

MIXED FINANCING MODES FOR CAPITAL-CONSTRAINED SUPPLY CHAIN WITH RISK-AVERSE MEMBERS

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Abstract. This paper considers a two-echelon supply chain consisting of a supplier and a capital-constrained retailer. Both the supplier and the retailer are risk-averse decision makers. The capital-constrained retailer may adopt two mixed financing modes: (1) bank credit and equity financing (BEF) and (2) trade credit and equity financing (TEF). Using a mean-variance framework, we analyze the supply chain members financing and ordering decisions in two cases: symmetric and asymmetric retailer risk aversion threshold information. In the case of symmetric information, we characterize the conditions under which both the supplier and the retailer prefer BEF or TEF. In the case of asymmetric information, we demonstrate that the retailer has an incentive to pretend to be less risk averse. To prevent this distortion behavior, we design a minimum quantity contract for the supplier. Finally, we extend our model to a bank loan-trade credit-equity mixed financing mode (BTEF) in which the retailer can borrow from the bank and the supplier and seeks financial support from investors. The numerical simulations support our results.

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1. INTRODUCTION

The shortage of capital has already seriously restricted the development of small and medium-sized enterprises (SMEs) [37,38]. For example, the World Bank Group survey shows that 62% of the 2700 enterprises interviewed are SMEs, and 54.5% of them need loans to meet their capital needs [29]. In practice, the most common financing methods for capital-constrained enterprises are debt financing and equity financing [30,35]. The most common forms of debt financing are bank loans and trade credit. For example, Bradley and Rubach [2] investigated 3561 representative U.S. businesses and found that more than 60 of them resorted to bank credit. Shi [11] indicates that 20% of all SMEs in China solve the problem of capital constraints through bank credit. Due to the information asymmetry between SMEs and banks and high operational risk, SMEs often have difficulty meeting the requirements of bank loans [31,46]. Then, SMEs attempt to use trade credit to address their financial constraints [8]. In addition, trade credit is gradually becoming one of the most popular financing

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modes between upstream and downstream enterprises [23, 26, 36]. For example, Yang and Birge [36] show that for large public retailers in North America, accounts payable (the amount of trade credit owed by buyers to suppliers) represent approximately one-third of their total liabilities. Another common financing method is equity financing. Enterprises do not need to repay principal and interest under equity financing and instead transfer a certain proportion of their profits as a return to investors. Equity financing can not only alleviate the capital constraints of enterprises but also partially share risk. Compared with Kouvelis and Zhao [15] and Yang *et al.* [40] in that only single debt financing, we find that equity financing reduces the supply chain members' risk. Thus, many enterprises choose equity financing [30]. For example, JD, Baidu, Alibaba and Yahoo promote the rapid growth of enterprises by introducing equity financing before listing. The retailer Qingyang adopted equity financing from its supplier Haier in 2018 [39]. Some scholars explore the impact of equity financing on supply chain members operation decisions [20, 37].

Many existing studies examine capital-constrained supply chain members financing and ordering decisions under a single financing mode [13, 15, 26, 43]. However, in practice, SMEs often adopt mixed financing modes, including debt financing and equity financing. For example, Dingdong Fresh, an e-commerce company and start-up founded in 2017 in Shanghai, directly provides users and households fresh produce, meat, and seafood and other daily necessities. Dingdong has carried out several rounds financing since its establishment, including a Series D round that reached 700 million. It is jointly invested by DST Global, Coatue and other shareholders. Furthermore, Dingdong signed a bank enterprise strategic cooperation agreement with the Bank of Shanghai. According to the agreement, the Bank of Shanghai will further expand the financial support for Dingdong to 8 billion yuan and will customize and develop various financial products according to the needs of Dingdong, including project loans, supply chain financing and other forms, to help Dingdong achieve high-quality development. Ofo, once as one of the two biggest bike-sharing firms in China, obtained 866 million from Alibaba, Trina Solar Capital, Ant Financial, and Junli Capital and declared announced that it had completed the E2 round financing. This round financing Ofo adopts mixed of equity and debt financing mode. Husk Power Systems borrowed 17.75 million from Cisco and the Overseas Private Investment Corporation. In addition, Husk Power Systems obtained 5 million through equity financing [30]. During Oct., 2016 to Jun., 2017, the institutional investors, such as JOY Capital, Vertex, Hillhouse Capital, and WI Harper Group, invested more than 1 billion dollars to fund the Mobike's business. (tech.caijing.com.cn) In addition, Mobike orders bikes from Foxconn. Foxconn not only offers bikes to Mobike and but also allows delay payment. It reduces the Mobike's ordering cost from RMB 3000 per unit to less than RMB 2000 [33]. Therefore, the Mobike adopts trade credit and equity financing. Some scholars study supply chain members decisions under mixed financing [33, 35].

Most of the above studies assume that supply chain members are risk neutral. However, in general, a capital-constrained supply chain will face operations risk and financial risk. Therefore, risk management is very important for enterprise operations. For example, Hewlett-Packard saved at least 100 million in costs through a procurement risk management system to manage supply chain risks in 2008 [26]. In the second quarter of 2001, Cisco Systems, Inc. wrote off 2.5 billion in inventory due to rapidly weakening demand and locked-in supply agreements [24]. Due to inventory shortages, Nike lost 100 million in sales revenue in the third quarter of 2001 [24]. Different enterprises may make different inventory decisions because of their different risk averse attitudes. Thus, the decision-making of enterprises is often related to their risk averse attitude. A survey conducted by McKinsey Company shows that most executives exhibit extreme levels of risk aversion regardless of the size of the investment, even if the expected value of the proposed project is positive [14]. In 2013, although sales in the overall market increased, Marks and Spencer's profits were declining. Retail experts think that the decline was caused by the company's risk-averse strategy. In addition, the risk averse attitude of cotton companies often leads to changes in the corresponding production plan [1]. The data from consulting firm AlixPartners show that the United States had 26 major retailers that went bankrupt in 2018 [27]. Therefore, enterprises' risk averse attitude has an important impact on their decision-making.

To answer these questions, we consider a two-echelon supply chain consisting of a supplier and a capital-constrained retailer. First, we use a mean-variance framework to characterize risk sharing between supply chain members under mixed financing modes. Second, we study risk-averse supply chain members' operational

decisions under mixed financing modes. Then, we explore the supply chain members' financing equilibrium. We find that the supplier is not always willing to provide trade credit and that the retailer's financing preference depends on their risk aversion threshold. In contrast, Kouvelis and Zhao [15] show that the supplier is always willing to provide trade credit. When the retailer is very poor, the retailer may prefer bank credit; otherwise, the retailer prefers trade credit. Next, we discuss the asymmetric information case in which the retailer's risk aversion threshold is private. We show that the capital-constrained retailer has an incentive to pretend to be less risk averse. We design a minimum quantity contract for the supplier to prevent the retailer from reporting false risk averse information. Finally, we extend our model to a bank loan-trade credit-equity mixed financing mode (BTEF) in which the retailer can borrow from the bank and the supplier and seeks financial support from investors.

The contributions of this paper are summarized as follows. First, we consider two kinds of mixed financing modes and explore the impact of risk attitudes on supply chain members' optimal decisions. Second, we characterize the conditions under which supply chain members with risk constraints choose either BEF or TEF. We find that the mixed financing strategy in equilibrium depends on supply chain members' risk aversion attitude. The results are different from the existing literature in which supply chain members without risk constraints prefer trade credit or trade credit financing as a unique financing equilibrium only when the supplier's risk aversion threshold is moderate under single financing modes [15, 40]. Third, we discuss the case of asymmetric information in which the capital-constrained retailer's risk aversion threshold is private information. We design a minimum quantity contract for the supplier to prevent the retailer from distorting information disclosure.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 describes the notation and assumptions. Sections 4 and 5 study the optimal decisions under two mixed financing modes under symmetric information. Section 6 analyzes supply chain members' financing equilibrium under symmetric information. Section 7 explores the decisions in the asymmetric information scenario. Section 8 studies the optimal decisions under bank loan-trade credit-equity financing. Section 9 concludes and discusses the management insights.

2. LITERATURE REVIEW

This paper focuses on the impact of risk constraints on supply chain members' financing and ordering decisions. Our work is closely related to two streams of literature: supply chain financing and risk management in the supply chain.

2.1. Supply chain financing

The vast literature on supply chain financing focuses on improving supply chain members' performance through different financing modes [26, 36, 43]. Many studies show that when the capital-constrained retailer borrows from a competitive bank market, supply chain members' decisions are the same as those of the unconstrained newsboy model [4, 13, 15]. From the supplier's perspective, Lee and Rhee [17] explore the influence of trade credit on improving supply chain performance and show that using trade credit in addition to the markdown allowance can coordinate the supply chain. From an empirical perspective, Lee *et al.* [17] investigate the impact of trade credit on firm performance under various types of competition in supply chains. Silaghi and Moraux [26] show that trade credit may be a tool for supply chain coordination. Our paper differs from the above studies, in which supply chain members are risk neutral, in that we assume that the supplier and the retailer are risk-averse decision makers.

Many studies compare the performance of different financing modes [3, 15, 22]. For example, Jing *et al.* [10] show that both the manufacturer and the retailer prefer trade credit when the production cost is relatively low; otherwise, they prefer bank credit. Kouvelis and Zhao [15] present that compared with bank credit, trade credit can improve both the supplier's and the capital-constrained retailer's profits. Cao *et al.* [3] study the capital-constrained retailer's financing strategy between bank credit and trade credit when consumers have low-carbon preferences. They show that trade credit is also a unique financing equilibrium. Yang and Birge [36] show that

compared with bank credit, trade credit benefits risk sharing between the supplier and the capital-constrained retailer. In our paper, we show that equity financing can not only alleviate the capital constraints of enterprises but also partially share risk. Yan *et al.* [35] investigate the capital-constrained supplier's financing schemes under retailer financing and retailer investment when the retailer is a loss aversion decision maker. Lu and Wu [22] explore the capital-constrained retailer's optimal financing strategy under bank credit and credit when a multinational firm invests in a low-tax jurisdiction. They find that bank credit is an optimal financing strategy under tax asymmetry. Our paper differs from the above literature, which mainly focuses on single financing modes, in that we assume that the capital-constrained retailer adopts mixed financing modes, including debt financing and equity financing and that both the supplier and the retailer are risk-averse decision makers.

The above literature mainly focuses on a single financing mode. However, an increasing number of SMEs want to expand financing channels to solve the problem of capital constraints [33, 36, 41]. Specifically, Yang and Birge [36] study the risk-sharing role of trade credit when the capital-constrained retailer adopts mixed financing, including trade credit and bank loans. Yan *et al.* [35] explore the capital-constrained retailer's optimal financing strategy under financing portfolios, including supplier finance and supplier investment. Zhang *et al.* [43] explore the preference of remanufacturing modes between outsourcing and authorization under original equipment manufacturers with capital constraints and adopt financing portfolios including trade credit and bank credit. Yan and Ye [33] investigate supply chain members' optimal financing and ordering decisions when a capital-constrained retailer adopts hybrid financing schemes, including bank credit and trade credit. Yang *et al.* [40] study the impact of supply chain members' risk averse attitude on their financing equilibrium under bank credit and trade credit. Different from the above literature, we assume that the capital-constrained retailer adopts mixed financing, including debt financing and equity financing. Debt financing includes bank credit and trade credit from banks and the supplier. The retailer transfers a certain proportion of his profit to investors to obtain equity financing from investors. Then, the capital-constrained retailer has two possible financing modes: bank credit and equity financing (BEF) and trade credit and equity financing (TEF).

2.2. Risk management in the supply chain

Many scholars have proposed various methods for measuring risk; among them, the mean-variance framework is one of most commonly used [6, 19, 27, 28]. Specifically, Choi *et al.* [6] investigate the supply chain coordination of a buyback contract under the mean-variance framework and show that buyback contracts do not always coordinate the supply chain under risk constraints. Under a wholesale price and a profit-sharing contract, Wei and Choi [28] study supply chain coordination under a mean-variance framework and obtain the necessary and sufficient conditions under which supply chain coordination is achieved. Using a mean-variance framework, Zhuo *et al.* [45] study supply chain coordination and supply chain member risk sharing under option contracts. They show that option contracts do not always coordinate the supply chain under risk constraints. Li *et al.* [19] study a risk-averse retailer purchasing consumption commodity futures contracts to conduct mean-variance financial hedging and obtain a closed-form, time-consistent financial hedging policy. Based on the mean-variance framework, Li and Jiang [20] study the influence of consumer return policy and the retailer's risk aversion on supply chain members' decisions in a dual-channel competitive market. Choi *et al.* [7] explore pricing decisions in a mass customization supply chain when both the manufacturer and two competing retailers are risk-averse decision makers. Wang *et al.* [27] investigate the incentive effect of trade credit when the capital-constrained retailer's sales cost is private information. They obtain the optimal trade credit contract configuration and the risk-averse retailer's optimal decisions by maximizing the mean-variance utility function. Our paper differs from the above literature in that we further consider both risk constraints and capital constraints in the supply chain and mainly focus on supply chain members' financing and ordering decisions.

Our work is most closely related to Kouvelis and Zhao [15], Yang and Birge [36] and Yang *et al.* [40]. Kouvelis and Zhao [15] study the interaction of short-term single debt financing and inventory decisions. They present that compared with bank credit, trade credit can improve both the supplier's and the capital-constrained retailer's profits. Yang and Birge [36] explore how trade credit improves supply chain performance by allowing the capital-constrained retailer to partially share risk with the supplier. Yang *et al.* [40] show that trade credit

TABLE 1. Comparisons of the three models.

Literature	Financing scheme	Risk attitude	Information structure
Kouvelis and Zhao [15]	BCF or TCF	Risk neutral	Symmetric information
Yang and Birge [36]	BCF+TCF	Risk neutral	Symmetric information
Yang <i>et al.</i> [40]	BCF or TCF	Risk averse	Symmetric information
This paper	BEF or TEF	Risk averse	Asymmetric risk information

Notes. BCF and TCF represent bank credit financing and trade credit financing, respectively. BEF and TEF represent bank credit and equity financing and trade credit and equity financing. Asymmetric risk information indicates that the retailer's risk aversion threshold is private.

financing is a unique financing equilibrium only when the supplier's risk aversion threshold is moderate under single financing modes. The proposed mean-variance model with two mixed financing schemes and two risk-averse members obviously differs from that with single debt financing presented by Kouvelis and Zhao [15] and Yang *et al.* [40]. First, we assume that the capital-constrained retailer adopts mixed financing, including debt financing and equity financing. Debt financing includes bank credit and trade credit from banks and the supplier. Second, the above literature shows that trade credit can achieve risk sharing between supply chain members, and we show that equity financing allows the retailer to partially share risk with investors. Third, we discuss the case of asymmetric information in which the capital-constrained retailer's risk aversion threshold is private information. Our results reveal that the capital-constrained retailer has an incentive to pretend to be less risk averse. We design a minimum quantity contract for the supplier to prevent the retailer from reporting false risk averse information. In addition, Kouvelis and Zhao [15] and Yang and Birge [36] assume that supply chain members are risk neutral. In our paper, we assume that supply chain members are risk averse and use variance to characterize their risk. Table 1 summarizes the differences between this paper and the relevant literature.

3. THE MODEL

We consider a two-echelon supply chain consisting of a risk-averse supplier and a risk-averse retailer. The capital-constrained retailer faces random demand. Demand is a positive random variable x . The probability density function is $f(x)$, the distribution function is $F(x)$ and the complementary distribution function is $\bar{F}(x)$. Assume that $z(x) = \frac{f(x)}{\bar{F}(x)}$ is increasing in x [15, 16, 36].

We assume that the supplier has sufficient capital to cover her production. However, the capital-constrained retailer only has initial capital B , which is insufficient to cover his orders. Due to the high operational risk and information asymmetry between the capital-constrained retailer and banks, the retailer's loan amount from the bank or borrowing from the supplier still cannot cover his orders. Hence, we assume that the capital-constrained retailer adopts a mixed financing mode of debt and equity financing [30, 35]. For debt financing, the retailer can borrow from external banks or an internal supplier to satisfy uncertain market demand. The retailer has two mixed financing modes: (1) bank credit and equity financing (BEF) and (2) trade credit and equity financing (TEF). We assume that the equity financing ratio is ϕ and the debt financing ratio is $1 - \phi$, where $0 \leq \phi < 0.5$ [5, 9, 10].

The sequence of events is as follows: the supplier, as the Stackelberg leader, first decides the wholesale price $w_j, j = B, T$, where the subscripts B and T represent bank credit and equity financing and trade credit and equity financing, respectively. Second, based on the wholesale price, the retailer determines the order quantity q_j . Then, the retailer borrows money from the bank, the supplier and investors. Under mixed financing, the retailer finances $\phi(w_j q_j - B)$ from investors and $(1 - \phi)(w_j q_j - B)$ from the bank or the supplier. At the end of the selling season, the retailer first repays the loans and interest to the bank or the supplier and then transfers ϕ of his profits to investors if his sales income is enough to cover his loans. Otherwise, the retailer goes bankrupt and pays all sales income to the bank or the supplier. To avoid triviality, we assume $c(1 + r_f)w_j(1 + r_j)p, j = B, T$,

TABLE 2. Summary of notation.

Notation	Definition
p	Retail price
c	Production cost
w_j	Wholesale price under financing mode j , where $j = B, T$
X	Random demand, $X \in [0, +\infty)$
$f(X)$	Probability density function of X
$F(X)$	Cumulative distribution function of X
ϕ	Equity financing ratio
q_j	Order quantity under financing mode j , where $j = B, T$
P_{ij}	Profit of member i under financing model j , where $i = S, R$ and $j = B, T$
EP_{ij}	Expected profit of member i under financing model j , where $i = S, R$ and $j = B, T$
SP_{ij}	Standard deviation of the profit of member i under financing model j , where $i = S, R$ and $j = B, T$
V_{ij}	Variance of the profit of member i under financing mode j , where $i = SC, S, R$ and $j = B, T$
K_i	Risk aversion threshold of member i , where $i = S, R$
q_{R,EP_j}	Optimal order quantity that maximizes the retailer's expected profit under financing mode j , where $i = S, R$ and $j = B, T$
q_{R,SP_j}	Retailer's maximum order quantity that satisfies risk constraint under financing mode j , where $j = B, T$
q_{R,MVP_j}	Retailer's optimal order quantity that maximizes the mean-variance optimization problem under financing mode j , where $i = S, R$ and $j = B, T$
r_B	Interest rate of bank loans
r_T	Interest rate of trade credit
r_f	Risk-free interest rate

Notes. For notation purposes, we use BEF and TEF to represent bank credit and equity financing and trade credit and equity financing, respectively.

where c is the supplier's production cost, p is the retailer's retail price, r_f is the risk-free interest rate, r_B and r_T represent the interest rate of bank loans and the interest rate of trade credit.

In this paper, we mainly consider the risk-averse members' financing and ordering decisions under two mixed financing modes. Therefore, we construct the mean-variance model to analyze the decisions in the presence of two mixed financing modes. The aim of supply chain member i is to maximize his or her own expected profits given risk constraints [6, 45, 46]. Therefore, the objective of the supply chain member i is formulated in (P) as follows:

$$\begin{aligned} & \max_{q_j} EP_{ij} \\ & \text{s.t. } SP_{ij} \leq K_j, \end{aligned} \tag{P}$$

where P_{ij} and EP_{ij} are the profit and the expected profit of member i under financing model j , where $i = S, R$ and $j = B, T$. $SP_{ij} = \sqrt{V_{ij}}$, where SP_{ij} and V_{ij} are the standard deviation and variance of the profit of member i under financing model j . $K_i \geq 0$ is member i 's risk aversion threshold. q_{R,MVP_j} is the retailer's optimal order quantity that maximizes the mean-variance optimization problem under financing mode j . q_{R,EP_j} is the retailer's optimal order quantity that maximizes his expected profit under financing mode j . q_{R,SP_j} is the retailer's maximum order quantity that satisfies risk constraint under financing mode j . To facilitate interpretation, we list the main notation in Table 2.

4. BANK CREDIT AND EQUITY FINANCING

In this section, the retailer borrows from the bank and seeks financial support from investors. Before the selling season, the supplier sets the wholesale price w_B . According to the wholesale price, the retailer determines the

order quantity q_B . The retailer's initial capital B cannot cover his order, and he should finance the amount $w_B q_B - B$ from the bank and investors. We assume that the retailer's equity financing ratio is ϕ . Under BEF, the retailer finances $\phi(w_B q_B - B)$ from investors and $(1 - \phi)(w_B q_B - B)$ from the bank. In return, the retailer transfers a fraction ϕ of his profits to investors. The interest rate charged by the bank is r_B . Fully competitive banks only earn the risk-free interest rate r_f [3, 4, 15]. At the end of the sales season, the retailer obtains sales income $p \cdot \min\{q_B, x\}$. If $p \cdot \min\{q_B, x\} < (1 - \phi)(w_B q_B - B)(1 + r_B)$, then the retailer pays all sales income to the bank and goes bankrupt. If $p \cdot \min\{q_B, x\} \geq (1 - \phi)(w_B q_B - B)(1 + r_B)$, the retailer first pays $(1 - \phi)(w_B q_B - B)(1 + r_B)$ to the bank and then transfers $\phi(p \cdot \min\{q_B, x\} - (1 - \phi)(w_B q_B - B)(1 + r_B))$ to investors. Thus, the retailer's profit is $P_{RB} = (1 - \phi)[p \cdot \min\{q_B, x\} - (1 - \phi)(w_B q_B - B)(1 + r_B)]^+$. The following Lemma 4.1 characterizes the variances of the members' profits under BEF.

