

A GAME THEORETIC MODEL FOR CELLULAR NETWORK OPERATORS' COOPERATION UNDER GOVERNMENT INTERVENTION

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Abstract. In the digital world today, cellular networks and their operators play a competitive and important role in communications. The bases of the competition of operators are the quality of provided services and the coverage level of their antennas, thereby attract customers. This paper studies cellular networks with two old and new operators and under the influence of government intervention in one area. Due to the high cost of building an antenna, the new operator participates in the cost of the infrastructure of the old operator to use the services of these antennas for their customers. On the other hand, the government considers incentive schemes to support mobile operators. The government plans to take part in the infrastructure costs of the old operator, and will receive the income tax from it. Hence, the new operator will go off from paying tax. The government subsidy contract with the old operator is based on the coverage level of the antenna and supports the operator to increase the coverage level. By doing so, the quality level of services and coverage development rate for the old operator increases, leading to increased demand and increased profits for this operator. On the other hand, as government support increases the demand for the old operator, the demand for the new operator decreases and the profit of the new operator decreases. Some numerical examples for Iranian telecommunication companies are applied to examine the applicability of the proposed models. Finally, sensitivity analysis on the main parameters is analyzed in-depth to extract some managerial implications.

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1. INTRODUCTION

In the past decade, the development of information and communication has brought about many changes in human life by providing services such as multimedia communications, high-quality videos, voice and video calls, the internet, and so on. This rapid growth in communications is due to the increase in mobile phone users, so that, regardless of income and lifestyle, smartphones have become an essential part of people's daily lives. Smartphones are equipped with a receiver and transmitter system that transmits data to the cellular network *via* this interface. The popularity and speed of access to services provided by cellular networks have made them one of the most potent players in communications. For this reason, in 2017, Global System for Mobile Communications or GSM Association reported that more than half of the world's population uses

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mobile services [3]. This has created a technological and economic competitive environment between cellular networks operators, seeking to provide better quality services and subsequently attract more demand. A higher level of quality, better services, and lower energy consumption indicate the performance of the cellular network, so managers are looking for a way to optimize network design. The infrastructure and antennas of any mobile network are owned by an operator.

A cellular network operator is a telecommunications service provider that provides voice and data-dependent services to users. Therefore, cellular network operators design a variety of mobile networks and offer different services based on geographical conditions and customer preferences in order to attract more demands in broader coverage. Bandwidth traffic is one of the parameters that plays an important role in customer dissatisfaction and the imposition of economic costs on operators, which is why operators seek to increase the quality of communications and reduce traffic volumes. The data and information have shown that messages, when transmitted to mobile users, occupy more than 33% of the signaling traffic passing through the bandwidth [2]. The first generation (1G) cellular system had a smaller bandwidth, so that supported 10 Kbps data rates and the second (2G) and third (3G) generations were produced with higher quality. However, through the evolution of wireless networks, the fourth generation (4G) has been released with the highest quality data transmission and peak data rates reached about 1 Gbps [28]. Thus, the quality and coverage have become competitive advantages for operators.

In recent years, due to the growing number of mobile operators, a competitive environment has been created between operators to absorb more demand and subsequently higher profits. For this reason, operators are trying to create an antenna coverage for their users in all geographic areas so that they cope with demand satisfaction. On the other hand, operators provide two types of SIM cards to answer all customers, one of which is a prepaid card and other SIM cards are permanent. The price of a prepaid SIM card is lower than the permanent SIM card and requires a periodic charge, but it costs more for services. Occasionally, operators are constructing a cellular antennas network jointly due to reduced infrastructure costs, intending to meet the demand of their customers [4]. The government subsidizes operators for the purpose of customer satisfaction and the raising of the quality and coverage of the mobile network; on the other hand, it receives tax on operators' income.

In this paper, a model is developed for cellular networks consisting of an old operator, a new operator, and a government with four kinds of deterministic demand. The demand for operators is divided into two parts, the first part is the demand for prepaid SIM cards and the second part is the demand for permanent SIM cards. The demand for operator A is affected by quality level and coverage development rate, which are two endogenous variables. Similarly, the demand for operator B is affected by permanent SIM card's price and prepaid SIM card's price, which are two endogenous variables. Because operator A is present in this area and provides services, operator A 's SIM cards prices are specified as parameters. Also, because operator A is older and the antennas are in its possession, the quality level is determined by operator A and operator B does not interfere. The main contributions of the current paper that distinguish it from the other similar investigations are stated below.

- Presenting a novel mathematical model for cellular networks members' cooperation with the view of government intervention.
- Analyzing the effect of coverage development rate and quality level in cellular networks due to their impacts on demand.
- Implementing a cooperation agreement between two operators for the participation of operator B in the infrastructure cost and use the services.
- Proposing a novel development rate based subsidies contract as the government tool to encourage operator A .

It is noteworthy to mention that in a few articles in the literature, the relationship between cellular networks has been considered as a game. One of these articles is presented by Srivastava *et al.* [24] and the other one is investigated by Kamal, Coupechoux, and Godlewski [14], but in none of both papers, these relations have not presented as a mathematical model. In the current paper, for the first time, relationships between members are

examined in the mathematical model, and the difference between the prices of c prepaid and permanent SIM cards as well as the price of their services has been addressed. Also, in the proposed model, the importance of the quality level of services and the coverage rate of the cellular network has been considered. It should be noted that no article in the literature has implemented a coverage development rate subsidiary from the government to encourage operators.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature. In Section 3, the problem description is presented. Section 4 contains the mathematical models of the centralized and decentralized systems. In Section 5, a novel coverage level based contract is suggested to coordinate operator A and government. In Section 6, some sensitivity analysis is conducted to illustrate the applicability of the proposed models. Concluding remarks are presented in the final section.

2. LITERATURE REVIEW

According to studies conducted in the field of cellular networks, it has become clear that most of the papers in this field have been devoted to the design of the cellular network, and few articles have focused on game theory in this issue. Initially, the review of the network design articles has been addressed. Due to the wide range of sensors, wireless networks have attracted many research efforts. In the same way, Cheng *et al.* [6] proposed two heuristic methods: power assignment based on minimum spanning tree and incremental power transmission levels to the sensors with aiming to minimizing energy cost. One of the problems with wireless networks is energy consumption in the transmission of information, which depends on the next node's intersection selection. More precisely, this can be called a routing problem. Chang and Tassiulas [5] considered the routing problem as a linear programming problem and, on the other hand, they sought to optimize network lifetime. Zhang and Hou [30] provided a cover model for wireless networks, with the least number of active sensors providing more coverage and connectives. Given the importance of designing a cellular network in mobile telecommunication systems, Talbi *et al.* [25] proposed a multi-objective model of the problem in its full practical complexity with the goal of improving the quality of networks and their robustness. In order to compute a set of diversified non-dominated networks, the Pareto approach is used in their paper. By using three realistic criteria, the efficiency and the reliability of hierarchical parallel hybrid models are shown. Liu *et al.* [18] maximized network lifetime, coverage, and connectives by applying deployment strategies based on voronoi polygons to increase coverage in wireless mobile sensor networks. Lanza-Gutierrez and Gomez-Pulido [17] provided a network model for assessing the efficiency deploy energy relay nodes in previously low-cost static wireless sensor networks with the aim of optimizing the average energy cost, sensitivity, and network reliability. Due to the scale and complexity of the model, the metaheuristics algorithms such as the two standard genetic Algorithms, the trajectory algorithm, and two novel swarm intelligence algorithms have been used as a solution method. Salih *et al.* [22] developed a non-cooperative game-theoretic model and strategy space according to user preference to solve network selection problems. Mohamadi *et al.* [20] developed a fuzzy scenario-based optimization model for telecommunication towers concerning humanitarian relief logistics in which the problem was formulated in a nonlinear and multi-objective model. Keskin [15] suggested a column generation heuristic approach for optimizing a mixed-integer linear programming model of wireless sensor network design with mobile sinks. Also, Ekmen and Altin-Kayhan [8] proposed a novel strategy for multi-copy and multi-path routing, regarding improve energy efficiency. Considering the reliability concept, they developed a mixed integer programming model. Additionally, Dahi *et al.* [7] have proposed a new low-complexity adaptive cellular genetic algorithm for optimizing the mobile phone tracking process, which it has implemented in 25 differently-sized realistic mobile networks and the results have been assessed.

