RAIRO-Oper. Res. 56 (2022) 649–687
https://doi.org/10.1051/ro/2022023

RAIRO Operations Research
www.rairo-ro.org

MALMQUIST-LUENBERGER PRODUCTIVITY INDEXES FOR DYNAMIC NETWORK DEA WITH UNDESIRABLE OUTPUTS AND NEGATIVE DATA

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Abstract. The data envelopment analysis (DEA) technique is well known for computing the Malmquist-Luenberger productivity index (MLPI) in measuring productivity change in the decision-making units (DMUs) over two consecutive periods. In this research, we detect infeasibility of the directional distance function (DDF) based DEA model of MLPI under the variable returns to scale technology when data takes on negative values. We address this problem by developing a novel DDF-based DEA model that computes an improved MLPI. We extend the DDF approach to the dynamic network structure and introduce the dynamic MLPI for analyzing the performance of DMUs over time. We also develop the dynamic sequential MLPI to detect shifts in the efficient frontiers due to random shocks or technological advancements over time. The dynamic network structure in the two indexes comprises multiple divisions in DMUs connected vertically by intermediate productivity links and horizontally over time by carryovers. The proposed models are feasible and bounded with undesirable features and negative and non-negative data values. Real data of 39 Indian commercial public and private banks from 2008 to 2019 used to illustrate the two indexes.

Mathematics Subject Classification. 90B10, 90C05, 90C08, 90C39, 90C90, 91B06.

Received July 7, 2021. Accepted February 7, 2022.

1. Introduction

Productivity improvement is essential for the long-run sustainability and well being of the firms. How well a firm stand in an environment of cut-throat competition hinges on how well the firm performs on the productivity front. Given the importance of productivity change, a large body of literature has emerged, aiming at measuring the productivity growth rates using a range of productivity measurement approaches. The earliest methodological developments is attributed to the work of Solow [66] who measured productivity growth using a growth accounting framework. Among the later methodological innovations that enticed a large number of practical applications are Malmquist productivity index (MPI) and Malmquist-Luenberger productivity index (MLPI). These indexes allow decomposing the productivity change into two distinct drivers: efficiency change (EC) and technical change (TC). Though both MPfI and MLPI have emerged as standard tools for computing productivity change, their computation using the non-parametric DEA-based framework attracted more attention both at methodological and application levels.

Keywords. Data envelopment analysis, directional distance function, dynamic network structure, productivity change, dynamic Malmquist-Luenberger productivity indexes.

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The parametric and non-parametric approaches co-exist to measure the efficiency and productivity of entities. However, the two approaches exhibit some advantages and limitations. While, on the one hand, the parametric methods (like stochastic frontier analysis) assume the functional form of the frontier and estimate the parameter in it, the non-parametric method avoids the same altogether. Moreover, unlike parametric techniques, which require parameter estimation in the regression model of the explanatory variables and the probability distribution of the error measure (or noise), the non-parametric methodology of DEA is deterministic. Hence, it does not suffer from estimation errors.

Our present research contributes to both the methodology and the applications. On the theoretical side, we discovered that in the case of negative data, the modified MLPI model in [18] is infeasible. Briec and Kerstens [7] pointed out that many of Malmquist productivity indices models are infeasible. In another work, Briec and Kerstens [8] highlighted the infeasibility of the directional distance function (DDF) based Luenberger productivity index model. It is thus natural to recommend a remedy for it. Taking a leaf from the work of Lin and Chen [46], we propose a novel DDF to build an upgraded MLPI model. We prove that the proposed model is feasible for any real data, thus fully resolving the MLPI DEA model's long-standing infeasibility issue.

Secondly, we note that the traditional MLPI model treats production technology as a black box, excluding intermediate goods or linking activities. We strive to delve deep and unlock the black box in this study in order to reveal the dynamic network structure's insight and complex function. In the dynamic network DEA framework, we propose the dynamic Malmquist-Luenberger productivity index (DMLPI) to assess productivity growth of a unit. However, we realized that, like MLPI, the DMLPI also suffers from the inconsistency issue pointed out by Aparicio *et al.* [2]. The inconsistency in MLPI referred to a situation when the numerical value did not measure the actual production frontier shifts properly. Aparicio *et al.* [2] put forward a new postulate, closely related to the notion of sequential frontiers, and showed that the models based on sequential frontiers provide a more reliable measure than the traditional model in computing MLPI. In the same spirit, we finally introduce the dynamic sequential MLPI (DSMLPI) in the dynamic network framework to evaluate the sequential productivity change across different periods and different divisions in the presence of desirable and undesirable outputs with negative data values. The DSMLPI overcomes the technical regress while computing productivity growth over consecutive periods in network dynamic DEA.

To the best of our knowledge, there is no research reported in the literature to cognate DMLPI and DSMLPI in the dynamic network structure under the variable returns to scale (VRS) technology in the presence of negative data-values. The proposed indexes capture the productivity growth or regress independently of the scale change under VRS. At the application level, we analyzed the data of Indian commercial banks from 2008 to 2019 to highlight the viability of DMLPI and DSMLPI.

The paper is structured as follows. In Section 2, we briefly review the relevant DEA literature including dynamic and network framework research, and models involving negative data values. We also present the literature on variants of Malmquist productivity indexes. Section 3 revisits the issue of infeasibility in MLPI in the presence of negative data values. We identify a new direction vector and apply it to formulate an improved MLPI to fix the specific case. Section 4 explains the dynamic network structure in the series. Section 5 introduces the DMLPI and formulates the DDF based DEA model to compute it. The model is shown to be feasible and bounded. Section 6 proposes the DSMLPI, utilizing the sequential DEA frontier composed by applying all data from the reference benchmark time. We also compare the two indexes on the synthetic data. Section 7 presents an application of the two proposed indexes in analyzing productivity change in 39 Indian commercial banks from 2008 to 2019. In Section 8, we present our findings as well as our plans for future research.

2. Literature review

A relevant literature on network and dynamic DEA along with a quick overview of Malmquist productivity index are presented in this section.

Dynamic DEA models

The dynamic DEA models study the efficiency change in a decision making unit (DMU) in the presence of inter-temporal dependence of input-output data over a multi-period scenario. The situations at which inter-temporal dependence may occur can be divided into two cases. The first case is when the output level is affected by the change in the capital stock over various production periods. The capital stock is the plant, equipment, and other assets that help with production. Emrouznejad and Thanassoulis [20] presented the case of dynamic DEA where the inter-temporal dependence takes place by changing the capital stock among various production periods. Ghobadi [32] introduced a generalized DEA model to pursue such an inter-temporal dependence and extend the questions from inverse DEA to the dynamic DEA.

The second situation for inter-temporal dependency refers to the case in which some outputs produced in a period are utilized as inputs in the next period. Such variables are termed carryovers, which are transmitted over different periods to establish interdependence. The performance of a unit in a given period can be affected by its performance in previous periods. Färe and Grosskopf [25] designed a dynamic DEA model to count the effect of carryovers in efficiency analysis. Since then, there is a burgeoning research [21, 40, 54, 57], to quote a few, in developing models and highlighting applications of dynamic DEA. A survey by Mariz *et al.* [52] records the evolution of the literature on dynamic DEA models from 1996 to 2016. Tone and Tsutsui [68] introduced a dynamic DEA model applying the SBM approach and classified carryovers into four categories, namely, desirable, undesirable, discretionary, and non-discretionary carryovers. Desirable (undesirable) carryovers are treated as desirable (undesirable) outputs at period t and desirable (undesirable) inputs to period $t+1$. Retained earnings, net earned surplus, carry forward losses, belated demands, non-performing loans, are a few examples of carryovers in different businesses.

Network DEA models

In the classical form, DEA treats the entire system as a single process black box and aggregate the data to evaluate the efficiency of DMU with no attention to the intermediate products linking the initial inputs to the final outputs. For instance, in an industrial plant composed of multiple independent divisions, a division can produce intermediate products for other divisions to use, giving rise to a network production system. A network DEA model allows investigating the structure and processes inside a DMU. The overall efficiency of the system in a network DEA model reflects the efficiency of divisions in contrast to the traditional DEA models. Charnes *et al.* [11] were the first to develop a two-stage network DEA model. Kao [41] developed the network DEA model with the classification of two-stage series and parallel structures. Subsequently several authors [13, 16, 37] made contributions in developing models and highlighting applications of network DEA in a variety of domains under different types of data-sets. One can refer to the text by Kao [42] for more on network DEA.

Integrating the dynamic and network structures into one, Tone and Tsutsui [69] applied the SBM approach to propose a dynamic network DEA (DNDEA) model and called it by a dynamic network slack based model (DNSBM). In this non-radial DEA model, they introduced the carryovers to connect the divisions of a DMU in different periods. Tavana *et al.* [67] proposed the RDM based dynamic two-stage DEA model for handling negative input-intermediate-output data assuming both the desirable and undesirable carryovers between different periods. Fernández *et al.* [28] reviewed the applications of the DNDEA in diverse fields. Avkiran [6] applied a DNDEA model in commercial banking. One can refer to [10, 35, 74], and references therein, for more insight on DNDEA.

Productivity indexes

The MPI, established by Caves *et al.* [9] after being introduced by Malmquist [51], analyses productivity change over time. This index is characterized *via* distance functions and defined as the geometric mean of indexes in two consecutive periods. Chen [12] extended MPI into a non-radial MPI in order to alleviate inefficiencies caused by the non-zero slacks. Some of the recent applications of MPI include studies by Falavigna *et al.* [24], Fernandez *et al.* [28], Walheer [71], Njuki *et al.* [55], and a few more. Conventional MPI fails to assess the actual productivity improvement in the presence of undesirable outputs and their influence on productivity (see, [18, 19]). Chung *et al.* [14] applied the DDF approach to define the MLPI to quantify productivity and environmental performance in the presence of undesirable outputs. Aparicio *et al.* [2] explored the issue of MLPI inconsistency and introduced a new postulate on undesirable outputs to fix it. Using their research, Aparicio *et al.* [4] examined the environmental performance of 39 nations from 1995 to 2007, concluding that technical advancement is the primary driver of productivity growth. Emrouznejad *et al.* [22] noted that the MLPI inherits the weakness of infeasibility while calculating the cross-period DDFs. Later on, Du *et al.* [18] raised the concern of infeasibility in computing MLPI and provided a modified variation that used the direction vectors described by Lin and Chen [45] to solve the long-standing problem.

