

SUPPLY CHAIN CONTRACTING CONSIDERING THE COST STRUCTURE BETWEEN CAPACITY AND QUALITY UNDER INFORMATION ASYMMETRY

WEI ZENG*

Abstract. This paper studies the supply chain contracting problem to ensure on-time delivery of high-quality products from the supplier to the buyer. The notions of complements and substitutes are utilized to describe the correlation among the supplier's production cost, production capacity, and product quality, *i.e.*, the cost structure between capacity and quality. A principal-agent model is proposed to solve the supply chain contracting considering the cost structure between capacity and quality under information asymmetry. We derive the conditions for complements and substitutes and the optimal menu of contracts. Our results indicate that the cost structure plays an important role in supply chain contracting.

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1. INTRODUCTION

Supply chain management is one of the effective means to enhance the core competitiveness of enterprises. Firms focus on their core strategic advantages by outsourcing a significant portion of production or construction activities. The goal of outsourcing is to promote supply chain members to operate efficiently and effectively to achieve competitive advantages and customer satisfaction [7, 10, 23]. There are many advantages to outsourcing and contract manufacturing, such as the benefits of cost savings, efficient production, and quality improvement due to the large-scale and specialized production of contract manufacturers [15]. But there are also issues associated with outsourcing and supply chain management:

- (1) Asymmetric information: Information asymmetry is a concern in supply chain management. There are two kinds of information asymmetry from the viewpoint of the members within a supply chain, *i.e.*, information asymmetry from the buyer's side and the supplier's side. From the buyer's side, demand information asymmetry is one of the major information asymmetries, while capacity, quality, and cost information are major sources of information asymmetry from the supplier's side. Due to information asymmetry, some supply chain members may have a motive to misstate their information, *e.g.*, overstate or understate demand information or cost information, to earn more interest and bring about moral hazard problems. There has been a growing concern for supply chain contract design to mitigate the negative effects of different types of information asymmetry and realize supply chain coordination.

Keywords. Supply chain contracting, cost structure, capacity, quality, information asymmetry.

School of Artificial Intelligence and Automation, Huazhong University of Science and Technology, Wuhan 430074, P.R. China.
*Corresponding author: zengwei@hust.edu.cn

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(2) Capacity and quality concerns: The contract manufacturer is expected to invest in production to build sufficient capacity and deliver high-quality products on time. Building production capacity and improving product quality are two concerns in production investment and supply chain management. Production investment includes investment in purchasing new equipment, hiring experienced workers, improving the quality management system, etc. In practice, production investment may bring about changes in capacity level and quality level at the same time. Especially, changes in capacity level and quality level caused by production investment depend on the correlation among production cost, capacity level, and quality level, *i.e.*, the cost structure between capacity and quality. For example, investment in purchasing new equipment or hiring experienced workers would improve both product quality level and production capacity level, *i.e.*, production investment or cost drive both capacity and quality in the same direction. On the other hand, if the supplier strengthens quality inspection to improve product quality, *e.g.*, increasing the sampling test of products, it is possible to decrease the production capacity because the quality inspection is often time-consuming, *i.e.*, production investment or cost drive capacity and quality in the opposite direction. These two phenomena can be described by the terms of complements and substitutes in economics [9], *i.e.*, complements or substitutes relationship between production capacity and product quality. Therefore, the cost structure between capacity and quality should be considered in supply chain contract design to encourage contract manufacturers to build capacity or improve quality.

Considering the above two concerns, this paper focuses on the supply chain contract design. In detail, we consider a one-buyer and one-supplier (or contract manufacturer) supply chain. The buyer can't monitor the supplier's product quality and the supplier's product quality is its private information. The supplier determines the quality level and capacity level. The buyer designs contract to encourage the on-time delivery of high-quality products from the supplier. The main contributions in this paper can be summarized as follows. Considering that correlation among production cost, capacity level, and quality level, *i.e.*, the production cost structure between capacity and quality, the production cost is expressed as a function of capacity level and quality level. Furthermore, we propose a principal-agent model to solve the supply chain contracting problem under information asymmetry. The analysis results show that the optimal contract depends on the production cost structure between capacity level and quality level, *i.e.*, whether the supplier's production capacity and product quality are complements or substitutes. Moreover, we derive that the monotonicity conditions for complements and substitutes, *i.e.*, the supplier's capacity level will increase in its quality level in the case of complements, while the supplier's capacity level will decrease in its quality level in the case of substitutes. These monotonicity conditions can explain the phenomenon of changes in capacity level and quality level in production investment mentioned above. Finally, we solve the optimal menu of contracts.

The rest of the paper is organized as follows: Section 2 presents the literature review. Section 3 introduces our problem description and model setting. In Section 4, the optimal contract solution under information symmetry and information asymmetry are proposed, respectively. Section 5 gives numerical examples to illustrate the proposed model. Section 6 gives a summary and future work. Some proofs are provided in the appendices.

2. LITERATURE REVIEW

Information asymmetry is a concern in supply chain management, which includes cost information asymmetry [8, 12], demand information asymmetry [12, 17, 26], supply information asymmetry [2, 6], quality information asymmetry [16], capacity information asymmetry [21], etc. In the paper, we focus on the information asymmetry from the supplier's side.

The capacity information is one kind of supply information. For the buyer, the capacity information is important to make order decisions in a supply chain. Pun and Heese [21] evaluate a contract design problem in a one-manufacturer N-supplier supply chain with the suppliers' capacity information asymmetry. Their results show that the manufacturer can conduct audits to reveal the supplier's type and use a menu of two-part tariffs to mitigate the issues caused by the limited knowledge of capacity information.