Due to the presence of different decision-makers in wireless networks, a mathematical analysis in the form of a game will create a competitive and attractive atmosphere among decision-makers. However, by reason of the complexity of mobility and traffic models, the dynamic topology, and the unpredictability of link quality, mathematical analysis of *ad-hoc* wireless networks has not been effective and successful. Srivastava *et al.* [24] developed a model that examines the various interactions in these networks as game theory and motivates

decision-makers to participate in social activities and increase profits. Ji and Liu [13] investigated a game-theoretical overview of dynamic spectrum sharing in an *ad-hoc* wireless network from three aspects: analysis of network users' behaviors, efficient dynamic distributed design, and optimality analysis. Kamal *et al.* [14] proposed a game-theoretical model to analyze the interaction among cellular operators with packet services in cellular networks. For this model, there are three objectives: user productivity, spectrum price, and the blocking probability into consideration. Trestian *et al.* [26] studied an overview of issues and challenges of cellular decision making, a comprehensive classification of related game-theoretic approaches, and a discussion on the application of game theory problems that the next generation of 4G deals with them. To improve the energy efficiency of green cellular networks, Zheng *et al.* (2015) presented a distributed cooperative approach by graphical game, where each base station acts as a game player with the constraint of traffic load. Qin *et al.* [21] developed a Probabilistic Routing scheme based on Game Theory (PRGT) to stimulate cooperation among selfish nodes. Also, they implemented a Markov-based probability prediction model to investigate selecting the relying node. It is worth mentioning that Fander and Yaghoubi [10] recently proposed the mathematical models for cooperation among mobile phone manufacturer, mobile application developers, and mobile operators based on a revenue sharing contract and without government intervention.

In the discussion of the game theory in the reviewed articles, the government intervention is shown as a supportive and motivational tool for organizations and customers. Sheu and Chen [23] investigated the impacts of financial government interventions on the competition of green Supply chains. Their analysis based on a three-stage game model showed that if the government becomes involved as a facilitator, the profitability of the green supply chains improves dramatically. Hafezalkotob [11] proposed a model with considering the price competition of two supply chains, one green and one regular, under the government financial intervention. He analyzed the impact of government tariffs on the players' optimal policies. Mahmoudi and Rasti-Barzoki [19] examined the contradiction between government goals and the goals of the manufacturers in the sustainable supply chain for the first time. Three different scenarios for the government are considered. In the first scenario, the government applies taxes and subsidies in order to maximize its profit with the highest overall environmental impact. In the second scenario, the government determines tariffs that calculate the overall environmental impact, at least with respect to their lower profit margin. Finally, in the third scenario, the government eliminates profit and environmental goals through a linear combination in a goal function. In addition, Hafezalkotob [12] presented twelve mathematical programming models using the Stackelberg game between government and supply chain. A comprehensive analysis of the supply chain shows some managerial insights, which have indicated that all interventions are beneficial because they lead to more social services than prohibitive policies. Zand *et al.* [29] analyzed direct government limitation on closed-loop supply chain members' optimal decisions and profit by determining certain thresholds for product greening level based on maximizing government social welfare. Recently, a three-echelon closed-loop solar cell supply chain including a supplier, a third-party logistics (3PL), and an assembler is presented by Kharaji Manouchehrabadi and Yaghoubi [16]. Fander and Yaghoubi [10] recently analyzed the government supervision in fuel-efficient technology on automotive and fuel supply chain with two policies of subsidy-taxation and granting loans to customers.

In none of the above-mentioned articles, the participation of two operators is considered in the construction of a cellular network with government support. Furthermore, due to the importance of the coverage level in customer satisfaction, the government allocates subsidies to operators on this basis. To the best of our knowledge, this paper is the first study that analyzes the decisions of operators and government besides the improvement of operators' profit and government utility.

3. PROBLEM DESCRIPTION

First, all of the notations applied in this paper are illustrated in Table 1. Then, consider a cellular network with an old operator *A* and a new operator *B*, in which operator *B* has participated in the infrastructure costs of operator *A*. In this network, the government supports operator *A* by considering subsidies on the infrastructure cost, in order to increase services quality and coverage development rate and, on the other hand, receives income

TABLE 1. Parameters and decision variables.

α	Coefficient of demand ratio
r	Coverage development rate
a_1	Potential demand for prepaid SIM cards
a_2	Potential demand for permanent SIM cards
b	Competitive services price coefficient
$p^{A,k,1}$	The selling price of the prepaid SIM card's services for operator A
$p^{B,k,1}$	The selling price of the prepaid SIM card's services for operator B
$p^{A,k,2}$	The selling price of the permanent SIM card's services for operator A
$p^{B,k,2}$	The selling price of the permanent SIM card's services for operator B
D	The demand average
c	Competitive SIM cards price coefficient
d	Coefficient of quality level
η	Coefficient of infrastructure rate
φ	Participation rate of operator B
γ	Government taxes rate
r_0	Coefficient of coverage development rate
q	Quality level of operator A
m	Coefficient of coverage development rate costs
k	Coefficient of quality level costs
λ_1	Coefficient of the government's economic satisfaction factor
λ_2	Coefficient of the government's quality level satisfaction factor
λ_3	Coefficient of the government's coverage development satisfaction factor
$D^{A,1}$	The demand of prepaid SIM card of operator A
$D^{A,2}$	The demand of permanent SIM card of operator A
$D^{B,1}$	The demand of prepaid SIM card of operator B
$D^{B,2}$	The demand of permanent SIM card of operator B
$p^{A,sim,1}$	The price of prepaid SIM card of operator A
$p^{A,sim,2}$	The price of permanent SIM card of operator A
$p^{B,sim,1}$	The price of prepaid SIM card of operator B
$p^{B,sim,2}$	The price of permanent SIM card of operator B
Π_A	The profit of operator A
Π_B	The profit of operator B
Π_g	The utility function of the government
Π_T	The total profit centralized system

tax. It is worth noting that operator B does not include government subsidiary support and has been exempted by taxes to support this operator. Figure 1 shows the structure of the cellular network.