Lemma 4.1. *Under BEF, the variances of members' profits are given by $V_{BB} = p^2 \left(2 \int_0^{k_B(q_B)} (k_B(w_B, q_B) - x) F(x) dx - \left(\int_0^{k_B(q_B)} F(x) dx \right)^2 \right)$, $V_{IB} = (\phi p)^2 \left(2 \int_{k_B(q_B)}^{q_B} (q_B - x) F(x) dx - \left(\int_{k_B(q_B)}^{q_B} F(x) dx \right)^2 \right)$, $V_{RB} = ((1 - \phi)p)^2 \left(2 \int_{k_B(q_B)}^{q_B} (q_B - x) F(x) dx - \left(\int_{k_B(q_B)}^{q_B} F(x) dx \right)^2 \right)$, $V_{SB} = 0$, where $k_B = \frac{(1 - \phi)(w_B q_B - B)(1 + r_B)}{p}$.*

Lemma 4.1 shows that the bank and investors take on some of the supply chain risk and that the supplier does not bear any risk under BEF. When $x < k_B$, the retailer's sales income is insufficient to cover his loans, and he goes bankrupt. Correspondingly, the bank and investors suffer losses and bear default risk. When $k_B \leq x < q_B$, the retailer's sales income is sufficient to cover his loans. The retailer first pays $p k_B$ to the bank and then transfers a fraction ϕ of his profits to investors. As demand increases, the profits of the retailer and investors increase. Investors share the retailer's risk, which means that the retailer adopts mixed financing, including equity financing, which can reduce his risk. These findings are different from those in Yang *et al.* [40] in that only the retailer and the bank bear risk under bank credit financing.

4.1. Retailer's decision

Before the selling season, the bank provides $(1 - \phi)(w_B q_B - B)$ to the retailer. At the end of the selling season, the bank receives $\min\{p \cdot \min\{q_B, x\}, (1 - \phi)(w_B q_B - B)(1 + r_B)\}$. In a fully competitive market, the bank only earns the risk-free interest rate r_f . Then, we have $(1 - \phi)(w_B q_B - B)(1 + r_f) = E[\min\{p \cdot \min\{q_B, x\}, (1 - \phi)(w_B q_B - B)(1 + r_B)\}]$. Thus, the retailer's expected profit is

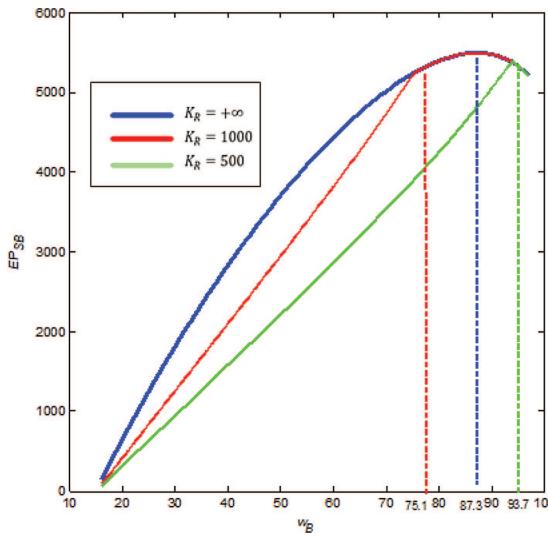
$$EP_{RB} = (E[\min\{p \cdot \min\{q_B, x\}\} - (1 - \phi)(w_B q_B - B)(1 + r_f)])(1 - \phi). \quad (4.1)$$

$$\begin{aligned} & \max_{q_B} EP_{RB}(q_B) \\ & \text{s.t. } SP_{RB}(q_B) \leq K_R, \end{aligned} \quad (P1)$$

where $SP_{RB} = \sqrt{V_{RB}}$, is the standard deviation of the retailer's profit. We define $q_{R,SPB}(w_B) = \operatorname{argmax}_{q_B} \{SP_{RB}(q_B) \leq K_R\}$, which gives the retailer's maximum quantity that satisfies $SP_{RB}(q_B) \leq K_R$.

Proposition 4.2. *Under BEF, (i) the retailer's optimal order quantity, $q_{R,MVB}(w_B) = \min\{q_{R,EPB}(w_B), q_{R,SPB}(w_B)\}$, where $q_{R,EPB}(w_B) = \bar{F}^{-1}\left(\frac{(1 - \phi)w_B(1 + r_f)}{p}\right)$, and (ii) $q_{R,MVB}(w_B)$ is increasing in w_B for $w_B \in (c, w_{B1})$ and decreasing in w_B for $w_B \in [w_{B1}, p]$, where $w_{B1} = \frac{p\bar{F}(q_{R,SPB}(w_{B1}))}{(1 - \phi)(1 + r_f)}$.*

Proposition 4.2 presents the retailer's optimal order quantity under BEF. When the retailer's risk constraint is active, the retailer's order quantity $q_{R,MVB}(w_B) = q_{R,SPB}(w_B)$. When the retailer's risk constraint is inactive, the retailer's order quantity $q_{R,MVB}(w_B) = q_{R,EPB}(w_B)$. When $w_B \in (c, w_{B1})$, the retailer's order quantity $q_{R,MVB}(w_B) = q_{R,SPB}(w_B)$ is increasing in the wholesale price. As the wholesale price increases, the retailer will borrow more from the bank, and investors will invest more. Thus, the risk of the bank and that of investors increase. Furthermore, the retailer's risk declines. For a given K_R , the retailer will raise the order quantity. When $w_B \in [w_{B1}, p]$, the retailer's order quantity $q_{R,MVB}(w_B) = q_{R,EPB}(w_B)$ is decreasing in the wholesale price w_B .

FIGURE 1. $EP_{SB}(w_B)$ changes with w_B .

4.2. Supplier's decision

From Proposition 4.2, we know that the retailer's order quantity is $q_{R,MVB}(w_B)$ under BEF. Before the selling season, the supplier's production cost is $cq_{R,MVB}(w_B)$. At the beginning of the selling season, the supplier receives $w_B q_B$. Then, the supplier's profit is

$$EP_{SB} = (w_B - c)q_{R,MVB}(w_B)(1 + r_f). \quad (4.2)$$

Lemma 4.1 shows that the supplier is risk-free under BEF. Therefore, under the mean-variance framework, the supplier's problem is

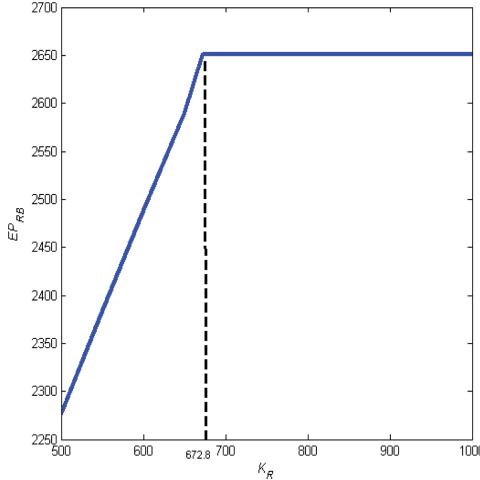
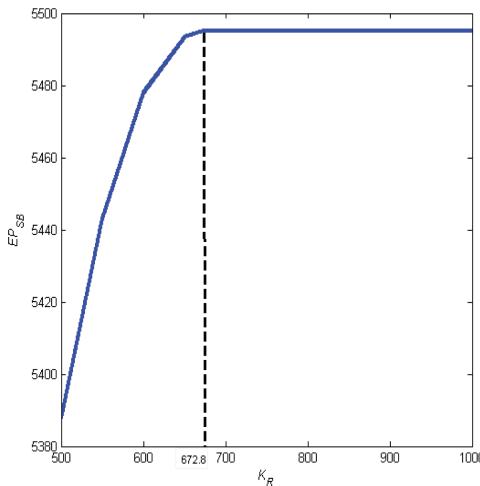
$$\max_{w_B} EP_{SB}(q_{R,MVB}(w_B)). \quad (P2)$$

The following Proposition 4.3 presents the supplier's optimal wholesale price.

Proposition 4.3. *Under BEF, the optimal wholesale price $w_B^* = \max\{w_{B1}, w_{B0}\}$, where $w_{B0} = \frac{p\bar{F}(q_{R,EPB}(w_{B0}))}{(1-\phi)(1+r_f)}$ and $p\bar{F}(q_{R,EPB}(w_{B0}))(1 - q_{R,EPB}(w_{B0})z(q_{R,EPB}(w_{B0}))) - (1 - \phi)c(1 + r_f) = 0$.*

Proposition 4.3 shows that with risk constraints, the supplier's optimal wholesale price is higher than that without risk constraints. The retailer with risk constraints reduces his order quantity. To obtain more profits, the supplier will raise the wholesale price. When K_R is small, i.e., $K_R < SP_{RB}(q_{R,EPB}(w_{B0}))$, the risk constraint for the retailer plays a role. The retailer orders conservatively $q_{R,MVB}(w_B) = q_{R,SPB}(w_B)$, and the supplier charges w_{B1} . When K_R is large, i.e., $K_R \geq SP_{RB}(q_{R,EPB}(w_{B0}))$, the risk constraint for the retailer does not play a role. The retailer orders aggressively $q_{R,MVB}(w_B) = q_{R,EPB}(w_B)$. The supplier correspondingly charges w_{B0} , which equals the wholesale price without risk constraints.

To clarify Proposition 4.3, we use numerical simulations, as shown in Figure 1. We maintain the following assumptions across the simulations: (1) the random demand X is normally distributed with a mean = 100 and a variance $\sigma = 30$; (2) $p = 100$, $c = 15$, $r_f = 0.03$, $\phi = 0.1$ and $B = 2000$ [28, 41]. We calculate $w_{B0} = 87.3$ and $K_{B0} = SP_{RB}(q_{R,EPB}(w_{B0})) = 672.8$. When $K_R < K_{B0} = 672.8$, the retailer's risk constraint is active, and his risk aversion plays a role in the supplier's wholesale price. From Figure 1, when $K_R = 500 < K_{B0}$, the retailer's risk constraint is active, and $w_{B1} = 93.7 > w_{B0} = 87.3$. Then, the supplier's optimal wholesale price

FIGURE 2. EP_{RB} changes with K_R .FIGURE 3. EP_{SB} changes with K_R .

$w_B^* = w_{B1} = 93.7$, and her profit is 5384.6. When $K_R > K_{B0} = 672.8$, the retailer's risk constraint is inactive. From Figure 1, when $K_R = 1000 > K_{B0}$, the retailer's risk constraint is inactive, and $w_{B1} = 75.1 < w_{B0} = 87.3$. Then, the supplier's optimal wholesale price $w_B^* = w_{B0} = 87.3$, and her profit is 5495.1.

Figures 2 and 3 present that the profit of the retailer and the supplier change with K_R under BEF. When $K_R < 672.8$, the profit of the retailer and the supplier increase with K_R . In this case, the retailer's risk constraint is active. As K_R increases, the retailer raises his order quantity. When $K_R \geq 672.8$, the profit of the retailer and the supplier remains unchanged. In this case, the retailer's risk constraint is inactive. The retailer's order quantity equals $q_{R,EPB}(w_B)$ without risk constraints and remains unchanged.

5. TRADE CREDIT AND EQUITY FINANCING

5.1. Retailer's decision

In this section, we assume that the capital-constrained retailer uses TEF to order products. Before the selling season, the supplier, as the Stackelberg leader, first charges the wholesale price w_T . Based on the supplier's wholesale price, the retailer determines his order quantity q_T . The retailer's initial capital is B , and he needs funds $w_T q_T - B$. We assume that the equity financing ratio is ϕ and the trade credit ratio is $1 - \phi$. Under TEF, the retailer finances $\phi(w_B q_B - B)$ from investors and $(1 - \phi)(w_B q_B - B)$ from the supplier. At the end of the selling season, if the retailer's sales income generates enough trade credit payments, the retailer first pays $(1 - \phi)(w_T q_T - B)(1 + r_T)$ to the supplier and then transfers $\phi[p \cdot \min\{q_T, x\} - (1 - \phi)(w_T q_T - B)(1 + r_T)]$ to investors, where r_T is the interest rate on trade credit. We assume that r_T is a constant; for example, trade credit rates may be determined by the industry benchmark interest rate [12, 15, 32]. If the retailer's sales income is not enough to cover his trade credit, the retailer pays all sales income $p \cdot \min\{q_T, x\}$ to the supplier and goes bankrupt. The retailer's profit is $P_{RT} = (1 - \phi)[p \cdot \min\{q_T, x\} - (1 - \phi)(w_T q_T - B)(1 + r_T)]^+$. Before the selling season, the supplier's production cost is c_{QT} . At the beginning of the selling season, the supplier receives $\phi(w_T q_T - B) + B$ from the retailer. At the end of the selling season, the supplier receives $\min\{p \cdot \min\{q_T, x\}, (1 - \phi)(w_T q_T - B)(1 + r_T)\}$ from the retailer. Thus, the supplier's profit is $P_{ST} = \min\{p \cdot \min\{q_T, x\}, (1 - \phi)(w_T q_T - B)(1 + r_T) + (\phi(w_T q_T - B) + B) - c_{QT}\}$. The following Lemma 5.1 characterizes the variances of the members' profits.

Lemma 5.1. *Under TEF, the variances of members' profits are given by $V_{IT} = (\phi p)^2 \left(2 \int_{k_T}^{q_T} (q_T - x) F(x) dx - \left(\int_{k_T}^{q_T} F(x) dx \right)^2 \right)$, $V_{RT} = ((1 - \phi)p)^2 \left(2 \int_{k_T}^{q_T} (q_T - x) F(x) dx - \left(\int_{k_T}^{q_T} F(x) dx \right)^2 \right)$, $V_{ST} = p^2 \left(2 \int_0^{k_T} (k_T - x) F(x) dx - \left(\int_0^{k_T} F(x) dx \right)^2 \right)$, where $k_T = \frac{(1 - \phi)(w_T q_T - B)(1 + r_T)}{p}$.*

Lemma 5.1 shows that under TEF, the supplier, the retailer and investors share supply chain risk. Thus, compared with single trade credit financing, the retailer can reduce his risk by adopting mixed financing, including equity financing. This is because the retailer transfers some risk to the investors. These findings are different from those in Yang *et al.* [40] in that only the retailer and supplier bear risk under trade credit financing. Since investors share the retailer's risk, mixed financing, including equity financing, can reduce the retailer's risk. When $x < k_T$, the retailer's sales income cannot cover his trade credit, and he goes bankrupt. Correspondingly, the supplier and investors suffer losses. As the demand increases, the supplier's profit increases. When $k_T \leq x < q_T$, the retailer's sales income is sufficient to repay his trade credit. Thus, the supplier has no loss. As demand increases, the retailer's and investors' profit increases.

Under TEF, the retailer's expected profit is

$$EP_{RT} = (1 - \phi)E[p \cdot \min\{q_T, x\} - (1 - \phi)(w_T q_T - B)(1 + r_T)]^+. \quad (5.1)$$

Under the mean-variance framework, the retailer's problem is

$$\begin{aligned} & \max_{q_T} EP_{RT}(q_T) \\ & \text{s.t. } SP_{RT}(q_T) \leq K_R, \end{aligned} \quad (P3)$$

where $SP_{RT} = \sqrt{V_{RT}}$, is the standard deviation of the retailer's profit. We define $q_{R,SPT}(w_T) = \operatorname{argmax}_{q_T} \{SP_{RT}(q_T) \leq K_R\}$, which gives the retailer's maximum quantity that satisfies $SP_{RT}(q_T) \leq K_R$.

Proposition 5.2. *Under TEF, (i) the retailer's optimal order quantity, $q_{R,MVT}(w_T) = \min\{q_{R,EPT}(w_T), q_{R,SPT}(w_T)\}$, where $q_{R,EPT}(w_T) = \bar{F}^{-1} \left(\frac{(1 - \phi)w_T(1 + r_T)}{p} \bar{F} \left(\frac{(1 - \phi)w_T q_{R,EPT}(w_T) - B}{p} (1 + r_T) \right) \right)$, and (ii) $q_{R,MVT}(w_T)$ is increasing in w_T for $w_T \in (c, w_{T1})$ and decreasing in w_T for $w_T \in [w_{T1}, p)$, where w_{T1} satisfies $q_{R,EPT}(w_{T1}) = q_{R,SPT}(w_{T1})$.*

Proposition 5.2 indicates that, with risk constraints, the retailer's optimal order quantity is $\min\{q_{R,EPT}(w_T), q_{R,SPT}(w_T)\}$. When the retailer's risk constraint is active, the retailer orders $q_{R,MVT}(w_T) = q_{R,SPT}(w_T)$. When the retailer's risk constraint is inactive, the retailer orders $q_{R,MVT}(w_T) = q_{R,EPT}(w_T)$. The retailer adopts a relatively aggressive ordering strategy. When $w_T \in (c, w_{T1})$, the retailer's order quantity $q_{R,MVT}(w_T) = q_{R,SPT}(w_T)$. The supplier raises the wholesale price, and the retailer will borrow more from the supplier and investors. Thus, the supplier and investors will take more risk. Correspondingly, the retailer's risk decreases. Hence, the retailer has an incentive to order more products. When $w_T \in [w_{T1}, \frac{p}{1+r_T})$, the retailer's order quantity is $q_{R,MVT}(w_T) = q_{R,EPT}(w_T)$. Obviously, the retailer's order quantity decreases with the wholesale price.

5.2. Supplier's decision

Based on the above analysis, under TEF, the supplier's expected profit is

$$EP_{ST} = E[\min\{p \cdot \min\{q_T, x\}, (1 - \phi)(w_T q_T - B)(1 + r_T)\}] + (\phi(w_T q_T - B) + B) - c q_T (1 + r_f). \quad (5.2)$$

Under TEF, the retailer's optimal order quantity is $q_{R,MVT}(w_T) = \min\{q_{R,EPT}(w_T), q_{R,SPT}(w_T)\}$. Therefore, under the mean-variance framework, the supplier's problem is

$$\begin{aligned} & \max_{w_T} EP_{ST}(q_{R,MVT}(w_T)) \\ & \text{s.t. } SP_{ST}(q_{R,MVT}(w_T)) \leq K_S, \end{aligned} \quad (P4)$$

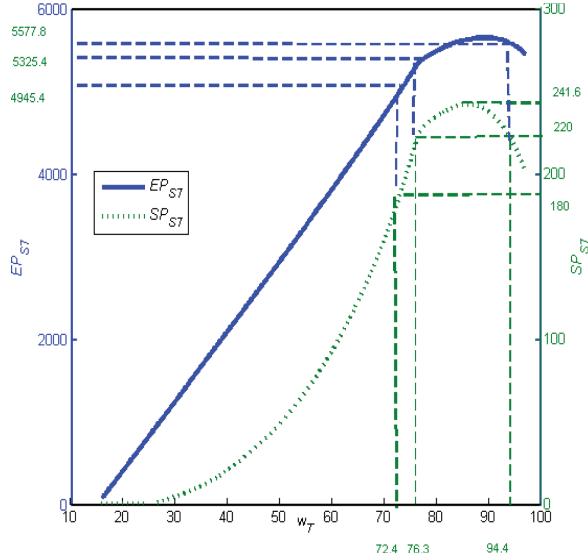
where $K_S \geq 0$ is the supplier's risk aversion threshold.

We next explore the supplier's optimal wholesale price under risk constraints. To obtain analytical results, we assume that $\frac{\partial z(x)}{\partial x} > 0$ [15, 44]. Many commonly used distributions can satisfy this assumption, such as truncated normal, uniform, exponential, and power distributions.