Apart from these studies, Oh *et al.* [56] discussed inappropriate consideration of the technical features in MLPI. The MLPI operates on the synchronous DEA frontier which assumes that the efficiency frontier in each period envelops the observations from that period only and does not depend on the data of the previous periods. In other words, the efficiency analysis in the subsequent periods does not take into account the technology of the previous periods, and hence, the inward movement of the efficient frontier may indicate some technical regress. Kumar [44] observed technical regress while employing the conventional MLPI. Shestalova [65] argued that technology should be considered in the state of progress or at least remaining unchanged with the progress in time. The technical regress registered in MLPI is rather uncommon in the industrial sector except for some specific areas. Oh *et al.* [56] came up with an alternative measure for the environmentally sensitive productivity growth, liberated from factitious technical regress. The new measure named the sequential Malmquist-Luenberger productivity index (SMLPI) is the amalgamation of successive sequential reference production sets and the DDF. The notable advantage of SMLPI is that it is circular and operates with sequential DEA frontier, which, in a specific period, envelops all the data points observed up to that time, leaving no possibility of registering any technical regress. Due to the incorporation of past periods information, the SMLPI is less sensitive than MLPI to the presence or absence of a specific observation in the data.

DEA models with negative data

Scheel [62] suggested to treat negative inputs (outputs) as positive outputs (inputs) to measure efficiency. Seiford and Zhu [63] pointed out that such an arrangement alters the original input-output framework of the process. Ali and Seiford [1] applied data transformation approach using a shifted negative transformation $\psi(x) = -x + \tau$, $\tau > 0$ (sufficiently large) to transform the data into positive values. This method is sensitive to the choice of the shift parameter τ, and can result in inconsistent outcomes [73].

Subsequent years witnessed some research suggestions to tackle negative data values directly. Among all such studies, the range directional measures (RDM) [59], modified slack based measure (SBM) model [64], and semi-oriented radial measure (SORM) model [23] proved to be useful. Matin *et al.* [53] and Jahanshahloo and Piri [38] presented the modified SORM models. Lin and Liu [47] analyzed the conditions satisfied by the direction vectors so that the super-efficiency model remains feasible and yields bounded super-efficiency scores in the presence of negative data.

In this paper, we adopt the DDF approach to propose MLPI in a dynamic network DEA framework. We stick to the DDF based DEA modeling for two reasons. One, the DDF aims to increase the desirable outputs while concurrently reducing the undesirable outputs [14]. And two, the findings of Russell and Schworm [61] showed that all path-based indexes, including DDF, satisfy continuity property and violate indication property. Continuity property ensures that a minor error in measuring the input-output data results in only a small error in the efficiency score, and indication property provides that the index value is equal to some specified value if and only if the evaluated DMU is efficient.

Another essential component of our study is adopting variable returns to scale (VRS) in production technology. Coelli *et al.* [15] and Kourtzidis *et al.* [43] argued that due to a variety of variables like imperfect competition, budgetary constraints, the decision making units (DMUs) may be forced to operate at a scale that is not necessarily optimal, while constant returns to scale (CRS) measures results in optimal scale operation by all DMUs. Wheelock *et al.* [72] suggested to assume VRS for better capturing the effects of technical advances and regulatory changes amongst DMUs of various sizes. Färe *et al.* [27] pointed out that CRS technology is inconsistent in the presence of negative valued data, and thus advocated using the VRS technology in productivity analysis. Lovell [48] presented decomposition of Malmquist index under CRS and VRS production technology following the decomposition of Ray and Desli [60].

Zofio *et al.* [75] claimed that any Malmquist index under VRS would not be regarded as a productivity index. Earlier, Grifell-Tatje and Lovell [33] and Lovell *et al.* [49] raised an issue of elucidation of Malmquist-type indices under VRS. In response, Portela *et al.* [58] asserted that the Malmquist index under VRS captures the productivity change net of the scale economies. We follow this assertion, and consider the productivity change to be independent of the scale efficiency change factor.

3. The MLPI and infeasibility concern

In this section, we highlight the persistent issue of infeasibility in MLPI while computing the cross-period DDFs in the presence of negative data values. We design a new direction vector to resolve this issue.

We shall be using the following notations in this section.

$j \in \{1,\ldots,n\}$: the index set of n DMUs
$i \in \{1,\ldots,m\}$: the index set of m inputs
$r \in \{1,\ldots,s\}$: the index set of s desirable outputs
$p \in \{1,\ldots,q\}$: the index set of q undesirable outputs
$X_j^t = \left(x_{1j}^t,\ldots,x_{mj}^t\right)'$: the vector of inputs consumed by DMU_j in period t
$Y_j^t = \left(y_{1j}^t,\ldots,y_{sj}^t\right)'$: the vector of desirable outputs produced by DMU_j in period t
$U_j^t = \left(u_{1j}^t,\ldots,u_{qj}^t\right)'$: the vector of undesirable outputs produced by DMU_j in period t
λ_j^t: the intensity scalar of DMU j in period t
o: the DMU under evaluation.

The MLPI measures the productivity change in two consecutive periods t and $t+1$ by noting the most substantial increase in the desirable outputs compatible with the reduction in the undesirable outputs. Under the CRS assumption, Chung *et al.* [14] defined the production possibility set (PPS) of period t considering the data of period $t+1$ as follows:

$$P^t = \left\{ \left(X^{t+1}, Y^{t+1}, U^{t+1}\right) \mid X^{t+1} \geq \sum_{j=1}^{n} \lambda_j^t X_j^t,\ Y^{t+1} \leq \sum_{j=1}^{n} \lambda_j^t Y_j^t,\ U^{t+1} = \sum_{j=1}^{n} \lambda_j^t U_j^t,\ \lambda_j^t \geq 0,\ j=1,\ldots,n \right\}, \tag{3.1}$$

where $X^{t+1} = \left(X_1^{t+1},\ldots,X_m^{t+1}\right)'$, $Y^{t+1} = \left(Y_1^{t+1},\ldots,Y_s^{t+1}\right)'$, $U^{t+1} = (U_1^{t+1},\ldots,U_q^{t+1})'$, are respectively the input, desirable output, and undesirable output vectors in period $t+1$.

The DDF $D^t\left(X^{t+1}, Y^{t+1}, U^{t+1}\right)$ of DMU_o under CRS assumption is described as follows:

$$\begin{aligned}
\text{(M1)} \qquad & D^t\left(X^{t+1}, Y^{t+1}, U^{t+1}\right) = \max\ \beta_o \\
\text{s.t.}\ & \sum_{j=1}^{n} \lambda_j^t\, y_{rj}^t \geq (1+\beta_o) y_{ro}^{t+1}, && r=1,\ldots,s, \\
& \sum_{j=1}^{n} \lambda_j^t\, x_{ij}^t \leq x_{io}^{t+1}, && i=1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j^t\, u_{pj}^t = (1-\beta_o) u_{po}^{t+1}, && p=1,\ldots,q,
\end{aligned}$$

$$\beta_o \in \mathbb{R}, \quad \lambda_j^t \geq 0, \qquad\qquad j = 1, \ldots, n.$$

The DDF $D^t(X^{t+1}, Y^{t+1}, U^{t+1})$ can be viewed as the function that evaluates data of DMU_o from period $t+1$ with respect to the production technology of period t. An explanatory graphical illustration of MLPI in Section 8.4 in [42] depicts the DDF for a DMU consuming the data and production technology from different periods.

Following Caves *et al.* [9], Chung *et al.* [14] defined the DDF based MLPI under CRS assumption as the geometric mean of indexes of two consecutive periods t and $t+1$ as follows:

$$\text{ML}^{(t,t+1)} = \left(\frac{1 + D^t(X^t, Y^t, U^t)}{1 + D^t(X^{t+1}, Y^{t+1}, U^{t+1})} \times \frac{1 + D^{t+1}(X^t, Y^t, U^t)}{1 + D^{t+1}(X^{t+1}, Y^{t+1}, U^{t+1})} \right)^{\frac{1}{2}}. \tag{3.2}$$

Note that the PPS under VRS assumption is the same as P^t with an additional constraint $\sum_{j=1}^n \lambda_j^t = 1$ in (3.1).

Du *et al.* [18] observed infeasibility in (3.2) under both CRS and VRS assumptions while calculating the cross-period DDFs namely, $D^t\big(X^{t+1}, Y^{t+1}, U^{t+1}\big)$ and $D^{t+1}(X^t, Y^t, U^t)$. They proposed to replace the equality constraints in the undesirable outputs to the inequalities $U^{t+1} \geq \sum_{j=1}^n \lambda_j^t U_j^t$ in the PPS P^t amounting to replacing the weak disposability by strong disposability in the undesirable outputs in (3.1). A different case is made under VRS assumption, where following Lin and Chen [45], Du *et al.* [18] chose $\big(-x_{io}^{t+1} - \max_{1\leq j\leq n}\{x_{ij}^t\},\, y_{ro}^{t+1},\, -u_{po}^{t+1} - \max_{1\leq j\leq n}\{u_{pj}^t\}\big)$, $i = 1, \ldots, m$, $r = 1, \ldots, s$, $p = 1, \ldots, q$, as a reference bundle to calculate the DDF of DMU_o in period t consuming the data of period $t+1$ as follows:

$$\begin{aligned}
\text{(M2)} \qquad & D^t\big(X^{t+1}, Y^{t+1}, U^{t+1}\big) = \max \ \beta_o \\
\text{s.t.} \quad & \sum_{j=1}^n \lambda_j^t \, y_{rj}^t \geq (1 + \beta_o) y_{ro}^{t+1}, && r = 1, \ldots, s, \\
& \sum_{j=1}^n \lambda_j^t \, x_{ij}^t \leq (1 - \beta_o) x_{io}^{t+1} - \beta_o \max_{1\leq j\leq n}\{x_{ij}^t\}, && i = 1, \ldots, m, \\
& \sum_{j=1}^n \lambda_j^t \, u_{pj}^t \leq (1 - \beta_o) u_{po}^{t+1} - \beta_o \max_{1\leq j\leq n}\{u_{pj}^t\}, && p = 1, \ldots, q, \\
& \sum_{j=1}^n \lambda_j^t = 1, \\
& \beta_o \in \mathbb{R}, \ \lambda_j^t \geq 0, && j = 1, \ldots, n.
\end{aligned}$$

Model (M2) thoroughly addresses the infeasibility issue in MLPI except in one case when the data involves negative values. The following exhibit demonstrates this situation.

