - \delta_2 p), \quad t_1 \leq t \leq T. \quad (5.2)$$

Equations (5.1) and (5.2) can be rewritten as

$$\frac{dq_1}{dt} + \theta q_1 = -D, \quad 0 \leq t \leq t_1 \quad (5.3)$$

$$\frac{dq_2}{dt} + \lambda(\alpha) q_2 = -D, \quad t_1 \leq t \leq T \quad (5.4)$$

where $D = \delta_1 - \delta_2 p$. Also $q_1(t_1) = M (< Q)$. Now after solving equation (5.3) by using the boundary condition $q_1(t_1) = M$, we get the inventory level at any time $t \in [0, t_1]$ and is given by

$$q_1(t) = -\frac{D}{\theta} + \left(M + \frac{D}{\theta} \right) e^{\theta(t_1-t)}. \quad (5.5)$$

The solution of equation (5.4) by using $q_2(T) = 0$ implies the inventory level at any time $t \in [t_1, T]$ is given by

$$q_2(t) = \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t)} - 1 \right). \quad (5.6)$$

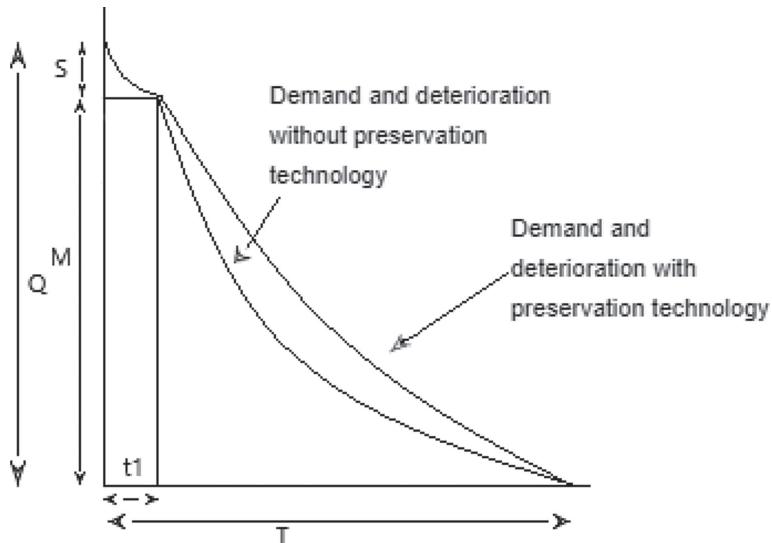


FIGURE 1. Pictorial presentation of the proposed problem.

Using the continuity at $t = t_1$, we get

$$M = \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right). \quad (5.7)$$

Also initially $q_1(0) = Q$. Therefore from the equation (5.5) we get

$$\begin{aligned} Q &= -\frac{D}{\theta} + \left(M + \frac{D}{\theta} \right) e^{\theta t_1} \\ &= -\frac{D}{\theta} + \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{D}{\theta} \right\} e^{\theta t_1}. \end{aligned} \quad (5.8)$$

We know that $S = Q - M$. Therefore,

$$S = \left[-\frac{D}{\theta} + \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{D}{\theta} \right\} e^{\theta t_1} \right] - \left[\frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) \right]. \quad (5.9)$$

The profit function of the inventory system involves the following components:

- (1) Ordering Cost (OC) = C .
- (2) Holding Cost (HC) = $\int_{t_1}^T c(g + ht) q_2(t) dt$

$$\begin{aligned} &= c \int_{t_1}^T (g + ht) \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t)} - 1 \right) \right\} dt \\ &= \frac{cDg}{\lambda(\alpha)} \int_{t_1}^T \left(e^{\lambda(\alpha)(T-t)} - 1 \right) dt + \frac{cDh}{\lambda(\alpha)} \int_{t_1}^T t \left(e^{\lambda(\alpha)(T-t)} - 1 \right) dt \\ &= \left[\frac{cDg}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + (t_1 - T) \right\} + \frac{cDh}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) \right] \\ &\quad + \left[\frac{cDh}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)^2} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} \right]. \end{aligned}$$

- (3) Purchasing Cost (PC) = cQ .

$$= c \left[-\frac{D}{\theta} + \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{D}{\theta} \right\} e^{\theta t_1} \right].$$

Defective product quantity due to carrying is $S = Q - M$, which is given by the equation (5.9).

