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JOINT REPLENISHMENT STRATEGY FOR DETERIORATING MULTI-ITEM THROUGH MULTI-ECHELON SUPPLY CHAIN MODEL WITH IMPERFECT PRODUCTION UNDER IMPRECISE AND INFLATIONARY ENVIRONMENT

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Abstract. As the industry environment becomes more competitive, the supply chain management for multi items has become an essential part of the industries. In this paper, a multi-echelon inventory model for deteriorating multi items with imperfect production has been developed under the environment of fuzzy and inflation. A single producer, multi-supplier, and multi-retailer are considered from the integrated point of view. Here, the producer only produces the retailer's need to have a tremendous advantage and minimum loss. It is observed that the inflation rate is almost uncertain for deteriorating goods in every supply chain. In this paper, the inflation rate is taken as a triangular fuzzy number, and the centroid method is used to defuzzify the profit function. The shortage is not allowed in any part, an imperfect production process is considered, but it is not reworkable in this supply chain. Different inflation rates are considered for additional items because inflation has strained the most vulnerable consumers (the daily wage earners), who mainly demand goods in short and small quantities. This entire model is developed based on the retailer's demand and due to which, the profit potential is maximized. The central premise of this study is to get maximum benefit by creating a production model for deterioration items. Finally, a numerical example and sensitivity analysis illustrate the present study. It is observed that if the number of shipments taken from the supplier increases during the production period, the total profit increases in crisp and fuzzy. If a positive change occurs in the number of shipments received through the producer to the retailer, then the fuzzy model has positive, but a slight negative change occurs in the crisp model. This paper shows the effect of a joint replenishment policy for multi-item compared with the independent approaches.

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1. Introduction

The multi-echelon supply chain model (MESCM) plays a vital role in the present era, acting as an organization. The objective of this organization is that all members of the supply chain should not have any inconvenience, and a consumer should get the maximum benefit of quality. The main aim behind the supply chain is that any product through this chain should be delivered to the consumer appropriately and carefully. The commodity price should be correct, and the probability of a mistake should be reduced. Clark and Scarf [3] were the first authors to discuss the concept of MESCM in inventory research. Many researchers developed an inventory model in different conditions through a supply chain system. When the products are destroyed in a short time, such as fruits and vegetables, the supply chain becomes very important, and many products are present in the market, which affects the consumer according to their quality. Therefore, this organization becomes an obligation to satisfy customers. Along with deterioration, inflation is an essential part of MESCM. Sometimes, inflation changes suddenly because inflation has strained the most vulnerable consumers (the daily wage earners). They mainly demand goods in the short run and small quantities.

Sarker *et al.* [31] developed the supply chain inventory model (SCIM) in an inflationary environment. He *et al.* [10] discussed the production-inventory model in multi-market demand for deteriorating items. Kugele *et al.* [29] introduced a multi-objective goal programming problem for smart production system under carbon emission reduction. Habib *et al.* [28] discussed the multi-stage innovative, sustainable biofuel production system. The world is currently affected by tremendous changes in inflation in the wake of the corona global pandemic. Most of the time, it is observed that the SCIM is developed for a single product, but it is not necessary to stabilize the value of any object, due to which the entire composition of the supply chain may have to suffer loss. Therefore, developing this model aims to maximize profit by trading many commodities in different inflation rates through the supply chain management (SCM), making this model different and unique. Moreover, the purpose of considering additional inflation for various items is from the most vulnerable consumers affected by a single rupee change in prices.

The rest of the paper is presented as follows: A comprehensive literature review is provided in Section 2. Problem definition, notation, and assumptions for the proposed model are presented in Section 3. In Section 4 describes the concepts used in the model with subsections of retailers, producers, and suppliers. In Section 5, the fuzzy model is developed, and solution methodology is given in Section 6. To understand the model's reality numerical example is provided in Section 7. Discussions and managerial insights are discussed in Sections 8 and 9, respectively. The conclusions of this model is shown in Section 10.

2. Literature review

2.1. Inventory model based on MESCM

MESCM is a streamline in every situation from producer's flow for natural disasters. MESCM can correctly diagnose problems and disruptions in the organization. MESCM plays an essential role in delivering goods to their destination quickly, safely, and efficiently. Inventory management (IM) is working in a queer and unpredictable atmosphere. Most of the researchers have discussed the inventory model with MESCM. Chou [2] developed the integrated two-stage inventory model for the deteriorating item. Rani and Kishan [25] described MESCM as having a variable demand rate for deteriorating items. Singh *et al.* [38] discussed MESCM for different types of products, and the model was formulated for more than one supplier and retailer with constant demand and production. Mohammadi *et al.* [19] developed a model to optimize integrated manufacturing and products inspection policy for a deteriorating manufacturing system with imperfect. Dai *et al.* [4] discussed the MESCM for three different types of demand, taking shortage and partial backlogging as the replenishment cycle to infinite. Mahapatra *et al.* [30] introduced an inventory model that used preservation technology with fuzzy learning. Moon *et al.* [7] discussed a model on machine reliability and found solutions based on degree of difficulty of geometric programing. Sarkar *et al.* [8] developed an inventory model which was solved by artificial neural network with multithreading. Sarkar and Bhuniya [18] discussed a sustainable flexible production system that provide improved service with green technology investment. Maihami *et al.* [17] examined the MESCM for deteriorating items under a probabilistic environment. Lu *et al.* [16] developed a two-stage SCM with carbon tax regulation for a total profit with price-dependent demand and applied the Stackelberg game. Sarkar *et al.* [49] discussed the development framework for a multi-stage production system for controlling defective products. Jauhari *et al.* [12] served on a supply chain model with a mutual understanding of single manufacturing, retailer, and single collector. They used two types of the recovery process: remanufacturing, refurbishing, and applied carbon emissions and Stackelberg game with green technology. Sebatjane and Adetunji [32] introduced a three-echelon supply chain model with price and freshness-dependent demand. Yadav *et al.* [53] discussed a sustainable supply chain model with preservation technology and cross-price elasticity of demand. Uthayakumar and Hemapriya [50] developed an integrated MESCM with setup cost reduction and variable lead time. Sarkar *et al.* [51] developed a model on innovative product design to reduce the environmental risks. Vandana *et al.* [52] discussed the impact of energy and carbon reduction with SCM's two-level trade credit policy.

2.2. Inventory model based on deteriorating items

Deterioration plays an essential role because deterioration occurs in almost all fruits, vegetables, and medicines. Many authors have deeply studied deteriorating inventory in recent years.

Singh *et al.* [37] discussed the MESCM with two warehouses, imperfect production, variable demand rate, and inflation. Tat *et al.* [47] developed an economic order quantity (EOQ) model with instantaneous deteriorating items for a vendor-managed inventory (VMI) system. Taleizadeh [40] formulated an EOQ model for deteriorating items in a purchasing system with multiple prepayments. Taleizadeh and Nematollahi [41] developed an inventory control problem for backordering and financial considerations for deteriorating items. Taleizadeh *et al.* [44] established a joint optimization model of price, replenishment frequency, replenishment cycle, and production rate in VMI systems with deteriorating items. Taleizadeh *et al.* [45] formulated a model based on optimal multi-discount selling prices schedule for the deteriorating product. Lashgari *et al.* [14] developed an inventory control problem for deteriorating items with back-ordering and financial considerations under two levels of trade credit linked to order quantity. Diabat *et al.* [9] established a lot-sizing model with partial downstream delayed payment, partial upstream advance payment, and partial backordering for deteriorating items. Tavakoli and Taleizadeh [48] developed an EOQ model for decaying things with advanced prices and conditional discounts. Taleizadeh and Rasuli-Baghban [42] derived a model on pricing and lot sizing of a decaying item under group dispatching with time-dependent demand and decay rates. Lashgari *et al.* [15] formulated a model for ordering non-instantaneous deteriorating items under hybrid partial prepayment, partial trade credit, and partial backordering. Taleizadeh *et al.* [46] established a partial linked-to-order delayed payment and lifetime effects on decaying items ordering. Nobil *et al.* [20] developed a single-machine lot-scheduling problem for deteriorating items with a negative exponential deterioration rate. Panda *et al.* [22] discussed an inventory model for deteriorating goods with storage stock problems and a stock price-dependent demand rate. Ali *et al.* [1] developed an inventory model by making a green supply chain with credit period-dependent demand for deteriorating products. Shaikh *et al.* [33] developed an inventory model for deteriorating items with ramp type demand under trade credit and preservation technology. Singh *et al.* [39] introduced two levels of storage model for deteriorating items with stock-dependent demand. Singh and Tayal [36] discussed an inventory model with a replenishment policy for deteriorating items with a trade credit policy.

2.3. Inventory model based on fuzzy logic

There are many products in the market whose price fluctuations create much uncertainty. Fuzzy set theory mainly deals with quantitative analysis of uncertainty and imprecision. Therefore, the fuzzy set theory plays an essential role in developing inventory management better. In the last few years, many researchers have developed an inventory model very well with the help of fuzzy set theory. Jaggi *et al.* [11] discussed an inventory model with the help of fuzzy theory, in which the demand is taken as time-dependent, and the shortage is permissible. Taleizadeh *et al.* [43] developed a revisiting a fuzzy rough EOQ model for deteriorating items considering quantity discount and prepayment. Sharmila and Uthayakumar [35] discussed the fuzzy theory with shortage

Table 1. Key features of the inventory model developed in earlier researches.

References	Model	Demand	No. of product	Inflation	Fuzzy
Ali *et al.* [1]	Green supply chain	Credit-period demand	Single	–	–
Dai *et al.* [4]	Multi-echelon	Trapezoidal, ramp type	Single	–	–
De and Mahata [6]	EOQ	Fuzzy sense	Single	–	(Yes) Cloudy
Singh *et al.* [38]	Three echelon supply chain	Constant	Multi	–	–
Jaggi *et al.* [11]	Fuzzy inventory	Time-dependent	Single	–	Yes
Karmakar *et al.* [13]	EOQ	Fuzzy sense	Single	–	(Yes) Cloudy
Panda *et al.* [22]	EOQ	Stock-dependent	Single	–	–
Saha [26]	Supply chain	Price-dependent	Single	–	Yes
Saranya and Varadarajan [27]	Supply chain	Exponential	Single	–	Yes
Lu *et al.* [16]	Multi-echelon supply chain	Price-dependent	Single	–	–
This Paper	Multi-echelon supply chain	Constant	Multi	Yes	Yes

and exponential demand, and they took all the costs affecting the inventory as a triangular fuzzy number. De and Beg [5] introduced the triangular dense fuzzy set theory in a new form to defuzzify the total cost function. Saha [26] developed an inventory model with fuzzy set theory for deteriorating products through the supply chain in which he did not take the shortage. Saranya and Varadarajan [27] discussed an inventory model with the help of the graded mean integration method (GMIM) for the deficiency. Padiyar *et al.* [21] presented a deteriorating item inventory model with price-dependent consumption rate under a fuzzy environment, where the total cost is defuzzify by the graded mean representation method. In their model, they allowed shortages that are completely backlogged.