Illustration 3.1. Consider the dataset of 7 DMUs having one input, one desirable output, and one undesirable output, in two periods t and $t+1$ as described in Table 1. Columns 2 and 5 report the input, columns 3 and 6 present the desirable output, and columns 4 and 7 show the undesirable output in periods t and $t+1$, respectively. Column 8 shows the cross-period efficiency score $D^t\big(X^{t+1}, Y^{t+1}, U^{t+1}\big)$ applying model (M2). The model corresponding to unit G is observed to be infeasible.

Infeasibility limits the broader applications of MLPI in problems involving negative data values such as the return of investment, the profit in business, and so on. Silva Portela *et al.* [59] documented several drawbacks of data transformation and proposed a DDF based DEA model. Lin and Chen [46] proposed a modified DDF based super-efficiency model to overcome the infeasibility issue with negative data in the super-efficiency analysis.

TABLE 1. The two period data of 7 DMUs and the cross-period efficiency scores by model (M2).

DMU	X^t	Y^t	U^t	X^{t+1}	Y^{t+1}	U^{t+1}	$D^t(X^{t+1}, Y^{t+1}, U^{t+1})$
A	2	4	4	0	2	−2	−0.0833
B	4	8	−3	2	7	−6	Infeasible
C	0	−4	2	5	4	1	0.1954
D	3	4	−2	7	2	1	0.3554
E	10	8	1	8	5	5	0.2762
F	5	7	−3	6	−6	2	0.3559
G	1	3	−5	3	9	−4	−0.3889

Taking motivation from their study, in this paper, we attempt to eliminate the infeasibility issue in MLPI by formulating the new direction vector.

Define the direction vectors $\mu^t = (\mu_1^t, \dots, \mu_m^t)'$, $\nu^t = (\nu_1^t, \dots, \nu_s^t)'$, and $\omega^t = (\omega_1^t, \dots, \omega_q^t)'$ with components

$$\begin{aligned} \mu_i^t &= \xi * \max_{1\le j\le n} \{|x_{ij}^t|\}, && i = 1, \dots, m, \\ \nu_r^t &= \min_{1\le j\le n} \{y_{rj}^t\} - \pi, && r = 1, \dots, s, \\ \omega_p^t &= \xi_1 * \max_{1\le j\le n} \{|u_{pj}^t|\}, && p = 1, \dots, q, \end{aligned}$$

where π, ξ, ξ_1 are positive reals chosen such that the vectors $(X^{t+1} + \mu^t)$, $(Y^{t+1} - \nu^t)$, and $(U^{t+1} + \omega^t)$ are strictly positive.

Based on these new reference directions, a DDF is obtained as follows:

$$D^t(X^{t+1}, Y^{t+1}, U^{t+1}) = \max\{\beta |\ ((1-\beta)X^{t+1} - \beta\mu^t, (1+\beta)Y^{t+1} - \beta\nu^t, (1-\beta)U^{t+1} - \beta\omega^t) \in P^t\} \tag{3.3}$$

Here, P^t is taken with strong disposability in the undesirable outputs and VRS condition in (3.1). We propose an improved VRS DEA model for calculating the DDF of DMU$_o$ in period $t+1$ *via* the reference technology of period t as follows:

$$\text{(M3)} \qquad D^t(X^{t+1}, Y^{t+1}, U^{t+1}) = \max\ \beta_o$$

$$\text{s.t.} \quad \sum_{j=1}^n \lambda_j^t\, y_{rj}^t \ge y_{ro}^{t+1} + \beta_o(y_{ro}^{t+1} - \nu_r^t), \qquad r = 1, \dots, s, \tag{3.4}$$

$$\sum_{j=1}^n \lambda_j^t\, x_{ij}^t \le x_{io}^{t+1} - \beta_o(x_{io}^{t+1} + \mu_i^t), \qquad i = 1, \dots, m, \tag{3.5}$$

$$\sum_{j=1}^n \lambda_j^t\, u_{pj}^t \le u_{po}^{t+1} - \beta_o(u_{po}^{t+1} + \omega_p^t), \qquad p = 1, \dots, q, \tag{3.6}$$

$$\sum_{j=1}^n \lambda_j^t = 1, \tag{3.7}$$

$$\beta_o \in \mathbb{R},\ \lambda_j^t \ge\ 0, \qquad j = 1, \dots, n.$$

To begin, we'll show that (3.3) is translation invariant. According to Aparicio *et al.* [3], a DDF is translation invariant if and only if the directional vector associated with the translated sample of units is equivalent to the one associated with the original DMUs.

Assume that the outputs of all DMUs are translated to $y_{rj}^{t'} = y_{rj}^{t} + \tau_r,\ r = 1,\dots,s,\ j = 1,\dots,n,\ t = 1,\dots,T$, where, τ_r, $r = 1,\dots,s$, are arbitrary constants. The translated $\nu_r^{t'}$ should be calculated by

$$\nu_r^{t'} = \min_{1\le j\le n}\left\{y_{rj}^{t'}\right\} - \pi = \min_{1\le j\le n}\left\{y_{rj}^{t} + \tau_r\right\} - \pi = \min_{1\le j\le n}\left\{y_{rj}^{t}\right\} + \tau_r - \pi = \nu_r^t + \tau_r.$$

Hence, the output directional vector of the translated DMUs $y_{rj}^{t+1'} - \nu_r^{t'} = y_{rj}^{t+1} + \tau_r - \nu_r^t - \tau_r = y_{rj}^{t+1} - \nu_r^t$, remain invariant.

If all the inputs of DMUs are translated to $x_{ij}^{t'} = x_{ij}^{t} + \eta_i,\ i = 1,\dots,m,\ j = 1,\dots,n,\ t = 1,\dots,T$, where, η_i, $i = 1,\dots,m$, are arbitrary constants. The translated $\mu_i^{t'}$ should be calculated by

$$\mu_i^{t'} = \xi * \max_{1\le j\le n}\left\{\left|x_{ij}^{t'}\right|\right\} = \xi * \max_{1\le j\le n}\left\{\left|x_{ij}^{t} + \eta_i\right|\right\} = \xi * \max_{1\le j\le n}\left\{\left|x_{ij}^{t}\right|\right\} - \eta_i = \mu_i^t - \eta_i,$$

note that, we translated μ_i^t by the opposite amount of the corresponding input translation to ensure translation invariance of the input directional vector, as specified by Lin and Chen [46]. Hence, the input directional vector of the translated DMUs $x_{ij}^{t+1'} - \mu_i^{t'} = x_{ij}^{t+1} + \eta_i - \mu_i^t - \eta_i = x_{ij}^{t+1} - \mu_i^t$, remain invariant.

On the similar lines, we can show that the directional vector of undesirable output is translation invariant. Hence, we conclude that the proposed DDF is tranlsation invariant.

Theorem 3.2. *Model (M3) is feasible and bounded, and its optimal value $\beta_o^* > -1$.*

Proof. The inequalities (3.4)–(3.6) can be written as follows:

$$\beta_o \le \frac{\sum_{j=1}^n \lambda_j^t y_{rj}^t - y_{ro}^{t+1}}{y_{ro}^{t+1} - \nu_r^t}, \qquad r = 1,\dots,s,$$
$$\beta_o \le \frac{x_{io}^{t+1} - \sum_{j=1}^n \lambda_j^t x_{ij}^t}{x_{io}^{t+1} + \mu_i^t}, \qquad i = 1,\dots,m,$$
$$\beta_o \le \frac{u_{po}^{t+1} - \sum_{j=1}^n \lambda_j^t u_{pj}^t}{u_{po}^{t+1} + \omega_p^t}, \qquad p = 1,\dots,q.$$

For a non-negative vector λ^t satisfying (3.7), we can choose β_o, as

$$\beta_o \le \min\left(\min_{1\le r\le s}\left\{\frac{\sum_{j=1}^n \lambda_j^t y_{rj}^t - y_{ro}^{t+1}}{y_{ro}^{t+1} - \nu_r^t}\right\}, \min_{1\le i\le m}\left\{\frac{x_{io}^{t+1} - \sum_{j=1}^n \lambda_j^t x_{ij}^t}{x_{io}^{t+1} + \mu_{ik}^t}\right\}, \min_{1\le p\le q}\left\{\frac{u_{po}^{t+1} - \sum_{j=1}^n \lambda_j^t u_{pj}^t}{u_{po}^{t+1} + \omega_p^t}\right\}\right)\cdot \tag{3.8}$$

Therefore, we can find $(\lambda^t = (\lambda_j^t,\ j = 1,\dots,n),\ \beta_o)$ in the feasible set of (M3), ensuring that the model is feasible. Moreover, by (3.8), the model is bounded above, hence, optimal solution exists with a finite optimal value β_o^*.

We next prove that $\beta_o^* > -1$. We show that, for any non-negative vector λ^t satisfying (3.7), we can find positive constants π, ξ, and ξ_1 such that each of the three ratios in (3.8) are strictly greater than -1.

From (3.4), for any $\pi > 0$,

$$\frac{\sum_{j=1}^n \lambda_j^t y_{rj}^t - y_{ro}^{t+1}}{y_{ro}^{t+1} - \nu_r^t} \ge \frac{\min_{1\le j\le n}\{y_{rj}^t\} - y_{ro}^{t+1}}{y_{ro}^{t+1} - \nu_r^t} > -1, \quad r = 1,\dots,s,\ k = 1,\dots,K.$$

If we choose ξ such that

$$\xi > \max\left\{0,\ \max_{1\le i\le m}\left\{-\frac{x_{io}^{t+1}}{\max_{1\le j\le n}\left\{\left|x_{ij}^t\right|\right\}}\right\},\ \max_{1\le i\le m}\left\{\frac{\max_{1\le j\le n}\left\{x_{ij}^t\right\} - 2x_{io}^{t+1}}{\max_{1\le j\le n}\left\{\left|x_{ij}^t\right|\right\}}\right\}\right\},$$

TABLE 2. The cross-period efficiency scores by model (M3) on the data in Table 1, with $\pi = 1,\ \xi = 3,\ \xi_1 = 3$.