We can sell the defective product with lesser price P . Therefore the revenue by selling defective item is $= P(Q - M) = PS$.

- (4) Sale Revenue (SR) = $p \int_{t_1}^T (\delta_1 - \delta_2 p) dt + PS$

$$\begin{aligned} &= p(\delta_1 - \delta_2 p)(T - t_1) + PS \\ &= p(\delta_1 - \delta_2 p)(T - t_1) + P \left[-\frac{D}{\theta} + \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{D}{\theta} \right\} e^{\theta t_1} - \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) \right] \\ &= p(\delta_1 - \delta_2 p)(T - t_1) - \frac{PD}{\theta} + P \left\{ \frac{D}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right) + \frac{D}{\theta} \right\} e^{\theta t_1} - \frac{PD}{\lambda(\alpha)} \left(e^{\lambda(\alpha)(T-t_1)} - 1 \right). \end{aligned}$$

We assumed that cost of preservation technology investment per unit item is α .

- (5) Preservation Technology Cost (PTC) = $\alpha(T - t_1)$.

Therefore the profit function is given by

$$\begin{aligned} TP(p, T) &= \frac{1}{T} [SR - OC - PC - HC - PTC] \\ &= \frac{1}{T} \left[pD(T - t_1) - \frac{PD}{\theta} + P \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} e^{\theta t_1} - \frac{PD}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) \right. \\ &\quad \left. - C + \frac{cD}{\theta} - ce^{\theta t_1} \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} - \frac{cDg}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + (t_1 - T) \right\} \right. \\ &\quad \left. - \frac{cDh}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) - \frac{cDh}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)^2} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} - \alpha (T - t_1) \right]. \end{aligned}$$

Our target is to find the optimal selling price p^* per unit item and the length of the cycle T^* to maximize the retailer's profit per unit time $TP(p, T)$.

6. THEORETICAL RESULTS

We formulated the concavity of the objective function for the above problem. To analyze the concavity for the models, we used some results from Combini and Martein [6]. Depending on the Theorems 3.2.9 and 3.2.10 in Combini and Martein [6], we know that the function of the form

$$\gamma(x) = \frac{f(x)}{g(x)}; \quad x \in \mathbb{R} \quad (6.1)$$

is strictly pseudo-concave if $f(x)$ is differentiable, non-negative and strictly concave function and $g(x)$ is positive, convex and differentiable function.

To represent the optimality of our problem by using the above results first, we determined the optimal value of p^* then we calculated the optimal value of replenishment T^* , which maximizes the retailer's total profit per unit time using the optimal selling price value p^* .

Theorem 6.1. *For a fixed $p > 0$, $TP(p, T)$ is a pseudo-concave function of T . Hence, there exist a unique T (Say T^*) such that $TP(p, T)$ attains the maximum value.*

Proof. As we consider the value of $p > 0$ is fixed. So, $TP(p, T)$ becomes function of T . □

Take,

$$TP(p, T) = \frac{f_1(T)}{g_1(T)}$$

where $g_1(T) = T$ and

$$f_1(T) = \left[pD(T - t_1) - \frac{PD}{\theta} + P \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} e^{\theta t_1} - \frac{PD}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) \right. \\ \left. - C + \frac{cD}{\theta} - ce^{\theta t_1} \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} - \frac{cDg}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + (t_1 - T) \right\} \right. \\ \left. - \frac{cDh}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) - \frac{cDh}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)^2} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} - \alpha (T - t_1) \right]$$

Aim: (i) $f_1(T)$ is strictly concave and (ii) $g_1(T)$ is convex.