Proposition 5.3. *Under TEF, (i) when $K_S \geq \max\{SP_{ST}(q_{R,MVT}(\tilde{w}_T)), SP_{ST}(q_{R,MVT}(w_{T1}))\}$, then $w_T^* = \max\{w_{T1}, w_{T0}\}$; (ii) when $K_S < \max\{SP_{ST}(q_{R,MVT}(\tilde{w}_T)), SP_{ST}(q_{R,MVT}(w_{T1}))\}$, then $w_T^* = \arg\max\{EP_{ST}(q_{R,MVT}(w_{Tl})), EP_{ST}(q_{R,MVT}(w_{Tu}))\}$; where w_{T0} satisfies equation $(\phi w_{T0}(1 + r_f) + (1 - \phi)w_{T0}(1 + r_T)\bar{F}\left(\frac{(1-\phi)w_{T0}q_{R,EPT}(w_{T0})-B(1+r_T)}{p}\right)\xi(w_{T0}) - c(1 + r_f) = 0$, $\xi(w_{T0}) = \frac{1-q_{R,EPT}(w_{T0})z(q_{R,EPT}(w_{T0}))}{(1-\phi)(w_{T0}q_{R,EPT}(w_{T0})(1+r_T))}z\left(\frac{(1-\phi)(w_{T0}q_{R,EPT}(w_{T0})-B)(1+r_T)}{p}\right)$ and w_{Ti} satisfies equation $SP_{ST}(q_{R,MVT}(w_{Ti})) = K_S$, $i = l, u$.*

When $K_S \geq \max\{SP_{ST}(q_{R,MVT}(\tilde{w}_T)), SP_{ST}(q_{R,MVT}(w_{T1}))\}$, the supplier's risk constraint is inactive under TEF. The supplier's wholesale price depends on K_R and is independent of K_S . Therefore, $w_T^* = \max\{w_{T1}, w_{T0}\}$. The result is different from Kouvelis and Zhao [15] and Yang *et al.* [40] in that the capital constrained retailer adopts trade credit financing. The supplier bears both the production costs and all the financing risks if the retailer goes bankrupt under trade credit financing. However, under TEF, the supplier only bears the production costs and partially bears the financing risks if the retailer goes bankrupt. Equity financing can not only alleviate the capital constraints of enterprises but also partially share risk. When $K_S < \max\{SP_{ST}(q_{R,MVT}(\tilde{w}_T)), SP_{ST}(q_{R,MVT}(w_{T1}))\}$, the supplier's risk constraint is active under TEF. From the proof of Proposition 5.3, we know that EP_{ST} and SP_{ST} are concave in w_T . Then, equation $SP_{ST}(q_{R,MVT}(w_T)) = K_S$ has at most two solutions w_{Tl} and w_{Tu} . Thus, the supplier's maximum expected profit may be in w_{Tl} or w_{Tu} . Therefore, $w_T^* = \arg\max\{EP_{ST}(q_{R,MVT}(w_{Tl})), EP_{ST}(q_{R,MVT}(w_{Tu}))\}$.

Based on the settings in Figure 1 and $K_R = 1000$, we examine how the supplier's EP and SP change with w_T under TEF in Figure 4. Based on Proposition 5.3, we can calculate that $w_{T0} = 89.3$, $w_{T1} = 76.8$. When $w_T \leq w_{T1} = 76.8$, the retailer's risk constraint is active, and his order quantity $q_{R,MVT}(w_T) = q_{R,SPT}(w_T)$. When $w_T > w_{T1} = 76.8$, the retailer's risk constraint is inactive, and his order quantity

FIGURE 4. $EP_{ST}(w_T)$ and $SP_{ST}(q_{R,MVT}(w_T))$.

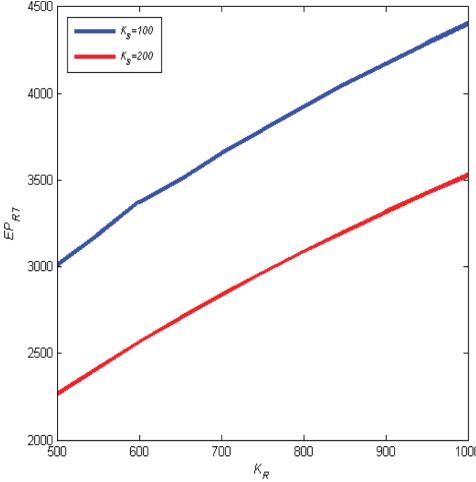
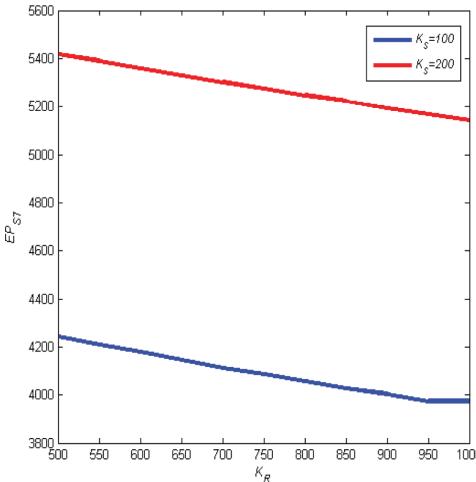
$q_{R,MVT}(w_T) = q_{R,EPT}(w_T)$. From Figure 2, we find that the supplier's expected profit and the standard deviation are concave in w_T . Then, we calculate that $\max EP_{ST}(q_{R,MVT}(w_T)) = 5654.9$ and $\max SP_{ST}(q_{R,MVT}(w_T)) = 241.6$. When $K_S \geq 241.6$, the supplier's risk constraint is inactive. For example, when $K_S = 260 > 241.6$, then the supplier's optimal wholesale price $w_T^* = w_{T0} = 89.3$, and her expected profit $EP_{ST}(q_{R,MVT}(89.3)) = 5654.9$. When $K_S < 241.6$, the supplier's risk constraint is active, and her risk aversion plays a role in the wholesale price. For example, when $K_S = 220 < 241.6$, then $w_{Tl} = 76.3$, and $w_{Tu} = 94.4$. Then, $EP_{ST}(q_{R,MVT}(w_{Tl})) = 5325.4$, and $EP_{ST}(q_{R,MVT}(w_{Tu})) = 5577.8$. Hence, $w_T^* = w_{Tu} = 94.4$. When $K_S = 180 < 241.6$, then $w_{Tl} = 72.4$. In this case, equation $SP_{ST}(q_{R,MVT}(w_T)) = K_S$ has only one solution. Hence, $w_T^* = w_{Tl} = 72.4$.

Figure 5 presents the retailer's profit changes with K_R under TEF. Obviously, the retailer's profit increases with K_R . In addition, we also find that when K_S increases, the retailer's profit decreases. Since the supplier's risk aversion threshold increases, the supplier will charge a large wholesale price to obtain more profit. Thus, the retailer's profit decreases. Figure 6 shows that the supplier's profit decreases with K_R . As K_R increases, the retailer will raise the order quantity. Since the supplier's risk aversion is active and remains unchanged, the supplier reduces the wholesale price to avoid risk. Thus, the supplier's profit decreases with K_R .

6. FINANCING EQUILIBRIUM

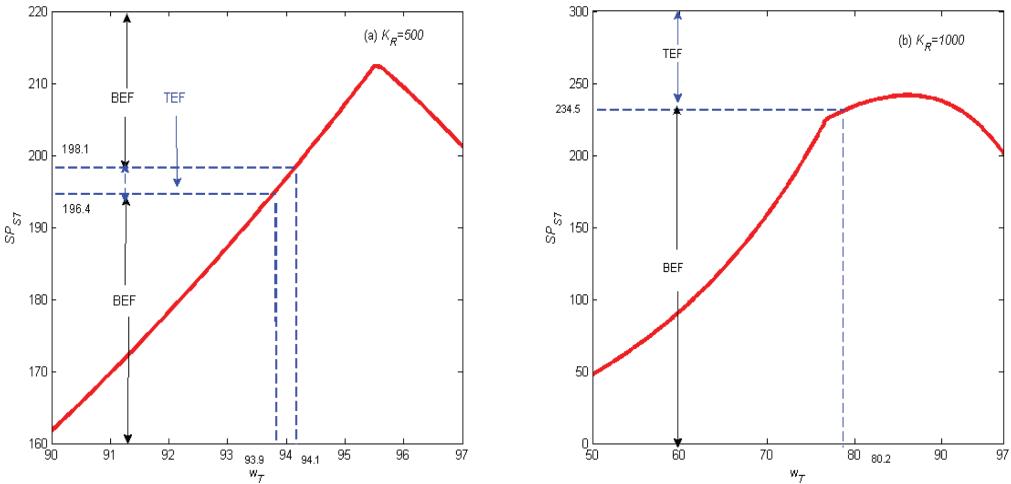
In this section, we investigate the impact of risk aversion on the financing equilibrium when the capital-constrained retailer can choose either BEF or TEF. Define \hat{w}_T as the wholesale price such that $EP_{RT}(q_{R,MVT}(\hat{w}_T)) = EP_{RB}(q_{R,MVB}(w_B^*))$. Define w_{T_i} such that $EP_{RT}(q_{R,MVT}(w_{T_i})) = EP_{RB}(q_{R,MVB}(w_B^*)), i = m, n$, where $w_{Tm} < w_{Tn}$. Denote by the corresponding risk aversion threshold that $K_{Si} = SP_{ST}(q_{R,MVT}(w_{T_i}))$.

Proposition 6.1. (i) *If $\bar{w}_T < \max\{\tilde{w}_T, w_{T1}\}$, when $K_{Sm} < K_S < \bar{K}_S$, TEF is the unique financing equilibrium; otherwise, BEF is the unique financing equilibrium.*
(ii) *If $\bar{w}_T > \max\{\tilde{w}_T, w_{T1}\}$, when $K_S > \min\{K_{Sm}, \bar{K}_S\}$, TEF is the unique financing equilibrium; otherwise, BEF is the unique financing equilibrium, where $\bar{K}_S = SP_{ST}(q_{R,MVT}(\bar{w}_T))$, $\bar{w}_T = \min\{\hat{w}_T, w_{Tn}\}$.*

FIGURE 5. EP_{RT} changes with K_R .FIGURE 6. EP_{ST} changes with K_R .

Proposition 6.1 shows the financing equilibrium under risk constraints. From the supplier's perspective, the supplier has no risk under BEF, while she may bear risk under TEF. Only when the supplier's expected profit under TEF is greater than that under BEF does the supplier prefer to provide trade credit and bear risk. The proof of Proposition 6.1 indicates that the supplier's expected profit and standard deviation are concave in the wholesale price. Thus, the wholesale price satisfies $w_{Tm} < w_T < w_{Tn}$. From the retailer's perspective, he will adjust his financing mode to obtain more profits while satisfying his risk constraint based on the supplier's decisions. The proof of Proposition 6.1 indicates that the retailer's expected profit under TEF is decreasing in the wholesale price. Thus, when $w_T < \hat{w}_T$, the retailer with a risk constraint will obtain more profits under TEF than under BEF. Therefore, only when $w_{Tm} < w_T < \hat{w}_T$, TEF is the unique financing equilibrium.

If $\bar{w}_T < \max\{\tilde{w}_T, w_{T1}\}$, only when $w_{Tm} < w_T < \hat{w}_T$ do both the supplier and the retailer prefer TEF. This is because the standard deviation of the supplier's profit is increasing in w_T in the interval (w_{Tm}, \hat{w}_T) . Correspondingly, the supplier's risk aversion threshold satisfies $K_{Sm} < K_S < \bar{K}_S$. When $K_S \leq K_{Sm}$, the

FIGURE 7. SP_{RT} changes with w_T .

supplier may charge a lower wholesale price than w_{Tm} to avoid risk under TEF. Then, in this case, the supplier is not willing to provide trade credit to the retailer. When $K_S \geq \bar{K}_S$, the supplier's risk aversion threshold is high, and the supplier will charge a relatively high wholesale price, $w_T \geq \hat{w}_T$, to obtain more profits. Then, the retailer will choose BEF to obtain more profits. Thus, only when $K_{Sm} < K_S < \bar{K}_S$ do both the supplier and the retailer prefer TEF. Otherwise, BEF is the unique financing equilibrium. If $\bar{w}_T > \max\{\tilde{w}_T, w_{T1}\}$, only when $w_{Tm} < w_T < \hat{w}_T$ do both the supplier and the retailer prefer TEF. This is because the standard deviation of the supplier's profit is concave in w_T in the interval (w_{Tm}, \hat{w}_T) . Thus, $SP_{ST}(q_{R,MVT}(w_T)) > \min\{K_{Sm}, \bar{K}_S\}$. When $K_S \leq \min\{K_{Sm}, \bar{K}_S\}$, then the supplier's risk aversion threshold is small, and the supplier will charge a relatively high wholesale price, $w_T \geq \hat{w}_T$, or a relatively low wholesale price, $w_T < w_{Tm}$, to avoid risk. When $w_T \geq \hat{w}_T$, the retailer will choose BEF to obtain more profits. When $w_T < w_{Tm}$, the supplier is not willing to provide trade credit to the retailer. Then, the retailer will choose BEF to obtain more profits. Thus, BEF is the unique financing equilibrium. When $K_S > \min\{K_{Sm}, \bar{K}_S\}$, the supplier's risk aversion threshold is high. To obtain more profits, the supplier is willing to bear some risk. Thus, the supplier charges a moderate wholesale price, *i.e.*, $w_{Tm} < w_T < \hat{w}_T$. In this case, TEF is the unique financing equilibrium. In contrast, Yang *et al.* [40] show that TCF is a unique financing equilibrium only when the supplier's risk aversion threshold is moderate under single debt financing modes.

For example, based on the settings in Figure 1, Figure 7 examines how the supplier's SP_{ST} changes with w_T under TEF. When $K_R = 500$, we calculate $EP_{SB} = 5384.6$, $EP_{RB} = 2274.4$, $w_{Tm} = 93.9$, $\hat{w}_T = 94.1$, $w_{T1} = 95.5$ and $\tilde{w}_T = 86.1$. Then, $\bar{w}_T < \max\{\tilde{w}_T, w_{T1}\}$. Furthermore, $K_{Sm} = 196.4$, and $\bar{K}_S = 198.1$. The retailer prefers TEF only when his profit is more than 2274.4 under TEF. Thus, the supplier's wholesale price $w_T \leq 94.1$. Only when $w_T > 93.9$ is the supplier willing to provide trade credit to the retailer under mixed financing. Thus, only when the wholesale price satisfies $93.9 < w_T \leq 94.1$ do the expected profits of both the supplier and the retailer under TEF exceed their profits under BEF. Therefore, when $196.4 < K_S < 198.1$, TEF is the financing equilibrium. Figure 7a presents the above results. Figure 7b illustrates that the supplier's SP_{ST} changes with w_T when $K_R = 1000$. We calculate $EP_{SB} = 5495.1$, $EP_{RB} = 2651.1$, and $w_{Tm} = 80.2 < \max\{\tilde{w}_T, w_{T1}\} = 86.1 < \bar{w}_T = 87.7$. Furthermore, $K_{Sm} = 234.5$, and $\bar{K}_S = 240.9$. When $K_S > 234.5$, then the supplier's wholesale price $80.2 < w_T < 91.1$. For a given $K_S (K_S > 234.5)$, the supplier sets a wholesale price $80.2 < w_T < 87.7$, and then the expected profits of both the supplier and the retailer under TEF are greater than those under BEF. Therefore, when $K_S > \min\{K_{Sm}, \bar{K}_S\} = 234.5$, TEF is the unique financing equilibrium. For example, when $K_S = 240$, the supplier sets a wholesale price $w_T = 83.4$. Then, we calculate $EP_{ST} = 5582.0$, $EP_{RT} = 2924.8$. Obviously, both the supplier and the retailer prefer TEF.

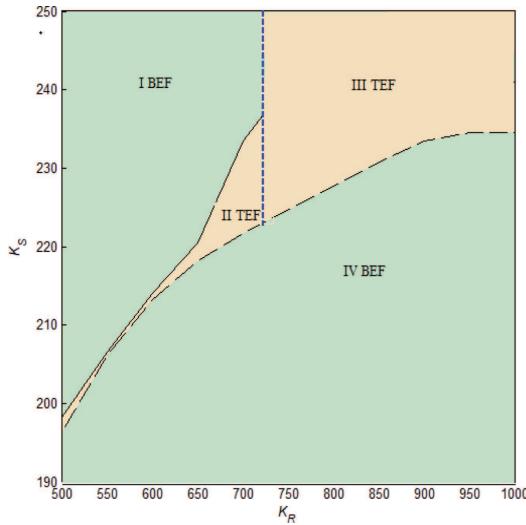


FIGURE 8. The supply chain members financing equilibrium.

Figure 8 presents the supply chain members' financing equilibrium with a risk constraint. Note that the black solid line denotes \bar{K}_S , and the black dotted line denotes K_{Sm} . When $K_R < 715.2$, we find that when the supplier's risk aversion threshold K_S falls into Area II ($K_{Sm} < K_S < \bar{K}_S$), TEF is the financing equilibrium. Otherwise, BEF is the financing equilibrium. When $K_R \geq 715.2$, the supplier's risk aversion threshold K_S falls into Area III ($K_S > \min\{K_{Sm}, \bar{K}_S\}$), and TEF is the financing equilibrium; otherwise, BEF is the financing equilibrium.

We also find that the supplier's critical risk aversion thresholds K_{Sm} and \bar{K}_S are non-decreasing in K_R . When $K_R < 672.8$, $w_{B0} < w_{B1}$ and $\hat{w}_T < w_{T1}$, the retailer's risk constraint is active under BEF and TEF. The retailer's expected profit increases with K_R under BEF. In this case, the upper bound of the wholesale price \hat{w}_T increases. Thus, the critical risk aversion threshold \bar{K}_S increases with K_R . When $672.8 \leq K_R < 715.2$, $w_{B0} \geq w_{B1}$ and $\hat{w}_T < w_{T1}$. In this case, the retailer's risk constraint is inactive under BEF. As K_R increases, the retailer's expected profit remains unchanged under BEF. When the supplier sets the wholesale price \hat{w}_T , the risk constraint for the retailer is active under TEF. As K_R increases, the retailer's order quantity increases under TEF. Thus, the critical risk aversion threshold \bar{K}_S increases with K_R . When $K_R \geq 715.2$, $w_{B0} > w_{B1}$, and $\hat{w}_T \geq w_{T1}$. The retailer's risk constraint is inactive under BEF. When the supplier sets wholesale price \hat{w}_T , the risk constraint for the retailer is inactive under TEF. Then, the retailer's expected profit remains unchanged under BEF and TEF. Hence, the critical risk aversion threshold \bar{K}_S remains unchanged.

When $K_R < 672.8$, then $w_{B0} < w_{B1}$, and $w_{Tm} < w_{T1}$. The retailer's risk constraint is active under BEF and TEF. The supplier is willing to provide trade credit when her risk aversion threshold increases. Thus, the critical risk aversion threshold K_{Sm} increases with K_R . When $672.8 \leq K_R < 912.3$, $w_{B0} \geq w_{B1}$, and $w_{Tm} < w_{T1}$. In this case, the retailer's risk constraint is inactive under BEF. Then, the supplier's expected profit remains unchanged under BEF. When the supplier sets wholesale price w_{Tm} , the risk constraint for the retailer is active under TEF. As K_R increases, the retailer's order quantity increases. Thus, the critical risk aversion threshold K_{Sm} increases with K_R . When $K_R \geq 912.3$, $w_{B0} > w_{B1}$, and $w_{Tm} \geq w_{T1}$. The retailer's risk constraint is inactive under BEF. Then, the supplier's expected profit remains unchanged under BEF. When the supplier sets wholesale price w_{Tm} , the risk constraint for the retailer is inactive under TEF. As K_R increases, the retailer's order quantity remains unchanged. Thus, as K_R increases, the critical risk aversion threshold K_{Sm} remains unchanged.

7. ASYMMETRIC INFORMATION: THE RETAILER'S RISK AVERSION THRESHOLD IS PRIVATE

In the previous sections, we focus on the symmetric information case in which the retailer's risk aversion threshold is common knowledge. However, in practice, the information might be asymmetric, and then, the retailer's risk aversion threshold is unknown to the supplier. If the supplier's risk aversion threshold is unknown to the retailer, then the retailer can infer it from the supplier's wholesale price. Then, the outcomes in equilibrium are the same as in the case of symmetric information. Therefore, in this section, we consider a setting where the capital-constrained retailer's risk aversion threshold is private information.

7.1. Bank credit and equity financing

7.1.1. Retailer's problem

Suppose that K'_R is the retailer's risk aversion threshold disclosed to the supplier. Correspondingly, the supplier adjusts the wholesale price w_{B1} to $w_{B1}(K'_R)$ based on Proposition 4.3.

Lemma 7.1. *The capital-constrained retailer has an incentive to pretend to be less risk averse under BEF.*

Lemma 7.1 shows that the capital-constrained retailer has an incentive to pretend to be less risk averse. If the risk constraints of both K'_R and K_R are active for the retailer, then $w_B^*(K'_R) = w_{B1}(K'_R) < w_B^*(K_R) = w_{B1}(K_R)$. If the risk constraint of K'_R is inactive and K_R is active for the retailer, then $w_B^*(K'_R) = w_{B0} < w_B^*(K_R) = w_{B1}(K_R)$. If the risk constraints of both K'_R and K_R are inactive for the retailer, then $w_B^*(K'_R) = w_B^*(K_R) = w_{B0}$. Thus, the retailer pretends to be less risk averse to induce the supplier to reduce the wholesale price under BEF. From the proof of Lemma 7.1, we know that the retailer's expected profit is decreasing in the wholesale price under BEF. Therefore, pretending to be less risk averse will create a greater expected profit for the capital-constrained retailer under BEF.

7.1.2. Supplier's problem

Lemma 7.1 shows that the retailer has an incentive to pretend that $K'_R > K_R$. This untruthful disclosure benefits the retailer. Can the supplier, as the Stackelberg leader, prevent the retailer's untruthful disclosure? Similar to Wei and Choi [28] and Zhuo *et al.* [45], we design a minimum quantity contract that includes the wholesale price and the minimum quantity according to the risk aversion threshold announced by the retailer. The minimum quantity contract is designed such that the retailer will be worse off if he provides false risk information. Therefore, by setting the appropriate wholesale price and a minimum quantity q_{\min} , the supplier can ensure that the retailer discloses true information. Proposition 7.2 shows how the minimum quantity contract is designed.