DMU	$D^t\left(X^{t+1}, Y^{t+1}, U^{t+1}\right)$
A	−0.0276
B	−0.1884
C	0.0842
D	0.1602
E	0.1318
F	0.1561
G	−0.0843

then, $\xi > 0$, and for all $i = 1, \ldots, m$,

$$\xi \max_{1\le j\le n}\left\{\left|x_{ij}^t\right|\right\} > -x_{io}^{t+1}, \tag{3.9}$$

$$\xi \max_{1\le j\le n}\left\{\left|x_{ij}^t\right|\right\} > \max_{1\le j\le n}\left\{x_{ij}^t\right\} - 2x_{io}^{t+1} \ge \sum_{j=1}^{n} \lambda_j^t x_{ij}^t - 2x_{io}^{t+1}. \tag{3.10}$$

From (3.9) and (3.10), we get that, for all $i = 1, \ldots, m$, $\quad x_{io}^{t+1} + \mu_i^t > 0$, and $\dfrac{x_{io}^{t+1} - \sum_{j=1}^{n} \lambda_j^t x_{ij}^t}{x_{io}^{t+1} + \mu_i^t} > -1$. Consequently, $X^{t+1} + \mu^t > 0$, and the second ratio in (3.8)

$$\min_{1\le i\le m} \frac{x_{io}^{t+1} - \sum_{j=1}^{n} \lambda_j^t x_{ij}^t}{x_{io}^{t+1} + \mu_i^t} > -1.$$

In a similar way, we can choose a constant $\xi_1 > 0$ such that $U^{t+1} + \omega^t > 0$, and the third ratio in (3.8), corresponding to the undesirable outputs, is strictly greater than −1.

Furthermore, at the optimal solution, there is at least one binding constraint in (3.4)–(3.6), else otherwise, we can apply (3.8) to find a feasible solution with β_o lower than the optimal value β_o^*. Thus, $\beta_o^* > -1$. □

The cross-period efficiency scores on the dataset described in Table 1 by model (M3) are reported in Table 2. The value of $D^t\left(X^{t+1}, Y^{t+1}, U^{t+1}\right)$ is less than the corresponding values in Table 1 indicating that, in comparison to model (M2), the DMUs at $t+1$ are closer to the efficient frontier at t by our proposed model (M3) resulting in better target values for the DMUs.

We use the data in Table 1 to present the graphical interpretation of the direction vector in computing the cross-period DDFs, omitting the data of undesirable output for geometrical convenience. In Table 3, Column 2 records the β value by model (M3). Columns 3 and 4 depict the input and output targets for DMUs in period $t+1$ when projected on the efficient frontier at a period t. The graphical representation of the efficient frontier at period t, data from period $t+1$, and their associated targets on the efficient frontier are shown in Figure 1.

DMUs B, C, E, and G are all efficient at t, forming an efficient frontier at t (shown in blue in Fig. 1). Rest three units A, D, F (the blue dots in the PPS of t) are inefficient at t. The goal is to project the seven units to this efficient frontier at period $t+1$ (as indicated by the green dots). To compute $D^t(X^{t+1}, Y^{t+1})$, we apply model (M3). We can observe that $D^t(X^{t+1}, Y^{t+1}) < 0$ for units A, B and G, because they are beyond the PPS of period t. To reach the efficient frontier at t, these three must raise their inputs and decrease their outputs at $t+1$. Our reasoning is supported by the results in columns 3 and 4. The remaining units C, D, E, and F are located within the PPS of period t and can meet their objectives as usual (by increasing output and reducing input). In Figure 1, the direction vectors for the DMUs at period $t+1$ to attain the efficient frontier at period t are red lines.

TABLE 3. Projections of $t+1$ period input-output data to the tth period efficient frontier, where $D^t(X^{t+1}, Y^{t+1})$, computed using model (M3) with $\pi = 1$ and $\xi = 3$.

DMUs	$D^t(X^{t+1}, Y^{t+1})$	X_t^{t+1}	Y_t^{t+1}
A	−0.0276	0.8295	1.8065
B	−0.0357	3.1429	6.5714
C	0.0842	2.0545	4.7574
D	0.1602	1.0728	3.1214
E	0.1318	2.9909	6.3182
F	0.1667	0	−6.1667
G	−0.0714	5.3571	8

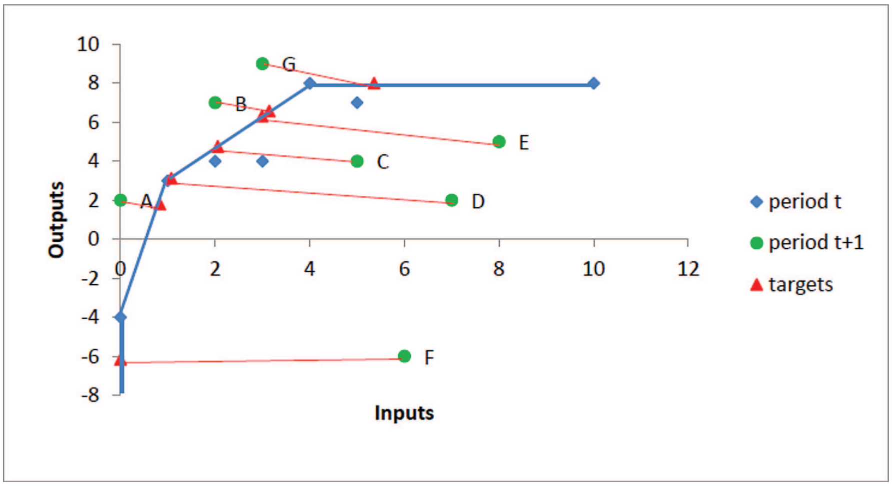


FIGURE 1. Projections of DMUs data at $t+1$ to the efficient frontier formed by the data of the DMUs at period t. The efficient frontier at t is depicted in blue, the green dots represent the data of DMUs at $t+1$, and projections of $t+1$ data on the efficient frontier at t along the proposed direction vectors are in red.

Model (M3) projects all units at $t+1$ on the efficient frontier at t; however, the projections are not necessarily strong efficient. For example, at $t+1$, units F and G are projected onto the weak efficient frontier at t. Our findings are consistent with those published by Portela *et al.* [59].

We are now in a position to introduce the improved MLPI as the geometric mean of two-period indexes as follows:

$$\mathrm{ML}^{(t,t+1)} = \left(\frac{1 + D^t(X^t, Y^t, U^t)}{1 + D^t(X^{t+1}, Y^{t+1}, U^{t+1})} \times \frac{1 + D^{t+1}(X^t, Y^t, U^t)}{1 + D^{t+1}(X^{t+1}, Y^{t+1}, U^{t+1})} \right)^{\frac{1}{2}}$$

$$= \left(\frac{1 + D^t(X^t, Y^t, U^t)}{1 + D^{t+1}(X^{t+1}, Y^{t+1}, U^{t+1})} \right) \times \left(\frac{1 + D^{t+1}\left(X^{t+1}, Y^{t+1}, U^{t+1}\right)}{1 + D^t(X^{t+1}, Y^{t+1}, U^{t+1})} \times \frac{1 + D^{t+1}(X^t, Y^t, U^t)}{1 + D^t(X^t, Y^t, U^t)} \right)^{\frac{1}{2}}.$$

The term in the first bracket from the left depicts efficiency change or shift in the DMU from the efficient frontier over the period. The term in second bracket measures the technical change, which itself is the product of two factors measuring the frontier shift in the DMU at periods $t+1$ and t, respectively. Note, $\mathrm{ML}^{(t,t+1)} > 1$

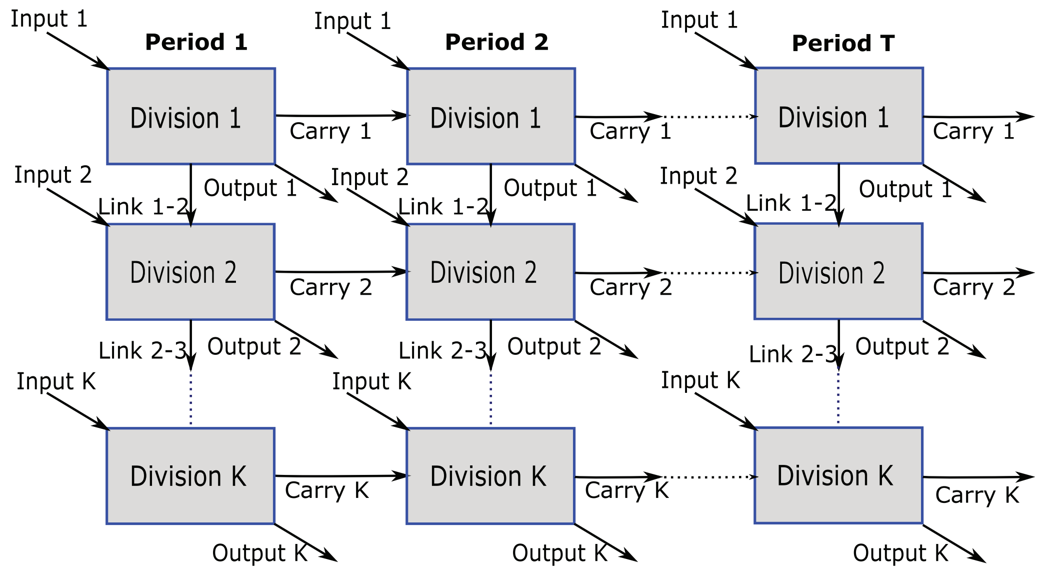


FIGURE 2. The dynamic network series structure with K divisions and T period of analysis. The divisions are connected vertically through intermediate links and horizontally across time by the carryover activities.

implies improvement in productivity while $\mathrm{ML}^{(t,t+1)} < 1$ indicates productivity regress due to its components, efficiency change and technical change, not subject to the scale change from period t to $t+1$.

4. Dynamic network DEA structure

The dynamic network DEA considers the internal network structure of a DMU over the period. In this paper, we assume that the DEA network possesses a series structure in which each DMU comprises of K divisions linked in series. Besides producing its outputs, the division also produces the intermediate products used by the next succeeding division to produce its outputs. We assume that the outputs of each division are different from their intermediate products. We also assume that each division is supplied with the exogenous inputs along with the intermediate inputs received from the former division. The multiple divisions of a DMU are connected vertically *via* the intermediate products and horizontally across the consecutive periods through the carryover activities. Figure 2 presents the architect of the dynamic network DEA for a DMU.

We shall be using the following notations for dynamic network DEA models in the discussion to follow.

