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EFFECTS OF GREEN IMPROVEMENT AND PRICING POLICIES IN A DOUBLE DUAL-CHANNEL COMPETITIVE SUPPLY CHAIN UNDER DECISION-MAKING POWER STRATEGIES

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Abstract. With the intensive growth of internet use, the customers choose the online market as the right preference. Hence, manufacturers are attracted to launch an online channel that includes a retail channel. To maintain the versatile demand types of products, a retailer is to stock more than one product of the same category, and consequently, he has to purchase products from different manufacturers. This article formulates a dual-channel supply chain model with two manufacturers and a standard retailer, where the optimal online prices, retail prices, wholesale prices, and level of green improvements are decided under different types of decision making power strategies such as Centralized, joint manufacturers Stackelberg, separate Stackelberg, Nash games are investigated. The optimal results are derived and compared with the help of a numerical example. Moreover, a sensitivity analysis is performed to scrutinize the effect of some important parameters. It is found that the green level is higher in a double dual-channel model than in a single dual-channel model. Moreover, the own-channel price sensitivity parameters affect the profit functions of the members negatively. The manufacturers must control the cost-coefficients of greening to increase the green level of the manufacturing products.

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1. Introduction

Sustainability is an essential aspect of today's world because of the strong links between it and the importance of environmental issues, and most countries are working on various directions to develop sustainability [1, 13]. Manufacturing innovative green products are one of the significant characteristics of these topics.

A green product is defined as comparatively less detrimental to the environment than a conventional product. This type of product is known as an eco-friendly product. The concept of eco-friendly products was first introduced by Navinchandra [27] to increase the balance of products with environmental aspects without harming the quality and the efficacy of the product. Nowadays, customers' consciousness of the environment allows the industries to manufacture more green innovative products. Also, the Government regulations pressure the industries to go for manufacture eco-friendly products [26]. In recent days, green supply chain (SCN) management has

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had an emerging positive effect on the environment because of its features of reducing carbon emission [9,39]. In the present study, two manufacturers are studied who can produce green improved products and supply them either through online channels (OC) or retail channels (RC).

The increasing convenience of assessing the internet, the customers' changing shopping behavior, and the people's busy lifestyle are the key factors behind expanding the online market. The possible advantages of an OC are as follows: firstly, the OC provides products directly to the customers to be very convenient for them. The online price is lower than the retail price since there is no third party involved. Secondly, the customers have more options to choose from than any retail shop, and thirdly they can get their desired accessories at doorsteps. The market is also open 24 hours a day, seven days a week [31]. The RCs also have some points to be chosen by the customers. Firstly, the retailers can provide retail services; for example, the customers can physically experience the products in a retail shop, experience the retailers' free demonstration, etc. Furthermore, in the RC, the customers get instant delivery of the products [15, 23]. Besides, customers can easily trust to buy products from the retail shop. These factors persuade manufacturers to open an OC and a RC, allowing the SCN to meet the needs of various types of customers. As a result, most reputed manufacturers have opened OCs and traditional RCs to supply their products [17].

Previous literature in SCN mainly focused on the dual-channel (DC) consisting of one retailer and a manufacturer and analyzing their optimal decisions and economic goals. Zhao *et al.* [45] recently developed a model with two suppliers and a traditional retailer to examine pricing decisions under various decision-making power strategies.

The present study considers two DC SCNs with two manufacturer and a common retailer, and assumes both the manufacturer produce products with different green improvement (GI) levels. Also, the members' decision-making power strategies are considered to analyze the competition market structures. This research analyzes how greenness and the option in substitute products affect the market's demands. The study examined how the greenness of products influences demand rates and players' pricing decisions and the most profitable strategy for SCN members (See Fig. 1). DC SCN can be applied to many real-world situations relevant to the proposed model. For example, two manufacturing companies, Maytag and GE, sell their products through common retailers BestBuy and Sears. Customers can find their respective wish-listed products from a common retailer that will increase the retailer's selling rate as well for the manufacturers [25]. Furthermore, two footwear manufacturing companies produce footwear and sell it either through their own online stores or through retail stores. The retail store may have the capability to store and sell both brands. Nowadays, manufacturers are mindful of the environment, and they try to use eco-friendly materials for packaging and use organic raw materials for production. Similar types of SCN can be observed in the textile industries. Assume that P and Q are two textile industries that produce yarns and sell them online or through the retail store. A retail store may have both the yarn from the industries P and Q. Consequently, retailers are common to industries P and Q. Then, they can decide which GI level and what price to set for the products in order to maximize profits. This type of SCN model can also be found in the electronics industry.

This model is presented here in order to answer the following research questions:

1. Under different leadership problems, what is the optimal price, the optimal GI level, and the optimal benefits in DCSC?
2. Under what circumstances do the players make their corresponding maximum profit in the presence of greenness of the products?
3. How do the cost coefficients of the GI and the pricing sensitivity parameters influence the decision variables (DVs) and the members' profits?
4. How does the presence of two DC SCN together impact the profits and decisions of the chain's members?

This article is arranged as follows. Section 2 attract the relative literature review. Then, Section 3 describes the problem and states the present study's assumptions. In Section 4, the model is formulated, and the methodology is also discussed. Section 5 deals with numerical examples and the sensitivity analysis of the parameters. After that, Section 6 states the managerial real insights and implementations. Finally, in Section 7, the findings and future directions of the research studies are presented.

2. Literature Review

This section mainly focused on the vital research conducted in this direction and visualized the research gaps. This section starts with the outlines of the recent literature on the DC SCN, observes various GI research, and finally proposes the overview and research gaps.

2.1. Dual-channel supply chain

Chen [4] calculates the effect of pricing decisions and advertising methods on the DC SCN under different power-making strategies. It also investigates the optimal level of advertisement, investment, and selling prices. Huang *et al.* [18] consider a DC SCN with disrupted production type and analyze the optimal pricing decisions, production quantity, and the corresponding profits. They also draw some managerial insights on the strength of numerical examples. Tiaojun and Shi [37] examine the channel priority approaches under shortage due to arbitrary yields. The optimal DVs and profit functions (PFs) are discussed under different channel coordinations. Batarfi *et al.* [2] examine the effect of introducing a DC with a two-level SCN, in which the manufacturer sells standard products through the RC, and a particular type of customized products are sold through the OC. Chen *et al.* [6] consider a DC supply along with pricing and quality decisions and also analyze the effect of adding a new channel on the existing SCN. Wang *et al.* [38] investigate the price and servicing decisions of products with a retailer and two manufacturers, and numerical examples are formed to examine the optimal results. All these articles have investigated a different kind of DC SCN consisting of non-green products. Jafari *et al.* [19] formulate a DC SCN with a manufacturer and more than one retailer where a linear discount policy is proposed. Also, they investigate the equilibrium decisions with respect to different decision-making powers of the members of the SCN. Nowadays, the awareness about the environment increases very fast, and consequently, some researchers [20, 22] take an interest in the Green products in the DC SCN. Heydari *et al.* [14] consider a SCN where the manufacturer conducts GI, and the optimal pricing decisions and the channel coordination is discussed. Pi *et al.* [29] consider a DC SCN and service strategies involving two retailers and a common manufacturer and construct a numerical example with a hypothetical data set and find some essential managerial insights. Ranjbar *et al.* [32] formulate a DC three-level closed-loop (CL) SCN with a manufacturer, a retailer, and a collector. They also assess the players' optimal decisions in various leadership game models. Rahmani and Yavari [30] consider a DC SCN in two decision-making structures. Then the pricing, GI level, and profits are calculated under demand disruption. Aslani and Heydari [1] discuss the issues of pricing, green level, and the coordination of the channels in a DC SCN. Moreover, transshipment contrast is proposed to analyze its applications and fulfillments. Ranjan and Jha [31] examines the pricing decisions and the coordination phenomena in a SCN where the retailer sells the non-green product, but the OC sells GI products, and finally evaluates the optimal values of the DVs. Gao *et al.* [10] consider a DC SCN where the government set a minimum GI level to be maintained. Furthermore, they discuss the impact of Greenness on the optimal DVs of the members of the chain. Chen *et al.* [7] examine the optimal decisions of the members in a DC SCN under different game-theoretic frameworks. Also, they focus on the retail service, manufacturer service, and quality effort and obtain some important insights. After that, a multi-channel SC is considered by Sarkar and Pal [33] where a single manufacturer deals with two retailers and also opens an OC. Also, the manufacturer has provided direct service to the customers. Then, the equilibrium decisions are obtained, and the best profitable strategy is detected. Cao *et al.* [3] study the production quantity and pricing decisions in a closed-loop DC SCN under two types of subsidy policies. Also, they investigate the best profitable policy. Esmailnezhad and Saidi-mehrabad [8] propose a mix-integer non-linear model to study the manufacturing systems in a three-stage SCN to act with the customers' demand functions. Sensitivity and a real case are analyzed to provide important insights. Also, a DC SCN is composed of a retailer and a supplier by Yan *et al.* [41] to discuss the optimal decisions of the SCN members under decentralized models with demand disruption. The results show that the performance of SCN can be improved by enriching the revenue-sharing contract. Taleizadeh *et al.* [36] investigate the effect of the pricing and quality of the products under the return policy. A solution algorithm is described and performed to solve the numerical example and analyze the parameters' sensitivity. On the other hand, [35] and [5] have also studied in DC SC models and investigated the optimal decisions of the members.

2.2. Green innovation

SCN articles on green innovation discuss strategies to increase competitiveness and maximize market share to produce environmentally friendly products. According to [43], the GI level of a product can be improved by a manufacturer. Papagiannakis *et al.* [28] state that the Green innovated products can increase the profit of the SCN and enhance the environmental concerns. Ghosh and Shah [11] investigate a SCN with a retailer and a manufacturer. Under different channel strategies, the pricing decisions, GI level, and profits are discussed. Zhou and Ye [46] demonstrated a carbon-neutral DC SCN with a producer and a retailer. The optimal strategies are being evaluated and compared between the single-channel and DC SCN. Wang and Song [40] consider a DC SCN with uncertain demand, and the retailer provides promotional effort. The manufacturer decides the optimal level of GI, and the optimal pricing decisions are calculated. Zhang *et al.* [44] develops a two-stage DC SCN and investigates the best pricing and greening strategies under two different decision scenarios. Gao *et al.* [10] studies two types of green products in a DC SCN and obtains the corresponding players' optimal Decisions. The significant findings reveal the impact of the eco-level policy set by the government on various products. Yan *et al.* [42] explores a CL SCN model with a socially responsible manufacturer, a retailer, and a third-party recycler under four different decision-making strategies. As a result of the findings, the recycling rate of the wastes is increased, as well as the manufacturer's performance to become more responsible corporate social life. Taleizadeh *et al.* [36] study the effects of carbon emission and remanufacturing on a DC SCN in both the direction of logistics. The optimal pricing and collection strategies are also investigated. Li *et al.* [24] investigate the optimal DV and PF of the members of a green SCN under the Stackelberg game with different information patterns. Here, a manufacturer produces green products and sells them through two competitive retailers. Li and Liu [21] consider a two-echelon green SCN with government interventions with a supplier and a retailer under fuzzy uncertainties. Then a numerical example is presented, and also the sensitivity analysis is conducted to obtain the important managerial insights.