Proposition 7.2. (i) Under BEF, (i) if $w_{B0} < w_{B1}(K'_R)$, then $q_{B\min} = q_{R,SPB}(w_{B1}(K'_R), K'_R)$, and $w_B^*(K'_R) = w_{B1}(K'_R)$; (ii) if $w_{B1}(K'_R) < w_{B0}$, then $q_{B\min} = q_{R,EPB}(w_{B0})$, and $w_B^*(K'_R) = w_{B0}$.

As discussed above, the supplier is risk-free under BEF. We therefore only consider the impact of the retailer's risk aversion threshold. If the risk constraints of both K'_R and K_R are active for the retailer, then $w_B^*(K'_R) = w_{B1}(K'_R) < w_B^*(K_R) = w_{B1}(K_R)$. The supplier's estimated order quantity is $q_{R,MVB}(w_B^*(K'_R), K'_R) = q_{R,SPB}(w_{B1}(K'_R), K'_R)$. Based on $w_B^*(K'_R)$ and the true K_R , the retailer's actual order quantity is $q_{R,MVB}(w_B^*(K'_R), K_R) = q_{R,SPB}(w_{B1}(K'_R), K_R)$. This is because $SP_{RB}(q_{R,SPB}(w_{B1}(K'_R), K'_R)) \geq SP_{RB}(q_{R,SPB}(w_{B1}(K'_R), K_R)) = K_R$. Therefore, the supplier setting the minimum order quantity as $q_{B\min} = q_{R,SPB}(w_{B1}(K'_R), K'_R)$ can prevent the retailer from untruthfully disclosing. If the risk constraint of K'_R is inactive and K_R is active for the retailer, then $w_B^*(K'_R) = w_{B0} < w_B^*(K_R) = w_{B1}(K_R)$. The supplier's estimated order quantity is $q_{R,MVB}(w_B^*(K'_R), K'_R) = q_{R,EPB}(w_{B0})$. Based on $w_B^*(K'_R)$ and the true K_R , the retailer's actual order quantity is $q_{R,MVB}(w_B^*(K'_R), K_R) = q_{R,SPB}(w_{B0}, K_R)$. This is because $SP_{RB}(q_{R,EPB}(w_{B0})) \geq SP_{RB}(q_{R,SPB}(w_{B0}, K_R)) = K_R$. Therefore, the supplier sets the minimum order quantity as $q_{\min} = q_{R,EPB}(w_{B0})$ to prevent the retailer from untruthfully disclosing. If the risk constraints of both K'_R and K_R are inactive for

the retailer, then $w_B^*(K'_R) = w_B^*(K_R) = w_{B0}$. The supplier's optimal wholesale price w_{B0} is independent of the retailer's risk aversion threshold. The retailer's distorted information K'_R has no impact on the supplier's decision. Hence, $q_{\min} = q_{R,EPB}(w_{B0})$.

Table 3 illustrates the supplier's minimum quantity contract. The scenario of $K'_R = K_R$ represents the case in which the retailer discloses the true risk aversion threshold. The scenario of $K'_R > K_R$ represents the case in which the retailer discloses the distorted risk aversion threshold. Compared with case (1a), case (1b) shows that the retailer pretends to be less risk averse ($K'_R = 600 > K_R = 500$). Furthermore, the supplier sets a wholesale price $w_B^*(K'_R) = 90.09$ smaller than $w_B^*(K_R) = 93.7$ based on the true information. This case is equivalent to $w_{B0} < w_{B1}(K'_R) < w_{B1}(K_R)$ in Proposition 7.2 ($w_{B0} = 87.3$). The supplier's estimated order quantity is $q_{R,SPB}(w_{B1}(K'_R), K'_R) = 70.9$. However, based on $w_B^*(K'_R) = 90.0$ and $K_R = 500$, the retailer's actual order quantity is $q_{R,SPB}(w_{B1}(K'_R), K_R) = 65.5$, which benefits the retailer but harms the supplier. To prevent this untruthful disclosure, the supplier sets the minimum order quantity $q_{\min} = q_{R,SPB}(w_{B1}(K'_R), K'_R) = 70.9$. Compared to case (1a), case (1c) is consistent with the case of $w_{B1}(K'_R) < w_{B0} < w_{B1}(K_R)$ in Proposition 7.2. The supplier sets the minimum order quantity $q_{\min} = q_{R,EPB}(w_{B0}) = 73.8$. Compared to case (2a), case (2b) is equivalent to the case of $w_{B1}(K'_R) < w_{B1}(K_R) < w_{B0}$. The supplier's optimal wholesale price w_{B0} is independent of the retailer's risk aversion threshold. Untruthful information K'_R therefore does not affect the supplier's decision. The supplier's estimated order quantity equals the retailer's actual order quantity; *i.e.*, $q_{R,MVB}(K'_R) = q_{R,MVB}(K_R) = q_{R,EPB}(w_{B0}) = 73.8$.

7.2. Trade credit and equity financing

7.2.1. Retailer's problem

Suppose that K'_R is the retailer's risk aversion threshold to be disclosed to the supplier. Correspondingly, the supplier adjusts the wholesale prices $w_{T1}(K_R)$, $w_{Tm}(K_R)$ and $w_{Tn}(K_R)$ to $w_{T1}(K'_R)$, $w_{Tm}(K'_R)$ and $w_{Tn}(K'_R)$ based on Propositions 5.2 and 5.3. Lemma 7.3 presents the retailer's information disclosure decision under TEF.

Lemma 7.3. *The capital-constrained retailer has an incentive to pretend to be less risk averse under TEF.*

Lemma 7.3 shows that the capital-constrained retailer has an incentive to pretend to be less risk averse under TEF. If the retailer discloses K'_R to the supplier. Then, the supplier's estimated order quantity is $q_{R,MVT}(w_T) = \min\{q_{R,EPT}(w_T), q_{R,SPT}(w_T, K'_R)\}$. When $K_S \geq \max\{\text{SP}_{ST}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{ST}(q_{R,MVT}(w_{T1}))\}$, the supplier's risk constraint is inactive. From Proposition 5.3, under TEF, the supplier's wholesale price $w_T^* = \max\{w_{T1}, w_{T0}\}$. This is because $q_{R,SPT}(w_T, K'_R) > q_{R,SPT}(w_T, K_R)$, $w_{T1}(K'_R) < w_{T1}(K_R)$. When $K_S < \max\{\text{SP}_{ST}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{ST}(q_{R,MVT}(w_{T1}))\}$, then $w_T^* = \text{argmax EP}_{ST}(q_{R,MVT}(w_{Tl}), \text{EP}_{ST}(q_{R,MVT}(w_{Tu}))$. Similarly, $w_{Tl}(K'_R) \leq w_{Tl}(K_R)$. Therefore, $w_T^*(K'_R) \leq w_T^*(K_R)$. Thus, the retailer pretends to be less risk averse and can induce the supplier to reduce the wholesale price. From the proof of Proposition 6.1, we know that the retailer's expected profit is decreasing in the wholesale price under TEF. Therefore, pretending to be less risk averse will create a greater expected profit for the capital-constrained retailer under TEF. Thus, the retailer has an incentive to pretend to be less risk averse under TEF.

7.2.2. Supplier's problem

The retailer has an incentive to pretend that $K'_R > K_R$. This untruthful disclosure benefits the retailer under TEF. Similar to the previous analysis under BEF, we design a minimum quantity contract to prevent the retailer from cheating. Proposition 7.4 indicates how to design such a minimum quantity contract.

Proposition 7.4. *Under TEF, (i) when $K_S \geq \max\{\text{SP}_{ST}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{ST}(q_{R,MVT}(w_{T1}))\}$, if $w_{T0} < w_{T1}(K'_R)$, then $q_{\min} = q_{R,SPT}(w_{T1}(K'_R), K'_R)$ and $w_T^*(K'_R) = w_{T1}(K'_R)$; if $w_{T1}(K'_R) < w_{T0}$, then $q_{\min} = q_{R,EPT}(w_{T0})$, and $w_T^*(K'_R) = w_{T0}$; (ii) when $K_S < \max\{\text{SP}_{ST}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{ST}(q_{R,MVT}(w_{T1}))\}$, if $w_T^* = w_{Tl}$, $q_{\min} = q_{R,SPT}(w_{Tl}(K'_R), K'_R)$, and $w_T^*(K'_R) = w_{Tl}(K'_R)$; if $w_T^* = w_{Tu}$, $q_{\min} = q_{R,EPT}(w_{Tu})$ and $w_T^*(K'_R) = w_{Tu}$.*

As discussed above, the supplier would bear the retailer's default risk under TEF. When $K_S \geq \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, the supplier's risk constraint is inactive. If the risk constraints of both K'_R and K_R are active for the retailer, then $w_T^*(K'_R) = w_{T1}(K'_R) < w_T^*(K_R) = w_{T1}(K_R)$. The supplier's estimated order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{SPT}}(w_{T1}(K'_R), K'_R)$. Based on $w_T^*(K'_R)$ and the true K_R , the retailer's actual order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{SPT}}(w_{T1}(K'_R), K_R)$. Then, $\text{SP}_{\text{RT}}(q_{R,\text{SPT}}(w_{T1}(K'_R), K'_R)) \geq \text{SP}_{\text{RT}}(q_{R,\text{SPT}}(w_{T1}(K'_R), K_R)) = K_R$. Therefore, the supplier sets the minimum order quantity as $q_{\min} = q_{R,\text{SPT}}(w_{T1}(K'_R), K'_R)$ to prevent the retailer from untruthfully disclosing. If the risk constraint of K'_R is inactive and K_R is active for the retailer, then $w_T^*(K'_R) = w_{T0} < w_T^*(K_R) = w_{T1}(K_R)$. The supplier's estimated order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{EPT}}(w_{T0})$. Based on $w_T^*(K'_R)$ and the true K_R , the retailer's actual order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{SPT}}(w_{T0}, K_R)$. This is because $\text{SP}_{\text{RT}}(q_{R,\text{EPT}}(w_{T0})) \geq \text{SP}_{\text{RT}}(q_{R,\text{SPT}}(w_{T0}, K_R)) = K_R$. Therefore, the supplier sets the minimum order quantity as $q_{\min} = q_{R,\text{EPT}}(w_{T0})$ can prevent the retailer from untruthfully disclosing. If the risk constraints of both K'_R and K_R are inactive for the retailer, the supplier's optimal wholesale price w_{T0} is independent of the retailer's risk aversion threshold. The retailer's distorted information K'_R has no impact on the supplier's decision. Hence, $q_{\min} = q_{R,\text{EPT}}(w_{T0})$.

When $K_S < \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, the supplier's risk constraint is active. If $w_T^* = w_{Tl}$, then $w_T^*(K'_R) = w_{Tl}(K'_R) \leq w_T^*(K_R) = w_{Tl}(K_R)$. The supplier's estimated order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{SPT}}(w_{Tl}(K'_R), K'_R)$. The retailer's actual order quantity is $q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{SPT}}(w_{Tl}(K'_R), K_R)$. Then, $\text{SP}_{\text{RT}}(q_{R,\text{SPT}}(w_{Tl}(K'_R), K'_R)) \geq \text{SP}_{\text{RT}}(q_{R,\text{SPT}}(w_{Tl}(K'_R), K_R)) \geq K_R$. Therefore, the supplier sets the minimum order quantity as $q_{\min} = q_{R,\text{SPT}}(w_{Tl}(K'_R), K'_R)$ can prevent the retailer from untruthfully disclosing. If $w_T^* = w_{Tu}$, then $w_T^*(K'_R) = w_T^*(K_R) = w_{Tu}$. The supplier's estimated order quantity equals the retailer's actual order quantity; *i.e.*, $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{EPT}}(w_{Tu})$. Hence, $q_{\min} = q_{R,\text{EPT}}(w_{Tu})$.

To gain further insights into the supplier's action under TEF, Table 4 illustrates how to set the minimum quantity contract. Compared to case (1a), case (1b) presents the case in which the retailer pretends to be less risk averse ($K'_R = 600 > K_R = 500$). Then, the supplier sets a wholesale price $w_T^*(K'_R) = 74.9$ smaller than $w_T^*(K_R) = 80.0$ based on the members' true risk aversion thresholds, which is consistent with the case of $w_T^* = w_{Tl}, w_{Tl}(K'_R) < w_{Tl}(K_R)$ in Proposition 7.4. The supplier's estimated order quantity is $q_{R,\text{SPT}}(w_{Tl}(K'_R), K'_R) = 67.9$. However, based on $w_T^*(K'_R) = 74.9$ and $K_R = 500$, the retailer's actual order quantity is $q_{R,\text{SPT}}(w_{Tl}(K'_R), K_R) = 62.9$. Obviously, this order quantity benefits the retailer but harms the supplier. To prevent the retailer from untruthfully disclosing, the supplier sets the minimum order quantity $q_{\min} = q_{R,\text{SPT}}(w_{Tl}(K'_R), K'_R) = 67.9$. Compared to case (2a), case (2b) is consistent with the case of $w_T^*(K_R) = w_T^*(K'_R) = w_{Tu}$ in Proposition 7.4. The supplier's optimal wholesale price w_{Tu} depends on her risk aversion threshold and is independent of the retailer's risk aversion threshold. The distorted risk aversion threshold K'_R therefore does not affect the supplier's decision. The supplier's estimated order quantity equals the retailer's actual order quantity; *i.e.*, $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{EPT}}(w_{Tu}) = 68.6$. Compared to case (3a), case (3b) is consistent with the case of $w_{T1}(K'_R) < w_{T1}(K_R) < w_{T0}$ in Proposition 7.4. In this case, $w_T^*(K'_R) = w_T^*(K_R) = w_{T0} = 89.3$, the supplier's optimal wholesale price w_{T0} is independent of the risk aversion threshold. The supplier's estimated order quantity equals the retailer's actual order quantity; *i.e.*, $q_{R,\text{MVT}}(w_T^*(K'_R), K'_R) = q_{R,\text{MVT}}(w_T^*(K'_R), K_R) = q_{R,\text{EPT}}(w_{T0}) = 74.3$. Compared to case (4a), case (4b) is consistent with the case of $w_{T0} < w_{T1}(K'_R) < w_{T1}(K_R)$ in Proposition 7.4. Then, the supplier sets the minimum order quantity $q_{\min} = q_{R,\text{SPT}}(w_{T1}(K'_R), K'_R) = 71.5$. Compared to case (4a), case (4c) is consistent with the case of $w_{T1}(K'_R) < w_{T0} < w_{T1}(K_R)$ in Proposition 7.4. Then, the supplier sets the minimum order quantity $q_{\min} = q_{R,\text{EPT}}(w_{T0}) = 74.3$.

8. BANK LOAN-TRADE CREDIT-EQUITY FINANCING

In this section, we assume that the retailer borrows from the bank and the supplier and seeks financial support from investors. Before the selling season, the supplier sets the wholesale price w_{BT} . According to the wholesale

TABLE 3. The optimal w_B^* and $q_{R,MVB}$ for different K_R under BEF when the retailer's risk aversion threshold is private information.

	K_R	K'_R	$w_{B1}(K'_R)(K_R)$	$w_B^*(K'_R)(K_R)$	$q_{R,MVB}(K'_R)(K_R)$	q_{\min}	EP_{SB}	EP_{RB}	SP_{RB}
(1a)	500	500	93.7(93.7)	93.7(93.7)	66.5(66.5)	66.5	5387.8	2274.4	500.0
(1b)	500	600	90.0(93.7)	90.0(93.7)	70.9(65.5)	70.9	5052.2	2477.7	500.0
(1c)	500	1000	75.1(93.7)	87.3(93.7)	73.8(64.8)	73.8	4823.4	2621.1	500.0
(2a)	1000	1000	75.1(75.1)	87.3(87.3)	73.8(73.8)	73.8	5495.1	2651.1	672.5
(2b)	1000	1200	67.5(75.1)	87.3(87.3)	73.8(73.8)	73.8	5495.1	2651.1	672.5

TABLE 4. The optimal w_T^* and $q_{R,MVT}$ for different K_S and K_R under TEF when the retailer's risk aversion threshold is private information.

	K_S	K_R	K'_R	$w_{T1}(K'_R)(K_R)$	$w_{T_i}^*(K'_R)(K_R)$	$w_T^*(K'_R)(K_R)$	$q_{R,MVT}(K'_R)(K_R)$	q_{\min}	EP_{ST}	EP_{RT}	SP_{RT}	SP_{ST}
(1a)	100	500	500	95.5(95.5)	80.0(80.0)	80.0(80.0)	63.5(63.5)	63.5	4245.1	3005.9	500.0	100.0
(1b)	100	500	600	92.0(95.5)	74.9(80.0)	74.9(80.0)	67.9(62.9)	67.9	3873.7	3264.4	500.0	77.6
(2a)	220	1000	1000	76.8(76.8)	94.4(94.4)	94.4(94.4)	68.6(68.6)	68.6	5577.8	2260.3	532.3	220.0
(2b)	220	1000	1200	68.6(76.8)	94.4(94.4)	94.4(94.4)	68.6(68.6)	68.6	5577.8	2260.3	532.3	220.0
(3a)	250	1000	1000	76.8(76.8)	—(—)	89.3(83.9)	74.3(74.3)	74.3	5654.9	2556.4	673.3	238.8
(3b)	250	1000	1200	68.6(76.8)	—(—)	89.3(89.3)	74.3(74.3)	74.3	5654.7	2556.4	673.3	238.8
(4a)	250	500	500	95.5(95.5)	—(—)	95.5(95.5)	67.0(67.0)	67.0	5530.9	2199.6	500.0	212.4
(4b)	250	500	600	92.0(95.5)	—(—)	92.0(95.5)	71.5(65.9)	71.5	5207.8	2387.8	500.0	178.3
(4c)	250	500	800	84.5(95.5)	—(—)	89.3(95.5)	74.3(65.2)	74.3	4974.8	2529.9	500.0	156.3

Notes. The symbols a, b and c represent the retailer's true and fake information disclosure, respectively. The symbol — indicates that it does not exist.

price, the retailer determines the order quantity q_{BT} . The retailer's initial capital B cannot cover his order, and he finances $\phi(w_{BT}q_{BT} - B)$ from investors, $\theta(w_{BT}q_{BT} - B)$ from the bank, and $(1 - \phi - \theta)(w_{BT}q_{BT} - B)$ from the supplier. The interest rate charged by the bank is r_B . Fully competitive banks earn only the risk-free interest rate r_f . We assume that the retailer's bank loan is senior to trade credit [21, 25, 36]. Let k_{B0} and k_{BT} denote the bank loan default threshold and trade credit default threshold, respectively. Then, we have $k_{B0} = \frac{\theta(w_{BT}q_{BT} - B)(1 + r_B)}{p}$ and $k_{BT} = \frac{(\theta(w_{BT}q_{BT} - B)(1 + r_B) + (1 - \phi - \theta)(w_{BT}q_{BT} - B)(1 + r_T))}{p}$. At the end of the sales season, the retailer obtains sales income $p \cdot \min\{q_{BT}, x\}$. If $p \cdot \min\{q_{BT}, x\} < pk_{B0}$, then the retailer pays all sales income to the bank and goes bankrupt. If $pk_{B0} < p \cdot \min\{q_{BT}, x\} < pk_{BT}$, then the retailer first pays pk_{B0} to the bank and then pays the remaining sales income $p \cdot \min\{q_{BT}, x\} - pk_{B0}$ to the supplier. If $p \cdot \min\{q_{BT}, x\} > pk_{BT}$, the retailer first pays pk_{B0} to the bank and $pk_{BT} - pk_{B0}$ to the supplier and then transfers $\phi(p \cdot \min\{q_{BT}, x\} - pk_{BT})$ to investors. Thus, the retailer's profit is $P_{RB} = [p \cdot \min\{q_{BT}, x\} - pk_{BT}]^+$. Lemma 8.1 characterizes the variances of the members' profits under the BTEF.

Lemma 8.1. *Under BTEF, the variances of members' profits are given by $V_{BBT} = p^2 \left(2 \int_0^{k_{BT}} (k_{BT} - x)F(x) dx - \left(\int_0^{k_{BT}} F(x) dx \right)^2 \right)$, $V_{IBT} = (\phi p)^2 \left(2 \int_{k_{BT}}^{q_{BT}} (q_{BT} - x)F(x) dx - \left(\int_{k_{BT}}^{q_{BT}} F(x) dx \right)^2 \right)$, $V_{RBT} = ((1 - \phi)p)^2 \left(2 \int_{k_{BT}}^{q_{BT}} (q_{BT} - x)F(x) dx - \left(\int_{k_{BT}}^{q_{BT}} F(x) dx \right)^2 \right)$, $V_{SBT} = p^2 \left(2 \int_{k_{B0}}^{k_{BT}} (k_{BT} - x)F(x) dx - \left(\int_{k_{B0}}^{k_{BT}} F(x) dx \right)^2 \right)$.*

Lemma 8.1 shows that under BTEF, the supplier, retailer, bank and investors share supply chain risk. When $x < k_{B0}$, the retailer's sales income cannot cover his bank credit, and he goes bankrupt. Since bank loan is senior to trade credit, the retailer only pays all sales income to the bank. Correspondingly, the bank, the supplier and

investors suffer losses. When $k_{B0} < x < k_{BT}$, the retailer's sales income cannot cover his trade credit, and he goes bankrupt. The retailer first pays pk_{B0} to the bank and then pays the remaining sales income to the supplier. In this case, the bank has no loss. Correspondingly, the supplier and investors suffer losses. When $k_{BT} < x < q_{BT}$, the retailer's sales income is sufficient to repay his loans. Thus, both the bank and the supplier have no loss. After payment of the loans, then the retailer transfers a fraction ϕ of his profits to investors. Compared with BEF and TEF, more members share supply chain risks under BTEF, which reduces the retailers risk.

