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STUDY ON THE COMPLEXITY OF CHANNEL PRICING GAME IN SHOWROOMING O2O SUPPLY CHAIN

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Abstract. An O2O supply chain consisting of a manufacturer with an online direct channel and a retailer who resells through a brick-and-mortar store is considered. Three channel power structures (vertical Nash, manufacturer Stackelberg, and retailer Stackelberg) and three pricing sequences (simultaneous pricing, manufacturer pricing early, retailer pricing early) are considered. Counter-intuitively, under the manufacturer Stackelberg structure, the retailer has a first-mover advantage and retailer-pricing-early achieves Pareto optimality. In the other cases, the manufacturer and the retailer have a late-mover advantage. Under the vertical Nash structure, both parties may get into a prisoner's dilemma. Extending the basic model to dynamic pricing, we found that the first mover of sequential pricing has better stability. The retailer Stackelberg structure has better stability than the vertical Nash structure and the manufacturer Stackelberg is the most unstable power structure. To avoid the negative impact of equilibrium price instability, the vertical and horizontal price matching mechanisms are proposed and the effectiveness in improving stability is proved.

Mathematics Subject Classification. 90B60, 91A06.

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1. Introduction

With the booming of e-commerce, more and more manufacturers have opened online channels such as official online shopping malls to earn extra profits. However, from the perspective of incumbent retailers, it is a great threat. Orders placed through the direct channel are switched from the traditional retail channel. Competition between retailers and manufacturers across offline and online channels is becoming more intense when customers engage in showrooming. Brick-and-mortar stores allow customers to experience and identify their "best-fit" products while online shopping has a low-priced advantage. Therefore, some strategic customers check out products first at brick-and-mortar (hereafter BM) stores but buy them online at a lower price. This free-riding behavior of customers is referred to as "showrooming". This phenomenon results in losing potential customers of BM store.

In practice, there exist three pricing sequences between the manufacturer and the retailer in O2O (online to offline) supply chains: manufacturer pricing first, retailer pricing first and simultaneous pricing. For each pricing sequence, the corresponding case can be found in the market. The details have been summarized in Table 1.

Keywords. Endogenous pricing timing, dynamic pricing, power structure, showrooming, price matching.

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Table 1. Different price sequences and examples.

Case	Pricing sequences	Examples	Corresponding implications
1	Manufacturer pricing early	Best Buy, the electrical goods retailer, makes price decisions for the computer of Dell after the PC manufacturer Dell set the price for the direct channel	Manufacturers first decide the price in the real market
2	Retailer pricing early	Case 2: Fast fashion apparel brands like H&M and Gap determine prices in an online mall after BM stores set their prices [18]	Retailers first decide the price in the real market
3	Simultaneous pricing	Case 3: GREE, the world's largest air conditioning company, simultaneously sets prices on mall.gree.com (direct channel), its offline independent retailers GOME and Suning	Manufacturers and retailers decide the price at the same time

Therefore, the following questions arise: (1) Are manufacturers and retailers prefer to price first or price later? (2) What will be the pricing sequences of the online and offline channels?

To solve these problems, we investigate an O2O supply chain composed of a manufacturer with a direct online channel and a retailer that resells through the BM store. Three supply chain power structures (manufacturer Stackelberg, retailer Stackelberg, and vertical Nash) and three pricing sequences (manufacturer pricing early, retailer pricing early and simultaneous pricing) are taken into account. We adopt two-stage game to figure out the endogenous pricing timing issue. In the first stage, both parties choose their pricing timing respectively: pricing early or late. In the second stage, the manufacturer and the retailer set retail prices to maximize profits according to the pricing sequence. If the manufacturer and the retailer make the same pricing timing decision, they will set prices simultaneously. Otherwise, they play a sequential pricing game. We find that the retailer always sets a higher price when pricing early. While the manufacturer sets a higher price when pricing early if the wholesale price is lower, otherwise, it sets a higher price when pricing late. When pricing simultaneously, the price and profit of the players are both the lowest. By comparing the equilibrium profits under the three pricing sequences, we obtain the pricing timing preferences of the manufacturer and the retailer. The manufacturer always tends to price late. Retailer tends to price late only when the wholesale price and the fraction of strategic customers are both sufficiently low, otherwise, it prefers to price early. We then obtain the equilibrium pricing sequences under the three power structures.

Considering the prices are not static, we further study the dynamic evolution of the equilibrium prices in three pricing sequences. We find that the first mover has better price stability in the sequential pricing game. When price adjustment parameters exceed the stability range, the system will enter chaos through bifurcation, accompanied by the dual-channel prices changes from fluctuation to disorder and unpredictable, and some abnormal market phenomena will appear. Therefore, we propose vertical and horizontal price matching to improve the stability of online and offline prices respectively. The results show that horizontal price matching can always improve the stability of prices, while vertical price matching can effectively improve price stability only when the fraction of customers who seek price matching is low.

The structure of this paper is organized as follows. Section 2 reviews related literature. Section 3 models basic games under three pricing sequences and three power structures. Section 4 analyzes equilibrium prices, profits and derives pricing timing preference and equilibrium pricing sequences under three power structures. Section 5 extends Bertrand games to dynamic pricing and analyzes the stability of dual-channel price. Price matching strategies are proposed to improve stability. Section 6 concludes this paper.

2. Literature review

Many scholars have researched the pricing of the O2O supply chain. Mehra *et al.* [22] investigated the conflict of interests between the brick-and-mortar stores and the competitive online retailers caused by consumers' free-riding behaviors. From the perspective of BM stores, the authors proposed two effective strategies to counter "showrooming". The short-term strategy is price matching and the long-term strategy is an exclusivity of product assortments. Li *et al.* [10] investigated the showrooming effect on pricing and service effort. The timing decision on service effort provided by the BM stores was considered. Zhang *et al.* [33] set up an optimal production model, which analyzed the manufacturer's production decisions, parameter sensitivity, and economic and environmental impact. Polat and Gungor [24] propose a mixed-integer programming model for decision-makers to manage their activities in the WEEE closed-loop supply chain network. However, few people have studied pricing sequences of online and offline channels.

Some papers have taken decision sequences into models. Wang *et al.* [28] investigated the Cournot games between an original equipment manufacturer (OEM) and a contract manufacturer (CM). The two players make quantity decisions sequentially or simultaneously. Niu *et al.* [23] studied the pricing sequences between an OEM and its original design manufacturer (ODM). Two market environments: the ODM market and the OEM market are considered. Chen *et al.* [4] studied three pricing sequences between an OEM and an ODM considering the operational risks. Liu and Ke [12] considered the pricing sequences when the manufacturer operates two channels *via* an e-commerce platform and explored the optimal pricing timing under different power structures. The power structure in this paper only affects the priority decision-making power of pricing timing, without considering the impact on wholesale prices. Chen *et al.* [5] and Guo and Wu [6] adopted Nash bargaining and embodied the impact of negotiation power on price. Zu and Zeng [36] set up a Stackelberg differential game model between manufacturers and retailers, and studied optimal pricing strategies and energy efficiency efforts. The models considered in these papers are static games and are in single period. This paper extends basic game to dynamic pricing.

The most commonly used method of analyzing dynamic pricing is to analyze the stability, bifurcation and chaotic behavior of equilibrium with the chaos theory of complex dynamic systems. Ma *et al.* [17] studied the dynamic price game in a multi-channel supply chain considering price discount sensitivities. Bao *et al.* [2] studied the short-term and long-term repeated game behaviors of two-vehicle supply chains. Lou and Ma [13] studied three dynamic games of the home appliance supply chain *via* stable region, bifurcation and maximum Lyapunov exponent. These papers only described the complex characteristics of dynamic pricing, but did not control the instability, or only control in the sense of mathematical theory, without proposing the actual economic mechanism. The vertical and horizontal price matching mechanisms proposed in this paper are effective theoretically and there are also practical. Ma and Xu [16] developed a new game model to study solar energy investment problem, and they developed a dynamic system to study the long-term behavior of players in that game and analyzed the stability region and optimal strategy. Ma *et al.* [21] studied the electric vehicle supply chain system under policy intervention, which consists of the government and two manufacturers producing electric vehicles and fuel vehicles respectively. This paper aims to study the effect of pricing time on profitability and stability. The results indicates that subsidies may have a negative impact on system stability. By comparison, Stackelberg pricing strategy with time delay is more suitable for system operation management, making the system more flexible and able to adjust the supply chain system status in time. Ma *et al.* [20] established the game model of duopoly automobile manufacturers and considered carbon emission reduction policy constraints. They discussed how electric vehicles and fuel vehicles compete in terms of product performance under delayed pricing decisions. They found that government intervention maximizes social welfare. Zhu *et al.* [34, 35] established a closed-loop supply chain model of used electrical appliances, considering two recycling methods: resale method and new trade-in method. It is found that the speed of manufacturer's decision adjustment has significant influence on the stability of the model. Manufacturers should adopt stable rather than radical price adjustment strategies to achieve stable profit growth throughout the supply chain and avoid price wars.