Γ: time horizon of study

$j \in \{1,\dots,n\}$: the index set of n DMUs

$k \in \{1,\dots,K\}$: the index set of K divisions

$t \in \{1,\dots,T\}$: the index set of T periods

$i \in \{1,\dots,m\}$: the index set of m inputs

$r \in \{1,\dots,s\}$: the index set of s desirable outputs

$p \in \{1,\dots,q\}$: the index set of q undesirable outputs

$\ell_1 \in \{1, \ldots, ngood\}$: the index set of *ngood* desirable carryover
$\ell_2 \in \{1, \ldots, nbad\}$: the index set of *nbad* undesirable carryover
$g \in \{1, \ldots, h\}$: the index set of h intermediate products
o: a DMU under evaluation
$X_{jk}^t = \left(x_{1jk}^t, \ldots, x_{mjk}^t\right)'$: the vector of inputs consumed by DMU_j for division k in period t
$Y_{jk}^t = \left(y_{1jk}^t, \ldots, y_{sjk}^t\right)'$: the vector of desirable outputs produced by DMU_j for divission k in period t
$U_{jk}^t = \left(u_{1jk}^t, \ldots, u_{qjk}^t\right)'$: the vector of undesirable outputs produced by DMU_j for division k in period t
$C_{jk}^{dt} = \left(c_{1jk}^{dt}, \ldots, c_{ngood\,jk}^{dt}\right)'$: the vector of desirable carryovers produced by DMU_j from period t to $t+1$ for division k
$C_{jk}^{ut} = \left(c_{1jk}^{ut}, \ldots, c_{nbad\,jk}^{ut}\right)'$: the vector of undesirable carryovers produced by DMU_j from period t to $t+1$ for division k
$Z_{jk}^t = \left(z_{1jk}^t, \ldots, z_{hjk}^t\right)'$: the vector of intermediate products by DMU_j from division k to $k+1$ in period t
λ_{jk}^t: the intensity scalar of DMU_j for division k in period t.

Tone *et al.* [69] classify the carryover activities into four categories namely, desirable, undesirable, discretionary, and non-discretionary. We have included only the desirable and undesirable carryover activities constraints in our present study; the remaining two could be added easily following the same procedure. Furthermore, Tone *et al.* [69] presented four cases of the linking constraints namely, free and non-discretionary, both as input and output links. Below, we explain the formulation of constraints corresponding to the input-output links connecting the divisions vertically.

Consider two divisions k and $k+1$ in succession. The linking activities are *outputs* from division k and hence a larger amount is considered favorable, so

$$\sum_{j=1}^{n} \lambda_{jk}^t z_{gjk}^t \geq z_{gok}^t, \quad g = 1, \ldots, h,\ t = 1, \ldots, T. \tag{4.1}$$

The linking activities are *inputs* to the division $k+1$ and hence a smaller amount is regarded favorable, so

$$\sum_{j=1}^{n} \lambda_{j\,k+1}^t z_{gjk}^t \leq z_{gok}^t, \quad g = 1, \ldots, h,\ t = 1, \ldots, T. \tag{4.2}$$

From (4.1) and (4.2), we have

$$\sum_{j=1}^{n} \lambda_{jk}^t z_{gjk}^t \geq \sum_{j=1}^{n} \lambda_{j\,k+1}^t z_{gjk}^t, \quad g = 1, \ldots, h,\ t = 1, \ldots, T. \tag{4.3}$$

The constraints in (4.3) will be imposed as the equality constraint to ensure that there is no loss and no new creation of intermediate products while passing from division k to division $k+1,\ k = 1, \ldots, K-1$.

5. Dynamic ML productivity index

The dynamic network DEA models generate relative period efficiency scores based on efficiency frontiers in each period, but they do not capture each frontier's absolute position in the study's time horizon. In this section, we propose a dynamic MLPI of DMUs. The proposed model is shown to be feasible in evaluating the cross-period DDFs in the presence of undesirable and desirable features taking negative and non-negative data values.

To measure the productivity change, liberated from scale change between two consecutive periods t and $t+1$, the dynamic network PPS in period t consuming the data of period $t+1$ under the VRS assumption is defined as follows:

$$P^{'t} = \Bigg\{ \big(X_k^{t+1}, Y_k^{t+1}, U_k^{t+1}, C_k^{d\,t+1}, C_k^{u\,t+1}\big) \mid X_k^{t+1} \geq \sum_{j=1}^{n} \lambda_{jk}^t X_{jk}^t,\ Y_k^{t+1} \leq \sum_{j=1}^{n} \lambda_{jk}^t Y_{jk}^t,\ U_k^{t+1} \geq \sum_{j=1}^{n} \lambda_{jk}^t U_{jk}^t,$$
$$C_k^{d\,t+1} \leq \sum_{j=1}^{n} \lambda_{jk}^t C_{jk}^{dt},\ C_k^{dt} \geq \sum_{j=1}^{n} \lambda_{jk}^t C_{jk}^{d\,t-1},\ C_k^{u\,t+1} \geq \sum_{j=1}^{n} \lambda_{jk}^t C_{jk}^{ut},\ C_k^{ut} \leq \sum_{j=1}^{n} \lambda_{jk}^t C_{jk}^{u\,t-1},\ k = 1, \ldots, K,$$
$$\sum_{j=1}^{n} \lambda_{jk}^t Z_{jk}^t = \sum_{j=1}^{n} \lambda_{j\,k+1}^t Z_{jk}^t,\ k = 1, \ldots, K-1,\ \sum_{j=1}^{n} \lambda_{jk}^t = 1,\ \lambda_{jk}^t \geq 0,\ j = 1, \ldots, n,\ k = 1, \ldots, K \Bigg\}, \quad (5.1)$$

where $X_k^t = (X_{1k}^t, \ldots, X_{mk}^t)'$, $Y_k^t = (Y_{1k}^t, \ldots, Y_{sk}^t)'$, $U_k^t = \left(U_{1k}^t, \ldots, U_{qk}^t\right)'$, $C_k^{dt} = \left(C_{1k}^{dt}, \ldots, C_{ngood\,k}^{dt}\right)'$, $C_k^{ut} = (C_{1k}^{ut}, \ldots, C_{nbad\,k}^{ut})'$, are respectively the input, desirable output, undesirable output, desirable carryover, and undesirable carryover vectors for division k at time t, and λ_{jk}^t are the intensity scalars.

For $t = 1, \ldots, T$, $k = 1, \ldots, K$, define vectors $\mu_k^t = (\mu_{1k}^t, \ldots, \mu_{mk}^t)'$, $\nu_k^t = (\nu_{1k}^t, \ldots, \nu_{sk}^t)'$, $\omega_k^t = \left(\omega_{1k}^t, \ldots, \omega_{qk}^t\right)'$, $\gamma_k^t = \left(\gamma_{1k}^t, \ldots, \gamma_{ngood\,k}^t\right)'$ $\sigma_k^t = (\sigma_{1k}^t, \ldots, \sigma_{nbad\,k}^t)'$, $\alpha_k^t = \left(\alpha_{1k}^t, \ldots, \alpha_{ngood\,k}^t\right)'$, and $\rho_k^t = (\rho_{1k}^t, \ldots, \rho_{nbad\,k}^t)'$, with components as follows:

$$\begin{aligned}
\mu_{ik}^t &= \xi * \max_{1 \leq j \leq n} \left\{ \left| x_{ijk}^t \right| \right\}, && i = 1, \ldots, m, \\
\nu_{rk}^t &= \min_{1 \leq j \leq n} \left\{ y_{rjk}^t \right\} - \pi, && r = 1, \ldots, s, \\
\omega_{pk}^t &= \xi' * \max_{1 \leq j \leq n} \left\{ \left| u_{pjk}^t \right| \right\}, && p = 1, \ldots, q, \\
\gamma_{\ell_1 k}^t &= \min_{1 \leq j \leq n} \left\{ c_{\ell_1 jk}^{dt} \right\} - \pi', && \ell_1 = 1, \ldots, ngood, \\
\sigma_{\ell_2 k}^t &= \xi'' * \max_{1 \leq j \leq n} \left\{ \left| c_{\ell_2 jk}^{ut} \right| \right\}, && \ell_2 = 1, \ldots, nbad, \\
\alpha_{\ell_1 k}^t &= \xi''' * \max_{1 \leq j \leq n} \left\{ c_{\ell_1 jk}^{dt} \right\}, && \ell_1 = 1, \ldots, ngood, \\
\rho_{\ell_2 k}^t &= \min_{1 \leq j \leq n} \left\{ \left| c_{\ell_2 jk}^{ut} \right| \right\} - \pi'', && \ell_2 = 1, \ldots, nbad.
\end{aligned}$$

The direction vectors for DMU_o, corresponding to the division k, $k = 1, \ldots, K$, are defined as $(X_k^{t+1} + \mu_k^t,\ Y_k^{t+1} - \nu_k^t,\ U_k^{t+1} + \omega_k^t,\ C_k^{d\,t+1} - \gamma_k^t,\ C_k^{u\,t+1} + \sigma_k^t,\ C_k^{d\,t} + \alpha_k^{t-1},\ C_k^{u\,t} - \rho_k^{t-1})$, where t and $t+1$ correspond to the periods of production technology and data under observation, respectively.

Using these direction vectors, the DDFs for DMU_o in period t consuming the data of period $t+1$ is defined as follows:

$$D^t\big(X^{t+1}, Y^{t+1}, U^{t+1}, C^{d\,t+1}, C^{u\,t+1}\big) = \max \Big\{ \beta_o \mid \Big(X_k^{t+1} - \beta_o\big(X_k^{t+1} - \mu_k^t\big),\ Y_k^{t+1} + \beta_o\big(Y_k^{t+1} - \nu_k^t\big),$$
$$U_k^{t+1} - \beta_o\big(U_k^{t+1} - \omega_k^t\big),\ C_k^{d\,t+1} + \beta_o\big(C_k^{d\,t+1} - \gamma_k^t\big),\ C_k^{u\,t+1} - \beta_o\big(C_k^{u\,t+1} - \sigma_k^t\big),\ C_k^{d\,t} - \beta_o\big(C_k^{d\,t} - \alpha_k^{t-1}\big),$$
$$C_k^{u\,t} + \beta_o\big(C_k^{u\,t} - \rho_k^{t-1}\big)\Big) \in P^{'t},\ k = 1, \ldots, K \Big\}.$$