8.1. Retailer's decision

Before the selling season, the bank provides $\theta(w_{BT}q_{BT} - B)$ to the retailer. At the end of the selling season, the bank receives $\min\{p \cdot \min\{q_{BT}, x\}, \theta(w_{BT}q_{BT} - B)(1 + r_B)\}$. In a fully competitive market, the bank earns only the risk-free interest rate r_f . Then, we have $\theta(w_{BT}q_{BT} - B)(1 + r_f) = E[\min\{p \cdot \min\{q_{BT}, x\}, \theta(w_{BT}q_{BT} - B)(1 + r_B)\}]$. Thus, the bank's interest rate r_B satisfies $\int_0^{k_{BT}} p\bar{F}(x) dx - \theta(w_{BT}q_{BT} - B)(1 + r_f) = 0$. Then, the retailer's expected profit is

$$EP_{RBT} = (1 - \phi)E[p \cdot \min\{q_{BT}, x\} - pk_{BT}]^+. \quad (8.1)$$

Under the mean-variance framework, the retailer's problem is

$$\begin{aligned} & \max_{q_{BT}} EP_{RBT}(q_{BT}) \\ & \text{s.t. } SP_{RBT}(q_{BT}) \leq K_R, \end{aligned} \quad (P5)$$

where $SP_{RBT} = \sqrt{V_{RBT}}$, is the standard deviation of the retailer's profit. We define $q_{R,SPBT}(w_T) = \operatorname{argmax}_{q_{BT}} \{SP_{RBT}(q_T) \leq K_R\}$, which gives the retailer's maximum quantity that satisfies $SP_{RBT}(q_{BT}) \leq K_R$.

Proposition 8.2. *Under BTEF, (i) the retailer's optimal order quantity, $q_{R,MVBT}(w_{BT}) = \min\{q_{R,EPBT}(w_{BT}), q_{R,SPBT}(w_{BT})\}$, where $q_{R,EPBT}(w_{BT}) = \bar{F}^{-1}\left(\frac{\theta w_{BT}(1+r_f)}{\bar{F}(k_{B0}) + \frac{(1-\phi-\theta)w_{BT}(1+r_T)}{p}}\bar{F}(k_{BT})\right)$, and (ii) $q_{R,MVBT}(w_T)$ is increasing in w_{BT} for $w_{BT} \in (c, w_{BT1})$ and decreasing in w_{BT} for $w_{BT} \in [w_{BT1}, p]$, where w_{BT1} satisfies $w_{BT1} = \frac{p\bar{F}(q_{R,SPBT}(w_{BT1}))}{\frac{\theta(1+r_f)}{\bar{F}(k_{B0})} + (1-\phi-\theta)(1+r_T)p\bar{F}(k_{BT})}$.*

Proposition 8.2 shows that the retailer's optimal order quantity is $\min\{q_{R,EPBT}(w_{BT}), q_{R,SPBT}(w_{BT})\}$ under BTEF. When the retailer's risk constraint is active, the retailer orders $q_{R,MVBT}(w_{BT}) = q_{R,SPBT}(w_{BT})$. When the retailer's risk constraint is inactive, the retailer orders $q_{R,MVBT}(w_{BT}) = q_{R,EPBT}(w_{BT})$. When $w_{BT} \in (c, w_{BT1})$, the retailer's order quantity $q_{R,MVBT}(w_{BT}) = q_{R,SPBT}(w_{BT})$ is increasing in the wholesale price. The supplier raises the wholesale price, and the retailer will borrow more from the bank, the supplier and investors. Thus, the bank, the supplier and investors will take more risk. Thus, the retailer's risk decreases. Hence, for a given K_R , the retailer has an incentive to order more products. When $w_{BT} \in [w_{BT1}, \frac{p}{1+r_T}]$, the retailer's order quantity $q_{R,MVBT}(w_{BT}) = q_{R,EPBT}(w_{BT})$ is decreasing in the wholesale price.

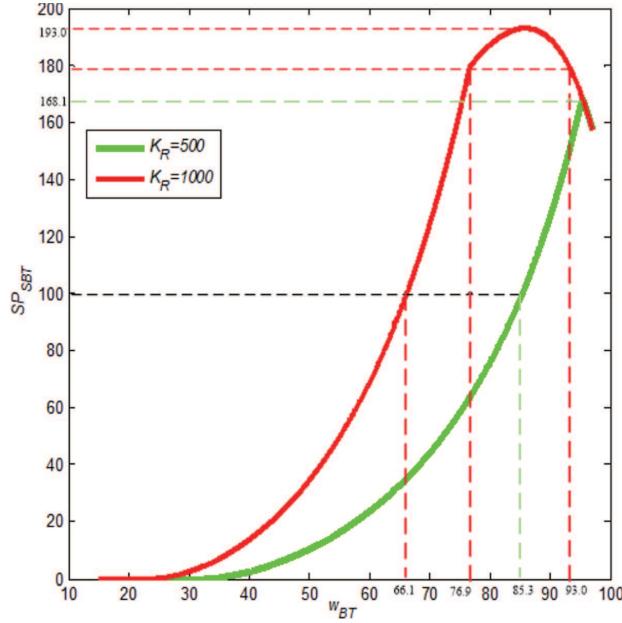
8.2. Supplier's decision

From the above analysis, the supplier's expected profit is

$$EP_{SBT} = E[p \cdot \min\{\max\{0, x - k_{B0}\}, k_{BT} - k_{B0}\}] + ((\theta + \phi)(w_{BT}q_{BT} - B) + B - cq_T)(1 + r_f). \quad (8.2)$$

Under BTEF, the retailer's optimal order quantity is $q_{R,MVBT}(w_{BT}) = \min\{q_{R,EPBT}(w_{BT}), q_{R,SPBT}(w_{BT})\}$. Therefore, under the mean-variance framework, the supplier's problem is

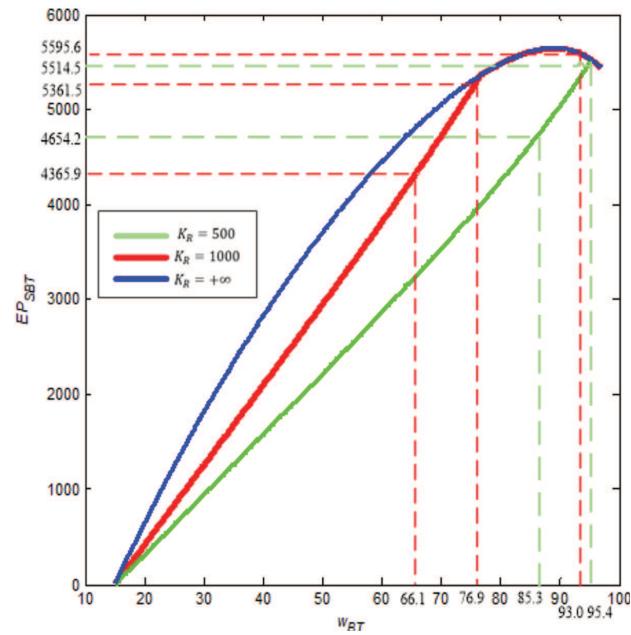
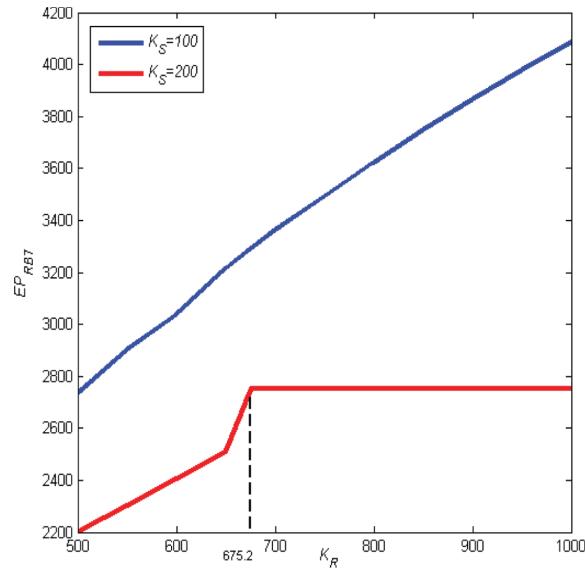
$$\begin{aligned} & \max_{w_{BT}} EP_{SBT}(q_{R,MVBT}(w_{BT})) \\ & \text{s.t. } SP_{SBT}(q_{R,MVBT}(w_{BT})) \leq K_S, \end{aligned} \quad (P6)$$

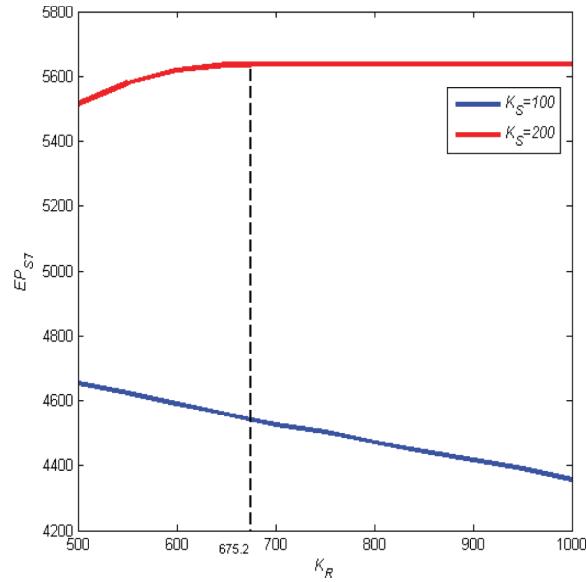
FIGURE 9. SP_{SBT} changes with w_{BT} .

where $K_S \geq 0$ is the supplier's risk aversion threshold. We next explore the supplier's optimal wholesale price under risk constraints. It is difficult to obtain analytical results for the supplier's wholesale price due to the complexity of the retailer's order quantity and the supplier's profit functions. Thus, based on the settings in Figure 1 and $\theta = 0.5$, we use numerical examples to examine the supplier's wholesale price and expected profit under BTEF.

Figures 9 and 10 present the supplier's standard deviation and expected profit change with w_{BT} under BTEF. We find that the supplier's standard deviation and expected profit are concave in w_{BT} . When $K_S < 168.1(193.0)$, the supplier's risk constraint is active under $K_R = 500(1000)$. When $K_S > 168.1(193.0)$, the supplier's risk constraint is inactive under $K_R = 500(1000)$. When $K_S = 100$, the supplier will set wholesale price $w_{BT}^* = 85.3(66.1)$ under $K_R = 500(1000)$ to control risk. Then, the supplier's expected profit is $EP_{SBT}(q_{R,MVBT}(85.3)) = 4654.2$ under $K_R = 500$ and $EP_{SBT}(q_{R,MVBT}(66.1)) = 4365.9$ under $K_R = 1000$. When $K_S = 180$ and $K_R = 500$, the supplier's risk constraint is inactive, and the retailer's risk constraint is active. Thus, the supplier's optimal wholesale price $w_{BT}^* = w_{BT1} = 95.4$ is independent of K_S and dependent on K_R . Correspondingly, the supplier's expected profit $EP_{SBT}(q_{R,MVBT}(95.4)) = 5514.5$. When $K_S = 180$ and $K_R = 1000$, the supplier's risk constraint is active, and the retailer's risk constraint is inactive. Then, the supplier will set a wholesale price of 76.9 or 93.0 to avoid risk. Since $EP_{SBT}(q_{R,MVT}(76.9)) = 5361.5 < EP_{SBT}(q_{R,MVT}(93.0)) = 5595.6$, the supplier's optimal wholesale price $w_{BT}^* = 93.0$.

Next, we analyze the effects of supply chain members' risk aversion threshold on their profits. Figures 11 and 12 present that the profit of the retailer and that of the supplier change with K_R under BTEF. When $K_S = 100$, the retailer's profit increases with K_R , while the supplier's profit decreases with K_R . In this case, the supplier's risk aversion threshold is small, and her risk constraint is active. As K_R increases, the retailer will raise the order quantity. Then, the supplier will reduce the wholesale price to avoid risk. Thus, the retailer's profit increases with K_R , and the supplier's profit decreases with K_R . When $K_S = 200$, the profit of both the retailer and the supplier does not decrease with K_R . In this case, the supplier's risk aversion threshold is large, and her risk constraint is inactive. When $K_R < 675.2$, the retailer's risk constraint is active. As K_R increases, the retailer will raise the order quantity. Thus, both the retailer's and the supplier's profit increase with K_R . When $K_R \geq 675.2$,

FIGURE 10. EP_{SBT} changes with w_{BT} .FIGURE 11. EP_{RBT} changes with K_R .

FIGURE 12. EP_{SBT} changes with K_R .

the retailer's risk constraint is inactive. The retailer's order quantity and the supplier's wholesale price are independent of K_R . Thus, both the retailer's profit and the supplier's profit remain unchanged.

9. CONCLUSION AND MANAGERIAL INSIGHTS

In this paper, we consider a simple supply chain consisting of a supplier and a capital-constrained retailer. The capital-constrained retailer adopts one of two mixed financing modes (BEF and TEF) to order. Under the mean-variance framework, we investigate how supply chain members' risk-averse attitudes affect their financing and ordering decisions under two mixed financing modes.

Our main results are as follows. First, we obtain the conditions under which both the supplier and the retailer prefer TEF or BEF. When the retailer's risk constraint is active, only when the supplier's risk aversion threshold is moderate is TEF the financing equilibrium. Otherwise, BEF is the financing equilibrium. When the retailer's risk constraint is inactive, only when the supplier's risk aversion threshold exceeds a critical risk aversion threshold is TEF the financing equilibrium. Otherwise, BEF is the financing equilibrium. The results are different from the results in Kouvelis and Zhao [15] in that the supplier is always willing to provide trade credit and the retailer's financing preference depends on his initial capital. Yang *et al.* [40] show that TCF is a unique financing equilibrium only when the supplier's risk aversion threshold is moderate under single debt financing modes. Second, we characterize the risk sharing between the supplier and the retailer under mixed financing. Compared with the single debt financing of Yang *et al.* [40], the risks of both the supplier and the retailer will be reduced under mixed financing since investors share partial risk through equity financing. Third, when the retailer's risk aversion threshold is private, the retailer has an incentive to pretend to be less risk averse under these two mixed financing modes. Minimum quantity contracts efficiently prevent the retailer's untruthful disclosure.

Our paper derived some managerial implications. The supplier with risk constraints sets the appropriate wholesale price to induce the retailer to choose the mixed financing mode that benefits her. When the retailer's risk constraint is active and the supplier's risk aversion threshold is moderate, the supplier charges a low wholesale price and provides trade credit to induce the retailer to choose TEF. When the retailer's risk constraint

is inactive and the supplier's risk aversion threshold is relatively high, the supplier charges the appropriate wholesale price and provides trade credit to induce the retailer to choose TEF. The retailer can adopt mixed financing, including equity financing, to reduce his risk.

In this paper, we use a mean-variance framework to explore how supply chain members' risk-averse attitudes affect their financing and ordering decisions under two mixed financing modes. In our model, we assume that a two-echelon supply chain consists of a supplier and a capital-constrained retailer. However, when a supply chain consists of a capital-constrained retailer and multiple suppliers, especially when one supplier provides trade credit to the capital constrained retailer, there are spillovers when the capital constrained retailer order products from other suppliers. Therefore, future research should consider the impact of the competition between suppliers. Second, in our model, both the supplier and the capital-constrained retailer have a good knowledge of the distribution function of random market demand. However, in practice, compared with the upstream supplier, the retailer is endowed with superior information about market demand. It may be interesting to explore the impact of demand information asymmetry on supply chain members financing and ordering decisions. Third, in our model, the retailer's equity financing ratio is exogenous; however, the retailer's equity financing ratio is the most important decision when there is a financial gap. It would be interesting to explore the endogeneity of the equity financing ratio.

APPENDIX A. PROOF OF LEMMA 4.1

Let $k_B = \frac{(1-\phi)(w_B q_B - B)(1+r_B)}{p}$. Based on the definition of variance,

$$\begin{aligned} V_{RB} &= \text{Var}\left((1-\phi)p[\min\{q_B, x\} - k_B]^+\right) - p(q_B - x)^+ \\ &= (1-\phi)^2 p^2 \text{Var}\left((q_B - x)^+\right) + \text{Var}\left((k_B - x)^+\right) - 2\text{Cov}\left((k_B - x)^+, (q_B - x)^+\right), \end{aligned}$$

where $\text{Var}(k_B - x)^+ = \left(2 \int_0^{k_B} (k_B - x)F(x) dx - \left(\int_0^{k_B} F(x) dx\right)^2\right)$ and $\text{Var}((q_B - x)^+) = \left[E((q_B - x)^2) - [E(q_B - x)]^2\right] = \left(2 \int_0^{q_B} (q_B - x)F(x) dx - \left(\int_0^{q_B} F(x) dx\right)^2\right)$.

Based on the definition of covariance,

$$\begin{aligned} \text{Cov}\left((k_B - x)^+, (q_B - x)^+\right) &= E\left[(q_B - x)^+(k_B - x)^+\right] - E\left[(q_B - x)^+\right]E\left[(k_B - x)^+\right] \\ &= \int_0^{k_B} (q_B - x)(k_B - x)f(x) dx - \int_0^{q_B} (q_B - x)f(x) dx \int_0^{k_B} (k_B - x)f(x) dx \\ &= k_B q_B F(k_B) - (q_B + k_B) \int_0^{k_B} x f(x) dx + \int_0^{k_B} x^2 f(x) dx \\ &\quad - \int_0^{q_B} F(x) dx \int_0^{k_B} F(x) dx \\ &= - \int_0^{k_B} F(x) dx - 2 \int_0^{k_B} x F(x) dx - \int_0^{q_B} F(x) dx \int_0^{k_B} F(x) dx \\ &= (q_B + k_B) \int_0^{k_B} F(x) dx - 2 \int_0^{k_B} x F(x) dx - \int_0^{q_B} F(x) dx \int_0^{k_B} F(x) dx \end{aligned}$$

Therefore,

$$V_{RB} = (1-\phi)^2 p^2 \left\{ 2 \int_0^{q_B} (q_B - x)F(x) dx - \left(\int_0^{q_B} F(x) dx\right)^2 + 2 \int_0^{k_B} (k_B - x)F(x) dx \right\}$$

$$\begin{aligned}
& - \left(\int_0^{k_B} F(x) dx \right)^2 - 2(q_B + k_B) \int_0^{k_B} F(x) dx + 4 \int_0^{k_B} x F(x) dx \\
& + 2 \int_0^{q_B} F(x) dx \int_0^{k_B} F(x) dx \Big\} \\
& = (1 - \phi)^2 p^2 \left\{ 2 \int_{k_B}^{q_B} (q_B - x) F(x) dx - \left(\int_{k_B}^{q_B} F(x) dx \right)^2 \right\}.
\end{aligned}$$

Similarly, we have $V_{BB} = p^2 \left(2 \int_0^{k_B(q_B)} (k_B(w_B, q_B) - x) F(x) dx - \left(\int_0^{k_B(q_B)} F(x) dx \right)^2 \right)$, $V_{IB} = (\phi p)^2 \left(2 \int_{k_B(q_B)}^{q_B} (q_B - x) F(x) dx - \left(\int_{k_B(q_B)}^{q_B} F(x) dx \right)^2 \right)$.

Since the supplier without any risk under BEF, thus, $V_{SB} = 0$.

APPENDIX B. PROOF OF PROPOSITION 4.2

The first derivative of EP_{RB} with respect to q_B yields $\frac{\partial EP_{RB}}{\partial q_B} = (1 - \phi)p \left(\bar{F} \left(\frac{(1-\phi)w_B(1+r_f)}{p} \right) \right)$. The second-order condition of EP_{RB} with respect to q_B yields $\frac{\partial^2 EP_{RB}}{\partial q_B^2} = -(1 - \phi)p f(q_B) < 0$. EP_{RB} is concave. From $\frac{\partial EP_{RB}}{\partial q_B} = 0$, $q_{R,EPB}(w_B) = \bar{F}^{-1} \left(\frac{(1-\phi)w_B(1+r_f)}{p} \right)$. The first derivative of $q_{R,EPB}(w_B)$ with respect to w_B yields $\frac{\partial q_{R,EPB}(w_B)}{\partial w_B} = \frac{(1-\phi)(1+r_f)}{-p f(q_B)} < 0$. Hence, $q_{R,EPB}(w_B)$ is decreasing in w_B .