TABLE 2. Literature review.

Literature	O2O	Show-rooming	Endogenous price timing	Power structure	Dynamic pricing
Mehra *et al.* [22]	✓	✓			
Balakrishnan *et al.* [1]	✓	✓			
Pun *et al.* [25]	✓	✓			
Chen *et al.* [4]	✓		✓	✓	
Niu *et al.* [23]	✓		✓	✓	
Yan *et al.* [31]			✓	✓	
Liu and Ke [12]	✓		✓	✓	
Lou and Ma [13]	✓			✓	✓
Ma and Xie [17]				✓	✓
Ma *et al.* [21]	✓			✓	✓
Li and Ma [9]	✓				✓
Ma and Xu [16]				✓	✓
Ma *et al.* [21]			✓	✓	✓
Ma *et al.* [20]			✓	✓	✓
Zhu *et al.* [34]				✓	✓
Zhu *et al.* [35]					✓
This paper	✓	✓	✓	✓	✓

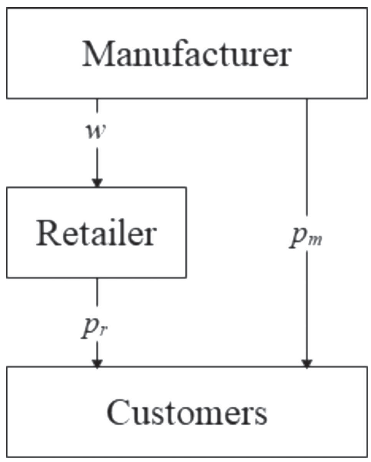


FIGURE 1. the structure of the O2O supply chain.

Table 2 shows the comparison between this paper and existing literature, there are few literatures considering all the elements.

3. MODEL SETTING

3.1. Notations and assumption

The O2O (online and offline) supply chain in this paper consists of a manufacturer with a direct online channel and a retailer that resells through the BM store. Figure 1 illustrates the structure of the dual-channel supply chain. The manufacturer and retailer are represented by the subscripts m and r, respectively. The corresponding online and offline prices are set at p_m and p_r, where $p_r > p_m$. Customers who visit the BM store can experience

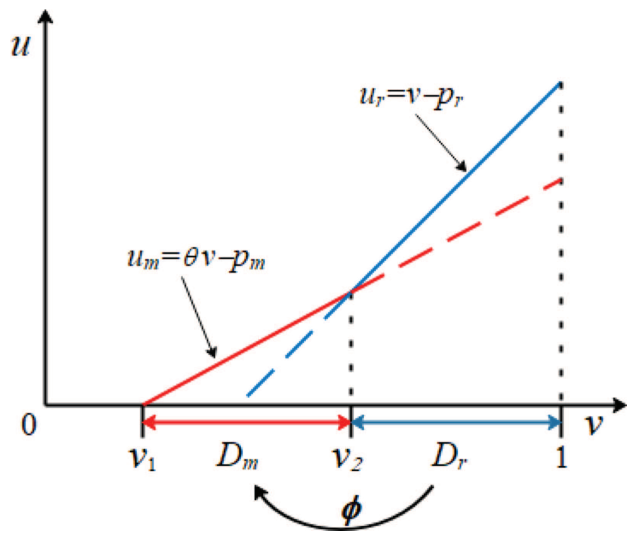


FIGURE 2. Utility and demand functions.

and choose the products that best fit their unique needs. While customers who evaluate products online cannot accurately assess non-digital attributes, so the product they select may not be their best fit. We introduce the parameter $\theta \in (0,1)$ to represent the value discount of an online product due to its non-digital attributes. The value of products purchased from BM stores is perceived as $v \in (0,1)$, and the value of products purchased online is perceived as θv. ϕ is the portion of consumers who have no traveling cost for showrooming. It means consumers prefer to showrooming behavior. $1-\phi$ is the portion of consumers who have very high traveling costs for showrooming and thus will never do so. x is strategic customers who engage in showrooming. Thus, the net utilities that customers obtain from these two channels are shown as equation (3.1).

$$\begin{cases} u_r = v - p_r - \phi x \\ u_m = \theta v - p_m - (1-\phi)x. \end{cases} \tag{3.1}$$

Because of the online price advantage and the offline advantage of accurately finding best-fit products, some strategic customers will first visit the BM store of the retailer to identify their best-fit products and then switch to online store with a lower price. This free-riding behavior of customers is called "showrooming". We denote the fraction of strategic customers engage in showrooming as ϕ, which is reflected in the demand functions equation (3.2). Then, combining with equation (3.1), we depict the demand function as Figure 2. To ensure that the demands in both channels are positive, we assume $0 < v_1 < v_2 < 1$ and derive $\frac{p_m}{p_r} \leq \theta \leq 1 + p_m - p_r$.

The corresponding demand function can be expressed as equation (3.2).

$$\begin{cases} D_r = (1-\phi)\left(1 - \frac{p_r - p_m}{1-\theta}\right) \\ D_m = \frac{p_r - p_m}{1-\theta} - \frac{p_m}{\theta} + \phi\left(1 - \frac{p_r - p_m}{1-\theta}\right). \end{cases} \tag{3.2}$$

Because the analysis result is robust to the production cost. To simplify the calculation, we set it to a constant and equal to zero [3, 8, 10, 11]. In Figure 1, the manufacturer has two sources of profit: retail online through a direct channel and wholesale to retailer. The retailer only acts as a reseller. Consequently, their respect profit functions can be expressed as follows:

$$\begin{cases} \Pi_m = p_m D_m + w D_r \\ \Pi_r = (p_r - w) D_r. \end{cases} \tag{3.3}$$

Table 3 is a summary of all the notations used in this paper.

TABLE 3. Notations.

Symbol	Description
(i, j)	Three pricing sequences: (S, S): simultaneous pricing; (E, L): manufacturer pricing early; (L, E): retailer pricing early
p_m^i, p_r^j	The manufacturer's direct online price and the retailer's offline price of the BM store under three pricing sequences
Π_m^i, Π_r^j	The profits of the manufacturer and the retailer under three pricing sequences
D_m, D_r	The manufacturer's online demand and the retailer's offline demand
u_m, u_r	Consumer surplus from the manufacturer's direct online channel and retailer's offline channel
v	Valuation obtained from best-fit product
θ	The value discount of an online product due to its non-digital attributes
ϕ	The fraction of strategic customers who engage in showrooming
w	Exogenous wholesale price
γ	The fraction of customers who seek vertical/historical price matching
λ	The fraction of customers who seek horizontal/inter-channel price matching

TABLE 4. Pricing game between manufacturer and retailer under three power structures.

Vertical Nash game / (Manufacturer/Retailer) Stackelberg pricing game		Manufacturer	
		Early	Late
Retailer	Early	Π_m^S, Π_r^S / Null	Π_m^L, Π_r^E
	Late	Π_m^E, Π_r^L	Π_m^S, Π_r^S / Null

TABLE 5. The process of pricing games.

Stage 1	The manufacturer and the retailer decide whether to price first or price late
Stage 2.1	The manufacturer and the retailer first bargain for wholesale prices based on their bargaining power
Stage 2.2	The manufacturer and the retailer set their respective retail price according to the pricing sequence determined in stage 1

3.2. Endogenous pricing game

To explore the pricing sequences under three power structures (manufacturer Stackelberg pricing game, retailer Stackelberg pricing game, and vertical Nash game) between the manufacturer and the retailer, we depict the game model in Table 4. The manufacturer and retailer can decide their pricing timing (early or late). When both players make the decision to price early or late, three equilibrium pricing sequences will be formed: simultaneous pricing, manufacturer pricing early, and retailer pricing early. In practice, pricing timing decisions are rarely made at exactly the same time, it can be regarded as simultaneous pricing if neither player has the other party's pricing information.