Our new VRS model for evaluating the DDF $D^{t+a}\big(X^{t+b},Y^{t+b},U^{t+b},C^{d\,t+b},C^{u\,t+b}\big)$, $a,b\in\{0,1\}$, for DMU$_o$ is described as follows:

$$
\begin{aligned}
&(\mathrm{M4})_{a,b} \qquad D^{t+a}\big(X^{t+b},Y^{t+b},U^{t+b},C^{d\,t+b},C^{u\,t+b}\big)=\max\ \beta_o^{a,b}\\
&\text{s.t.}\ \sum_{j=1}^{n}\lambda_{jk}^{t+a}\,y_{rjk}^{t+a}\geq y_{rok}^{t+b}+\beta_o^{a,b}\big(y_{rok}^{t+b}-\nu_{rk}^{t+a}\big), && r=1,\ldots,s,\ k=1,\ldots,K, && (5.2)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}\,x_{ijk}^{t+a}\leq x_{iok}^{t+b}-\beta_o^{a,b}\big(x_{iok}^{t+b}+\mu_{ik}^{t+a}\big), && i=1,\ldots,m,\ k=1,\ldots,K, && (5.3)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}\,u_{pjk}^{t+a}\leq u_{pok}^{t+b}-\beta_o^{a,b}\Big(u_{pok}^{t+b}+\omega_{pk}^{t+a}\Big), && p=1,\ldots,q,\ k=1,\ldots,K, && (5.4)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_1jk}^{d\,t+a}\geq c_{\ell_1ok}^{d\,t+b}+\beta_o^{a,b}\big(c_{\ell_1ok}^{d\,t+b}-\gamma_{\ell_1k}^{t+a}\big), && \ell_1=1,\ldots,ngood,\ k=1,\ldots,K, && (5.5)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_1jk}^{d\,t+a-1}\leq c_{\ell_1ok}^{d\,t+b-1}-\beta_o^{a,b}\big(c_{\ell_1ok}^{d\,t+b-1}+\alpha_{\ell_1k}^{t+a-1}\big), && \ell_1=1,\ldots,ngood,\ k=1,\ldots,K, && (5.6)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_2jk}^{u\,t+a}\leq c_{\ell_2ok}^{u\,t+b}-\beta_o^{a,b}\big(c_{\ell_2ok}^{u\,t+b}+\sigma_{\ell_2k}^{t+a}\big), && \ell_2=1,\ldots,nbad,\ k=1,\ldots,K, && (5.7)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_2jk}^{u\,t+a-1}\geq c_{\ell_2ok}^{u\,t+b-1}+\beta_o^{a,b}\big(c_{\ell_2ok}^{u\,t+b-1}-\rho_{\ell_2k}^{t+a-1}\big), && \ell_2=1,\ldots,nbad,\ k=1,\ldots,K, && (5.8)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}z_{gjk}^{t+a}=\sum_{j=1}^{n}\lambda_{j,k+1}^{t+a}z_{gjk}^{t+a}, && g=1,\ldots,h,\ k=1,\ldots,K-1, && (5.9)\\
&\sum_{j=1}^{n}\lambda_{jk}^{t+a}=1, && k=1,\ldots,K, && (5.10)\\
&\beta_o^{a,b}\in\mathbb{R},\ \lambda_{jk}^{t}\geq 0, && j=1,\ldots,n,\ k=1,\ldots,K,\ t=1,\ldots,T,
\end{aligned}
$$

where $X^{t+1}=\big(X_1^{t+1},\ldots,X_K^{t+1}\big)'$, $Y^{t+1}=\big(Y_1^{t+1},\ldots,Y_K^{t+1}\big)'$, $U^{t+1}=\big(U_1^{t+1},\ldots,U_K^{t+1}\big)'$, $C^{d\,t+1}=\big(C_1^{d\,t+1},\ldots,C_K^{d\,t+1}\big)'$, $C^{u\,t+1}=\big(C_1^{u\,t+1},\ldots,C_K^{u\,t+1}\big)'$, are respectively the input, desirable output, undesirable output, desirable carryover, and undesirable carryover vectors at time $t+1$. Note that while evaluating the cross- period DDFs $D^{t+a}\big(X^{t+b},Y^{t+b},U^{t+b},C^{d\,t+b},C^{u\,t+b}\big)$, $a\neq b$, $a,b\in\{0,1\}$, the constants $\pi,\ \pi',\ \pi'',\ \xi,\ \xi',\ \xi'',\ \xi'''$ are chosen such that $\big(y_{rok}^{t+b}-\nu_{rk}^{t+a}\big)$, $\big(x_{iok}^{t+b}+\mu_{ik}^{t+a}\big)$, $\Big(u_{pok}^{t+b}+\omega_{pk}^{t+a}\Big)$, $\big(c_{\ell_1ok}^{d\,t+b}-\gamma_{\ell_1k}^{t+a}\big)$, $\big(c_{\ell_1ok}^{d\,t+b-1}+\alpha_{\ell_1k}^{t+a-1}\big)$, $\big(c_{\ell_2ok}^{u\,t+b}+\sigma_{\ell_2k}^{t+a}\big)$, $\big(c_{\ell_2ok}^{u\,t+b-1}-\rho_{\ell_2k}^{t+a-1}\big)$ are positive for all $i,\ r,\ p,\ \ell_1,\ell_2,\ k,\ t$. We can prove the translation invariance of these direction vectors in the same way we did in Section 3. The DMLPI in period t to $t+1$ is defined as follows:

$$
\mathrm{DML}^{(t,t+1)}=\left(\frac{1+D^{t}\big(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\big)}{1+D^{t}(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1})}\times\frac{1+D^{t+1}\big(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\big)}{1+D^{t+1}(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1})}\right)^{\frac{1}{2}}.
$$

Recalling that the CRS technology assumption is not compatible with the negative data [26], it is not appropriate to decompose DMLPI into the factors consisting of scale efficiency change factor as in [22]. The decomposition of DMLPI, independent of scale efficiency change, is described as follows:

$$
\mathrm{DML}^{(t,t+1)}=\left(\frac{1+D^{t}\big(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\big)}{1+D^{t+1}(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1})}\right)
$$

$$\times \left(\frac{1 + D^{t+1}\big(X^{t+1}, Y^{t+1}, U^{t+1}, C^{d\,t+1}, C^{u\,t+1}\big)}{1 + D^{t}(X^{t+1}, Y^{t+1}, U^{t+1}, C^{d\,t+1}, C^{u\,t+1})} \times \frac{1 + D^{t+1}\big(X^{t}, Y^{t}, U^{t}, C^{dt}, C^{ut}\big)}{1 + D^{t}(X^{t}, Y^{t}, U^{t}, C^{dt}, C^{ut})} \right)^{\frac{1}{2}}$$
$$= \mathrm{EC}^{(t,t+1)} \times \mathrm{TC}^{(t,t+1)}, \tag{5.11}$$

where efficiency change component $\mathrm{EC}^{(t,t+1)}$ represents the movement of a DMU towards the best practice frontier and technical change component $\mathrm{TC}^{(t,t+1)}$ measures the shift in the efficient frontier composed by all divisions in period t to $t{+}1$. $\mathrm{DML}^{(t,t+1)} = 1$ indicates no change in productivity over the two consecutive periods, while $\mathrm{DML}^{(t,t+1)} > 1$ and $\mathrm{DML}^{(t,t+1)} < 1$ indicate improvement and regress respectively in productivity in all the divisions of DMU_o from period t to $t+1$.

We need to solve linear problems $(\mathrm{M4})_{a,b}$, for $a, b \in \{0,1\}$, to compute the DMLPI of DMU_o between periods t and $t+1$.

Theorem 5.1. *For $a, b \in \{0,1\}$, model $(\mathrm{M4})_{a,b}$ is feasible and bounded, and its optimal value $\beta_o^{*\,a,b} > -1$.*

Proof. If $a = b$, then model $(\mathrm{M4})_{a,b}$ is feasible by taking $\beta_o^{a,b} = 0$, $\lambda_{ok}^{t+a} = \lambda_{o\,k+1}^{t+a} = 1$ and $\lambda_{jk}^{t+a} = \lambda_{j\,k+1}^{t+a} = 0$, $\forall\, j = 1,\dots,n$, $j \neq o$. Let $a \neq b$. The inequalities (5.2)–(5.8) can be written as follows:

$$\left.\begin{array}{ll}
\beta_o^{a,b} \le \frac{\sum_{j=1}^n \lambda_{jk}^{t+a} y_{rjk}^{t+a} - y_{rok}^{t+b}}{y_{rok}^{t+b} - \nu_{rk}^{t+a}}, & r = 1,\dots,s,\\
\beta_o^{a,b} \le \frac{x_{iok}^{t+b} - \sum_{j=1}^n \lambda_{jk}^{t+a} x_{ijk}^{t+a}}{x_{iok}^{t+b} + \mu_{ik}^{t+a}}, & i = 1,\dots,m,\\
\beta_o^{a,b} \le \frac{u_{pok}^{t+b} - \sum_{j=1}^n \lambda_{jk}^{t+a} u_{pjk}^{t+a}}{u_{pok}^{t+b} + \omega_{pk}^{t+a}}, & p = 1,\dots,q,\\
\beta_o^{a,b} \le \frac{\sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_1 jk}^{d\,t+a} - c_{\ell_1 ok}^{d\,t+b}}{c_{\ell_1 ok}^{d\,t+b} - \gamma_{\ell_1 k}^{t+a}}, & \ell_1 = 1,\dots,ngood,\\
\beta_o^{a,b} \le \frac{c_{\ell_2 ok}^{u\,t+b} - \sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_2 jk}^{u\,t+a}}{c_{\ell_2 ok}^{u\,t+b} + \sigma_{\ell_2 k}^{t+a}}, & \ell_2 = 1,\dots,nbad.\\
\beta_o^{a,b} \le \frac{c_{\ell_1 ok}^{d\,t+b-1} - \sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_1 jk}^{d\,t+a-1}}{c_{\ell_1 ok}^{d\,t+b-1} + \alpha_{\ell_1 k}^{t+a-1}}, & \ell_1 = 1,\dots,ngood,\\
\beta_o^{a,b} \le \frac{\sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_2 jk}^{u\,t+a-1} - c_{\ell_2 ok}^{u\,t+b-1}}{c_{\ell_2 ok}^{u\,t+b-1} - \rho_{\ell_2 k}^{t+a-1}}, & \ell_2 = 1,\dots,nbad.
\end{array}\right\} \tag{A}$$

We firstly intend to show that by choosing the six positive constants π, π', π'', ξ, ξ', ξ'', ξ''' appropriately, the seven ratios in block (A) are greater than -1.

