A COMMON WEIGHT CREDIBILITY DATA ENVELOPMENT ANALYSIS MODEL FOR EVALUATING DECISION MAKING UNITS WITH AN APPLICATION IN AIRLINE PERFORMANCE

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Abstract. Data envelopment analysis (DEA) model has been widely applied for estimating efficiency scores of decision making units (DMUs) and is especially used in many applications in transportation. In this paper, a novel common weight credibility DEA (CWCDEA) model is proposed to evaluate DMUs considering uncertain inputs and outputs. To develop a credibility DEA model, a credibility counterpart constraint is suggested for each constraint of DEA model. Then, the weights generated by the credibility DEA (CDEA) model are considered as ideal solution in a multi-objective DEA model. To solve the multi-objective DEA model, a goal programming model is proposed. The goal programming model minimized deviations from the ideal solutions and found the common weights of inputs and outputs. Using the common weights generated by goal programming model, the final efficiency scores for decision making are calculated. The usefulness and applicability of the proposed approach have been shown using a data set in the airline industry.

1. Introduction

In the last decade, transportation has become an important issue for all people and policymakers around the world. All countries and their policymakers decide to develop this sector to increase the satisfaction of the people. There are lots of benefits in development of transportation such as economic growth, increase in tourists, export and import as trade and culture. Due to different human needs, there are different ways named air, sea, rail and road transportation to travel to another city or send cargos. Among the various transportation systems, air transportation is the most favorable one because it has the highest speed compared to other ways. Air transportation has specific importance due to time-saving and a considerable reduction in costs [5].

Nowadays, the growth of the tourism industry, development of technology and importance of comfort and welfare of passengers brings up a new discussion for policymakers like how to make decisions for improvement and change the current trend, which will lead to the development of this industry. Airlines' competition has

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increased considerably since this issue has been bold. According to these factors, airline companies will survey with considering the input and output indicators for making better decisions.

Due to the importance of air transportation, it is necessary to rank airlines to find out the efficient ones. The results of efficiency analysis can be used by policymakers to make the right decision for the development of the airlines. If an airline gets the first rank, it means that it is an efficient unit and must take a step forward. In many cases, managers compare the inefficient units with the efficient airlines to improve them.

In this paper, the efficiency scores of airlines in Iran under fuzzy conditions have been calculated. For this purpose, 14 airlines including Iranair, Iranairtour, Ata, Atrak, Aseman, Taban, Pouyaair, Zagros, Gheshm, Kaspian, Kishair, Mahan, Meraj and Naft have been evaluated. In the real world applications, there are uncertainty in data which should be considered in the analysis. In order to handle uncertain data, several approaches are applied such as interval programming, fuzzy theory, robust optimization and so on [38]. In this study, the data are considered as L-R fuzzy numbers and the credibility theory is used. Based on credibility theory, the counterpart of each constraint in the DEA model is developed and then, a credibility DEA (CDEA) model is proposed. Furthermore, this study doesn't allow flexibility in choosing weights for variables without considering other units. To deal with this issue, this paper applies the common weight approach. In other words, in addition to considering uncertainty in data, policymakers want to measure the efficiency of airlines with common weights for inputs and outputs. For finding a set of common weights for inputs and outputs, the solution of CDEA model is considered as ideal solution. Then, the deviations of ideal solutions for all DMUs are minimized using a goal programming approach. In fact, the final common weight CDEA (CWCDEA) model, which is based on goal programming, is run once, and a set of common weights is found. Finally, using generated common weights, the efficiency score of each DMU is calculated.

The rest of this paper is organized as follows. Section 2 reviews the literature. In Section 3, the DEA, common weight DEA, fuzzy DEA-based credibility theory, and common weight credibility DEA models are presented. Section 4 includes of the gathered data and selected indicators. In Section 5, the results are presented, and the ranks of airlines are determined. The conclusion of this paper is summarized in Section 6.

2. LITERATURE REVIEW

There are several methods to evaluate airlines, such as analytical hierarchy process (AHP), balanced scorecard (BSC), productivity indexes, stochastic frontier analysis (SFA), data envelopment analysis (DEA), etc. [29]. For instance, See and Rashid [45] used Törnqvist index method to calculate the total factor productivity (TFP) growth of Malaysia airlines (MAS) between 1980 and 2013. Gutiérrez and Lozano [15] used DEA to research about the potential output increase scenarios and operational efficiency of 21 small and medium-sized airports (SMA). Yu *et al.* [58] used a dynamic network DEA approach to examine the efficiency score of Indian and Chinese air carrier companies.

Data envelopment analysis (DEA) is a non-parametric mathematical tool for measuring the relative efficiency of homogeneous decision-making units (DMUs) DEA, which was presented by Charnes *et al.* [8], is one of the most popular models to evaluate DMUs such as airlines [32]. Chang *et al.* [7] calculated the economic and ecological efficiency of 27 global airlines in 2010. They used slacks-based measure DEA (SBM-DEA) for calculating economic and ecological efficiencies. For fully ranking, Wanke and Barros [54] used VDRAM-DEA (Virtual Frontier Dynamic Range Adjusted Model – DEA) to assess efficiency in Latin American airlines.

Several researchers have considered airlines as multi-stage systems and applied network DEA to evaluate them. Zhu [62] used a two-stage network DEA approach to compute airlines' performance. Lozano and Gutiérrez [29] introduced a slacks-based network DEA method for analyzing the efficiency of European airlines. Omrani and Soltanzadeh [37] introduced a dynamic network DEA (DNDEA) model to evaluate Iran's airlines.

In real-world applications, data are not crystal clear. In other words, there is uncertainty in inputs and outputs. Hence, different models have been proposed to handle the uncertainty in DEA models. In fact, data are not always accessible and reliable, so it be said it is approximately impractical to have data with transparency for the inputs and outputs [44]. To deal with uncertainty and achieve more accurate results, different approaches

such as imprecise DEA (IDEA), chance constraint DEA (CCDEA), fuzzy DEA (FDEA), robust DEA (RDEA) and bootstrap DEA have been suggested [36].

One of the most important approaches to deal with uncertainty in DEA is the fuzzy theory. Zadeh [60] have introduced the concept of fuzzy set. Wanke *et al.* [55] used a fuzzy DEA model to deal with uncertainty of input and output of Nigerian airports and assessed productive efficiency of them. Wanke *et al.* [56] proposed a new fuzzy DEA model to assess the uncertainty underlying. By applying a fuzzy dynamic network DEA model, Olfat *et al.* [35] studied the sustainability of airports through a multi-perspective, multi-system, and multi-process operation.

Izadikhah and Khoshroo [19] introduced a novel non-radial DEA model based on a modification of the enhanced Russell model (ERM model) in the presence of an undesirable output in a fuzzy environment. Soltanzadeh and Omrani [46] evaluated Iranian airlines using a dynamic network DEA model with fuzzy inputs and outputs. Heydari *et al.* [18] proposed a fully fuzzy network DEA-RAM model for evaluating airlines. They extended the network DEA-RAM model in the fully fuzzy environment. Tavassoli *et al.* [49] proposed a novel super-efficiency DEA model to appraise the relative efficiency of DMUs with stochastic and zero data. Arana-Jiménez *et al.* [3] proposed a fuzzy DEA slacks-based approach to evaluate efficiency score when the inputs and outputs are fuzzy numbers.

In the mentioned studies, the α-cut approach has been used to handle and solve uncertainty problem in the DEA model. In some new works, the credibility approach has been applied instead of the α-cut in the FDEA model. In order to measure a fuzzy case, Zadeh [61] proposed the concept of possibility. Although possibility measure has been widely used, it has no self-duality, which is needed in both practice and theory aspects [26]. In order to define a self-dual measure, Liu and Liu [27] presented the concept of credibility measure. Credibility theory was founded by Liu [25] as a branch of mathematics for studying the behavior of fuzzy phenomena. Credibility theory allows for modelling kinds of situations to give answers to the question of how to combine specific and collective claims information to reach the best evaluation of the particular final claim amount. Credibility theory has been used in several cases and developed by several researchers. Cukierman and Meltzer [11] developed a positive theory of credibility, ambiguity, and inflation under the discretion and asymmetric information. Pourrahmani *et al.* [41] worked on the optimization of an evacuation plan with uncertain demands. They designed a genetic algorithm based on fuzzy credibility theory to optimize the problem. The optimum parameter set for the genetic algorithm is obtained by Taguchi experimental design. Bai *et al.* [4] used the parametric interval-valued fuzzy variable-based inputs and outputs to deal with uncertainty problem in the real-world. Ahmadvand and Pishvaee presented a credibility-based fuzzy common weights DEA approach to handle the Kidney allocation problems under inherent uncertainty. Wardana *et al.* [57] proposed a combined p-robust technique, fuzzy credibility constrained programming and DEA. Gupta *et al.* [14] proposed a credibility Fuzzy DEA approach for portfolio selection based on their efficiency score. Yue *et al.* [59] presented a full fuzzy-interval credibility-constrained nonlinear programming approach for irrigation water allocation under uncertainty. Mahla and Agarwal [31] used a fuzzy slacks-based measure (SBM) DEA model to measure each DMUs efficiency in the presence of uncertain data. The credibility approach has been used to convert fuzzy SBM DEA model into a crisp linear programming model. Omrani *et al.* [40] introduced a robust credibility DEA (RCDEA) model to deal with uncertainty in data. They proposed two new types of RCDEA models: RCDEA model with exact perturbation in fuzzy inputs and outputs and RCDEA model with fuzzy perturbation in fuzzy inputs and outputs. They used a bi-objective BWM-RCDEA model to deal with flexibility of weights. The proposed model has been solved by min–max approach.