In the case of cost information asymmetry, several types of cost information are considered in the existing literature, including production cost information [1, 20], holding cost information [3, 4, 25], quality cost information [11, 24], etc. Different contract types, including a menu of contracts, quantity discount contracts, price commitment contracts, wholesale price contracts, etc., are used to examine their impacts on mitigating the cost information asymmetry. For example, Kaya and Özer [11] investigate the effects of a price commitment contract to mitigate quality cost information asymmetry in a one-OEM and one-contract manufacturer supply chain. Their results show that the price commitment contract can be an effective way to mitigate the quality risk caused by information asymmetry. Yang *et al.* [24] design a menu of contracts (including order quantity, quality level, and transfer payment) in a supply chain with one buyer and one supplier. In their model setting, the supplier's quality cost information is private and the supplier's market value concern is considered. Their results indicate that the supplier's market value concern is an important factor to mitigate the distortion of quality information.

It is shown that relatively little attention is paid to quality information asymmetry [22]. Lim [16] studies the supply chain contracting problem to mitigate the negative effects of quality information asymmetry. The author proposes two quality-control schemes embedded in the contract and finds that a contract with an inspection scheme is preferred for the supplier with low product quality, while a contract with a warranty scheme is preferred for the supplier with high product quality.

From the above literature review, we find there exists extant literature that focuses on supply chain contracting under information asymmetry. It is worth pointing out that the existing literature usually does not consider the cost structure or cost is only affected by a single factor, for example, the quality cost is affected by the single factor of quality level. In practice, the supplier's production cost is affected by several factors, such as investment in increasing production capacity, improving product quality, etc., *i.e.*, the production cost is affected by factors such as capacity, quality, etc. One of the missing in the study of supply chain contracting is how the cost structure affects supply chain contracting.

In this paper, our model setting is like Feng *et al.* [5], which considers the supplier's quality, capacity, and correlated cost in the service outsourcing problem. Feng *et al.* [5] assume that the supplier's cost is the linear sum of capacity cost and quality cost, and these two costs are both private and functions of capacity level and quality level respectively. Moreover, these two costs may be positively, or negatively, correlated with each other to capture the correlation structures between the two costs. Different from Feng *et al.* [5], in the paper, considering it is difficult to explicitly split up the production cost into quality cost and capacity cost, the production cost is expressed as a function of quality level and capacity level. This more general expression can not only cover the case that the production cost is the linear sum of capacity cost and quality cost in Feng *et al.* [5], but also cover the complements or substitutes relationship between capacity and quality mentioned above. We propose a principal-agent model to solve the supply chain contracting problem under information asymmetry. Our analysis results show that the optimal contract depends on whether the supplier's production capacity and product quality are complements or substitutes. Moreover, we derive the monotonicity conditions for complements and substitutes and solve the optimal menu of contracts.

3. PROBLEM DESCRIPTION AND ASSUMPTION

In the paper, a supply chain with one buyer and one supplier is considered. The buyer is the focal company in the supply chain and determines transfer payment P according to the supplier's product quality level x and on-time product delivery rate, which depends on the supplier's capacity level q . The quality level x and the capacity level q are determined by the supplier. Specifically, we define the quality level x as a defective rate of delivering products and assume that x is continuous ($x \in [0, 1]$) [13]. The smaller the value of x is, the higher the quality level is. We define the capacity level q as the proportion of delivering products that cannot meet the buyer's deadline, *i.e.*, delay rate of product delivery. Like the definition of quality level, we assume that the capacity level q is continuous ($q \in [0, 1]$). The smaller the value of q is, the higher the capacity level is.

TABLE 1. Notations.

Symbol	Definition
x	Supplier's truthful product quality level, <i>i.e.</i> , defective rate, $x \in [0, 1]$
y	Supplier's announced product quality level, $y \in [0, 1]$
q	Supplier's capacity level, <i>i.e.</i> , delay rate of a batch, $q \in [0, 1]$
Π_b	Buyer's costs
$P(q, x)$	Buyer's transfer payment to the supplier
$K(q, x)$	Buyer's penalty for the defective delivery products or products not delivered on time
$C(q, x)$	Supplier's production cost
Π_s	Supplier's profit
Π	Supply chain's total costs

Furthermore, we assume the buyer's purchase price $P(q, x)$ is a convex function of the quality level x and the capacity level q , and is decreasing in x or q , respectively. The supplier's defective products or delayed delivery brings a penalty $K(q, x)$ to the buyer, which is the incurred cost or loss caused by repair or replacement of products, damage to reputation, etc. It is natural to assume that the penalty $K(q, x)$ is a convex function of q and x , which is increasing in q or x , respectively.

In the paper, we do not split up the production cost into quality cost and capacity cost and assume that the production cost $C(q, x)$ is a convex function of the capacity level q and the quality level x , which is decreasing in q or x , respectively.

The buyer incurs the following costs, including the transfer payment $P(q, x)$ and the penalty $K(q, x)$:

$$\Pi_b(x) = P(q, x) + K(q, x). \quad (3.1)$$

The supplier's payoff is listed as follows, including the transfer payment $P(q, x)$ and the production cost $C(q, x)$:

$$\Pi_s(x) = P(q, x) - C(q, x). \quad (3.2)$$

Table 1 lists the notations used in the paper.

In the paper, we assume that the supplier's product quality x is private information with a support $[0, 1]$ and the buyer has an unbiased belief about its distribution. Let $F(x)$ and $f(x)$ be the distribution and density functions of the random variable, respectively. $F(x)$ is assumed to be log-concave and the failure rate, *i.e.*, $\frac{f(x)}{(1-F(x))}$, is assumed to be increasing. It is well known that distributions like normal, uniform, gamma, exponential, etc. satisfy the assumption [14].

Assume that both the buyer and the supplier are risk-neutral. The buyer expects the supplier to improve its quality level and capacity level to realize on-time high-quality product delivery. There exist conflicts of interest in the supply chain because both sides want to minimize their expected cost or maximize their expected profit. Due to the information asymmetry, the supplier has a motive to misstate its information and brings about the moral hazard problem.