From (5.2), for any $\pi > 0$,

$$\frac{\sum_{j=1}^n \lambda_{jk}^{t+a} y_{rjk}^{t+a} - y_{rok}^{t+b}}{y_{rok}^{t+b} - \nu_{rk}^{t+a}} \ge \frac{\min\limits_{1\le j\le n}\left\{y_{rjk}^{t+a}\right\} - y_{rok}^{t+b}}{y_{rok}^{t+b} - \nu_{rk}^{t+a}} > -1, \quad r = 1,\dots,s,\ k = 1,\dots,K.$$

Similarly, for any $\pi' > 0$ and $\pi'' > 0$,

$$\frac{\sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_1 jk}^{d\,t+a} - c_{\ell_1 ok}^{d\,t+b}}{c_{\ell_1 ok}^{d\,t+b} - \gamma_{\ell_1 k}^{t+a}} > -1, \quad \frac{\sum_{j=1}^n \lambda_{jk}^{t+a} c_{\ell_2 jk}^{u\,t+a-1} - c_{\ell_2 ok}^{u\,t+b-1}}{c_{\ell_2 ok}^{u\,t+b-1} - \rho_{\ell_2 k}^{t+a-1}} > -1.$$

Next, consider the ratio in the second inequality in block (A). If we choose

$$\xi > \max\left\{0,\ \max_{1\le i\le m,\ 1\le k\le K}\left\{-\frac{x_{iok}^{t+b}}{\max\limits_{1\le j\le n}\left\{\left|x_{ijk}^{t+a}\right|\right\}}\right\},\ \max_{1\le i\le m,\ 1\le k\le K}\left\{\frac{\max\limits_{1\le j\le n}\left\{x_{ijk}^{t+a}\right\} - 2x_{iok}^{t+b}}{\max\limits_{1\le j\le n}\left\{\left|x_{ijk}^{t+a}\right|\right\}}\right\}\right\},$$

then, $\xi > 0$, and

$$\xi \max_{1\le j\le n}\left\{\left|x_{ijk}^{t+a}\right|\right\} > -x_{iok}^{t+b},$$

$$\xi \max_{1\le j\le n}\left\{\left|x_{ijk}^{t+a}\right|\right\} > \max_{1\le j\le n}\left\{x_{ijk}^{t+a}\right\} - 2x_{iok}^{t+b} \ge \sum_{j=1}^{n}\lambda_{jk}^{t+a}x_{ijk}^{t+a} - 2x_{iok}^{t+b}.$$

Hence, $x_{iok}^{t+b}+\mu_{ik}^{t+a} > 0$, and $\dfrac{x_{iok}^{t+b}-\sum_{j=1}^{n}\lambda_{jk}^{t+a}x_{ijk}^{t+a}}{x_{iok}^{t+b}+\mu_{ik}^{t+a}} > -1.$

By the similar arguments, we can choose ξ', ξ'', and ξ''' to ensure that the other three ratios in third, fifth, and sixth inequalities in block (A) are greater than -1.

Feasibility of the model

To evaluate the feasible region of model $(\mathrm{M4})_{a,b}$, we are required to find $\beta_o^{a,b}$ and λ_{jk}^{t}, $j=1,\dots,n$, $k=1,\dots,K$, $t=1,\dots,T$, which simultaneously satisfy (5.9), (5.10), and the inequalities in block (A). Note that the subsystem (5.9) and (5.10) is always feasible. Hence, for one such feasible vector λ, we can choose $\beta_o^{a,b}$, as

$$\begin{aligned}-1 < \beta_o^{a,b} \le \min\Bigg(&\min_{1\le r\le s}\left\{\frac{\sum_{j=1}^{n}\lambda_{jk}^{t+a}y_{rjk}^{t+a}-y_{rok}^{t+b}}{y_{rok}^{t+b}-\nu_{rk}^{t+a}}\right\},\ \min_{1\le i\le m}\left\{\frac{x_{iok}^{t+b}-\sum_{j=1}^{n}\lambda_{jk}^{t+a}x_{ijk}^{t+a}}{x_{iok}^{t+b}+\mu_{ik}^{t+a}}\right\},\\ &\min_{1\le p\le q}\left\{\frac{u_{pok}^{t+b}-\sum_{j=1}^{n}\lambda_{jk}^{t+a}u_{pjk}^{t+a}}{u_{pok}^{t+b}+\omega_{pk}^{t+a}}\right\},\ \min_{1\le \ell_1\le ngood}\left\{\frac{\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_1 jk}^{d\,t+a}-c_{\ell_1 ok}^{d\,t+b}}{c_{\ell_1 ok}^{d\,t+b}-\gamma_{\ell_1 k}^{t+a}}\right\},\\ &\min_{1\le \ell_1\le ngood}\left\{\frac{c_{\ell_1 ok}^{d\,t+b-1}-\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_1 jk}^{d\,t+a-1}}{c_{\ell_1 ok}^{d\,t+b-1}+\alpha_{\ell_1 k}^{t+a-1}}\right\},\ \min_{1\le \ell_2\le nbad}\left\{\frac{c_{\ell_2 ok}^{u\,t+b}-\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_2 jk}^{u\,t+a}}{c_{\ell_2 ok}^{u\,t+b}+\sigma_{\ell_2 k}^{t+a}}\right\},\\ &\min_{1\le \ell_2\le nbad}\left\{\frac{\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_2 jk}^{u\,t+a-1}-c_{\ell_2 ok}^{u\,t+b-1}}{c_{\ell_2 ok}^{u\,t+b-1}-\rho_{\ell_2 k}^{t+a-1}}\right\}\Bigg).\end{aligned} \tag{5.12}$$

From (5.12), it is clear that the feasible set of $(\mathrm{M4})_{a,b}$ is bounded above, and the optimal value $\beta_o^{*\,a,b} > -1$. □

Illustration 5.2. For testing and validation purposes, we consider the synthetic dataset of 30 DMUs for three periods; each DMU comprises two divisions connected in the series. There are two inputs, one desirable output and one undesirable output in division 1, and one input and one desirable output in division 2. There is one intermediate link between divisions and one desirable carryover in each division and each period. Figure 3 shows the dynamic network structure of a DMU for three periods. Tables 4 and 5 present the randomly generated data for three periods from a uniform distribution in the intervals given in the last row of the Table 4. We apply the proposed models $(\mathrm{M4})_{a,b}$, for $a,\, b \in \{0,1\}$, to evaluate DMLPI and its components efficiency change and technical change for $t=1$ to $t=3$ in consecutive periods and the results are reported in Table 6.

In Table 6, columns 2 and 5 depict the efficiency change between $t = 1\,\&\,2$, and $t = 2\,\&\,3$, respectively. We observe that 6 DMUs namely $\{1, 10, 14, 17, 18, 20\}$ show no change in their efficiency scores and hence remain efficient in the entire period of analysis. Out of the remaining 25 DMUs, 9 units show improvement and 15 units exhibit regress in efficiency while moving from $t = 1\,\&\,2$ to $t = 2\,\&\,3$.

Columns 3 and 6 in Table 6 report the technical change measured by the shift of an efficient frontier between two consecutive periods. We observe that 13 units report technical regress both in periods $t = 1\,\&\,2$ and $t = 2\,\&\,3$.

Columns 4 and 7 display the DMLPI of DMUs represented by equation (5.11) as a product of efficiency change and technical change. We found that 18 units show productivity gain and 12 units show productivity

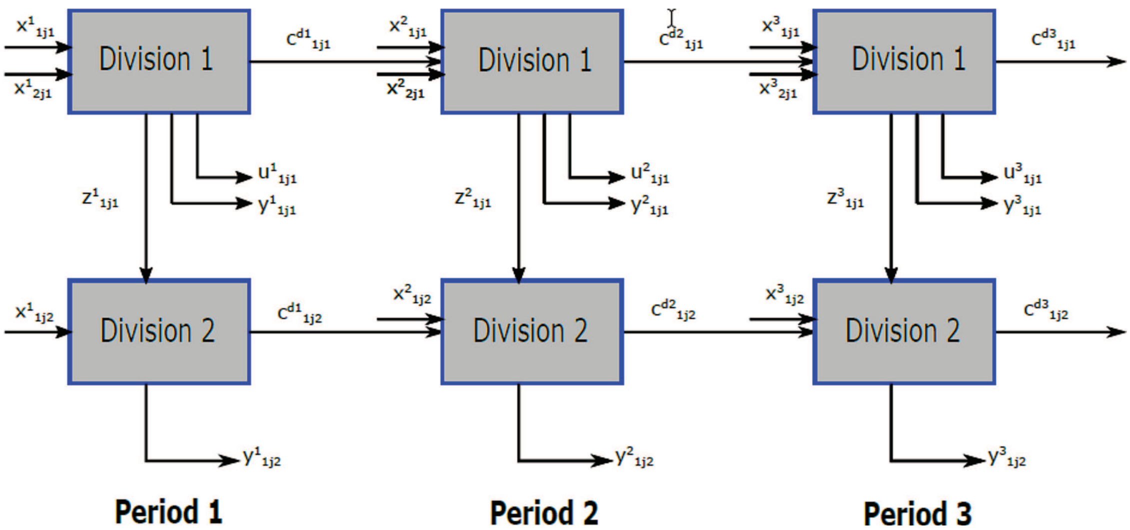


FIGURE 3. The dynamic network structure of a DMU with 2 divisions and 3 periods of analysis.

loss in period $t = 1\,\&\,2$ whereas 15 units record productivity gain and 15 record productivity loss in period $t = 2\,\&\,3$. Note that the productivity change captured by DMLPI is independent of the scale efficiency change factor.

From Figure 4, we observe that several DMUs report technical regress between two consecutive periods under consideration. The reason for it can be attributed to the fact that while computing $D^t\big(X^f, Y^f, U^f, C^{df}, C^{uf}\big)$, $f = t,\ t+1$, the technology considered is only of period t and does not include the existing technology from the previous periods. In other words, the earlier periods' technology becomes unworkable in the subsequent periods leading to a technical regress, a situation uncommon in the industrial sector. The DMLPI thus fails to integrate the technology feature in it correctly. The sequential frontier analysis in DEA [70] provides an alternative to overcome the limitation of DMLPI.

6. Dynamic sequential ML productivity index

In this section, we introduce the dynamic sequential ML productivity index.

The sequential PPS at t establishes a reference production set formed by all the DMUs upto time t. It is defined by $\overline{P^t} = co(P^1 \cup P^2 \cup \ldots \cup P^t),\ t = 1, \ldots, T$, that is, it assumes that the technology of all the preceding periods are available in period t. Here, $co(\cdot)$ denotes the convex hull. The sequential production technology frontier for the dynamic DEA with network structures is described as follows:

$$\overline{P^t} = \Bigg\{ \big(X_k^t, Y_k^t, U_k^t, C_{\ell_1 k}^{dt}, C_{\ell_2 k}^{ut}, Z_k^t\big) \mid X_k^t \geq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} X_{jk}^{\tau},\ Y_k^t \leq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} Y_{jk}^{\tau},\ U_k^t \geq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} U_{jk}^{\tau},$$

$$C_k^{dt} \leq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} C_{jk}^{d\tau},\ C_k^{d\,t-1} \geq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} C_{jk}^{d\tau-1},\ C_k^{ut} \geq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} C_{jk}^{u\tau},$$

$$C_k^{u\,t-1} \leq \sum_{\tau=1}^{t}\sum_{j=1}^{n} \lambda_{jk}^{\tau} C_{jk}^{u\tau-1},\ k = 1, \ldots, K,\ \sum_{j=1}^{n} \lambda_{jk}^{t} Z_j^t = \sum_{j=1}^{n} \lambda_{j\,k+1}^{t} Z_j^t,\ k = 1, \ldots, K-1,$$

TABLE 4. Synthetic data set 30 DMUs for period $t = 1$, and divisions $k = 1, 2$.