In the DEA model, there isn't any limitation for DMUs to choose weights inputs and outputs. There is an issue that exists in DEA is that because of the flexibility in the weight selection for variables, it is possible that many DMUs may become efficient. Thus, each DMU's aim is achieving to maximum efficiency, without considering other units. In many cases, DEA is unable to compare the efficient DMUs. Selecting weights freely for inputs and outputs is the strength and weakness of the DEA model. If a DMU cannot be placed on the frontier by selecting free weights for inputs and outputs, the inefficiency of DMU is very meaningful [36]. Unlike,

it is possible for the different DMUs to select very small weights (close to zero) for the inputs and outputs, which will not be acceptable for the decision maker.

There are different methods to reduce flexibility in weights and find a common set of weights for inputs and outputs. Afsharian *et al.* [1] analyzed 135 different common weight DEA publications. They focused on DEA approaches which used common set of weights for inputs and outputs for centralized management situation. In common weights models, the weights are not flexible. In fact, it is found a set of common weights for inputs and outputs of all DMUs. Roll *et al.* [43] introduced the common weights models for first time. Then, Kao and Hung [22] presented a developed nonlinear common weight DEA (CWDEA) model. Zohrehbandian *et al.* [64] expanded the CWDEA model proposed by Kao and Hung [22] and presented a linear goal programming approach to find out common weights of inputs and outputs. Liu and Peng ranked DMUs based on their efficiency score which obtained from common weights method. They have been ranked according to the optimization of the group's efficiency. Jahanshahloo *et al.* [20] proposed two new methods for finding a common weights by comparing an ideal line and the special line. Sun and Guo [47] considered ideal and anti-ideal DMUs to determine common weights for fully ranking efficient DMUs. Hatami-Marbini *et al.* [17] applied goal programming approach to obtain common weights for inputs and outputs variables. Toloo [50] proposed a new minimax mixed integer linear programming (MILP) model that can recognize the most efficient DMU. Numerical examples from different studies have been used to compare result of the proposed model with other models. Also, Toloo and Salahi [51] introduced a new nonlinear mixed integer programming model with higher discriminating power in compare with previous studies in the literature. They used a linearization technique to formulate an equivalent mixed integer linear programming model that decreases the computational burden.

Carrillo and Jorge [6] presented a novel model by combining common weight DEA and multi-objective optimization approach. Hatami-Marbini *et al.* [17] employed a common weights DEA model for centralized resource reduction and target setting. They defined the amount of reduction needed for each DMU's inputs and outputs that leads to increasing the efficiency score of all the DMUs. Puri *et al.* [42] presented a new common weights methodology where in interval calculations and unit production boundaries are considered to provide unique weights to calculate interval efficiencies. Wang *et al.* [53] proposed a novel approach to define a common set of weights based on the degree of consensus. Hatami-Marbini and Saati [16] presented a common-weights method for two-stage systems that allows to consider equality of opportunity in a fuzzy environment when evaluating the system efficiency. Also, Toloo *et al.* [52] developed a novel classifier non-radial directional distance method to deal with flexible measures. They considered input contraction and output expansion, simultaneously, in the presence of flexible measures. Mavi *et al.* [33] used common weights in the two-stage network DEA method for joint analysis of eco-efficiency and eco-innovation. Gharakhani *et al.* [13] generated common weights in dynamic network DEA by using a goal programming approach. They applied it on 30 non-life insurance companies in Iran. An *et al.* [2] developed common-weight DEA model to consider quantitative and qualitative criteria at the same time with establishing a comprehensive index system that includes inputs and outputs. Contreras *et al.* [10] introduced a bargaining approach with two players per DMU to determine common weights in DEA. Chu *et al.* [9] proposed a new common-weight multi-criteria decision-making (MCDM). They used a new algorithm to improve existing technology. Omrani *et al.* [38] presented an integrated (DEA) best-worst method (BWM) for considering DMs' preferences in DEA and reducing flexibility in weights of inputs and outputs. Kazemi *et al.* [23] proposed a common weights model with fuzzy data and non-discretionary inputs. They also ranked homogenous DMUs with considering environmental criteria in the presence of uncertainty. In a recent study, Omrani *et al.* [39] developed a new multi-objective best worst method (BWM)-robust DEA (RDEA) to incorporate decision makers' preferences into analysis under uncertainty. They used min-max technique to solve the proposed model and applied the it on 28 hospitals in northwestern region of Iran.

3. Methodology

In this section, the proposed common weight credibility DEA (CWCDEA) model is presented. In the first step, based on a lemma, a credibility DEA (CDEA) model is introduced. Then, to develop the CDEA to a common weight model, the objective function of CDEA model is considered as ideal solution. In the next step, to find a set of common weights for inputs and outputs, by using a goal programming model, the deviations of the ideal solution for all DMUs are minimized, simultaneously. In the following, the definitions and models are described as follows:

3.1. Preliminaries

First, a brief review of fuzzy sets definitions and terms have been explained in the following. For more details, the readers can refer to Dubois and Prade [12], Zimmermann [63], Liu and Liu [27, 28] and Li and Liu [24].

Definition 3.1. Let U be a universe set. A fuzzy set $\widetilde{A}$ of U is defined by a membership function $\mu_{\widetilde{A}}(x) \to [0, 1]$, $\forall x \in U$.

Definition 3.2. The α-cut of fuzzy set $\widetilde{A}$, $\widetilde{A}_\alpha$, is the crisp set $\widetilde{A}_\alpha = \{x | \mu_{\widetilde{A}}(x) \geq \alpha\}$.

Definition 3.3. A fuzzy number L-R type is expressed as $\widetilde{A} = (m, \alpha, \beta)_{\mathrm{LR}}$ with below membership function:

$$\mu_{\widetilde{A}}(r) = \begin{cases} \mathrm{L}\left(\frac{m-r}{\alpha}\right) & r \leq m \\ \mathrm{R}\left(\frac{r-m}{\beta}\right) & r \geq m \end{cases} \tag{3.1}$$

where L and R are the left and right functions, respectively, and α and β are the (non-negative) left and right spreads, respectively.

Definition 3.4. An L-R fuzzy number, $\widetilde{A} = (m, \alpha, \beta)_{\mathrm{LR}} = (m, \alpha, \beta)$ is a triangular fuzzy number if

$$\mathrm{L}(x) = \mathrm{R}(x) = \begin{cases} 1 - x & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases} \tag{3.2}$$

Definition 3.5. Let $\widetilde{A} = (m, \alpha, \beta)_{\mathrm{LR}}$ and $\widetilde{B} = \left(\bar{m}, \bar{\alpha}, \bar{\beta}\right)_{\mathrm{LRR}}$ be two positive triangular fuzzy numbers. The addition, subtraction and multiplication of $\widetilde{A}$ and $\widetilde{B}$ are as follows:

Addition: $\widetilde{A} + \widetilde{B} = (m, \alpha, \beta)_{\mathrm{LR}} + \left(\bar{m}, \bar{\alpha}, \bar{\beta}\right)_{\mathrm{LR}} = (m + \bar{m}, \alpha + \bar{\alpha}, \beta + \bar{\beta})_{\mathrm{LR}}$.

Subtraction: $\widetilde{A} - \widetilde{B} = (m, \alpha, \beta)_{\mathrm{LR}} - \left(\bar{m}, \bar{\alpha}, \bar{\beta}\right)_{\mathrm{LR}} = \left(m - \bar{m}, \alpha + \bar{\beta}, \beta + \bar{\alpha}\right)_{\mathrm{LR}}$.

Multiplication (approximation):

$$\widetilde{A} \otimes \widetilde{B} = (m, \alpha, \beta)_{\mathrm{LR}} \otimes \left(\bar{m}, \bar{\alpha}, \bar{\beta}\right)_{\mathrm{LR}} = \left(m\bar{m}, m\bar{\alpha} + \bar{m}\alpha - \alpha\bar{\alpha}, m\bar{\beta} + \bar{m}\beta + \beta\bar{\beta}\right)_{\mathrm{LR}}.$$

Definition 3.6. A possibility space is defined as $(\Theta, P(\Theta), Pos)$ where Θ is nonempty set, $P(\Theta)$ is the power set of Θ and Pos is the possibility measure. The possibility measure satisfies the below axioms:

(1) $Pos(\oslash) = 0$, $Pos(X) = 1$;
(2) $\forall A, B \in P(\Theta)$, if $A \subseteq B \to Pos(A) \leq Pos(B)$;
(3) $Pos(A_1 \cup A_2 \ldots \cup A_k) = Sup\,Pos_j(A_j)$

where X is the universe set.

Definition 3.7. The necessity measure is defined as $Nec(A) = 1 - Pos(A^c)$ where A^c is the complementary set of A set. The necessity measure satisfies the below axioms:

(1) $Nec(\oslash) = 0$, $Nec(X) = 1$;

(2) $\forall\, A, B \in P(\Theta)$, if $A \subseteq B \rightarrow Nec(A) \leq Nec(B)$;

(3) $Nec(A_1 \cap A_2 \ldots \cap A_k) = Inf\, Nec_j(A_j)$.