The process of pricing games is detailed in Table 5. In the first stage, both parties decide their respective pricing timing: early or late. Under the vertical Nash structure, the manufacturer and retailer have equal power and independently determine their respective pricing timing. When making the same choice, they play simultaneous games; otherwise, they play sequential games. Under the manufacturer (retailer)-Stackelberg structure, when making the same pricing timing choice, the follower will follow the leader's decision. In the second stage, the manufacturer and the retailer first bargain for wholesale prices based on their bargaining power. Under

TABLE 6. Equilibrium price.

Manufacturer	Retailer
$p_m^S = \dfrac{\theta(3(1-\phi)w + (1+\phi)(1-\theta))}{4-\theta-3\phi\theta}$	$p_r^S = \dfrac{(2+\theta-3\phi\theta)w + (2-\phi\theta)(1-\theta)}{4-\theta-3\phi\theta}$
$p_m^E = \dfrac{\theta(2(1-\phi)w + (1+\phi)(1-\theta))}{2(2-\theta-\theta\phi)}$	$p_r^L = \dfrac{4w(1-\theta\phi) + (1-\theta)(4-\theta-\theta\phi)}{4(2-\theta-\theta\phi)}$
$p_m^L = \dfrac{\theta\big(2w(1-\phi)(3-\theta-2\theta\phi) + (1-\theta)\big(2+2\phi-3\phi\theta-\phi^2\theta\big)\big)}{4(1-\theta\phi)(2-\theta-\theta\phi)}$	$p_r^E = \dfrac{(1-\theta)(2-\theta\phi) + 2w(1-\theta\phi)}{2(2-\theta-\theta\phi)}$

the Stackelberg structure, the leader has complete wholesale price bargaining power. Under the vertical Nash structure, both parties have the same bargaining power, and the wholesale price is determined according to Nash bargaining. Then, both parties set their respective retail price according to the pricing sequence determined in the first stage.

4. Equilibrium analysis

Note that subscripts m, r represent players manufacturer and retailer, and superscripts (S, S), (E, L) and (L, E) represent three pricing sequences: simultaneous pricing, a manufacturer pricing early and retailer pricing early respectively. We use backward induction to solve the pricing games described in Table 5.

4.1. Stage 2: Price competition

4.1.1. Stage 2.2: Retail price competition

We solve the games under three pricing sequences and derive the equilibrium prices of the manufacturer and the retailer are as shown in Table 6.

We check for the condition $p_r \geq p_m \geq w \geq 0$ in Table 6 and derive that $\max\left\{\frac{3\theta(1+\phi)}{4} - 1, 0\right\} \leq w \leq \frac{\theta(1+\phi)}{4}$. Let $\underline{w}$ and $\overline{w}$ represent the lower bound and upper bound respectively. Suppose the wholesale price w has a lower bound $\underline{w} = \frac{3\theta(1+\phi)}{4} - 1 > 0$ and $\overline{w} = \frac{\theta(1+\phi)}{4}$. By comparing the equilibrium prices of the manufacturer and the retailer under the three pricing sequences, we get the results as Proposition 4.1.

Proposition 4.1. *The comparison results of equilibrium prices:*

(1) *For the retailer,* $p_r^E > p_r^L > p_r^S$.
(2) *For the manufacturer,* $p_m^E > p_m^L > p_m^S$ *if* $\underline{w} \leq w < \frac{\theta\phi}{2}$*; otherwise,* $p_m^L > p_m^E > p_m^S$.

Proposition 4.1 shows that, whether for the manufacturer or retailer, the price is always the lowest when simultaneous pricing. The main intuition behind it is that the competition in simultaneous pricing is more fierce than that in sequential pricing, which has more shared price information and can be regarded as a form of collusion [23,32]. Proposition 4.1 also shows that when $w < \frac{\theta\phi}{2}$, pricing late leads to a lower price than pricing early. The late-mover can undercut the earlier price. Counter intuitively, when $\frac{\theta\phi}{2} \leq w \leq \overline{w}$, the manufacturer charges a higher price when the manufacturer prices late. When the wholesale price is higher, the retailer's price will rise accordingly, because of $\frac{\partial p_r^S}{\partial w} > 0$, $\frac{\partial p_r^L}{\partial w} > 0$, $\frac{\partial p_r^E}{\partial w} > 0$. If the manufacturer prices late, the information of the retailer's high price is obtained, the manufacturer will also increase its price. When the manufacturer price early, without grasping the other retailer's pricing information and set price just based on profit margin, the price will be lower.

TABLE 7. The boundary values of θ.

Power structure	Wholesale price	Pricing sequence	$\underline{\theta} = \frac{p_m}{p_r}$	$\overline{\theta} = 1 + p_m - P_r$
MS	$\frac{\theta(1+\phi)}{4}$	M	0	1
		R	0	1
RS	$\frac{3\theta(1+\phi)}{4} - 1$	M	1	1
		R	$\frac{-3+19\phi^2-\sqrt{9-50\phi^2+64\phi^3-23\phi^4}}{4\left(\phi^2+3\phi^3\right)}$	1
VN	$\frac{\theta(1+\phi)-1}{2}$	M	0	1
		R	$\frac{-1-\phi+6\phi^2-\sqrt{1+2\phi-11\phi^2+12\phi^3-4\phi^4}}{4\phi^3}$	1
		S	$\frac{3-5\phi}{1-3\phi^2}$	1
	$\frac{\theta(1+\phi)}{8}$	M	0	$\frac{4(1-\phi)}{2+\phi-\phi^2}$
		R	0	$\frac{1+10\phi-7\phi^2-\sqrt{1+20\phi-106\phi^2+116\phi^3-15\phi^4}}{4\left(3\phi^2-\phi^3\right)}$
		S	0	$\frac{8(1-\phi)}{1+6\phi-3\phi^2}$

4.1.2. Stage 2.1: Wholesale price bargaining

The equilibrium wholesale price depends on the power structures between the manufacturer and the retailer. Under the Stackelberg structure, the proposed offer will be accepted by the follower. Therefore, the wholesale price will be $w^* = \overline{w}$ under the manufacturer Stackelberg structure and $w^* = \underline{w}$ under the retailer Stackelberg structure. Because we assume the maximum value of wholesale price in manufacturer Stackelberg structure and minimum value of wholesale price in retailer Stackelberg structure. We further set the wholesale price as the middle value of two sequential pricing schemes. That means, in the vertical Nash structure, the manufacturer and the retailer would reach a wholesale price $w^* = \frac{\underline{w}+\overline{w}}{2}$.

4.1.3. The boundary values of θ

To ensure the coexistence of dual channels, we get the constraint $\frac{p_m}{p_r} \leq \theta \leq 1 + p_m - p_r$ from Figure 1. The upper and lower bounds of θ are represented by $\underline{\theta} = \frac{p_m}{p_r}$ and $\overline{\theta} = 1 + p_m - p_r$ respectively. Substituting the optimal wholesale prices derived in Section 4.1.2 under three power structures into the equilibrium prices in Table 6, we obtain boundary values of θ as shown in Table 7. We abbreviate the power structures: manufacturer Stackelberg, retailer Stackelberg and vertical Nash as MS, RS, and VN respectively and abbreviate the pricing sequences: manufacturer-pricing-early, retailer-pricing-early and simultaneous pricing as M, R, S respectively.

4.2. Stage 1: Pricing sequence

4.2.1. Pricing timing preference

The equilibrium profits of the manufacturer and the retailer under three pricing sequences are summarized in Table 8.

Proposition 4.2. *The comparison results of equilibrium profits*

(1) *For the manufacturer,* $\Pi_m^L > \Pi_m^E > \Pi_m^S$.
(2) *For the retailer,* $\Pi_r^L > \Pi_r^E > \Pi_r^S$ *under the retailer Stackelberg and vertical Nash structures;* $\Pi_r^E > \Pi_r^L > \Pi_r^S$ *under the manufacturer Stackelberg structure, which is the only case with a first-mover advantage.*
(3) *For the whole supply chain,* $\Pi(L,E) > \Pi(E,L) > \Pi(S,S)$.