2.3. Research gaps and contributions

Beyond these literature reviews, the comparison table (see Tab. 1) is constructed to identify the research gaps. As shown in Table 1, the main research gaps and contributions of the present study are

1. A few articles exist considering the vertical and horizontal competition in a DC SCN. However, there is no literature considering both the competitions in a DC green SCN. This study tries to fill the gap by assuming both channel competition in a green SCN.
2. There are multiple kinds of literature on the concept of discount policy on green products, but no such literature discussed the effect of discount policy in a DC SCN consisting of two substitute products. In order to fill this gap in research, the DC green SCN is formulated where two manufacturers manufacture green products, and the manufacturers set the discount policy.
3. No literature exists that discusses all of the game-theoretic approaches the members may take. In this model, all the possible strategies are formulated and compared the profits of the members, which helps them find the best profitable strategy.

3. Problem description

The present research investigates a DC green SCN, including two manufacturers and one identical retailer (See Fig. 1). The manufacturers produce their desirable green improved products (say Product 1 (P1) and Product 2 (P2)) and sell them through either individual OC or the RC.

TABLE 1. Comparison table.

Papers	Assumption and Model formulation								DVs and Strategies						
	SCN		DP [(a)]	GI	Players		Competitions		DV		Strategies				
	DC	Green			M [(b)]	R [(c)]	V [(d)]	H [(e)]	Pr [(f)]	GI Level	CP	MS	M1S	M2S	NG
Wang and Song [40]	✓	✓	×	✓	1	1	✓	×	✓	✓	✓	✓	×	×	×
Wang *et al.* [38]	✓	×	×	×	2	1	✓	✓	✓	×	×	✓	✓	✓	×
Aslani and Heydari [1]	✓	✓	×	✓	1	1	✓	×	✓	✓	✓	✓	×	×	×
Ranjan and Jha [31]	✓	✓	×	✓	1	1	✓	×	✓	✓	✓	✓	×	×	×
Heydari *et al.* [14]	✓	✓	×	✓	1	1 [(g)]	✓	×	✓	✓	✓	✓	×	×	×
Rahmani and Yavari [30]	✓	✓	×	✓	1	1	✓	×	✓	✓	✓	✓	×	×	×
Pi *et al.* [29]	✓	×	×	✓	1	2	✓	✓	✓	×	✓	✓	×	×	×
Ranjbar *et al.* [32]	✓	×	×	×	1	1 [(h)]	✓	×	✓	×	✓	✓	×	×	×
Huang *et al.* [18]	✓	×	×	×	1	1	✓	×	✓	×	✓	✓	×	×	×
Batarfi *et al.* [2]	✓	×	×	×	1	1	✓	×	✓	×	✓	✓	×	×	×
Zhou and Ye [46]	✓	×	×	×	1	1	✓	×	✓	×	×	✓	×	×	×
Chen *et al.* [6]	✓	×	×	×	1	1	✓	×	✓	×	✓	✓	×	×	×
Zhao *et al.* [45]	✓	×	×	×	2	1	✓	✓	✓	×	×	✓	✓	✓	✓
This Paper	✓	✓	✓	✓	2	1	✓	✓	✓	✓	✓	✓	✓	✓	✓

Notes. [(a)]Discount policy, [(b)]Manufacturer, [(c)]Retailer, [(d)]Vertical, [(e)]Horizontal, [(f)]Pricing, [(g)]with a distributor, [(h)]with a collector.

TABLE 2. Notations.

	Parameters
a_i $(i = 1, 2, 3, 4)$	Maximum possible demand of the channels
α_i $(i = 1, 2, 3, 4)$	Own-channel price sensitivities on demand rates
γ_j $(j = 1, 2)\,(> 0)$	Demand sensitivity co-efficient of the GI
$\beta\,(> 0)$	Fractional part of recyclable product of one unit used product
	Decision variables
$\theta_j\,(j = 1, 2)$	GI levels of P1 and P2 $(0 < \theta_j < \theta_{max})$
$p_{rj}\,(j = 1, 2)$	Retail price for P1 and P2 ($ per unit).
$w_j\,(j = 1, 2)$	Wholesale price for P1 and P2 ($ per unit)
$p_j\,(j = 1, 2)$	Online prices for P1 and P2 ($ per unit)
	Dependent variables
$D_j\,(j = 1, 2)$	customer demand of P1 and P2 in OC
$D_{rj}\,(j = 1, 2)$	Customer demand of P1 and P2 in RC
Π_{m1}, Π_{m2} and Π_r	M1's, M2's and retailer's PFs ($)

Consequently, the retailer sells both the products (*i.e.*, P1 and P2) at different retail prices. Manufacturers have the power to control the level of GI of their respective products. In either case, the customer will select the RC or the OC and then choose whether to purchase P1 or P2.

Following that, different decision-making mechanisms were considered, such as Centralized policy(CP), Nash game(NG), and three decentralized models (Manufacturers Stackelberg (MS), Manufacturer 1 Stackelberg (M1S), Manufacturer 2 Stackelberg (M2S)). Under the decentralized models, players take decisions separately, and the sequence of decisions is also different. In CP, the members will make their decisions collaboratively, and consequently, the total profit (TP) of the SCN would be maximum under CP. Furthermore, we will compare the DV and the profits of the members in CP with the different decentralized models. Thus, the five other leadership models are CP, MS, M1S, and M2S and NG. Here, Table 2 describes the used notation of the parameters, DVs, PFs throughout the paper. The following assumptions have been made to validate the proposed model.

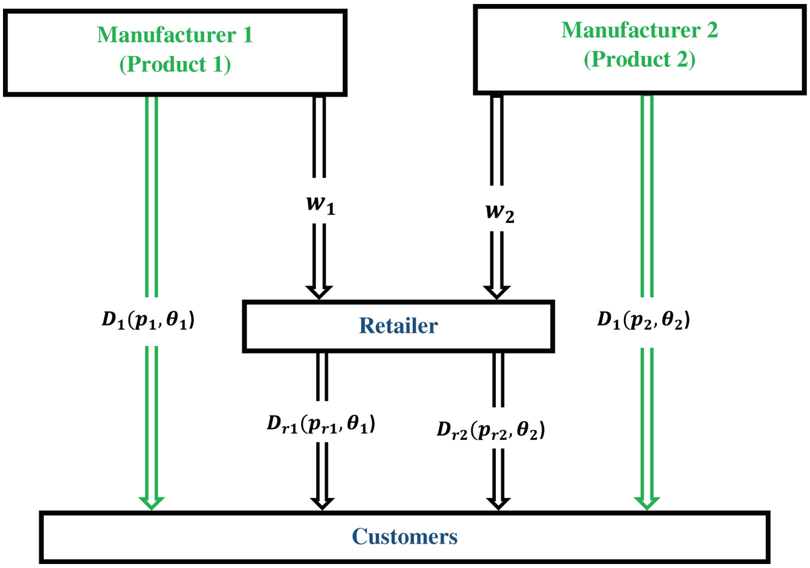

FIGURE 1. Diagram of dual-channel supply chain of the proposed model.

3.1. Assumptions

- Manufacturer 1 (M1) produces P1 with the GI level θ_1 and sells it through the RC at wholesale price w_1 and sells directly to the customers through OC at a price p_1. Consequently, P2 is produced by Manufacturer 2 (M2). M2 imposed the GI level θ_2 while producing the product and sold it at wholesale price w_2 to the retailer and fixed OC price p_2. The retail prices of P1 and P2 are p_{r1} and p_{r2} respectively.
- All the demands are taken to be linear dependence pricing decisions, namely online prices (p_1 and p_2) as well as retail prices (p_{r1} and p_{r2}) and level of GIs [24, 38, 43, 46]. It is assumed that the demands are downward directing of the own channel pricing, upward directing of the cross channel pricing, and upward directing of the products' GI. For the demand for simplicity, the production costs of M_1 and M_2 are considered 0. Let D_1 and D_2 be the customer demands through the OC of P1 and P2, respectively. The customer demands of P1 and P2 through RC be D_{r1} and D_{r2} respectively, and the demand functions can be formed as follows:

$$D_1 = a_1 - \alpha_1 p_1 + \beta(p_2 + p_{r1} + p_{r2}) + \gamma_1\theta_1 \tag{3.1}$$
$$D_2 = a_2 - \alpha_2 p_2 + \beta(p_1 + p_{r1} + p_{r2}) + \gamma_2\theta_2 \tag{3.2}$$
$$D_{r1} = a_3 - \alpha_3 p_{r1} + \beta(p_1 + p_2 + p_{r2}) + \gamma_1\theta_1 \tag{3.3}$$
$$D_{r2} = a_4 - \alpha_4 p_{r2} + \beta(p_1 + p_2 + p_{r1}) + \gamma_2\theta_2. \tag{3.4}$$

  The self-price sensitivity parameters of each of the demand function are more effective than the cross channel price sensitivity parameter i,e., $\alpha_i > \beta$, $i = 1, 2, 3, 4$ [12, 16, 31]. Furthermore, the impact of self-price sensitivity parameters is greater than the sensitivity parameters of GIs of demand rates ($i.e.$, $\gamma_j, j = 1, 2$) $i.e.$, $\alpha_i > \gamma_j$. It is also considered that both the manufacturer maintain a fixed ratio of OC and RC so that online prices remain lesser than the retail price. Mathematically, $\frac{p_i}{p_{ri}} = k_i$, where $k_i < 1$ is the fixed number determined by the corresponding manufacturer.
- The manufacturers are not subjected to the effects of marginal cost. Moreover, it has some fixed cost for imposing the GI on the manufacturers. The cost functions of the GI is considered as a convex function $c(\theta_i) = \frac{1}{2}\zeta_i\theta_i^2$ $(i = 1, 2)$, where ζ_i is the green cost coefficient [11, 38].

3.2. Notation

The notations are used to develop the model are list in the following table.

4. Model formulation

With the help of above assumptions, we obtain the PFs of the players of the model as follows:

$$\Pi_{m1} = p_1 D_1 + w_1 D_{r1} - \frac{1}{2}\zeta_1\theta_1^2 \tag{4.1}$$

$$\Pi_{m2} = p_2 D_2 + w_2 D_{r2} - \frac{1}{2}\zeta_2\theta_2^2 \tag{4.2}$$

$$\Pi_r = (p_{r1} - w_1)D_{r1} + (p_{r2} - w_2)D_{r2}, \tag{4.3}$$

where the notations are described previously and D_1, D_2, D_{r1} and D_{r2} are taken from (3.1), (3.2), (3.3) and (3.4) respectively. The subscripts $m1$, $m2$ and r represent the M1, the M2 and the retailer respectively.

The TP of the SCN is the addition of the individual players' profits and is obtained as

$$\begin{aligned}\Pi_t &= \Pi_{m1} + \Pi_{m2} + \Pi_r \\ &= p_1 D_1 + p_2 D_2 + p_{r1} D_{r1} + p_{r2} D_{r2} - \frac{1}{2}(\zeta_1\theta_1^2 + \zeta_2\theta_2^2).\end{aligned} \tag{4.4}$$

Now, We consider the following game theoretic models:

1. Centralized policy
2. Manufacturers Stackelberg policy
3. Manufacturer 1 Stackelberg policy
4. Manufacturer 2 Stackelberg policy
5. Nash policy

Under each game-theoretic approach, the procedure to determine optimal decisions and consequently the profits of the players are discussed.