Definition 3.8. The credibility measure is defined as $Cre(A) = \frac{1}{2}\{Pos(A) + Nec(A)\}$. The credibility measure satisfies the below axioms:

(1) $Cre(\oslash) = 0$, $Cre(X) = 1$;

(2) $\forall\, A, B \in P(\Theta)$, if $A \subseteq B \rightarrow Cre(A) \leq Cre(B)$;

(3) $Cre(A) + Cre(A^c) = 1$, $\forall A \subseteq P(X)$.

Definition 3.9. Let λ be a fuzzy variables. The possibility, necessity and credibility of the fuzzy event $(\lambda \geq r)$ are defined as:

$$Pos(\lambda \geq r) = \underset{t \geq r}{Sup}\, \mu_\lambda(t) \tag{3.3}$$

$$Nec(\lambda \geq r) = 1 - Pos(\lambda < r) = 1 - \underset{\lambda < r}{Sup}\, \mu_\lambda(t) \tag{3.4}$$

$$Cre(\lambda \geq r) = \frac{1}{2}\{Pos(\lambda \geq r) + Nec(\lambda \geq r)\}. \tag{3.5}$$

3.2. Data envelopment analysis (DEA)

DEA is a nonparametric method that uses linear programming to measure the efficiency of DMUs with multiple inputs and multiple outputs. In DEA, efficiency is defined as a ratio of weighted sum of outputs to a weighted sum of inputs. The data form a frontier, DMUs which are on the frontier are evaluated as efficient. Output-oriented DEA models maximize output for a given quantity of input factors, contrariwise input-oriented models minimize input factors required for a given level of output. The envelopment form of input-oriented DEA-CCR model is as follows [8]:

$$\theta_o^{\text{CCR}} = \min\, \theta$$

s.t.

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \qquad i = 1, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \ldots, s$$

$$\lambda_j \geq 0, \qquad\qquad j = 1, \ldots, n$$

free θ. \qquad (3.6)

In model (3.6), jth DMU uses m inputs $x_{1j}, \ldots, x_{mj}$ for producing s outputs $y_{1j}, \ldots, y_{rj}$. Also, the efficiency score of DMU under evaluation is θ_o^{CCR}.

3.3. Credibility DEA (CDEA) model

In this section, the credibility DEA (CDEA) model is described. For developing DEA using fuzzy credibility theory, first following lemma is proven.

Lemma. *Let* $\widetilde{\lambda_1} = (m_1, \alpha_1, \beta_1)_{\text{LR}}$ *and* $\widetilde{\lambda_2} = (m_2, \alpha_2, \beta_2)_{\text{LR}}$ *be two L-R fuzzy numbers with continuous membership functions. For a given confidence level* $\gamma \in [0,1]$ *it is proven that:*

(I) *If* $\gamma \leq 0.5$, *then* $Cr\left(\widetilde{\lambda_1} \geq \widetilde{\lambda_2}\right) \geq \gamma \Leftrightarrow m_1 + \beta_1 \text{R}^{-1}(2\gamma) \geq m_2 - \alpha_2 \text{R}^{-1}(2\gamma).$

(II) *If* $\gamma > 0.5$, *then* $Cr\left(\widetilde{\lambda_1} \geq \widetilde{\lambda_2}\right) \geq \gamma \Leftrightarrow m_1 - \alpha_1 \text{L}^{-1}(2(1-\gamma)) \geq m_2 + \beta_2 \text{L}^{-1}(2(1-\gamma)).$

Proof. Suppose that

$$\widetilde{\lambda} = \widetilde{\lambda}_1 - \widetilde{\lambda}_2 = (m_1, \alpha_1, \beta_1)_{\mathrm{LR}} \oplus (-m_2, \beta_2, \alpha_2)_{\mathrm{LR}} = (m_1 - m_2, \alpha_1 + \beta_2, \alpha_2 + \beta_1)_{\mathrm{LR}} = \left(\overline{m}, \overline{\alpha}, \overline{\beta}\right)_{\mathrm{LR}} \qquad (3.7)$$

According to Definition 3.9, we have:

$$Cr\left(\widetilde{\lambda}_1 \geq \widetilde{\lambda}_2\right) = Cr\left(\widetilde{\lambda}_1 - \widetilde{\lambda}_2 \geq 0\right) = Cr\left(\widetilde{\lambda} \geq 0\right) = \frac{1}{2}\left[P_0, \left(\widetilde{\lambda} \geq 0\right) + Nec\left(\widetilde{\lambda} \geq 0\right)\right]$$

$$= \frac{1}{2}\left[P_0, \left(\widetilde{\lambda} \geq 0\right) + 1 - P_0, \left(\widetilde{\lambda} \geq 0\right)\right] = \frac{1}{2}\left[\sup_{t \geq 0} \mu(t) + 1 - \sup_{t < 0} \mu(t)\right]. \qquad (3.8)$$

It is clear that the equation (3.8) can be expressed as follows:

$$Cr\left(\widetilde{\lambda} \geq 0\right) = \begin{cases} 1 & 0 \leq \overline{m} - \overline{\alpha} \\ \frac{1}{2}\left[1 + 1 - \mathrm{L}\left(\frac{\overline{m}}{\overline{\alpha}}\right)\right] = 1 - \frac{1}{2}\mathrm{L}\left(\frac{\overline{m}}{\overline{\alpha}}\right) & \overline{m} - \overline{\alpha} \leq 0 \leq \overline{m} \\ \frac{1}{2}\left[\mathrm{R}\left(-\frac{\overline{m}}{\overline{\beta}}\right) + 1 - 1\right] = \frac{1}{2}\mathrm{R}\left(\frac{\overline{m}}{\overline{\beta}}\right) & \overline{m} \leq 0 \leq \overline{m} + \overline{\beta} \\ 0 & \overline{m} + \overline{\beta} < 0. \end{cases} \qquad (3.9)$$

If $\gamma \leq 0.5$, then

$$C_r\left(\widetilde{\lambda} \geq 0\right) \geq \gamma \Leftrightarrow \frac{1}{2}\mathrm{R}\left(\frac{-\overline{m}}{\overline{\beta}}\right) \geq \gamma \Leftrightarrow \mathrm{R}\left(\frac{-\overline{m}}{\overline{\beta}}\right) \geq 2\gamma \Leftrightarrow \frac{-\overline{m}}{\overline{\beta}} \leq \mathrm{R}^{-1}(2\gamma) \Leftrightarrow -\frac{m_1 - m_2}{\alpha_2 + \beta_1} \leq \mathrm{R}^{-1}(2\gamma)$$

$$\Leftrightarrow m_1 - m_2 \leq (\alpha_2 + \beta_1)\mathrm{R}^{-1}(2\gamma) \Leftrightarrow m_1 + \beta_1\mathrm{R}^{-1}(2\gamma) \geq m_2 - \alpha_2\mathrm{R}^{-1}(2\gamma).$$

If $\gamma > 0.5$, then

$$C_r\left(\widetilde{\lambda} \geq 0\right) \geq \gamma \Leftrightarrow 1 - \frac{1}{2}\mathrm{L}\left(\frac{\overline{m}}{\overline{\alpha}}\right) \geq \gamma \Leftrightarrow \frac{1}{2}\mathrm{L}\left(\frac{\overline{m}}{\overline{\alpha}}\right) \leq 1 - \gamma \Leftrightarrow \mathrm{L}\left(\frac{\overline{m}}{\overline{\alpha}}\right) \leq 2(1 - \gamma)$$

$$\Leftrightarrow \frac{\overline{m}}{\overline{\alpha}} \geq \mathrm{L}^{-1}(2(1 - \gamma)) \Leftrightarrow \frac{m_1 - m_2}{\alpha_1 + \beta_2} \geq \mathrm{L}^{-1}(2(1 - \gamma)) \Leftrightarrow m_1 - m_2 \geq (\alpha_1 + \beta_2)\mathrm{L}^{-1}(2(1 - \gamma))$$

$$\Leftrightarrow m_1 - \alpha_1\mathrm{L}^{-1}(2(1 - \gamma)) \geq m_2 + \beta_2\mathrm{L}^{-1}(2(1 - \gamma)).$$

The credibility counterpart of model (3.6) can be expressed as follows:

$$\theta_o^{\mathrm{CDEA}} = \min \theta$$

$$\text{s.t.}$$

$$Cre\left(\sum_{j=1}^{n} \lambda_j \widetilde{x}_{ij} \leq \theta \widetilde{x}_{io}\right) \geq \gamma_i, \qquad i = 1, \ldots, m$$

$$Cre\left(\sum_{j=1}^{n} \lambda_j \widetilde{y}_{rj} \geq \widetilde{y}_{ro}\right) \geq \gamma_r, \qquad r = 1, \ldots, s$$

$$\lambda_j \geq 0, \qquad j = 1, \ldots, n$$

$$\text{free } \theta \qquad (3.10)$$

where $\widetilde{x}_{ij} = (x_{ij}^m, x_{ij}^\alpha, x_{ij}^\beta)_{\mathrm{LR}}$ and $\widetilde{y}_{ij} = (y_{ij}^m, y_{ij}^\alpha, y_{ij}^\beta)_{\mathrm{LR}}$ are the L-R fuzzy numbers. According to Definition 3.3, the membership functions of inputs and outputs $j = 1, \ldots, n$ can be expressed as follows, respectively:

$$\mu_{\widetilde{x}_{ij}}(t) = \begin{cases} \mathrm{L}\left(\frac{x_{ij}^m - t}{x_{ij}^\alpha}\right) & t \leq x_{ij}^m, \ i = 1, \ldots, m \\ \mathrm{R}\left(\frac{t - x_{ij}^m}{x_{ij}^\beta}\right) & t \geq x_{ij}^m, \ i = 1, \ldots, m \end{cases} \qquad (3.11)$$

$$\mu_{\widetilde{y}_{ij}}(t) = \begin{cases} \mathrm{L}\left(\frac{y_{ij}^m - t}{y_{ij}^\alpha}\right) & t \le y_{ij}^m, \ i = m+1, \ldots, m+s \\ \mathrm{R}\left(\frac{t - y_{ij}^m}{y_{ij}^\beta}\right) & t \ge y_{ij}^m, \ i = m+1, \ldots, m+s. \end{cases} \tag{3.12}$$