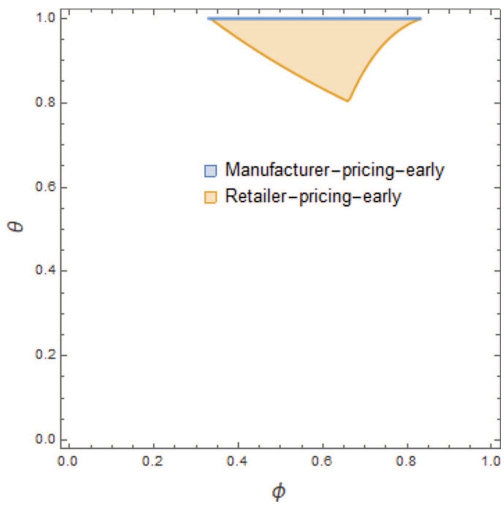
By comparing the equilibrium profits under three pricing sequences presented in Proposition 4.2, the pricing timing preferences of the manufacturer and the retailer are clear. The manufacturer and the retailer obtain lowest profits when pricing simultaneously. This result attributes to fierce competition, which conforms to the

TABLE 8. The profits of the manufacturer and the retailer.

Power structure	Wholesale price	Profit	Manufacturer stackelberg pricing game		Retailer stackelberg pricing game	Vertical nash game
MS	$\frac{\theta(1+\phi)}{4}$	Π_m	$\frac{1}{16}\theta(3-\phi)(1+\phi)$	$<$	$\frac{16\theta(-3+\phi)(1+\phi)-16\theta^2(-3+\phi)(1+\phi)(1+2\phi)+\theta^3(-13+\phi(-76+\phi(-110+\phi(-12+19\phi))))+\theta^4\left(1+\phi\left(8+\phi\left(38+\phi\left(20+\phi-4\phi^2\right)\right)\right)\right)}{64(-1+\theta\phi)(-2+\theta+\theta\phi)^2}$	
		Π_r	$\frac{1}{4}(1-\theta)(1-\phi)$	$<$	$\frac{(-1+\theta)(-1+\phi)(-4+\theta+3\theta\phi)^2}{32(-1+\theta\phi)(-2+\theta+\theta\phi)}$	
RS	$\frac{3\theta(1+\phi)}{4}-1$	Π_m	$-1+\phi+\frac{1}{16}\theta(11+(2-9\phi)\phi)$	$<$	$\frac{1}{64}\left(11-\frac{29}{\theta}-2\theta+(47+25\theta)\phi-36\theta\phi^2+\frac{9(-1+\theta)^3}{\theta(-1+\theta\phi)}-\frac{16(-1+\theta)^4}{\theta(-2+\theta+\theta\phi)^2}+\frac{48(-1+\theta)^3}{\theta(-2+\theta+\theta\phi)}\right)$	
		Π_r	$(1-\theta)(1-\phi)$	$>$	$\frac{(-1+\theta)(-1+\phi)(-8+\theta(3+5\phi))^2}{32(-1+\theta\phi)(-2+\theta+\theta\phi)}$	
VN	$\frac{\theta(1+\phi)-1}{2}$	Π_m	$\frac{1}{8}(3(-1+\phi)+\theta\left(3+\phi-2\phi^2\right)$	$<$	$\frac{1}{16}\left(4-\frac{4}{\theta}+4(1+\theta)\phi-4\theta\phi^2+\frac{(-1+\theta)^3}{\theta(-1+\theta\phi)}-\frac{4(-1+\theta)^4}{\theta(-2+\theta+\theta\phi)^2}+\frac{8(-1+\theta)^3}{\theta(-2+\theta+\theta\phi)}\right)$	$\frac{\left(35+34\phi-53\phi^2\right)\theta-24(1-\phi)+2\theta^2(-6+\phi(-25+\phi(-4+19\phi)))-\theta^3(1+\phi)(-1+\phi(-9+\phi(-7+9\phi)))}{4(-4+\theta+3\theta\phi)^2}$
		Π_r	$\frac{9}{16}(-1+\theta)(-1+\phi)$	$>$	$\frac{(-1+\theta)(-1+\phi)(-3+\theta+2\theta\phi)^2}{8(-1+\theta\phi)(-2+\theta+\theta\phi)}$	$\frac{(-1+\theta)(-1+\phi)(-3+\theta+2\theta\phi)^2}{(-4+\theta+3\theta\phi)^2}$
	$\frac{\theta(1+\phi)}{8}$	Π_m	$-\frac{\theta(1+\phi)(16+\theta(-9+\phi)(1+\phi))}{64(-2+\theta+\theta\phi)}$	$<$	$\frac{-128\theta(1+\phi)-24\theta^2(1+\phi)(-5+(-12+\phi)\phi)+\theta^3(-21+\phi(-232+\phi(-414+\phi(-120+19\phi))))+\theta^4(1+\phi(12+\phi(142+\phi(88+(17-4\phi)\phi))))}{256(-1+\theta\phi)(-2+\theta+\theta\phi)^2}$	$\frac{\theta(1+\phi)\left(-128+\theta(72-8(-24+\phi)\phi)+\theta^2(-7+\phi(-57+\phi(-73+9\phi)))\right)}{64(4-\theta-3\theta\phi)^2}$ The lowest
		Π_r	$\frac{(-1+\theta)(-1+\phi)(8-3\theta(1+\phi))^2}{64(-2+\theta+\theta\phi)^2}$	$>$	$\frac{(-1+\theta)(-1+\phi)(-8+\theta+5\theta\phi)^2}{128(-1+\theta\phi)(-2+\theta+\theta\phi)}$	$\frac{(-1+\theta)(-1+\phi)(-8+\theta+5\theta\phi)^2}{16(-4+\theta+3\theta\phi)^2}$ The lowest

TABLE 9. Equilibrium pricing sequence under the manufacturer Stackelberg structure.

Manufacturer Stackelberg $w = \frac{\theta(1+\phi)}{4}$		Manufacturer	
		Early	Late
Retailer	Early	/	Π_m^L, Π_r^E **Pareto optimality**
	Late	Π_m^E, Π_r^L	/



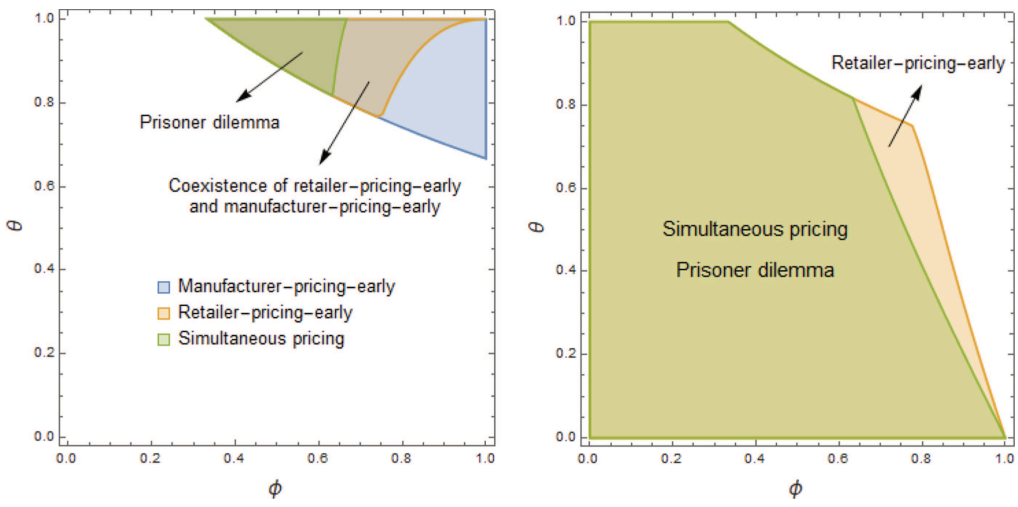
$$w = \frac{3\theta(1+\phi)}{4} - 1$$

FIGURE 3. Pricing sequence under the retailer Stackelberg structure.

lowest prices in Proposition 4.1. The manufacturer always gets higher profits when pricing late. The retailer has the same results under the retailer Stackelberg and vertical Nash structures. The late-mover advantage derived from the pricing timing allows the manufacturer to undercut the retailer's price. However, the retailer has a first-mover advantage under the manufacturer Stackelberg structure. The wholesale price is so high that equal to the online retail price, which means that the retailer has a much greater cost than the manufacturer. Even if pricing late, the retailer cannot undercut the online price. Therefore, the retailer prefers to price early to charge a higher price rather than pursuing market share at low prices. Proposition 4.2 also shows that supply chain efficiency is highest when the manufacturer price early, followed by retailer-pricing-early, and lowest when price simultaneously.