4.1. Centralized policy

All SCN players operate as a single-player under this strategy, *i.e.*, there is a central decision-maker to make the decisions for the whole system. The results of this model are usually used as a benchmark for comparing with the decentralized models, and there is only one overall profit for the entire SCN. The TP of the SCN is obtained from the equation (4.4), and it depends on the retail prices (p_{r1} and p_{r2}) and the GI levels (θ_1 and θ_2) of the P1 and P2. Simplifying the equation (4.4), we have

$$\Pi_t = C_1 p_{r1}^2 + C_2 p_{r2}^2 + C_3 p_{r1} p_{r2} + C_4 p_{r1}\theta_1 + C_5 p_{r2}\theta_2 + C_6 p_{r1} + C_7 p_{r2} - \frac{(\zeta_1\theta_1^2)}{2} - \frac{(\zeta_2\theta_2^2)}{2}, \tag{4.5}$$

where
$C_1 = (-k_1^2\alpha_1 - \alpha_3 + 2k_1\beta); C_2 = (-k_2^2\alpha_2 - \alpha_4 + 2k_2\beta); C_3 = \beta(2 + 2k_1 + 2k_2 + 2k_1k_2); C_4 = (\gamma_1 + k_1\gamma_1); C_5 = (\gamma_2 + k_2\gamma_2); C_6 = a_3 + a_1k_1; C_7 = a_4 + a_2k_2$.

Proposition 4.1. *The PF of the CP* (4.5) *is maximum at the point:*

$$\begin{cases} p_{r1}^* = \frac{\zeta_1\left(C_5^2C_6 + 2C_2C_6\zeta_2 - C_3C_7\zeta_2\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2 + 2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2 + 2C_2\zeta_2\right)} \\ p_{r2}^* = \frac{\left(-C_4^2C_7 + C_3C_6\zeta_1 - 2C_1C_7\zeta_1\right)\zeta_2}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2 + 2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2 + 2C_2\zeta_2\right)} \\ \theta_1^* = -\frac{C_4\left(C_5^2C_6 + 2C_2C_6\zeta_2 - C_3C_7\zeta_2\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2 + 2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2 + 2C_2\zeta_2\right)} \\ \theta_2^* = -\frac{C_5\left(C_4^2C_7 - C_3C_6\zeta_1 + 2C_1C_7\zeta_1\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2 + 2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2 + 2C_2\zeta_2\right)} \end{cases}$$

if the following conditions holds:

1. $2C_2\zeta_2 + C_5^2 < 0$
2. $(4C_1C_2 - C_3^2)\zeta_1\zeta_2 > 2C_1C_5^2\zeta_1 + C_4^2(2C_2\zeta_2 + C_5^2)$

Proof. See Appendix A □

The optimal profit of the SCN can be obtained by substituting the values of the DVs from the proposition 4.1 in the equation (4.5).

4.2. Manufacturers' Stackelberg

In the Stackelberg model, all the members of the SCN optimize their corresponding decisions one by one according to the decision-making power. Both the manufacturers act as a single-player in this game and lead the SCN. In the end, the retailer makes his decision following both the manufacturer. Member leadership mainly depends on the members' ability to make decisions and has a truthful effect on the SCN. In this study, the level of GI is controlled by the manufacturers to offer a new eco-friendly product to the customers, so the manufacturers have more ability to make decisions and are considered as a leader. According to the Stackelberg model principle, the optimal response to the follower (*i.e.*, the retailer) is derived. Then, using these optimal responses in the leader's PFs (*i.e.*, manufacturers), the optimal decisions of the leaders are determined. Therefore the formulation of the model according to the decision making power is as follows:

$$\begin{cases} L1 : \text{Profit of M1 + M2} \\ L2 : \text{Profit of retailer} \end{cases}$$

i.e.,

$$\begin{aligned} L1 : \Pi_{ms} =& \Pi_{m1} + \Pi_{m2} \\ =& p_1D_1 + w_1D_{r1} + p_2D_2 + w_2D_{r2} - \frac{1}{2}\zeta_1\theta_1^2 - \frac{1}{2}\zeta_2\theta_2^2 \quad (4.6) \\ L2 : \Pi_r =& (p_{r1} - w_1)D_{r1} + (p_{r2} - k_2w_2)D_{r2} \quad (4.7) \end{aligned}$$

Proposition 4.2. *The PF of the retailer Π_r is maximum at the point:*

$$\begin{cases} p_{r1}^* = \frac{2(\alpha_4-k_2\beta)(-a_3-w_1\alpha_3+k_1w_1\beta+w_2\beta+k_1w_2\beta-\gamma_1\theta_1)+(2+k_1+k_2)\beta(-a_4-w_2\alpha_4+w_1\beta+k_2w_1\beta+k_2w_2\beta-\gamma_2\theta_2)}{(2+k1+k2)^2\beta^2+4(\alpha_3-k_1\beta)(-\alpha_4+k_2\beta)} \\ p_{r2}^* = \frac{(2+k_1+k_2)\beta(-a_3-w_1\alpha_3+k_1w_1\beta+w_2\beta+k_1w_2\beta-\gamma_1\theta_1)+2(\alpha_3-k_1\beta)(-a_4-w_2\alpha_4+w_1\beta+k_2w_1\beta+k_2w_2\beta-\gamma_2\theta_2)}{(2+k_1+k_2)^2\beta^2+4(\alpha_3-k_1\beta)(-\alpha_4+k_2\beta)} \end{cases}$$

if the following conditions holds:

1. $(-2\alpha_3 + 2k_1\beta)(-2\alpha_4 + 2k_2\beta) > (2 + k_1 + k_2)^2\beta^2$

Then, substituting the values of p_{r1} and p_{r2} in Π_{ms}, the PFs can be maximized.

Proof. See Appendix B □

Using the optimal decisions of the members, the profits of each of the member can be maximized.

4.3. Manufacturer 1 Stackelberg

Here, we assume that M1 has the greatest ability to decide first, and that M2 is the follower. The retailer will follow both of the manufacturers. Therefore the M1 sets his/her corresponding DVs, and then other players will optimize their related profits accordingly by setting their DVs. We formulate the model as follows:

$$\begin{cases} L1 : \Pi_{m1} = p_1D_1 + w_1D_{r1} - \frac{1}{2}\zeta_1\theta_1^2 \\ L2 : \Pi_{m2} = p_2D_2 + w_2D_{r2} - \frac{1}{2}\zeta_2\theta_2^2 \\ L3 : \Pi_r = \quad (p_{r1} - w_1)D_{r1} + (p_{r2} - k_2w_2)D_{r2}. \end{cases}$$

The solution procedure of this model is discussed in Appendix D.

4.4. Manufacturer 2 Stackelberg

This section simply interchanges the manufacturer's decision-making powers of the previous Section 4.3 and then repeats the similar procedure as given above.

4.5. Nash game

To validate the NG in our proposed model, we have to construct an assumption as follows: the retail price is depended on the wholsale price and the relation is $w_i = k_j p_{ri}$ for $i = 1, 2$ and $j = 3, 4$ provided $k_1, k_2 > k_3, k_4$ (using the assumption, $w_i < p_i$). Putting the values $w_i = k_j p_{ri}$ in the equations (4.1), (4.2) and (4.3), the following equations are obtained:

$$\Pi_{m1} = p_1 D_1 + k_3 p_{r1} D_{r1} - \frac{1}{2}\zeta_1 \theta_1^2 \tag{4.8}$$

$$\Pi_{m2} = p_2 D_2 + k_4 p_{r2} D_{r2} - \frac{1}{2}\zeta_2 \theta_2^2 \tag{4.9}$$

$$\Pi_r = (p_{r1} - k_3 p_{r1}) D_{r1} + (p_{r2} - k_4 p_{r2}) D_{r2}. \tag{4.10}$$

When the players have same decision power and have set their respective decisions independently and simultaneously, then the game is called NG. We formulate the model as follows:

$$\begin{cases} L1: \Pi_{m1} = p_1 D_1 + k_1 p_{r1} D_{r1} - \frac{1}{2}\zeta_1 \theta_1^2 \\ L1: \Pi_{m2} = p_2 D_2 + k_2 p_{r2} D_{r2} - \frac{1}{2}\zeta_2 \theta_2^2 \\ L1: \Pi_r = \quad (p_{r1} - k_1 p_{r1}) D_{r1} + (p_{r2} - k_2 p_{r2}) D_{r2} \end{cases}$$

Proposition 4.3. *The maximum profits of the players are maximum under NG when the values of the DVs are as follows:*

$$\begin{cases} p_{r1} = \dfrac{\zeta_1(-1+k_4)(a_4\zeta_2(-2+k_2(-1+k_3)+k_3+k_1(-1+k_4)+k_4)\beta - a_3(-1+k_3)(-2\zeta_2(\alpha_4-k_2\beta) + (k_2+k_4)\gamma_2^2))}{(-1+k_3)(k_1+k_3)(-1+k_4)\gamma_1^2(-2\zeta_2(\alpha_4-k_2\beta)+(k_2+k_4)\gamma_2^2) - \zeta_1(\zeta_2(-2+k_2(-1+k_3) + k_3+k_1(-1+k_4)+k_4)^2\beta^2 + 2(-1+k_3)(-1+k_4)(\alpha_3-k_1\beta)(-2\zeta_2(\alpha_4-k_2\beta)+(k_2+k_4)\gamma_2^2))} \\[2ex] p_{r2} = \dfrac{\zeta_2(1-k_3)(a_3\zeta_1(-2+k_2(-1+k_3)+k_3+k_1(-1+k_4)+k_4)\beta + a_4(-1+k_4)(2\zeta_1(\alpha_3-k_1\beta)-(k_1+k_3)\gamma_1^2))}{(1-k_3)(k_1+k_3)(-1+k_4)\gamma_1^2(-2g_2(\alpha_4-k_2\beta)+(k_2+k_4)\gamma_2^2)+\zeta_1(-4\zeta_2(-1+k_3)(-1+k_4)\alpha_3(\alpha_4-k_2\beta) + \zeta_2\beta(k_1^2(-1+k_4)^2\beta + (-2+k_2(-1+k_3)+k_3+k_4)^2\beta + 2k_1(-1+k_4)(2(-1+k_3)\alpha_4 + (-2+k_2+k_3-k_2k_3+k_4)\beta)) + 2(-1+k_3)(-1+k_4)(k_2+k_4)(\alpha_3-k_1\beta)\gamma_2^2)} \\[2ex] \theta_1 = \dfrac{-((k_1+k_3)(-1+k_4)\gamma_1(a_4\zeta_2(-2+k_2(-1+k_3)+k_3+k_1(-1+k_4)+k_4)\beta + a_3(-1+k_3)(2\zeta_2(\alpha_4-k_2\beta) - (k_2+k_4)\gamma_2^2)))}{((1-k_3)(k_1+k_3)(-1+k_4)\gamma_1^2(-2\zeta_2(\alpha_4-k_2\beta)+(k_2+k_4)\gamma_2^2)+\zeta_1(-4\zeta_2(-1+k_3)(-1+k_4)\alpha_3(\alpha_4-k_2\beta) + \zeta_2\beta(k_1^2(-1+k_4)^2\beta + (-2+k_2(-1+k_3)+k_3+k_4)^2\beta + 2k_1(-1+k_4)(2(-1+k_3) \alpha_4 + (-2+k_2+k_3-k_2k_3+k_4)\beta)) + 2(-1+k_3)(-1+k_4)(k_2+k_4)(\alpha_3-k_1\beta)\gamma_2^2))} \\[2ex] \theta_2 = \dfrac{-((-1+k_3)(k_2+k4_)(a_3\zeta_1(-2+k_2(-1+k_3)+k_3+k_1(-1+k_4)+k_4)\beta + a_4(-1+k_4)(2\zeta_1(\alpha_3-k_1\beta) - (k_1+k_3)\gamma_1^2))\gamma_2)}{((1-k_3)(k_1+k_3)(-1+k_4)\gamma_1^2(-2\gamma_2(\alpha_4-k_2\beta)+(k_2+k_4)\gamma_2^2)+\zeta_1(-4\zeta_2(-1+k_3)(-1+k_4) \alpha_3(\alpha_4-k_2\beta) + \zeta_2\beta(k_1^2(-1+k_4)^2\beta + (-2+k_2(-1+k_3)+k_3+k_4)^2\beta + 2k_1(-1+k_4) (2(-1+k_3)\alpha_4 + (-2+k_2+k_3-k_2k_3+k_4)\beta)) + 2(-1+k_3)(-1+k_4)(k_2+k_4)(\alpha_3-k_1\beta)\gamma_2^2))} \end{cases}$$

if the following conditions holds:

1. $(-2\alpha_3 + 2k_1\beta)(-2\alpha_4 + 2k_2\beta) > (2 + k_1 + k_2)^2\beta^2$

Proof. See Appendix C □

The optimal decisions of the members are listed in the proposition 4.3 and the profits are maximum at that point.