According to Zadeh extension principle, the membership functions of left-hand and right-hand sides for constraints of model (3.10) can be expressed as follows:

$$\mu_{\sum_{j=1}^n \lambda_j \widetilde{x}_{ij}}(t) = \begin{cases} \mathrm{L}\left(\frac{\sum_{j=1}^n \lambda_j x_{ij}^m - t}{\sum_{j=1}^n \lambda_j x_{ij}^\alpha}\right) & t \le \sum_{j=1}^n \lambda_j x_{ij}^m, \ i = 1, \ldots, m \\ \mathrm{R}\left(\frac{t - \sum_{j=1}^n \lambda_j x_{ij}^m}{\sum_{j=1}^n \lambda_j x_{ij}^\beta}\right) & t \ge \sum_{j=1}^n \lambda_j x_{ij}^m, \ i = 1, \ldots, m \end{cases} \tag{3.13}$$

$$\mu_{\sum_{j=1}^n \lambda_j \widetilde{y}_{ij}}(t) = \begin{cases} \mathrm{L}\left(\frac{\sum_{j=1}^n \lambda_j y_{ij}^m - t}{\sum_{j=1}^n \lambda_j y_{ij}^\alpha}\right) & t \le \sum_{j=1}^n \lambda_j y_{ij}^m, \ i = m+1, \ldots, m+s \\ \mathrm{R}\left(\frac{t - \sum_{j=1}^n \lambda_j y_{ij}^m}{\sum_{j=1}^n \lambda_j y_{ij}^\beta}\right) & t \ge \sum_{j=1}^n \lambda_j y_{ij}^m, \ i = m+1, \ldots, m+s \end{cases} \tag{3.14}$$

$$\mu_{\theta \widetilde{x}_{io}}(t) = \begin{cases} \mathrm{L}\left(\frac{\theta x_{io}^m - t}{\theta x_{io}^\alpha}\right) & t \le \theta x_{io}^m, \ i = 1, \ldots, m \\ \mathrm{R}\left(\frac{t - \theta x_{io}^m}{\theta x_{io}^\beta}\right) & t \ge \theta x_{io}^m, \ i = 1, \ldots, m. \end{cases} \tag{3.15}$$

Based on the membership functions (3.11)–(3.15), the fuzzy numbers $\sum_{j=1}^n \lambda_j \widetilde{x}_{ij}$, $\sum_{j=1}^n \lambda_j \widetilde{y}_{ij}$, $\theta \widetilde{x}_{io}$ and $\widetilde{y}_{ro}$ are shown as below L-R fuzzy numbers:

$$\sum_{j=1}^n \lambda_j \widetilde{x}_{ij} = \left(\sum_{j=1}^n \lambda_j x_{ij}^m, \sum_{j=1}^n \lambda_j x_{ij}^\alpha, \sum_{j=1}^n \lambda_j x_{ij}^\beta\right)_{\mathrm{LR}}, \qquad i = 1, \ldots, m \tag{3.16}$$

$$\sum_{j=1}^n \lambda_j \widetilde{y}_{ij} = \left(\sum_{j=1}^n \lambda_j y_{ij}^m, \sum_{j=1}^n \lambda_j y_{ij}^\alpha, \sum_{j=1}^n \lambda_j y_{ij}^\beta\right)_{\mathrm{LR}}, \qquad i = m+1, \ldots, m+s \tag{3.17}$$

$$\theta \widetilde{x}_{io} = \left(\theta x_{io}^m, \theta x_{io}^\alpha, \theta x_{io}^\beta\right)_{\mathrm{LR}}, \qquad\qquad i = 1, \ldots, m \tag{3.18}$$

$$\widetilde{y}_{io} = \left(y_{io}^m, y_{io}^\alpha, y_{io}^\beta\right)_{\mathrm{LR}}, \qquad\qquad i = m+1, \ldots, m+s. \tag{3.19}$$

In this study, the data are considered as triangular fuzzy numbers. Hence, according to Definition 3.4, we have:

$$\mathrm{L}(x) = \mathrm{R}(x) = \mathrm{L}^{-1}(x) = \mathrm{R}^{-1}(x) = 1 - x. \tag{3.20}$$

According to above lemma, for $\gamma_i \le 0.5$, the first constraint of model (3.10) is expressed as follows:

$$\theta x_{io}^m + \theta x_{io}^\beta \mathrm{R}^{-1}(2\gamma_i) \ge \sum \lambda_j x_{ij}^m - \sum \lambda_j x_{ij}^\alpha \mathrm{R}^{-1}(2\gamma_i)$$
$$\Rightarrow \theta x_{io}^m + \theta x_{io}^\beta (1 - 2\gamma_i) \ge \sum \lambda_j x_{ij}^m - \sum \lambda_j x_{ij}^\alpha (1 - 2\gamma_i)$$
$$\Rightarrow \sum_{j=1}^n \lambda_j \left[x_{ij}^m - (1 - 2\gamma_i) x_{ij}^\alpha\right] \le \theta \left[x_{io}^m + (1 - 2\gamma_i) x_{io}^\beta\right], \qquad i = 1, \ldots, m. \tag{3.21}$$

In addition, the second constraint of model (3.10) is converted to a linear constraint as follows:

$$\sum \lambda_j y_{rj}^m + \sum \lambda_j y_{rj}^\beta \mathrm{R}^{-1}(2\gamma_r) \ge y_{ro}^m - y_{ro}^\alpha \mathrm{R}^{-1}(2\gamma_r)$$
$$\Rightarrow \sum \lambda_j y_{rj}^m + \sum \lambda_j y_{rj}^\beta (1 - 2\gamma_r) \ge y_{ro}^m - y_{ro}^\alpha (1 - 2\gamma_r)$$

$$\Rightarrow \sum_{j=1}^{n} \lambda_j \Big[y_{rj}^m + (1 - 2\gamma_r) y_{rj}^{\beta} \Big] \geq y_{ro}^m - (1 - 2\gamma_r) y_{ro}^{\alpha}, \qquad r = 1, \ldots, s. \quad (3.22)$$

By considering the constraints (3.21) and (3.22), the final CDEA model for $\gamma_i, \gamma_r \leq 0.5$ is expressed follows:

$$\text{Min } \theta$$
$$\text{s.t.}$$
$$\sum_{j=1}^{n} \lambda_j \Big[x_{ij}^m - (1 - 2\gamma_i) x_{ij}^{\alpha} \Big] \leq \theta \Big[x_{io}^m + (1 - 2\gamma_i) x_{io}^{\beta} \Big] \qquad i = 1, \ldots, m$$
$$\sum_{j=1}^{n} \lambda_j \Big[y_{ij}^m + (1 - 2\gamma_i) y_{ij}^{\beta} \Big] \geq y_{io}^m - (1 - 2\gamma_i) y_{io}^{\alpha}, \qquad r = 1, \ldots, s$$
$$\lambda_j \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad \theta \text{ free.} \quad (3.23)$$

The multiplier CDEA dual model of (3.23) is written as (3.24).

$$\max_{\gamma_r \leq 0.5} \sum_{r=1}^{s} u_r [y_{ro}^m - (1 - 2\gamma_r) y_{ro}^{\alpha}]$$
$$\text{s.t.}$$
$$\sum_{i=1}^{m} v_i \Big[x_{io}^m + (1 - 2\gamma_i) x_{io}^{\beta} \Big] = 1$$
$$\sum_{r=1}^{s} u_r \Big[y_{rj}^m + (1 - 2\gamma_r) y_{rj}^{\beta} \Big] - \sum_{i=1}^{m} v_i \Big[x_{ij}^m - (1 - 2\gamma_i) x_{ij}^{\alpha} \Big] \leq 0, \qquad j = 1, \ldots, n$$
$$v_i, u_r \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad i = 1, \ldots, m; r = 1, \ldots, s. \quad (3.24)$$

Similar to above way and according to above lemma, the multiplier CDEA for $\gamma_i, \gamma_r > 0.5$ is expressed as follows:

$$\max_{\gamma_r > 0.5} \sum_{r=1}^{s} u_r \Big[y_{ro}^m + (2\gamma_r - 1) y_{ro}^{\beta} \Big]$$
$$\text{s.t.}$$
$$\sum_{i=1}^{m} v_i [x_{io}^m - (2\gamma_i - 1) x_{io}^{\alpha}] = 1$$
$$\sum_{r=1}^{s} u_r \Big[y_{rj}^m - (2\gamma_r - 1) y_{rj}^{\alpha} \Big] - \sum_{i=1}^{m} v_i \Big[x_{ij}^m + (2\gamma_i - 1) x_{ij}^{\beta} \Big] \leq 0, \qquad j = 1, \ldots, n$$
$$v_i, u_r \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad i = 1, \ldots, m; r = 1, \ldots, s. \quad (3.25)$$