Operator B receives a φ rate of its customers' demand free from the network due to its participation in costs, and the rest of its demand is purchased from operator A with a discount price based on the percentage of participation.

The main assumptions of this paper are stated as follows:

- Market demand for operator A is a function of its coverage development rate and technology level, which it has two part for permanent and prepaid SIM cards. Let $D^{A,1}(r, q) = \alpha r_0 a_1 - b(p^{A,k,1} - p^{B,k,1}) - c(p^{A,sim,1} - p^{B,sim,1}) + d(q)$ and $D^{A,2}(r, q) = \alpha r_0 a_2 - b(p^{A,k,2} - p^{B,k,2}) - c(p^{A,sim,2} - p^{B,sim,2}) + d(q)$, where $a > 0$, $b > 0$, $c > 0$, $d > 0$, and $r_0 > 0$. Note that $D^{A,1}(r, q), D^{A,2}(r, q) > 0$ is always satisfied [1, 10].
- Market demand for operator B is a function of its sale prices for prepaid and permanent SIM cards' services, which it has two part for permanent and prepaid SIM cards. Let $D^{B,1}(p^{B,k,1}) = (1 - \alpha)r_0 a_1 - b(p^{B,k,1} - p^{A,k,1}) - c(p^{B,sim,1} - p^{A,sim,1}) + d(q)$ and $D^{B,2}(p^{B,k,2}) = (1 - \alpha)r_0 a_2 - b(p^{B,k,2} - p^{A,k,2}) - c(p^{B,sim,2} - p^{A,sim,2}) + d(q)$.

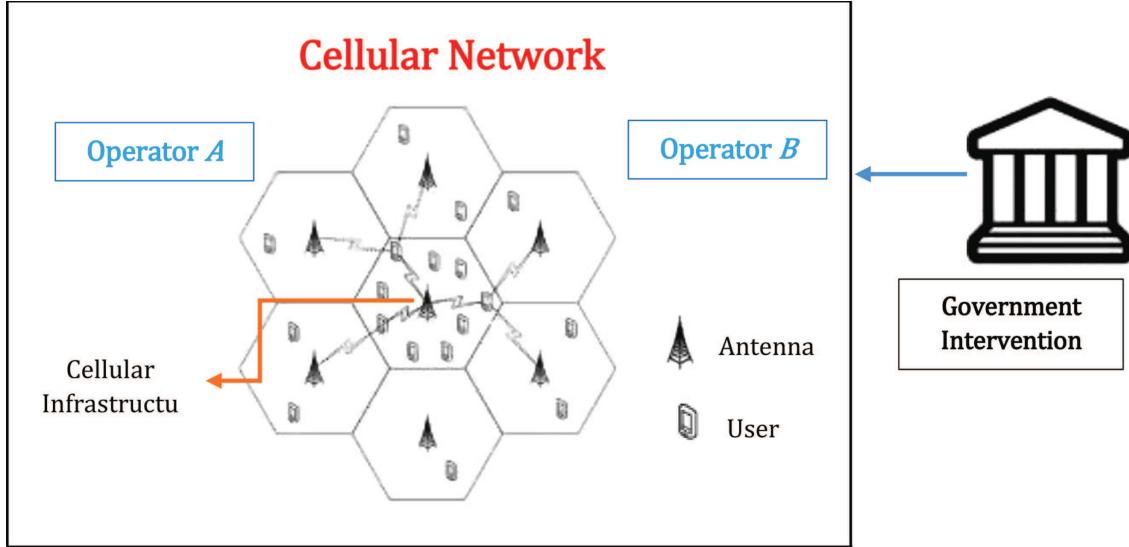


FIGURE 1. The structure of the cellular network.

$b(p^{B,k,2} - p^{A,k,2}) - c(p^{B,\text{sim},2} - p^{A,\text{sim},2}) + d(q)$, where $a > 0, b > 0, c > 0, d > 0$, and $r_0 > 0$. Note that $D^{B,1}(p^{B,k,1}), D^{B,2}(p^{B,k,1}) > 0$ is always satisfied [1, 10].

- Energy costs are included in the operating cost [27]. Energy costs are costs related to energy consumption when providing services to operators, the main of which is the cost of electricity consumption. Since operating costs and energy costs are both per unit of demand and are considered as parameters in this model, energy costs are considered as a subset of operating costs.
- Operator B does not receive subsidies from the government and is exempt from taxes. Operator B is a new operator in the market that does not have the infrastructure and meets the demand of its customers by purchasing services from operator A . However, the presence of this operator in the market and making the price of services and SIM cards competitive can be a threat to the demand and profit of operator A . As a result, in order to support the old operator and help its survival, the government participates in the infrastructure costs of this operator and receives taxes in return. In contrast, the government does not provide financial support to the new operator and only exempts it from taxes to help.

4. MATHEMATICAL MODELING

According to the problem descriptions, the behavior of the cellular network and the government can be examined as follows. Operator B participates in infrastructure costs of operator A as φ rate and uses the services of the cellular network. Operator A wants to improve its coverage development rate and quality level to increase its profit and operator B tries to decrease its services price to have more profit. The government intervenes in this network and contributes to infrastructure costs as rr_0 rate and receives revenue taxes as γ rate from operator A .

4.1. Centralized system

In this case, there is no centralized model between operator A and operator B . Due to the point that customers of operators are independent of each other, each of the operators provides the right service to request their customers. The two operators also have different SIM cards and only use a common antenna in this area. If these two operators are integrated, their business goals will deal with many challenges. Both parties are

concerned about their own business condition, and they decide to participate with each other to raise profits and reduce costs.

4.2. Decentralized system

In this section, operator A seeks to increase its own profit. Operator A faces a demand, which varies with quality level and coverage development rate. Indeed, as quality level (q) and coverage development rate (r) increase, demand increases; likewise, an increase in the demand value of operator A leads to an increase in operator A 's profit. On the other hand, Operator B does not have infrastructure and antenna at this location, but because of the considerable demand, it wants to provide services at this location. Operator B copes with a demand, which varies with prepaid SIM card services price ($p^{B,k,1}$) and permanent SIM card services price ($p^{B,k,2}$). Actually, by increasing prepaid SIM card services price ($p^{B,k,1}$) and permanent SIM card services price ($p^{B,k,2}$), demand decreases; so, increase in the demand value of operator B leads to a decrease in operator A 's profit. Therefore, it is economical for operator B to participate in part of the infrastructure costs of operator A instead of building a new network, thereby keeping prices low and benefiting from services. However, due to the huge costs of creating the necessary infrastructure to provide services, it proposes to operator A to participate in the infrastructure costs of improving the quality and developing the coverage level of operator A by (φ). Accordingly, operator B contributes $(1 - \varphi)(kq^2 + m(rr_0)^2)$ of the infrastructure costs of operator A and in contrast responds to $(\varphi)(D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2}))$ of its demand for free. According to the joint agreement between the two operators, operator B buys the rest of its service demand from the first operator at a discounted price $((1 - \varphi)p^{A,k,1})$. By implementing this agreement, operator A with the benefits obtained can improve the quality of its services and expand the level of coverage of its services. On the other hand, operator B can provide services to its customers in this area and satisfy customers without the need for investment to create infrastructure. It should be noted that operator A is the old operator in this region and has sold its prepaid and permanent SIM cards according to market demand, and after that, it only offers services related to these SIM cards to customers in this region. Proceeds from the sale of permanent and prepaid SIM cards for operator A exist only for the part that is added to the areas covered by operator A after the development of the coverage level. Therefore, the number of prepaid and permanent SIM cards sold in the coverage development sector is in the form of dividing the demand potential depending on the coverage level rate on the average consumption of the region and as $(\frac{\alpha r a_1}{D})$ and $(\frac{\alpha r a_2}{D})$. On the other hand, operator B , because it has recently entered this region, has the potential of the market to sell permanent and prepaid SIM cards in this region, and therefore the number of SIM cards sold is related to the total demand and as $(\frac{D^{B,1}(p^{B,k,1})}{D})$ and $(\frac{D^{B,2}(p^{B,k,2})}{D})$. In this Stackelberg game, which is a non-cooperative game, operator A is the leader and operator B is the follower. The total profit generated by operator A is as follows:

$$\begin{aligned} \Pi_A = & \left(p^{A,k,1} D^{A,1}(r, q) + p^{A,k,2} D^{A,2}(r, q) + p^{A,\text{sim},1} \left(\frac{\alpha r a_1}{D} \right) + p^{A,\text{sim},2} \left(\frac{\alpha r a_2}{D} \right) \right) \\ & - (1 - \varphi)(kq^2 + m(rr_0)^2) + (1 - \varphi)(D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2})) (1 - \varphi) p^{A,k,1} \\ & - \eta(D^{A,1}(r, q) + D^{A,2}(r, q) + D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2})). \end{aligned} \quad (4.1)$$

In equation (4.1), the first and second terms indicate the sales revenue from prepaid and permanent SIM cards services. The third and fourth terms state the sales revenue from prepaid and permanent SIM cards according to the point that the ratio of the demand is calculated by dividing the coverage into the average demand, which indicates the number of SIM cards. The fifth term refers to the Infrastructure costs based on the quality level and coverage development rate. The sixth term depends on the sales revenue of the rest of operator B 's service demand, which is satisfied by the first operator A . The seventh term concerns to operational and energy costs.

Theorem 4.1. *The operator A 's profit function is concave in r and q . Thus, the optimal and exclusive (r, q) can be obtained to maximize the operator A 's expected profit. The optimal coverage development rate and quality*

level are calculated as follows, respectively.

$$r = \frac{-2\alpha(a_1 p^{A,\text{sim},1} + a_2 p^{A,\text{sim},2}) + (2a_1\eta + a_1(-2 + (-1 + \alpha)(-2 + \varphi)\varphi)p^{A,k,1})}{4(-1 + \varphi)mr_0^2D} \\ + \frac{a_2(2\eta + (-1 + \alpha)(-1 + \varphi)^2 p^{A,k,1} - (1 + \alpha)p^{A,k,2})r_0D}{4(-1 + \varphi)mr_0^2D} \quad (4.2)$$

$$q = \frac{d(8\eta + (-5 - 2(-2 + \varphi)\varphi)p^{A,k,1} - 3p^{A,k,2})}{4(-1 + \varphi)k}. \quad (4.3)$$

Proof. Since the first principal minor is negative $(-2(1 - \varphi)mr_0^2 < 0)$ and the second principal minor is positive $(4(1 - \varphi)^2mr_0^2k > 0)$, the operator A 's expected profit function is concave in both r and q . Therefore, the optimal solution can be achieved if the corresponding parameters satisfy the conditions mentioned for concavity.

$$H = \begin{bmatrix} \frac{\partial^2 \Pi_A}{\partial r^2} & \frac{\partial^2 \Pi_A}{\partial r \partial q} \\ \frac{\partial^2 \Pi_A}{\partial r \partial q} & \frac{\partial^2 \Pi_A}{\partial q^2} \end{bmatrix} \\ H = \begin{bmatrix} -2(1 - \varphi)mr_0^2 & 0 \\ 0 & -2(1 - \varphi)k \end{bmatrix}.$$

The profit generated by operator B is as follows:

$$\Pi_B = \left(p^{B,k,1}D^{B,1}(p^{B,k,1}) + p^{B,k,2}D^{B,2}(p^{B,k,2}) + p^{B,\text{sim},1}\left(\frac{D^{B,1}(p^{B,k,1})}{D}\right) + p^{B,\text{sim},2}\left(\frac{D^{B,2}(p^{B,k,2})}{D}\right) \right) \\ - \varphi\left(kq^2 + m(rr_0)^2\right) - (1 - \varphi)(D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2}))(1 - \varphi)p^{A,k,1}. \quad (4.4)$$

In equation (4.4), the first and second terms indicate the sales revenue from prepaid and permanent SIM cards services. The third and fourth terms state the sales revenue from prepaid and permanent SIM cards according to the ratio of the demand multiplied by services prices to the average demand, which indicates the number of SIM cards. The fifth term refers to the participation rate cost in infrastructure costs based on the quality level and coverage development rate. The sixth term is the purchases cost of the demand. \square

Theorem 4.2. *The operator B 's profit function is concave in $p^{B,k,1}$ and $p^{B,k,2}$. Thus, the optimal and exclusive $(p^{B,k,1}, p^{B,k,2})$ can be obtained to maximize operator B 's expected profit. The optimal prepaid SIM cards services price and permanent SIM cards services price are calculated as follows,*

$$p^{B,k,1} = \frac{1}{8b(-1 + \varphi)kmr_0D} \left(a_1^2(-1 + \alpha)k \left(2\alpha p^{A,\text{sim},1} + (-2\eta + (2 - (-1 + \alpha)(-2 + \varphi)\varphi)p^{A,k,1})r_0D \right) \right. \\ \left. + a_1a_2(-1 + \alpha)k \left(2\alpha p^{A,\text{sim},1} + (-2\eta + (-1 + \alpha)(-1 + \varphi)^2 p^{A,k,1} + p^{A,k,2} + \alpha p^{A,k,2})r_0D \right) \right. \\ \left. + mr_0 \left(\left(-d^2((5 + 2(-2 + \varphi)\varphi)p^{A,k,1}) + 3p^{A,k,2} \right) + 4c(2d^2\eta + (-1 + \varphi)k(p^{A,\text{sim},1} - p^{B,\text{sim},1})) \right) D \right. \\ \left. + 4b(-1 + \varphi)k \left(-p^{B,\text{sim},1} + (2 + (-2 + \varphi)\varphi)p^{A,k,1}D \right) \right) \quad (4.5)$$

$$p^{B,k,2} = \frac{1}{8b(-1 + \varphi)kmr_0D} \left(a_1^2(-1 + \alpha)k \left(2\alpha p^{A,\text{sim},1} + \varphi(-\alpha(-2 + \varphi) + \varphi)p^{A,k,1}r_0D - 2(\eta + (-1 + \varphi)p^{A,k,1})r_0D \right) \right. \\ \left. + a_2^2(-1 + \alpha)k \left(2\alpha p^{A,\text{sim},2} + \alpha(-(-1 + \varphi)^2 p^{A,k,1} + p^{A,k,2})r_0D + (-2\eta + (-1 + \varphi)^2 p^{A,k,1} + p^{A,k,2})r_0D \right) \right. \\ \left. + mr_0 \left(\left(-d^2((5 + 2(-2 + \varphi)\varphi)p^{A,k,1} + 3p^{A,k,2}) + 4c(2d^2\eta + (-1 + \varphi)k(p^{A,\text{sim},2} - p^{B,\text{sim},2})) \right) D \right. \right. \\ \left. \left. + 4b(-1 + \varphi)k \left(-p^{A,\text{sim},2} + ((-1 + \varphi)^2 p^{A,k,1} + p^{A,k,2})D \right) \right) \right). \quad (4.6)$$