Karmakar *et al.* [13] discussed the cloudy fuzzy in the EOQ model for the uncertainty of demand. De and Mahata [6] developed the EOQ model in cloudy fuzzy to produce poor quality in which proportionate discount allowed. Rajput *et al.* [23] proposed an inventory model with different demand function types and discussed the importance of fuzzy parameters in healthcare industries. They used a triangular fuzzy number for the demand, defuzzified the model with the signed distance method, and got the maximum profit for all three models. Rajput *et al.* [24] proposed an inventory model for the crisp and the fuzzy environment. They included reliability-induced demand and fuzzy parameters with a graded mean integration method for defuzzification for the model in a vague sense. Sharma *et al.* [34] described a reverse logistics inventory model in which production and remanufacturing are both discussed in a fuzzy environment. They provided an example to satisfy the result in both environments.

2.4. Research gap and contribution of this paper

The difference between the work done and the work of this paper can be understood in Table 1. Many researchers have developed inventory models very effectively in different conditions. Still, very few have developed the relationship between the producer, supplier, and retailer, where the probability of total profit is maximum. With this, very few researchers have expressed transport expenditure for a period in every supply chain as fixed

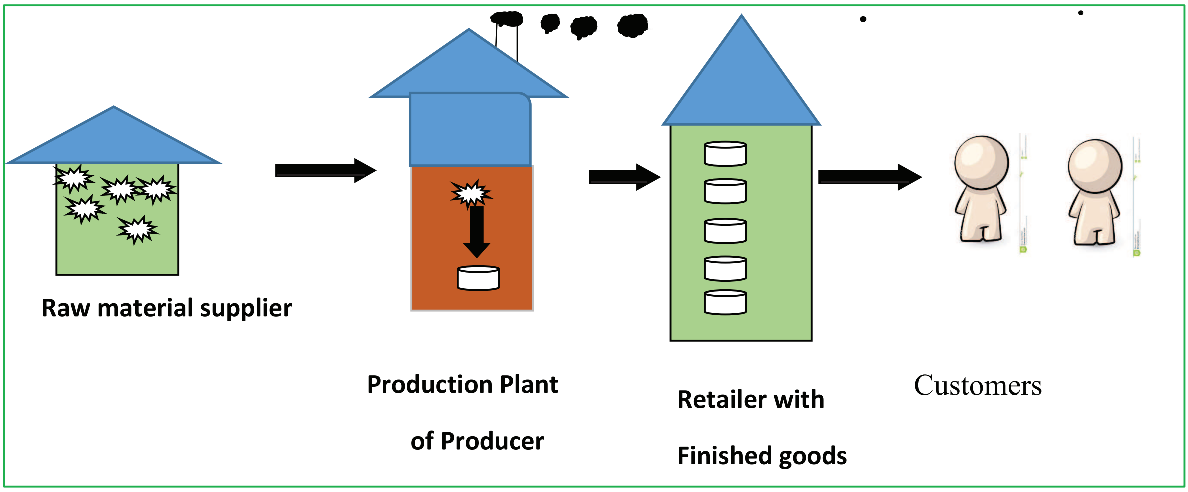


FIGURE 1. Proposed model of production management of multi-items.

and variable costs. At present, no research paper has been found in which a multi-echelon system has been developed for a single producer with multi-supplier and multi-retailer with multi-product under an inflationary and fuzzy environment. In contrast to the above research model, some essential elements make this model unique and fulfill the expectations of the current environment.

(1) Due to the ongoing competition in the market at present, it is not necessary for the member of the supply chain system from each product to benefit. This model has been developed so that more than one product has been used to be replenished with another product if it is not satisfied with one, and the model will not be affected.
(2) Different inflation rates for different items and deterioration rates for different items are considered because inflation has strained the most vulnerable section of the consumers (the daily wage earners), who mainly demand goods in the short run and small quantities.
(3) Multi-supplier and multi-retailer have been engaged to bring more strength in the mobilization and promptly meet the consumer's needs.
(4) To maximize the business benefit, the product is produced as per the retailer's requirement to be used properly. Holding cost is reduced.
(5) Inflation rate for many products keeps oscillating. There is no certainty in it, and to solve the problem of this uncertainty, the inflation rate for the entire product has been taken as the triangular fuzzy number is different for each product.

3. Problem description, notation, and assumptions

3.1. Problem description

This proposed joint replenishment strategy is for deteriorating multi-item through MESCM with imperfect production under an imprecise and inflationary environment. A three-level supply chain process is taken in which multi-supplier, single producer, and multi-retailer are considered. Figure 1 shows the production process, items delivery, and sale process. In this supply chain, firstly producer buys raw material from the supplier. It is produced in the production plant as per the retailer's requirement, after which the items are sold to the retailer according to his demand. Figure 2 shows that in the model, multi-supplier is arranging raw material for producer and producer is delivered the finished goods to multi-retailer to deliver the items to customers.

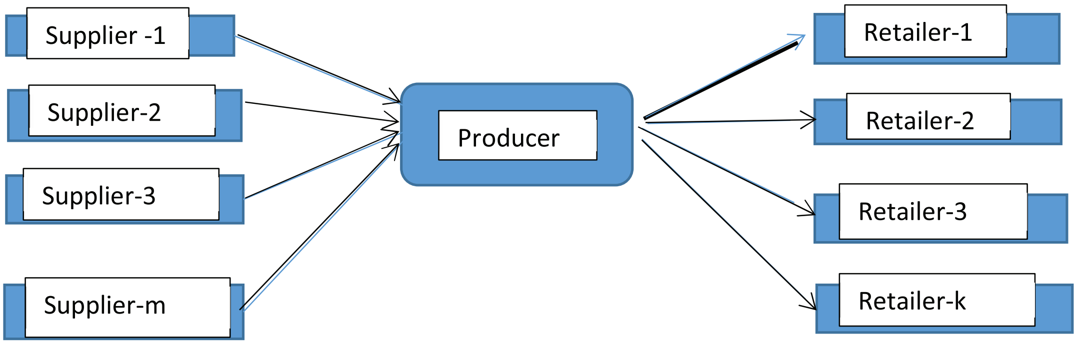


FIGURE 2. Proposed model of production management of multi-supplier's and multi-retailer's with a producer.

3.2. Notation

In this subsection, the notation can be used to develop the model.

3.2.1. Indices

m	Total number of suppliers
k	Total number of retailers
n	Total number of delivery from producer to retailer in the cycle time
l	Total number of items

3.2.2. Supplier's parameters

Deterioration rate of raw material for ith item	θ_{1i}
Supplier sale revenue cost for ith item for βth-supplier (\$/unit)	$A_{S\beta i}$
Inflation rate for ith item	r_i
Ordering cost of ith items for the βth-supplier (\$/order)	$O_{S\beta i}$
Holding cost of ith items for the βth-supplier (\$/unit)	$h_{S\beta i}$
Deterioration cost for the βth-supplier (\$/unit)	$d_{S\beta i}$
Demand rate for producer of finished goods which is transferred from supplier	$P_{i\beta}$
Fixed transportation cost for each supplier (\$/delivery)	d_T
Variable transportation cost of ith item for the βth-supplier (\$/unit)	$d_{ti\beta}$
Inventory level for a supplier where $Q_{Si} = \sum_{\beta=1}^{m} \frac{QksP_{i\beta}}{qm\theta_{1i}} \left(e^{\theta_{1i}(T_s-t)} - 1\right)$	Q_{si}

3.2.3. Producer's parameters

Deterioration rate of finished goods for ith item	θ_{2i}
Production rate for ith items	p_i
Demand rate for producer of finished goods which is transferred from supplier	$P_{i\beta}$
Variable transportation cost of ith item for the βth-supplier (\$/unit)	$d_{ti\beta}$
Sales revenue cost of ith items for the producer's (\$/unit)	V_i
Producer's non-production time period	T_2
Producer's setup cost for ith items (\$/setup)	S_{Pi}
Producers ordering cost for ith items (\$/order)	O_{pi}.
Producers holding cost for ith items (\$/unit)	C_{pi}
Producers deteriorating cost for ith items (\$/unit)	d_i
Inflation rate for ith items	r_i
Production rate	P_i
Inventory level for producer where $Q_{p1} = \left(\frac{P_i - L\lambda}{\theta_{2i}}\right)\left(1 - e^{-\theta_{2i}T_1}\right)$	Q_{Pi}

3.2.4. Retailer's parameters

Deterioration rate of finished goods for ith item	θ_{2i}
Inflation rate for ith items	r_i
Ordering cost for ith items and jth retailers (\$/order)	O_{Rij}
Sale revenue cost for ith items, and jth retailers (\$/unit)	A_{Rij}
Holding cost for ith items and jth retailers (\$/unit)	h_{rij}
Purchasing cost for ith items and jth retailers (\$/cycle)	U_{Rij}
Fixed transportation costs for every retailer (\$/delivery)	R_T
Variable transportation cost for ith items and jth retailers (\$/vehicle unit)	R_{tij}
Deteriorating cost for ith items and jth retailers (\$/unit)	d_{Rij}
Demand rate for ith items and jth retailer, where $j = 1, 2, 3, \ldots, k$,	D_{ij}
Inventory level for the retailer where $Q_{ri} = \sum_{j=1}^{k} D_{ij} \left(\frac{e^{\theta_{2i}(T_R - t)} - 1}{\theta_{2i}} \right)$	Q_{ri}

3.2.5. Decision variable

Production period	T_1

3.3. Assumptions

- Sometimes the situation becomes such in which it becomes essential for any product to reach every customer. Still, only one retailer cannot contact every customer in this situation, and one producer produces many products. All types of raw materials will be available from a single supplier are unnecessary. In this model, multi-supplier, multi-retailer, and single producer have been taken.
- Different inflation rates and different deterioration rates for various goods are considered because inflation has put the most vulnerable section of consumers (daily wage earners) under stress, who mainly demand goods in less time and smaller quantities, and each product has its different period of survival due to which their damage time may be extra.
- This model has been developed keeping in mind the situations that include serious problems like disaster situations, global pandemics. In these circumstances, the shortage of items cannot be given importance due to which deficiency is not allowed in this model.
- Once the item is produced from the production house, the retailer gets the thing as per his requirement. In many situations, the whole case depends on the customer's demand, and in those circumstances, the holding cost for the safety and maintenance of the item depends on that time. Therefore, in this model, the retailer's holding cost is taken as time-dependent.