According to the Revelation Principle [18, 19], the buyer has a direct and truthful mechanism to obtain the optimal expected payoff. Let $(P(q(x), x), q(x))$ denote a menu of contracts intended for a type- x supplier, *i.e.*, the supplier chooses from this menu by announcing its product quality level to be x and its production capacity to be $q(x)$, then the buyer pays the supplier transfer payment $P(q(x), x)$. The supply chain contracting problem is that the buyer determines a menu of contracts $(P(q(\cdot), \cdot), q(\cdot))$ to minimize its expected costs while the supplier satisfies incentive compatibility (IC) constraint and individual rationality (IR) constraint. The IC constraint denotes the supplier truthfully announcing its product quality and the IR constraint denotes the supplier wants to participate in the trade and take the contract.

4. MODELING AND ANALYSIS

4.1. Optimal contract under information symmetry

We first consider the optimal (first-best) contract $(P^*(q^*(\cdot), \cdot), q^*(\cdot))$ under information symmetry as a benchmark. In the base case, the supplier's quality information is common knowledge for both the buyer and the supplier. A centralized supply chain, which is assumed to be managed by one central entity, is usually considered as the case of information symmetry. To derive the optimal contract, a principal-agent theory is utilized to build the model. The principal-agent model under information symmetry can be written as follows:

$$\min_{(P(q(\cdot), \cdot), q(\cdot))} P(q, x) + K(q, x) \quad (4.1)$$

$$\text{s.t. } \begin{cases} (\text{IR}) \quad \Pi_s(x) \geq 0 \\ 0 \leq q(x) \leq 1, \quad \forall x \in [0, 1] \end{cases} \quad (4.2)$$

Here the supplier's opportunity cost is assumed to be zero for simplicity. Therefore, the supplier participates in the trade only if its profit is non-negative, *i.e.*, the individual compatibility (IR) constraint given in equation (4.2). The IR constraint means that it is in the best interest of the supplier to take the contract.

Proposition 4.1. *In the optimal (first-best) contracts under information symmetry, the optimal capacity q^* satisfies $\frac{\partial C(q^*(x), x)}{\partial q} + \frac{\partial K(q^*(x), x)}{\partial q} = 0$, and the optimal transfer payment $P^*(q^*(x), x) = C(q^*(x), x)$.*

Proof. According to the IR constraint, the following equation is satisfied, otherwise, the buyer may reduce the transfer payoff P .

$$\Pi_s(x) = P(q(x), x) - C(q(x), x) = 0. \quad (4.3)$$

Therefore, the objective function of equation (4.1) is rewritten as $\Pi = C(q, x) + K(q, x)$, *i.e.*, the total costs of the supply chain. Since the objective function is a convex function of q , the optimal production capacity q^* can be obtained by Karush–Kuhn–Tucker conditions. The optimal payment $P^*(q^*(x), x)$ can be obtained by equation (4.3). \square

4.2. Optimal contract under information asymmetry

In this subsection, we focus on the optimal (second-best) contract $(P^{**}(q^{**}(\cdot), \cdot), q^{**}(\cdot))$ under information asymmetry. In the case of information asymmetry, the supplier's quality information is private. In the principal-agent model under information asymmetry, the buyer minimizes its expected total costs, while the supplier should satisfy the IC constraint and the IR constraint. The principal-agent model can be written as follows:

$$\min_{(P(q(\cdot), \cdot), q(\cdot))} P(q, x) + K(q, x) \quad (4.4)$$

$$\text{s.t. } \begin{cases} (\text{IC}) \quad \Pi_s(x) \geq \Pi_s(y), \quad \forall x, y \in [0, 1] \\ (\text{IR}) \quad \Pi_s \geq 0, \quad \forall x \in [0, 1] \\ 0 \leq q(x) \leq 1, \quad \forall x \in [0, 1], \end{cases} \quad (4.5)$$

where $\Pi_s(y) = P(q(y), y) - C(q(y), x)$ denotes the supplier's profit with product quality x but announcing y . The IC constraint means the supplier announces the true information.

4.2.1. Conditions for complements and substitutes

To make the problem of the optimal contract easier to deal with, we consider an equivalent form of the IC constraint and derive the conditions for complements and substitutes to represent the correlation among quality level, capacity level, and production cost.

Proposition 4.2. *The IC constraint is equivalent to the following conditions:*

(1) *The first-order condition:*

$$\frac{\partial P(q, x)}{\partial q} \frac{dq}{dx} + \frac{\partial P(q, x)}{\partial x} - \frac{\partial C(q, x)}{\partial q} \frac{dq}{dx} = 0, \quad \forall x \in [0, 1]. \quad (4.6)$$

(2) *The monotonicity condition:*

$$\frac{dq}{dx} \geq 0, \quad \forall x \in [0, 1], \text{ when } \frac{\partial^2 C(q, x)}{\partial q \partial x} \leq 0 \text{ (i.e., complements)} \quad (4.7)$$

$$\frac{dq}{dx} \leq 0, \quad \forall x \in [0, 1], \text{ when } \frac{\partial^2 C(q, x)}{\partial q \partial x} \geq 0 \text{ (i.e., substitutes).} \quad (4.8)$$

Proof. See proof of Proposition 4.2 in Appendix A. \square

In Proposition 4.2, two cases of the supplier's cost structure, *i.e.*, the correlation among the production cost C , the quality level x , and the capacity level q , is proposed, namely, complements and substitutes.

The cross partials $\frac{\partial^2 C(q, x)}{\partial q \partial x} \leq 0$ for all q and x . This case represents that the supplier's production capacity and its product quality are complements: a higher value of q (*i.e.*, a lower production capacity level) decreases $\frac{\partial C(q, x)}{\partial x}$ (*i.e.*, the supplier's marginal production cost w.r.t. quality). The complements relationship between production capacity and product quality implies that the level of each enhances the effect on the marginal production cost of the other. Given complements, suppliers with different quality levels can be separated by the menu of contracts. In particular, the supplier with a higher-quality level has a higher capacity level than the supplier with a lower-quality level.