Proof. Since the first principal minor is negative ($-2b < 0$) and the second principal minor is positive ($4b^2 > 0$), the operator B 's expected profit function is concave in both $p^{B,k,1}$ and $p^{B,k,2}$. Therefore, the optimal solution can be achieved if the corresponding parameters satisfy the conditions mentioned for concavity.

$$H = \begin{bmatrix} \frac{\partial^2 \Pi_B}{\partial p^{B,k,1} \partial p^{B,k,2}} & \frac{\partial^2 \Pi_B}{\partial p^{B,k,1} \partial p^{B,k,2}} \\ \frac{\partial^2 \Pi_B}{\partial p^{B,k,1} \partial p^{B,k,2}} & \frac{\partial^2 \Pi_B}{\partial p^{B,k,2} \partial p^{B,k,2}} \end{bmatrix}$$

$$H = \begin{bmatrix} -2b & 0 \\ 0 & -2b \end{bmatrix}.$$

□

5. GOVERNMENT INTERVENTION

To support the business, in many cases, government considers subsidies. Hence their costs will decrease, and as a result, the profitability will decrease. In this model, the presence of the government in the network and its involvement in the decisions of operator A has been used as a mechanism to coordinate and increase profits in the network. However, since the government has not been involved in the network before, it was not possible to determine the upper and lower limits of the contract parameter in this model. Therefore, a new contract was implemented between the government and operator A based on the value of one of the decision variables that both the government and the operator wanted to increase. In this case, the government decides to take part in infrastructure costs in order to support the old operator. On the other hand, to encourage operator A to improve the quality level and increase the coverage development rate, the rate of its partnership is based on the increased coverage development rate. The contract is that the government contributes to operator A 's infrastructure costs based on the level of coverage-development rate of operator A (rr_0) before the contract is executed, and in return receives a tax from the operator at a fixed rate (γ). The purpose of the government is to support operator A in costs, and from the other side, by increasing the quality level and the coverage development rate, it will enhance customer satisfaction and improve the social responsibility aspects. The government does not receive taxes from operator B to support him and does not subsidize his costs. In this Stackelberg model, the government is the leader and operator A and operator B are the followers. The total profit generated by operator A after government support is calculated as follows:

$$\begin{aligned} \Pi_A = & (1 - \gamma) \left(p^{A,k,1} D^{A,1}(r, q) + p^{A,k,2} D^{A,2}(r, q) + p^{A,\text{sim},1} \left(\frac{\alpha r a_1}{D} \right) + p^{A,\text{sim},2} \left(\frac{\alpha r a_2}{D} \right) \right) \\ & - (1 - \varphi - rr_0) \left(kq^2 + m(rr_0)^2 \right) + (1 - \gamma)(1 - \varphi) (D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2})) (1 - \varphi) p^{A,k,1} \\ & - \eta (D^{A,1}(r, q) + D^{A,2}(r, q) + D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2})). \end{aligned} \quad (5.1)$$

The optimal values of the decision variables before the placement in the utility function of the government are as follows.

$$\begin{aligned} r^* = & \frac{1}{4(\varphi - 1)(rr_0 - 1)m r_0^2 D} \left(2a_1 \alpha (1 - \gamma) p^{A,\text{sim},1} + 2a_2 \alpha (1 - \gamma) p^{A,\text{sim},2} \right. \\ & + a_1 (-2\eta + (-2 + (\alpha - 1)(\varphi - 2)\varphi)(\gamma - 1)p^{A,k,1}) r_0 D \\ & \left. + a_2 (-2\eta + (\gamma - 1)((\alpha - 1)(\varphi - 1)^2 p^{A,k,1} - (\alpha - 1)p^{A,k,2})) r_0 D \right) \end{aligned} \quad (5.2)$$

$$q^* = \frac{d(8\eta + (\gamma - 1)((5 + 2(\varphi - 2)\varphi)p^{A,k,1} + 3p^{A,k,2}))}{4(1 - \varphi)k(rr_0 - 1)} \quad (5.3)$$

$$p^{B,k,1*} = \frac{1}{8b(\varphi - 1)(\gamma - 1)kmr_0(rr_0 - 1)D} \left(a_1^2 (-1 + \alpha)(\gamma - 1)k \left(2\alpha(\gamma - 1)p^{A,\text{sim},1} \right. \right.$$

$$\begin{aligned}
& + (2\eta - (-2 + (\alpha - 1)(\varphi - 2)\varphi)(\gamma - 1)p^{A,k,1})r_0D) + a_1a_2(\alpha - 1)(\gamma - 1)k(2\alpha(\gamma - 1)p^{A,\text{sim},2} \\
& + (2\eta + (\gamma - 1)(-(\alpha - 1)(\varphi - 1)^2p^{A,k,1} + p^{A,k,2} + \alpha p^{A,k,2}))r_0D) \\
& - mr_0((\gamma - 1)(d^2(\gamma - 1)((5 + 2(\varphi - 2)\varphi)p^{A,k,1} + 3p^{A,k,2}) \\
& + 4c(2d^2\eta + (\varphi - 1)k(p^{A,\text{sim},1} - p^{B,\text{sim},1})(1 - rr_0)))D \\
& + 4b(\varphi - 1)k(rr_0 - 1)((\gamma - 1)p^{B,\text{sim},1} + (2 + (\varphi - 2)\varphi - \gamma)p^{A,k,1}D)) \quad (5.4)
\end{aligned}$$

$$\begin{aligned}
p^{B,k,2*} = & \frac{1}{8b(\varphi - 1)(\gamma - 1)kmr_0(rr_0 - 1)D} \left(-mr_0((\gamma - 1)(8cd^2\eta + d^2(\gamma - 1)((5 + 2(\varphi - 2)\varphi)p^{A,k,1} \right. \\
& \left. + 3p^{A,k,2}) - 4c(\varphi - 1)k(p^{A,\text{sim},2} - p^{B,\text{sim},2})(rr_0 - 1))D + 4b(\varphi - 1)k(rr_0 - 1)((\gamma - 1)p^{B,\text{sim},2} \right. \\
& \left. + ((\varphi - 1)^2p^{A,k,1} + p^{A,k,2} - \gamma p^{A,k,2})D) + a_1a_2(\alpha - 1)(\gamma - 1)k((2\eta + (2 + (\varphi - 2)\varphi)(\gamma - 1) \right. \\
& \times p^{A,k,1})r_0D + \alpha(\gamma - 1)(2p^{A,\text{sim},1} - (\varphi - 2)\varphi p^{A,k,1}r_0D) + a_2^2(\alpha - 1)(\gamma - 1)k((2\eta + (\gamma - 1) \right. \\
& \times ((\varphi - 1)^2p^{A,k,1} + p^{A,k,2}))r_0D + \alpha(\gamma - 1)(2p^{A,\text{sim},2} + ((\varphi - 1)^2p^{A,k,1} + p^{A,k,2})r_0D)) \right) \quad (5.5)
\end{aligned}$$