Table 3. Optimality test.

	Eigenvalues of HM with respect to [(a)]				
	Π_t	Π_{ms}	Π_{m1}	Π_{m2}	Π_r
CP	-24.9 , -10.6, -1.8, -1.6	–	–	–	–
MS	–	-9.8, -4.4, -1.8, -1.6	–	–	-14.3, -6.6
M1S	–	–	-6.4 , -1.7	-7.2, -1.7	-14.3, -6.6
M2S	–	–	-6.9, -1.7	-6.6 , -1.7	-14.3, -6.6
NG	–	–	-1.8	-1.8	-3, -1.4

Notes. [(a)]In the above table, the eigenvalues are rounded up to one decimal point.

5. Discussion of Results

We evaluate the sensitivity of the model's important parameters and explain the behavior of the proposed model with a numerical illustration.

5.1. Numerical Example

Here, a numerical example illustrates and validates the proposed model. Due to the difficulty of collecting real-life industrial data, some of the data is gathered from the previous literature, and the remainder is assumed hypothetically to verify the proposed model. In this study, the hypothetical data are consistent with the published literature [45], and [38] to the largest extent possible; however, it is impossible to assume an identical data set with any previous literature as our model is uniquely formulated and have studied never before. Therefore, the hypothetical example has been constructed as follows:
Let us consider a market with two manufacturers (M1 and M2) and a common retailer. The M1 and M2 produce P1 and P2 respectively and sells them through either their personal OC or common RC. Let the market potential of OCs of M1 and M2 are 425 units/unit time (*i.e.*, $a_1 = 425$) and 410 units/unit time (*i.e.*, $a_2 = 410$) respectively and the market potential of the RCs of M1 and M2 are 475 units/unit time (*i.e.*, $a_3 = 475$) and 490 units/ unit time (*i.e.*, $a_4 = 490$) respectively. The own channel price sensitivity of the OCs of M1 and M2 are 5.6 unit/ unit \$ (*i.e.*, $\alpha_1 = 5.6$) and 5.5 unit/ unit \$ (*i.e.*, $\alpha_2 = 5.5$) respectively and the RCs are 6 unit/ unit \$ (*i.e.*, $\alpha_3 = 6$) and 6.25 unit/ unit \$ (*i.e.*, $\alpha_4 = 6.25$) respectively. Let also consider the cross channel pricing sensitivity of the channels is 1 unit/ unit \$. Next, let both the manufacturer provide a discount 10% on the retail price to sell the similar products through their corresponding OCs (*i.e.*, $k_1 = 0.9$ and $k_2 = 0.9$). The demand sensitivity of coefficient of θ_1 and θ_2 be 0.75 and 0.73 (*i.e.*, $\gamma_1 = 0.75$ and $\gamma_2 = 0.73$) respectively and the cost coefficients of the level of GI θ_1 and θ_2 are \$ 1.85 and \$ 1.82 (*i.e.*, $\zeta_1 = 1.85$ and $\zeta_2 = 1.82$) respectively. Hence, the data set is as follows: $k_1 = 0.9$; $k_2 = 0.9$; $a_1 = 425$; $a_2 = 410$; $a_3 = 475$; $a_4 = 490$; $\alpha_1 = 5.6$; $\alpha_2 = 5.5$; $\alpha_3 = 6$; $\alpha_4 = 6.25$; $\beta = 1$; $\gamma_1 = 0.75$; $\gamma_2 = 0.73$; $\zeta_1 = 1.85$; $\zeta_2 = 1.82$; Now we check the conditions of optimality and also find the optimal results under each strategies.

In Table 3, the eigenvalues of HMs of the PFs are listed, and the optimal results are stored in Table 4 under each strategy.

In Table 3, all the eigenvalues are negative, *i.e.*, the optimality condition holds. Therefore, the values of DVs are optimal, and the PFs are maximum at the values. The optimal values of the DVs and each member's maximized PF have been listed in Table 4 for each strategic game structure.

5.1.1. Discussion regarding Numerical analysis:

In Table 4, it can be observed that the total SCN's profit is maximum under CP, and the TP can be arranged as $\Pi_t^{CP} > \Pi_t^{NG} > \Pi_t^{M1S} > \Pi_t^{M2S} > \Pi_t^{MS}$. The level of GI is maximum under CP. Therefore the products become more eco-friendly if the members make decisions jointly; consequently, the demands under CP are higher than the others. According to both manufacturers, the best profitable strategy would be NG. Retailer able to make

TABLE 4. DVs and optimal profits.

Decision variables	CP	MS game	M1S game	M2S game	NG
p_{r1}	92.57	117.17	101.87	100.2	79.53
p_{r2}	91.17	115.33	98.76	100.43	77.65
w_1	74.06 [(a)]	74.93	47.51	44.23	63.62
w_2	72.94	74.89	44.77	48.07	62.12
θ_1	71.3	56	41.98	41.61	54.81
θ_2	69.48	55.45	41.66	42	52.95
Π_{m1}	35438.4	29919.7	27990.6	28466.3	34564
Π_{m2}	34985	29974	28403.9	27930.2	34347.6
Π_r	8421.6	11355.7	19510.7	19508.1	8209.42
Π_t	78845	71249.4	75905.2	75904.6	77121

Notes. [(a)]Here we consider that both the manufacturer gives a discount of 20% on the retail price

the best profit under MS2 and the retailer's profit obeys the inequality $\Pi_r^{M1S} > \Pi_r^{M2S} > \Pi_r^{MS} > \Pi_r^{NG}$. The retail prices are maximum under MS and lowest under NG. The CP always gives maximum profit for the SCN. When the players settle a contract on the wholesale price, the manufacturers provide a discount on the selling price to the retailer and agree to jointly make the decision, *i.e.*, they accept the CP strategy. The discussion on individual profits behavior is stated in Corollary 5.1.

Corollary 5.1. *The profit of each member under CP may not be higher than that of under the other strategies.*

The total channel's profit of CP is larger than any other strategy, but the members' profits in CP do not need to be always greater than their individual profits in other strategies, which may be less or equal.

5.2. Discussion on parameters' sensitivity

In this section, the sensitivity of the parameters on the DVs and profits are analyzed. In tables 5 and 6, we have listed the change of the values of the variables with respect to different values of the parameters under each gaming strategy. In the Figures 2, 3, 4 and 5, the sensitivity of the parameters on the different PFs have been shown. From the respective tables and figures, the observations have been made in the following subsections:

5.2.1. Impact of the Parameters γ_1 and γ_2

To analyze the impacts of the parameters γ_1 and γ_2, the parameters are taken fixed in the numerical example 5.1 and only the value of γ_i $(i = 1, 2)$ will vary at a time. Figure 2 demonstrates the change profits of the Manufacturers, retailer, and the SCN with γ_i $(i = 1, 2)$. In Figures 2a and 2b, one can observe that the TP of the SCN is larger in CP than any other strategies. In addition, the PF rises with each increment of γ_i.

The individual profit of each of the manufacturer is maximum under CP and the curve of the PFs are in upward direction with increasing values of γ_i (See Fig. 2c, 2e, 2d, 2f). But it is shown in Figures 2g and 2h that the retailer's profit is higher under strategy- M2S than any other strategies. Hence it is more profitable to join in the M2S strategy from in retailer's point of view, but the manufacturers would like to join in CP to gain more profits.

The demand coefficients of the level of GIs (*i.e.*, γ_i) have positive impacts on each channel's demands. Table 5 describes that if γ_i is high, then the members would like to fix their DVs to an upper value. Thus if the demand coefficients of GI are large, then the manufacturer will develop the level of GI, which improves the demand rates and allows the manufacturers and retailer to set higher prices of the products. Since γ_1 is directly connected with θ_1, so with the increment of γ_1, the optimal value of θ_1 increases faster than the increment of θ_2. Also, the level of GI is always higher under CP than any other strategy. Thus, CP performs better as an eco-friendly product.