The cross partials $\frac{\partial^2 C(q, x)}{\partial q \partial x} \geq 0$ for all q and x . This case represents that the supplier's capacity and its product qualities are substitutes: a higher value of q (*i.e.*, a lower production capacity level) increases $\frac{\partial C(q, x)}{\partial x}$. The substitutes relationship between production capacity and product quality implies that the effect of each on the marginal production cost varies inversely with the level of the other. Given substitutes, compared to the supplier with a lower-quality level, the supplier with a higher-quality level has a lower capacity level.

From Proposition 4.2, the supplier's capacity level q increases in its quality level x given complements. This case indicates such a phenomenon in practice mentioned above: investment in purchasing new equipment or hiring experienced workers can improve the supplier's capacity level, in the meantime, it is helpful to improve the supplier's product quality. Given substitutes, the supplier's capacity level decreases in its quality level. There also exists such a phenomenon in practice mentioned above: strengthening quality inspection may bring about a decrease in production capacity, in the meantime, it is helpful to improve product quality.

4.2.2. Optimal menu of contracts

Next, we give the following equivalent form of the aforementioned principal-agent model (4.4) and (4.5).

Proposition 4.3. *The principal-agent model (4.4) and (4.5) is equivalent to*

$$\min_{(q(\cdot), \Pi_s(\cdot))} \int_0^1 ((\Pi_s(x) + C(q, x) + K(q, x))f(x) dx, \quad (4.9)$$

$$\text{s.t. } \begin{cases} (\text{IC}) \quad \Pi'_s(x) = -\frac{\partial C(q, x)}{\partial x}, & x \in [0, 1] \\ (\text{IR}) \quad \Pi_s \geq 0, & \forall x \in [0, 1] \\ 0 \leq q(x) \leq 1, & \forall x \in [0, 1]. \end{cases} \quad (4.10)$$

Proof. See proof of Proposition 4.3 in Appendix B. \square

According to Proposition 4.3, optimal control theory is used to solve the optimal menu of contracts and get the following proposition.

Proposition 4.4. (1) *If the supplier's capacity and quality are complements, then the optimal (second-best) menu of contracts $(P^{**}(q^{**}(x), x), q^{**}(x))$ satisfy*

$$\begin{cases} \frac{dq}{dx} \geq 0 \\ \left(\frac{\partial C(q^{**}(x), x)}{\partial q} + \frac{\partial K(q^{**}(x), x)}{\partial q} \right) f(x) + (F(x) - 1) \frac{\partial^2 C(q^{**}(x), x)}{\partial x \partial q} = 0 \\ P^{**}(q^{**}(x), x) = C(q^{**}(x), x) - \int_0^x \frac{\partial C(q^{**}(s), s)}{\partial s} ds \\ 0 \leq q^{**}(x) \leq 1, \end{cases} \quad \forall x \in [0, 1]. \quad (4.11)$$

(2) *If the supplier's capacity and quality are substitutes, then the optimal (second-best) menu of contracts $(P^{**}(q^{**}(x), x), q^{**}(x))$ satisfy*

$$\begin{cases} \frac{dq}{dx} \leq 0 \\ \left(\frac{\partial C(q^{**}(x), x)}{\partial q} + \frac{\partial K(q^{**}(x), x)}{\partial q} \right) f(x) + (F(x) - 1) \frac{\partial^2 C(q^{**}(x), x)}{\partial x \partial q} = 0 \\ P^{**}(q^{**}(x), x) = C(q^{**}(x), x) - \int_0^x \frac{\partial C(q^{**}(s), s)}{\partial s} ds \\ 0 \leq q^{**}(x) \leq 1, \end{cases} \quad \forall x \in [0, 1] \quad (4.12)$$

Proof. See proof of Proposition 4.4 in Appendix C. \square

According to Proposition 4.4, in the case of complements, both capacity and quality are driven in the same direction. Therefore, in extreme cases, capacity and quality may reach the maximum or minimum at the same time. The optimal menu of contracts can separate the suppliers with different quality levels, in particular, the supplier with a higher-quality level has a higher capacity level than the supplier with a lower-quality level. In the case of substitutes, capacity and quality are driven in the opposite direction. Therefore, in extreme cases, when the capacity reaches the maximum (minimum), the quality level reaches the minimum (maximum). The optimal menu of contracts can separate the suppliers with different quality levels, in particular, the supplier with a higher (lower)-quality level has a lower (higher) capacity level than the supplier with a lower (higher)-quality level.

According to Proposition 4.4, the supplier's information rent is $\Pi_s(x) = - \int_0^x \frac{\partial C(q^{**}(s), s)}{\partial s} ds$. In the case of complements, *i.e.*, $\frac{\partial^2 C(q, x)}{\partial q \partial x} \leq 0$, the increase of q will increase the information rent $- \int_0^x \frac{\partial C(q^{**}(s), s)}{\partial s} ds$. Therefore, the buyer has a motive to induce the supplier to decrease q , *i.e.*, increasing the production capacity level to decrease the information rent. In the case of substitutes, *i.e.*, $\frac{\partial^2 C(q, x)}{\partial q \partial x} \geq 0$, the decrease of q will increase the information rent, and the buyer has a motive to induce the supplier to increase q , *i.e.*, decreasing the production capacity level to decrease the information rent. Therefore, we can have the following proposition.

