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GLOBAL MULTI-PERIOD PERFORMANCE EVALUATION - NEW MODEL AND PRODUCTIVITY INDEX

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Abstract. In this paper, we introduce a novel multi-period data envelopment analysis (MDEA) model that attempts to circumvent the limitations of the existing MDEA models. The proposed global MDEA model is essentially based on major modifications of fundamental DEA axioms to enable a decision making unit (DMU), defined with inputs and outputs of period t, to be evaluated within the production possibility set (PPS) of another period $l,\ t \neq l$. Building on the properties of the global MDEA model, we also introduce a global productivity index, identified as Global Progress and Regress index (GPRI), that render possible the evaluation of a DMU's extent of progress or regress over multi-period time horizons under variable returns to scale (VRS) production technologies. This lifts the restrictions to two successive periods and constant returns to scale (CRS) of existing productivity indices. The most salient features of the new MDEA model as well as the GPRI are highlighted using an application that involves a real-life sample of 25 bank branches considered over 4 years.

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1. Introduction

Data envelopment analysis (DEA) is an optimization approach proven for its strength in evaluating performance of decision making units (DMUs) that employ multiple inputs to produce multiple outputs ([43,44]). In standard DEA models, the efficiency of a DMU is evaluated using input and output data bundles related to a specific time period ([31,34]). However, when the fluctuation of inputs and outputs is important over different time periods, more advanced DEA models may be needed.

The Malmquist Productivity Index (MPI), introduced by Malmquist [22] and improved by Caves *et al.* [7], is the first known metric for evaluating a DMU's performance between two time periods. Later on, Grifell-Tatjé and Lovell [11] defined the quasi-MPI conjunctly with the "one-sided" efficiency concept. In spite of its importance in informing managers on the progress or the regress of a DMU over the time-axis, the MPI is not concerned with estimating the DMU's efficiency scores. Therefore, a new class of DEA models was introduced to assess performance for multi-period production systems.

Keywords. Data envelopment analysis, Multi-period production systems, Global efficiency, Productivity index.

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In multi-period data envelopment analysis (MDEA), the relative efficiency of a set of DMUs can be evaluated by considering time serial data. Park and Park [37] presented a two-phase DEA approach for measuring the aggregative efficiency of a DMU over multi-period production systems. Sohn and Kim [42] considered as a ground for performance analysis the simple average of the efficiency scores of multiple periods, while Esmaeilzadeh and Hadi-Vencheh [10] proposed a super-efficiency model to measure the efficiency scores without setting weighs on the inputs and the outputs over the time periods. The production possibility sets (PPSs) of all individual periods are aggregated into a single PPS, which may cause overestimation of the efficiency scores besides having observations, that do not belong to any individual PPS, included into the aggregated PPS.

In overall efficiency evaluation, standard DEA models can be used with the data aggregated over multiple time periods. Since such approach ignores the specific situation of each time period, Kao and Liu [17] suggested a relational network model that allows the efficiency to be assessed under each individual period. Oh and Shin [24] used Monte Carlo analysis based on stochastic frontier analysis (SFA) and DEA to remedy the mis-measurement that affects budget allocation and system performance when frontier estimation models are applied. The authors considered measurement errors for scenarios with different multi-period budgeting strategies and different production frontier estimation models. While considering the operations of individual periods, Kou and Wang [16] introduced a multi-period two-stage DEA model that evaluates simultaneously overall and period efficiencies, where the overall efficiency is expressed as a weighted average of the period efficiencies. In an attempt to measure and decompose the overall efficiency of multi-period and multi-division systems (MPMDS), Kou *et al.* [18] proposed a new formulation for dynamic network DEA (DNDEA) models based on system thinking to assess performance of MPMDS for complex investment and management decisions. Razavi Hajiagha *et al.* [40] noted that, in a single time point, DEA is a cross-sectional approach for relative efficiency evaluation and, hence, considering the input-output data over multi-period time series provides an unnecessary compensating impact and makes the efficiency appraisal unrealistic. Therefore, the authors proposed a two-stage approach based on Chebyshev inequality bounds to avoid the negative impact of data accumulation. Li *et al.* [21] extended the cross-sectional DEA model to time-varying Malmquist DEA for dynamic financial distress prediction. Lee *et al.* [20] proposed a DEA model with consistent weights for time lag effects throughout the time periods, where the inputs of a period can be used to produce the outputs of several subsequent periods. Chen *et al.* [9] presented a model for the evaluation of the overall efficiency of multi-period regional R&D investment activities, which accounts for the dynamic interdependence between these activities over different periods.

In a bid to generalize the MPI and overcome its shortcommings, Pastor and Lovell [38] and Portela [39] introduced the global and the circular MPIs, which can provide more accurate results for productivity analysis while considering the convex combination of the PPSs.

In a similar vein, Munisamy and Arabi [23] and Wang *et al.* [45] addressed the multi-period evaluation of DMUs within a meta-frontier that considers the union of several PPSs. It is noteworthy that all MDEA models developed in the aforementioned studies are based on the union of PPSs, which allows implicitly observations, that belong to none of the individual PPSs, to enter the global PPS (convex combination of the PPSs or meta-frontier). Thus, the efficiency evaluation is likely to be conducted with reference to "fictive" observations and the outcomes might not reflect the true performance of the assessed DMUs. In this paper, we introduce a novel MDEA model that aims to circonvent these limitations of the existing MDEA models. Although the proposed approach adopts the union of the PPSs too, its key distinctive feature stems in its ability to guarantee that the reference set includes solely observations that belong effectively to the individual PPSs. As such, the efficiency evaluation process becomes more reliable and more rational under the new approach. The proposed global MDEA model involves major modifications of fundamental DEA axioms so that to render possible for a DMU, defined with inputs and outputs of period t, to be evaluated within the PPS of another period $l,\ t \neq l$. Furthermore, building on the properties of the global MDEA model, we introduce a global productivity index, termed Global Progress and Regress index (GPRI). Unlike the existing productivity indices that are restricted to two successive periods for only constant returns to scale (CRS) production technologies, the GPRI enables the DMU's efficiency as well as the extent of its progress or regress to be evaluated over multi-period time horizons under variable returns to scale (VRS) assumption. The most salient features of the new MDEA model as well as the GPRI are highlighted using a real-life sample of 25 bank branches considered over 4 years.

This paper sets out as follows. The methodological background relating to the proposed approach is presented in Section 2. Section 3 will be dedicated to the global MDEA model, where new axioms will be defined, followed by an explicit formulation of the new approach with a numerical illustration. Next, we introduce the GPRI, before applying the new concepts on a real-life problem in the the banking sector in Section 4. Finally, conclusions and new research venues can be found in Section 5.

2. Methodological background

2.1. Standard DEA models

Assume that we have n DMUs to be evaluated. Let $X \in \mathbb{R}_+^{m\times n}$ and $Y \in \mathbb{R}_+^{s\times n}$ represent the matrices of inputs and outputs of the n DMUs, with x_j and y_j denoting the jth column of X and Y, respectively. Each DMU_j $(j = 1,\ldots,n)$, consumes m inputs x_{ij} $(i = 1,\ldots,m)$ to produce s outputs y_{rj} $(r = 1,\ldots,s)$.

The CCR model, initially proposed by Charnes, Cooper and Rhodes [8], is a standard DEA model that assumes constant returns to scale (CRS) for the production technology and considers the following axioms for its PPS T_c.

A_1. Feasibility: $\forall\ j,\ (x_j, y_j) \in T_c$.
A_2. Convexity: T_c is a convex set.
A_3. Constant returns to scale: $\forall\ (x_j, y_j) \in T_c,\ (\alpha\ x_j, \alpha\ y_j) \in T_c, \quad \alpha \in \mathbb{R}$.
A_4. Free disposability: If $(x, y) \in T_c,\ y \geq y' \geq 0$ and $x \leq x'$ then $x', y') \in T_c$.
A_5. Closedness: T_c is a closed set.
A_6. Minimum extrapolation: T_c is the minimal set that satisfies axioms A_1-A_5.