Also,

$$f_1''(T) = \left[\begin{aligned} & - (c - P) D \lambda(\alpha) e^{\theta t_1} e^{\lambda(\alpha)(T-t_1)} - P \lambda(\alpha) D e^{\lambda(\alpha)(T-t_1)} - c D g e^{\lambda(\alpha)(T-t_1)} \\ & - \frac{c D h}{\lambda(\alpha)} \{ (e^{\lambda(\alpha)(T-t_1)} - 1) + t_1 \lambda(\alpha) e^{\lambda(\alpha)(T-t_1)} \} \end{aligned} \right]$$

Since we know that $c > P$. So, $(c - P) D \lambda(\alpha) e^{\theta t_1} e^{\lambda(\alpha)(T-t_1)} > 0$.

Also as $(e^{\lambda(\alpha)(T-t_1)} - 1) > 0$, this implies $\{(e^{\lambda(\alpha)(T-t_1)} - 1) + t_1 \lambda(\alpha) e^{\lambda(\alpha)(T-t_1)}\} > 0$
 $\therefore f_1''(T) < 0$.

Therefore, $f_1(T)$ is strongly concave function for all $p > 0$. Also, $f_1(T)$ is a positive differentiable concave function of T . Additionally, $g_1(T) = T$ is a differentiable convex function and $g_1(T) > 0$.

Hence, for any fixed p the total profit function $TP(p, T)$ is strongly pseudo-concave function of T . So, there exists an unique T^* such that $TP(p, T)$ attains the maximum value.

Theorem 6.2. *For any specified value of the cycle length $T > 0$, $TP(p, T)$ is a concave function of p . Hence there exists a unique p (say p^*) such that $TP(p, T)$ attains the maximum value.*

Proof. As we consider the value of $T > 0$ is fixed. So, $TP(p, T)$ becomes function of p . \square

Take,

$$TP(p, T) = f_2(p)$$

$$\therefore f_2'(p) = \frac{1}{T} \left[\begin{array}{l} (\delta_1 - 2\delta_2 p)(T - t_1) + \frac{\delta_2 P}{\theta} \\ -Pe^{\theta t_1} \left\{ \frac{\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{\delta_2}{\theta} \right\} + \frac{P\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) - \frac{c\delta_2}{\theta} + \\ ce^{\theta t_1} \left\{ \frac{\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{\delta_2}{\theta} \right\} + \frac{c\delta_2 g}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + (t_1 - T) \right\} \\ + \frac{c\delta_2 h}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) + \frac{c\delta_2 h}{\lambda(\alpha)} \left\{ \frac{1}{\lambda^2(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} - 0 \end{array} \right]$$

$$\therefore f_2''(p) = \frac{-2\delta_2(T - t_1)}{T}$$

As $(T - t_1) > 0$ so, $f_2''(p) < 0$.

Therefore, for any specified value of $T > 0$ $TP(p, T)$ is a concave function. So, there exists a unique p (say p^*) such that $TP(p, T)$ attains the maximum value. This completes the proof.

The necessary condition to find the optimal selling price (p^*) can be found by equating the first order partial derivative of $TP(p, T)$ with respect to p of $TP(p, T)$ equal to zero. After simplifying, the necessary condition is given by

$$\left[\begin{array}{l} (\delta_1 - 2\delta_2 p)(T - t_1) + \frac{\delta_2 P}{\theta} \\ -Pe^{\theta t_1} \left\{ \frac{\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{\delta_2}{\theta} \right\} + \frac{P\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) - \frac{c\delta_2}{\theta} + \\ ce^{\theta t_1} \left\{ \frac{\delta_2}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{\delta_2}{\theta} \right\} + \frac{c\delta_2 g}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + (t_1 - T) \right\} \\ + \frac{c\delta_2 h}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) + \frac{c\delta_2 h}{\lambda(\alpha)} \left\{ \frac{1}{\lambda^2(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} \end{array} \right] = 0$$