4. Model formulation

In the proposed MESCM, the producer required s unit of raw material for one unit requirement of the retailer, there is a total k retailer, and each retailer need a Q unit, so the producer needs Qks units of raw material from the supplier and that too till the time of production, here m suppliers are there, and they delivered the items in q times till the time of production to the producer, that is every supplier has to deliver $\frac{Qks}{qm}$ units of raw material per delivery. A producer is giving finished goods to the retailer in n shipments, and producer is delivering λ times in the time interval $[0, T_1]$ to the retailer, and $n - \lambda$ times in the time interval $[0, T_2]$.

4.1. Model formation for the retailer

The retailer's inventory for finished goods at time t is shown in Figure 3. The retailer has maximum inventory L at time $t = 0$, during the time interval $[0, T_R]$ inventory decreases due to the combined effect of deterioration as well as demand, then the retailer's inventory system can be represented by the following differential equation:

$$\frac{\mathrm{d}Q_{ri}(t)}{\mathrm{d}t} = -\sum_{j=1}^{k} D_{ij} - \theta_{2i} Q_{ri}(t), \qquad 0 \leq t \leq T_R \tag{4.1}$$

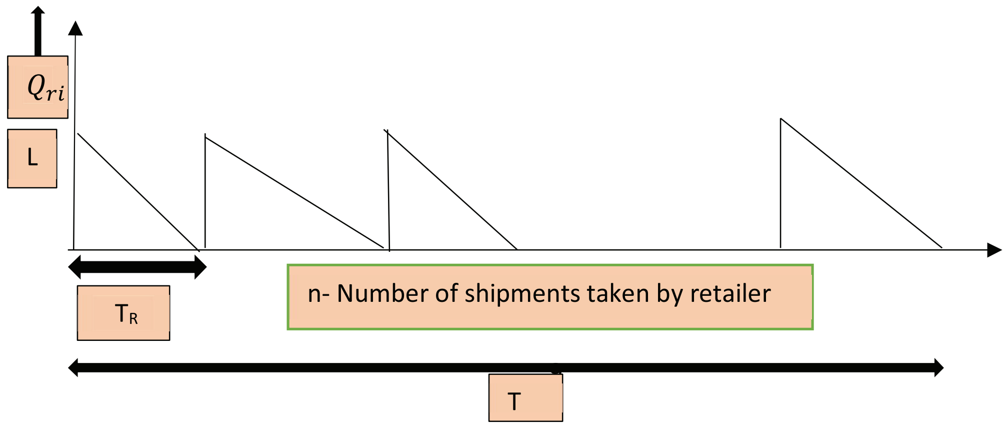


FIGURE 3. Retailer's inventory level.

with the boundary condition $Q_{ri}(0) = L$, $Q_{ri}(T_R) = 0$, solution of Equation (4.1) is

$$Q_{ri}(t) = \sum_{j=1}^{k} D_{ij}\left(\frac{e^{\theta_{2i}(T_R - t)} - 1}{\theta_{2i}}\right), \tag{4.2}$$

where maximum inventory level is

$$L = \sum_{j=1}^{k} D_{ij}\left(\frac{e^{\theta_{2i}T_R} - 1}{\theta_{2i}}\right). \tag{4.3}$$

For a retailer, profit function depends on the following factors.

4.1.1. *Ordering cost*

Ordering cost is the total cost involved in ordering the inventory, including the cost of finding a producer and inspection of the inventory

$$\mathrm{OC}_R = \sum_{j=1}^{k} O_{Rij}. \tag{4.4}$$

4.1.2. *Sales revenue*

Sales revenue is the total revenue generated by selling and delivering a product (finished goods) to consumers, and total sales revenue for the retailer is

$$\mathrm{SRS}_R = \sum_{j=1}^{k} A_{Rij} \int_0^{T_R} D_{ij} e^{-r_i t}\mathrm{d}t = \sum_{j=1}^{k} A_{Rij} D_{ij}\left(\frac{1 - e^{-r_i T_R}}{r_i}\right). \tag{4.5}$$

4.1.3. *Holding cost*

Holding costs involved in carefully storing and maintaining inventory, including hardware equipment, material handling equipment, and IT software applications. Finally, the total holding cost for the retailer is

$$\begin{aligned}\mathrm{HC}_R &= \sum_{j=1}^{k} h_{rij} \int_0^{T_R} Q_{ri}(t) e^{-r_i t}\mathrm{d}t \\ &= \sum_{j=1}^{k} \int_0^{T_R} (a_{ij} + b_{ij}t) D_{ij}\left(\frac{e^{\theta_{2i}(T_R - t)}}{\theta_{2i}}\right) e^{-r_i t}\mathrm{d}t\end{aligned}$$

$$= \frac{D_{ij}}{\theta_{2i}} \begin{bmatrix} a_{ij}\left(\frac{e^{\theta_{2i}T_R}-e^{-r_iT_R}}{r_i+\theta_{2i}}\right) + \frac{a_{ij}}{r_i}(e^{-r_iT_R}-1) \\ -b_{ij}\left[\frac{T_Re^{-r_iT_R}}{r_i+\theta_{2i}} + \frac{e^{-r_iT_R}-e^{\theta_{2i}T_R}}{(r_i+\theta_{2i})^2}\right] \\ +b_{ij}\left(T_R\frac{e^{-r_iT_R}}{r_i} + \frac{e^{-r_iT_R-1}}{r_i^2}\right) \end{bmatrix}. \tag{4.6}$$

4.1.4. Purchasing cost

The purchasing cost is the total cost involved in purchasing the inventory from the producer, and in this, the purchasing cost for the retailer is

$$\mathrm{PC}_R = \sum_{j=1}^{k} \frac{L}{k} U_{ij}. \tag{4.7}$$

4.1.5. Deterioration cost

Different deterioration rates for an additional item and costs differently after the item is damaged, for example, returning a mild bad item to the producer or destroying the item that became useless. Hence, the deteriorating cost for the retailer is

$$\begin{aligned} \mathrm{DTC}_R &= \sum_{j=1}^{k} \theta_{2i} d_{Rij} \int_0^{T_R} Q_{ri}(t) e^{-r_i t} \mathrm{d}t, \\ &= \sum_{j=1}^{k} \theta_{2i} d_{Rij} \frac{D_{ij}}{\theta_{2i}} \left\{ \frac{e^{\theta_{2i}T_R} - e^{-r_iT_R}}{\theta_{2i}+r_i} - \frac{1-e^{-r_iT_R}}{r_i} \right\}. \end{aligned} \tag{4.8}$$

4.1.6. Transportation cost

Transportation cost is mainly divided into two categories: fixed transportation cost and variable transportation cost. The variable transportation cost depends on the actual movement of traffic. This cost is included in the cost of fuel and vehicle maintenance. It is entirely dependent on the delivery of the item, quantity, and location, and fixed transportation costs include the monthly installment payment of the vehicle, insurance of the vehicle, driver's salary, and administrative, managerial costs. The transportation cost is

$$\mathrm{TPC}_R = KR_T + \sum_{j=1}^{k} R_{tij}\left(\frac{L}{K}\right). \tag{4.9}$$

Retailer's profit function for different l items is

$$\mathrm{TPFR} = \frac{n}{T}\sum_{i=1}^{l}(\mathrm{SRC}_R - \mathrm{OC}_R - \mathrm{HC}_R - \mathrm{PC}_R - \mathrm{TPC}_R - \mathrm{DTC}_R). \tag{4.10}$$

4.2. Model formation for producer

Each retailer needs Q units of finished goods, and the producer gives finished goods to the retailer in total n shipments. Here, T_1 is the production period and T_2 is non-production period. Finished goods are being sent λ times to the retailer along with the production and $(n-\lambda)$ times in the non-production period. The requirement of regular shipment of fixed quantity L units of retailer are made in time T_s. The producer's inventory model is shown in Figure 4. Here, complete cycle length T is divided into two parts T_1 and T_2.

The inventory level becomes zero at $t = T_2$, the producer's inventory level of finished goods over the cycle time T for ith. The following linear differential equations give items

$$\frac{\mathrm{d}Q_{p1i}(t)}{\mathrm{d}t} = P_i - L\lambda - \theta_{2i}Q_{p1i}(t), \qquad 0 \le t \le T_1 \tag{4.11}$$

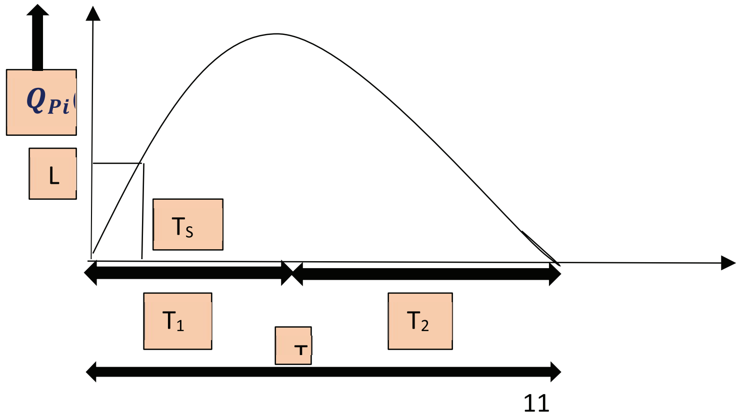


FIGURE 4. Producer's inventory level.

$$\frac{\mathrm{d}Q_{p2i}(t)}{\mathrm{d}t} = -(n-\lambda)L - \theta_{2i}Q_{p1i}(t), \qquad 0 \le t \le T_2 \tag{4.12}$$

with conditions $Q_{P1}(0) = 0$ and $Q_{p2}(T_2) = 0$.

Solution of above equations are

$$Q_{p1i}(t) = \left(\frac{P_i - L\lambda}{\theta_{2i}}\right)\left(1 - e^{-\theta_{2i}t}\right) \tag{4.13}$$

$$Q_{p2i}(t) = \left(\frac{(n-\lambda)L}{\theta_{2i}}\right)\left(-1 + e^{(T_2-t)\theta_{2i}}\right). \tag{4.14}$$

For the producer, profit function depends on the following factors.