Proposition 4.5. *In the case of complements, the optimal (second-best) capacity level $q^{**}(x)$ is not greater than the optimal (first-best) capacity level $q^*(x)$, *i.e.*,*

$$q^{**}(x) \leq q^*(x), \quad (4.13)$$

*while in the case of substitutes, the optimal (second-best) capacity level $q^{**}(x)$ is not lower than the optimal (first-best) capacity level $q^*(x)$, *i.e.*,*

$$q^{**}(x) \geq q^*(x). \quad (4.14)$$

5. NUMERICAL EXAMPLES

In this section, numerical examples are given to illustrate the proposed models. In the numerical examples, we assume that the buyer has a belief about the distribution of the product quality which obeys the uniform distribution due to information asymmetry.

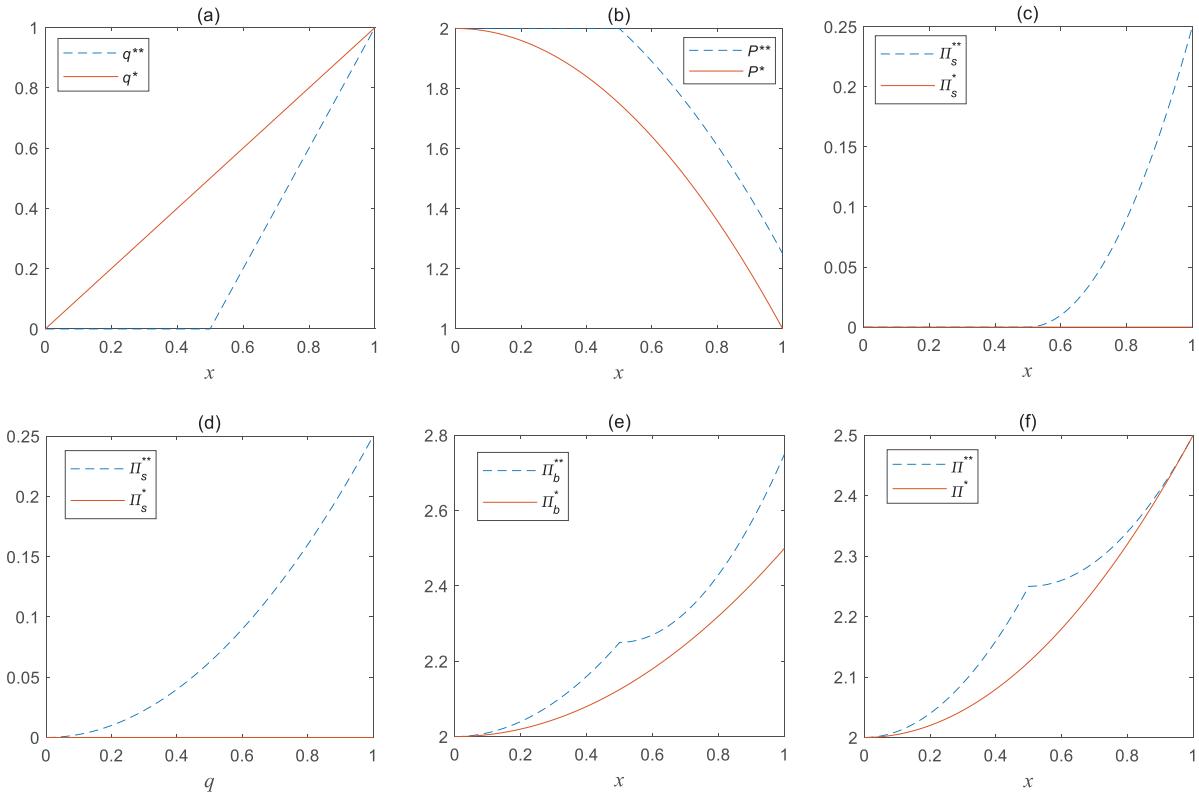


FIGURE 1. The optimal menu of contracts in the case of complements.

5.1. The case of complements

In the case of complements, we assume the production cost function $C(q, x) = 2 - qx$, and the penalty function $K(q, x) = \frac{1}{2}q^2(x) + x^2$. The cross partials $\frac{\partial^2 C(q, x)}{\partial q \partial x} \leq 0$. According to Proposition 4.4, the optimal menu of contracts is given as follows:

$$q^{**}(x) = \begin{cases} 0, & x \in [0, \frac{1}{2}) \\ 2x - 1, & x \in [\frac{1}{2}, 1], \end{cases}$$

$$P^{**}(q, x) = C(q, x) - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds = \begin{cases} 2, & x \in [0, \frac{1}{2}) \\ 2\frac{1}{4} - x^2, & x \in [\frac{1}{2}, 1]. \end{cases}$$

$$\text{The information rent } \Pi_s = - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds = \int_0^x q(s) ds = \begin{cases} 0, & x \in [0, \frac{1}{2}) \\ x^2 - x + 1/4, & x \in [\frac{1}{2}, 1] \end{cases}.$$

For the numerical example under information symmetry, we can derive optimal (first-best) quality $q^*(x) = x$ according to Proposition 4.1, and other optimal parameters, as shown in Figure 1.

From Figure 1a, $q^*(x) \geq q^{**}(x)$ holds, as Proposition 4.5 points out.

Figure 1a shows that q^{**} is increasing in x . This is the monotonicity condition for complements.