$\square$

3.4. Common weights DEA (CWDEA) model

In this section, the DEA model (3.6) is developed to common weight version. First, the multiplier form of DEA model (3.6) which is the dual program is written as follows:

$$\max \sum_{r=1}^{s} u_r y_{ro}$$

s.t.

$$\sum_{i=1}^{m} v_i x_{io} = 1$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1, \ldots, n$$

$$v_i, u_r \geq 0, \qquad\qquad i = 1, \ldots, m; r = 1, \ldots, s. \qquad (3.26)$$

In model (3.26), the efficiency score of DMU j is expressed as $\theta_j^{\text{CCR}} = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$. It is assumed that all DMUs want to maximize their scores, simultaneously. Hence, the DEA model (3.26) should be solved.

$$\max \left\{ \frac{\sum_{r=1}^{s} u_r y_{r1}}{\sum_{i=1}^{m} v_i x_{i1}}, \frac{\sum_{r=1}^{s} u_r y_{r2}}{\sum_{i=1}^{m} v_i x_{i2}}, \ldots, \frac{\sum_{r=1}^{s} u_r y_{rn}}{\sum_{i=1}^{m} v_i x_{in}} \right\}$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad\qquad j = 1, \ldots, n$$

$$v_i, u_r \geq 0, \qquad\qquad i = 1, \ldots, m; r = 1, \ldots, s. \qquad (3.27)$$

The DEA model (3.26) is a multi-objective programming. There are several approaches such as global criterion, constraint-ε, goal programming, goal attainment and etc. to solve the model (3.26). In this paper, goal programming approach is used to find non-dominated solution of model (3.26). It is clear that the ideal solution for jth objective function of model (3.26) is θ_j^{CCR} which can be obtained from model (3.26). As mentioned before, θ_j^{CCR} can be calculated as $\theta_j^{\text{CCR}} = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}}$. This equation can be re-expressed as $\sum_{r=1}^{s} u_r y_{rj} - \theta_j^{\text{CCR}} \sum_{i=1}^{m} v_i x_{ij} = 0$. In order to find common weights, any deviation from ideal solution should be minimized. Hence, the model (3.26) is rewritten as follows:

$$\min \sum_{j=1}^{n} (d_j^+ + d_j^-)$$

s.t.

$$\sum_{r=1}^{s} u_r y_{rj} - \theta_j^{\text{CCR}} \sum_{i=1}^{m} v_i x_{ij} + d_j^- - d_j^+ = 0, \qquad j = 1, \ldots, n$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad\qquad j = 1, \ldots, n$$

$$v_i, u_r, d_j^-, d_j^+ \geq 0, \qquad\qquad i = 1, \ldots, m; r = 1, \ldots, s. \qquad (3.28)$$

Assume that the optimal solution of model (3.28) is (u_r^*, v_i^*). Then, the efficiency score of jth DMU using common weights (u_r^*, v_i^*) is calculated as follows:

$$\theta_j^{\text{CWDEA}} = \frac{\sum_{r=1}^{s} u_r^* y_{rj}}{\sum_{i=1}^{m} v_i^* x_{ij}}. \qquad (3.29)$$

3.5. Common weight CDEA (CWCDEA) model

In this section, the common weight credibility DEA (CWCDEA) is introduced. For $\gamma \leq 0.5$, the CDEA model (3.24) is developed. According to model (3.24), the efficiency score of jth DMU can be calculated as follows:

$$\theta_{j, \gamma \leq 0.5}^{\text{CDEA}} = \frac{\sum_{r=1}^{s} u_r [y_{ro}^m - (1 - 2\gamma_r) y_{ro}^\alpha]}{\sum_{i=1}^{m} v_i \left[x_{io}^m + (1 - 2\gamma_i) x_{io}^\beta \right]}. \qquad (3.30)$$

In fact, the efficiency score generated by model (3.24) is ideal solution for DMU j. The equation (3.30) can be shown as $\sum_{r=1}^{s} u_r[y_{ro}^m - (1-2\gamma_r)y_{ro}^\alpha] - \theta_{j,\gamma\leq0.5}^{\text{CDEA}} \sum_{i=1}^{m} v_i\left[x_{io}^m + (1-2\gamma_i)x_{io}^\beta\right] = 0$. All DMUs search to combination weights which minimize any deviations from their ideal solutions. To develop model (3.24) to a CWCDEA model, the multi-objective model (3.31) can be considered:

$$\max \left\{ \frac{\sum_{r=1}^{s} u_r[y_{r1}^m - (1-2\gamma_r)y_{r1}^\alpha]}{\sum_{i=1}^{m} v_i\left[x_{i1}^m + (1-2\gamma_i)x_{i1}^\beta\right]}, \frac{\sum_{r=1}^{s} u_r[y_{r2}^m - (1-2\gamma_r)y_{r2}^\alpha]}{\sum_{i=1}^{m} v_i\left[x_{i2}^m + (1-2\gamma_i)x_{i2}^\beta\right]}, \dots, \frac{\sum_{r=1}^{s} u_r[y_{rn}^m - (1-2\gamma_r)y_{rn}^\alpha]}{\sum_{i=1}^{m} v_i[x_{rn}^m + (1-2\gamma_i)x_{rn}^\beta]} \right\}$$

s.t.

$$\sum_{r=1}^{s} u_r\left[y_{rj}^m + (1-2\gamma_r)y_{rj}^\beta\right] - \sum_{i=1}^{m} v_i\left[x_{ij}^m - (1-2\gamma_i)x_{ij}^\alpha\right] \leq 0, \qquad j = 1,\dots,n$$

$$v_i, u_r \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad i = 1,\dots,m; r = 1,\dots,s. \qquad (3.31)$$

The ideal solution for each DMU can be calculated using the equation (3.30). In order to find a set of common weights for inputs and outputs, any deviations from ideal solution for all DMUs should be minimized. Therefore, goal programming model (3.32) can be used for generating the common weights for all DMUs.

$$\min \sum_{j=1}^{n} (d_j^+ + d_j^-)$$

s.t.

$$\sum_{r=1}^{s} u_r[y_{ro}^m - (1-2\gamma_r)y_{ro}^\alpha] - \theta_{j,\gamma\leq0.5}^{\text{CDEA}} \sum_{i=1}^{m} v_i\left[x_{io}^m + (1-2\gamma_i)x_{io}^\beta\right] + d_j^- - d_j^+ = 0, \; j = 1,\dots,n$$

$$\sum_{r=1}^{s} u_r\left[y_{rj}^m + (1-2\gamma_r)y_{rj}^\beta\right] - \sum_{i=1}^{m} v_i\left[x_{ij}^m - (1-2\gamma_i)x_{ij}^\alpha\right] \leq 0, \qquad\qquad j = 1,\dots,n$$

$$v_i, u_r, d_j^-, d_j^+ \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i = 1,\dots,m; r = 1,\dots,s. \qquad (3.32)$$

Assume that the optimal solution of model (3.32) is (u_r^*, v_i^*). Then, the efficiency score of jth DMU using common weights (u_r^*, v_i^*) is calculated as follows:

$$\theta_{j,\gamma\leq0.5}^{\text{CWCDEA}} = \frac{\sum_{r=1}^{s} u_r[y_{r2}^m - (1-2\gamma_r)y_{r2}^\alpha]}{\sum_{i=1}^{m} v_i[x_{i2}^m + (1-2\gamma_i)x_{i2}^\beta]}. \qquad (3.33)$$

In the similar way, the goal programming CWCDEA model for $\gamma_i, \gamma_r > 0.5$ is expressed as follows:

$$\min \sum_{j=1}^{n} (d_j^+ + d_j^-)$$

s.t.

$$\sum_{r=1}^{s} u_r\left[y_{ro}^m + (2\gamma_r-1)y_{ro}^\beta\right] - \theta_{j,\gamma>0.5}^{\text{CDEA}} \sum_{i=1}^{m} v_i[x_{io}^m - (2\gamma_i-1)x_{io}^\alpha] + d_j^- - d_j^+ = 0, \quad j = 1,\dots,n$$

$$\sum_{r=1}^{s} u_r\left[y_{rj}^m - (2\gamma_r-1)y_{rj}^\alpha\right] - \sum_{i=1}^{m} v_i\left[x_{ij}^m + (2\gamma_i-1)x_{ij}^\beta\right] \leq 0, \qquad\qquad j = 1,\dots,n$$

$$v_i, u_r, d_j^-, d_j^+ \geq 0, \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad i = 1,\dots,m; r = 1,\dots,s. \qquad (3.34)$$

Based on the (u_r^*, v_i^*) generated by model (3.33), the efficiency score of DMUj is calculated using equation (3.34):

$$\theta_{j,\gamma>0.5}^{\text{CWCDEA}} = \frac{\sum_{r=1}^{s} u_r \left[y_{ro}^m + (2\gamma_r - 1)y_{ro}^\beta \right]}{\sum_{i=1}^{m} v_i [x_{io}^m - (2\gamma_i - 1)x_{io}^\alpha]}. \tag{3.35}$$