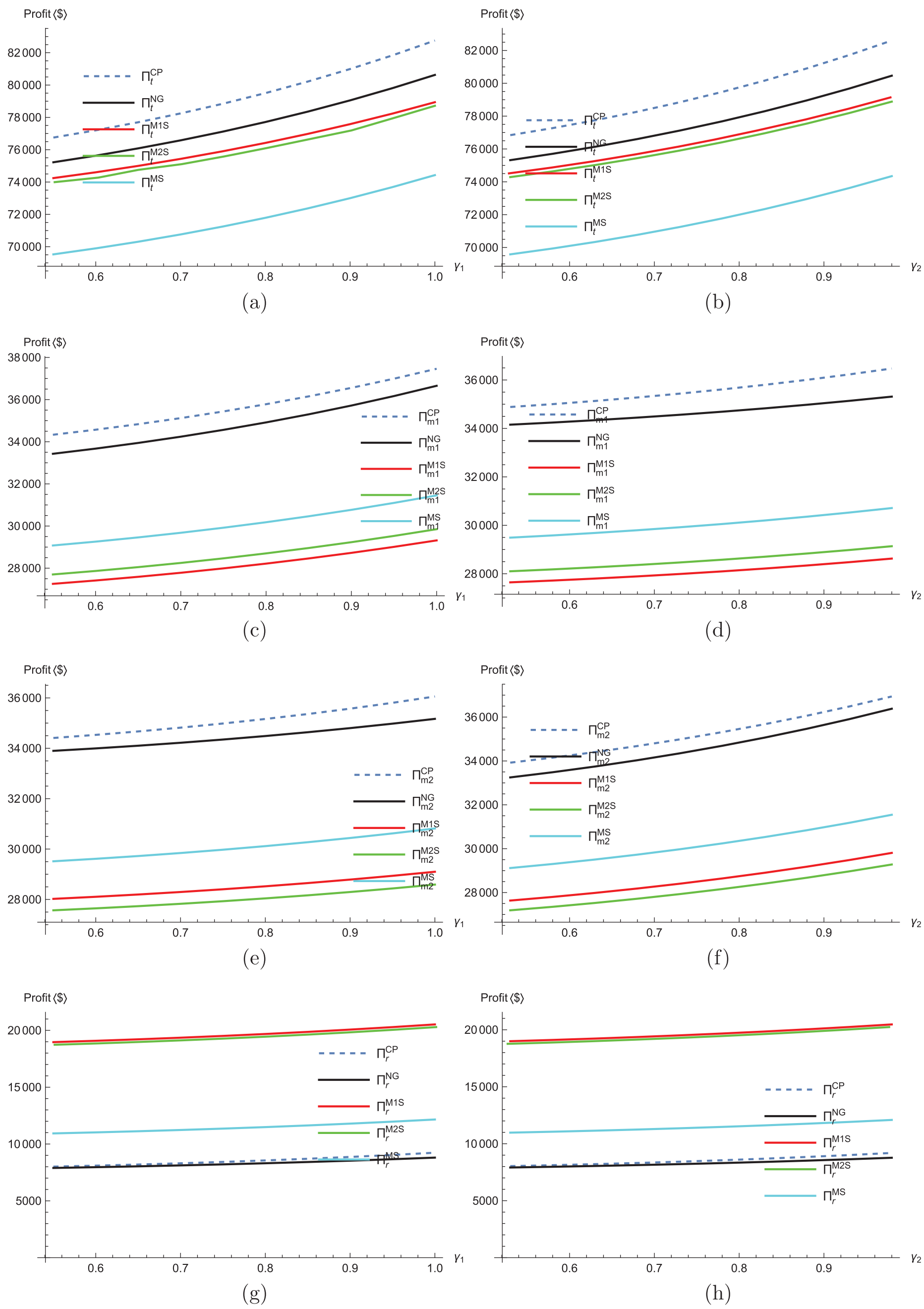

FIGURE 2. Effect of the parameters γ_1 and γ_2 on the optimal profits. (a) TP *versus* γ_1. (b) TP *versus* γ_2. (c) Manufacturer-1's profit *versus* γ_1. (d) Manufacturer-1's profit *versus* γ_2. (e) Manufacturer-2's profit *versus* γ_1. (f) Manufacturer-2's profit *versus* γ_2. (g) Retailer's profit *versus* γ_1. (h) Retailer's profit *versus* γ_2.

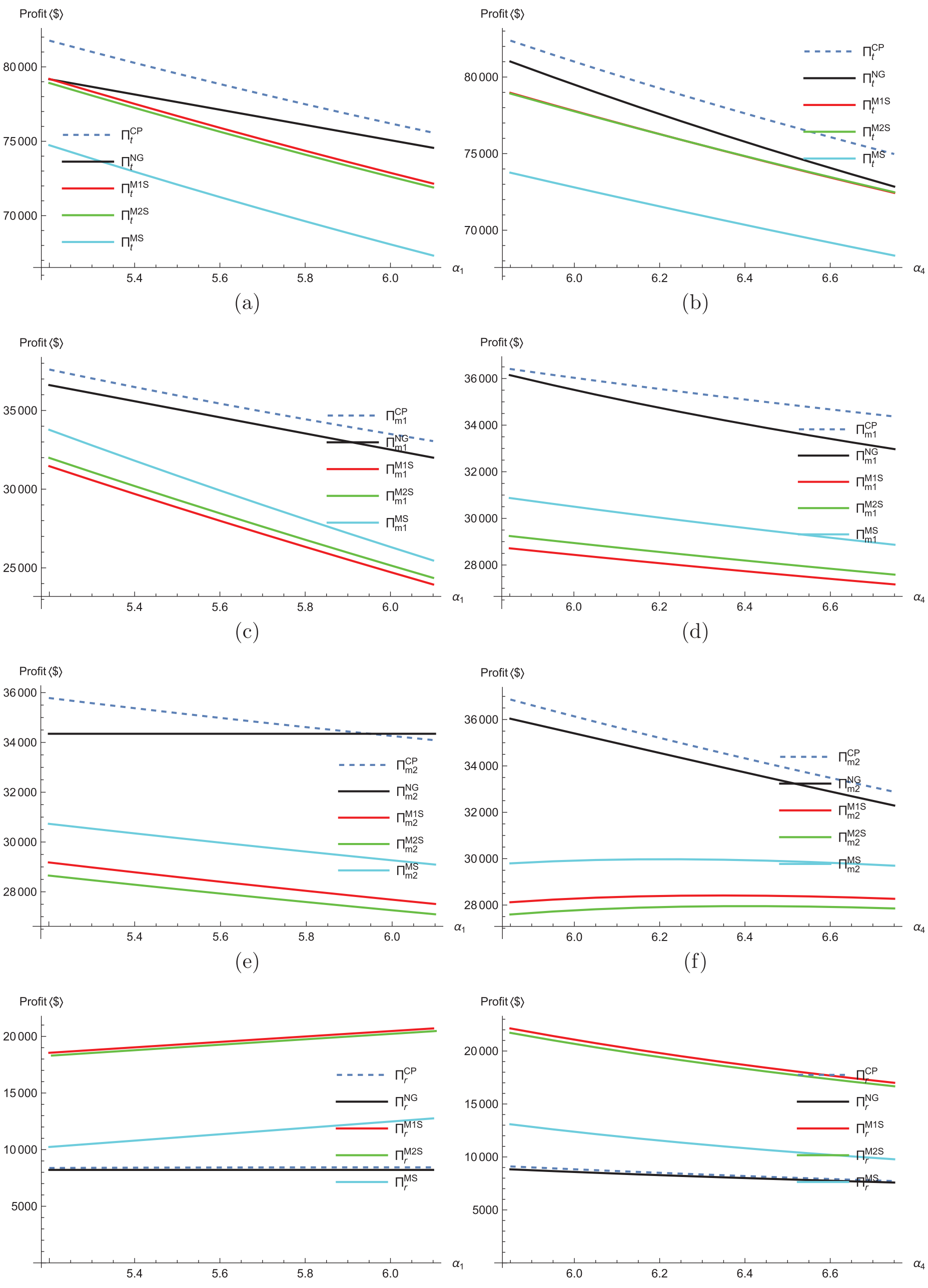

FIGURE 3. Effect of the parameters α_1 and α_4 on the optimal profits. (a) Total PF *versus* α_1. (b) Total PF *versus* α_4. (c) Manufacturer-1's profit *versus* α_1. (d) Manufacturer-1's profit *versus* α_4. (e) Manufacturer-2's profit *versus* α_1. (f) Manufacturer-2's profit *versus* α_4. (g) Retailers profit *versus* α_1. (h) Retailer's profit *versus* α_4.

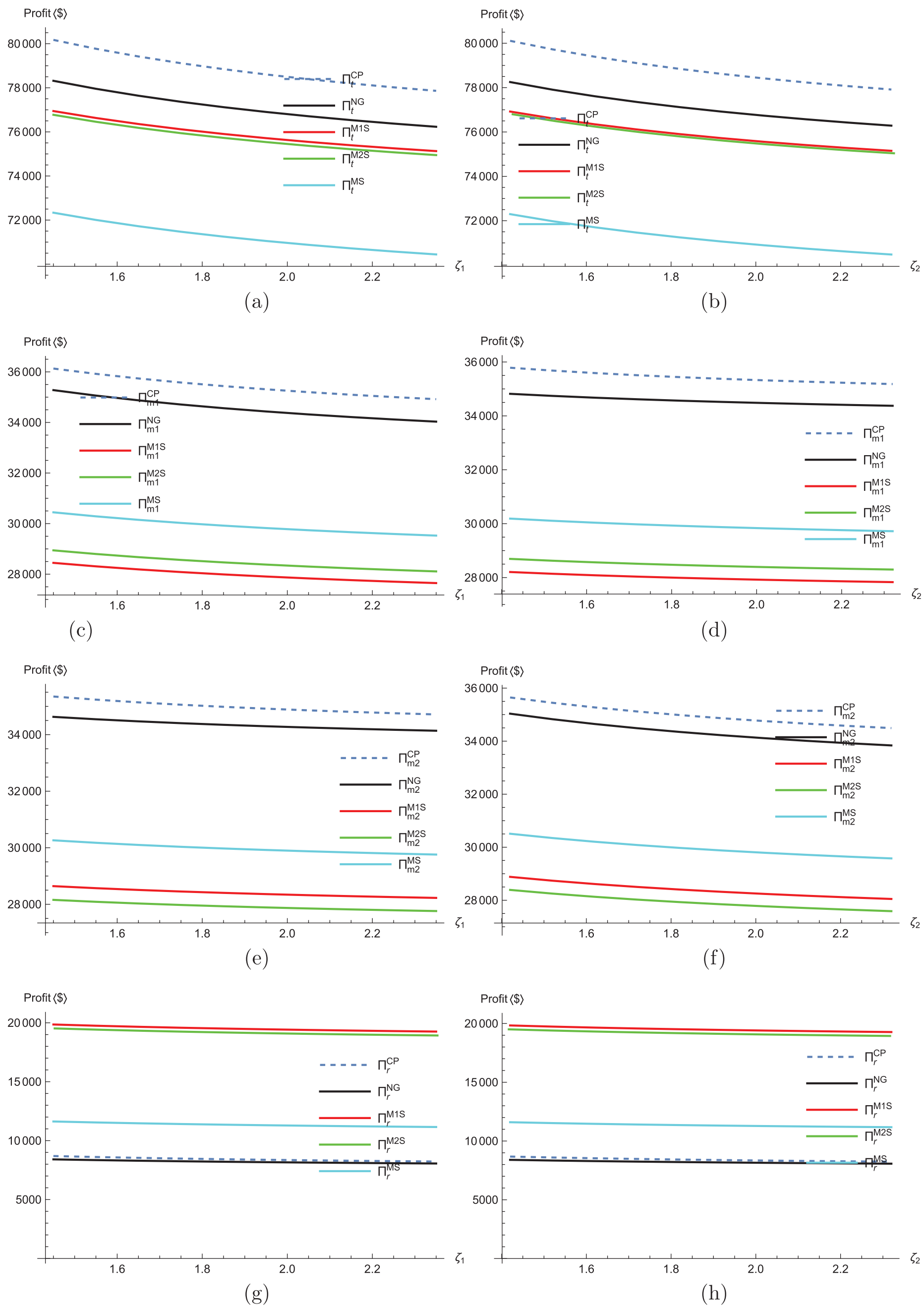

FIGURE 4. Effect of the parameters ζ_1 and ζ_2 on the optimal profits. (a) TP *versus* ζ_1. (b) TP *versus* ζ_2. (c) Manufacturer-1's profit *versus* ζ_1. (d) Manufacturer-1's profit *versus* ζ_2. (e) Manufacturer-2's profit *versus* ζ_1. (f) Manufacturer-2's profit *versus* ζ_2. (g) Retailers profit *versus* ζ_1. (h) Retailer's profit *versus* ζ_2.