4.2.1. Sales revenue

Sales revenue is the total revenue generated by selling and delivering a product (finished goods) to consumers. Total sales revenue for the producer is

$$\mathrm{SRC}_P = V_i kQ. \tag{4.15}$$

4.2.2. Setup cost

The total cost incurred in setup production plant before production is

$$\mathrm{SUC}_P = S_{Pi}. \tag{4.16}$$

4.2.3. Ordering cost

Ordering cost is the total cost involved in ordering the inventory, including purchasing the inventory and inspection of the inventory. Finally, the total ordering cost for the producer is

$$\mathrm{OC}_P = O_{Pi}. \tag{4.17}$$

4.2.4. Holding cost

Holding cost involved in careful storage and maintenance of the inventory, including hardware equipment, material handling equipment, IT software applications, and the holding cost for the producer is

$$\mathrm{HC}_P = C_{Pi}\begin{bmatrix} \int_0^{T_1} Q_{P1i}(t)e^{-r_i t}\mathrm{d}t + e^{-r_i T_1}\int_0^{T_2} Q_{P2i}(t)e^{-r_i t}\mathrm{d}t \\ -\left((n-\lambda)e^{-r_i T_1} - \lambda\right)\int_0^{T_R} Q_{ri}(t)e^{-r_i t}\mathrm{d}t \end{bmatrix}$$

$$= C_{Pi}\begin{bmatrix} \frac{P_i - L\lambda}{\theta_{2i}}\left(\frac{1-e^{-r_iT_1}}{r_i} + \frac{e^{-(\theta_{2i}+r_i)T_1}-1}{\theta_{2i}+r_i}\right) \\ +\frac{e^{-r_iT_1}(n-\lambda)L}{\theta_{2i}}\left[e^{\theta_{2i}T_2}\left(\frac{1-e^{-(r_i+\theta_{2i})T_2}}{r_i+\theta_{2i}} + \frac{e^{-r_iT_2}-1}{r_i}\right)\right] \\ -\left((n-\lambda)e^{-r_iT_1}-\lambda\right)\sum_{j=1}^{k}\frac{D_{ij}}{\theta_{2i}}\left(\frac{e^{\theta_{2i}T_R}-e^{-r_iT_R}}{\theta_{2i}+r_i} + \frac{e^{-r_iT_R}-1}{r_i}\right) \end{bmatrix}. \tag{4.18}$$

4.2.5. Deterioration cost

Each item has a different deterioration rate, and the deteriorating cost for the producer is

$$\mathrm{DC}_P = d_i\theta_{2i}\begin{bmatrix} \int_0^{T_1} Q_{P1i}(t)e^{-r_it}\mathrm{d}t + e^{-r_iT_1}\int_0^{T_2} Q_{P2i}(t)e^{-r_it}\mathrm{d}t \\ -\left((n-\lambda)e^{-r_iT_1}-\lambda\right)\int_0^{T_R} Q_{ri}(t)e^{-r_it}\mathrm{d}t \end{bmatrix} \tag{4.19}$$

$$= d_i\theta_{2i}\begin{bmatrix} \frac{P_i - L\lambda}{\theta_{2i}}\left(\frac{1-e^{-r_iT_1}}{r_i} + \frac{e^{-(\theta_{2i}+r_i)T_1}-1}{\theta_{2i}+r_i}\right) \\ +\frac{e^{-r_iT_1}(n-\lambda)L}{\theta_{2i}}\left[e^{\theta_{2i}T_2}\left(\frac{1-e^{-(r_i+\theta_{2i})T_2}}{r_i+\theta_{2i}} + \frac{e^{-r_iT_2}-1}{r_i}\right)\right] \\ -\left((n-\lambda)e^{-r_iT_1}-\lambda\right)\sum_{j=1}^{k}\frac{D_{ij}}{\theta_{2i}}\left(\frac{e^{\theta_{2i}T_R}-e^{-r_iT_R}}{\theta_{2i}+r_i} + \frac{e^{-r_iT_R}-1}{r_i}\right) \end{bmatrix}. \tag{4.20}$$

Producer's total profit function for different l items is

$$\mathrm{TPFP} = \frac{1}{T}\sum_{i=1}^{l}[\mathrm{SRC}_P - \mathrm{SUC}_P - \mathrm{OC}_P - \mathrm{HC}_P - \mathrm{DC}_P]. \tag{4.21}$$

4.3. Model formation for supplier

To meet the total requirement for the retailers, every supplier has to give $\frac{kQs}{m}$ units of raw material to the producer in a fixed time interval, since m suppliers supply the raw material in total q shipment during the production period. Therefore in a fixed time interval T_s, the total $\frac{Qks}{qm}$ units of raw material will reach the producer from one supplier. The producer in the production house will procure raw material from the supplier keeping a closed watch on imperfect production and deterioration. The producer takes particular care that the retailer's total needs are fulfilled.

Supplier's inventory model is shown in Figure 5 and can be represented by the following first-order linear differential equation

$$\frac{\mathrm{d}Q_{si}(t)}{\mathrm{d}t} = -\sum_{\beta=1}^{m}\left(\frac{Qks}{qm}\right)P_{i\beta}(t) - \theta_{1i}Q_{si}(t), \qquad 0 \le t \le T_s \tag{4.22}$$

with condition $Q_{si}(T_s) = 0$. Solution of Equation (4.21) is

$$Q_{Si}(t) = \sum_{\beta=1}^{m}\frac{QksP_{i\beta}}{qm\theta_{1i}}\left(e^{\theta_{1i}(T_s-t)} - 1\right). \tag{4.23}$$

For a supplier, profit function depends on the following factors.

4.3.1. Sales revenue

Sales revenue is the total revenue of delivering/selling a product (raw materials) or service to the producer. Total sales revenue for the supplier is

$$\mathrm{SRC}_S = \sum_{\beta=1}^{m} A_{S\beta i}\left(\frac{kQs}{m}\right). \tag{4.24}$$

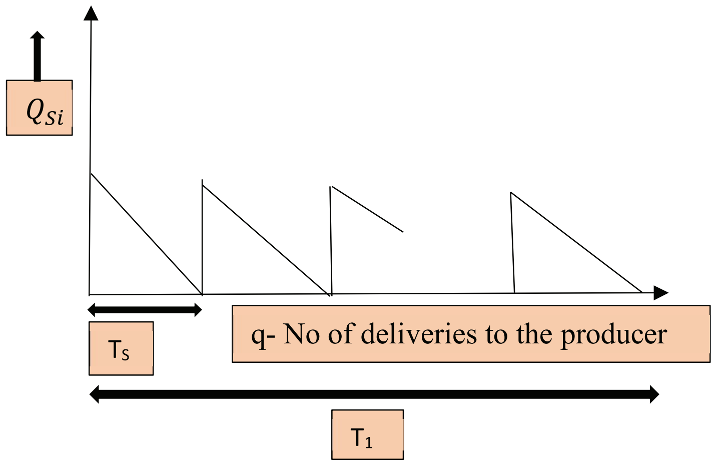


FIGURE 5. Supplier's inventory level.

4.3.2. Ordering cost

Ordering cost is the total cost of ordering the inventory, including order of inventory and finding an outsider supplier. Therefore, the total ordering cost for the supplier is

$$\mathrm{OC}_S = \sum_{\beta=1}^{m} O_{Si\beta}. \tag{4.25}$$

4.3.3. Holding cost

Holding cost involved in careful storage and maintenance of inventory, and the holding cost for the supplier is

$$\begin{aligned}\mathrm{HC}_S &= \sum_{\beta=1}^{M} h_{si\beta} \int_0^{T_S} Q_{si}(t) e^{-r_i t} \mathrm{d}t,\\ &= \sum_{\beta=1}^{m} h_{si\beta} \frac{QksP_{i\beta}}{qm\theta_{1i}} \left\{ \frac{e^{\theta_{1i}T_s} - e^{-\theta_{1i}r_i}}{\theta_{1i} + r_i} + \frac{e^{-r_i T_s} - 1}{r_i} \right\}.\end{aligned} \tag{4.26}$$

4.3.4. Deterioration cost

Different deterioration rates for different items and deteriorating cost involves due to the deterioration of items that become useless. Thus, the deteriorating cost for the supplier is

$$\begin{aligned}\mathrm{DC}_S &= \sum_{\beta=1}^{M} d_{si\beta}\theta_{1i} \int_0^{T_S} Q_{si}(t) e^{-r_i t} \mathrm{d}t,\\ &= \sum_{\beta=1}^{m} d_{si\beta} \frac{QksP_{i\beta}}{qm} \left\{ \frac{e^{\theta_{1i}T_s} - e^{-\theta_{1i}r_i}}{\theta_{1i} + r_i} + \frac{e^{-r_i T_s} - 1}{r_i} \right\}.\end{aligned} \tag{4.27}$$

4.3.5. Transportation cost

Transportation cost is mainly divided into two categories in which one is fixed transportation cost and the other is variable transportation cost. In which variable transportation cost depends on the demand transfer to producer and fixed transportation cost includes the monthly installment payment of the vehicle, insurance of the vehicle and administrative, managerial costs. Thus, the transportation cost of supplier is

$$\mathrm{TPC}_S = d_T + \sum_{\beta=1}^{m} d_{ti\beta}\left(\frac{Qks}{m}\right)\cdot \tag{4.28}$$

Supplier's total profit function for different l items is

$$\mathrm{TPFS} = \sum_{i=1}^{l} \frac{q}{T}[\mathrm{SRC}_S - \mathrm{OC}_S - \mathrm{HC}_S - \mathrm{DC}_S - \mathrm{TPC}_S]. \tag{4.29}$$

The total profit of this model per replenishment of all l items is the sum of TPFR, TPFS, and TPFP.