The first derivative of V_{RB} with respect to q_B yields $\frac{\partial V_{RB}}{\partial q_B} = 2(1 - \phi)^2 p^2 \left(\bar{F}(q_B) \int_{k_B}^{q_B} F(x) dx - \frac{\partial k_B}{\partial q_B} F(k_B) \int_{k_B}^{q_B} \bar{F}(x) dx \right)$. Since $(1 - \phi)(w_B q_B - B)(1 + r_f) = E[\min\{p \cdot \min\{q_B, x\}, (1 - \phi)(w_B q_B - B)(1 + r_B)\}]$, then $\frac{\partial k_B}{\partial q_B} = \frac{(1-\phi)w_B(1+r_f)}{p \bar{F}(k_B)}$. Thus,

$$\begin{aligned}
\frac{\partial V_{RB}}{\partial q_B} & = 2(1 - \phi)^2 p^2 \left(\bar{F}(q_B) \int_{k_B}^{q_B} F(x) dx - \frac{(1 - \phi)w_B(1 + r_f)}{p \bar{F}(k_B)} F(k_B) \int_{k_B}^{q_B} \bar{F}(x) dx \right) \\
& \geq 2(1 - \phi)^2 p^2 \left(\bar{F}(q_B) \int_{k_B}^{q_B} F(x) dx - \frac{(1 - \phi)w_B(1 + r_f)}{p} F(k_B)(q_B - k_B) \right).
\end{aligned}$$

Since the retailer's order quantity with risk constraints is not more than the optimal order quantity without risk constraint, i.e., $q_B q_{R,EPB}(w_B)$. Thus, $\bar{F}(q_B) \geq \frac{(1-\phi)w_B(1+r_f)}{p}$. Hence,

$$\frac{\partial V_{RB}}{\partial q_B} \geq 2(1 - \phi)^3 p w_B (1 + r_f) \left(\int_{k_B}^{q_B} (F(x) - F(k_B)) dx \right) > 0.$$

Since $SP_{RB} = \sqrt{V_{RB}}$, $SP_{RB}(q_B)$ is increasing in q_B . Since $q_{R,SPB}(w_B) = \operatorname{argmax}_{q_B} \{SP_{RB}(q_B) \leq K_R\}$, $((1 - \phi)p)^2 \left(2 \int_{k_B}^{q_{R,SPB}(w_B)} (q_{R,SPB}(w_B) - x) F(x) dx - \left(\int_{k_B}^{q_{R,SPB}(w_B)} F(x) dx \right)^2 \right) = K_R^2$ for given K_R . The first-order condition of $q_{R,SPB}(w_B)$ with respect to w_B yields

$$\bar{F}(q_{R,SPB}(w_B)) \int_{k_B}^{q_{R,SPB}(w_B)} F(x) dx \frac{\partial q_{R,SPB}(w_B)}{\partial w_B} - F(k_B) \int_{k_B}^{q_{R,SPB}(w_B)} \bar{F}(x) dx \frac{\partial k_B}{\partial w_B} = 0.$$

Since $(1 - \phi)(w_B q_B - B)(1 + r_f) = E[\min\{p \cdot \min\{q_B, x\}, (1 - \phi)(w_B q_B - B)(1 + r_B)\}]$, then The first-order condition of $\frac{w_B q_{R,SPB}(w_B)(1+r_B)}{p}$ with respect to w_B yields

$$p \bar{F}(k_B) \frac{\partial k_B}{\partial w_B} = (1 - \phi)(1 + r_f) \left(q_{R,SPB}(w_B) + (w_B) \frac{\partial q_{R,SPB}(w_B)}{\partial w_B} \right).$$

Further,

$$\begin{aligned} \frac{\partial q_{R,SPB}(w_B)}{\partial w_B} &= \frac{\frac{(1-\phi)q_{R,SPB}(w_B)(1+r_f)}{p}F(k_B)\int_{k_B}^{q_{R,SPB}(w_B)}\bar{F}(x)dx}{\left\{\bar{F}(q_{R,SPB}(w_B))\int_{k_B}^{q_{R,SPB}(w_B)}F(x)dx - \frac{(1-\phi)w_B(1+r_f)}{p}F(k_B)\int_{k_B}^{q_{R,SPB}(w_B)}\bar{F}(x)dx\right\}} \\ &> 0. \end{aligned}$$

Thus, $\frac{\partial q_{R,SPB}(w_B)}{\partial w_B} \geq 0$.

Let $q_{R,EPB}(w_{B1}) = q_{R,SPB}(w_{B1})$. When $w_B < w_{B1}$, then $q_{R,SPB}(w_B) < q_{R,EPB}(w_B)$ and $SP_R(q_{R,EPB}(w_B)) > K_R$. The optimal order quantity in (P1) is $q_{R,MVB}(w_B) = q_{R,SPB}(w_B) < q_{R,EPB}(w_B)$. Hence, $q_{R,MVB}(w_B)$ is increasing in w_B in the interval (c, w_{B1}) . Similarly, $\frac{\partial V_{RB}}{\partial q_{R,EPB}(w_B)} > 0$. Therefore, $SP_R(q_{R,EPB}(w_B))$ is increasing in $q_{R,EPB}(w_B)$. When $w_B \geq w_{B1}$, then $q_{R,SPB}(w_B) \geq q_{R,EPB}(w_B)$ and $SP_R(q_{R,EPB}(w_B)) \leq K_R$. The optimal order quantity in (P1) is $q_{R,MVB}(w_B) = q_{R,EPB}(w_B)$. Thus, $q_{R,MVB}(w_B)$ is decreasing in w_B . Further, $q_{R,MVB}(w_B) = \min\{q_{R,EPB}(w_B), q_{R,SPB}(w_B)\}$.

APPENDIX C. PROOF OF PROPOSITION 4.3

From the proof of Proposition 4.2, when $w_B \geq w_{B1}$, $SP_{RB}(q_{R,EPB}(w_B)) \leq K_R$. The supplier's profit is $EP_{SB} = (w_B - c)q_{R,MVB}(w_B)(1 + r_f) = (w_B - c)q_{R,EPB}(w_B)(1 + r_f)$. From Proposition 4.2, $q_{R,EPB}(w_B) = \bar{F}^{-1}\left(\frac{(1-\phi)w_B(1+r_f)}{p}\right)$. Further, $w_B = \frac{p\bar{F}(q_{R,EPB})}{(1-\phi)(1+r_f)}$. Therefore, $EP_{SB} = \left(\frac{p\bar{F}(q_{R,EPB})}{(1-\phi)(1+r_f)} - c\right)q_{R,EPB}(1 + r_f)$. Thus, the supplier's problem is equivalent to choosing $q_{R,EPB}$. The first derivative of EP_{SB} with respect to $q_{R,EPB}$ yields $\frac{\partial EP_{SB}}{\partial q_{R,EPB}} = \left(\frac{p\bar{F}(q_{R,EPB}) - pf(q_{R,EPB})q_{R,EPB}}{(1-\phi)(1+r_f)} - c\right)(1 + r_f) = \left(p\bar{F}(q_{R,EPB})\frac{(1-q_{R,EPB}z(q_{R,EPB}))}{(1-\phi)(1+r_f)} - c\right)(1 + r_f)$. Let \tilde{q} solve $qz(q) = 1$. Assume $z(q)$ is the increasing failure rate. Then, $qz(q)$ is increasing in q . When $q_{R,EPB} > \tilde{q}$, then $1 - q_{R,EPB}z(q_{R,EPB}) < 0$. Further, $\frac{\partial EP_{SB}}{\partial q_{R,EPB}} < 0$. Hence, the supplier chooses $q_{R,EPB} \leq \tilde{q}$. When $q_{R,EPB} \leq \tilde{q}$, $1 - q_{R,EPB}z(q_{R,EPB}) \geq 0$. Further, $\frac{\partial^2 EP_{SB}}{\partial q_{R,EPB}^2} = -pf(q_{R,EPB})\frac{(1-q_{R,EPB}z(q_{R,EPB}))}{(1-\phi)(1+r_f)} - p\bar{F}(q_{R,EPB})\frac{\left(z(q_{R,EPB}) + q\frac{\partial z(q_{R,EPB})}{\partial q_{R,EPB}}\right)}{(1-\phi)(1+r_f)} < 0$. From $\frac{\partial EP_{SB}}{\partial q_{R,EPB}} = 0$, $p\bar{F}(q_{R,EPB})(1 - q_{R,EPB}z(q_{R,EPB})) - (1 - \phi)c(1 + r_f) = 0$. The optimal wholesale price $w_B^* = w_{B0} = \frac{p\bar{F}(q_{R,EPB}(w_{B0}))}{(1-\phi)(1+r_f)}$.

When $w_B \leq w_{B1}$, $q_{R,MVB}(w_B) = q_{R,SPB}(w_B)$. The first-order condition of EP_{SB} with respect to w_B yields $\frac{\partial EP_{SB}}{\partial w_B} = \left(q_{R,SPB}(w_B) + (w_B - c)\frac{\partial q_{R,SPB}(w_B)}{\partial w_B}\right)(1 + r_f)$. Since $\frac{\partial q_{R,SPB}(w_B)}{\partial w_B} > 0$, $\frac{\partial EP_{SB}}{\partial w_B} > 0$. EP_{SB} is increasing in w_B . Hence, $w_B^* = w_{B1}$. Further, we have $w_B^* = \max\{w_{B0}, w_{B1}\}$.

APPENDIX D. PROOF OF LEMMA 5.1

For concision, let $k_T = \frac{(1-\phi)(w_T q_T - B)}{p}$. Based on the definition of variance,

$$\begin{aligned} V_{RT} &= \text{Var}((1 - \phi)(p - k_T)(q_T - k_T - (q_T - x)^+ + (\rho - x)^+)) \\ &= (1 - \phi)^2 p^2 (\text{Var}(q_T - x)^+ + \text{Var}((k_T - x)^+) - 2\text{Cov}((k_T - x)^+, (q_T - x)^+)), \end{aligned}$$

where $\text{Var}((k_T - x)^+) = 2 \int_0^{k_T} (k_T - x)F(x)dx - \left(\int_0^{k_T} F(x)dx\right)^2$ and $\text{Var}((q_T - x)^+) = [E((q_T - x)^2) - E(q_T - x)^2]^2 = 2 \int_0^{q_T} (q_T - x)F(x)dx - \left(\int_0^{q_T} F(x)dx\right)^2$.

Based on the definition of covariance,

$$\text{Cov}((k_T - x)^+, (q_T - x)^+) = E((q_T - x)^+) - E((q_T - x)^+)E((k_T - x)^+)$$

$$\begin{aligned}
&= \int_0^{k_T} (q_T - x)(k_T - x)f(x) dx - \int_0^{q_T} (q_T - x)(k_T - x)f(x) dx \int_0^{k_T} (k_T - x)f(x) dx \\
&= \frac{w_T q_T^2}{p} F(k_T) - (q_T + k_T) \int_0^{k_T} x f(x) dx + \int_0^{k_T} x^2 f(x) dx - \int_0^{q_T} F(x) dx \int_0^{k_T} F(x) dx \\
&= \frac{w_T q_T^2}{p} F(k_T) - (q_T + k_T) \left(k_T F(k_T) - \int_0^{k_T} F(x) dx \right) + k_T^2 F(k_T) - 2 \int_0^{k_T} x F(x) dx \\
&\quad - \int_0^{q_T} F(x) dx \int_0^{k_T} F(x) dx \\
&= (q_T + k_T) \int_0^{k_T} F(x) dx - 2 \int_0^{k_T} x F(x) dx - \int_0^{q_T} F(x) dx \int_0^{k_T} F(x) dx.
\end{aligned}$$

Therefore,

$$\begin{aligned}
V_{RT} &= (1 - \phi)^2 P^2 \left\{ \left(2 \int_0^{q_T} (q_T - x) F(x) dx - \left(\int_0^{q_T} F(x) dx \right)^2 \right) + 2 \int_0^{k_T} (k_T - x) F(x) dx \right. \\
&\quad \left. - \left(\int_0^{k_T} F(x) dx \right)^2 - 2(q_T + k_T) \int_0^{k_T} F(x) dx + 4 \int_0^{k_T} x F(x) dx \right. \\
&\quad \left. + 2 \int_0^{q_T} F(x) dx \int_0^{k_T} F(x) dx \right\} \\
&= (1 - \phi)^2 P^2 (2 \int_{k_T}^{q_T} (q_T - x) F(x) dx - \left(\int_0^{q_T} F(x) dx \right)^2 - \left(\int_0^{k_T} F(x) dx \right)^2 \\
&\quad + 2 \left(\int_0^{k_T} F(x) dx + \int_{k_T}^{q_T} F(x) dx \right) \int_0^{k_T} F(x) dx) \\
&= (1 - \phi)^2 P^2 \left\{ 2 \int_{k_T}^{q_T} (q_T - x) F(x) dx - \left(\int_{k_T}^{q_T} F(x) dx \right)^2 \right\}.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
V_{ST} &= (1 - \phi)^2 P^2 \left(2 \int_0^{k_T} (k_T - x) F(x) dx - \left(\int_0^{k_T} F(x) dx \right)^2 \right). \\
V_{ST} &= \phi^2 P^2 \left(2 \int_{k_T}^{q_T} (q_T - x) F(x) dx - \left(\int_{k_T}^{q_T} F(x) dx \right)^2 \right).
\end{aligned}$$

APPENDIX E. PROOF OF PROPOSITION 5.2

The first derivative of EP_{RT} with respect to q_T yields $\frac{\partial EP_{RT}}{\partial q_T} = (1 - \phi)p \left(\bar{F}(q_T) - \frac{(1 - \phi)w_T(1 + r_T)}{p} \bar{F}(k_B) \right)$. Let $h(q_T) = \frac{\bar{F}(q_T)}{\frac{(1 - \phi)w_T(1 + r_T)}{p} \bar{F}(k_T)}$. Further, $\ln h(q_T) = \ln \bar{F}(q_T) - \ln \frac{(1 - \phi)w_T(1 + r_T)}{p} \bar{F}(k_T)$. The first derivative of $\ln h(q_T)$ with respect to q_T yields $\frac{\partial \ln h(q_T)}{\partial q_T} = - \left(z(q_T) - \frac{(1 - \phi)w_T(1 + r_T)}{p} z(k_T) \right)$. Since $z(q_T)$ is increasing in q_T , $z(q_T) > \frac{(1 - \phi)w_T(1 + r_T)}{p} z(k_T)$. Further, $\frac{\partial \ln h(q_T)}{\partial q_T} < 0$ and $h(q_T)$ is decreasing in q_T . Therefore, $h(q_T) = 1$ exists a unique root of $\hat{q}_T = \bar{F}^{-1} \left(\frac{(1 - \phi)w_T(1 + r_T)}{p} \bar{F}(k_T) \right)$. When $q_T \leq \hat{q}_T$, $h(q_T) \geq 1$ and

$\frac{\partial \text{EP}_{\text{RT}}}{\partial q_T} \geq 0$. When $q_T > \hat{q}_T$, $h_{q_T} < 1$ and $\frac{\partial \text{EP}_{\text{RT}}}{\partial q_T} < 0$. Therefore EP_{RT} is concave. From $\frac{\partial \text{EP}_{\text{RT}}}{\partial q_T} = 0$, $q_{R,\text{EPT}}(w_T) = \bar{F}^{-1}\left(\frac{(1-\phi)w_T(1+r_T)}{p}\bar{F}\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right)\right)$.

The first derivative of $q_{R,\text{EPT}}(w_T)$ with respect to w_T yields $\frac{\partial q_{R,\text{EPT}}(w_T)}{\partial w_T} = \frac{1 - \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right)}{w_T\left(\frac{(1-\phi)w_T(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right) - z(q_{R,\text{EPT}}(w_T))\right)}$. Obviously, the denominator is nonnegative. Next, we show that $1 - \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right) > 0$. Since, $\frac{\partial q_{R,\text{EPT}}(w_T)}{\partial B} = \frac{\frac{(1-\phi)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right)}{w_T\left(\frac{(1-\phi)w_T(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right) - z(q_{R,\text{EPT}}(w_T))\right)} < 0$. Then, $\frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right) < \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\right)$. In the case of $B = 0$, $q_{R,\text{EPT}}(w_T,0)$ solves $\bar{F}(q_{R,\text{EPT}}(w_T,0)) = \frac{(1-\phi)w_T(1+r_T)}{p}\bar{F}\left(\frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\right)$. Thus, $q_{R,\text{EPT}}(w_T,0)\bar{F}(q_{R,\text{EPT}}(w_T,0)) = \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\bar{F}\left(\frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\right)$. Since, $\frac{\partial q_T \bar{F}(q_T)}{\partial q_T} = \bar{F}(q_T)(1 - q_T z(q_T))$ and $z(q_T)$ is increasing in q_T , $1 - q_T z(q_T)$ is decreasing in q_T . Let \tilde{q} solve $\tilde{q}z(\tilde{q}) = 1$. When $q_T \leq \tilde{q}$, $\frac{\partial q_T \bar{F}(q_T)}{\partial q_T} \geq 0$. When $q_T > \tilde{q}$, $\frac{\partial q_T \bar{F}(q_T)}{\partial q_T} < 0$. Therefore, $q_T \bar{F}(q_T)$ is concave in q_T . The maximum value of $q_T \bar{F}(q_T)$ is achieved at \tilde{q} . Since $q_{R,\text{EPT}}(w_T,0)\bar{F}(q_{R,\text{EPT}}(w_T,0)) = \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\bar{F}\left(\frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\right)$ and $\frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p} < q_{R,\text{EPT}}(w_T,0)$, $q_{R,\text{EPT}}(w_T,0) < \tilde{q}$. Thus, $1 - \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T,0)(1+r_T)}{p}\right) > 0$. Further, $1 - \frac{(1-\phi)w_T q_{R,\text{EPT}}(w_T)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_{R,\text{EPT}}(w_T) - B)(1+r_T)}{p}\right) > 0$. Therefore, $\frac{\partial q_{R,\text{EPT}}(w_T)}{\partial w_T} < 0$.

The first-order condition of V_{RT} with respect to q_T yields $\frac{\partial V_{\text{RT}}}{\partial q_T} = 2(1-\phi)^2 p^2 \left(\bar{F}(q_T) \int_{k_T}^{q_T} F(x) dx - \frac{(1-\phi)w_T(1+r_T)}{p} F(k_T) \int_{k_T}^{q_T} \bar{F}(x) dx \right)$. The retailer's order quantity with risk constraints is not more than the optimal order quantity without risk constraint, i.e., $q_{R,\text{SPT}}(w_T) \leq q_{R,\text{EPT}}(w_T)$. There exist a w'_T such that $q_{R,\text{EPT}}(w'_T) = q_{R,\text{SPT}}(w_T) \leq q_{R,\text{EPT}}(w_T)$ and $w'_T \geq w_T$. Since $\bar{F}(q_{R,\text{EPT}}(w'_T)) = \frac{(1-\phi)w'_T(1+r_T)}{p}\bar{F}\left(\frac{(1+r_T)(w'_T q_{R,\text{EPT}}(w'_T) - B)}{p}\right)$, Then, $\frac{\partial \frac{(1-\phi)w_T(1+r_T)}{p}F\left(\frac{(1+r_T)(w_T q_T - B)}{p}\right)}{\partial w_T} = \frac{(1-\phi)(1+r_T)}{p}$, $\bar{F}\left(\frac{(1+r_T)(w_T q_T - B)}{p}\right) \left(1 - \frac{(1-\phi)w_T q_T(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_T - B)(1+r_T)}{p}\right)\right) > \frac{(1-\phi)(1+r_T)}{p}\bar{F}\left(\frac{(1+r_T)(w_T q_T - B)}{p}\right)$, $\left(1 - \frac{(1-\phi)(w_T q_T - B)(1+r_T)}{p}z\left(\frac{(1-\phi)(w_T q_T - B)(1+r_T)}{p}\right)\right)$. Since $w'_T \geq w_T$, $\frac{(1-\phi)w_T(1+r_T)}{p}F\left(\frac{(1+r_T)(w_T q_T - B)(1+r_T)}{p}\right) < \frac{(1-\phi)w'_T(1+r_T)}{p}\bar{F}\left(\frac{(1+r_T)(w'_T q_T - B)(1+r_T)}{p}\right)$. Thus, $\frac{(1-\phi)w_T(1+r_T)}{p}\bar{F}\left(\frac{(1+r_T)(w_T q_{R,\text{SPT}}(w_T) - B)(1+r_T)}{p}\right) < \frac{(1-\phi)w'_T(1+r_T)}{p}\bar{F}\left(\frac{(1+r_T)(w'_T q_{R,\text{SPT}}(w_T) - B)(1+r_T)}{p}\right) = \bar{F}(q_{R,\text{SPT}})$.