With axioms A_1–A_6, T_c can be defined as follows:

$$T_c = \left\{(x,y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s | \sum_{j=1}^n \lambda_j x_j \leq x, \sum_{j=1}^n \lambda_j y_j \geq y,\ \lambda_j \geq 0,\ j = 1,\ldots,n\right\} \tag{2.1}$$

Assuming variable returns to scale (VRS), Banker, Charnes and Cooper [5] introduced BCC model, whose PPS discards A_3 and is defined as follows:

$$T_v = \left\{(x,y) \in \mathbb{R}_+^m \times \mathbb{R}_+^s | \sum_{j=1}^n \lambda_j x_j \leq x, \sum_{j=1}^n \lambda_j y_j \geq y, \sum_{j=1}^n \lambda_j = 1, \lambda_j \geq 0, j = 1,\ldots,n\right\} \tag{2.2}$$

Under a DEA framework, the relative efficiency of a DMU can be assessed through input contraction and/or output increment [30]. The latter approach in known as output orientation while the former is an input orientation of the DEA model [35]. The output oriented CCR model that estimates the efficiency φ of DMU_o can be formulated as the following linear programming (LP).

$$\begin{aligned}
\max\ & \varphi \\
s.t.\ & \sum_{j=1}^n \lambda_j x_{ij} \leq x_{io} && i = 1,\ldots,m \\
& \sum_{j=1}^n \lambda_j y_{rj} \geq \varphi y_{ro} && r = 1,\ldots,s \\
& \lambda_j \geq 0 && j = 1,\ldots,n
\end{aligned} \tag{2.3}$$

$\varphi^* = 1$ if DMU_o is located on the efficiency frontier. Otherwise, $\varphi^* > 1$ and DMU_o is declared inefficient. Similarly, the input oriented BCC model can write as follows.

$$
\begin{aligned}
& \min \theta \\
& s.t. \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} && i = 1, \ldots, m \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} && r = 1, \ldots, s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0 && j = 1, \ldots, n
\end{aligned}
\tag{2.4}
$$

Here, $\theta^* = 1$ if DMU_o is efficient, and $\theta^* < 1$ for an inefficient DMU_o.

In models (2.3) and (2.4), the potential benchmark(s) of DMU_o can be obtained from the equations $(x_o, \varphi^* y_o)$ and $(\theta^* x_o, y_o)$, respectively, which define the radial projections of DMU_o on the corresponding efficiency frontiers [1].

It is noteworthy that the standard DEA models, such as (2.3) and (2.4), assume that the input and output data belong to the same time period. In most real world problems, several events do potentially affect a production system's performance over multi-period time horizons. Under these conditions, conducting performance analysis with the standard DEA models may undermine the DMU's true performance.

In the next section, we present a brief review of two models that dealt with multi-period performance evaluation in the DEA literature, prior to developing a new alternative DEA model.

2.2. Multi-period efficiency evaluation

Consider n DMUs to be evaluated over L different time periods t, $t \in \{t_1, \ldots, t_L\}$. Let $X^t \in \mathbb{R}_+^{m\times n}$ and $Y^t \in \mathbb{R}_+^{s\times n}$ be the matices of observed input and output measures of the n DMUs in time period t. With x_j^t and y_j^t denoting the jth column of X^t and Y^t, respectively, each DMU_j $(j = 1, \ldots, n)$ consumes m inputs x_{ij}^t $(i = 1, \ldots, m)$ to produce s outputs y_{rj}^t $(r = 1, \ldots, s)$.

2.2.1. Multi aggregative efficiency model

In order to evaluate the efficiency of DMU_o within a multi-period production system, Park and Park [37] proposed the Multi Aggregative Efficiency (MAE) model, which writes as follows under VRS:

$$
\begin{aligned}
& \max \varphi \\
& s.t. \sum_{j=1}^{n} \lambda_j^t x_{ij}^t \le x_{io}^t && i = 1, \ldots, m;\ t = t_1, \ldots, t_L \\
& \sum_{j=1}^{n} \lambda_j^t y_{rj}^t \ge \varphi y_{ro}^t && r = 1, \ldots, s;\ t = t_1, \ldots, t_L \\
& \sum_{j=1}^{n} \lambda_j^t = 1 && t = 1, \ldots, L \\
& \lambda_j^t \ge 0 && j = 1, \ldots, n;\ t = t_1, \ldots, t_L
\end{aligned}
\tag{2.5}
$$

In model (2.5), Park and Park [37] adopt an optimistic viewpoint for assessing DMU_o by considering its best efficiency score over L aggregated time periods.

2.2.2. Multi-period super-efficiency evaluation

Esmaeilzadeh and Hadi-Vencheh [10] proposed a modified version of the super-efficiency model [3] that takes into account time serial data. Instead of evaluating DMU_o from an optimistic viewpoint, as the MAE model, the authors claim that it is more rational to consider the average of the efficiency scores over all periods. The VRS form of this model follows.

$$
\begin{aligned}
&\max\ \varphi_o = \frac{1}{L}\sum_{t=1}^{L}\varphi^t \\
&s.t.\ \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j^t x_{ij}^t \le x_{io}^t && i=1,\dots,m;\ t=1,\dots,L \\
&\qquad \sum_{\substack{j=1\\ j\neq o}}^{n}\lambda_j^t y_{rj}^t \ge \varphi^t y_{ro}^t && r=1,\dots,s;\ t=t_1,\dots,t_L \\
&\qquad \sum_{j=1}^{n}\lambda_j^t = 1 && t=1,\dots,L \\
&\qquad \lambda_j^t \ge 0 && j=1,\dots,n;\ t=t_1,\dots,t_L
\end{aligned} \tag{2.6}
$$

Model (2.6) enables the measurement of the efficiency score φ^t of each DMU_o in each period $t, t\in\{t_1,\dots,t_L\}$, while providing its aggregative efficiency φ_o over L time periods. The important feature of model (2.6) resides in the fact that it does not require any information on price or preferential weightings of data.

3. Global multi-period DEA model

Consider a DMU p to be evaluated over three time periods t_1, t_2 and t_3. Assume that the efficiency score of DMU p is 0.8 in period t_1. Due to possible changes of the production technology in periods t_2 and t_3, the efficiency of DMU p can also change, that is, it may improve or deteriorate, or remain unchanged. Consider a situation where DMU p preserves the efficiency score 0.8 in period t_2, despite better production conditions, and the same score in period t_3, even with worse production conditions. Under such scenarios, the decision maker (DM) would certainly face controversial issues, such as:

– Is the efficiency score 0.8 suitable for evaluating the performance of DMU p in each period?
– Does the score 0.8 have the same meaning in each period?

To answer these questions, Hosseinzadeh *et al.* [14] introduced the periodic efficiency (PE) model, which considers the data of different periods simultaneously for the evaluation. Building on the PE model, this paper aims to bring more plausible answers to the aforementioned questions through a global multi-period approach.

3.1. Illustrative example

Given a set of DMUs that employ one input x to produce one output y, Figure 1 shows in red and blue lines, respectively, the efficiency frontiers at different time periods t_1 and t_2 under an output oriented DEA setting. Let PPS^{t_1} and PPS^{t_2} refer to their respective PPSs. On the same graph, the dotted lines represent the PE frontier induced by PPS^{t_1} and PPS^{t_2}, and whose PPS will be denoted PPS^{PE}. It can be said that PPS^{t_1} and PPS^{t_2} are enveloped by PPS^{PE}. In Figure 1, the gray area appears as part of PPS^{PE}, although it belongs to neither PPS^{t_1} nor PPS^{t_2}.

In Figure 2, with PPS^{PE} shown in dotted lines, the inefficient DMU p can improve its efficiency by reducing its input through its horizontal projection p' on the PE frontier. Here, it is important to note that p' does not belong to PPS^{t_1} nor PPS^{t_2}. Therefore, p' cannot be reached by using standard DEA models with the data of times t_1 and t_2. In order to overcome such an obstacle, we consider the projection p'' of DMU p on the union of the original efficiency frontiers, as depicted in Figure 3. Accordingly, we develop an axiomatic approach for

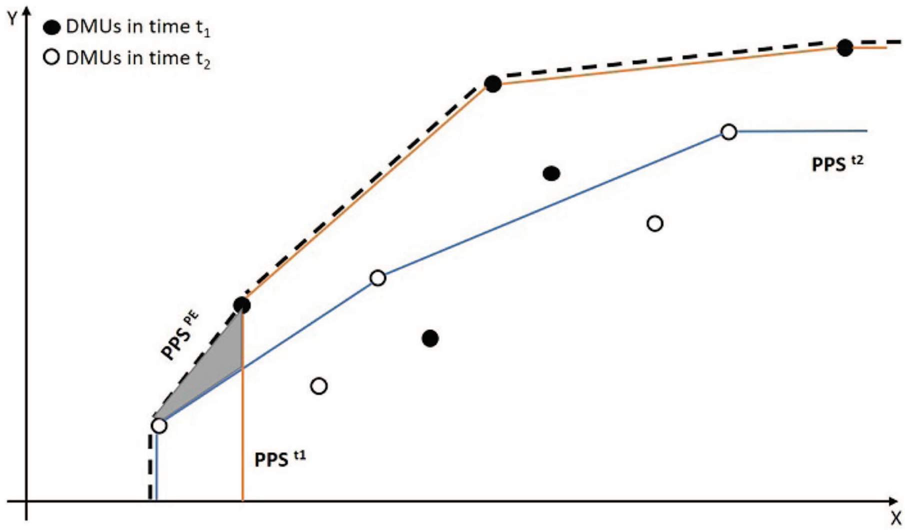

FIGURE 1. Original PPSs at time periods t_1 and t_2 with Induced PE frontier.