$$\mathrm{TPC} = \mathrm{TPFR} + \mathrm{TPFP} + \mathrm{TPFS}$$

$$= \sum_{i=1}^{l} \left[\begin{array}{l} \frac{q}{T}\left[\begin{array}{l} -\sum_{\beta=1}^{m} O_{Si\beta} - d_T \\ +\sum_{\beta=1}^{m}\sum_{j=1}^{k}\left\{\begin{array}{l} \frac{sA_{Si\beta}D_{ij}T}{m} - d_{ti\beta}D_{ij}T\frac{s}{m} \\ -\frac{D_{ij}r_iP_{i\beta}T_1^2Ts}{2mq^3}\left(\frac{h_{si\beta}}{\theta_{1i}} + d_{si\beta}\right) \end{array}\right\} \end{array}\right] \\ +\frac{1}{T}\left[\begin{array}{l} V_iT\sum_{j=1}^{k} D_{ij} - S_{Pi} - O_{Pi} \\ -(d_i\theta_{2i} + C_{Pi})\left\{\begin{array}{l} \frac{\sum_{j=1}^{k} TD_{ij}\lambda - nP_i}{n\theta_{2i}}\left(\frac{r_iT_1^2}{2}\right) \\ +(1 - r_iT_1)(n-\lambda)\sum_{j=1}^{k}\frac{D_{ij}T}{2\theta_{2i}n}(T - T_1)^2 \\ -[(n-\lambda)(1 - r_iT_1) - \lambda]\sum_{j=1}^{k}\frac{D_{ij}r_iT^2}{2n^2\theta_{2i}} \end{array}\right\} \end{array}\right] \\ +\frac{n}{T}\left[\begin{array}{l} \sum_{j=1}^{k}\left\{\begin{array}{l} A_{Rij}D_{ij}\frac{T}{n} - O_{Rij} - \frac{D_{ij}U_{ij}T}{nk} \\ -\frac{R_{tij}D_{ij}T}{nK} - \frac{d_{Rij}D_{ij}r_iT^2}{2n^2} \end{array}\right\} - kR_T \\ -\sum_{j=1}^{k}\frac{D_{ij}}{\theta_{2i}}\left\{\begin{array}{l} \frac{a_{ij}r_iT^2}{2n^2} + \frac{b_{ij}r_iT^2}{n^2(r_i+\theta_{2i})} \\ -\frac{b_{ij}T^2}{n^2} \end{array}\right\} \end{array}\right] \end{array}\right]. \tag{4.30}$$

5. Fuzzy model

In order to discuss the fuzzy model in this supply chain, there is a need for some important definitions, which are as follows:

Definition 5.1. A fuzzy set $\widetilde{a}$ on the interval $(-\infty, \infty)$ is called a fuzzy point if its membership function (MF) is

$$\mu_{\widetilde{a}}(x) = \left\{\begin{array}{ll} 1, & x = a \\ 0, & x \neq a \end{array}\right\},$$

where a is the support point of the fuzzy set.

Definition 5.2. A fuzzy set $[p_a, q_a]$ where $0 \le a \le 1$, p, $q \in R$ and $p < q$, is called a level of the fuzzy interval if its MF is

$$\mu_{[p_a,q_a]}(x) = \left\{\begin{array}{ll} a, & p \le x \le q \\ 0, & \text{otherwise} \end{array}\right\}.$$

Definition 5.3. A fuzzy number $S = (p, q, \zeta)$ where $p < q < \zeta$ and p, q, $\zeta \in R$, is called triangular fuzzy number (TFN) if its MF is

$$\mu_L = \left\{\begin{array}{ll} \frac{x-p}{q-p}, & p \le x \le q \\ \frac{\zeta - x}{\zeta - q}, & q \le x \le \zeta \\ 0, & \text{otherwise} \end{array}\right\},$$

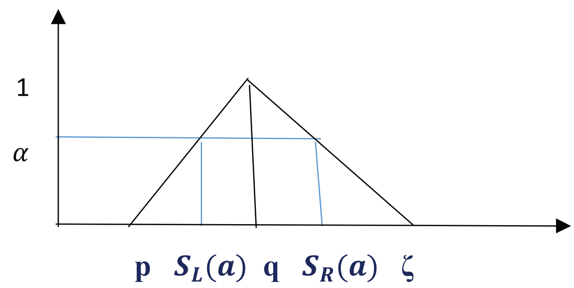


FIGURE 6. a-cut of triangular fuzzy number.

when $p = q = \zeta$, we have a fuzzy point, $(\zeta, \zeta, \zeta) = \widetilde{\zeta}$.

The family of all TFN on R is denoted as

$$E_N = \{(p, q, \zeta) | p < q < \zeta \qquad \forall p, q, \zeta \in R\}.$$

The a cut of $S = (p, q, \zeta) \in E_N$, $0 \le a \le 1$ is

$$S(a) = [S_L(a), S_R(a)]$$

where $S_L(a) = p + (q - p)a$ and $S_R(a) = \zeta - (\zeta - q)a$ are the left and right endpoint of $S(a)$.

Definition 5.4. The centroid method on the triangular fuzzy number $S = (p, q, \zeta)$ is define as (Fig. 6)

$$C(S) = \frac{p + q + \zeta}{3}.$$

Due to uncertainly it is not easy to define the parameters precisely, we assumed inflation rate r_i may change within some limit.

Let $r_i = (r_{i_1}, r_{i_2}, r_{i_3})$ as a triangular fuzzy number.

Total profit function in a fuzzy sense is

$$\widetilde{\mathrm{TPC}} = \sum_{i=1}^{l} \left[\begin{array}{l} \frac{q}{T} \left[\begin{array}{l} -\sum_{\beta=1}^{m} O_{Si\beta} - d_T \\ +\sum_{\beta=1}^{m} \sum_{j=1}^{k} \left\{ \begin{array}{l} \frac{sA_{Si\beta} D_{ij} T}{m} - d_{ti\beta} D_{ij} T \frac{s}{m} \\ -\frac{D_{ij} \widetilde{r}_i P_{i\beta} T_1^2 Ts}{2mq^3} \left(\frac{h_{si\beta}}{\theta_{1i}} + d_{si\beta} \right) \end{array} \right\} \end{array} \right] \\ +\frac{1}{T} \left[\begin{array}{l} V_i T \sum_{j=1}^{k} D_{ij} - S_{Pi} - O_{Pi} \\ -(d_i \theta_{2i} + C_{Pi}) \left\{ \begin{array}{l} \frac{\sum_{j=1}^{k} T D_{ij} \lambda - n P_i}{n \theta_{2i}} \left(\frac{\widetilde{r}_i T_1^2}{2} \right) \\ +(1 - \widetilde{r}_i T_1)(n - \lambda) \sum_{j=1}^{k} \frac{D_{ij} T}{2\theta_{2i} n} (T - T_1)^2 \\ -[(n - \lambda)(1 - \widetilde{r}_i T_1) - \lambda] \sum_{j=1}^{k} \frac{D_{ij} \widetilde{r}_i T^2}{2n^2 \theta_{2i}} \end{array} \right\} \end{array} \right] \\ +\frac{n}{T} \left[\begin{array}{l} \sum_{j=1}^{k} \left\{ A_{Rij} D_{ij} \frac{T}{n} - O_{Rij} - \frac{D_{ij} U_{ij} T}{nk} - \frac{R_{tij} D_{ij} T}{nK} - \frac{d_{Rij} D_{ij} \widetilde{r}_i T^2}{2n^2} \right\} - kR_T \\ -\sum_{j=1}^{k} \frac{D_{ij}}{\theta_{2i}} \left\{ \frac{a_{ij} \widetilde{r}_i T^2}{2n^2} + \frac{b_{ij} \widetilde{r}_i T^2}{n^2 (\widetilde{r}_i + \theta_2 i)} - \frac{b_{ij} T^2}{n^2} \right\} \end{array} \right] \end{array} \right]. \tag{5.1}$$

Defuzzify the total profit function in fuzzy sense by the centroid method and it is represented by $\mathrm{TPC_{CM}}$ as

$$\mathrm{TPC_{CM}} = \frac{1}{3}[\mathrm{TPC_{CM_1}}(T_1) + \mathrm{TPC_{CM_2}}(T_1) + \mathrm{TPC}(T_1)],$$

where

$$
\mathrm{TPC}_{\mathrm{CM}_\pi} = \sum_{i=1}^{l} \left[\begin{array}{l} \frac{q}{T} \left[\begin{array}{l} -\sum_{\beta=1}^{m} O_{Si\beta} - d_T \\ +\sum_{\beta=1}^{m} \sum_{j=1}^{k} \left\{ \begin{array}{l} \frac{sA_{Si\beta} D_{ij} T}{m} - d_{ti\beta} D_{ij} T \frac{s}{m} \\ -\frac{D_{ij} \widetilde{r_{i_\pi}} P_{i\beta} T_1^2 T s}{2mq^3} \left(\frac{h_{si\beta}}{\theta_{1i}} + d_{si\beta} \right) \end{array} \right\} \end{array} \right] \\ +\frac{1}{T} \left[\begin{array}{l} V_i T \sum_{j=1}^{k} D_{ij} - S_{Pi} - O_{Pi} \\ -(d_i \theta_{2i} + C_{Pi}) \left\{ \begin{array}{l} \frac{\sum_{j=1}^{k} T D_{ij} \lambda - n P_i}{n\theta_{2i}} \left(\frac{\widetilde{r_{i_\pi}} T_1^2}{2} \right) \\ +(1 - \widetilde{r_{i_\pi}} T_1)(n-\lambda) \sum_{j=1}^{k} \frac{D_{ij} T}{2\theta_{2i} n} (T - T_1)^2 \\ -[(n-\lambda)(1 - \widetilde{r_{i_\pi}} T_1) - \lambda] \sum_{j=1}^{k} \frac{D_{ij} \widetilde{r_{i_\pi}} T^2}{2n^2 \theta_{2i}} \end{array} \right\} \end{array} \right] \\ +\frac{n}{T} \left[\begin{array}{l} \sum_{j=1}^{k} \left\{ A_{Rij} D_{ij} \frac{T}{n} - O_{Rij} - \frac{D_{ij} U_{ij} T}{nk} - \frac{R_{tij} D_{ij} T}{nK} - \frac{d_{Rij} D_{ij} \widetilde{r_{i_\pi}} T^2}{2n^2} \right\} - kR_T \\ -\sum_{j=1}^{k} \frac{D_{ij}}{\theta_{2i}} \left\{ \frac{a_{ij} r_{i_\pi} T^2}{2n^2} + \frac{b_{ij} \widetilde{r_{i_\pi}} T^2}{n^2 (\widetilde{r_{i_\pi}} + \theta_2 i)} - \frac{b_{ij} T^2}{n^2} \right\} \end{array} \right] \end{array} \right] \tag{5.2}
$$

where $\pi = 1, 2, 3$.