The total utility generated by the government is calculated as follows:

$$\begin{aligned}
\Pi_g = & \lambda_1\gamma \left[\left(p^{A,k,1}D^{A,1}(r, q) + p^{A,k,2}D^{A,2}(r, q) + p^{A,\text{sim},1}\left(\frac{\alpha r a_1}{D}\right) + p^{A,\text{sim},2}\left(\frac{\alpha r a_2}{D}\right) \right) \right. \\
& \left. + (1 - \varphi)(D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2}))(1 - \varphi)p^{A,k,1} \right] \\
& + \lambda_2q[D^{A,1}(r, q) + D^{A,2}(r, q) + D^{B,1}(p^{B,k,1}) + D^{B,2}(p^{B,k,2})] + \lambda_3rr_0(a_1 + a_2). \quad (5.6)
\end{aligned}$$

In equation (5.6), the first term indicates the taxes revenue from operator A . The second term states government satisfaction with quality. The third term refers to government satisfaction with coverage development rate.

6. NUMERICAL EVALUATION

In this section, a numerical example based on the case study with sensitivity analysis is presented to illustrate the above theoretical results and to gain some managerial insights.

6.1. Numerical examples

In this section, inspired by the case study in the Iran market, the numerical examples are designed to illustrate theoretical results of coverage development rate contract of cellular networks. To conduct numerical evaluations, some of the parameters are assumed to be constant in all experiments. Based on the mobile phone operators in the Iranian market the price of the permanent and prepaid SIM cards are $p^{A,\text{sim},1} = 20\,000$, $p^{A,\text{sim},2} = 150\,000$, $p^{B,\text{sim},1} = 20\,000$, $p^{B,\text{sim},2} = 60\,000$ (mci.ir, Irancell.ir). Other data have been estimated by experts in this field in Iranian market as $a_1 = 100\,000$, $a_2 = 50\,000$, $D = 7000$, $b = 500$, $k = 60$, $m = 14\,000\,000$, $\lambda_1 = 0.75$, $\lambda_2 = 1$, $\lambda_3 = 1$.

The rest of the parameter values for all five examples are given in Table 2.

In Example #1, in the decentralized model, when the first principal minors for operator A and operator B are negative ($-2(1 - \varphi)mr_0^2 = -3\,686\,144 < 0$, $-2b = -1000 < 0$) and the second principal minors for them are positive ($4(1 - \varphi)^2mkr_0^2 = 300\,789\,350.4 > 0$, $4b^2 = 4\,000\,000 > 0$), the optimal coverage development rate of operator A is placed at 1.012 and the optimal quality level of the operator A is 0.52. The optimal selling price of the prepaid and permanent SIM cards' services for operator B are placed at 74.75 and 62.30. The optimal profits for operator A and the operator B in the decentralized model are $\Pi_A = 778\,998.370$ and $\Pi_B = 1.381 \times 10^6$ thousand tomans². After the implementation of the cooperation plan, the optimal coverage development rate of

²Iranian toman is equal to 10 Iranian rials (IRR).

TABLE 2. Parameter values for the numerical experiments.

No.	c	φ	d	r_0	α	$p^{A,k,1}$	$p^{A,k,2}$	η
1	0.1	0.38	0.2	0.44	0.3	65	59	4.3
2	0.1	0.4	0.21	0.44	0.32	63	57	4.4
3	0.11	0.3	0.29	0.44	0.4	67	60	2.75
4	0.1	0.3	0.3	0.4	0.4	67	57	4
5	0.13	0.29	0.28	0.43	0.45	66	61	3.5

TABLE 3. The results of the numerical examples before/after government intervention.

No.	Before/ After contract	Operator A 's profit	Operator B 's profit	Development rate	Quality level	Permanent SIM's price	Prepaid SIM's price
1	Before	778 998.370	1.381×10^6	1.012	0.52	62.30	74.75
	After	1.457×10^6	1.146×10^6	2.07	1.06	78.63	107.41
2	Before	726 589.700	1.274×10^6	1.017	0.54	59.77	71.86
	After	1.640×10^6	423 105.876	2.32	1.22	79.31	110.93
3	Before	1.413×10^6	1.039×10^6	1.05	0.73	65.94	76.30
	After	1.585×10^6	891 746.59	1.76	1.22	75.29	95.01
4	Before	1.406×10^6	941 261.13	1.13	0.72	63.28	75.79
	After	1.467×10^6	819 005.39	1.82	1.15	71.57	92.38
5	Before	1.452×10^6	971 332.	1.08	0.68	67.36	73.83
	After	1.567×10^6	716 523.	1.77	1.11	75.48	90.07

TABLE 4. The results of the numerical examples for government.

No.	γ	Government's profit
1	0.391	1.955×10^6
2	0.386	2.206×10^6
3	0.416	2.00×10^6
4	0.409	1.872×10^6
5	0.414	2.00×10^6

operator A is placed at 2.07 and the optimal quality level of operator A is 1.06. The optimal selling prices of the prepaid and permanent SIM cards' services for operator B are placed at 107.41 and 78.63. After the contract, the optimal profit for operator A increases to $\Pi_A = 1.457 \times 10^6$ and the optimal profit for operator B decreases to $\Pi_B = 1.146 \times 10^6$ thousand tomans. The optimal utility of government is 1.955×10^6 and the tax rate coefficient is 0.391. The results of the model indicate that after the contract between government and operators, operator A increases its coverage development rate and quality level; with increasing these factors, the demand of operator A increases and subsequently, the profit of operator A increases. On the other hand, operator B gains less profit than the decentralized system, due to the lack of government support and participation in infrastructure cost, sustaining high costs. In order to counteract the decline in profits, operator B increases its sales prices of the prepaid and permanent SIM cards' services. Tables 2–4 summarize the parameter values related to the five presented numerical examples.