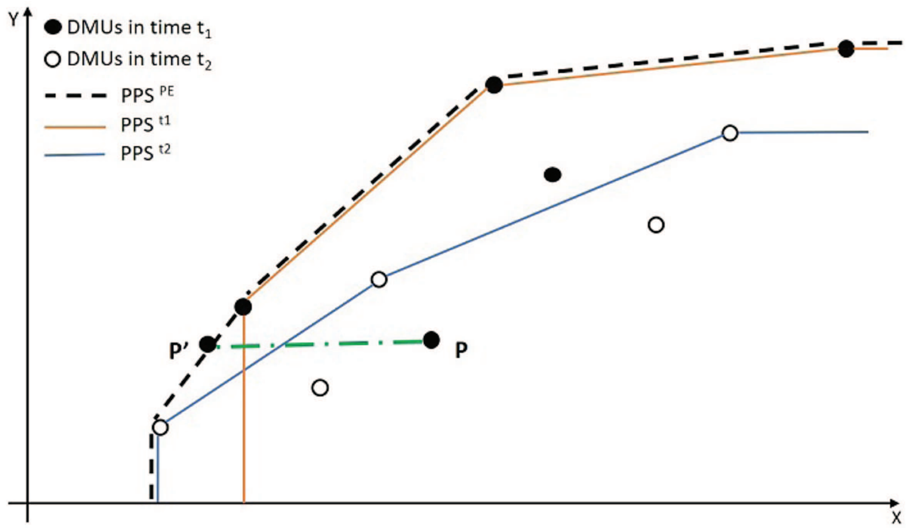

FIGURE 2. Setting p' as initial target for efficiency improvement.

a global multi-period model, which adopts as PPS the union of the PPSs over all the time periods (t_1 and t_2), identified as the global multi-period PPS (PPS^G). Indeed, p' belongs neither to PPS^{t_1} nor to PPS^{t_2}. Hence, it is more rational to find the coordinates of p'' as a valid reference point instead of p'. It is not possible to find the coordinates of p'' by using the standard DEA models over PPS^{t_1} or PPS^{t_2}.

Unlike the global multi-period methods reviewed in the literature (*e.g.*, [23,38,39,45]), the proposed approach ensures that the reference point considered for the evaluation of p belongs effectively to the existing PPSs, which exempts explicitly those areas that are not part of these PPSs. In Figure 4, the Global Efficient frontier depicted with purple lines can be interpreted as the union of PPS^{t_1} and PPS^{t_2}. As such, the input-projection p'' of DMU p will be located on PPS^G.

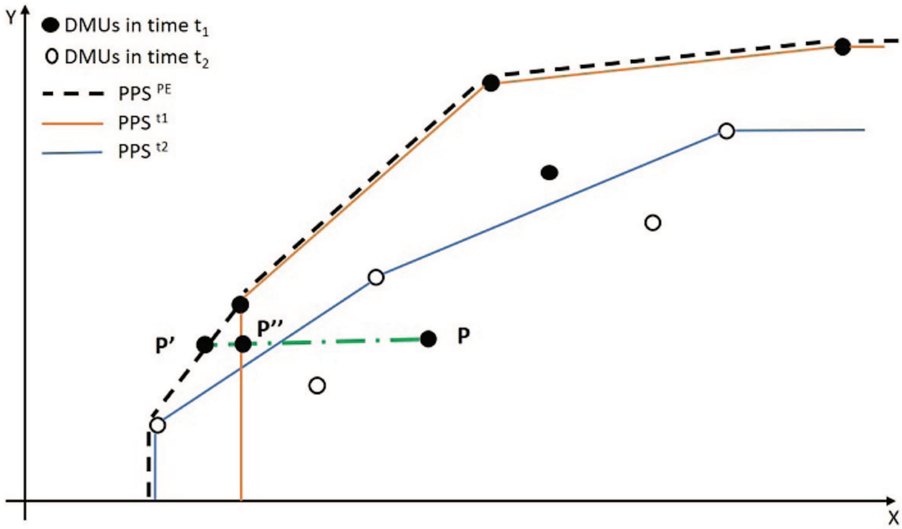

FIGURE 3. Establishing p'' as a desired target for efficiency improvement.

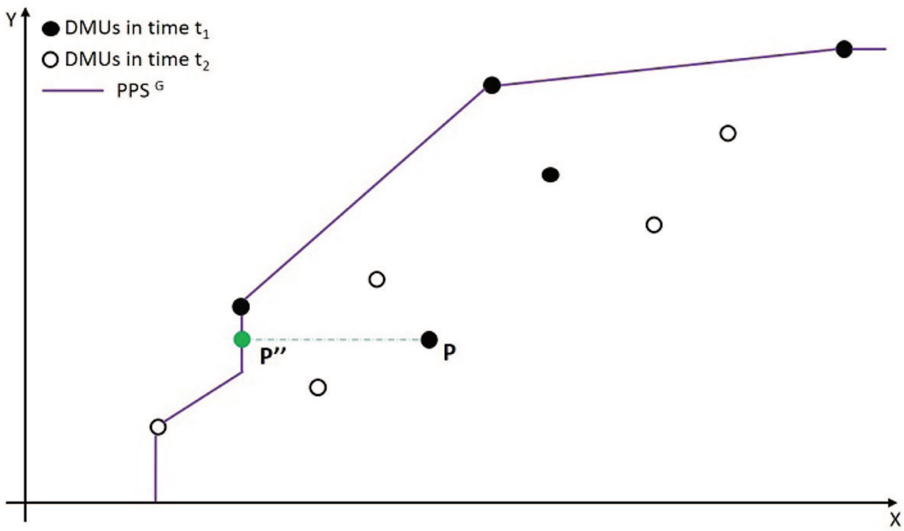

FIGURE 4. PPS of Global Efficiency.

3.2. Axioms for the Global multi-period approach

The GMP-PPS can be defined as follows:

$$T_G = \left\{(x^t, y^t)|\ x^t \quad \textit{can produce} \quad y^t, \quad t = t_1, \ldots, t_L\right\}.$$

The construction of the VRS form of T_G requires the following axioms.
Non emptiness: Considering the data sets in successive years t_1 to t_L, we have:

$$\forall j \quad (x_j^l, y_j^l)^t \in T_G^v, \quad l \in \{t_1, \ldots, t_L\}$$

This axiom emphasizes that, in the GMP-PPS, each year contains its own observations (DMUs). **Convexity**: Under the convexity assumption, in successive years, we have, for each year l:

$$(x^l, y^l)^t = \sum_{j=1}^{n} \mu_j (x_j^l, y_j^l)^t, \ \sum_{j=1}^{n} \mu_j = 1, \ \mu_j \geq 0, \ j = 1, \ldots, n \text{ and } l \in \{t_1, \ldots, t_L\}$$

The convexity assumption holds for each data set, from t_1 to t_L. In this situation, T_G^v is defined as the union of the convex combinations of each separate data set, from t_1 to t_L.

$$T_G^v = \left\{ (x,y) | \bigcup_{l=t_1}^{t_L} \left(y \geq \sum_{j=1}^{n} \lambda_j^l x_j^l, \ y \leq \sum_{j=1}^{n} \lambda_j^l y_j^l, \ \sum_{j=1}^{n} \lambda_j^l = 1, \ \lambda_j^l \geq 0, \ j = 1, \ldots, n \right) \right\}$$

It should be noted that T_G^v does not consider the convexity assumption for the entire data set from t_1 to t_L.

Disposability: If $(x,y) \in T_G^v$ and $x' \geq x$ and $y \leq y'$ then $(x', y') \in T_G^v$.