4.2.2. Equilibrium pricing sequence

Based on the pricing timing preferences of the manufacturer and the retailer presented in Proposition 4.2, we can derive the pricing sequences under three power structures in Table 9, Figures 3 and 4 respectively. In Table 9, we underline the preferred pricing timing of the manufacturer and retailer, and circle the equilibrium pricing sequence. Based on the constraints of θ in Table 7 and the profits comparison in Table 8, we have obtained the pricing sequences of the manufacturer and the retailer under the retailer Stackelberg and vertical Nash structures in Figures 2 and 3 respectively. We use three colors: ▢, ▢, ▢, to indicate the three pricing sequences: manufacturer-pricing-early, retailer-pricing-early and simultaneous pricing.



$w = \frac{\theta(1+\phi)-1}{2}$ $\qquad$ $w = \frac{\theta(1+\phi)}{8}$

FIGURE 4. Pricing sequence under the vertical Nash structure.

Proposition 4.3. *We obtain the equilibrium pricing sequences under three power structures as follows.*

Under the manufacturer Stackelberg structure the unique equilibrium is retailer-pricing-early and achieves Pareto optimality.

Under the retailer Stackelberg structure, shown as Figure 3.

When $\theta < \frac{-3+19\phi^2-\sqrt{9-50\phi^2+64\phi^3-23\phi^4}}{4(\phi^2+3\phi^3)}$, *the manufacturer prices early; otherwise, the retailer prices early.*

Under the vertical Nash structure, shown as Figure 4.

When $\theta > \frac{3-5\phi}{1-3\phi^2}$, *the manufacturer and the retailer price simultaneously, which traps them into prison's dilemma.*

When $\frac{-1-\phi+6\phi^2-\sqrt{1+2\phi-11\phi^2+12\phi^3-4\phi^4}}{4\phi^3} < \theta < \frac{3-5\phi}{1-3\phi^2}$, *two sequential pricing games coexist.*

When $\theta < \frac{-1-\phi+6\phi^2-\sqrt{1+2\phi-11\phi^2+12\phi^3-4\phi^4}}{4\phi^3}$, *only manufacturer-pricing-early exists.*

5. DYNAMIC PRICING GAME

5.1. Model construction

With the complex changes in the market, prices are not static in the long run. We further study the evolution of the basic games under three pricing sequences considering dynamic pricing. Because it is impossible to have complete information about the market and competitors, the rationality limitation is unavoidable and the assumption of complete rationality is unrealistic. So the manufacturer and the retailer are considered bounded rational. Bounded rationality means that the strategic equilibrium between players is often not the result of a one-time choice, but the result of continuous adjustment. The dynamic adjustment mechanism of prices for the manufacturer and the retailer is based on current marginal profits. Raise/reduce prices in the next retail period if the marginal profit is positive/negative. The corresponding dynamic pricing model can be shown in equation (5.1).

$$\begin{cases} p_m^i(t+1) = p_m^i(t) + \alpha_i p_m^i(t)\frac{\partial \Pi_m^i}{\partial p_m^i} \\ p_r^j(t+1) = p_r^j(t) + \beta_j p_r^j(t)\frac{\partial \Pi_r^j}{\partial p_r^j} \end{cases}$$

$$(i,j) = (S,S),(E,L),(L,E) \tag{5.1}$$

where α_i and β_j are the price adjustment parameters of the manufacturer and the retailer respectively.

5.2. Model analysis

When $(i,j) = (S,S)$, the equilibrium prices are $(0,0)$, $(0,p_r^S)$, $(p_m^S,0)$, (p_m^S,p_r^S). When $(i,j) = (E,L)$, the equilibrium prices are $(0,0)$, $(0,p_r^L)$, $(p_m^E,0)$, (p_m^E,p_r^L). When $(i,j) = (L,E)$, the equilibrium prices are $(0,0)$, $(0,p_r^E)$, $(p_m^L,0)$, (p_m^L,p_r^E). The values of (p_m^i,p_r^j) are equal to the values in Table 6. $(0,0)$, $(0,p_r^j)$, $(p_m^i,0)$ are boundary equilibrium solutions without any practical significance. Only the Nash/Stackelberg equilibrium has economic valuation with both positive prices. The Jacobian matrix at (p_m^i,p_r^j) can be calculated as equations (5.2)–(5.4).

$$J^{(S,S)} = \begin{pmatrix} j_{11}^{(S,S)} & \frac{p_m^S\alpha_S(-1+\phi)}{-1+\theta} \\ \frac{p_r^S\beta_S(-1+\phi)}{-1+\theta} & \frac{-1+\theta+\beta_S\left(1+p_m^S-4p_r^S+w-\theta\right)(-1+\phi)}{-1+\theta} \end{pmatrix} \tag{5.2}$$

$$j_{11}^{(S,S)} = \frac{-4p_m^S\alpha_S(-1+\theta\phi)+\theta\left(-1-\left(p_r^S+w\right)\alpha_S+\theta+\alpha_S\left(-1+p_r^S+w+\theta\right)\phi\right)}{(-1+\theta)\theta}$$

$$J^{(E,L)} = \begin{pmatrix} 1+\frac{-4p_m^E\alpha_E(-2+\theta+\theta\phi)+\theta(2w\alpha_E(-1+\phi)+\alpha_E(-1+\theta)(1+\phi))}{2(-1+\theta)\theta} & 0 \\ j_{21}^{(E,L)} & j_{22}^{(E,L)} \end{pmatrix} \tag{5.3}$$

$$j_{21}^{(E,L)} = \frac{p_r^L\beta_L(-1+\phi)\left(-4p_m^E\alpha_E(-2+\theta+\theta\phi)+\theta(2(-1+\theta)+2w\alpha_E(-1+\phi)+\alpha_E(-1+\theta)(1+\phi))\right)}{2(-1+\theta)^2\theta}$$

$$j_{21}^{(E,L)} = 1-\frac{\beta_L(-1+\phi)\begin{pmatrix} 2(-1+\theta)\theta\left(-1+4p_r^L-w+\theta\right)+2p_m^{E2}\alpha_E(-2+\theta+\theta\phi) \\ +p_m^E\theta(2-2\theta-2w\alpha_E(-1+\phi)-\alpha(-1+\theta)(1+\phi)) \end{pmatrix}}{2(-1+\theta)^2\theta}$$

$$J^{(L,E)} = \begin{pmatrix} j_{11}^{(L,E)} & j_{12}^{(L,E)} \\ 0 & 1+\frac{\beta_E(1-\phi)\left(2(1+w-2p_r^L(2-\theta)-\theta)+\theta\left(-1+4p_r^L-2w+\theta\right)\phi\right)}{2(-1+\theta)(-1+\theta\phi)} \end{pmatrix} \tag{5.4}$$

$$j_{11}^{(L,E)} = \frac{\begin{array}{c} 2(-1+\theta)(-1+\theta\phi)(-1-w\alpha+\theta+\alpha_L(-1+w+\theta)\phi)-2p_r^{E2}\alpha_L\beta_E(-1+\phi)^2(-2+\theta+\theta\phi) \\ +\frac{8p_m^L\alpha_L(1-\theta)(-1+\theta\phi)^2}{\theta}-p_r^E\alpha_L(1-\phi)(2(1-\theta)(1-\theta\phi)-\beta_E(1-\phi)((1+2w-\theta)\theta\phi-2(1+w-\theta))) \end{array}}{2(-1+\theta)^2(-1+\theta\phi)}$$

$$j_{12}^{(L,E)} = \frac{\begin{array}{l} p_m^L\alpha_L(1-\phi)\Big(2(1-\theta)(1-\theta\phi)+\beta_E(1-\phi)\Big(2+2w+4p_r^E(-2+\theta)-2\theta \\ +\theta\left(-1+4p_r^E-2w+\theta\right)\phi\Big)\Big) \end{array}}{2(-1+\theta)^2(1-\theta\phi)}.$$