There exists a threshold $\frac{1}{2}$ of the quality level x with regard to the capacity level q , as shown in Figure 1a. When $x \leq \frac{1}{2}$, q^{**} keeps zero, and the transfer payment P^{**} and information rent $(\Pi_s^{**} = - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds)$

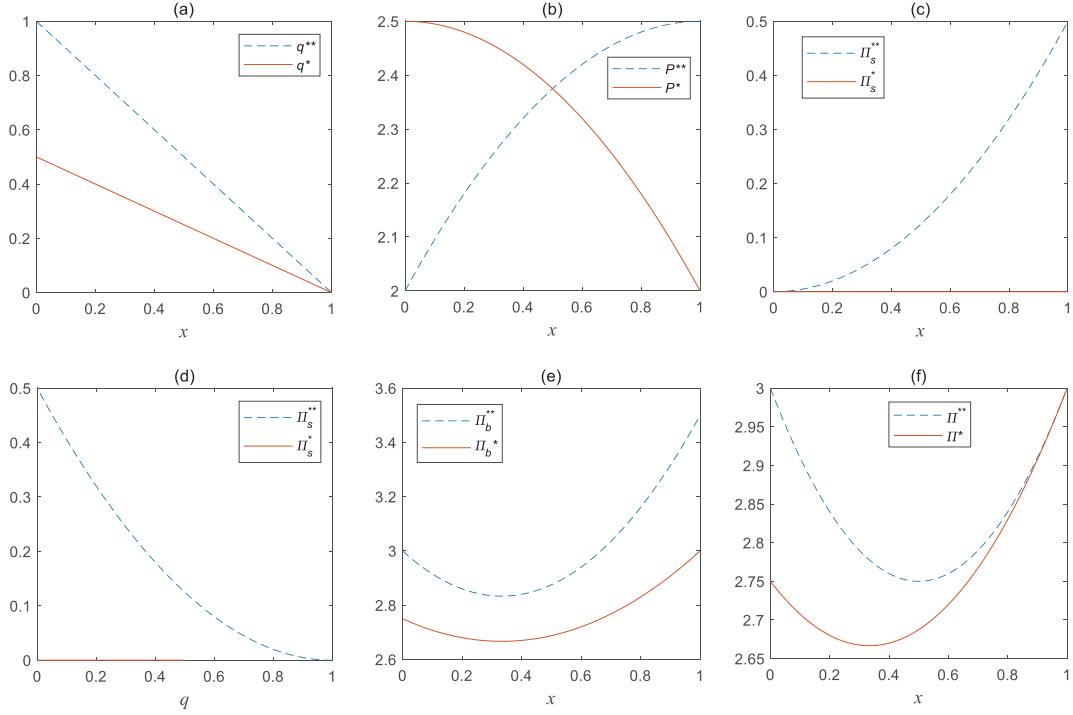


FIGURE 2. The optimal contract in the case of substitutes.

also keep unchanged, as shown in Figures 1b and 1c, respectively. When $x \geq \frac{1}{2}$, q^{**} and information rent Π_s^{**} are increasing in x , as shown in Figures 1a and 1c, respectively, and the transfer payment P^{**} is decreasing in x because the increase of x brings about the reduction of production cost, although the information rent is increasing in x , as shown in Figure 1b.

The information rent under information symmetry Π_s^* keeps zero, while the information rent under information asymmetry Π_s^{**} is increasing in x or q , as shown in Figures 1c and 1d, respectively.

From Figures 1e and 1f, the buyer's production cost and the total costs of the supply chain under information symmetry are lower than that under information asymmetry, respectively.

5.2. The case of substitutes

In the case of substitutes, we assume the production cost function $C(q, x) = 2 + (1 - q)(1 - x)$, and the penalty function $K(q, x) = q^2 + x^2$. The cross partials $\frac{\partial^2 C(q, x)}{\partial q \partial x} \geq 0$. According to Proposition 4.4, the optimal menu of contracts is given as follows:

$$q^{**}(x) = 1 - x, x \in [0, 1]$$

$$P^{**}(q, x) = C((q), x) - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds = 2 + (1 - q)(1 - x) + \int_0^x (1 - q(s)) ds = 2 + x - \frac{1}{2}x^2.$$

$$\text{Information rent} - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds = \int_0^x (1 - q(s)) ds = \frac{1}{2}x^2.$$

We can derive the optimal (first-best) quality $q^*(x) = (1 - x)/2$ under information symmetry according to Proposition 4.1, and other optimal parameters, as shown in Figure 2.

From Figure 2a, $q^*(x) \leq q^{**}(x)$ holds, as Proposition 4.5 points out.

Figure 2a shows that q^{**} (or q^*) is decreasing in x . This is the monotonicity condition for substitutes.

The transfer payment P^{**} is increasing in x , while P^* is decreasing in x . This is because the information rent Π_s^{**} is increasing in x under information asymmetry, while the information rent Π_s^* keeps zero under information symmetry, as shown in Figure 2c.

The information rent Π_s^{**} is decreasing in x , which is the opposite of the case of complements, as shown in Figure 2d.

From Figures 2e and 2f, the buyer's production cost and the total costs of the supply chain under information symmetry are lower than that under information asymmetry, respectively.

6. CONCLUSION

Capacity and quality are two dimensions focused on in the field of supply chain management. The buyer usually expects the buyer can provide fast, high-quality product delivery. But capacity and quality are correlated. In practice, we often see the phenomenon that the improvement of capacity (quality) may lead to the decrease of quality (capacity), or capacity and quality increase simultaneously. This is because there exists a complex correlation among the production cost, capacity, and quality, *i.e.*, the cost structure between capacity and quality. In the paper, two cases of the supplier's cost structure are considered, namely, complements and substitutes. In the case of complements, the supplier's production capacity level increases with the quality level, while the supplier's production capacity level decreases with the quality level in the case of substitutes. These two cost structures reflect the interaction between production capacity and product quality in practice.

Based on the cost structure between capacity and quality, this paper focused on the supply chain contracting problem with information asymmetry. Our results suggest that the cost structure plays an important role in the contract design problem. In the case of complements, the information rent is increasing in the capacity level q , and the second-best capacity is not greater than the first-best one; While in the case of substitutes, the information rent is decreasing in the capacity level q , and the second-best capacity is not lower than the first-best one.

This research could be extended in several directions. First, future work could extend the dimensions of information asymmetry, not just the information asymmetry on the supplier's side, but also on the buyer's side, *e.g.*, the demand information asymmetry, which will affect the determination of the supplier's capacity level. Second, we do not consider any quality control method in our proposed models. In practice, quality inspection, quality certification, quality warranty, and related incentive measures are often used. The proposed model could be extended to consider quality control methods in the contract design problem.