4. Data

In evaluating the performance of airlines, it is important to recognize all effective items to achieve the desired result. Selecting suitable inputs and outputs is one of the most important steps in evaluating airlines' performance. In this paper, the inputs and outputs are chosen based on previous studies. In order to evaluate the airlines, Merkert and Hensher [34] used labor (FTE), available ton-kilometer (ATK), ATK price and FTE price as first stage inputs and also revenue passenger-kilometers and revenue ton-kilometers indicators as first stage outputs. Also, they selected airline size, stage length, aircraft size, fleet age, aircraft families and aircraft manufacturers as explanatory variables in the second stage. Zhu [62] selected cost per available seat mile (ASM), salaries per ASM, wages per ASM and benefits per ASM, fuel expense per ASM, fuel cost and gallons of fuel as inputs. In addition, load factor and fleet size as intermediate variables and further used revenue passenger miles and passenger revenue have been chosen as outputs. Lu *et al.* [30] considered number of employees, fuel consumed, seating capacity, flight equipment, maintain expense, ground property and equipment as inputs, available seat miles and available ton miles as intermediate indicators. Also, they used revenue passenger miles and non-passenger revenue indicators as outputs in their study. Ha *et al.* used variables such as runway length, terminal size and employees as inputs. Also, passenger and cargo indicators are considered as outputs. Tavassoli *et al.* [48] used the number of passenger, labor and the number of cargo planes as inputs, passenger plane-kilometer and cargo plane-kilometer as intermediate indicators and finally, passenger-kilometer and ton-kilometer as outputs. Lozano and Gutiérrez [29] selected fuel cost, non-current assets, salary and other operating cost as input variables and available seat-kilometer and available ton-kilometer as intermediate indicators. Moreover, passenger-kilometer revenue and ton-kilometer revenue are chosen as output variables. Jain and Natarajan [21] selected total available ton kilometer and operating cost indicators as inputs and revenue passenger kilometer performed and non-passenger revenue as output indicators. Omrani and Soltanzadeh [37] used the number of employees as input, number of scheduled flights, available seat-km, available ton-km, number of the fleet seats as intermediate indicators. They also applied passenger-km performed and passenger ton-km performed as output variables.

In this study, to focus on the operational efficiency of airlines, inputs and outputs are selected as follows:

Inputs
– The number of employees (x_1): number of staff working in the airline such as engineering, attendant, pilot, flight and etc.
– Available seat-kilometer (x_2): sum of the products obtained by multiplying the number of passenger seats available for sale on each flight by the flight distance.
– Available ton-kilometer (x_3): sum of the products obtained by multiplying the number of tons available for the carriage of revenue load on each flight by the flight distance.
– Fleet seat (x_4): number of available seats in the fleet.

Outputs
– Number of flights (y_1): number of perfumed flights.
– Passenger-kilometer perfumed (y_2): sum of products obtained by multiplying the number of revenue passengers carried on each flight by the flight distance.
– Ton-kilometer perfumed (y_3): multiplication of carried weight (ton) in every origin and destination of flight by the distance between the same origin and destination.

TABLE 1. The data of Iranian airlines for year 2016.

Airline	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$y(1)$	$y(2)$	$y(3)$
Iranair	11 118	1 838 923	210 542	2 694 694	20 212	1 511 100	155 781
Iranairtour	683	1 216 026	109 171	1 471 151	9439	1 068 524	96 164
Ata	606	1 371 597	135 969	2 134 435	13 294	1 183 086	130268
Atrak	225	153 794	15 468	175 835	1009	122 514	13 079
Aseman	3158	1 925 524	181 954	2 578 760	25 697	1 682 751	152 382
Taban	832	1 248 996	124 233	1 535 950	10 225	1 161 512	113 569
Zagros	684	2 210 951	221 093	2 598 240	16 239	1 559 286	141 894
Pouyaair	103	11 509	44 190	21 440	567	8860	17 043
Gheshm	832	662 626	56 329	940 416	9913	556 859	47 212
Kaspian	539	1 070 398	131 057	1 319 386	8849	861 441	89 139
Kishair	826	1 517 847	166 213	1 843 682	14 005	1 260 049	126 949
Mahan	4363	3 295 267	518 048	3 782 430	20 730	2 591 769	234 907
Meraj	334	26 319	30 477	363 640	2164	18 679	23 304
Naft	590	493 163	48 814	813 830	10 255	395 334	34 808
Max	11 118	3 295 267	518 048	3 782 430	25 697	2 591 769	234 907
Min	103	11 509	15 468	21 440	567	8860	13 079
Mean	1778.071	1 217 352.857	142 397	1 590 992.071	11 614.143	998 697.429	98 321.357
Standard deviation	2835.198	893 046.886	122 513.380	1 050 881.340	7211.786	707 224.454	62 727.366

The real data have been collected from the Civil Aviation Organization (CAO) of Iran for the 2016 and belongs to 14 airlines of Iran. Table 1 shows the summary of the datasets. The max, min, mean and standard deviation of the variables are added to bottom rows of the Table 1. Also, Figure 1 shows radar graph for all airlines. To draw this chart, all data are normalized between 0 and 1 by dividing each column to the maximum value. As can be seen, in viewpoint of number of employees, Iranair, as a public airline, is the biggest one in Iran. In addition, Mahan has the best situation in viewpoint of ton-kilometer performed.

In order to consider uncertainty in data, L-R fuzzy numbers have been used according to Definition 3.4. The left and right spreads, α and β have considered as 10% of the modal value m. The fuzzy L-R fuzzy numbers of data are reported in Table 2.

5. RESULTS

In this section, the results of efficiency estimating of 14 airlines in Iran are evaluated. First, DEA model is applied to measure the efficiency scores of airlines. Then, for evaluating airlines based on common weights, the goal programming approach is used to find out common weights. Due to considering uncertainty in data, the credibility DEA (CDEA) model is used to handle the uncertainty in inputs and outputs. Finally, to find the common weights in fuzzy conditions, the proposed common weight credibility DEA (CWCDEA) model is applied to recalculate efficiency of airlines. In the following, the results of DEA, CWDEA, CDEA and CWCDEA models are discussed, separately.

5.1. DEA results

In this section, overall efficiency of Iranian airlines is calculated. The results are reported in Table 3 and Figure 2.

According to the results, ten airlines including, Iranairtour, Ata, Aseman, Taban, Zagros, Pouyaair, Gheshm, Kishair, Meraj and Naft are located on the efficient frontier and they are technically efficient. In the other word, it can be said, they are favorable airlines among 14 airline companies. Other four airlines have gained efficiency
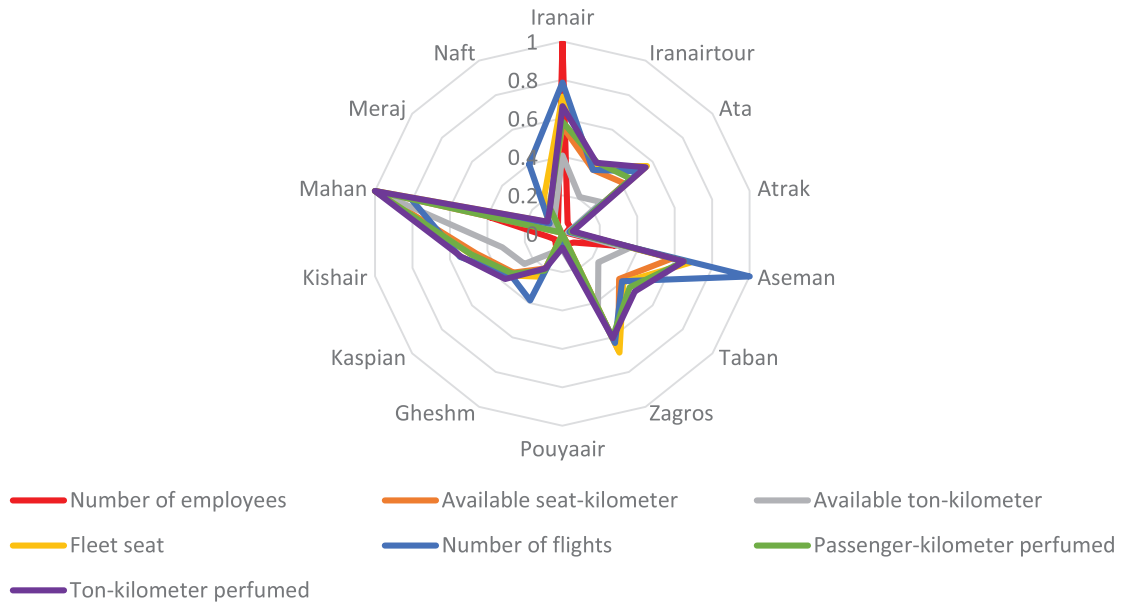
FIGURE 1. Radar graph for the airlines based on the normalized data.

scores above 0.90. According to the DEA results, Mahan, Iranair, Atrak and Kaspian are the least efficient airlines with scores 0.90, 0.91, 0.97 and 0.99, respectively. Notably some efficient airlines are not the ones that have famous in Iran and their names are less heard than other airlines. The results show that the DEA model is not suitable for fully ranking of airlines and it gives ten airlines the first rank. Hence, the results are somehow difficult to interpret.