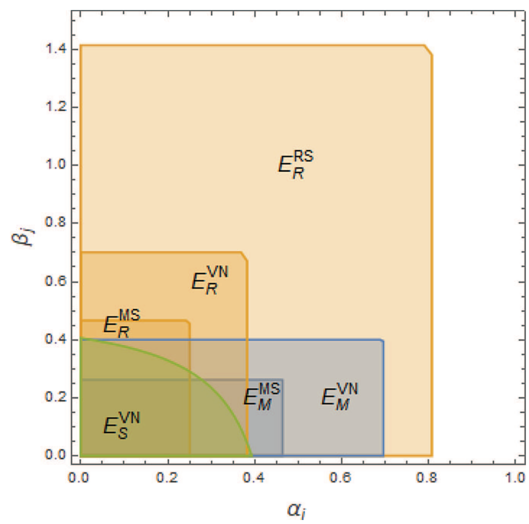
According to the Jury criterion, the conditions for the system (5.1) keeping equilibrium (p_m^i,p_r^j) locally stable are listed as the following set of inequalities (5.5).

$$\begin{cases} 1+Tr(J^{(i,j)})+Det(J^{(i,j)})>0 \\ 1-Tr(J^{(i,j)})+Det(J^{(i,j)})>0 \\ 1-Det(J^{(i,j)})>0 \end{cases} \tag{5.5}$$

where $Tr(J^{(i,j)})$ and $Det(J^{(i,j)})$ are the trace and determinant of $J^{(i,j)}$. The condition (5.5) offers a stable region in the place of the price adjustment parameters α_i, β_j.

TABLE 10. Equilibrium when $\phi = 0.5, \theta = 0.95$.

(p_m, p_r) Pricing sequence / Power structure	Manufacturer-pricing-early	Retailer-pricing-early	Simultaneous pricing
Manufacturer Stackelberg $w = 0.356$	$E_M^{\mathrm{MS}}(0.356, 0.381)$	$E_R^{\mathrm{MS}}(0.361, 0.392)$	
Retailer Stackelberg $w = 0.069$		$E_R^{\mathrm{RS}}(0.112, 0.129)$	
Vertical Nash $w = 0.213$	$E_M^{\mathrm{VN}}(0.2375, 0.25)$	$E_R^{\mathrm{VN}}(0.237, 0.260)$	$E_S^{\mathrm{VN}}(0.230, 0.246)$



$\phi = 0.5, \theta = 0.95$

FIGURE 5. The stability region of equilibrium (p_m^i, p_r^j) with respect to parameters α_i and β_j under the three pricing sequences.

5.3. Numerical simulation

Due to the complexity of the system, it is too complicated to analyze dynamic properties analytically. So we use numerical simulation instead. In practice, the customer's extent of value discount for a product online and the fraction of strategic customers are determined through econometric methods with mass data. The exact actual values of these parameters are difficult to obtain, so we refer to related literature [10, 12]. We set $\phi = 0.5, \theta = 0.95$ and to satisfy the conditions $\frac{3\theta(1+\phi)}{4} - 1 > 0$. Substituting two sets of parameter values into (p_m^i, p_r^j), the equilibria are listed in Table 10 respectively.

5.3.1. Local stability analysis

According to the Jury condition (5.5), the stable region of the price adjustment parameters α_i and β_j can be plotted as Figure 5. If the value of (α_i, β_j) is in the stable region, system (5.1) will be stable at corresponding equilibrium point, if it exceeds this region, the system will be unstable.

It can be seen from Figure 5 that the stability region of the equilibrium E_S^{VN} with respect to either single parameter α_i or β_j is the smallest when price simultaneously. Under the retailer stackelberg pricing game, the stable range of its price adjustment parameter $\alpha_L(\beta_L)$ becomes obviously larger.

Proposition 5.1. (1) *The sequential game has a better performance in keeping the system stable than the simultaneous game under the vertical Nash structure.*
(2) *For the manufacturer or the retailer, the retailer Stackelberg structure has better stability than the vertical Nash structure better than the manufacturer Stackelberg structurer.*
(3) *For the manufacturer or the retailer, pricing early provides them with greater flexibility to adjust the retail prices without causing system instability.*

5.3.2. Instability analysis: bifurcation and chaos

How the system (5.1) evolves when the parameter α_i or β_j exceeds the stable region depicted in Figure 5? We use bifurcation diagrams in Figure 6 to describe the changes in dual-channel retail prices under three pricing sequences when the system (5.1) is unstable. We select the vertical Nash structure as an example, but the corresponding conclusion is robust in Stackelberg structures. Values that meet the Jury condition are assigned to the price adjustment parameters, so that the system instability caused by the change of a special parameter can be investigated. In order to ensure the robustness of the results, we choose the value of parameter α_i or β_j near the left endpoint of the stable region. For example, in the case of E_M^{VN}, we discuss the change of dual-channel price with parameter β_j or α_i when $\alpha_i = 0.05$ or $\beta_j = 0.05$. In the appendix, we discuss the value of α_i or β_j near the right endpoint of the stable region. For instance, in the case of E_M^{VN}, we discuss the change of dual-channel price with parameter β_j or α_i when $\alpha_i = 0.65$ or $\beta_j = 0.35$. The results in the appendix are consistent with the main conclusions in Figure 6.

In Figure 6a, with the increase of the adjustment parameters, when $\alpha_E > 0.708$, the dual-channel prices simultaneously bifurcate from the unique equilibrium in the stability region into 2, 4, and 8 prices and then enter chaos through flip bifurcation, which is consistent with the stable region in Figure 5. Similar situations can also be seen in Figures 6b and 6c. When entering chaos, dual-channel prices become disorderly and unpredictable. The chaotic behavior of this system can be understood as the confusion of the market competition, and the player maybe even out of the market with an increasing price adjustment of one player. These phenomena will harm the healthy operation of the market and the development of supply chain members.

Proposition 5.2. (1) *When price adjustment parameters exceed the stability range, the system will enter chaos through bifurcation, accompanied by the dual-channel prices changes from fluctuation to disorder and unpredictable, and some abnormal market phenomena ($p_m > p_r, w > p_m, w > p_r, p_m = 0, p_r = 0$) will appear.*
(2) *Under the Stackelberg structure, in the sequential pricing game, when the leader prices early, the first-mover can be not affected and keep price stable when the late-mover loses stability due to price adjustment.*

5.4. Chaos control by price matching

It can be seen from Proposition 5.2 that instability is not expected. To improve stability of the system, we propose vertical and horizontal price matching to improve the stability of online and offline prices respectively. The vertical price matching policy, whereby a firm promises to reimburse the price difference to a customer who purchases a product before the firm marks it down [7]. A markdown strategy provides a seller with the flexibility to reach customers with different valuations (θv) is often used in practice by online stores. The horizontal price matching implemented by BM stores entails a credible commitment to match the price of an online channel, the efficacy of which is greater when consumers engage in showrooming. Retailers such as Best Buy, Target, Nordstrom, and Walmart are using this strategy to compete with Amazon [27]. When the manufacturer and retailer adopt the price matching mechanism, the profit function is shown in equation (5.6).

$$\begin{cases} \Pi_m = p_m D_m + wD_r - \gamma(x - p_m)\Big(\frac{y-x}{1-\theta} - \frac{x}{\theta} + \phi\Big(1 - \frac{y-x}{1-\theta}\Big)\Big) \\ \Pi_r = (p_r - w)D_r - \lambda(p_r - p_m)D_r \end{cases} \tag{5.6}$$