Minimality: Under VRS assumption, $T_G^v)$ is the smallest set satisfying the above axioms, defined as: $T_G^v = \Big\{(x,y) | \Big(x \geq \sum_{j=1}^{n} \lambda_j^{t_1} x_j^{t_1}, \ y \leq \sum_{j=1}^{n} \lambda_j^{t_1} y_j^{t_1}, \ \sum_{j=1}^{n} \lambda_j^{t_1} = 1, \ \lambda_j^{t_1} \geq 0, \ j = 1, \ldots, n \Big)$
$\vee \ldots \vee \Big(x \geq \sum_{j=1}^{n} \lambda_j^{t_L} x_j^{t_L}, \ y \leq \sum_{j=1}^{n} \lambda_j^{t_L} y_j^{t_L}, \ \sum_{j=1}^{n} \lambda_j^{t_L} = 1, \ \lambda_j^{t_L} \geq 0, \ j = 1, \ldots, n \Big) \Big\}$

3.3. Modeling the Global multi-period approach

Considering T_G^v for the efficiency evaluation of DMU_o, the corresponding input oriented model for each period $l \in \{t_1, \ldots, t_L\}$ writes as follows:

$$\begin{aligned} & \min \theta^l \\ & s.t. \ \ (\theta^l x_o^l, y_o^l)^t \in T_G^v, \quad l = t_1, \ldots, t_L. \end{aligned} \tag{3.1}$$

For a data set involving L time periods $t_1, \ldots, t_L$, model (3.1) can be written as:

$$\begin{aligned} & \min \theta^l \\ & s.t. \ \ \sum_{j=1}^{n} \lambda_j^f x_{ij}^f \leq \theta^l x_{io}^l + V_f M \ \ i = 1, \ldots, m; \ f = t_1, \ldots, t_L \\ & \qquad \sum_{j=1}^{n} \lambda_j^f y_{ij}^f \geq y_{ro}^l - V_f M \qquad r = 1, \ldots, s; \ f = t_1, \ldots, t_L \\ & \qquad \sum_{j=1}^{n} \lambda_j^f = 1, \qquad\qquad\qquad f = t_1, \ldots, t_L \\ & \qquad \sum_{f=t_1}^{t_L} V_f = L - 1 \\ & \qquad \lambda_j^f \geq 0, \qquad\qquad\qquad j = 1, \ldots, n; \ f = t_1, \ldots, t_L \\ & \qquad V_f \in \{0, 1\} \end{aligned} \tag{3.2}$$

Model (3.2) is a mixed integer linear program (MILP), where V_f is a binary variable and M is a large positive number. The big M and V_f ensure that the global multi-period efficiency score of DMU_o is minimal at a specific time period $t \in \{t_1, \ldots, t_L\}$. Indeed, constraint $\sum_{f=t_1}^{t_L} V_f = L - 1$ allows for the minimum efficiency score of DMU_o to be reached as a result of its evaluation with respect to each of the L efficiency frontiers t using its observations $(x_o, y_o)^t, t = t_1, \ldots, t_L$. Thus, the constraints corresponding to $V_f = 0$ are active and enable DMU_o to be assessed based on the efficiency frontier at time f, while the constraints for which $V_f = 1$ are redundant.

To avoid the computational complexity that may result out of the binary constraint in model (3.2), we develop model (3.3) to evaluate DMU_o at time l ($l \in \{t_1, \ldots, t_L\}$) with reference to all the efficiency frontiers $t_1, \ldots, t_L$.

$$\begin{aligned} &\min\ \theta^l \\ &s.t.\ \sum_{j=1}^{n} \lambda_j^f x_{ij}^f \leq \theta^l x_{io}^l\ i = 1, \ldots, m;\ f = t_1, \ldots, t_L, \\ &\qquad \sum_{j=1}^{n} \lambda_j^f y_{ij}^f \geq y_{ro}^l \quad r = 1, \ldots, s;\ f = t_1, \ldots, t_L, \\ &\qquad \sum_{j=1}^{n} \lambda_j^f = 1, \qquad f = t_1, \ldots, t_L \\ &\qquad \lambda_j^f \geq 0, \qquad j = 1, \ldots, n;\ f = t_1, \ldots, t_L \end{aligned} \tag{3.3}$$

Model (3.3) can be viewed as a set of separable models in periods $t_1, \ldots, t_L$, contrary to the unified model (3.2). Model (3.3) evalutes DMU_o with reference to each separate efficiency frontier, t_1 through t_L, for each set of inputs and outputs $(x_o, y_o)^l$ observed in period l ($l \in \{t_1, \ldots, t_L\}$).

As a result, L efficiency scores, $\theta^{l*} \in \{\theta^{*t_1},\ \theta^{*t_2}, \ldots, \theta^{*t_L}\}$ are produced for each evaluation involving $(x_o, y_o)^l$ over L time periods. Ultimately, the global multi-period efficiency score θ^{*j} corresponding to DMU_j ($j = 1, \ldots, n$) is derived after L series of L evaluation runs. Accordingly, model (3.3) requires less computational effort than model (3.2) as each θ^{l*} is a result of a series of L efficiency evaluations conducted over a single solution run. However, model infeasibility may occur in situations where DMU_j falls outside the intended temporal PPS and, hence, cannot be assessed within the PPS's efficiency frontier.

Theorem 3.1. *Model (3.3) is always feasible and yields a minimum efficiency score.*

Proof. Noting that l and f vary from t_1 to t_L, setting, *e.g.*, $l = t_1$ means DMU_o as well as the PPS are at time t_1 and solving model (3.3) under these conditions produces a regular efficiency score for DMU_o. If model (3.3) is formulated for DMU_o using a PPS corresponding to a different time, its efficiency score can be less than the one obtained while it is being evaluated in time t_1.

Assuming $\lambda_o^{t_1} = 1,\ \lambda_j^{t_1} = 0,\ j \neq o$, and $\theta^{t_1} = 1,\ \theta^{*t_1}$ can be obtaind by solving a standard DEA model. Therefore, $min_{t_1 \leq l \leq t_L}\{\theta^{*t_1}, \ldots, \theta^{*t_L}\}$ is not an empty set and it is always possible to have a global multi-period efficiency score as the minimum of this set. □

3.4. Global progress and regress index (GPRI)

Measuring the efficiency scores of a DMU over serial time periods is not sufficient to support informed decisions since production technologies may change over time, leading ineluctably to variations of the DMU's performance. To keep track of these changes, the MPI [7] enables the productivity status of a DMU to be assessed between two successive time periods t_1 and t_2 (see, *e.g.*, [12, 13] and [15]). However, managers would also be interested in evaluating productivity over several time periods, which prompts the following questions:

1. Which metrics can better quantify a DMU's change (progress or regress)?
2. How can productivity be evaluated over a time horizon of more than two periods?

To answer these questions, we propose a global progress and regress index (GPRI) that is based on the global multi-period efficiency. The proposed index enables the progress or the regress of a DMU to be assessed under VRS form of technology rather than assuming CRS, as the case of MPI.

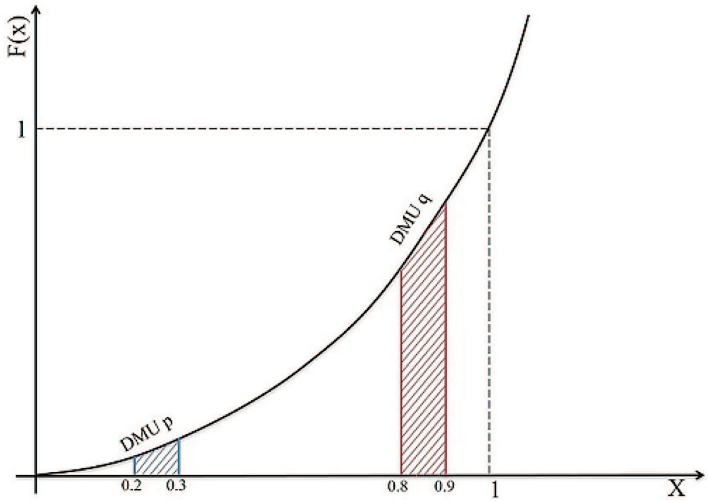

FIGURE 5. Profitability challenge.

Assume that the efficiency scores of DMU_p and DMU_q are, respectively, 0.2 and 0.8 in time t_1 and reach 0.3 and 0.9 in time t_2. Accordingly, we have:

$$Index_p = \frac{0.3}{0.2} = 1.5 \qquad Index_q = \frac{0.9}{0.8} = 1.125$$

In real life situations, increasing the efficiency level of a DMU from 0.8 to 0.9 is much harder than an increase from 0.2 to 0.3. Because this aspect is not explicitly reflected in the above index values, we develop a procedure that handles more formally the relative change and its extent. Consider the following function:

$$F(x) = x^2 \text{ where } F(0) = 0 \text{ and } F(1) = 1$$

Based on the area under the curve of F, Figure 5 shows that the progress is more substantial for DMU_q as compared to DMU_p, which translates better the real-life scenario. As such, the usage of function F can be seen as a more practical way to measure a DMU's change (progress or regress) from one period of time to another. Subsequently, the GPRI can be defined as follows.