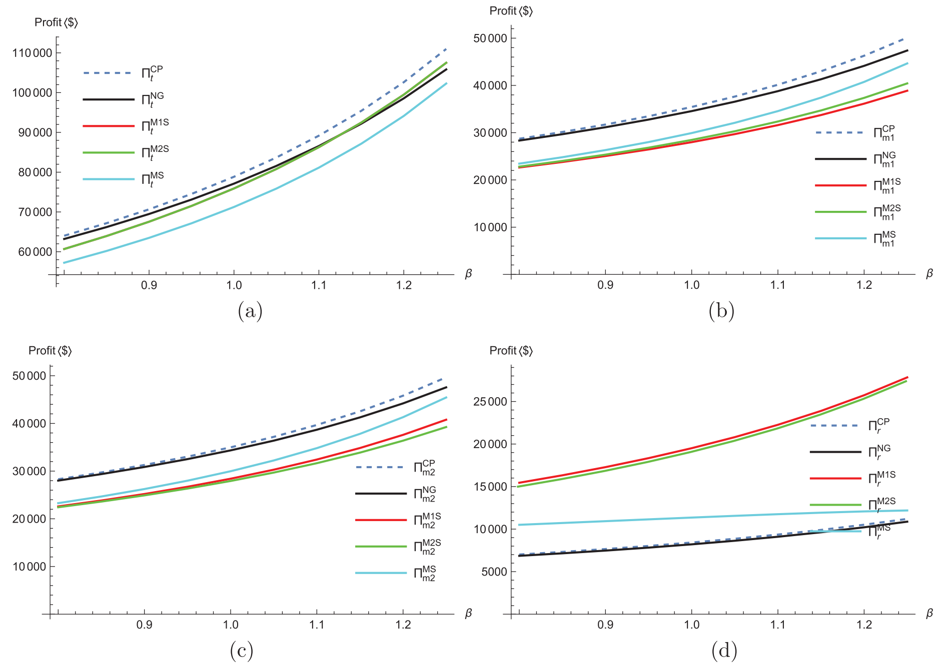

FIGURE 5. Effect of the parameters β on the optimal profits. (a) TP *versus* β. (b) Manufacturer-1's profit *versus* β. (c) Manufacturer-2's profit *versus* β. (d) Retailer's profit *versus* β.

5.2.2. Impact of the Parameters α_1 and α_4

This section examines the sensitivity of two parameters based on the channels' own price sensitivity ($\alpha_i, i = 1, 2, 3, 4$). In this model, α_1 and α_4 have some remarkable effects on the DVs and the profits of the players.

The effect of α_1 and α_4 are shown in Tables 5, 6 and Figure 3. It can be observed that the values of most of the DVs are decreasing with the increment of α_1 and α_4. Under the increment of α_1, $p_{r1}^{NS}, p_{r2}^{NG}, w_1^{NG}, w_2^{NG}, \theta_1^{NG}$ and θ_2^{NG} remain constant. Again, w_i^{MS} and w_i^{M1S} increases slightly with the increasing values of α_4. In the Figure 3, the PFs of the players are plotted with respect to increment of α_1 and α_4. According to the Figures 3a and 3b, the TP of the chain decreases with the increment of α_1 and α_4 and the profit is maximum under CP. In M1's point of view, the curve of the PFs are in downward direction with the increment of α_1 and α_4. Also M1 gains more under CP than any other strategies (See the Figs. 3c, 3d). The effect of α_4 on the M2's profit is same as M1's profit (See the Fig. 3f). The effect of α_1 on the profit of M2 is described in Corollary 5.2.

Corollary 5.2. *According to the Figure 3e, there exist a constant value α_1^0 of α_1 such that the following holds:*

1. *the profit of M2 is maximum under CP whenever $\alpha_1^0 < \alpha_1$ and*
2. *the profit of M2 is maximum under NG whenever $\alpha_1 > \alpha_1^0$.*

Under all strategies, M2 profits decrease as α_1 increases, except NG, where profits remain constant.

The retailer's PF increases with the increasing values of α_1 under all the cases of Stackelberg settings. However, under NG and CP, the retailer's profit remains fixed with the change of α_1. Again, the PF of the retailer decreases whenever the values of α_4 increase. In both cases, the retailer gains maximum and equal profits under M1S and M2S (See Figs. 3g and 3h). The profits are very sensitive with these two own-channel price sensitivity parameters α_1 and α_4. With the increment of α_1 and α_4, the demand rates negatively impact. As a result, the

TABLE 5. Effect of the parameters on the pricing DVs.

		p_{r1}^*					p_{r2}^*					w_1^*				w_2^*			
		CP	NG	MS	M1S	M2S	CP	NG	MS	M1S	M2S	NG	MS	M1S	M2S	NG	MS	M1S	M2S
γ_1	0.55	89.16	77.29	99.47	97.87	113.8	89.7	76.82	98.05	99.65	113.9	61.83	46.14	42.94	72.75	61.45	44.58	47.75	73.86
	0.65	90.7	78.31	100.6	98.93	115.3	90.36	77.2	98.37	100.	114.6	62.65	46.76	43.53	73.74	61.76	44.67	47.9	74.33
	0.75	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	0.85	94.8	80.97	103.4	101.7	119.3	92.13	78.19	99.22	100.9	116.2	64.77	48.4	45.06	76.34	62.55	44.89	48.27	75.56
	0.95	97.44	82.65	105.2	103.5	121.9	93.27	78.81	99.76	101.5	117.3	66.12	49.43	46.03	77.98	63.05	45.03	48.5	76.34
γ_2	0.53	91.14	78.73	101.1	99.49	115.8	87.93	75.61	96.51	98.12	112.1	62.98	47.21	44.05	73.91	60.49	43.47	46.68	72.73
	0.63	91.78	79.09	101.4	99.82	116.4	89.39	76.53	97.53	99.16	113.6	63.27	47.35	44.13	74.37	61.23	44.06	47.31	73.71
	0.73	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	0.83	93.51	80.05	102.4	100.7	118.1	93.3	78.97	100.2	101.9	117.4	64.04	47.71	44.35	75.59	63.18	45.61	48.96	76.3
	0.93	94.62	80.65	102.9	101.2	119.1	95.82	80.52	101.9	103.7	119.9	64.52	47.93	44.48	76.37	64.41	46.6	50.02	77.95
α_1	5.2	97.32	79.53	105.1	103.3	121.3	93.22	77.65	99.5	101.2	117.2	63.62	53.2	49.59	82.19	62.12	45.88	49.29	77.88
	5.4	94.89	79.53	103.5	101.7	119.2	92.17	77.65	99.13	100.8	116.2	63.62	50.31	46.87	78.5	62.12	45.32	48.67	76.36
	5.6	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	5.8	90.36	79.53	100.3	98.73	115.2	90.22	77.65	98.41	100.	114.5	63.62	44.8	41.67	71.49	62.12	44.24	47.48	73.48
	6.	88.26	79.53	98.81	97.3	113.3	89.31	77.65	98.07	99.67	113.6	63.62	42.16	39.18	68.16	62.12	43.73	46.92	72.11
		p_{r1}^*					p_{r2}^*					w_1^*				w_2^*			
		CP	NG	MS	M1S	M2S	CP	NG	MS	M1S	M2S	NG	MS	M1S	M2S	NG	MS	M1S	M2S
α_4	5.85	95.09	82.61	104.4	102.7	120.6	96.88	85.5	105.6	107.4	123.7	66.09	47.16	43.65	76.09	68.4	43.9	47.49	76.71
	6.05	93.79	80.99	103.1	101.4	118.8	93.94	81.39	102.1	103.8	119.4	64.8	47.36	43.97	75.52	65.11	44.43	47.88	75.86
	6.25	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	6.45	91.42	78.19	100.7	99.1	115.6	88.56	74.24	95.67	97.26	111.5	62.55	47.62	44.45	74.32	59.39	44.95	48.11	73.86
	6.65	90.33	76.96	99.63	98.07	114.1	86.09	71.12	92.76	94.29	108.	61.57	47.7	44.62	73.7	56.89	45.	48.02	72.76
ζ_1	1.45	94.73	80.92	103.4	101.7	119.3	92.1	78.17	99.21	100.9	116.2	64.74	48.37	45.04	76.29	62.54	44.89	48.26	75.54
	1.65	93.51	80.13	102.5	100.8	118.1	91.57	77.88	98.96	100.6	115.7	64.11	47.89	44.58	75.52	62.3	44.82	48.15	75.18
	1.85	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	2.05	91.83	79.04	101.4	99.7	116.4	90.85	77.47	98.61	100.3	115.	63.24	47.22	43.95	74.46	61.98	44.73	48.	74.67
	2.25	91.23	78.65	100.9	99.29	115.8	90.59	77.32	98.48	100.1	114.8	62.92	46.97	43.73	74.08	61.86	44.7	47.94	74.49
ζ_2	1.42	93.47	80.03	102.3	100.6	118.	93.21	78.92	100.2	101.9	117.3	64.02	47.7	44.34	75.57	63.14	45.58	48.93	76.24
	1.62	92.96	79.74	102.1	100.4	117.5	92.05	78.2	99.37	101.	116.2	63.79	47.59	44.28	75.21	62.56	45.12	48.44	75.48
	1.82	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	2.02	92.26	79.36	101.7	100.1	116.9	90.47	77.21	98.28	99.93	114.6	63.48	47.45	44.19	74.71	61.77	44.49	47.77	74.43
	2.22	92.01	79.22	101.6	99.93	116.6	89.91	76.86	97.89	99.53	114.1	63.37	47.4	44.16	74.53	61.49	44.27	47.53	74.06
β	0.8	75.17	67.19	87.45	86.56	96.49	74.01	65.73	85.39	86.27	95.08	53.75	42.54	40.78	59.01	52.58	41.1	42.86	58.99
	0.9	82.97	72.84	94.08	92.85	105.8	81.7	71.19	91.55	92.78	104.2	58.28	44.85	42.42	66.08	56.95	42.84	45.27	66.05
	1.	92.57	79.53	101.9	100.2	117.2	91.17	77.65	98.76	100.4	115.3	63.62	47.51	44.23	74.93	62.12	44.77	48.07	74.89
	1.1	104.7	87.55	111.2	108.9	131.3	103.1	85.41	107.3	109.6	129.2	70.04	50.63	46.24	86.32	68.32	46.93	51.35	86.26
	1.2	120.5	97.36	122.4	119.5	149.4	118.7	94.89	117.7	120.7	146.9	77.89	54.34	48.51	101.5	75.92	49.37	55.26	101.3

players have to decrease their corresponding pricing decisions. Depending on the value of α_1, the M2 chooses the most profitable approach (See Cor. 5.2). Furthermore, the level of GI decreases with the increasing values of α_1 and α_4. Therefore, to make more eco-friendly products, the values of the own-channel pricing sensitivity should reduce as much as possible.

5.2.3. Impact of the Parameters ζ_1 and ζ_2

Figure 4 depicts the variation of the profits with respect to the costs coefficients of GI (*i.e.*, ζ_1 and ζ_2) and the changing behavior of the DVs are listed in Tables 5 and 6. All the DVs decreases with the increment of ζ_1 and ζ_2. In CP, the SCN's TP, M1's profit and M2's profit are maximum compare to the other strategies (See the Figs. 4a, 4b, 4c, 4d, 4e, 4f). In Figures 4g and 4h, it is clear that the retailer's profit are equal in M1S and M2S and higher than the other strategies. With the inclusion of ζ_1 and ζ_2, the retailer's PFs decrease slightly, it is not very sensitive with ζ_1 and ζ_2. Also all members' PFs decreases with the increment of these two parameters.

The increment of coefficients of GI (*i.e.*, ζ_i) reduces the optimal level of GI, which has a direct negative impact on the demands of both the product (See the Tab. 5). Therefore to reduce the negative impact of the lesser level of GI, the players (the manufacturers and the retailer) have to reduce their corresponding pricing

Table 6. Effect of the parameters on the pricing DVs.