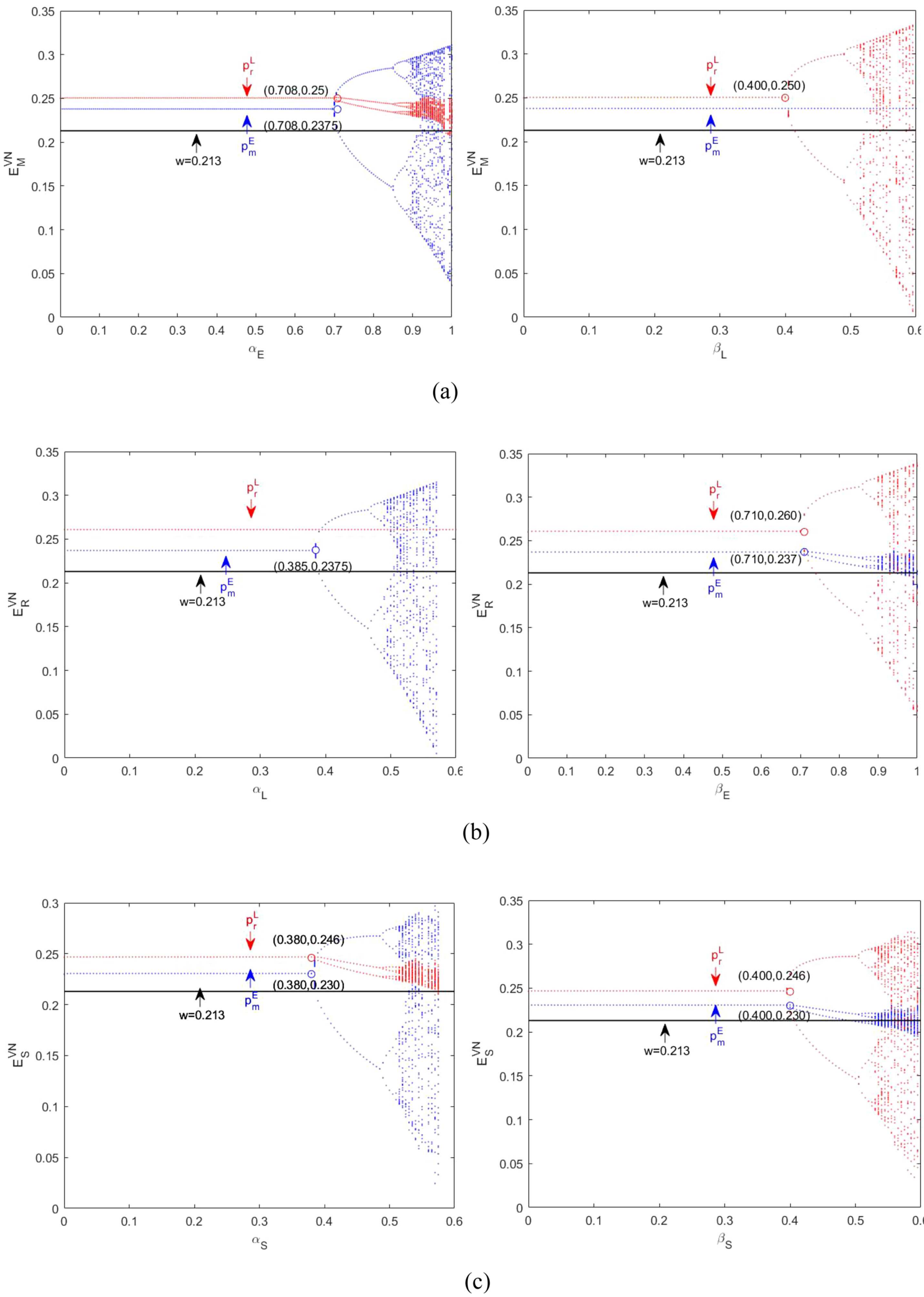


FIGURE 6. The impact of adjustment parameters on price evolution. (a) Manufacturer-pricing-early ($\alpha_i = 0.05, \beta_i = 0.05$). (b) Retailer-pricing-early ($\alpha_i = 0.05, \beta_i = 0.05$). (c) Simultaneous pricing ($\alpha_i = 0.05, \beta_i = 0.05$).

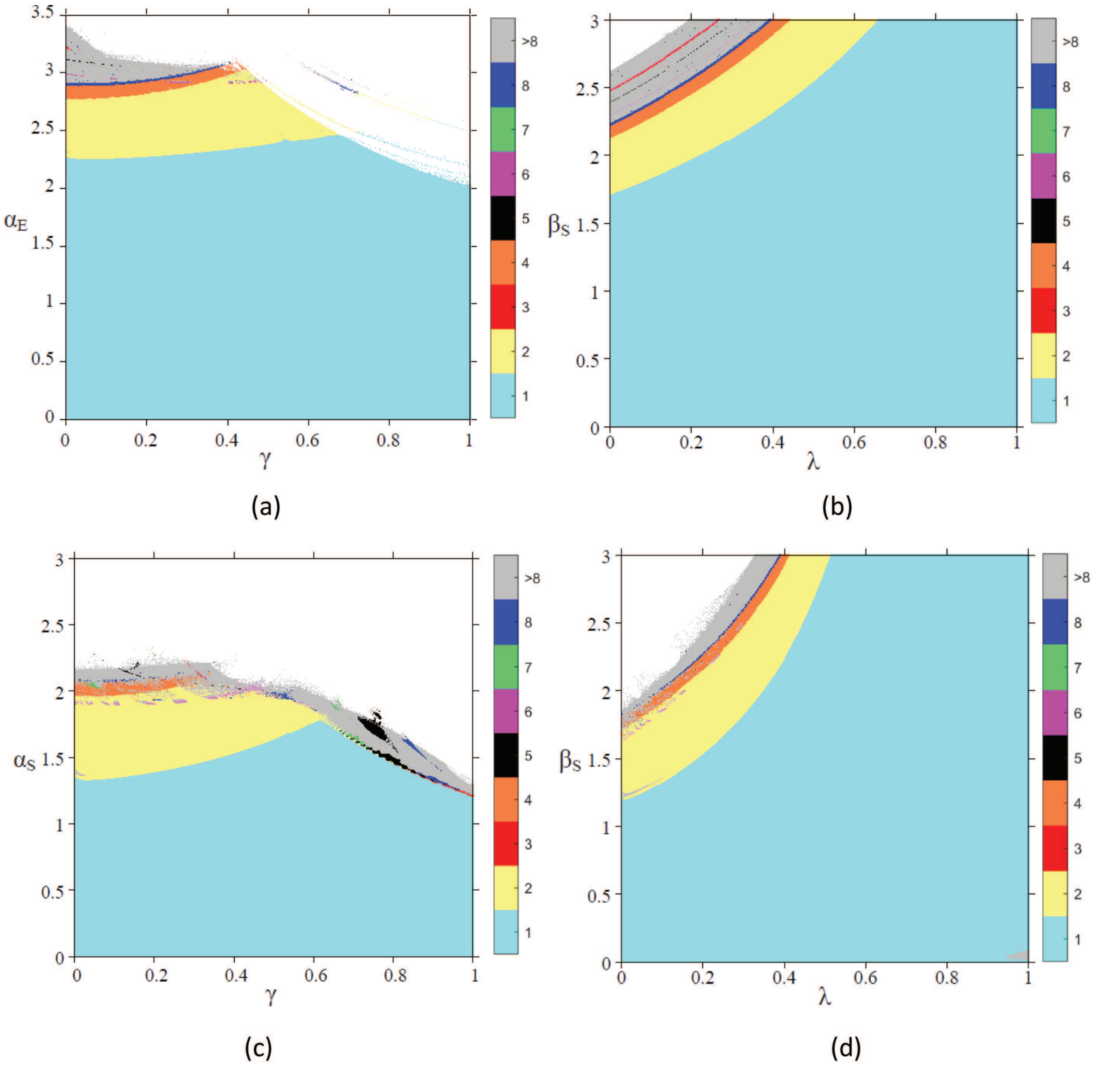


FIGURE 7. The parameter basins between the price adjustment parameter and the fraction of customers who seek price matching under three pricing sequences. The different colors represent the number of equilibrium prices. (a) Manufacturer-pricing-early. (b) Retailer-pricing-early. (c) Simultaneous pricing. (d) Simultaneous pricing.

where, $x = p_m(t-1)$, $y = p_r(t-1)$. $\gamma \in (0,1)$ and $\lambda \in (0,1)$ refer to the proportion of customers who seek vertical and horizontal price matching respectively to all customers in this period.