Definition 3.2. Given DMU_p to be evaluated between two successive time periods t_1 and t_2,

$$\mathrm{GPRI}_p = \int_{\theta_p^{t_1}}^{\theta_p^{t_2}} x^2 \; dx \tag{3.4}$$

GPRI_p is the extent of progress (regress) of DMU_p if $\mathrm{GPRI}_p > 0$ ($\mathrm{GPRI}_p < 0$).
There is neither a progress nor a regress if $\mathrm{GPRI}_p = 0$.

In addition to its potential to determine the change status of a DMU as well as the extent of that change, GPRI can also compare the performance of two DMUs between two successive time periods. For instance, if $GPRI_p = 0.4$ and $GPRI_q = 0.8$, one can conclude that DMU_q's progress is twice DMU_p's.

TABLE 1. Data of five DMUs.

DMU	I_{t_1}	O_{t_1}	I_{t_2}	O_{t_2}	I_{t_3}	O_{t_3}	I_{t_4}	O_{t_4}
1	6	7	3	10	2	4	5	11
2	2	5	4	3	2	6	12	9
3	5	8	7	8	3	9	5	7
4	1	4	7	9	3	8	9	12
5	4	9	3	11	8	10	8	9

It is important to note that the GPRI has the merit to be the first index that enables such comparisons, which certainly enhances the manager's toolbox for more informed decisions. Moreover, the stated features of the GPRI are not restricted to two successive time periods but can be easily extended to multiple period horizons.

3.5. Numerical illustration

To better illustrate the proposed model, we use the example presented in Table 1. There are five $DMUs$ to evaluate over four successive time periods, $t_1, \ldots, t_4$. Each DMU employs a single input to produce a single output.

If we solve model (3.3) for DMU_1 in four time-periods, t_1, t_2, t_3, and t_4, we obtain the best efficiency score of DMU_1 for each of these periods.

For instance, in time $l = t_1$, model (3.3) writes as shown in (3.5) and produces the optimal efficiency score $\theta^{*t_1} = 0.32$.

$$
\begin{aligned}
&\min\ \theta^{t_1}\\
&s.t.\ \ 6\lambda_1^{t_1} + 2\lambda_2^{t_1} + 5\lambda_3^{t_1} + 1\lambda_4^{t_1} + 4\lambda_5^{t_1} \le 6\theta^{t_1}\\
&\quad 3\lambda_1^{t_2} + 4\lambda_2^{t_2} + 7\lambda_3^{t_2} + 7\lambda_4^{t_2} + 3\lambda_5^{t_2} \le 6\theta^{t_1}\\
&\quad 2\lambda_1^{t_3} + 2\lambda_2^{t_3} + 3\lambda_3^{t_1} + 3\lambda_4^{t_3} + 8\lambda_5^{t_3} \le 6\theta^{t_1}\\
&\quad 5\lambda_1^{t_4} + 12\lambda_2^{t_4} + 5\lambda_3^{t_4} + 9\lambda_4^{t_4} + 8\lambda_5^{t_4} \le 6\theta^{t_1}\\
&\quad 7\lambda_1^{t_1} + 5\lambda_2^{t_1} + 8\lambda_3^{t_1} + 4\lambda_4^{t_1} + 9\lambda_5^{t_1} \ge 7\\
&\quad 10\lambda_1^{t_2} + 3\lambda_2^{t_2} + 8\lambda_3^{t_2} + 9\lambda_4^{t_2} + 11\lambda_5^{t_2} \ge 7\\
&\quad 4\lambda_1^{t_3} + 6\lambda_2^{t_3} + 9\lambda_3^{t_1} + 8\lambda_4^{t_3} + 10\lambda_5^{t_3} \ge 7\\
&\quad 11\lambda_1^{t_4} + 9\lambda_2^{t_4} + 7\lambda_3^{t_4} + 12\lambda_4^{t_4} + 9\lambda_5^{t_4} \ge 7\\
&\quad \lambda_j^f \ge 0, \quad j = 1, \ldots, 5 \quad f = t_1, \ldots, t_4
\end{aligned}
\tag{3.5}
$$

In time $l = t_2$, DMU_1 is evaluated using model (3.6), which yields $\theta^{*t_2} = 0.97$. Idem for times $l = t_3$ and $l = t_4$.

$$
\begin{aligned}
&\min\ \theta^{t_2}\\
&s.t.\ \ 6\lambda_1^{t_1} + 2\lambda_2^{t_1} + 5\lambda_3^{t_1} + 1\lambda_4^{t_1} + 4\lambda_5^{t_1} \le 3\theta^{t_2}\\
&\quad 3\lambda_1^{t_2} + 4\lambda_2^{t_2} + 7\lambda_3^{t_2} + 7\lambda_4^{t_2} + 3\lambda_5^{t_2} \le 3\theta^{t_2}\\
&\quad 2\lambda_1^{t_3} + 2\lambda_2^{t_3} + 3\lambda_3^{t_1} + 3\lambda_4^{t_3} + 8\lambda_5^{t_3} \le 3\theta^{t_2}\\
&\quad 5\lambda_1^{t_4} + 12\lambda_2^{t_4} + 5\lambda_3^{t_4} + 9\lambda_4^{t_4} + 8\lambda_5^{t_4} \le 3\theta^{t_2}\\
&\quad 7\lambda_1^{t_1} + 5\lambda_2^{t_1} + 8\lambda_3^{t_1} + 4\lambda_4^{t_1} + 9\lambda_5^{t_1} \ge 10\\
&\quad 10\lambda_1^{t_2} + 3\lambda_2^{t_2} + 8\lambda_3^{t_2} + 9\lambda_4^{t_2} + 11\lambda_5^{t_2} \ge 10\\
&\quad 4\lambda_1^{t_3} + 6\lambda_2^{t_3} + 9\lambda_3^{t_1} + 8\lambda_4^{t_3} + 10\lambda_5^{t_3} \ge 10\\
&\quad 11\lambda_1^{t_4} + 9\lambda_2^{t_4} + 7\lambda_3^{t_4} + 12\lambda_4^{t_4} + 9\lambda_5^{t_4} \ge 10\\
&\quad \lambda_j^f \ge 0, \quad j = 1, \ldots, 5 \quad f = t_1, \ldots, t_4
\end{aligned}
\tag{3.6}
$$

TABLE 2. Global multi-period efficiency scores.

Time	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
t_1	0.32	0.75	0.44	1.00	0.64
t_2	0.97	0.29	0.30	0.34	1.00
t_3	0.67	1.00	1.00	0.89	0.43
t_4	1.00	0.31	0.60	0.56	0.47

TABLE 3. Global Multi-period indices.

times	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
$t_1 - t_2$	0.30 (Prog)	−0.13 (Reg)	−0.02 (Reg)	−0.32 (Reg)	0.24 (Prog)
$t_2 - t_3$	−0.21 (Reg)	0.33 (Prog)	0.32 (Prog)	0.22 (Prog)	−0.31 (Reg)
$t_3 - t_4$	0.23 (Prog)	−0.32 (Reg)	−0.26 (Reg)	−0.18 (Reg)	0.01 (Prog)

TABLE 4. Malmquist productivity index.

Times	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5
$t_1 - t_2$	1.03 (Prog)	0.34 (Reg)	0.17 (Reg)	0.14 (Reg)	0.70 (Reg)
$t_2 - t_3$	0.423 (Reg)	0.75 (Reg)	1.23 (Prog)	0.37 (Reg)	0.55 (Reg)
$t_3 - t_4$	1.49(Prog)	0.96(Reg)	1.22(Prog)	1.29(Prog)	0.65(Reg)

We repeat the same process for DMU_2 and the remaining DMUs. The full results for all model runs are given in Table 2.

Next, we compute the GPRI between each pair of successive time periods for each DMU. The results are exhibited in Table 3.