6. Solution methodology

Total profit function in crisp model denoted by TPC, total profit function in a fuzzy sense is denoted by $\mathrm{TPC}_{\mathrm{CM}}$, and both are a function of a single variable T_1. The necessary condition for TPC and $\mathrm{TPC}_{\mathrm{CM}}$ to be maximized in both crisp and fuzzy models is

$$
\frac{\mathrm{d}\,\mathrm{TPC}(T_1)}{\mathrm{d}T_1} = 0, \tag{6.1}
$$

$$
\frac{\mathrm{d}\,\mathrm{TPC}_{\mathrm{CM}}(T_1)}{\mathrm{d}T_1} = 0. \tag{6.2}
$$

Provided both TPC and $\mathrm{TPC}_{\mathrm{CM}}$ satisfy the condition

$$
\frac{\mathrm{d}^2\mathrm{TPC}(T_1)}{\mathrm{d}T_1^2} < 0, \quad \frac{\mathrm{d}^2\mathrm{TPC}_{\mathrm{CM}}(T_1)}{\mathrm{d}T_1^2} < 0. \tag{6.3}
$$

To maximize the total profit function, the optimum value of T_1 for both crisp and fuzzy sense can be obtained by Equations (6.1) and (6.2), respectively. The Mathematica 5.2 Software is used to solve all these complex equations.

Lemma. *When the cycle time T and T_2 are fixed, then the total cost function is concave with respect to cycle time T_1.*

Proof. See Appendix A. □

7. Numerical analysis and sensitivity analysis

7.1. Numerical analysis

In the practical way, the following value of various parameters are given for this model

$m = 2$, $k = 2$, $n = 5$, $l = 2$, $q = 5$, $s = 4$, $\theta_{11} = 0.01$, $\theta_{12} = 0.02$, $\theta_{21} = 0.02$, $\theta_{22} = 0.025$, $A_{S11} = \$40$ per unit, $A_{S12} = \$50$ per unit, $A_{S21} = \$60$ per unit, $A_{S22} = \$55$ per unit, $r_1 = 0.005$, $r_2 = 0.007$, $O_{S21} = \$20$ per order, $O_{S22} = \$10$ per order, $O_{S11} = \$10$ per order, $O_{S12} = \$15$ per order, $h_{S21} = \$0.02$ per unit, $h_{S22} = \$0.012$ per unit, $h_{S11} = \$0.01$ per unit, $h_{S12} = \$0.03$ per unit, $d_{S11} = \$0.1$ per unit, $d_{S12} = \$0.1$ per unit, $d_{S21} = \$0.2$ per unit, $d_{S22} = \$0.2$ per unit, $d_T = \$25$ per unit, $d_{t11} = \$10$ per unit, $d_{t12} = \$15$ per unit, $d_{t21} = \$15$ per unit, $d_{t22} = \$25$ per unit, $V_1 = \$10$ per unit, $V_2 = \$15$ per unit, $S_{P1} = \$100$ per setup, $S_{P2} = \$200$ per setup, $O_{P1} = \$20$ per order, $O_{P2} = \$30$ per order, $C_{P1} = \$0.02$ per unit, $C_{P2} = \$0.03$ per unit, $d_1 = \$0.05$ per unit,

d_2 = \$0.07 per unit, O_{R11} = \$5 per order, O_{R12} = \$7 per order, O_{R21} = \$8 per order, O_{R22} = \$6 per order, A_{R11} = \$100 per unit, A_{R12} = \$150 per unit, A_{R21} = \$150 per unit, A_{R22} = \$250 per unit, U_{11} = \$0.10 per unit, U_{12} = \$0.15 per unit, U_{21} = \$0.15 per unit, U_{22} = \$0.25 per unit, R_T = \$15 per unit, R_{t11} = \$5 per unit, R_{t12} = \$6.5 per unit, R_{t21} = \$6 per unit, R_{t22} = \$7 per unit, d_{R11} = 0.\$10 per unit, d_{R12} = \$0.15 per unit, d_{R21} = \$0.12 per unit, d_{R22} = \$0.25 per unit, P_1 = 300 unit, P_2 = 400 unit, D_{11} = 15 unit, D_{12} = 17 unit, D_{21} = 10 unit, D_{22} = 20 unit, P_{11} = 25 unit, P_{12} = 35 unit, P_{21} = 30 unit, P_{22} = 40 unit, $a_{11} = 0.1$, $a_{12} = 0.2$, $a_{21} = 0.12$, $a_{22} = 0.15$, $b_{11} = 0.5$, $b_{12} = 0.35$, $b_{21} = 0.3$, $b_{22} = 0.4$, T = 150 days, $(r_{1_1}, r_{1_2}, r_{1_3}) = (0.007, 0.015, 0.02)$ and $(r_{2_1}, r_{2_2}, r_{2_3}) = (0.006, 0.018, 0.02)$.

7.1.1. Optimality representation of profit function with optimal values in crisp model

The optimal values of T_1^* is 120.54 days, TPC is \$59 221.02. T_R is 50 days, and T_S is 24.108 days.

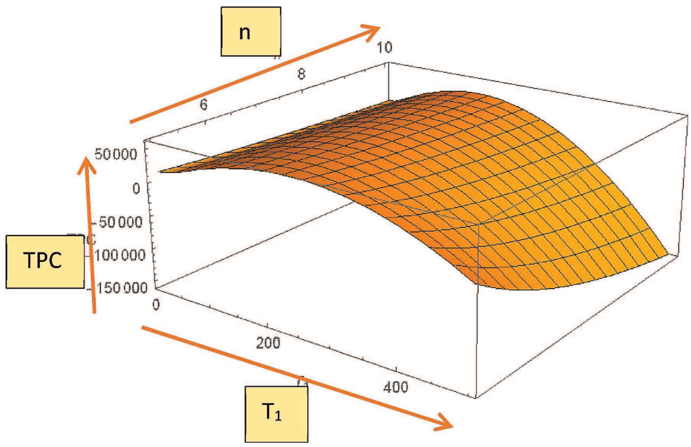


FIGURE 7. Concavity of total profit function in crisp model with respect to T_1.

7.1.2. Optimality representation of profit function with optimal values in fuzzy model

The optimal values of T_1^* is 83.694 days, TPC_{CM} is \$64 993.724, T_R is 50 days and T_S is 16.7388 days.

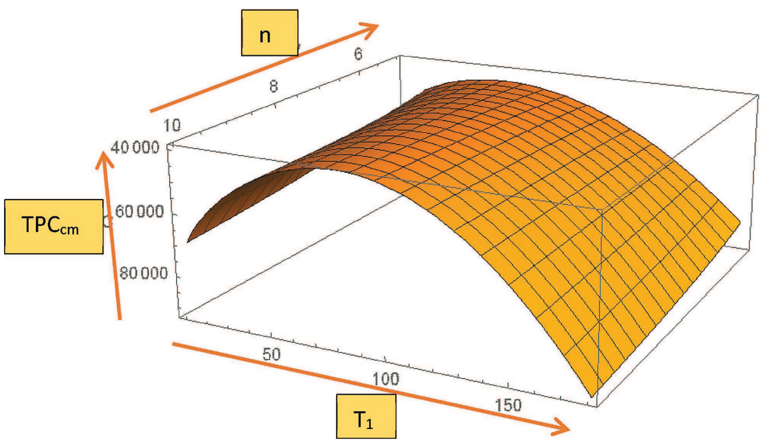


FIGURE 8. Concavity of total cost function in fuzzy sense with respect to T_1.

7.2. Sensitivity analysis

Taking the numerical values described above, sensitivity analysis for this model is shown below.

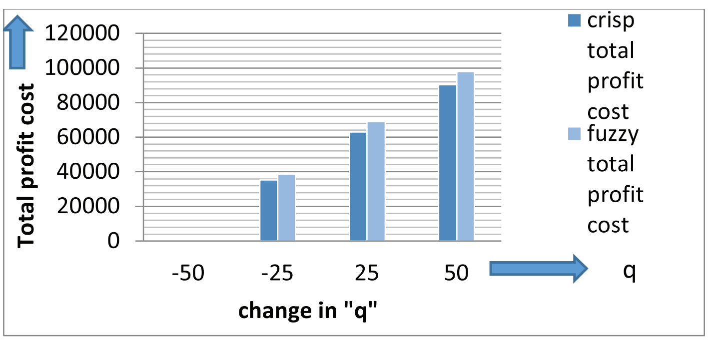
Parameter	% change in parameter	Crisp total cost	% change in crisp cost	Fuzzy total cost	% change in fuzzy cost
Q	−50	Not exist	–	Not exist	–
	−25	35 286.3	−0.4	38 434.7	−0.4
	+25	62 953	+0.06	69 039.4	+0.06
	+50	90 233.9	+0.52	97 881	+0.52
N	−50	88 755.5	+0.49	63 504.7	−0.002
	−25	69 067.2	+0.16	64 499.3	−0.008
	+25	57 814.3	−0.023	65 064.2	+0.002
	+50	49 373	−0.16	65 485.8	+0.008
S	−50	45 999	−0.22	53 458.9	−0.177
	−25	52 610	−0.112	59 226.3	−0.088
	+25	65 832	+0.112	70 761.1	+0.088
	+50	72 443	+0.22	76 528.5	+0.177
P_1	−50	59 144.7	−0.0015	64 890.8	−0.0015
	−25	59 183	−0.0006	64 942.2	−0.0008
	+25	59 259	+0.0006	65 045.2	+0.0007
	+50	59 297.3	+0.0015	65 096.7	+0.0015
P_2	−50	59 005.7	−0.003	64 819.8	−0.002
	−25	59 070.3	−0.002	64 906.7	−0.001
	+25	59 199.5	−0.0003	65 080.7	+0.001
	+50	59 264	+0.0007	65 167.7	+0.002
V_1	−50	59 061	−0.003	64 833.7	−0.002
	−25	59 141	−0.001	64 913.7	−0001
	+25	59 301	+0.001	65 073.7	+0.001
	+50	59 381	+0.003	65 153.7	+0.002
V_2	−50	58 996	−0.0037	64 768.7	−0.003
	−25	59 108.5	−0.0012	64 881.2	−0.002
	+25	59 333.5	+0.0012	65 106.2	+0.002
	+50	59 446	+0.0037	65 218.7	+0.003
SP_1	−50	59 221.4	0	64 994	0
	−25	59 221.2	0	64 993.9	0
	+25	59 221.9	0	64 993.6	0
	+50	59 220.7	0	64 993.4	0
SP_2	−50	59 221.7	0	64 994.4	0
	−25	59 221.3	0	64 994.1	0
	+25	59 221.4	0	64 993.4	0
	+50	59 221.2	0	64 993.1	0
OP_1	−50	59 221.1	0	64 993.1	0
	−25	59 221.1	0	64 994	0
	+25	59 221	0	64 994	0
	+50	59 221.1	0	64 994.1	0
OP_1	−50	59 221.1	0	64 993.6	0
	−25	59 220.7	0	64 994.1	0
	+25	59 221	0	64 993.9	0
	+50	59 220.9	0	64 994	0
CP_1	−50	60 601	+0.023	62 953.3	−0.032
	−25	59 911	+0.0116	63 975.5	−0.015
	+25	58 531	−0.0116	66 014	+0.015
	+50	57 841	−0.023	67 034	+0.032
CP_2	−50	60 069.6	+0.014	60 096.6	−0.075
	−25	59 658.8	+0.007	59 658.8	−0.082
	+25	58 783	−0.007	58 783.2	−0.095
	+50	58 345.4	−0.014	58 345.4	−0.102
d_1	−50	59 290	+0.0012	64 891.7	−0.001
	−25	59 255.5	+0.0005	64 942.7	−0.0007
	+25	59 186.5	−0.0005	65 044.7	+0.0007
	+50	59 152	−0.0012	65 095.7	+0.001
d_2	−50	59 272	+0.0008	64 806.6	−0.003
	−25	59 246.6	+0.0004	64 900	−0.001
	+25	59 195.5	−0.0004	65 087.3	+0.001
	+50	59 170	−0.0008	65 180	+0.003