Calculate the result of the profit function (5.6) with respect to the first-order derivation of the prices, and substitute into the system (5.1), a new system depicts the price dynamic adjustment process under the price matching mechanism will be established. Figure 7 depicts changes in the price adjustment parameters as the fraction of customers who seek price matching increases. When $\alpha_E = 3$, the dynamic pricing system is in a state of chaos. With the increase in price-matching consumers, retail prices have changed from chaos to 8, 4, and eventually 2 prices fluctuation. In Figure 7b, as the proportion of consumers seeking price matching between channels increases, the stable region of retailers' price adjustments is getting larger and larger. When $\beta_S = 2.5$, retail prices gradually restore to be stable from chaos to fluctuation as the proportion of consumers who matched prices increases. If the proportion of consumers with price matching is low, vertical price matching can improve the stability of the pricing system. However, if the proportion of consumers seeking price matching is sufficiently large, price matching will reduce the stability of the pricing system. When deciding whether to use historical price matching strategies, online stores should have a more accurate expectation of the proportion of consumers seeking price matching. The horizontal price matching strategy of physical stores can always improve price stability. Figures 7c and 7d can also draw the same conclusion about price matching.

Proposition 5.3. (1) *If the fraction of customers who seek price matching is low, the vertical price matching can effectively improve price stability; otherwise, weaken the price stability.*
(2) *The horizontal price matching can always improve the stability of prices.*

6. Conclusion

This paper considers a dual-channel supply chain with a manufacturer with a direct online channel and a BM retailer. Strategic customers who engage in showrooming are also considered in the duopoly Bertrand game. Demand functions for dual-channel are derived based on the consumer value. We consider three supply chain power structures (vertical Nash, manufacturer Stackelberg and retailer Stackelberg) and three pricing sequences (simultaneous pricing, manufacturer pricing early, retailer pricing early). By comparing the equilibrium prices under the three pricing sequences, we found that retailer always sets a higher price when pricing early. While the manufacturer sets a higher price when pricing early if the wholesale price is low, otherwise, sets a higher price when pricing late. When pricing simultaneously, the price and profit of the players are both the lowest. By comparing the equilibrium profits under the three pricing sequences, we obtain the pricing timing preferences of the manufacturer and retailer. The manufacturer always tends to price late. The retailer tends to price late only when the wholesale price and the fraction of strategic customers are both small, otherwise, the retailer prefers to price early. We then obtain the equilibrium pricing sequences under the three power structures. Considering the prices are not static, we further study the evolution of the basic games of three pricing sequences considering dynamic pricing and find that sequential pricing has a better performance in keeping the system stable than simultaneous pricing. When price adjustment parameters exceed the stability range, the system will enter chaos through bifurcation, accompanied by the dual-channel prices changes from fluctuation to disorder and unpredictable, and some abnormal market phenomena will appear. To improve stability of the system, we propose vertical and horizontal price matching to improve stability of online and offline prices respectively. The results show that the horizontal price matching can always improve the stability of prices. While vertical price matching can effectively improve price stability when the fraction of customers who seek price matching is low, otherwise, it will weaken the price stability.

Our research can also be extended from two directions. First, this paper treats the wholesale price as exogenously given. However, the manufacturer can adjust the wholesale price strategically according to the retailer's actions. Therefore, investigating the impact of the wholesale price as an endogenous variable on the equilibrium outcome is meaningful. Second, we regard the fraction of customers who seek vertical and horizontal price matching as an exogenous parameter, but it depends on the price gap between historical and present and between channels.

Appendix A.

In Figure A.1, we discuss the change of dual-channel price with parameter β_j or α_i, when the value of α_i or β_j is near the right endpoint of the stable region. Similar to the case of left endpoint, in Figure A.1a, with the increase of the adjustment parameters, when $\alpha_E > 0.708$, the dual-channel prices simultaneously bifurcate from the unique equilibrium in the stability region into 2, 4, and 8 prices and then enter chaos through flip bifurcation, which is also consistent with the stable region in Figure 5. Similar situations can also be seen in

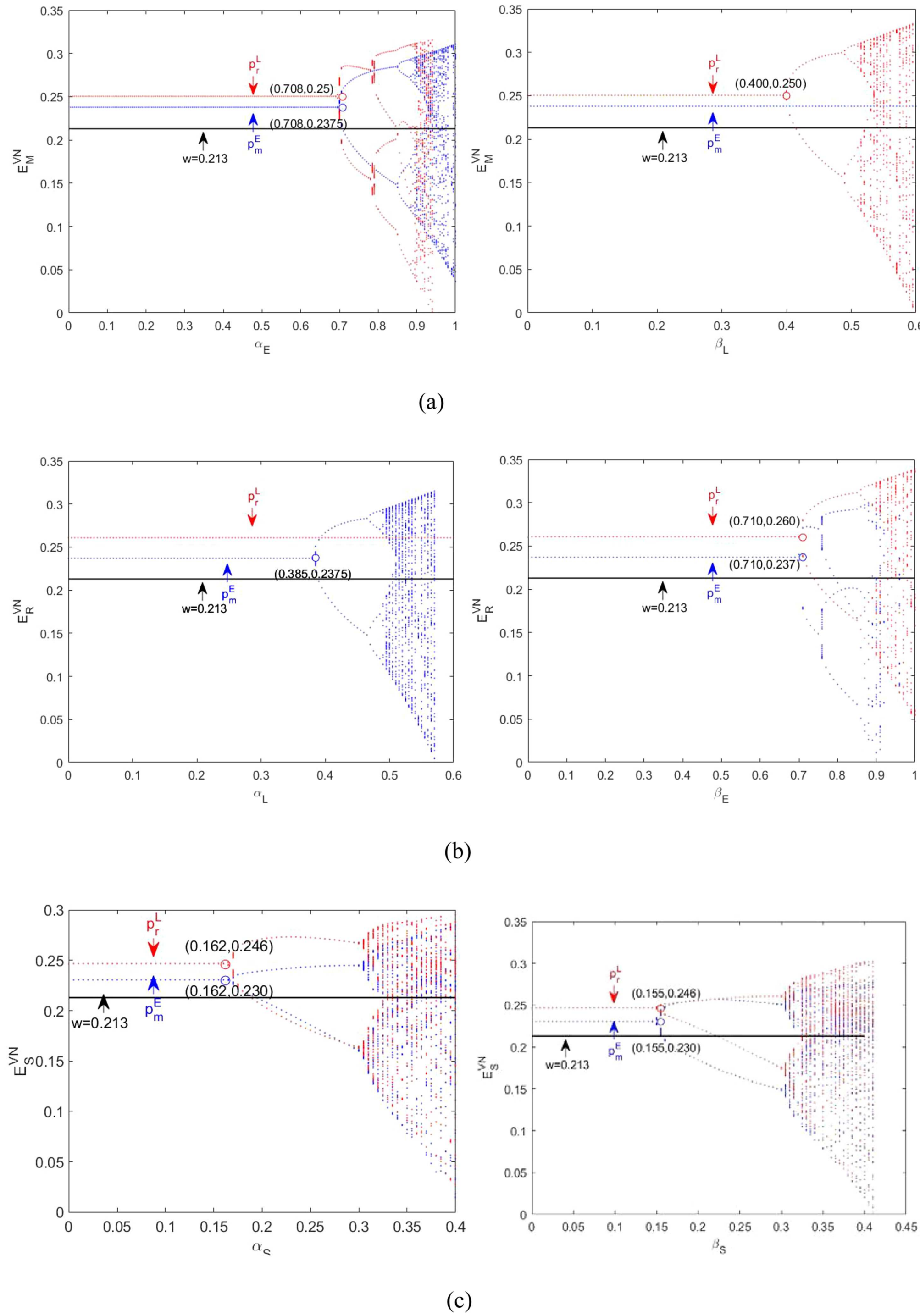


FIGURE A.1. The impact of adjustment parameters on price evolution. (a) Manufacturer-pricing-early ($\alpha_i = 0.65, \beta_i = 0.35$). (b) Retailer-pricing-early ($\alpha_i = 0.35, \beta_i = 0.65$). (c) Simultaneous pricing ($\alpha_i = 0.35, \beta_i = 0.35$).

Figures A.1b and A.1c. The unpredictability of chaotic behavior will affect the pricing strategies of both sides, and the strategic uncertainty will affect the profits of both sides.

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