Also, the necessary condition for finding the optimal cycle length (T^*) for a given selling price (p) can be found by equating the partial derivative $\frac{\partial TP(p, T)}{\partial T} = 0$. So the equation is

$$\begin{aligned} & \left[pD(T - t_1) - \frac{PD}{\theta} + P \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} e^{\theta t_1} - \frac{PD}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) \right. \\ & - \frac{1}{T} \left[-C + \frac{cD}{\theta} - ce^{\theta t_1} \left\{ \frac{D}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{D}{\theta} \right\} - \frac{cDg}{\lambda(\alpha)} \left\{ \frac{1}{\lambda(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + (t_1 - T) \right\} \right. \\ & \left. \left. - \frac{cDh}{\lambda(\alpha)} \left(\frac{t_1^2}{2} - \frac{T^2}{2} \right) - \frac{cDh}{\lambda(\alpha)} \left\{ \frac{1}{\lambda^2(\alpha)} (e^{\lambda(\alpha)(T-t_1)} - 1) + \frac{1}{\lambda(\alpha)} (t_1 e^{\lambda(\alpha)(T-t_1)} - T) \right\} - \alpha(T - t_1) \right] \right] \\ & + \left[pD + PDe^{\lambda(\alpha)(T-t_1)}e^{\theta t_1} - PDe^{\lambda(\alpha)(T-t_1)} - cDe^{\theta t_1}e^{\lambda(\alpha)(T-t_1)} - \frac{cDg}{\lambda(\alpha)} \left\{ e^{\lambda(\alpha)(T-t_1)} - 1 \right\} + \frac{cDhT}{\lambda(\alpha)} \right] = 0. \end{aligned}$$

After getting the value of p^* and T^* , we can calculate the optimal economic order quantity Q^* from the equation (5.8). Also we can find the optimal values of the defective item quantity (S^*) and (M^*) (optimal

quantity which is arrived at retailer's warehouse) from the equation (5.9), (5.7) respectively by using optimal values of p^* and T^* .

7. ALGORITHM FOR THE MODEL

Step-1: Initialized $TP_{\max}(p, T) = 0$ and $i = n$.

Step-2: Input all the value of given parameters.

Step-3: Set $j = 1$, $p^j = p_1$; where p_1 is the solution of $\delta_1 - 2\delta_2 p = 0$.

Step-4: Solve the equation $\frac{\partial TP}{\partial T} = 0$ to find the $T = T^{(j)}$ by using $p = p^{(j)}$.

Step-5: Substitute the value of $T^{(j)}$ in the equation $\frac{\partial TP}{\partial p} = 0$ and solve for $p_1^{(j)}$,

$$\text{set } p^{(j+1)} = p_1^{(j+1)}.$$

Step-6: If $|p^{(j+1)} - p^{(j)}| \leq 10^{-5}$ then go to step-7 else go to step-4; $j = j + 1$.

Step-7: Substitute the value of $p = p^{(j)}$ and $T = T^{(j)}$ in the equation $TP(p, T)$,

$$\text{Set } TP_i(p, T) = TP(p, T). \text{ If } TP_i(p, T) > TP_{\max}(p, T) \text{ then } TP_{\max}(p, T) = TP_i(p, T).$$

Step-8: Find the total profit per unit time $TP_{\max}(p, T)$ by using the optimal value of p and T .

Step-9: Find the optimal order quantity per cycle Q^* , S , M by using the value of p^* , T^* .

8. COMPUTATIONAL RESULTS

To inspect the applicability of our theoretical results we have demonstrate a numerical example using MATLAB software with the help of our developed algorithms.

8.1. Numerical examples

Let, us assumed that the purchasing cost of an item is \$40/unit. Along with $\delta_1 = 80$, $\delta_2 = 0.9$, $g = 0.2$ (\$/unit/week), $h = 0.05$ (\$/unit/week), $\theta = 0.5$, $\eta = 0.8$, $\bar{\alpha} = 10$, $\alpha = 5 \in [0, \bar{\alpha}]$, $\lambda(\alpha) = e^{-\eta\alpha} = 0.0183 (< \theta)$, $c = 40$ (\$/unit), $P = 35$ (\$/unit), $t_1 = \frac{3}{7}$ week = 0.42857 week, $C = 10$ (\$).

Solution:

Step-1: Initialized $TP_{\max}(p, T) = 0$, $i = 1$.

Step-2: Input all the values of given parameters.

Step-3: Set $j = 1$, $p^{(j)} = p_1 = \frac{a}{2b} = 44.4444$.

Step-4: After solving the equation $\frac{\partial TP}{\partial T} = 0$, we get $T = T^{(1)} = 0.9783$ by using $p = p^{(1)}$.