5.2. Common weight DEA (CWDEA) results

As shown, the efficiency scores of ten airlines are equal to one and the DEA model is unable to distinguish among them. Also, the policymakers want to evaluate the airlines based on the common weights. Hence, in this section, the CWDEA model is applied for fully ranking of all airlines. The common weight-DEA model is used to determine common weight for each input and output of 14 DMU. For generating common weights for inputs and outputs, the model (3.28) is applied. The results are presented in Table 4 and Figure 2.

The results of CWDEA model are very favorable in compared to the DEA model. As can be seen, five airlines Aseman, Meraj, Pouyaair, Taban, Naft have been top ranked and others have ranked between 6 and 14. Since, the results of the CWDEA model are based on common weights for all airlines, hence, policymakers can identify and resolve corporate weaknesses and strengths of airlines, fairly. Table 4 shows that Zagros with efficiency score 0.77 is at the bottom of the ranking. Subsequently, Mahan and Atrak are least efficient airlines after Zagros with scores 0.83 and 0.85.

5.3. Credibility DEA (CDEA) results

To handle the uncertainty, fuzzy credibility theory was used and CDEA models (3.24) and (3.25) are developed. In models (3.24) and (3.25), there is credibility parameter λ which can be set by decision-maker. As mentioned before, for $\lambda \leq 0.5$, CDEA model (3.24) and for $\lambda > 0.5$, CDEA model (3.25) are applied. The results of models (3.24) and (3.25) for $\lambda = 0.2, 0.4, 0.6, 0.8$ are reported in Table 3. It is notable that the efficiencies reported in Table 3 are normalized between 0 and 1. Also, the scores for $\lambda = 0.8$ are shown in Figure 2.

TABLE 2. The data in form of L-R fuzzy numbers $\widetilde{A} = (m, \alpha, \beta)_{\mathrm{LR}}$.

Airline	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$y(1)$	$y(2)$	$y(3)$
IranAir	(11 118, 1111.8, 1111.8)	(1 838 923, 183 892.3, 183 892.3)	(1 838 923, 183 892.3, 183 892.3)	(210 542, 21 054.2, 21 054.2)	(2 694 694, 269 469.4, 269 469.4)	(20 212, 2021.2, 2021.2)	(1 511 100, 151 110, 151 110)
IranAirtour	(683, 68.3, 68.3)	(1 216 026, 121 602.6, 121 602.6)	(1 216 026, 121 602.6, 121 602.6)	(109 171, 10 917.1, 10 917.1)	(1 471 151, 147 115.1, 147 115.1)	(9439, 943.9, 943.9)	(1 068 524, 106 852.4, 106 852.4)
Ata	(606, 60.6, 60.6)	(1 371 597, 137 159.7, 137 159.7)	(1 371 597, 137 159.7, 137 159.7)	(135 969, 13 596.9, 13 596.9)	(2 134 435, 213 443.5, 213 443.5)	(13 294, 1329.4, 1329.4)	(1 183 086, 118 308.6, 118 308.6)
Atrak	(225, 22.5, 22.5)	(153 794, 15 379.4, 15 379.4)	(153 794, 15 379.4, 15 379.4)	(15 468, 1546.8, 1546.8)	(175 835, 17 583.5, 17 583.5)	(1009, 100.9, 100.9)	(122 514, 12 251.4, 12 251.4)
Aseman	(3158, 315.8, 315.8)	(1 925 524, 192 552.4, 192 552.4)	(1 925 524, 192 552.4, 192 552.4)	(181 954, 18 195.4, 18 195.4)	(2 578 760, 257 876, 257 876)	(25 697, 2569.7, 2569.7)	(1 682 751, 168 275.1, 168 275.1)
Taban	(832, 83.2, 83.2)	(1 248 996, 124 899.6, 124 899.6)	(1 248 996, 124 899.6, 124 899.6)	(124 233, 12 423.3, 12 423.3)	(1 535 950, 153 595, 153 595)	(10 225, 1022.5, 1022.5)	(1 161 512, 116 151.2, 116 151.2)
Zagros	(684, 68.4, 68.4)	(2 210 951, 221 095.1, 221 095.1)	(2 210 951, 221 095.1, 221 095.1)	(221 093, 22 109.3, 22 109.3)	(2 598 240, 259 824, 259 824)	(16 239, 1623.9, 1623.9)	(1 559 286, 155 928.6, 155 928.6)
Pouyaair	(103, 10.3, 10.3)	(11 509, 1150.9, 1150.9)	(11 509, 1150.9, 1150.9)	(44 190, 4419, 4419)	(21 440, 2144, 2144)	(567, 56.7, 56.7)	(8860, 886, 886)
Gheshm	(832, 83.2, 83.2)	(662 626, 66 262.6, 66 262.6)	(662 626, 66 262.6, 66 262.6)	(56 329, 5632.9, 5632.9)	(940 416, 94 041.6, 94 041.6)	(9913, 991.3, 991.3)	(556 859, 55 685.9, 55 685.9)
Kaspian	(539, 53.9, 53.9)	(1 070 398, 107 039.8, 107 039.8)	(1 070 398, 107 039.8, 107 039.8)	(131 057, 13 105.7, 13 105.7)	(1 319 386, 131 938.6, 131 938.6)	(8849, 884.9, 884.9)	(861 441, 86 144.1, 86 144.1)
Kishair	(826, 82.6, 82.6)	(1 517 847, 151 784.7, 151 784.7)	(1 517 847, 151 784.7, 151 784.7)	(166 213, 16 621.3, 16 621.3)	(1 843 682, 184 368.2, 184 368.2)	(14 005, 1400.5, 1400.5)	(1 260 049, 126 004.9, 126 004.9)
Mahan	(4363, 436.3, 436.3)	(3 295 267, 329 526.7, 329 526.7)	(3 295 267, 329 526.7, 329 526.7)	(518 048, 51 804.8, 51 804.8)	(3 782 430, 378 243, 378 243)	(20 730, 2073, 2073)	(2 591 769, 259 176.9, 259 176.9)
Meraj	(334, 33.4, 33.4)	(26 319, 2631.9, 2631.9)	(26 319, 2631.9, 2631.9)	(30 477, 3047.7, 3047.7)	(363 640, 36 364, 36 364)	(2164, 216.4, 216.4)	(18 679, 1867.9, 1867.9)
Naft	(590, 59, 59)	(493 163, 49 316.3, 49 316.3)	(493 163, 49 316.3, 49 316.3)	(48 814, 4881.4, 4881.4)	(813 830, 81 383, 81 383)	(10 255, 1025.5, 1025.5)	(395 334, 39 533.4, 39 533.4)

The results of DEA and CDEA show that all DMUs ranked same in all specified λ. By increasing the value of λ, the efficiency of each airline changes and becomes larger a little. In the CDEA model, for $\lambda \le 0.5$, the scores are different from the DEA model. On the other hand, for $\lambda > 0.5$, the scores generated by DEA and CDEA models are the same. For instance, for $\lambda = 0.8$ the scores and ranks are similar. Similar to the DEA model, for $\lambda = 0.6, 0.8$, the scores of airlines are really close together and there is no significant difference. Therefore, analyzing this system based on these data is very difficult and even impossible.

TABLE 3. The results of DEA and CDEA model with different λ.