APPENDIX A. PROOF OF PROPOSITION 4.2

If the supplier's true quality is x but announced y , then its profit is given as follows:

$$\pi_s(y) = P(q(y), y) - C(q(y), x),$$

where $\pi_s(y)$ denotes the supplier's profit with product quality x but announcing y . When $y = x$, *i.e.*, the supplier truthfully reports its quality level, the supplier obtains its maximal profit. Therefore, the first-order condition for the supplier to report quality truthfully is shown as follows:

$$\left. \frac{\partial \pi_s(y)}{\partial y} \right|_{y=x} = \frac{\partial P(q(x), x)}{\partial q} \frac{dq}{dx} + \frac{\partial P(q(x), x)}{\partial x} - \frac{\partial C(q(x), x)}{\partial q} \frac{dq}{dx} = 0, \quad \forall x \in [0, 1]. \quad (\text{A.1})$$

The second-order condition for truthful announcing its quality is

$$\begin{aligned} \frac{\partial^2 \pi_s(y)}{\partial y^2} \bigg|_{y=x} &= \left(\frac{\partial^2 P(q, x)}{\partial q^2} \left(\frac{dq}{dx} \right)^2 + 2 \frac{\partial^2 P(q, x)}{\partial q \partial x} \frac{dq}{dx} + \frac{\partial P(q, x)}{\partial q} \frac{d^2 q}{dx^2} + \frac{\partial^2 P(q, x)}{\partial x^2} \right. \\ &\quad \left. - \frac{\partial^2 C(q, x)}{\partial q^2} \left(\frac{dq}{dx} \right)^2 - \frac{\partial^2 C(q, x)}{\partial q \partial x} \frac{dq}{dx} - \frac{\partial C(q, x)}{\partial q} \frac{d^2 q}{dx^2} \right) \leq 0. \end{aligned} \quad (\text{A.2})$$

Differentiating (A.1) and plugging into (A.2) gives

$$\frac{\partial^2 C(q, x)}{\partial q \partial x} \frac{dq}{dx} \leq 0 \quad \text{for all } x. \quad (\text{A.3})$$

According to equation (A.3), we can get the monotonicity conditions for complements and substitutes, respectively.

According to the IC constraint, it should satisfy

$$\pi_s(x) \geq \pi_s(y) \text{ for all } y \neq x.$$

$$\begin{aligned} \pi_s(x) - \pi_s(y) &= P(q(x), x) - C(q(x), x) - P(q(y), y) + C(q(y), x) \\ &= [P(q(s), s) - C(q(s), s)]_y^x + C(q(y), x) - C(q(y), y) \\ &= \int_y^x [P(q(s), s) - C(q(s), s)]' ds + C(q(y), x) - C(q(y), y) \\ &= \int_y^x \left(-\frac{\partial C(q(s), s)}{\partial s} \right) ds + C(q(y), x) - C(q(y), y), \end{aligned}$$

where the last equality is by applying equation (4.6). In the case of complements, by $\frac{dq}{dx} \geq 0$ and $\frac{\partial^2 C(q, x)}{\partial q \partial x} \leq 0$, it follows that

$$\pi_s(x) - \pi_s(y) = \int_y^x \left(\frac{\partial C(q(y), s)}{\partial s} - \frac{\partial C(q(s), s)}{\partial s} \right) ds \geq 0.$$

Therefore, the incentive constraint (IC) in equation (4.5) is satisfied, and the proposition in the case of complements holds. Similarly, the proposition in the case of substitutes also holds. \square

APPENDIX B. PROOF OF PROPOSITION 4.3

The supplier's truthful announcing its quality implies that

$$\Pi_s(x) = P(q(x), x) - C(q(x), x) \quad \text{for all } x.$$

Taking the derivative w.r.t. x and according to equation (A.1), we have

$$\Pi'_s(x) = -\frac{\partial C(q, x)}{\partial x}. \quad (\text{B.1})$$

Since $\Pi_s(x) = P(q, x) - C(q, x)$, we have $\Pi_b(x) = P(q, x) + K(q, x) = \Pi_s(x) + C(q, x) + K(q, x)$. Therefore, the proposition holds. \square

APPENDIX C. PROOF OF PROPOSITION 4.4

According to the monotonicity condition in Proposition 4.2, we know that $q(x)$ is nondecreasing in the case of complements, and $q(x)$ is nonincreasing in the case of substitutes.

According to Proposition 4.3, the optimal menu of contract can be solved by optimal control theory, where Π_s as the state variable and q as the control variable. The Hamiltonian function is defined as follows:

$$H(\Pi_s, q, \eta, x) = -[\Pi_s(x) + C(q, x) + K(q, x)]f(x) - \lambda(x)\frac{\partial C(q, x)}{\partial x} \quad (\text{C.1})$$

where $\lambda(x)$ is the costate variable. The optimal q^* satisfies the following necessary condition for minimizing Hamiltonian H :

$$\frac{\partial H}{\partial q} = \left(\frac{\partial C(q, x)}{\partial q} + \frac{\partial K(q, x)}{\partial q} \right) f(x) + \lambda(x)\frac{\partial^2 C(q, x)}{\partial x \partial q} = 0. \quad (\text{C.2})$$

Since C and K are all convex functions of q , Hamiltonian H is concave in q . Therefore, equation (C.2) is enough conditions for optimality.

According to the Pontryagin principle, the costate variable $\lambda(x)$ satisfies the following necessary condition:

$$\lambda'(x) = -\frac{\partial H}{\partial \Pi_s} = f(x). \quad (\text{C.3})$$

The following discussion is conducted based on the value of $\frac{\partial^2 C(q, x)}{\partial q \partial x}$:

(1) When $\frac{\partial^2 C(q, x)}{\partial q \partial x} \geq 0$.