Therefore, $\bar{F}(q_{R,\text{SPT}}(w_T)) \int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} F(x) dx - \frac{(1-\phi)w'_T(1+r_T)}{p}F(k_T(q_{R,\text{SPT}})) \int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} \bar{F}(x) dx \geq (q_{R,\text{SPT}}(w_T) - k_T(q_{R,\text{SPT}}))F(k_T(q_{R,\text{SPT}}))\left(\bar{F}(q_{R,\text{SPT}}(w_T)) - \frac{(1-\phi)w_T(1+r_T)}{p}F(k_T(q_{R,\text{SPT}}))\right) > 0$. Hence, $\frac{\partial V_{\text{RT}}}{\partial q_T} > 0$. Further, $\text{SP}_{\text{RT}}(q_T)$ is increasing in q_T . Since $q_{R,\text{SPT}}(w_T) = \text{argmax}_{q_T} \{\text{SP}_{\text{RT}}(q_T) \leq K_R\}$,

$$(1-\phi)^2 p^2 \left(2 \int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} (q_{R,\text{SPT}}(w_T) - x)F(x) dx - \left(\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} (q_{R,\text{SPT}}F(x) dx \right)^2 \right) = K_R^2$$

for a given K_R . Further,

$$\frac{\partial q_{R,SPT}(w_T)}{\partial w_T} = \frac{\frac{(1-\phi)q_{R,SPT}(w_T)(1+r_T)}{p}F(k_T(q_{R,SPT})) \int_{k_T(q_{R,SPT})}^{q_{R,SPT}} \bar{F}(x) dx}{\left\{ \bar{F}(q_{R,SPT}(w_T)) \int_{k_T(q_{R,SPT})}^{q_{R,SPT}} F(x) dx \right\} - \frac{(1-\phi)w_T(1+r_T)}{p}F(k_T(q_{R,SPT})) \int_{k_T(q_{R,SPT})}^{q_{R,SPT}} \bar{F}(x) dx}.$$

Similarly,

$$\begin{aligned} \frac{\partial V_{RT}}{\partial q_{R,EPT}(w_T)} &= 2(1-\phi)^2 p^2 \left(\bar{F}(q_{R,EPT}(w_T)) \int_{k_T(q_{R,EPT})}^{q_{R,EPT}(w_T)} F(x) dx - \frac{w_T}{p} F(k_T(q_{R,EPT})) \right. \\ &\quad \left. \int_{k_T(q_{R,EPT})}^{q_{R,EPT}(w_T)} \bar{F}(x) dx \right) \\ &= 2(1-\phi)^3 p w_T (1+r_T) \left(\bar{F}(k_T(q_{R,EPT})) \int_{k_T(q_{R,EPT})}^{q_{R,EPT}(w_T)} F(x) dx - F(k_T(q_{R,EPT})) \right. \\ &\quad \left. \int_{k_T(q_{R,EPT})}^{q_{R,EPT}(w_T)} \bar{F}(x) dx \right) \\ &\geq 2(1-\phi)^3 p w_T (1+r_T) (\bar{F}(k_T(q_{R,EPT}))(q_{R,EPT}(w_T) - k_T(q_{R,EPT})) \\ &\quad - F(k_T(q_{R,EPT})) \bar{F}(k_T(q_{R,EPT}))(q_{R,EPT}(w_T) - k_T(q_{R,EPT}))) \\ &= 0. \end{aligned}$$

Hence, $\frac{\partial V_{RT}}{\partial q_T} > 0$. $SP_{RT}(q_T)$ is increasing in q_T . Let $q_{R,EPT}(w_{T1}) = q_{R,SPT}(w_{T1})$. When $w_T < w_{T1}$, $q_{R,SPT}(w_T) < q_{R,EPT}(w_T)$ and $SP_{RT}(q_{R,EPT}(w_T)) > K_R$. The optimal order quantity in (P2) is $q_{R,MVT}(w_T) = q_{R,SPT}(w_T) < q_{R,EPT}(w_T)$. Hence, $q_{R,MVT}(w_T)$ is increasing in w_T in the interval (c, w_{T1}) . When $w_T > w_{T1}$, $q_{R,EPT}(w_T) \leq q_{R,EPT}(w_{T1}) = q_{R,SPT}(w_{T1}) < q_{R,SPT}(w_T)$. Therefore, $SP_{RT}(q_{R,EPT}(w_T)) \leq SP_{RT}(q_{R,SPT}(w_{T1})) = K_R$. The optimal order quantity in (P2) is $q_{R,MVT}(w_T) = q_{R,EPT}(w_T)$. Hence, $q_{R,MVT}(w_T)$ is decreasing in w_T in the interval $[w_{T1}, p]$. Further, we have $q_{R,MVT}(w_T) = \min\{q_{R,EPT}(w_T), q_{R,SPT}(w_T)\}$.

APPENDIX F. PROOF OF PROPOSITION 5.3

The first-order condition of EP_{ST} with respect to w_T yields $\frac{\partial EP_{ST}}{\partial w_T} = \left(\phi(1+r_f) + (1-\phi)(1+r_T) \bar{F} \left(\frac{(1-\phi)(w_T q_{R,MVT}(w_T) - B)(1+r_T)}{p} \right) \right) \frac{\partial w_T q_{R,MVT}(w_T)}{\partial w_T} - c(1+r_f) \frac{\partial q_{R,MVT}(w_T)}{\partial w_T}$.

(a) When $w_T \leq w_{T1}$, $q_{R,MVT}(w_T) = q_{R,SPT}(w_T)$ and $q_{R,SPT}(w_T)$ is increasing in w_T . The first-order condition of EP_{ST} with respect to w_T yields $\frac{\partial EP_{ST}}{\partial w_T} = \left((1-\phi)(1+r_T) \bar{F} \left(\frac{(1-\phi)(w_T q_{R,SPT}(w_T) - B)(1+r_T)}{p} \right) - c(1+r_f) \right) \frac{\partial q_{R,SPT}(w_T)}{\partial w_T} + (\phi(1+r_f) + (1-\phi)(1+r_T) \bar{F}(k_T(q_{R,SPT})) q_{R,SPT}(w_T)) + \phi w_T (1+r_f) \frac{\partial q_{R,SPT}(w_T)}{\partial w_T}$. Since $q_{R,SPT}(w_T) \leq q_{R,EPT}(w_T)$, $(1-\phi)w_T(1+r_T)\bar{F}(k_T(q_{R,SPT})) > (1-\phi)w_T(1+r_T)\bar{F}(k_T(q_{R,EPT}))$. Then, $(1-\phi)w_T(1+r_T)\bar{F}(k_T(q_{R,SPT})) > p\bar{F}(q_{R,EPT})$. Since $p\bar{F}(q_{R,EPT}(w_T)) > c(1+r_f)$, otherwise, the trade will not occurs. Thus, $(1-\phi)w_T(1+r_T)\bar{F}(k_T(q_{R,SPT})) > c(1+r_f)$. Therefore, $\frac{\partial EP_{ST}}{\partial w_T} > 0$. The first-order condition of SP_{ST} with respect to w_T yields $\frac{\partial SP_{ST}}{\partial w_T} = \frac{\frac{\partial V_{ST}}{\partial k_T(q_{R,SPT})}}{2\sqrt{V_{ST}}} \frac{(1-\phi)(1+r_T)}{p} \left(q_{R,SPT}(w_T) + \frac{\partial q_{R,SPT}(w_T)}{\partial w_T} \right)$, where $\frac{\partial V_{ST}}{\partial k_T(q_{R,SPT})} > 2p^2 \bar{F}(k_T(q_{R,SPT})) \int_0^{k_T(q_{R,SPT})} dx > 0$. From Proposition 5.2, $\frac{\partial q_{R,SPT}(w_T)}{\partial w_T} > 0$. Hence, $\frac{\partial SP_{ST}}{\partial w_T} > 0$.

(b) When $w_T > w_{T1}$, $q_{R,MVT}(w_T) = q_{R,EPT}(w_T)$. The first derivative of EP_{ST} with respect to w_T yields $\frac{\partial EP_{ST}}{\partial w_T} = ((\phi(1+r_f) + (1-\phi)(1+r_T)\bar{F}(k_T(q_{R,SPT})))\frac{1-q_{R,EPT}(w_T)z(q_{R,EPT}(w_T))}{1-\frac{(1-\phi)(w_T q_{R,EPT}(w_T)(1+r_T))}{p}-z(k_T(q_{R,EPT}))} - c(1+r_f))\frac{\partial q_{R,EPT}(w_T)}{\partial w_T}$. Letting $\xi(w_T) = \frac{1-q_{R,EPT}(w_T)z(q_{R,EPT}(w_T))}{1-\frac{(1-\phi)(w_T q_{R,EPT}(w_T)(1+r_T))}{p}-z(k_T(q_{R,EPT}))}$. Then, $\frac{\partial EP_{ST}}{\partial w_T} = ((\phi(1+r_f) + (1-\phi)(1+r_T)\bar{F}(k_T(q_{R,SPT})))\xi(w_T) - c(1+r_f))\frac{\partial q_{R,EPT}(w_T)}{\partial w_T}$.

Let $q_{R,EPT}(\tilde{w}_T) = \tilde{q}$. $M(w_T) = (\phi w_T(1+r_f) + (1-\phi)w_T(1+r_T)\bar{F}(k_T(q_{R,EPT})))\xi(w_T) - c(1+r_f)$.

When $w_T \leq \tilde{w}_T$, $q_{R,EPT}(w_T) \geq \tilde{q}$. Then, $1 - q_{R,EPT}(w_T)z(q_{R,EPT}(w_T)) \leq 0$ and $\xi(w_T) < 0$. Thus, $\frac{\partial EP_{ST}}{\partial w_T} > 0$. When $w_T > \tilde{w}_T$, $q_{R,EPT}(w_T) < \tilde{q}$. Then, $1 - q_{R,EPT}(w_T)z(q_{R,EPT}(w_T)) > 0$.

Let $L = \frac{(1-\phi)(w_T q_{R,EPT}(w_T)(1+r_T))}{p}$, $Z(q_{R,EPT}(w_T)) = q_{R,EPT}(w_T)z(q_{R,EPT}(w_T))$. The first-order condition of $\xi(w_T)$ with respect to w_T yields $\frac{\partial \xi(w_T)}{\partial w_T} = \frac{1}{(1-Lz(k_T(q_{R,EPT})))^2} \left(\frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} \frac{\partial q_{R,EPT}(w_T)}{\partial w_T} (1 - Lz(k_T(q_{R,EPT}))) - (1 - Z(q_{R,EPT}(w_T))) \left(-\frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})} - Z(k_T(q_{R,EPT})) \right) \frac{\partial Z(k_T(q_{R,EPT}))}{\partial w_T} \right) > \frac{-\frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} \frac{\partial q_{R,EPT}(w_T)}{\partial w_T} + \left(\frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})} + Z(k_T(q_{R,EPT})) \right) \frac{\partial Z(k_T(q_{R,EPT}))}{\partial w_T}}{(1 - Z(q_{R,EPT}(w_T))) \frac{(1-Lz(k_T(q_{R,EPT})))^2}{(1-Lz(k_T(q_{R,EPT})))^2}}$.

$\frac{\partial Z(k_T(q_{R,EPT}))}{\partial w_T} = \frac{(1-\phi)w_T(1+r_T)}{p} \frac{1-q_{R,EPT}(w_T)z(q_{R,EPT}(w_T))}{1-Lz(k_T(q_{R,EPT}))} \frac{\partial q_{R,EPT}}{\partial w_T}$. Since $k_T(q_{R,EPT}) < L < q_{R,EPT}(w_T)$, $Lz(k_T(q_{R,EPT})) < Z(q_{R,EPT}(w_T))$. Then, $\frac{\partial Z(k_T(q_{R,EPT}))}{\partial w_T} > \frac{\partial q_{R,EPT}}{\partial w_T}$. Thus,

$\frac{\partial \xi(w_T)}{\partial w_T} > (1 - Z(q_{R,EPT}(w_T))) \frac{-\frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} \frac{\partial q_{R,EPT}(w_T)}{\partial w_T} + \left(\frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})} + Z(k_T(q_{R,EPT})) \right) \frac{\partial Z(k_T(q_{R,EPT}))}{\partial w_T}}{(1 - Lz(k_T(q_{R,EPT})))^2}$. Since $-\frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} \frac{\partial q_{R,EPT}(w_T)}{\partial w_T} + \frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})} + Z(k_T(q_{R,EPT})) = -q_{R,EPT}(w_T) \frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} - z(q_{R,EPT}(w_T)) + \frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})} + Z(k_T(q_{R,EPT}))$. Our analysis in Section 5 assumes demand distributions with increasing and convex failure rates, then $\frac{\partial Z(q_{R,EPT}(w_T))}{\partial q_{R,EPT}(w_T)} > \frac{\partial Z(k_T(q_{R,EPT}))}{\partial k_T(q_{R,EPT})}$. Further, $\frac{\partial \xi(w_T)}{\partial w_T} > 0$. Thus, $\frac{\partial M(w_T)}{\partial w_T} > 0$. Let w_{T0} satisfies $M(w_{T0}) = 0$. When $\tilde{w}_T < w_T < w_{T0}$, then $M(w_T) < 0$. Thus, $\frac{\partial EP_{ST}}{\partial w_T} > 0$. When $w_T = w_{T0}$, then $M(w_{T0}) = 0$. Thus, $\frac{\partial EP_{ST}}{\partial w_T} = 0$. When $w_T > w_{T0}$, then $M(w_T) > 0$. Thus, $\frac{\partial EP_{ST}}{\partial w_T} > 0$.

When $w_{T1} < w_{T0}$, then EP_{ST} is increasing in w_T in interval (c, w_{T0}) and is decreasing in w_T in $(w_{T0}, \frac{p}{1+r_T})$. When $w_{T1} > w_{T0}$, then EP_{ST} is increasing in w_T in interval (c, w_{T1}) and is decreasing in w_T in $(w_{T1}, \frac{p}{1+r_T})$.

Therefore, EP_{ST} is increasing in w_T in $(c, \max\{w_{T0}, w_{T1}\})$ and is decreasing in w_T in $(\max\{w_{T0}, w_{T1}\}, \frac{p}{1+r_T})$.

The first-order condition of SP_{ST} with respect to w_T yields $\frac{\partial SP_{ST}}{\partial w_T} = \frac{\frac{\partial V_{ST}}{\partial k_T(q_{R,EPT})} (1-\phi)(1+r_T)}{2\sqrt{V_{ST}}} \left(q_{R,EPT}(w_T) + \frac{\partial q_{R,EPT}(w_T)}{\partial w_T} \right) = \frac{\frac{\partial V_{ST}}{\partial k_T(q_{R,EPT})} (1-\phi)(1+r_T)}{2\sqrt{V_{ST}}} \frac{1 - \frac{(1-\phi)w_T q_{R,EPT}(w_T)(1+r_T)}{p} z\left(\frac{(1-\phi)(w_T q_{R,EPT}(w_T)-B)(1+r_T)}{p}\right)}{\left(\frac{(1-\phi)w_T(1+r_T)}{p} z\left(\frac{(1-\phi)(w_T q_{R,EPT}(w_T)-B)(1+r_T)}{p}\right) - z(q_{R,EPT}(w_T))\right)}$.

When $w_T \leq \tilde{w}_T$, $q_{R,EPT}(w_T) \geq \tilde{q}$. Then, $1 - q_{R,EPT}(w_T)z(q_{R,EPT}(w_T)) \leq 0$. Thus, $\frac{\partial SP_{ST}}{\partial w_T} \geq 0$. When $w_T > \tilde{w}_T$, $q_{R,EPT}(w_T) < \tilde{q}$. Then, $1 - q_{R,EPT}(w_T)z(q_{R,EPT}(w_T)) > 0$. Thus, $\frac{\partial SP_{ST}}{\partial w_T} < 0$.

When $w_T \leq \tilde{w}_T$, then SP_{ST} is increasing in w_T in interval (c, \tilde{w}_T) and is decreasing in w_T in $(\tilde{w}_T, \frac{p}{1+r_T})$.

When $w_{T1} > \tilde{w}_T$, then SP_{ST} is increasing in w_T in interval (c, w_{T1}) and is decreasing in w_T in $(w_{T1}, \frac{p}{1+r_T})$.

Therefore, SP_{ST} is increasing in w_T in $(c, \max\{\tilde{w}_T, w_{T1}\})$ and is decreasing in w_T in $(\max\{\tilde{w}_T, w_{T1}\}, \frac{p}{1+r_T})$.

- (1) If $\tilde{w}_T \leq w_{T0} \leq w_{T1}$, EP_{ST} and SP_{ST} are increasing in w_T in the interval (c, w_{T1}) and decreasing in w_T in the interval $[w_{T1}, \frac{p}{1+r_T})$. When $K_S \geq SP_{ST}(q_{R,MVT}(w_{T1}))$, then $w_T^* = w_{T1}$; When $K_S < \max SP_{ST}(q_{R,MVT}(w_{T1}))$, then $w_T^* = \text{argmax} \{EP_{ST}(q_{R,MVT}(w_{Tl})), EP_{ST}(q_{R,MVT})(w_{Tu})\}$, where w_{Tl} satisfies $SP_{ST}(q_{R,MVT}(w_{Tl})) = K_S$, $i = l, u$.

- (2) If $\tilde{w}_T \leq w_{T1} < w_{T0}$, EP_{ST} is increasing in w_T in the interval (c, w_{T0}) and decreasing in w_T in the interval $[w_{T0}, \frac{p}{1+r_T})$. $SP_{ST}(q_{R,MVT}(w_T))$ is increasing in w_T in the interval (c, w_{T1}) and decreasing

in w_T in the interval $\left[w_{T1}, \frac{p}{1+r_T}\right)$. When $K_S \geq \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, then $w_T^* = w_{T0}$; When $K_S < \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, then $w_T^* = \text{argmax}\{\text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tl})), \text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tu}))\}$.

(3) If $w_{T1} < \tilde{w}_T \leq w_{T0}$, EP_{ST} is increasing in w_T in the interval (c, w_{T0}) and decreasing in w_T in the interval $\left[w_{T0}, \frac{p}{1+r_T}\right)$. $\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_T))$ is increasing in w_T in the interval (c, \tilde{w}_T) and decreasing in w_T in the interval $[\tilde{w}_T, \frac{p}{1+r_T})$. When $K_S \geq \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T))$, then $w_T^* = w_{T0}$; When $K_S < \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T))$, then $w_T^* = \text{argmax}\{\text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tl})), \text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tu}))\}$.

To summarize the proofs of (1), (2) and (3), we have that when $K_S \geq \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, then $w_T^* = \max\{w_{T1}, w_{T0}\}$; when $K_S < \max\{\text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{T1}))\}$, then $w_T^* = \text{argmax}\{\text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tl})), \text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tu}))\}$.

APPENDIX G. PROOF OF PROPOSITION 6.1

(i) The first-order condition of EP_{RT} with respect to w_T yields $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} = p\bar{F}(q_{R,\text{MVT}}(w_T))\frac{\partial q_{R,\text{MVT}}(w_T)}{\partial w_T} - (1 - \phi)(1 + r_T)\bar{F}(k(q_{R,\text{MVT}}))\frac{\partial w_T q_{R,\text{MVT}}(w_T)}{\partial w_T}$.

(a) When $w_T < w_{T1}$, $q_{R,\text{MVT}}(w_T) = q_{R,\text{SPT}}(w_T) < q_{R,\text{EPT}}(w_T)$. From Proposition 5.2(ii),

$$\frac{\partial q_{R,\text{SPT}}(w_T)}{\partial w_T} = \frac{\frac{(1-\phi)q_{R,\text{SPT}}(w_T)(1+r_T)}{p}F(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}} \bar{F}(x) dx}{\left\{ \frac{(1-\phi)w_T(1+r_T)}{p}F(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}} F(x) dx - \right\}}.$$

Thus,

$$\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} = \frac{(1-\phi)(1+r_T)\bar{F}(k(q_{R,\text{SPT}}))\left\{ F(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} \bar{F}(x) dx - \bar{F}(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} F(x) dx \right\}}{k_T(q_{R,\text{SPT}})F(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} \bar{F}(x) dx}.$$

The numerator of $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T}$ namely $F(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} \bar{F}(x) dx - \bar{F}(k_T(q_{R,\text{SPT}}))\int_{k_T(q_{R,\text{SPT}})}^{q_{R,\text{SPT}}(w_T)} F(x) dx \leq F(k_T(q_{R,\text{SPT}}))\bar{F}(k_T(q_{R,\text{SPT}}))(q_{R,\text{SPT}}(w_T) - k_T(q_{R,\text{SPT}})) - F(k_T(q_{R,\text{SPT}}))\bar{F}(k_T(q_{R,\text{SPT}}))(q_{R,\text{SPT}}(w_T) - k_T(q_{R,\text{SPT}})) = 0$. Thus, $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} < 0$.

(b) When $w_T \geq w_{T1}$, $q_{R,\text{MVT}}(w_T) = q_{R,\text{EPT}}(w_T) < q_{R,\text{EPT}}(w_T)$. The first-order condition of EP_{RT} with respect to w_T yields $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} = (p\bar{F}(q_{R,\text{EPT}}(w_T)) - (1 - \phi)w_T(1 + r_T)\bar{F}(k(q_{R,\text{EPT}}))\frac{\partial q_{R,\text{EPT}}(w_T)}{\partial w_T} - (1 - \phi)q_{R,\text{EPT}}(w_T)(1 + r_T)\bar{F}(k(q_{R,\text{EPT}})))\frac{\partial w_T q_{R,\text{EPT}}(w_T)}{\partial w_T}$.