Looking at the results related to DMU_1, its multi-period efficiency scores are, respectively, 0.32 and 0.97 in t_1 and t_2, with a GPRI of 0.3 between these two time-periods, reflecting a progress. In the meantime, the efficiency score drops to 0.67 in t_3, resulting in a regress from t_2 to t_3, duly quantified with a GPRI of −0.21. The increase of the multi-period efficiency score to 1 in t_4 is associated with a GPRI of $0.32(Prog)$ between t_1 and t_4. Noticeably, DMU_1's GPRI is $0.03(Prog)$ from t_2 to t_4 where the efficiency scores are 0.97 and 1, respectively. Such a gap between the latter GPRI (0.03) and the former (0.32) stresses the fact that the competition for reaching 1 is much harder from 0.32 than from 0.97. Similarly, if we consider DMU_5, whose efficiency scores are 0.43 and 0.47 in t_3 and t_4, respectively, its GPRI is $0.008(Prog)$. Although the efficiency score difference here is almost the same as with DMU_1 (between t_2 and t_4), the GPRI for the latter scenario is higher ($0.03(Prog)$).

Table 4 presents the values of the MPI as calculated for the above example.
The results show that there is a conflict in the change status (progress/regress) for 40% of the reported cases. Meanwhile, based on the efficiency scores given in Table 2, the results may suggest that the MPI does not reflect the changes correctly.

4. Application

We consider 25 bank branches over 4 years (2007 - 2010). Each bank branch is defined with three inputs and three outputs. The inputs consist of Personal and administrative costs, Deposits, and Profit payments. The outputs include Income from loans, Wage, and Revenue. The full data set is provided in Appendix A.

TABLE 5. Standard CCR efficiency scores.

Bank	2007	2008	2009	2010
1	1	1	0.937	1
2	0.749	0.793	1	0.536
3	1	1	0.528	0.611
4	0.81	0.91	0.86	0.743
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	0.801
9	0.676	1	1	1
10	0.986	1	0.861	0.723
11	1	0.879	1	1
12	0.947	1	1	1
13	1	0.761	1	1
14	1	1	1	1
15	0.619	1	0.879	0.725
16	0.975	0.793	0.674	0.755
17	0.704	0.519	0.789	1
18	1	1	1	1
19	1	1	1	1
20	1	1	1	1
21	0.749	0.694	0.747	0.712
22	1	1	1	1
23	1	1	1	1
24	0.391	0.526	0.7	0.952
25	0.568	0.616	0.624	0.752

4.1. Global multi-period model

In the first step of the application, the bank's performance is evaluated for each year through the standard input oriented CCR model. The corresponding efficiency scores are displayed in Table 5.

Based on these results, $Bank_8$ is fully efficient in the first three years but its efficiency score decreases to 80% in the last year. Thus, $Bank_8$ cannot be considered globally efficient. Global multi-period efficiency evaluation models are more suitable to investigate the multi-period performance of each bank over four years simultaneously. Using the input oriented forms of models (2.5) and (2.6) under VRS assumption, we obtain the banks' efficiency scores listed in the first two columns of Table 6. In regard to model (2.5), although Esmaeilzadeh and Hadi-Venche [10] mentioned possible occurrence of infeasibility under VRS, as also noted in Seiford and Zhu [41], we have not faced this problem in this application.

Next, we use model (3.3) to evaluate the global multi-period efficiency. The results are presented in the last columns of Table 6 for each evaluation period.

The most significant difference between model (3.3) and the two previous models resides in its potential to evaluate the efficiency of each bank with reference to the efficiency frontier that is induced by the union of the PPSs corresponding to all time periods, as shown in Figure 6. Thus, model (3.3) produces global multi-period efficiency scores that are more accurate since it does not require the efficiency scores to be computed for each bank within each period as the case of model (2.5). In addition, model (3.3) encompasses the super efficiency reflected in model (2.6).

More importantly, model (3.3) succeeded to overcome the inability of standard DEA models to compare the efficiency scores of different periods with different frontiers. For instance, let us consider the evaluation of

TABLE 6. Efficiency scores with multi-period efficiency models.

Bank	Model (5)	Model (6)	Model (9)			
			2007	2008	2009	2010
1	1	1.683	0.637	1	0.937	0.464
2	1	1.155	0.683	0.793	1	0.397
3	1	1.051	1	1	0.245	0.199
4	0.910	0.831	0.642	0.910	0.860	0.552
5	1	3.844	1	1	0.109	1
6	1	2.266	1	1	1	0.936
7	1	2.644	1	1	1	0.598
8	1	2.043	0.915	0.351	0.351	0.209
9	1	1.355	0.580	1	1	0.850
10	1	0.899	0.424	1	0.861	0.310
11	1	1.749	1	0.879	1	0.485
12	1	1.353	0.721	1	1	0.384
13	1	1.056	1	0.761	1	0.66
14	1	2.076	0.984	1	1	0.394
15	1	0.877	0.619	1	0.879	0.316
16	0.970	0.799	0.777	0.793	0.674	0.451
17	1	0.791	0.581	0.519	0.789	0.712
18	1	1.195	0.902	1	1	0.703
19	1	1.521	0.851	1	1	0.987
20	1	1.412	0.61	1	1	0.800
21	0.749	0.725	0.536	0.694	0.747	0.484
22	1	2.414	1	0.129	1	0.656
23	1	1.905	0.735	1	1	0.772
24	0.952	0.642	0.354	0.526	0.700	0.691
25	0.752	0.64	0.495	0.616	0.624	0.300

TABLE 7. Standard efficiency scores for three banks

Reference year	$Bank_1$	$Bank_2$	$Bank_3$
2007	1	0.748	1
2008	0.637	0.683	infeasible
2009	3.709	1.139	infeasible
2010	7.306	1.636	infeasible
θ^*	0.637	0.683	1

$Bank_1$, $Bank_2$, and $Bank_3$ over years 2007 to 2010 with the observations (inputs and outputs) of 2007 using standard DEA model. The results are summarized in Table 7 along the global multi-period efficiency scores θ^* obtained *via* model (3.3). These are depicted geometrically in the two dimensional simulated diagram presented in Figure 5.

As illustrated in Figure 6, $Bank_1$ falls on the efficiency frontier of year 2007 but under the efficiency frontier of 2008, which explains the scores 1 and 0.637, respectively. In 2009 and 2010, $Bank_1$ is outside these years' PPSs and, hence, performs as a super-efficient unit with scores 3.709 and 7.307, respectively. With regard to the four-year evaluation, the projection of $Bank_1$ on the global efficiency frontier (dotted line), which coincides with the frontier of year 2008, and, hence, its global efficiency score is 2008's standard efficiency, *i.e.*, 0.637. A similar discussion may apply to $Bank_2$. Meanwhile, the efficiency score of $Bank_3$ is 1 when it is assessed in 2007

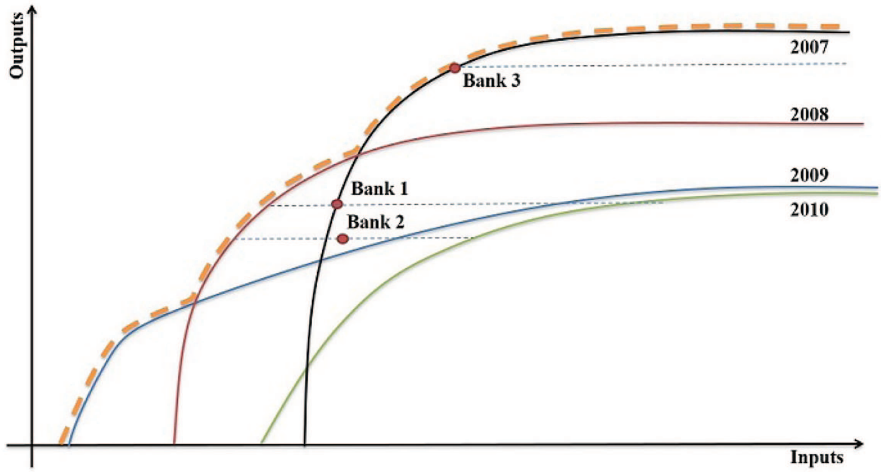

FIGURE 6. Standard and global efficiency evaluations.

TABLE 8. Super-efficiency scores of selected banks with model (2.6)

Reference year	$Bank_1$	$Bank_2$	$Bank_6$
2007	2.090	0.749	1.771
2008	1.712	0.793	2.991
2009	0.936	2.543	2.142
2010	1.993	0.535	2.162
Average	1.683	1.155	2.266

but its standard evaluation leads to infeasibility for the remaining years as it falls outside the corresponding PPSs.