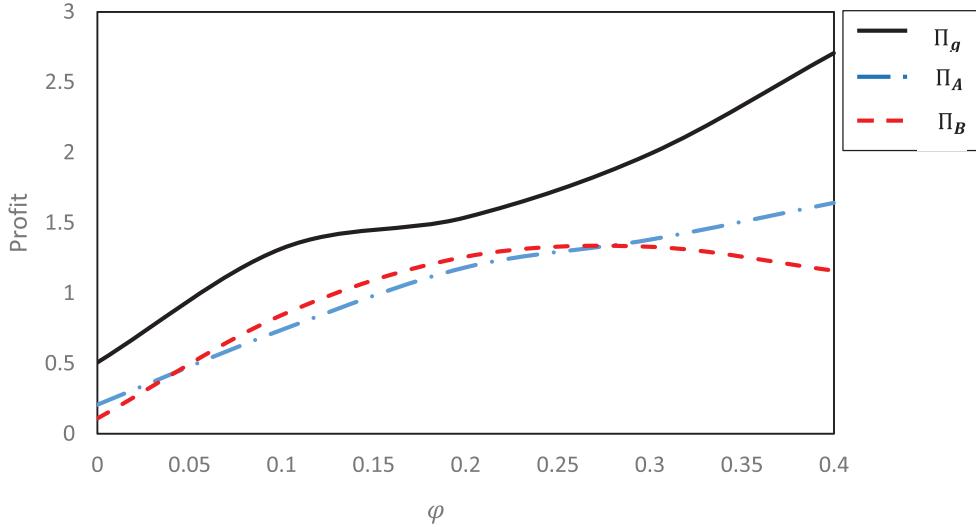


FIGURE 2. Alteration of φ vs. Beneficiary's profit function.

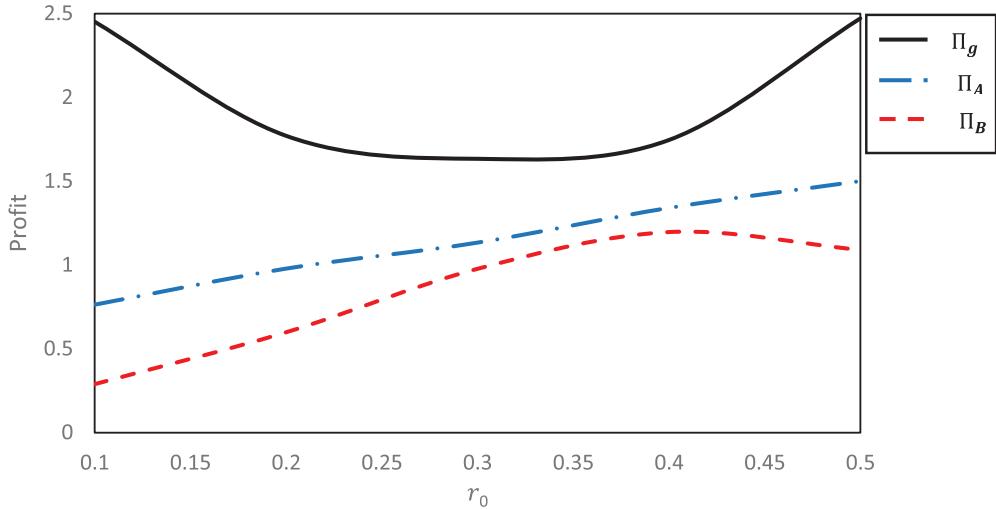
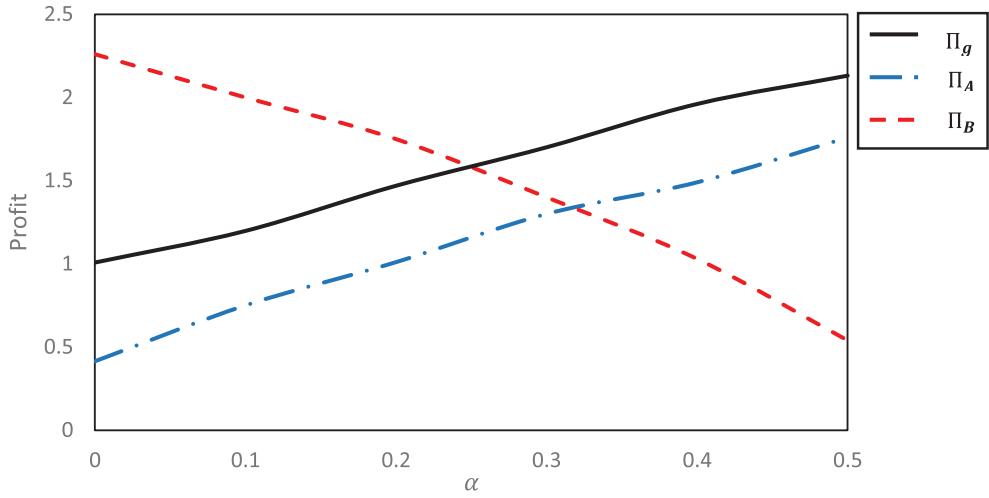
6.2. Sensitivity analysis

Some parameters have significant impacts on the performance of cellular networks and the results of the contract system. First, the impact of φ on the profits of operator A , operator B , and government are investigated and the results are shown in Figure 2. As can be seen in Figure 2, φ has a positive effect on the profits of operator A and the government; on the other hand, on the profit of operator B applies a two-part effect, in the first part, more profitable and in the second part reducing profits. As the participation rate φ increases, the participation of operator A and the government decreases in the cost of building a cellular network. As a result, with the increase in rate φ , the profit of operator A and the government increases and the profit of operator B decreases. However, it is important to note that operator B purchases a part of its demand from operator A with a discount based on this rate, thus, the profit of operator B first increases and then decreases with increasing φ . If operator B contributes more than 0.4 in the costs, then it will lose a lot, so it will not be more involved in any circumstances.

The major insights can be derived by Figure 2 are as follows:

- It is important that operator B seeks the optimal equilibrium point to determine the rate of cooperation in the infrastructure costs of operator A , because as the amount of this partnership increases, the costs of operator B exceed its revenue and the profit of this operator decreases.
- By increasing the percentage of operator B 's participation in the infrastructure costs of operator A , the share of government participation decreases and as a result, the government utility function increases. Hence, the government can spend the benefits of reducing its share of infrastructure costs on support in other issues, such as variety of operator services.

The coefficient effects of coverage development rate r_0 on network members' profits are also investigated and the results are shown in Figure 3. As illustrated in Figure 3, r_0 has a positive effect on operator A 's profit because as r_0 increases the coverage rate increases, and consequently, the demand of operator A increases; on the other hand, the government participation in costs increases and as a result, operator A 's profit increases. The effect of this parameter on the profit of operator B is twofold, initially profit increases with increasing r_0 , and then decreases. By increasing this parameter, demand increases and benefits operator B , but due to increased coverage, the cost of constructing the developed segment rises and reduces operator B 's profits. The government's profit decreases and then increases with this parameter's increase, because as the rate of coverage