Step-5: By using $T = T^{(1)} = 0.9783$ after solving $\frac{\partial TP}{\partial p} = 0$, we get $p_1^{(1)} = 68.5914$.

$$\therefore p^{(2)} = p_1^{(1)} = 68.5914$$

Step-6: $|p^{(2)} - p^{(1)}| = |68.5914 - 44.4444| \not\leq 10^{-5}$ go to step-4, $j = j + 1 = 2$

Step-4: $p^{(2)} = 68.5914$ By using $p = p^{(2)}$ from the equation $\frac{\partial TP}{\partial T} = 0$,

we get $T = T^{(2)} = 1.6660$.

Step-5: By using $T = T^{(2)} = 1.6660$ after solving $\frac{\partial TP}{\partial p} = 0$, we get $p_1^{(2)} = 69.2601$.

$$\therefore p^{(3)} = p_1^{(2)} = 69.2601$$

Step-6: $|p^{(3)} - p^{(2)}| = |69.2601 - 68.5914| \not\leq 10^{-5}$ go to step-4, $j = j + 1 = 2 + 1 = 3$

Step-4: $p^{(3)} = 69.2601$ By using $p = p^{(3)}$ and after solving the equation $\frac{\partial TP}{\partial T} = 0$,

we get $T = T^{(3)} = 1.6806$

Step-5: By using $T = T^{(3)}$ after solving the equation $\frac{\partial TP}{\partial p} = 0$, we get $p_1^{(3)} = 69.2919$.

$$\therefore p^{(4)} = p_1^{(3)} = 69.2919$$

Step-6: $|p^{(4)} - p^{(3)}| = |69.2919 - 69.2601| \not\leq 10^{-5}$ go to step-4, $j = j + 1 = 3 + 1 = 4$.

Step-4: By using $p^{(4)} = 69.2919$ and after solving the equation $\frac{\partial TP}{\partial T} = 0$,

we get $T = T^{(4)} = 1.6812$.

Step-5: By using $T = T^{(4)}$ after solving the equation $\frac{\partial TP}{\partial p} = 0$, we get $p_1^{(4)} = 69.2920$.

$$\therefore p^{(5)} = p_1^{(4)} = 69.2920$$

Step-6: $|p^{(5)} - p^{(4)}| = |69.2920 - 69.2919| \not\leq 10^{-5}$ go to step-4, $j = j + 1 = 4 + 1 = 5$.

Step-4: By using $p^{(5)} = 69.2920$ and after solving the equation $\frac{\partial TP}{\partial T} = 0$,

we get $T = T^{(5)} = 1.6812$.

Step-5: By using $T = T^{(5)}$ after solving the equation $\frac{\partial TP}{\partial p} = 0$, we get $p_1^{(5)} = 69.2920$.

$$\therefore p^{(6)} = p_1^{(5)} = 69.2920.$$

Step-6: $|p^{(6)} - p^{(5)}| = |69.2920 - 69.2920| \leq 10^{-5}$ go to step-7.

Step-7: Using $p = p^{(6)} = 69.2920$ and $T = T^{(5)} = 1.6812$, we get

$$TP_i(p, T) = 247.8542 > TP_{\max}.$$

Step-8: The optimal solution is

$$TP(p, T) = \$247.8542$$

$$p^* = \$69.2920$$

and

$$T^* = 1.6812 \text{ weeks.}$$

Step-9: The economic order quantity

$$Q^* = 457.5266 \text{ units.}$$

$$\text{Also, } S^* = 95.0523 \text{ units} \quad \text{and} \quad M^* = 362.4743 \text{ units.}$$

8.2. Sensitivity analysis

Here, we have described the sensitivity analysis of the optimal solutions with respect to different parameters such as deterioration rate during carrying (θ), deterioration rate when preservation technology applied $\lambda(\alpha)$, constant part of holding cost (g), co-efficient of linearly time dependent holding cost (h), time at which stock arrived at retailer's warehouse, purchasing cost per unit time (c), constant part of demand rate (δ_1), co-efficient of the price in the demand rate (δ_2) etc. We substitute the value of one parameter in a steps of 10% ($-20\%, -10\%, +10\%, +20\%$) but once at a time. From the sensitivity analysis Table 4, we can observe the following results:

(1) The total profit (TP^*) for our model decreases if the value of t_1 , $\lambda(\alpha)$, θ , h , g , δ_2 are enhanced. So, if the values of the mentioned parameters increase the retailer will earn less amount of profit. Also, the value of profit function (TP^*) increases with respect to the increasing values of δ_1 and P . Hence, the retailer can earn more profit if the values of δ_1 and P increase. Additionally, the parameter δ_1 has a powerful influence on the profit function TP^* as the higher value of δ_1 can help to extend the profit.

(2) The optimal selling price of the item (p^*) increases if we increase the values of t_1 , $\lambda(\alpha)$, θ , g , δ_1 , c . Also the selling price (p^*) reduces with respect to the higher values of P and δ_2 . So, if the holding cost and the purchasing cost of a unit item rises then the retailer will increase the unit selling price to obtain the maximum profit. Additionally, the parameter δ_2 has a powerful influence on increasing p^* but it also has a negative impact

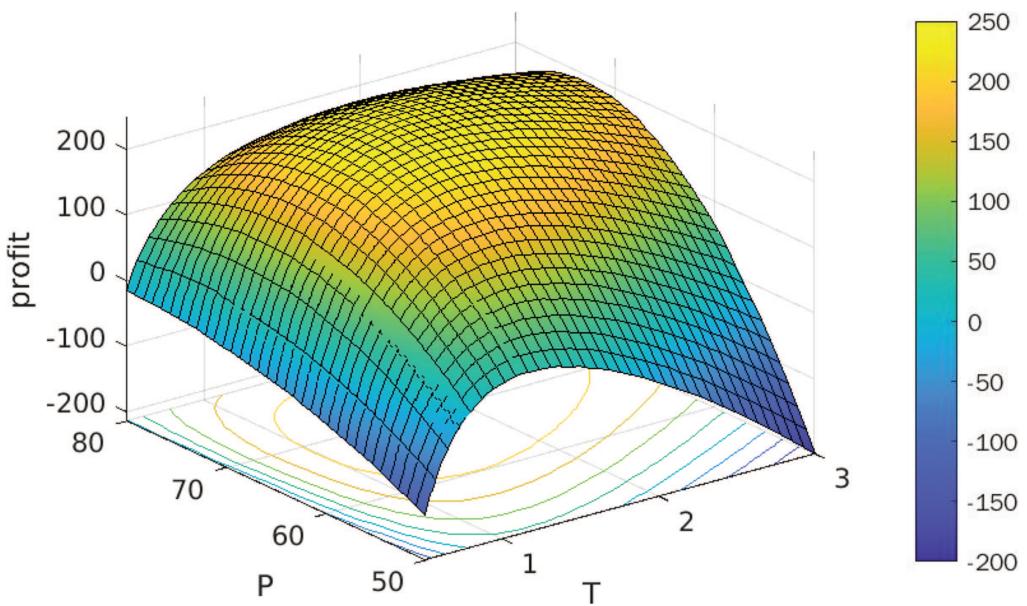
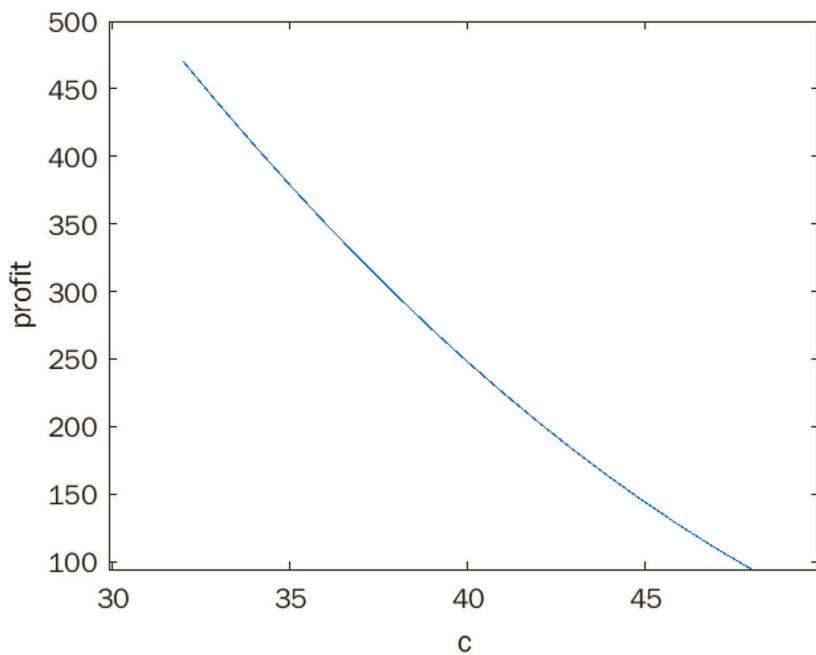
TABLE 4. Sensitivity analysis of different parameters.