		θ_1^*					θ_2^*				
		CP	NG	MS	M1S	M2S	CP	NG	MS	M1S	M2S
	0.55	50.36	39.06	29.99	29.73	39.88	68.36	52.38	41.38	41.67	54.68
	0.65	60.55	46.77	35.87	35.55	47.77	68.86	52.64	41.51	41.82	55.03
γ_1	0.75	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	0.85	82.76	63.24	48.37	47.94	64.66	70.21	53.31	41.84	42.22	55.95
	0.95	95.07	72.15	55.1	54.6	73.83	71.08	53.74	42.05	42.48	56.54
	0.53	70.2	54.26	41.66	41.34	55.23	48.65	37.43	29.47	29.7	39.1
	0.63	70.7	54.51	41.8	41.46	55.58	58.79	45.04	35.45	35.74	47.1
γ_2	0.73	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	0.83	72.02	55.17	42.19	41.78	56.51	80.84	61.22	48.16	48.56	64.23
	0.93	72.88	55.58	42.44	41.98	57.1	93.03	69.94	55.	55.46	73.53
		CP	NG	MS	M1S	M2S	CP	NG	MS	M1S	M2S
	5.2	74.96	54.81	46.64	46.15	61.72	71.04	52.95	42.2	42.66	57.78
	5.4	73.09	54.81	44.27	43.84	58.81	70.24	52.95	41.93	42.33	56.59
α_1	5.6	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	5.8	69.6	54.81	39.76	39.44	53.29	68.75	52.95	41.41	41.69	54.35
	6.	67.98	54.81	37.6	37.33	50.67	68.06	52.95	41.16	41.39	53.29
		θ_1^*					θ_2^*				
		CP	NG	MS	M1S	M2S	CP	NG	MS	M1S	M2S
	5.85	73.24	56.93	42.67	42.39	56.81	73.83	58.3	41.79	42.09	56.58
	6.05	72.24	55.82	42.32	41.99	56.42	71.59	55.5	41.79	42.11	56.06
α_4	6.25	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	6.45	70.42	53.89	41.66	41.25	55.57	67.49	50.62	41.45	41.8	54.77
	6.65	69.58	53.04	41.34	40.91	55.13	65.61	48.49	41.16	41.53	54.04
	1.45	93.1	71.16	54.43	53.94	72.76	70.19	53.3	41.84	42.22	55.94
	1.65	80.75	61.92	47.4	46.98	63.29	69.79	53.1	41.74	42.1	55.66
ζ_1	1.85	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	2.05	63.83	49.16	37.67	37.34	50.22	69.23	52.82	41.6	41.93	55.29
	2.25	57.78	44.57	34.17	33.87	45.52	69.04	52.73	41.55	41.87	55.15
	1.42	72.	55.15	42.19	41.78	56.49	91.05	68.97	54.26	54.71	72.36
	1.62	71.6	54.96	42.07	41.68	56.21	78.81	59.91	47.13	47.52	62.79
ζ_2	1.82	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	2.02	71.06	54.69	41.91	41.55	55.84	62.12	47.44	37.33	37.64	49.65
	2.22	70.87	54.6	41.86	41.51	55.7	56.17	42.96	33.81	34.09	44.95
	0.8	57.9	46.31	35.75	35.53	43.94	56.4	44.82	35.43	35.64	43.51
	0.9	63.91	50.2	38.59	38.3	49.29	62.26	48.54	38.27	38.54	48.81
β	1.	71.3	54.81	41.98	41.61	56.	69.48	52.95	41.66	42.	55.45
	1.1	80.64	60.34	46.1	45.64	64.67	78.58	58.24	45.81	46.23	64.01
	1.2	92.78	67.1	51.22	50.66	76.21	90.44	64.71	50.98	51.5	75.4

decisions. Also, the members' profits decrease with the increment of ζ_i. Therefore, the manufacturers should decrease their corresponding cost coefficients of GI so that the level of GI and the profits increase.

5.2.4. Impact of the Parameters β

In Tables 5 and 6, it can be shown that as the cross-channel pricing sensitivity parameter is increased, all of the DVs increase. Then, Figure 5 illustrates that the curves of the members' PFs arise upward very gradually for the increment of β, *i.e.*, the PFs are very sensitive with the parameter β, and they are proportional to β. The TP of the SCN is maximum under CP strategies. On the contrary, for M1 and M2, the profits under NG and CP are equal until the value of β is less than some fixed value, but the profits are maximum under CP (See Figs. 5b and 5c).

According to Figure 5d, the retailer gains more profit under M1S and M2S than the rest strategies.

As β positively impacts the demands, the members' profits are proportional to β. Also, increasing β helps manufacturers to impose more GI on their related products. So the members should try to increase the value of β to gain more and make products more eco-friendly.

6. Managerial insights

The present study discusses the theoretical results and the sensitivity of the parameters. Based on them, the following managerial insights can be drawn:

The TP of the SCN is maximum under CP, and the products' GI level is higher in CP than any other strategy. However, it does not imply that CP is the most lucrative strategy for each of the members. For example, in the given numerical example, the Manufacturer's best strategy is NG, and the retailer makes its best in M2S. The GI level is affected by the demand coefficient of GI (*i.e.*, γ_1 and γ_2) and the cost coefficient of GI (*i.e.*, ζ_1 and ζ_2). If the value of ζ_i $(i = 1, 2)$ increases, the GI's optimal level decreases, and the demands also decrease. Consequently, the selling prices should also drop down to push the demand up, and as a result, the members' profits decrease.

Furthermore, the best profitable strategy for individual members may differ on the values of the own-price sensitivity parameters. So, it is crucial to investigate the parameters' values to choose the best profitable strategy and obtain optimal decisions. The SCN's profit is lowest in MS-game. Therefore, the SCN's TP can be increased by introducing double DC with a retailer. An important observation is that the optimal GI level of the products in double DC is greater than the single DC. Therefore, it is necessary to introduce a double DC in a SCN to reduce the product's harmfulness compared to a single DC.

7. Conclusion

This paper formulates a DC SCN with the concept of green improved products, where two manufacturers produce green products and sells through either by an identical retailer or through their corresponding OCs. The demand rates are linear functions depending on the retail prices, online prices, and product's GI levels. The individual manufacturer determines the level of GIs and wholesale prices, but the retailer's DV is the retail price. The manufacturer provides a fixed discount on the retail price to sell products through an OC.

Five decision models (CP, MS, M1S, M2S, NG) are formulated considering the GI to answer the research queries mentioned in the introduction. The decision models are distingue according to the power of decision-making of the members. The Stackelberg theory and the classical optimization technique obtain the members' optimal decisions under each decision model.

The discussion of the results expresses that CP is the best profitable strategy for the SCN, so the CP helps to accomplish the economic targets. The products' GI level is maximum in the CP, which is beneficial to the climate. If a manufacturer offers a high GI level, the optimal pricing decisions increase, and the optimal profits increase faster than the other players.

The own-channel price sensitivity parameters negatively impact the profits of the corresponding members and TP of the SCN. Also, there exists a fixed value of α_1, which determines the best profitable strategies. Also, the increment of the GI's cost coefficients decreases the optimal level of GIs and the players' profits.

Furthermore, an important conclusion is that the single DC SCN's profit is lesser than any strategies in a double DC SCN. Moreover, the optimal GIs of both the products is higher in double DC than a single one. The relation between the parameters is determined under the strategies so that the members' can achieve their corresponding optimal profits. With the help of a hypothetical data set, the optimal results of the various models will be compared, and the sensitivity of the parameters will be analyzed, and key insights will be gained.

Although the proposed model has some limitations, such as the demand functions are deterministic and linear, but in reality, it would be uncertain. In this article, a retailer with multiple manufacturers is considered within a DC SCN, but it is possible that the manufacturers have multiple retailers, which makes channel coordination more difficult. Finally, the SCN's return policies or the promotional effort is not introduced in this model. Future research can be done in several directions where this topic can be explored. Firstly, one can modify the present model by assuming an uncertain demand function. Secondly, it would be advisable for further study to consider the traditional off-line competitive situations where multiple retailers compete for the customers. Lastly, it would be interesting to study the effects of the return policies of the SCN members and the promotional effort on the member's optimal decisions and PFs.

Appendix A.

The 1st order conditions of (4.5) is given by:

$$\begin{cases} \frac{\partial \Pi_t}{\partial p_{r1}} = 0 \implies C_6 + 2C_1 p_{r1} + C_3 p_{r2} + C_4\theta_1 = 0 \\ \frac{\partial \Pi_t}{\partial p_{r2}} = 0 \implies C_7 + C_3 p_{r1} + 2C_2 p_{r2} + C_5\theta_2 = 0 \\ \frac{\partial \Pi_t}{\partial \theta_1} = 0 \implies C_4 p_{r1} - \zeta_1\theta_1 = 0 \\ \frac{\partial \Pi_t}{\partial \theta_2} = 0 \implies = C_5 p_{r2} - \zeta_2\theta_2 = 0. \end{cases}$$

Solving the above equation with respect to the DVs p_{r1}, p_{r2}, θ_1 and θ_2, we have

$$\begin{cases} p^*_{r1} = \frac{\zeta_1\left(C_5^2 C_6 + 2C_2C_6\zeta_2 - C_3C_7\zeta_2\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2+2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2+2C_2\zeta_2\right)} \\ p^*_{r2} = \frac{\left(-C_4^2C_7 + C_3C_6\zeta_1 - 2C_1C_7\zeta_1\right)\zeta_2}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2+2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2+2C_2\zeta_2\right)} \\ \theta^*_1 = -\frac{C_4\left(C_5^2C_6 + 2C_2C_6\zeta_2 - C_3C_7\zeta_2\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2+2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2+2C_2\zeta_2\right)} \\ \theta^*_2 = -\frac{C_5\left(C_4^2C_7 - C_3C_6\zeta_1 + 2C_1C_7\zeta_1\right)}{-C_3^2\zeta_1\zeta_2 + C_4^2\left(C_5^2+2C_2\zeta_2\right) + 2C_1\zeta_1\left(C_5^2+2C_2\zeta_2\right)}. \end{cases}$$

Now, we calculate the Hessian matrix(HM) of the function $\Pi_t(p_{r1}, p_{r2}, \theta_1, \theta_2)$ in (4.5) as follows:

$$H_1 = \begin{pmatrix} \frac{\partial^2\Pi_t}{\partial p_{r1}^2} & \frac{\partial^2\Pi_t}{\partial p_{r1}\partial p_{r2}} & \frac{\partial^2\Pi_t}{\partial p_{r1}\partial\theta_1} & \frac{\partial^2\Pi_t}{\partial p_{r1}\theta_2} \\ \frac{\partial^2\Pi_t}{\partial p_{r2}\partial p_{r1}} & \frac{\partial^2\Pi_t}{\partial p_{r2}^2} & \frac{\partial^2\Pi_t}{\partial p_{r2}\partial\theta_1} & \frac{\partial^2\Pi_t}{\partial p_{r2}\partial\theta_2} \\ \frac{\partial^2\Pi_t}{\partial\theta_1\partial p_{r1}} & \frac{\partial^2\Pi_t}{\partial\theta_1\partial p_{r2}} & \frac{\partial^2\Pi_t}{\partial\theta_1^2} & \frac{\partial^2\Pi_t}{\partial\theta_1\partial\theta_2} \\ \frac{\partial^2\Pi_t}{\partial\theta_2\partial\theta_1} & \frac{\partial^2\Pi_t}{\partial\theta_2\partial p_{r1}} & \frac{\partial^2\Pi_t}{\partial\theta_2\partial p_{r2}} & \frac{\partial^2\Pi_t}{\partial\theta_2^2} \end{pmatrix} = \begin{pmatrix} 2C_1 & C_3 & C_4 & 0 \\ C_3 & 2C_2 & 0 & C_5 \\ C_4 & 0 & -\zeta_1 & 0 \\ 0 & C_5 & 0 & -\zeta_2 \end{pmatrix}$$

Hence the function Π_t is maximum at the point $(p^*_{r1}, p^*_{r2}, \theta^*_1, \theta^*_2)$ if the HM, H_1 is '-ve' definite at the same point. Since H_1 is independent of the DVs, we have to show that the principle minors of H_1 are alternatively negative and positive. Hence, if the following conditions hold, then Π_t is maximum at that point:

1. $-\zeta_2 < 0$ (which is always true as ζ_2 is assumed to be positive),
2. $\zeta_1\zeta_2 > 0$ (which is always true as ζ_1 and ζ_2 are assumed to be positive),
3. $2C_2\zeta_2 + C_5^2 < 0$,
4. $C_4^2C_5^2 + 2C_1C_5^2\zeta_1 + 2C_2C_4^2\zeta_2 + 4C_1C_2\zeta_1\zeta_2 - C_3^2\zeta_1\zeta_2 > 0$.