7.3. Analysis and discussions

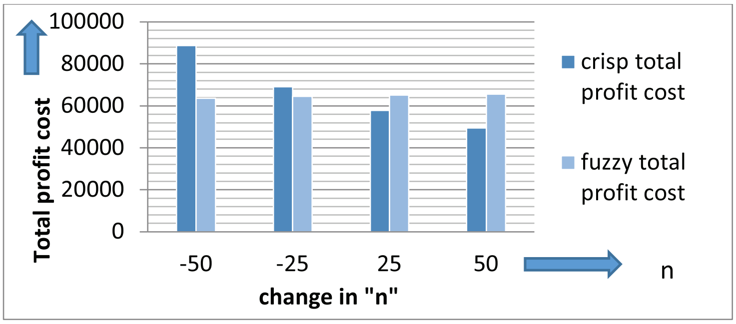
(1) It is seen that if the change occurs in parameter q, the profit function is increased in both crisp and fuzzy environments, but with the -50% change in q, the profit function does not exist.
(2) If 25% and 50% change occurs in the number of shipments received through the producer to the retailer, then the fuzzy model has positive, but in the crisp model, a slight negative change occurs. It is a different result when -25% and -50% change is done in parameter n.
(3) If the change occurs in parameter s, then profit function increases in both crisp and fuzzy environments, and there are some minor changes analyzed with respect to the change in parameter s. In each case, the fuzzy profit function is maximum as compared to the crisp model.
(4) If parameter P_1 changes, then the profit function for crisp and fuzzy models increases.
(5) If the change in parameter P_2 occurs, then the profit function increases. The total changes in both the crisp profit function and the fuzzy profit function are approximately the same. The fuzzy profit function is maximum with the -50%, -25%, $+25\%$, and $+50\%$ changes in P_2 as compare to the crisp profit function.
(6) The total percentage change in both crisp and fuzzy profit functions is approximate zero. While there are increasing changes in both profit functions due to change in V_1. In each case, the fuzzy profit function is maximum compared to the crisp profit function, which happens with the parameter V_2.
(7) There is a particular case of no effect on both profit functions with the change in S_{P1}, S_{P2}, O_{P1} and O_{P2}. The profit function remains the same; therefore, the percentage change is zero with each case of -50%, -25%, $+25\%$, and $+50\%$.
(8) Some minor decreasing changes in the crisp profit function occurs due to change in C_{P1}. While fuzzy profit function is increased with the change in C_{P1} but in each case, the fuzzy profit function is maximum than the crisp profit function.
(9) There are some minor decreasing changes in the crisp profit function due to changes in C_{P2}. While the changes in the fuzzy profit function are negative for the changes in C_{P2}. The fuzzy profit function continuously decreases due to percentage changes in the parameter.
(10) The sensitivity analysis in both the crisp profit function and the fuzzy profit function with respect to the parameter d_1 and d_2 has been analyzed. The total percentage change in both the profit function is approximate zero. While there are decreasing changes in the crisp profit function and increasing changes in the fuzzy profit function due to changes in both parameters.

7.4. Graphical illustration of the model

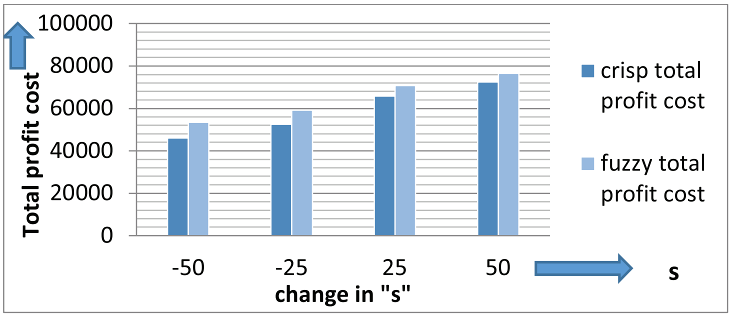
The variation in the profit function with respect to the several parameters in the crisp and fuzzy environment are shown through graphs (a) to (o).



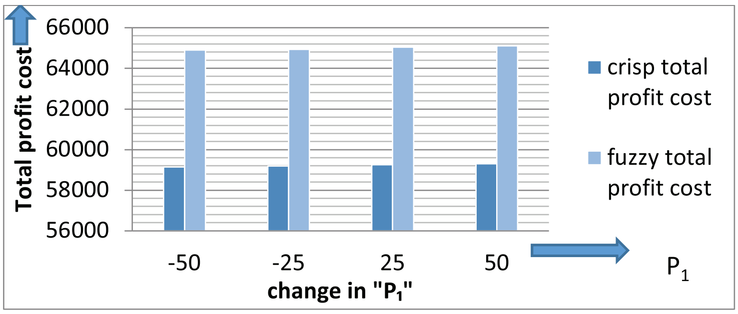
(a) Nature of profit function in a crisp and fuzzy environment with respect to parameter q.



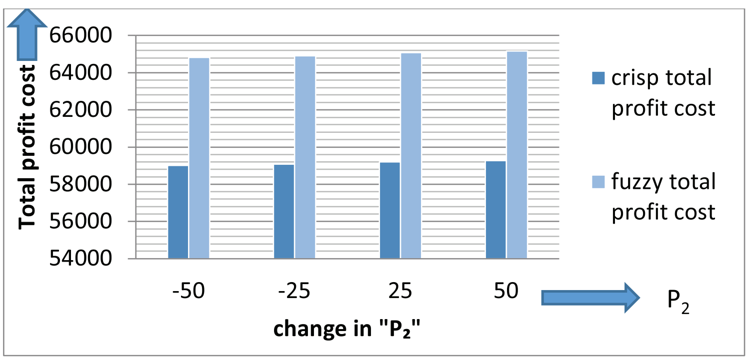
(b) Nature of profit function in a crisp and fuzzy environment with respect to parameter n.



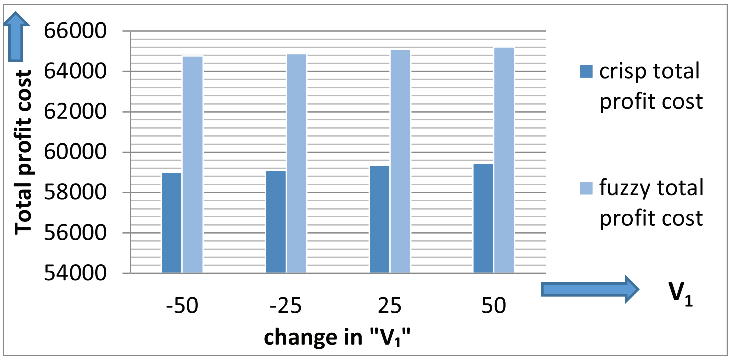
(c) Nature of profit function in a crisp and fuzzy environment with respect to parameter s.



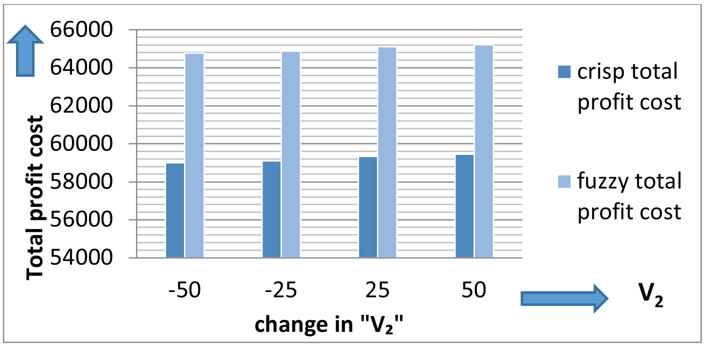
(d) Nature of profit function in crisp and fuzzy environment with respect to parameter P_1.



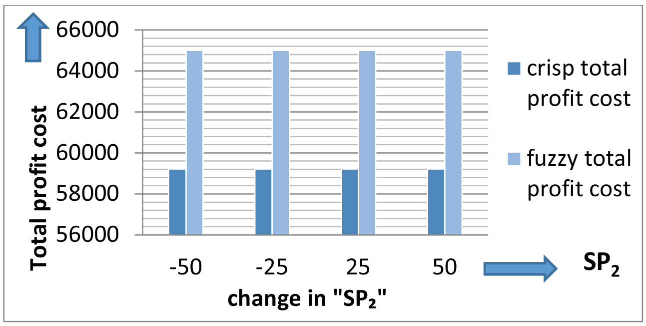
(e) Nature of profit function in a crisp and fuzzy environment with respect to parameter P_2.



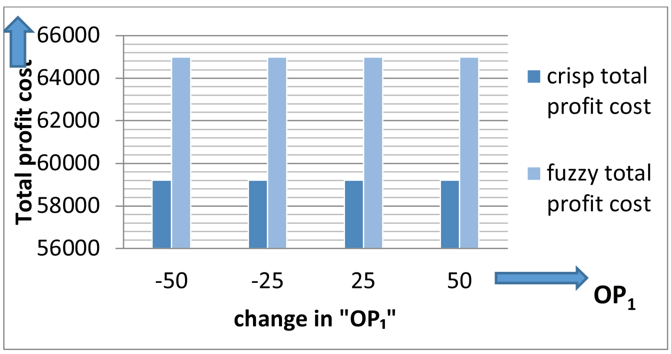
(f) Nature of profit function in a crisp and fuzzy environment with respect to parameter V_1.