Parameter	Original value	New value	p^*	T^*	TP^*
δ_1	80	64	60.0410	1.4937	68.3880
		72	64.6599	1.5884	144.6982
		88	73.9285	1.7699	378.5140
		96	78.5653	1.8541	537.1993
δ_2	0.9	0.7200	80.4905	1.8977	499.9213
		0.8100	74.4479	1.7813	354.3366
		0.9900	65.0753	1.5943	169.2436
		1.0800	61.5662	1.5185	111.1003
g	0.2	0.1600	68.9755	1.7876	261.9281
		0.1800	69.1375	1.7318	254.6832
		0.2200	69.4440	1.6351	241.3997
		0.2400	69.5820	1.5928	235.2838
h	0.05	0.0400	69.2464	1.7119	250.7218
		0.0450	69.2693	1.6962	249.2720
		0.0550	69.3145	1.6669	246.4669
		0.0600	69.3368	1.6532	245.1085
θ	0.5	0.4000	69.1393	1.6811	251.8729
		0.4500	69.2149	1.6812	249.8803
		0.5500	69.3707	1.6813	245.7945
		0.6000	69.4510	1.6814	243.7006
$\lambda(\alpha)$	0.0183	0.0147	69.2623	1.6923	249.2435
		0.0165	69.2772	1.6867	248.5470
		0.0201	69.3068	1.6758	247.1651
		0.0220	69.3215	1.6704	246.4794
P	35	28	71.6863	1.8468	195.2757
		31.5000	70.5265	1.7673	220.4265
		38.5000	67.9558	1.5866	277.8926
		42	66.4711	1.4801	311.0286
t_1	0.42857	0.3429	68.7113	1.5086	272.6559
		0.3857	68.0056	1.5970	259.8221
		0.4714	69.5721	1.7619	236.6280
		0.5143	69.8474	1.8394	226.0461
c	40	32	62.1864	1.6978	470.1099
		36	65.7328	1.6880	350.4032
		44	72.8604	1.6772	162.4548
		48	76.4358	1.6763	94.2004
C	10	8	69.2800	1.6755	249.0459
		9	69.2860	1.6784	248.4496
		11	69.2980	1.6841	247.2600
		12	69.3040	1.6869	246.667

on the demand function. So, It is important to maintain the balance between the higher selling price and the demand of the item.

(3) The optimal cycle length (T^*) increases with respect to the expanding values of t_1 and P . If the deterioration rate ($\theta, \lambda(\alpha)$) increases then the cycle length decreases. We observed that for the other parameters such as c, g, h etc. the cycle length is increasing first and then reduces.

(4) The optimal profit $TP^*(p, T)$ increases if we increase the value of the demand constant δ_1 that is the optimal profit $TP^*(p, T)$ varies proportionally with the demand constant δ_1 and it is shown in the Figure 3. Also, the optimal profit $TP^*(p, T)$ decreases with respect to the increasing the purchasing cost c and it is shown in the Figure 4.

FIGURE 2. Concavity of $TP(p, T)$ against p and T .FIGURE 3. Profit function $TP^*(p, T)$ vs. purchasing cost c graph.