Based on the standard DEA model (Tab. 2.5), $Bank_1$ performs efficiently in all periods except 2009 where its efficiency score drops to 0.937. However, the corresponding global multi-period efficiency scores are quite different due to the fact that model (3.3) tries to achieve the minimum possible efficiency score through projecting $Bank_1$ on the farthest efficiency frontier. Accordingly, its global multi-period efficiency scores are 0.637 and 0.464 when evaluated using its inputs and outputs of 2007 and 2010. Meanwhile, Table 6 reports, for $Bank_1$, multi-period efficiency scores of 1 and 1.683 with models (2.5) and (2.6), respectively, indicating successively full and super efficiency.

Consider the super-efficiency scores in Table 8 produced by model (2.6) for three selected banks. $Bank_1$ performs efficiently in years 2007, 2008, and 2010 and ends super-efficient on average. In the meantime, $Bank_2$ performs efficiently only in 2009 but it is also declared globally super-efficient. Finally, $Bank_6$ is the only bank among these three that preserves overall the same efficiency status every year. On the other hand, one can see in Table 6 that, based on model (2.5), all three banks are efficient.

4.2. Global Progress & Regress index

The MPI considers only two successive time periods for assessing a bank's performance progress, regress, or neutrality. One of the great features of the GPRI is its ability to address such objectives over several time periods. In order to show the potential of the later index, we calculate the GPRIs and thw MPIs for the 25

TABLE 9. Global Progress & Regress index and malmquist productivity index.

	GPRI			*MPI*		
Bank	2007–2008	2008–2009	2009–2010	2007–2008	2008–2009	2009–2010
1	0.247	−0.060	−0.241	2.840	1.276	0.754
2	0.060	0.167	−0.313	2.421	1.876	0.531
3	0.000	−0.328	−0.002	0.680	0.457	0.420
4	0.163	−0.039	−0.156	1.867	1.671	0.669
5	0.000	−0.333	0.333	1.105	2.353	0.903
6	0.000	0.000	−0.060	1.996	2.586	1.061
7	0.000	0.000	−0.262	0.942	1.808	0.855
8	−0.241	0.319	−0.330	0.999	1.951	0.420
9	0.268	0.000	−0.129	1.860	2.154	1.033
10	0.308	−0.120	−0.203	1.134	1.903	0.550
11	−0.107	0.107	−0.295	0.565	2.775	0.800
12	0.208	0.000	−0.314	1.560	2.019	0.697
13	−0.186	0.186	−0.237	1.170	1.960	0.717
14	0.016	0.000	−0.313	2.249	1.688	0.691
15	0.254	−0.107	−0.216	1.138	2.533	0.463
16	0.010	−0.064	−0.072	1.021	1.352	0.765
17	−0.019	0.117	−0.043	1.652	1.636	1.041
18	0.089	0.000	−0.217	2.401	1.658	0.749
19	0.128	0.000	−0.013	1.530	1.508	1.118
20	0.258	0.000	−0.163	3.151	1.756	0.833
21	0.060	0.027	−0.101	1.940	1.569	0.676
22	−0.333	0.333	−0.239	0.967	2.060	0.918
23	0.201	0.000	−0.180	3.152	1.662	0.834
24	0.034	0.066	−0.004	2.266	1.356	1.044
25	0.037	0.003	−0.072	2.000	1.408	0.615

banks and the results are presented in Table 9. Negative, positive, and nil GPRIs refer to regress, progress, and neurality status, respectively.

As can be seen, GPRI and MPI reveal the same productivity status for 24% of time sequences. It is also noteworthy that the MPI is calculated under CRS assumption to avoid infeasibility in DEA models, while GPRI is evaluated for VRS. In this application, attempts to compute MPI with VRS based DEA models led to about 60% infeasible cases. For example, the GPRI status of $Bank_1$ over the three successive time sequences is progress, regress, and regress, successively. With MPI, the only difference is the status of the second time sequence. Meanwhile, if we consider $Bank_5$ and $Bank_6$, it is evident that the associated productivity statuses are completely different with the two indices. While showing neural, regress, and progress for GPRI, $Bank_5$'s status is progress, progress, and regress for MPI. Similarly, the status of $Bank_6$ is neutral, neutral, and regress for GPRI and progress, progress, and progress for MPI.

Let θ_{τ}^{t} denote the global efficiency of $Bank_1$ when it is defined with its observations in time t and assessed within the efficiency frontier of time τ. The results for four time sequences are shown in Table 10 alongside the associated MPIs.

Note that θ_{2008}^{2008} and θ_{2009}^{2009} rerepresent here the standard efficiency scores of $Bank_1$ in years 2008 and 2009, respectively, since $t = \tau$. Meanwhile, $\theta_{2009}^{2008} = 24.6962$ and $\theta_{2008}^{2009} = 0.8344$ are the efficiency scores of $Bank_1$ obtained by solving model (3.3).

Figure 7 illustrates in dotted and solid lines, respectively, the CCR and the BCC efficiency frontiers in 2008 and 2009 with the positions of $Bank_1$ in each year.

TABLE 10. Global multi-period efficiency score and MPI for $Bank_1$.

Time sequence $t-\tau$	θ_τ^t	MPI
2008−2008	1.0000	1.0000
2008−2009	24.6962	0.6200
2009−2008	0.8344	2.4900
2009−2009	0.9365	0.9100

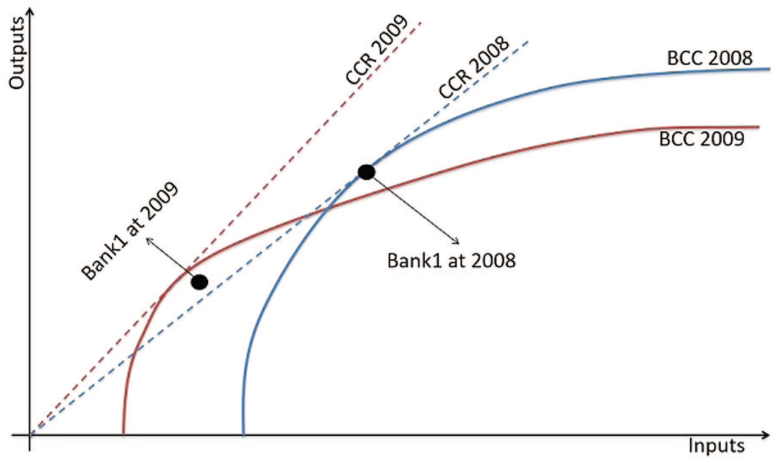

FIGURE 7. Comparison of BCC frontier and CCR frontier in time 2008.

$Bank_1$ is BCC-efficient in 2008 and it is CCR and BCC-inefficient in 2009, which justifies the regress noted by a value of $GPRI = -0.060$ as opposed an MPI value of 1.276, indicating a "wrong" progress. The same explanation applies to other cases that exhibit similar differences. Therefore, the results produced by the GPRI appear much more accurate than those obtained using MPI.

5. CONCLUSION

Multi-period evaluation of production systems using time serial data are of utmost importance for managers. In this paper, we introduced a new DEA model that allows the computation of global multi-period efficiency scores for DMUs as well as the extent of their progress and regress over multi-period time horizons. The new multi-period DEA (MDE) model is based upon underlying axioms for the DEA technique that enable handling the efficiency assessment within a production possibility set (PPS) induced from the union of all periodic PPSs. Accordingly, each DMU, defined with its observations in time period t, can easily be evaluated under the production technology of a different time period l before deciding on its global efficiency status. Moreover, exploiting the features of the new model, we developed a new productivity index, identified as global progress and regress index (GPRI), for measuring the extend of progress or regress of a DMU over two or more successive time periods under variable returns to scale (VRS) assumption. With these features, the new productivity index gains its merit of overcoming known limitations of the Malmquist productivity index (MPI) which is restricted to constant returns to scale (CRS) and only two successive time periods.

The proposed MDEA model as well as related GPRI can be utilized in banking and insurance sectors where periodic evaluation of units' performance is extremely important for decision making. As such, we have chosen a real-life sample of 25 banks with four-year data set to highlight, through an extensive discussion, the theoretical and the practical advantages of the proposed concepts.

Possible future research may include the extension of the proposed MDEA model to cross-efficiency DEA settings ([27]–[36]) where the evaluation will engage not only the DMUs but also the associated time periods. Under such a framework, more investigation might also be necessary to developping new metrics for a global ranking of the DMUs. Also, sensitivity analysis of extreme efficient DMUs can be considered in the models presented in this study.

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References

[1] N.K. Al-Mezeini, A. Oukil and A.M. Al-Ismaili, Investigating the efficiency of greenhouse production in Oman: A two-stage approach based on Data Envelopment Analysis and double bootstrapping. *J. Clean. Prod.* **247** (2020) 119160.