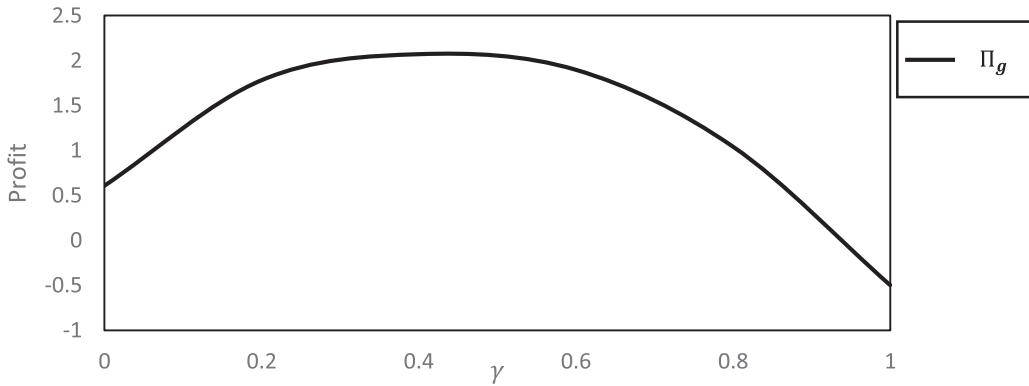
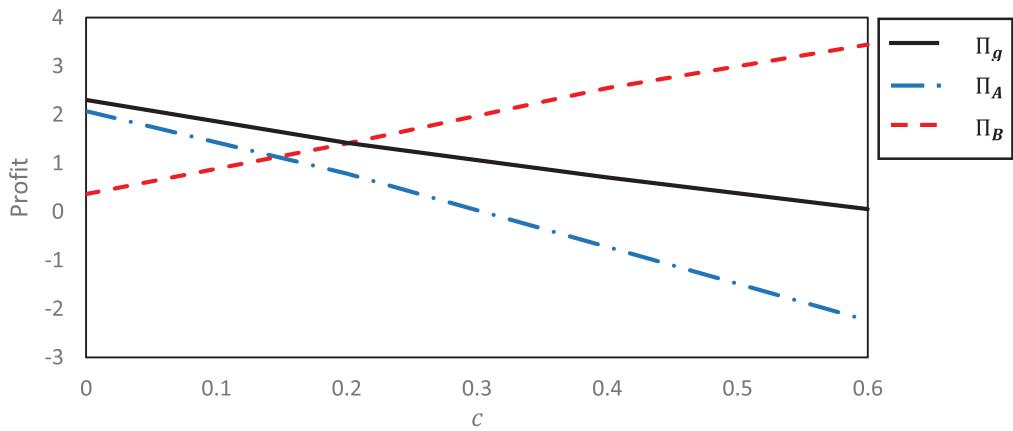
FIGURE 3. Alteration of r_0 vs. Beneficiary's profit function.FIGURE 4. Alteration of α vs. Beneficiary's profit function.

increases, government subsidy and cost participation will increase, but by increasing demand and gaining profit with operators, the government tax increases, which makes it profitable.

Some managerial insights can be derived by Figure 3 are as follows:

- As this parameter increases, the level of coverage of operator A increases, the participation rate of operator B in infrastructure costs increases. As it is clear, this increase from one level to the next reduces the profit of operator B due to the increase in costs, so operator B must seek to compensate for the decrease in revenue and increase its profit by other means.
- The government should seek to increase the sensitivity of this coefficient in the market so that by increasing this coefficient, it can increase its utility From participation costs.

The effect of α on cellular network members is investigated in Figure 4, which indicates that α has an adverse effect on operator B 's profit. On the other side, α has a positive effect on operator A 's and government's profit.

FIGURE 5. Alteration of γ vs. Beneficiary's profit function.FIGURE 6. Alteration of c vs. Beneficiary's profit function.

As α increases, the demand of operator A increases and the demand of operator B decreases. Consequently the profit of operator A increases and the profit of operator B decreases; as a result, the income from government tax increases and the government's profit rises.

The main findings of Figure 4 are as follows:

- Since α is the market share potential of market demand between two operators, increasing this share in favor of each operator increases the profit of that operator. Each of the operators can make themselves more famous in the market and gain more market potential by providing better quality or by doing things such as advertising.
- Another noteworthy point is that, given that the government supports operator A , increasing this share in favor of the first operator also increases the government's satisfaction. As a result, the government can seize more market share in favor of operator A by taking supportive measures.

The impact of the government tax coefficient is also investigated in Figure 5. The behavior of the government's utility function with the tax coefficient is a dual behavior, initially as γ increases the government's utility increase and reaching 0.4 in its own afterward; after that as γ increases, the government's utility decreases. Due to increasing γ , the taxes received from operator A increases, so that the operator A cannot increase the

quality and coverage; thus customer satisfaction decreases and consequently the government utility decreases. Therefore, the government should keep γ between 0.3 and 0.4, in order to both sides have optimal profit.

The major insight can be derived by Figure 5 is as follows:

- Since increasing the tax received from operator A , on the other hand, increases the government's participation in the operator's infrastructure costs, the government must seek a balance between the rate of participation in costs and the tax collection rate.

Finally, the effect of c is investigated in Figure 6. As can be seen in Figure 6, c has a positive effect operator B 's profit and has a negative effect on operator A and government's profit. As c increases, because of the sale prices of operator A 's SIM cards are more than operator B 's SIM cards, operator B attracts more demand and its profit increases; conversely, the demand of operator A decreases and its profit decreases. On the other hand, the government's profit is related to operator A 's profit; consequently when the profit of operator A decreases, the government's profit decreases.

Some managerial insight can be derived by Figure 6 is as follows:

- Since the price of the SIM cards of operator A is higher than the price of operator B , by increasing this sensitivity coefficient, the competitive price of the SIM cards increases and the profit of operator A and consequently the government decreases. In order to avoid this decrease in demand, operator A must reduce the price of its SIM cards.

7. CONCLUSIONS

In recent years, mobile smartphones have become an integral part of people's lives. For this reason, there is a space suitable for the business of mobile operators, each offering better and cheaper services to attract more demand. In the competitive environment between mobile operators, the service level includes the quality level of service provided and the coverage level as a competitive advantage. Sometimes, due to the remote and fragile nature of some of the areas, operators are working collaboratively in order to build antennas to serve customers. In this study, we considered two old and new cellular operators under the influence of government intervention in one area. Because of the high cost of infrastructure and with the aim of responding to customer demand, the new operator opted to participate in the infrastructure costs of the old operator. The new operator in the competitive situation in the area decides to price the sales price of SIM cards and services. The old operator aims to increase its profits by seeking to raise the quality level of service and coverage development rate in the region in order to be able to respond to more requests. On the other hand, the government considers incentive schemes to support mobile operators. The government plan is that to take part in the infrastructure cost of the old operator and receive an income tax, and to support the new operator, declare it exempt from tax. The government subsidy contract with the old operator is based on the coverage level of the antenna and supports the operator to increase the coverage level. Hence, after the contract between the government and operators, operator A increases its coverage development rate and quality level; with increasing these factors the demand of operator A increases and subsequently the profit of operator A increases. On the other hand, operator B gains less profit than decentralized system, due to the lack of government support and participation in infrastructure cost, sustaining high costs. In order to counteract the decline in profits, operator B increases its sales prices of the prepaid and permanent SIM cards' services.

Further studies may be extended to consider the energy parameter as one of the decision variables of operators to reduce costs. Also, to reduce environmental pollution, laws such as cap and trade can be implemented to reduce carbon emissions. On the other hand, the relationship between the two partners can be linked to a competitive and commercial relationship such as the purchase and sale of antennas and services, and contracts are being made to coordinate these two operators. In addition, due to the efforts of operators to reduce energy costs, energy can be considered as a decision variable.

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