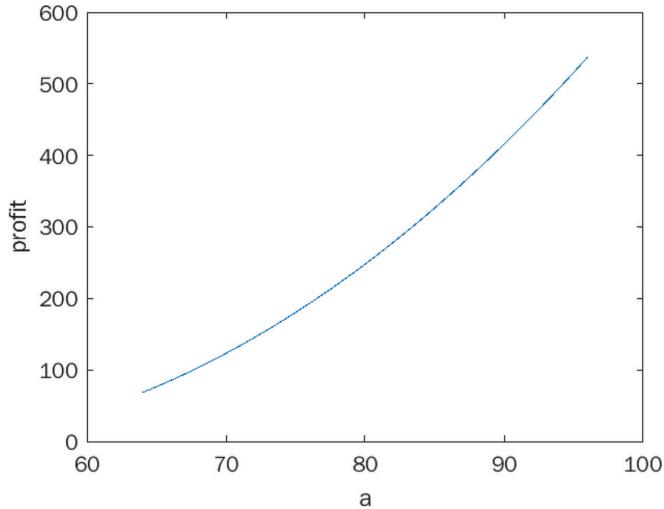


FIGURE 4. Profit function $TP^*(p, T)$ vs. demand constant (δ_1) graph.

9. CONCLUSION AND FUTURE RESEARCH DIRECTION

In our paper, we solved a profit maximization problem under several realistic assumptions such as deterioration during carrying, price sensitivity of the demand function, time-dependent holding cost, uses of preservation technology for deteriorating product, etc. Here we consider the deterioration rate due to carrying is constant but after using preservation technology the deterioration rate depends only on the preservation technology investment, also it is an increasing function of preservation technology investment. In addition, the selling price of the product (p) and the total inventory cycle length (T) are decision variables in the total profit function of our model. By using the optimal values of the selling price of the product (p^*) & the total inventory cycle length (T^*) we calculate the Economic Order Quantity (Q^*), optimal quantity of the defective product due to carrying (S^*) and the optimal inventory level (M^*) at the time t_1 . Finally, the numerical problem solved by using our modified algorithm as well as the sensitivity analysis of the various parameters, which are involved in the profit function are discussed. The numerical findings and approach of our model are applicable to several types of industries or companies that handling various types of deteriorating product because of realistic assumption of our model and mathematical generalization of various parameters. Therefore from our model and numerical findings the following conclusions can be drawn

1. The optimal cycle length increases concerning the expanding values of t_1 . So, the total profit decreases due to production increasing inventory cycle length. Therefore to make the maximum profit, the retailer has to reduce the time taken to bring the deteriorating items from the supplier's warehouse to retailer's warehouse by using various fast transportation modes.
2. The parameter δ_2 has significant impact on increasing p^* but it also has a negative influence on the demand function. So, It is important for a retailer to maintain the balance between the higher selling price and the demand of the item by taking various marketing strategies.
3. Also, the value of profit function increases with respect to the increasing values of P . So, the retailer can earn more profit by selling the less defective product to the customers at highest possible price.
4. The total profit (TP^*) for our model decreases when the value of $\lambda(\alpha), h, g$, holding times are enhanced. It indicates that the retailer can earn maximum amount of profit by decreasing the values of $\lambda(\alpha)$ by investing more on preservation technology and also by reducing the values of constant coefficients of the holding cost (h, g) as well as reducing the holding time of the products.
5. When the purchasing cost of the product increases(C), the total profit decreases but selling price of the product (p^*) and the optimal cycle length (T^*) increases. Also, selling price of the product (p^*) has negative

impact on demand. Hence, retailer can earn more profit by negotiating the purchasing cost of the product with wholesaler and it also gives an opportunity to a retailer to give the customer a discount on the selling price of the product.

The limitations of our considered models are

1. In this paper, we used hypothetical data for numerical illustration instead of real data due to the company's secret business policy they denied to share their real data.
2. Our model is useful only for those companies that work with deteriorating items.

Our proposed model one can easily extend for future research in many aspects such as

1. By introducing expiry date-dependent deterioration rate during carrying time.
2. This model can be extended by applying a two-level trade credit policy or two warehouse systems or introducing several types of variables such as fuzzy types, interval-valued types.

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