Hence the proof.

Appendix B.

First, we evaluate the 1st order conditions of Π_r, and equating them with zero as follows:

$$\begin{cases} \frac{\partial \Pi_r}{\partial p_{r1}} = 0 \\ \frac{\partial \Pi_r}{\partial p_{r2}} = 0 \end{cases}$$

Solving the above two equations, we get

$$p^*_{r1} = \frac{2(\alpha_4-k_2\beta)(-a_3-w_1\alpha_3+k_1w_1\beta+w_2\beta+k_1w_2\beta-\gamma_1\theta_1)+(2+k_1+k_2)\beta(-a_4-w_2\alpha_4+w_1\beta+k_2w_1\beta+k_2w_2\beta-\gamma_2\theta_2)}{(2+k1+k2)^2\beta^2+4(\alpha_3-k_1\beta)(-\alpha_4+k_2\beta)}$$
$$= \mathfrak{F}_1(w_1, w_2, \theta_1, \theta_2) \text{ (say)}$$
$$p^*_{r2} = \frac{(2+k_1+k_2)\beta(-a_3-w_1\alpha_3+k_1w_1\beta+w_2\beta+k_1w_2\beta-\gamma_1\theta_1)+2(\alpha_3-k_1\beta)(-a_4-w_2\alpha_4+w_1\beta+k_2w_1\beta+k_2w_2\beta-\gamma_2\theta_2)}{(2+k_1+k_2)^2\beta^2+4(\alpha_3-k_1\beta)(-\alpha_4+k_2\beta)}$$
$$= \mathfrak{F}_2(w_1, w_2, \theta_1, \theta_2) \text{ (say)}.$$

If the Hessian matrix (HM) of the function Π_r is '-ve' definite at the point (p^*_{r1}, p^*_{r2}), then Π_r is maximum at the same point. The HM of Π_r is

$$H^{MS}_r = \begin{pmatrix} \frac{\partial^2\Pi_r}{\partial p^2_{r1}} & \frac{\partial^2\Pi_r}{\partial p_{r1}\partial p_{r2}} \\ \frac{\partial^2\Pi_r}{\partial p_{r2}\partial p_{r1}} & \frac{\partial^2\Pi_r}{\partial\partial p^2_{r2}} \end{pmatrix} = \begin{pmatrix} -2\alpha_3 + 2k_1\beta & (2 + k_1 + k_2)\beta \\ (2 + k_1 + k_2)\beta & -2\alpha_4 + 2k_2\beta \end{pmatrix}.$$

According to the assumption, we have $\alpha_3, \alpha_4 > \beta$ and $k_1, k_2 < 1$. Hence $\alpha_3 > k_1\beta$ and $\alpha_4 > k_2\beta$. Consequently, $-2\alpha_3 + 2k_1\beta < 0$ and $-2\alpha_4 + 2k_2\beta < 0$. Since, a matrix is said to be negative definite if the determinant of its principal minors are alternatively negative and positive. Therefore, the HM is negative definite if

1. $-2\alpha_3 + 2k_1\beta < 0$.
2. $(-2\alpha_3 + 2k_1\beta)(-2\alpha_4 + 2k_2\beta) - (2 + k_1 + k_2)^2\beta^2 > 0$.

The first condition holds from the assumption. From the second condition, we have $(-2\alpha_3 + 2k1\beta)(-2\alpha_4 + 2k2\beta) > (2 + k1 + k2)^2\beta^2$.

Now, the values $p_{r1} = p^*_{r1}$ and $p_{r2} = p^*_{r2}$ are taken from 4.2 and substituted in Π_{ms}, it transforms to Π^1_{ms}. Then we equate the 1st order condition of renewed function Π^1_{ms} to 0, we obtain

$$\begin{cases} \frac{\partial\Pi^1_{ms}}{\partial\theta_1} = 0 \\ \frac{\partial\Pi^1_{ms}}{\partial\theta_2} = 0 \\ \frac{\partial\Pi^1_{ms}}{\partial w_1} = 0 \\ \frac{\partial\Pi^1_{ms}}{\partial w_2} = 0. \end{cases}$$

Then, we solve the equations and obtain the values of θ_1, θ_2, w_1 and w_2 as θ^*_1, θ^*_2, w^*_1 and w^*_2 respectively. If the HM of the function Π^1_{ms} is negative definite at $\theta_1 = \theta^*_1, \theta_2 = \theta^*_2, w_1 = w^*_1$ and $w_2 = w^*_2$, then the function is maximum at the same values of θ_1, θ_2, w_1 and w_2. Using these values we get the optimal values of p^*_{r1} and p^*_{r2}. Hence for $p_{r1} = p^*_{r1}, p_{r2} = p^*_{r2}, \theta_1 = \theta^*_1$, $\theta_2 = \theta^*_2$, $w_1 = w^*_1$ and $w_2 = w^*_2$, the functions Π_{ms} and Π_r are maximum. Hence the proof.

Appendix C.

The 1st order conditions of the functions Π_{m1}, Π_{m2} and Π_r listed above are:

$$\begin{cases} \frac{\partial\Pi_{m1}}{\partial\theta_1} = 0 \implies k_1p_{r1}\gamma_1 + w_1\gamma_1 - \zeta_1\theta_1 = 0 \\ \frac{\partial\Pi_{m2}}{\partial\theta_2} = 0 \implies k_2p_{r2}\gamma_2 + w_2\gamma_2 - \zeta_2\theta_2 = 0 \\ \frac{\partial\Pi_r}{\partial p_{r1}} = 0 \implies a_3 - p_{r1}\alpha_3 + (k_1p_{r1} + (1 + k_2)p_{r2} + (1 + k_1)(p_{r2} - w_2))\beta + (p_{r1} - w_1)(k_1\beta - \alpha_3) + \gamma_1\theta_1 = 0 \\ \frac{\partial\Pi_r}{\partial p_{r2}} = 0 \implies a_4 - p_{r2}\alpha_4 + ((1 + k_1)p_{r1} + k_2p_{r2} + (1 + k_2)(p_{r1} - w_1))\beta + (p_{r2} - w_2)(k_2\beta - \alpha_4) + \gamma_2\theta_2 = 0. \end{cases}$$

Solving all the equations, we get the DV's values as listed in proposition 4.3. Let us assume the result is as follows:

$p_{r1} = p^*_{r1}$, $p_{r2} = p^*_{r2}$, $\theta_1 = \theta^*_1, \theta_2 = \theta^*_2$. Now, we calculate the HMs of Π_{m1}, Π_{m2} and Π_r are H_{n1}, H_{n2} and H_{n3} respectively.

$$\begin{cases} H_{n1} = \begin{pmatrix} \frac{\partial^2\Pi_{m1}}{\partial\theta^2_1} \end{pmatrix} = \begin{pmatrix} -\zeta_1 \end{pmatrix} \\ H_{n2} = \begin{pmatrix} \frac{\partial^2\Pi_{m2}}{\partial\theta^2_2} \end{pmatrix} = \begin{pmatrix} -\zeta_2 \end{pmatrix} \\ H_{n3} = \begin{pmatrix} \frac{\partial^2\Pi_r}{\partial p^2_{r1}} & \frac{\partial^2\Pi_r}{\partial p_{r1}p_{r2}} \\ \frac{\partial^2\Pi_r}{\partial p_{r2}p_{r1}} & \frac{\partial^2\Pi_r}{\partial p^2_{r2}} \end{pmatrix} = \begin{pmatrix} -2\alpha_3 + 2k_1\beta & (2 + k_1 + k_2)\beta \\ (2 + k_1 + k_2)\beta & -2\alpha_4 + 2k_2 \end{pmatrix}. \end{cases}$$

Since H_{n1}, H_{n2} and H_{n3} are negative definite matrices then the obtained values of the variables are optimal and in these values the profits of the players are maximum.

Appendix D.

From the proposition 4.2, it can be concluded that the PF of the retailer Π_r is maximum at the point $p_{r1}^* = \mathfrak{F}_1(w_1, w_2, \theta_1, \theta_2)$ and $p_{r2}^* = \mathfrak{F}_2(w_1, w_2, \theta_1, \theta_2)$.

Then we put the values of p_{r1} and p_{r2} in the function Π_{m2} and it transforms to Π_{m2}^*. Then we equate the 1st order derivatives of Π_{m2}^* to 0 as follows:

$$\begin{cases} \frac{\partial \Pi_{m2}^*}{\partial w_2} = 0 \\ \frac{\partial \Pi_{m2}^*}{\partial \theta_2} = 0 \end{cases}$$

and then solving them we get the values $w_2 = \mathfrak{F}_3(w_1, \theta_1)$ and $\theta_2 = \mathfrak{F}_4(w_1, \theta_1,)$. If the HM is negative definite then the values of w_2 and θ_2 are optimum. Then we substitute $p_{r1} = \mathfrak{F}_1(w_1, w_2, \theta_1, \theta_2)$, $p_{r2} = \mathfrak{F}_2(w_1, w_2, \theta_1, \theta_2)$, $w_2 = \mathfrak{F}_3(w_1, \theta_1)$ and $\theta_2 = \mathfrak{F}_4(w_1, \theta_1)$ in the M1's PF Π_{m1} and this function transforms to Π_{m1}^*. Then the 1st order conditions of the function are given below:

$$\begin{cases} \frac{\partial \Pi_{m1}^*}{\partial w_1} = 0 \\ \frac{\partial \Pi_{m1}^*}{\partial \theta_1} = 0 \end{cases}$$

These two equations is solved and obtained the values of w_1 and θ_1 as w_1^* and θ_1^* respectively. If the HM of Π_{m1}^* is negative definite then the values are optimum and at these values of w_1 and θ_1, the profit of M1 Π_{m1} is maximum. Then putting the values $w_1 = w_1^*$ and $\theta_1 = \theta_1^*$ in the functions $\mathfrak{F}_3$ and $\mathfrak{F}_4$, we get the optimal values of w_2 and θ_2 respectively. After that all the optimal values of w_1, w_2, θ_1 and θ_2 are substituted in the functions $\mathfrak{F}_1$ and $\mathfrak{F}_2$ to obtain the optimal values of p_{r1} and p_{r2} respectively.

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