From Proposition 5.2, $p\bar{F}(q_{R,\text{EPT}}(w_T)) = (1 - \phi)w_T(1 + r_T)\bar{F}(k_T(q_{R,\text{EPT}}))$. Thus, $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} = -(1 - \phi)q_{R,\text{EPT}}(w_T)(1 + r_T)\bar{F}(k(q_{R,\text{EPT}}))\frac{\partial w_T q_{R,\text{EPT}}(w_T)}{\partial w_T}$. Since $\frac{\partial q_{R,\text{EPT}}(w_T)}{\partial w_T} < 0$, $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} < 0$. To summarize the proofs of (a) and (b), we have that $\frac{\partial \text{EP}_{\text{RT}}}{\partial w_T} < 0$. Let \hat{w}_T satisfy $\text{EP}_{\text{RT}}(\hat{w}_T) = \text{EP}_{\text{RB}}(w_B^*)$. Under TEF, the supplier sets wholesale price $w_T \leq \hat{w}_T$, otherwise the retailer chooses BEF. From Proposition 5.3(i), $\text{EP}_{\text{ST}}(w_T)$ is concave in w_T . Define by w_{Ti} the corresponding wholesale price such that $\text{EP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Ti})) = \text{EP}_{\text{SB}}(q_{R,\text{MVB}}(w_B^*))$, where $i = m, n$. Under TEF, the supplier sets wholesale price $w_{Tm} \leq w_T \leq w_{Tn}$, otherwise, the supplier's profit less than under BEF. Hence, under TEF, the supplier sets wholesale price $w_{Tm} \leq w_T \leq \min\{w_{Tn}, \hat{w}_T\}$. Let $\bar{w}_T = \min\{w_{Tn}, \hat{w}_T\}$. Let $K_{Sm} = \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(w_{Tm}))$ and $\bar{K}_S = \text{SP}_{\text{ST}}(q_{R,\text{MVT}}(\bar{w}_T))$. From the Proposition 5.3, we have SP_{ST} is increasing in w_T in the interval $(c, \max\{\hat{w}_T, w_{T1}\})$ and decreasing in w_T in the interval $(\max\{\hat{w}_T, w_{T1}\}, \frac{p}{1+r_T})$. If $\bar{w}_T \leq \max\{\hat{w}_T, w_{T1}\}$, then $K_{Sm} < \text{SP}_{\text{ST}} \leq \bar{K}_S$. Therefore, when $K_{Sm} < \bar{K}_S$, TEF is the unique financing equilibrium; otherwise BEF is the unique financing equilibrium. If $\bar{w}_T > \max\{\hat{w}_T, w_{T1}\}$, there exist the following two cases.

Cases 1. $K_{Sm} \leq \bar{K}_S$. When $K_S \leq K_{Sm}$, from Proposition 5.3, the supplier's wholesale price $w_T^* = w_{Tl}$ or $w_T^* = w_{Tu}$. Since $K_S \leq K_{Sm}$ and $K_{Sm} \leq \bar{K}_S$, then $w_{Tl} \leq w_{Tm}$ and $w_T > \bar{w}_T$. When $w_T^* = w_{Tl}$, then supplier's profit under TEF less than under BEF. The supplier is not willing to provide trade credit to the retailer. Thus, BEF is the financing equilibrium. When $w_T^* = w_{Tu}$, then the supplier's profit or the retailer's profit under TEF less than under BEF. Hence, TEF is not the financing equilibrium and BEF is the financing equilibrium. When $K_S > K_{Sm}$, the supplier's wholesale price $w_T \geq w_{Tm}$. For a given K_S , the supplier can set wholesale price $w_{Tm} < w_T^* < \bar{w}_T$. In this case, TEF is the financing equilibrium.

Cases 2. $K_{Sm} > \bar{K}_S$. When $K_S \leq \bar{K}_S$, from Proposition 5.3, the supplier's wholesale price $w_T^* = w_{Tl}$ or $w_T^* = w_{Tu}$. Since $K_S \leq \bar{K}_S$ and $\bar{K}_S < K_{Sm}$, then $w_{Tl} \leq w_{Tm}$ and $w_T > \bar{w}_T$. Thus, either the supplier's profit or the retailer's profit under TEF less than under BEF. Hence, TEF is not the financing equilibrium and BEF is the financing equilibrium. When $K_S > \bar{K}_S$, the supplier's wholesale price $w_T < \bar{w}_T$. For a given K_S , the supplier can set wholesale price $w_{Tm} < w_T^* < \bar{w}_T$. In this case, TEF is the financing equilibrium.

To summarize the proofs of the two cases, we have when $K_S > \min\{K_{Sm}, \bar{K}_S\}$, TEF is the unique financing equilibrium; otherwise BEF is the unique financing equilibrium.

APPENDIX H. PROOF OF LEMMA 7.1

Since $K'_R > K_R$, $q_{R,SPB}(w_B, K'_R) > q_{R,SPB}(w_B, K_R)$. Furthermore, $w_{B1}(K'_R) < w_{B1}(K_R)$. There exist the following three subcases.

Subcase 1. If $w_{B0} < w_{B1}(K'_R) < w_{B1}(K_R)$, then the estimated wholesale price is $w_B^*(K'_R) = w_{B1}(K'_R)$, while the optimal wholesale price based on the true information is $w_B^*(K_R) = w_{B1}(K_R)$. Thus, $w_B^*(K'_R) < w_B^*(K_R)$.

Subcase 2. If $w_{B1}(K'_R) < w_{B0} < w_{B1}(K_R)$, then the estimated wholesale price is $w_B^*(K'_R) = w_{B0}$, while the optimal wholesale price based on the true information is $w_B^*(K_R) = w_{B1}(K_R)$. Thus, $w_B^*(K'_R) < w_B^*(K_R)$.

Subcase 3. If $w_{B1}(K'_R) < w_{B1}(K_R) < w_{B0}$, then the estimated wholesale price is equal to the optimal wholesale price based on the true information, i.e., $w_B^*(K'_R) = w_B^*(K_R) = w_{B0}$.

To summarize the proofs of the three subcases, we have $w_B^*(K'_R) \leq w_B^*(K_R)$. Taking the first derivative of EP_{RB} with respect to w_B yields $\frac{\partial EP_{RB}}{\partial w_B} = (p\bar{F}(q_{R,MVB}(w_B)) - (1 - \phi)w_B(1 + r_f))\frac{\partial q_{R,MVB}(w_B)}{\partial w_B} - (1 - \phi)q_{R,MVB}(w_B)(1 + r_f)$. When $q_{R,MVB}(w_B) = q_{R,SPB}(w_B)$, $\frac{\partial EP_{RB}}{\partial w_B} = (p\bar{F}(q_{R,SPB}(w_B)) - (1 - \phi)w_B(1 + r_f))\frac{\partial q_{R,SPB}(w_B)}{\partial w_B} - (1 - \phi)q_{R,SPB}(w_B)(1 + r_f) = (1 - \phi)q_{R,SPB}(w_B)(1 + r_f)\bar{F}(q_{R,SPB}(w_B))\int_{k_B(q_{R,SPB})}^{q_{R,SPB}(w_B)}(F(x) - F(k_B(q_{R,SPB})))dx$

$$\left\{ \begin{array}{l} \bar{F}(k_B(q_{R,SPB}))\bar{F}(q_{R,SPB})\int_{k_B(q_{R,SPB})}^{q_{R,SPB}(w_B)}F(x)dx \\ - \frac{(1 - \phi)(w_B)(1 + r_f)}{p}F(k_B(q_{R,SPB}))\int_{k_B(q_{R,SPB})}^{q_{R,SPB}(w_B)}\bar{F}(x)dx \end{array} \right\}.$$

When $q_{R,MVB}(w_B) = q_{R,EPB}(w_B)$, $\bar{F}(q_{R,EPB}(w_B)) = \frac{(1 - \phi)(w_B)(1 + r_f)}{p}$. Thus, $\frac{\partial EP_{RB}}{\partial w_B} = (p\bar{F}(q_{R,MVB}(w_B)) - (1 - \phi)w_B(1 + r_f))\frac{\partial q_{R,MVB}(w_B)}{\partial w_B} - (1 - \phi)q_{R,MVB}(w_B)(1 + r_f) = -(1 - \phi)q_{R,MVB}(w_B)(1 + r_f) < 0$. Therefore, EP_{RB} is decreasing in w_B . Since $w_B^*(K'_R) \leq w_B^*(K_R)$, $EP_{RB}(K'_R) \geq EP_{RB}(K_R)$. Therefore, the retailer has incentives to pretend to be less risk averse.

APPENDIX I. PROOF OF PROPOSITION 7.2

Since $K'_R > K_R$, $q_{R,SPB}(w_B, K'_R) > q_{R,SPB}(w_B, K_R)$. Furthermore, $w_{B1}(K'_R) < w_{B1}(K_R)$. There exist the following three subcases.

Subcase 1. If $w_{B0} < w_{B1}(K'_R) < w_{B1}(K_R)$, then $w_B^*(K'_R) = w_{B1}(K'_R) < w_B^*(K_R) = w_{B1}(K_R)$. Therefore, the supplier's estimated order quantity is $q_{R,MVB}(w_B^*(K'_R), K'_R) = q_{R,SPB}(w_{B1}(K'_R), K'_R)$. Based on $w_B^*(K'_R)$ and true K_R , the retailer's actual order quantity is $q_{R,MVB}(w_B^*(K'_R), K_R) = q_{R,SPB}(w_{B1}(K'_R), K_R)$. Since $K'_R > K_R$, $q_{R,SPB}(w_{B1}(K'_R), K'_R) > q_{R,SPB}(w_{B1}(K'_R), K_R)$. Hence, $q_{\min} = q_{R,SPB}(w_{B1}(K'_R), K'_R)$.

Subcase 2. If $w_{B1}(K'_R) < w_{B0} < w_{B1}(K_R)$, then $w_B^*(K'_R) = w_{B0} < w_B^*(K_R) = w_{B1}(K_R)$. Therefore, the supplier's estimated order quantity is $q_{R,MVB}(w_B^*(K'_R), K'_R) = q_{R,EPB}(w_{B0})$. Based on $w_B^*(K'_R)$ and true K_R , the retailer's actual order quantity is $q_{R,MVB}(w_B^*(K'_R), K_R) = q_{R,SPB}(w_{B0}, K_R)$. From the proof of Proposition 4.2, $q_{R,SPB}(w_B)$ is increasing in w_B , while $q_{R,EPB}(w_B)$ is decreasing in w_B . Furthermore, $q_{R,SPB}(w_{B0}, K_R) < q_{R,SPB}(w_{B1}(K_R), K_R) = q_{R,EPB}(w_{B1}(K_R)) < q_{R,EPB}(w_{B0})$. Hence, $q_{\min} = q_{R,EPB}(w_{B0})$.

Subcase 3. If $w_{B1}(K'_R) < w_{B1}(K_R) < w_{B0}$, then $w_B^*(K'_R) = w_B^*(K_R) = w_{B0}$. Therefore, the supplier's estimated order quantity equals the retailer's actual order quantity, *i.e.*, $q_{R,MVB}(w_B^*(K'_R), K'_R) = q_{R,MVB}(w_B^*(K_R), K_R) = q_{R,EPB}(w_{B0})$. Furthermore, the supplier's optimal wholesale price w_{B0} is independent of the retailer's risk aversion threshold. The retailer's distorted information K'_R has no impact on the supplier's decision. Hence, $q_{\min} = q_{R,EPB}(w_{B0})$.

APPENDIX J. PROOF OF LEMMA 7.3

Since $K'_R > K_R$, $q_{R,SPT}(w_T, K'_R) > q_{R,SPT}(w_T, K_R)$. Furthermore, $w_{T1}(K'_R) < w_{T1}(K_R)$.

(i) When $K_S \geq \max\{\text{SP}_{\text{ST}}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,MVT}(w_{T1}))\}$, the supplier's risk constraint is inactive. the supplier's wholesale price $w_T^* = \max\{w_{T1}, w_{T0}\}$. There exist the following three subcases.

Subcase 1. If $w_{T0} < w_{T1}(K'_R) < w_{T1}(K_R)$, then the estimated wholesale price is $w_T^*(K'_R) = w_{T1}(K'_R)$, while the optimal wholesale price based on the true information is $w_T^*(K_R) = w_{T1}(K_R)$. Then, $w_T^*(K'_R) < w_T^*(K_R)$.

Subcase 2. If $w_{T1}(K'_R) < w_{T0} < w_{T1}(K_R)$, then the estimated wholesale price is $w_T^*(K'_R) = w_{T0}$, while the optimal wholesale price based on the true information is $w_T^*(K_R) = w_{T1}(K_R)$. Then, $w_T^*(K'_R) < w_T^*(K_R)$.

Subcase 3. if $w_{T1}(K'_R) < w_{T1}(K_R) < w_{T0}$, then the estimated wholesale price is equal to the optimal wholesale price based on the true information, *i.e.*, $w_T^*(K'_R) = w_T^*(K_R) = w_{T0}$.

(ii) When $K_S < \max\{\text{SP}_{\text{ST}}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,MVT}(w_{T1}))\}$, then $w_T^* = \text{argmax}\{\text{EP}_{\text{ST}}(q_{R,MVT}(w_{Tl})), \text{EP}_{\text{ST}}(q_{R,MVT}(w_{Tu}))\}$.

If $w_T^* = w_{Tl}$, then the estimated wholesale price is $w_T^*(K'_R) = w_{Tl}(K'_R)$, while the optimal wholesale price based on the true information is $w_T^*(K_R) = w_{Tl}(K_R)$. Since $K'_R > K_R$, $q_{R,SPT}(w_T, K'_R) > q_{R,SPT}(w_T, K_R)$.

Letting $U(K_R) = \frac{(1-\phi)(w_T q_{R,SPT}(w_T, K_R) - B)(1+r_T)}{p}$, then $U(K'_R) \geq U(K_R)$. Since $\text{SP}_{\text{ST}}(q_{R,SPT}(w_T, K_R)) = p\sqrt{2 \int_0^{U(K_R)} (U(K_R) - x) F(x) dx - \left(\int_0^{U(K_R)} F(x) dx\right)^2}$ and $\text{SP}_{\text{ST}}(q_{R,SPT}(w_T, K_R))$ is increasing in $U(K_R)$, $\text{SP}_{\text{ST}}(q_{R,SPT}(w_T, K_R)) \leq \text{SP}_{\text{ST}}(q_{R,SPT}(w_T, K'_R))$. From Proposition 5.3, we have $\text{SP}_{\text{ST}}(q_{R,SPT}(w_T, K_R))$ is increasing in w_T . Since $K_S = \text{SP}_{\text{ST}}(q_{R,MVT}(w_{Tl}(K_R), K_R)) = \text{SP}_{\text{ST}}(q_{R,MVT}(w_{Tl}(K'_R), K'_R))$, $w_{Tl}(K'_R) \leq w_{Tl}(K_R)$.

If $w_T^* = w_{Tu}$, then the estimated wholesale price is equal to the optimal wholesale price based on the true information, *i.e.*, $w_T^*(K'_R) = w_T^*(K_R) = w_{Tu}$.

To summarize the above proofs, we have $w_T^*(K'_R) \leq w_T^*(K_R)$. From Proposition 6.1(i), we have $\text{EP}_{\text{RT}}(w_T)$ is decreasing in w_T . Since $w_T^*(K'_R) \leq w_T^*(K_R)$, $\text{EP}_{\text{RT}}(K'_R) \geq \text{EP}_{\text{RT}}(K_R)$.

APPENDIX K. PROOF OF PROPOSITION 7.4

(i) When $K_S \geq \max\{\text{SP}_{\text{ST}}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{\text{ST}}(q_{R,MVT}(w_{T1}))\}$, the supplier's wholesale price $w_T^* = \max\{w_{T1}, w_{T0}\}$. There exist the following three subcases.

If $w_{T0} < w_{T1}(K'_R) < w_{T1}(K_R)$, then $w_T^*(K'_R) = w_{T1}(K'_R) < w_T^*(K_R) = w_{T1}(K_R)$. Thus, the supplier's estimated order quantity is $q_{R,MVT}(w_T^*(K'_R), K'_R) = q_{R,SPT}(w_{T1}(K'_R), K'_R)$. Based on $w_T^*(K'_R)$ and true K_R , the retailer's actual order quantity is $q_{R,MVT}(w_T^*(K'_R), K_R) = q_{R,SPT}(w_{T1}(K'_R), K_R)$. Since $K'_R > K_R$, $q_{R,SPT}(w_{T1}(K'_R), K'_R) > q_{R,SPT}(w_{T1}(K'_R), K_R)$. Hence, $q_{\min} = q_{R,SPT}(w_{T1}(K'_R), K'_R)$.

If $w_{T1}(K'_R) < w_{T0} < w_{T1}(K_R)$, then $w_T^*(K'_R) = w_{T0} < w_T^*(K_R) = w_{T1}(K_R)$. Thus, the supplier's estimated order quantity is $q_{R,MVT}(w_T^*(K'_R), K'_R) = q_{R,EPT}(w_{T0})$. Based on $w_T^*(K'_R)$ and true K_R , the

retailer's actual order quantity is $q_{R,MVT}(w_T^*(K'_R), K_R) = q_{R,SPT}(w_{T0}, K_R)$. Since $K'_R > K_R$, $q_{R,EPT}(w_{T0}) > q_{R,EPT}(w_{T1}, K_R) = q_{R,SPT}(w_{T1}, K_R)$. Hence, $q_{\min} = q_{R,EPT}(w_{T0})$.

If $w_{T1}(K'_R) < w_{T1}(K_R) < w_{T0}$, then $w_T^*(K'_R) = w_T^*(K_R) = w_{T0}$. Therefore, the supplier's estimated order quantity equals the retailer's actual order quantity, i.e., $q_{R,MVT}(w_T^*(K'_R), K'_R) = q_{R,MVT}(w_T^*(K'_R), K_R) = q_{R,EPT}(w_{T0})$. Furthermore, the supplier's optimal wholesale price w_{T0} is independent of the retailer's risk aversion threshold. The retailer's distorted information K'_R has no impact on the supplier's decision. Hence, $q_{\min} = q_{R,EPT}(w_{T0})$.

(ii) When $K_S < \max\{\text{SP}_{ST}(q_{R,MVT}(\tilde{w}_T)), \text{SP}_{ST}(q_{R,MVT}(w_{T1}))\}$, then $w_T^* = \text{argmax}\{\text{EP}_{ST}(q_{R,MVT}(w_{Tl})), \text{EP}_{ST}(q_{R,MVT}(w_{Tu}))\}$. There exist the following three subcases.

If $w_T^* = w_{Tl}$, then $w_T^*(K'_R) = w_{Tl}(K'_R) \leq w_T^*(K_R) = w_{Tl}(K_R)$. Thus, the supplier's estimated order quantity is $q_{R,MVT}(w_T^*(K'_R), K'_R) = q_{R,SPT}(w_{Tl}(K'_R), K'_R)$. Based on $w_T^*(K'_R)$ and true K_R , the retailer's actual order quantity is $q_{R,MVT}(w_T^*(K'_R), K_R) = q_{R,SPT}(w_{Tl}(K'_R), K_R)$. Since $\text{SP}_{ST}(q_{R,SPT}(w_{Tl}(K'_R), K'_R)) = \text{SP}_{ST}(q_{R,SPT}(w_{Tl}(K'_R), K_R)) = K_S$,

$$\frac{(1-\phi)(w_{Tl}(K'_R)q_{R,SPT}(w_{Tl}(K'_R), K_R) - B)(1+r_T)}{p} = \frac{(1-\phi)(w_{Tl}(K_R)q_{R,SPT}(w_{Tl}(K_R), K_R) - B)(1+r_T)}{p}.$$

Thus, $w_{Tl}(K'_R)q_{R,SPT}(w_{Tl}(K'_R), K'_R) = w_{Tl}(K_R)q_{R,SPT}(w_{Tl}(K_R), K_R)$. Since $w_{Tl}(K'_R) \leq w_{Tl}(K_R)$, $q_{R,SPT}(w_{Tl}(K'_R), K'_R) > q_{R,SPT}(w_{Tl}(K_R), K_R)$. Hence, $q_{\min} = q_{R,SPT}(w_{Tl}(K'_R), K'_R)$.

If $w_T^* = w_{Tu}$, then $w_T^*(K'_R) = w_T^*(K_R) = w_{Tu}$. Therefore, the supplier's estimated order quantity equals the retailer's actual order quantity, i.e., $q_{R,MVT}(w_T^*(K'_R), K'_R) = q_{R,MVT}(w_T^*(K'_R), K_R) = q_{R,EPT}(w_{Tu})$. Hence, $q_{\min} = q_{R,EPT}(w_{Tu})$.

APPENDIX L. PROOF OF LEMMA 8.1

Proofs of Lemma 8.1 is similar to that of Lemma 4.1 and, hence, are omitted.

APPENDIX M. PROOF OF PROPOSITION 8.2

Proofs of Proposition 8.2 is similar to that of Lemma 4.1 and, hence, are omitted.

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