[2] G.R. Amin and A. Oukil, Gangless cross-evaluation in DEA: An application to stock selection. *RAIRO: OR* **53** (2019) 645–655.

[3] P. Andersen and N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis. *Manag. Sci.* **39** (1993) 1261–1265.

[4] S. Avils Sacoto, W.D. Cook, H. Cant Delgado and D. Gemes Castorena, Time-staged Outputs in DEA. *Omega* **55** (2015) 1–9.

[5] R.D. Banker, A. Charnes and W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Manag. Sci.* **30** (1984) 1078–1092.

[6] M.S. Bazaraa, H.D. Sherali and C.M. Shetty, Non-linear programming: Theory and algorithms. Published by John Wiley, Sons, Inc., Hoboken, New Jersey, Published simultaneously in Canada, 3rd-ed. (2006).

[7] D.W. Caves, L.R. Christensen and W.E. Diewert, The economic theory of index numbers and the measurement of input, output and productivity. *Econometrica* **50** (1982) 1393–1414.

[8] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units. *Eur. J. Oper. Res.* **2** (1978) 429–444.

[9] K. Chen, M. Kou and X. Fu, Evaluation of multi-period regional R&D effciency: An application of dynamic DEA to China's regional R&D systems. *Omega* **74** (2017) 103–114.

[10] A. Esmaeilzadeh and A. Hadi-Vencheh, A super-efficiency model for measuring aggregative efficiency of multi-period production systems. *Measurement* **46** (2013) 3988–3993.

[11] E. Grifell-Tatjé and C.A.K. Lovell, A generalized malmquist productivity index. *Sociedad de Estadistica e Investigacion Operative* **7** (1999) 81–101.

[12] F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, M. Vaez-Ghasemi and Z. Moghaddas, Evaluation progress and regress of balanced scorecards by multi-stage Malmquist Productivity Index. *J. Ind. Prod. Eng.* **30** (2013) 345–354.

[13] F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, M. Vaez-Ghasemi and Z. Moghaddas, Modified Malmquist Productivity Index Based on Present Time Value of Money. *J. Appl. Math.* **2013** (2013) 8.

[14] F. Hosseinzadeh Lotfi, G.R. Jahanshahloo, M. Veaz-Ghasemi and Z. Moghaddas, Periodic efficiency measurement for achieving correct efficiency among several terms of evaluation. *Int. J. Oper. Res.* **18** (2013) 1–15.

[15] M.A. Jahantighi, Z. Moghaddas and M. Vaez-Ghasemi, Two-stage Malmquist Productivity Index with Intermediate Products. *Int. J. Ind. Math.* **4** (2012) 31–40.

[16] C. Kao and S.N. Hwang, Multi-period efficiency and Malmquist productivity index in two-stage production systems. *Eur. J. Oper. Res.* **232** (2014) 512–521.

[17] C. Kao and S. Liu, Multi-period efficiency measurement in data envelopment analysis: The case of Taiwanese commercial banks. *Omega* **47** (2014) 90–98.

[18] M. Kou, K. Chen, S. Wang and Y. Shao, Measuring efficiencies of multi-period and multi-division systems associated with DEA: An application to OECD countries national innovation systems. *Exp. Syst. Appl.* **46** (2016) 494–510.

[19] Y. Kristianto and A. Gunasekaran, A global optimization for sustainable multi-domain global manufacturing. *Comp. Oper. Res.* **89** (2018) 307–323.

[20] T. Lee, Y. Zhang and B.H. Jeong, A multi-period output DEA model with consistent time lag effects. *Comp. Ind. Eng.* **93** (2016) 267–274.

[21] Z. Li, J. Crook and G. Andreeva, Dynamic Prediction of Financial Distress Using Malmquist DEA. *Exp. Syst. Appl.* **80** (2017) 94–106.

[22] S. Malmquist, Index numbers and indifference surfaces. *Trabajos de Estatistica* **4** (1953) 209–242.

[23] S. Munisamy and B. Arabi, Eco-efficiency change in power plants: using a slacks-based measure for the meta-frontier Malmquis-teLuenberger productivity index. *J. Clean. Prod.* **105** (2015) 218–232.

[24] S.C. Oh and J. Shin, The impact of mismeasurement in performance benchmarking: A monte carlo comparison of SFA and DEA with different multi-period budgeting strategies. *Eur. J. Oper. Res.* **240** (2015) 518–527.

[25] M. Oral, G.R. Amin and A. Oukil, Cross-efficiency in DEA: A maximum resonated appreciative model. *Measurement* **63** (2015) 159–167.
[26] A. Oukil, Ranking *via* composite weighting schemes under a DEA cross-evaluation framework. *Comput. Ind. Eng.* **117** (2018) 217–224.
[27] A. Oukil, Embedding OWA under preference ranking for DEA crossefficiency aggregation: Issues and procedures. *Int. J. Intell. Syst.* **34** (2019) 947–965.
[28] A. Oukil, Exploiting value system multiplicity and Preference voting for Robust ranking. *Omega Int. J. Manag. Sci.* **94** (2020) 102048.
[29] A. Oukil and G.R. Amin, Maximum appreciative cross-efficiency in DEA: A new ranking method. *Comput. Ind. Eng.* **81** (2015) 14–21.
[30] A. Oukil and S.M. Govindaluri, A systematic approach for ranking football players within an integrated DEA-OWA framework. *Manage. Decis. Econ.* **38** (2017) 1125–1136.
[31] A. Oukil and A. Al-Zidi, Benchmarking the hotel industry in Oman through a three-stage DEA based procedure. *J. Arts Soc. Sci.* **9** (2018) 5–23.
[32] A. Oukil and S. Zekri, Investigating farming efficiency through a two stage analytical approach: Application to the agricultural sector in Northern Oman. Preprint arXiv:2104.10943 (2021).
[33] A. Oukil and A. El-Bouri, Ranking dispatching rules in multi-objective dynamic flow shop scheduling: a multi-faceted perspective. *Int. J. Prod. Res.* **59** (2022) 388-411.
[34] A. Oukil, N. Channouf and A. Al-Zidi, Performance evaluation of the hotel industry in an emerging tourism destination: Case of Oman. *J. Hosp. Tour. Manag.* **29** (2016) 60–68.
[35] A. Oukil, A.A. Soltani, H. Boutaghane, O. Abdalla, A. Bermad, M. Hasbaia and M.R. Boulassel, A Surrogate Water Quality Index to assess groundwater using a unified DEA-OWA framework. *Environ. Sci. Pollut. Res.* **28** (2021) 56658–56685.
[36] A. Oukil, A. El-Bouri and A. Emrouznejad, Energy-aware job scheduling in a multi-objective production environment: An integrated DEA-OWA model. *Comput. Ind. Eng.* **168** (2022) 108065.
[37] K.S. Park and K. Park, Measurement of multi-period aggregative efficiency. *Eur. J. Oper. Res.* **193** (2009) 567–580.
[38] J.T. Pastor and C.A.K. Lovell, A global Malmquist productivity index. *Econ. Lett.* **88** (2005) 266–271.
[39] M. Portela and E. Thanassoulis, A circular Malmquist-type index for measuring productivity. Aston University, UK Working Paper RP08-02 (2008).
[40] S.H. Razavi Hajiagha, S.S. Hashemi, H.A Mahdiraji and J. Azaddel, Multi-period data envelopment analysis based on Chebyshev inequality Bounds. *Exp. Syst. Appl.* **42** (2016) 7759–7767.
[41] L.M. Seiford and J. Zhu, Infeasibility of super-efficiency data envelopment analysis models. *INFOR* **37** (1999) 174–187.
[42] S.Y. Sohn and Y. Kim, DEA based multi-period evaluation system for research in academia. *Exp. Syst. Appl.* 39 (2012) 8274–8278.
[43] A.A. Soltani, A. Oukil, H. Boutaghane, A. Bermad and M.R. Boulassel, A new methodology for assessing water quality, based on data envelopment analysis: Application to Algerian dams. *Ecol. Indic.* **121** (2021) 106952.
[44] O. Sow, A. Oukil, B.M. Ndiaye and A. Marcos, Efficiency analysis of public transportation subunits using DEA and Bootstrap Approaches-Dakar Dem Dikk case study. *J. Math. Res.* **8** (2016) 114–127.
[45] N. Wang, J. Chen, S. Yao and Y.C. Chang, A meta-frontier DEA approach to efficiency comparison of carbon reduction technologies on project level. *Renew. Sustain. Energy Rev.* **82** (2018) 2606–2612.