| Airline | DEA | | CDEA | | | | | | | | | |
			0.2		0.4		0.6		0.8		1.0	
	Score	Norm	Score	Norm	Score	Norm	Score	Norm	Score	Norm	Score	Norm
Iranair	0.916	0.916	0.805	0.805	0.877	0.877	0.957	0.916	1.042	0.916	1.1308	0.916
Iranairtour	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Ata	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Atrak	0.972	0.972	0.855	0.855	0.931	0.931	1.015	0.972	1.105	0.972	1.2001	0.972
Aseman	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Taban	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Zagros	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Pouyaair	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Gheshm	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Kaspian	0.995	0.995	0.875	0.875	0.953	0.953	1.039	0.995	1.131	0.995	1.2284	0.995
Kishair	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Mahan	0.906	0.906	0.797	0.797	0.867	0.867	0.946	0.906	1.030	0.906	1.1186	0.906
Meraj	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
Naft	1.000	1.000	0.879	0.879	0.957	0.957	1.045	1.000	1.137	1.000	1.2346	1.000
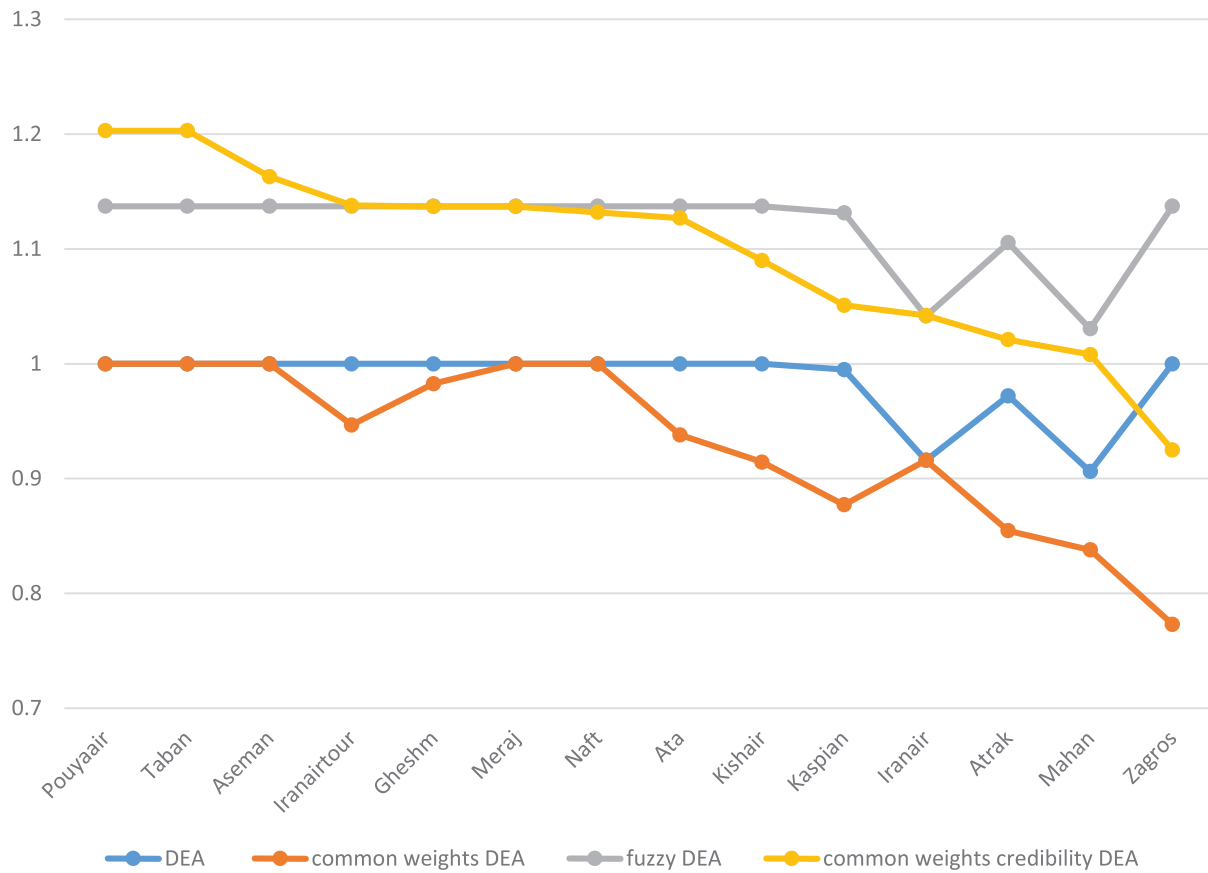
FIGURE 2. The results of different DEA models.
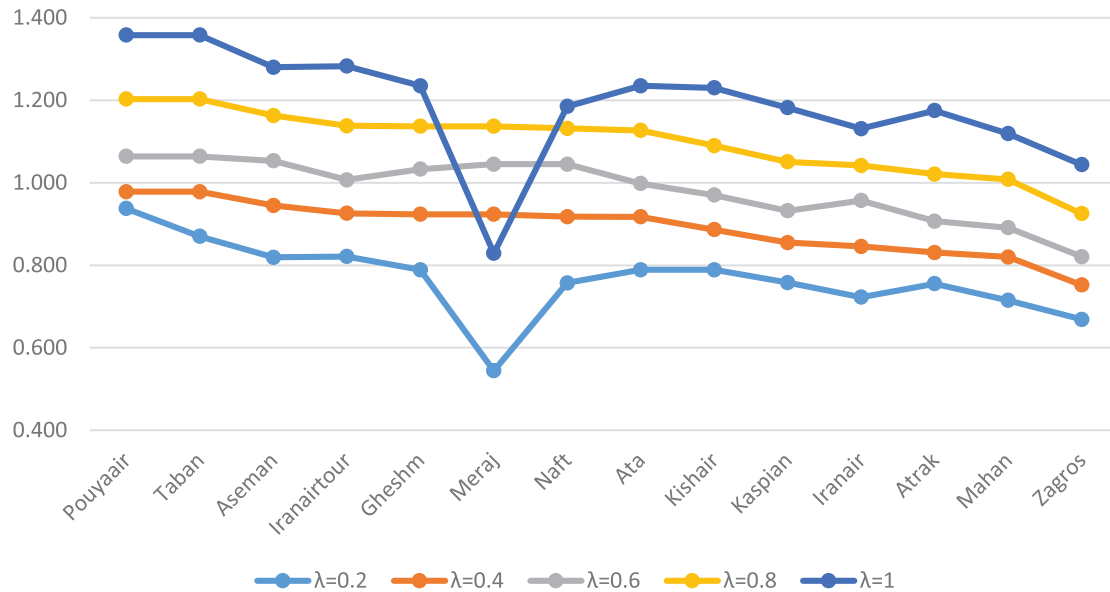
FIGURE 3. The efficiencies generated by CWCDEA in different values of λ.

TABLE 4. The results of CWDEA and CWCDEA models with different λ.

Airline	CWDEA		CWCDEA									
			0.2		0.4		0.6		0.8		1.0	
	Score	Normalized	Score	Normalized	Score	Normalized	Score	Normalized	Score	Normalized	Score	Normalized
Iranair	0.916	0.916	0.723	0.723	0.846	0.846	0.957	0.899	1.042	0.866	1.131	0.833
Iranairtour	0.947	0.947	0.821	0.821	0.926	0.926	1.007	0.947	1.138	0.946	1.283	0.945
Ata	0.938	0.938	0.789	0.789	0.917	0.917	0.998	0.938	1.127	0.938	1.235	0.909
Atrak	0.855	0.855	0.755	0.755	0.831	0.831	0.907	0.853	1.021	0.849	1.175	0.865
Aseman	1.000	1.000	0.819	0.819	0.945	0.945	1.053	0.990	1.163	0.967	1.280	0.943
Taban	1.000	1.000	0.870	0.870	0.978	0.978	1.064	1.000	1.203	1.000	1.358	1.000
Zagros	0.773	0.773	0.669	0.669	0.752	0.752	0.821	0.772	0.925	0.769	1.044	0.768
Pouyaair	1.000	1.000	0.938	0.938	0.978	0.978	1.064	1.000	1.203	1.000	1.358	1.000
Gheshm	0.983	0.983	0.789	0.789	0.923	0.923	1.033	0.971	1.137	0.946	1.235	0.909
Kaspian	0.877	0.877	0.758	0.758	0.855	0.855	0.932	0.876	1.051	0.874	1.182	0.871
Kishair	0.914	0.914	0.789	0.789	0.886	0.886	0.970	0.912	1.090	0.906	1.230	0.906
Mahan	0.838	0.838	0.715	0.715	0.820	0.820	0.891	0.838	1.008	0.838	1.119	0.824
Meraj	1.000	1.000	0.544	0.544	0.923	0.923	1.045	0.982	1.137	0.946	0.829	0.610
Naft	1.000	1.000	0.757	0.757	0.918	0.918	1.045	0.982	1.132	0.941	1.185	0.873

5.4. Common weight credibility DEA (CWCDEA) results

In this section, the results of the CWCDEA model are reported. For $\lambda \leq 0.5$, CWCDEA model (3.32) and for $\lambda > 0.5$, CWCDEA model (3.34) are applied. The scores are normalized and shown in Table 4 and Figure 3. Also, the result of model (3.34) for $\lambda = 0.8$ is presented in Figure 2.

For $\lambda = 0.8$, Pouyaair and Taban airlines are in the first rank. Also, Aseman and Iranairtour are in the third and fourth rank, respectively. On the other side, Zagros is the last airline according to scores generated by the CWCDEA model (3.34). Mahan and Atrak with scores 1.008 and 1.021 are ranked as 12th and 13th airlines. It is clear that the CWCDEA model has increased the distinguish power of the DEA model. CWCDEA model uses to find out better and completely ranking of all DMUs which can make decision making easier for policymakers

by considering same value for each indicator for all airlines. The CWCDEA model has been implemented with five different λ which results have been reported in Table 4 and Figure 3. When λ has been increased to 0.8, noticeable changes have been seen in scores. Totally, there is a decreasing trend between efficiencies.

6. CONCLUSION

In this paper, 14 airlines in Iran were assessed based on the common weight credibility DEA (CWCDEA) model. To develop a CWCDEA model, first, the DEA model was extended using fuzzy credibility theory. Actually, for each constraint of DEA model, a credibility counterpart constraint was written. Then, the scores generated by credibility DEA model were considered as ideal solution in a multi-objective programming framework. To solve the proposed multi-objective DEA, goal programming approach was used. The goal programming model minimized deviation from the ideal solutions and found the common weights of inputs and outputs. Based on generated common weights, 14 airlines were evaluated and ranked. According to the results of the CWCDEA model, Pouyaair and Taban were the first ranks and Zagros was the least efficient airline. To analyze the results of the proposed CWCDEA model, this paper examined the results with different λs. The results showed that with increasing λ, noticeable changes occurred in scores. As a conclusion, there was an increasing trend in efficiency scores with increasing λ.

One of the limitations and challenges of this research was the lack of financial data. Most of the airlines didn't report them due to the sake of confidentiality. Also, the data was not available for all Iranians' airlines so that only fourteen airlines have been considered in this study. For future research, the proposed models can be developed by other uncertain approaches like robust optimization. Also, the credibility theory can be considered in other DEA models such as SBM, RAM and so on. Furthermore, more study such as Meraj airline's break in Fiqure 3.3 for different amounts of λ or calculating total variability of the efficiency scores can be done.

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