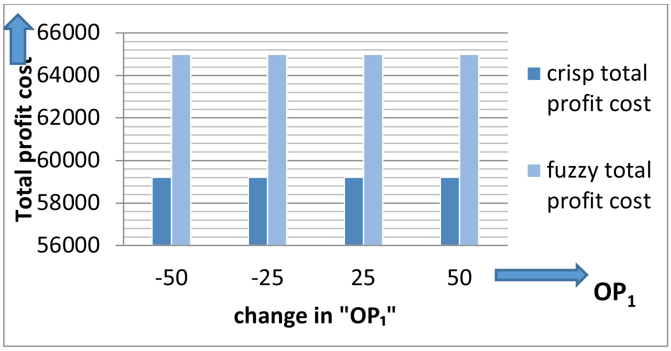
(g) Nature of profit function in a crisp and fuzzy environment with respect to parameter V_2.



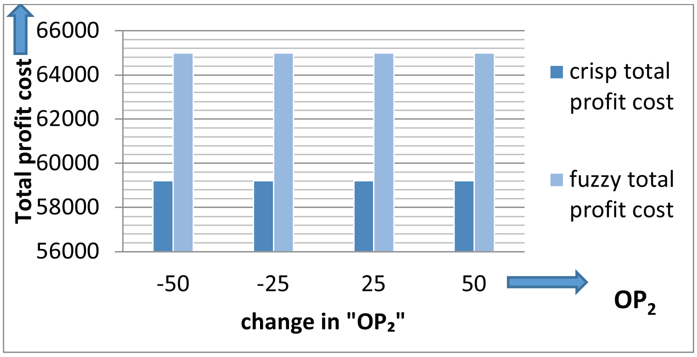
(h) Nature of profit function in a crisp and fuzzy environment with respect to parameter SP_1.



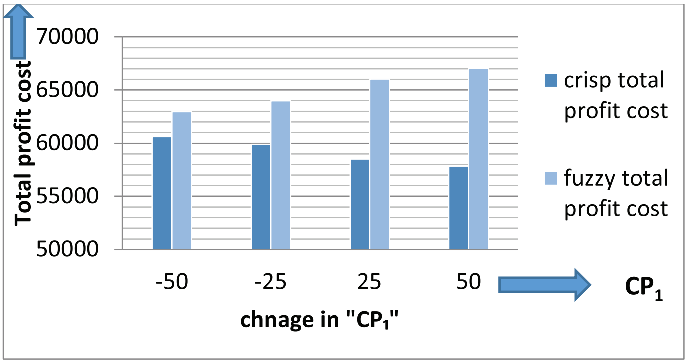
(i) Nature of profit function in a crisp and fuzzy environment with respect to parameter SP_2.



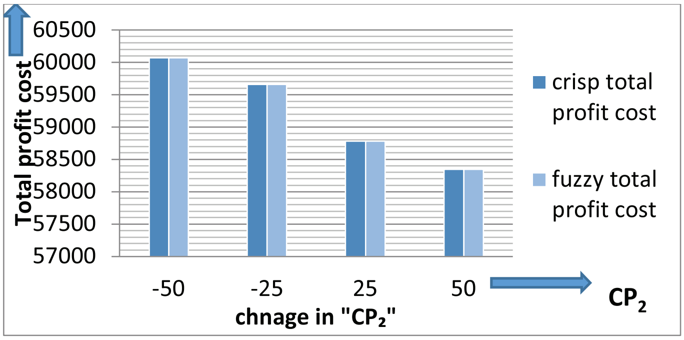
(j) Nature of profit function in a crisp and fuzzy environment with respect to parameter OP_1.



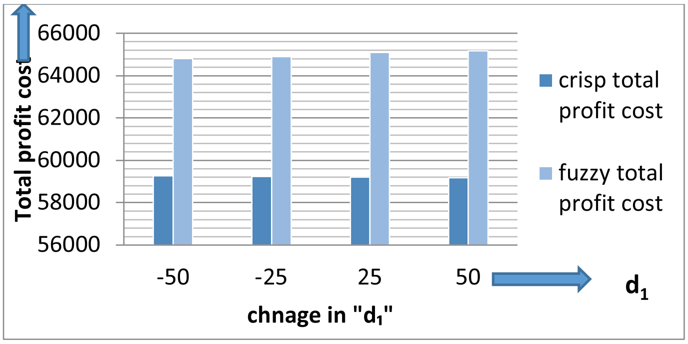
(k) Nature of profit function in a crisp and fuzzy environment with respect to parameter OP_2.



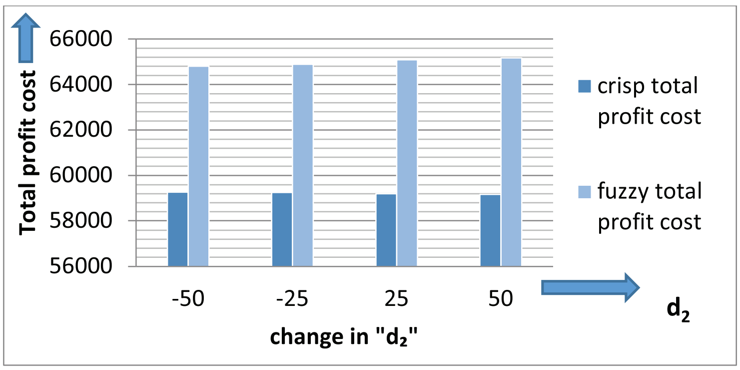
(l) Nature of profit function in crisp and fuzzy environment with respect to parameter CP_1.



(m) Nature of profit function in crisp and fuzzy environment with respect to parameter CP_2.



(n) Nature of profit function in crisp and fuzzy environment with respect to parameter d_1.



(o) Nature of profit function in a crisp and fuzzy environment with respect to parameter d_2.

8. Discussions

- Due to the current scenario in the market, the member of the supply chain system does not need to benefit from every product. However, by using more than one product, the profit from one product can be filled with another product, and the model is not affected. Different inflation rates and different deterioration rates for different goods are considered because inflation has put the most vulnerable section of consumers (daily wage earners) under stress, who mainly demand goods in less time and smaller quantities, and each product has its different period of survival due to which their damage time may be different.
- Figures 7 and 8 show the concavity of profit function with respect to the decision parameter T_1.

9. Managerial insights

- Through the graphical representation of profit function in crisp and fuzzy environment, it has been analyzed that profit is maximum in the fuzzy environment compared to crisp, but with respect to a parameter n profit is minimum in fuzzy environment.
- In this model, it is observed that the number of shipments taken from the supplier is increased during the production period. The total profit is increasing in both crisp and fuzzy, and if a positive change occurs in the number of shipments received through the producer to the retailer, then the fuzzy model has positive, but in the crisp model, a slight negative change occurs.
- In this model, every item has a different time duration to stay safe, due to which the rate of spoilage of the item is different. It impacts deterioration cost and the inflation rate, and it has been observed that as deterioration cost increases, there is a slight decrement in the profit function in the crisp model but, profit function increases in the fuzzy model.
- During the graphical representation of total profit cost with respect to the various parameters, it is observed that profit is maximum in the fuzzy environment compared to a crisp in most of the graphs.

10. Conclusions

This paper is formulated for the imperfect production process, but the goods damaged cannot be reused during the imperfect production process. This model has been developed for the multi-item (raw form), which is transferred from multi-retailer to single producer and then producer converts the raw material into finished goods delivered to the multi-supplier. This complete model has been developed in an environment of fuzzy and inflation. This model is illustrated through the numerical example, and the variation in the various parameters with respect to the profit function is analyzed through sensitivity analysis. It has been found that the supplier is giving raw material to the producer in as much shipment and the producer is giving the finished goods to the retailer for as low shipment, and the profit is increasing. It is observed that the number of shipments taken from the supplier is increased during the production period. The total profit is increasing in both crisp and fuzzy, and if a positive change occurs in the number of shipments received through the producer to the retailer, then the fuzzy model has positive, but in the crisp model, a slight negative change occurs. This model is not applicable in the case of a single supplier, single retailer, and multi-producer. The model can be extended with time-dependent demand, variable production in multi-producer, multi-supplier, multi-retailer, and remanufacturing batches in the green supply chain under carbon emissions reduction, variable deteriorating, and preservation technology with cloudy fuzzy and replenishment policy for deteriorating items with allowable shortage and trade credit policy.

Appendix A.

Both the first and second order derivatives of total cost function TPC with respect to T_1 are given below

$$\frac{\mathrm{d}\,\mathrm{TPC}}{\mathrm{d}T_1} = \sum_{i=1}^{l} \left[\begin{array}{l} \frac{q}{T}\left[\sum_{\beta=1}^{m}\sum_{j=1}^{k}\left\{\frac{-D_{ij}r_iP_{i\beta}T_1TS}{mq^3}\left(\frac{h_{si\beta}}{\theta_{1i}}+d_{si\beta}\right)\right\}\right] \\ +\frac{1}{T}\left[-(d_i\theta_{2i}+C_{pi})\left\{ \begin{array}{l} \frac{\sum_{j=1}^{k}TD_{ij}\lambda-nP_i}{n\theta_{2i}}r_iT_1 \\ +\left((n-\lambda)\sum_{j=1}^{k}\frac{D_{ij}T}{2\theta_{2i}n}(-r_i)(T-T_1)^2\right) \\ +\left((n-\lambda)\sum_{j=1}^{k}\frac{D_{ij}T}{\theta_{2i}n}(1-r_iT_1)(T-T_1)(-1)\right) \\ +(n-\lambda)r_i\sum_{j=1}^{k}\frac{D_{ij}r_iT^2}{2\theta_{2i}n^2} \end{array} \right\}\right] \end{array} \right] \tag{A.1}$$

$$\frac{\mathrm{d}^2\mathrm{TPC}}{\mathrm{d}T_1^2} = -\sum_{i=1}^{l}\left[\begin{array}{l}\frac{q}{T}\left[\sum_{\beta=1}^{m}\sum_{j=1}^{k}\left\{\frac{D_{ij}r_iP_{i\beta}TS}{mq^3}\left(\frac{h_{si\beta}}{\theta_{1i}}+d_{si\beta}\right)\right\}\right]\\ +\frac{1}{T}\left[(d_i\theta_{2i}+C_{pi})\left\{\begin{array}{l}\frac{\sum_{j=1}^{k}TD_{ij}\lambda-nP_i}{n\theta_{2i}}r_i\\ +\left((n-\lambda)\sum_{j=1}^{k}\frac{D_{ij}r_i(T-T_1)}{\theta_{2i}n}\right)\end{array}\right\}\right]\end{array}\right] \tag{A.2}$$

$$\Rightarrow \frac{\mathrm{d}^2\mathrm{TPC}}{\mathrm{d}T_1^2} < 0.$$

Hence TPC is concave with respect to T_1.

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