Since $\Pi'_s(x) = -\frac{\partial C(q, x)}{\partial x} \geq 0$ for all x , the supplier's profit increases in x . Therefore, in the optimal contract, the supplier's profit $\Pi_s(x)$ (i.e., the state variable) is zero at $x = 0$. That is, we have the transversality condition $\lambda(1) = 0$. Thus, the costate $\lambda(x) = F(x) - 1$.

Once the costate $\lambda(x)$ is determined, the optimal (second-best) q^{**} can be solved by equation (C.2).

Integrating equation (4.6) yields

$$\begin{aligned} P(q(x), x) - C(q(x), x) + \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds - P(q(0), 0) + C(q(0), 0) &= 0 \\ P(q(x), x) &= C(q(x), x) - \int_0^x \frac{\partial C(q(s), s)}{\partial s} ds. \end{aligned} \quad (\text{C.4})$$

Once q^{**} is determined, we can solve P^{**} using equation (C.4).

(2) When $\frac{\partial^2 C(q, y)}{\partial q \partial y} \leq 0$.

Similarly, we can solve the optimal menu of contracts when $\frac{\partial^2 C(q, y)}{\partial q \partial y} \leq 0$.

Based on the above analysis, the optimal menu of contracts under information asymmetry is given as Proposition 4.4. \square

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REFERENCES

- [1] M. Cakanyildirim, Q. Feng, X. Gan and S. Sethi, Contracting and coordination under asymmetric production cost information. *Prod. Oper. Manage.* **21** (2012) 345–360.
- [2] Y. Chen and G. Vulcano, Effects of information disclosure under first- and second-price auctions in a supply chain setting. *Manuf. Serv. Oper. Manage.* **11** (2009) 299–316.
- [3] C.J. Corbett and X. de Groot, A supplier's optimal quantity discount policy under asymmetric information. *Manage. Sci.* **46** (2000) 444–450.
- [4] C.J. Corbett, D. Zhou and C.S. Tang, Designing supply contracts: contract type and information asymmetry. *Manage. Sci.* **50** (2004) 550–559.

- [5] T. Feng, Z.J. Ren and F. Zhang, Service outsourcing: capacity, quality and correlated costs. *Prod. Oper. Manage.* **28** (2019) 682–699.
- [6] F. Firouzi, M.Y. Jaber and E. Baglieri, Trust in supply forecast information sharing. *Int. J. Prod. Res.* **54** (2016) 1322–1333.
- [7] C.M. Harland, Supply chain management: relationships, chains and networks. *Br. J. Manage.* **7** (1996) S63–S80.
- [8] S. Huang and J. Yang, Information acquisition and transparency in a supply chain with asymmetric production cost information. *Int. J. Prod. Econ.* **182** (2016) 449–464.
- [9] A.V. Iyer, L.B. Schwarz and A.Z. Stefanos, A principal-agent model for product specification and production. *Manage. Sci.* **51** (2005) 106–119.
- [10] U. Jüttner, M. Christopher and S. Baker, Demand chain management-integrating marketing and supply chain management. *Ind. Market. Manage.* **36** (2007) 377–392.
- [11] M. Kaya and Ö. Özer, Quality risk in outsourcing: noncontractible product quality and private quality cost information. *Naval Res. Logistics* **56** (2009) 669–685.
- [12] D. Kostamis and I. Duenyas, Purchasing under asymmetric demand and cost information: when is more private information better? *Oper. Res.* **59** (2011) 914–928.
- [13] Y. Lan, R. Zhao and W. Tang, An inspection-based price rebate and effort contract model with incomplete information. *Comput. Ind. Eng.* **83** (2015) 264–272.
- [14] T. Lewis and D. Sappington, Countervailing incentives in agency problems. *J. Econ. Theory* **49** (1989) 294–313.
- [15] L. Li, *Capacity and quality contract design in supply chain*. Ph.D. dissertation, Northwestern University (2012).
- [16] W. Lim, Producer-supplier contracts with incomplete information. *Manage. Sci.* **47** (2001) 709–715.
- [17] B.K. Mishra, S. Raghunathan and X. Yue, Information sharing in supply chains: incentives for information distortion. *IIE Trans.* **39** (2007) 863–877.
- [18] R.B. Myerson, Optimal coordination mechanisms in generalized principal-agent problems. *J. Math. Econ.* **10** (1981) 67–81.
- [19] R.B. Myerson, Mechanism design by an informed principal. *Econometrica* **51** (1983) 1767–1797.
- [20] Ö. Özer and G. Raz, Supply chain sourcing under asymmetric information. *Prod. Oper. Manage.* **20** (2011) 92–115.
- [21] H. Pun and H.S. Heese, Outsourcing to suppliers with unknown capabilities. *Eur. J. Oper. Res.* **234** (2014) 108–118.
- [22] B. Shen, T.M. Choi and S. Minner, A review on supply chain contracting with information considerations: information updating and information asymmetry. *Int. J. Prod. Res.* **57** (2019) 4898–4936.
- [23] K.C. Tan, V.R. Kannan and R.B. Hadfield, Supply chain management: supplier performance and firm performance. *Int. J. Purchas. Mater. Manage.* **34** (1998) 2–9.
- [24] J. Yang, W. Lu and H. Xu, Positive implications of market valuation under asymmetric quality information. *Int. J. Prod. Res.* **54** (2016) 2057–2074.
- [25] Q. Zhang and J. Luo, Coordination of a buyer-vendor supply chain for a perishable product under symmetric and asymmetric information. *Asia-Pac. J. Oper. Res.* **28** (2011) 673–688.
- [26] Y. Zhou, A comparison of different quantity discount pricing policies in a two-echelon channel with stochastic and asymmetric demand information. *Eur. J. Oper. Res.* **181** (2007) 686–703.

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