DMU$_j$	x^1_{1j1}	x^1_{2j1}	c^{d1}_{1j1}	y^1_{1j1}	u^1_{1j1}	z^1_{1j1}	x^1_{1j2}	y^1_{1j2}	c^{d1}_{1j2}
1	46	−5	51	22	19	44	16	92	10
2	40	−10	32	10	17	70	12	88	7
3	35	−4	31	40	17	46	20	118	37
4	20	15	24	46	8	70	6	138	4
5	10	3	48	45	12	65	12	110	40
6	44	−5	37	1	2	48	8	55	−3
7	34	−10	36	29	19	35	20	66	−1
8	45	9	34	37	18	46	12	97	3
9	45	11	36	45	19	51	18	116	0
10	37	−13	22	12	18	32	6	97	10
11	37	0	28	9	13	77	20	139	17
12	26	15	31	29	10	68	5	63	27
13	38	5	52	17	18	50	15	132	16
14	14	−9	30	35	13	63	8	125	18
15	49	5	34	8	18	40	8	69	22
16	69	4	48	45	20	30	5	75	34
17	31	11	41	48	0	31	20	82	19
18	40	−13	20	19	18	71	20	136	25
19	54	16	32	20	6	61	19	83	36
20	55	15	59	24	6	42	14	136	15
21	19	0	46	16	1	61	11	109	12
22	26	18	43	31	13	75	6	102	13
23	36	−6	37	49	11	65	11	80	34
24	51	−4	27	18	17	45	16	91	−1
25	64	0	28	15	20	76	19	118	5
26	54	12	23	47	13	75	13	101	−4
27	33	18	17	16	18	73	16	101	5
28	35	12	29	34	10	75	11	114	1
29	62	−5	28	11	7	80	15	58	30
30	24	18	24	50	4	60	15	84	−3
	[10, 70]	[−15, 20]	[15, 60]	[0, 50]	[0, 20]	[30, 80]	[5, 20]	[50, 140]	[−5, 40]

$$\left.\sum_{\tau=1}^{t}\sum_{j=1}^{n}\lambda^{\tau}_{jk}=1,\ \lambda^{t}_{jk}\geq 0, j=1,\ldots,n,\ k=1,\ldots,K,\ t=1,\ldots,T\right\}, \tag{6.1}$$

where $X^t_k = (X^t_{1k},\ldots,X^t_{mk})'$, $Y^t_k = (Y^t_{1k},\ldots,Y^t_{sk})'$, $U^t_k = \left(U^t_{1k},\ldots,U^t_{qk}\right)'$, $C^{dt}_k = \left(C^{dt}_{1k},\ldots,C^{dt}_{ngood\,k}\right)'$, $C^{ut}_k = \left(C^{ut}_{1k},\ldots,C^{ut}_{nbad\,k}\right)'$, are respectively the input, desirable output, undesirable output, desirable carryover, and undesirable carryover vectors for division k in period t.

Compared to the sequential PPS defined by Oh *et al.* [56] for the conventional DEA models, our proposed definition includes the carryover activities between t and $t+1$ and also the intermediate products in the vertical links between different divisions.

For $k = 1,\ldots,K$ and $t = 1,\ldots,T$, there exist constants η_1, η'_1, η''_1, η'''_1, δ_1, δ'_1, δ''_1 to define the vectors $\mu^t_k = (\mu^t_{1k},\ldots,\mu^t_{mk})'$, $\nu^t_k = (\nu^t_{1k},\ldots,\nu^t_{sk})'$, $\omega^t_k = \left(\omega^t_{1k},\ldots,\omega^t_{qk}\right)'$, $\gamma^t_k = \left(\gamma^t_{1k},\ldots,\gamma^t_{ngood\,k}\right)'$, $\sigma^t_k = (\sigma^t_{1k},\ldots,\sigma^t_{nbad\,k})'$,

TABLE 5. Synthetic data set of 30 DMUs for period $t = 2$, 3, and divisions $k = 1$, 2.

DMU$_j$	x^2_{1j1}	x^2_{2j1}	c^{d2}_{1j1}	y^2_{1j1}	u^2_{1j1}	z^2_{1j1}	x^2_{1j2}	y^2_{1j2}	c^{d2}_{1j2}	x^3_{1j1}	x^3_{2j1}	c^{d3}_{1j1}	y^3_{1j1}	u^3_{1j1}	z^3_{1j1}	x^3_{1j2}	y^3_{1j2}	c^{d3}_{1j2}
1	10	4	34	9	13	80	8	135	10	70	12	39	47	6	41	18	125	30
2	60	16	21	20	1	38	13	86	18	11	−10	23	25	13	56	20	72	11
3	65	−3	32	41	16	66	13	79	5	66	15	55	45	2	72	7	80	10
4	46	−10	29	30	14	64	13	71	29	17	3	16	9	8	51	7	108	26
5	48	−6	53	50	18	32	18	107	27	46	20	30	8	12	71	12	96	−2
6	64	17	41	33	17	32	11	79	−5	68	−12	48	9	8	64	13	57	3
7	21	−8	19	4	17	73	16	76	23	13	−11	37	30	0	30	17	105	14
8	65	1	20	38	19	64	7	138	9	65	13	21	41	3	57	10	87	38
9	20	16	58	12	2	69	18	88	10	18	−4	38	28	12	46	10	66	13
10	42	−14	16	44	1	42	17	97	10	37	1	27	17	8	50	14	69	26
11	37	17	18	20	13	74	12	52	4	15	17	24	30	15	55	6	87	8
12	39	19	29	42	6	64	6	52	22	49	16	34	20	11	31	13	95	6
13	64	18	60	35	17	38	11	64	17	12	−9	27	26	1	42	16	64	1
14	69	7	32	47	2	79	8	87	33	68	−3	36	23	1	51	5	129	2
15	66	20	41	14	17	60	7	95	4	25	−6	21	20	19	59	8	100	6
16	29	−1	49	23	13	67	16	60	5	46	8	53	15	5	54	20	132	25
17	53	−14	55	26	10	35	19	108	1	45	14	44	41	9	55	18	107	38
18	41	16	32	48	11	60	13	128	7	12	20	25	49	16	47	12	137	38
19	63	9	43	30	18	65	6	72	26	15	8	23	11	7	34	16	70	−4
20	67	−8	60	30	5	73	15	123	35	42	7	39	11	8	72	5	100	25
21	30	−5	39	11	0	56	13	90	3	45	3	23	32	13	48	16	64	4
22	41	16	34	14	10	33	15	65	37	51	−13	20	27	14	63	5	135	38
23	30	−4	46	4	19	62	16	102	14	64	−6	15	3	20	78	17	114	35
24	59	18	25	1	15	38	6	81	22	49	11	55	41	6	49	10	52	27
25	18	0	40	37	2	57	19	88	12	68	−9	58	30	11	55	17	50	17
26	18	19	48	33	16	73	14	134	0	51	−6	56	30	2	60	18	57	18
27	25	−13	21	41	13	58	14	70	26	35	12	28	27	16	48	15	105	14
28	43	−9	25	24	5	55	19	96	27	46	18	32	2	11	70	12	52	25
29	24	1	24	1	15	78	11	67	12	61	−8	16	43	16	37	17	63	−3
30	19	−6	52	9	13	69	17	125	5	27	−2	33	32	7	54	11	65	2

$\alpha^t_k = \left(\alpha^t_{1k}, \ldots, \alpha^t_{ngood\,k}\right)'$, and $\rho^t_k = (\rho^t_{1k}, \ldots, \rho^t_{nbad\,k})'$, with components

$$
\begin{aligned}
\mu^t_{ik} &= \eta_1 * \max_{1\le j\le n, 1\le\tau\le t}\left\{\left|x^\tau_{ijk}\right|\right\}, && i = 1, \ldots, m,\ k = 1, \ldots, K,\\
\nu^t_{rk} &= \min_{1\le j\le n, 1\le\tau\le t}\left\{y^\tau_{rjk}\right\} - \delta_1, && r = 1, \ldots, s,\ k = 1, \ldots, K,\\
\omega^t_{pk} &= \eta'_1 * \max_{1\le j\le n, 1\le\tau\le t}\left\{\left|u^\tau_{pjk}\right|\right\}, && p = 1, \ldots, q,\ k = 1, \ldots, K,\\
\gamma^t_{\ell_1 k} &= \min_{1\le j\le n, 1\le\tau\le t}\left\{c^{d\tau}_{\ell_1 jk}\right\} - \delta'_1, && \ell_1 = 1, \ldots, ngood,\ k = 1, \ldots, K,\\
\sigma^t_{\ell_2 k} &= \eta''_1 * \max_{1\le j\le n, 1\le\tau\le t}\left\{\left|c^{u\tau}_{\ell_2 jk}\right|\right\}, && \ell_2 = 1, \ldots, nbad,\ k = 1, \ldots, K,\\
\alpha^t_{\ell_1 k} &= \eta'''_1 * \max_{1\le j\le n, 1\le\tau\le t}\left\{c^{d\tau}_{\ell_1 jk}\right\}, && \ell_1 = 1, \ldots, ngood,\ k = 1, \ldots, K,\\
\rho^t_{\ell_2 k} &= \min_{1\le j\le n, 1\le\tau\le t}\left\{\left|c^{u\tau}_{\ell_2 jk}\right|\right\} - \delta''_1, && \ell_2 = 1, \ldots, nbad,\ k = 1, \ldots, K.
\end{aligned}
$$

TABLE 6. The DMLPI and its components for 30 DMUs on the synthetic data setting parameters $\xi = \xi' = \xi'' = \xi''' = 3$, and $\pi = \pi' = \pi'' = 1$.

DMU	$\mathrm{EC}^{(1,2)}$	$\mathrm{TC}^{(1,2)}$	$\mathrm{DML}^{(1,2)}$	$\mathrm{EC}^{(2,3)}$	$\mathrm{TC}^{(2,3)}$	$\mathrm{DML}^{(2,3)}$
1	1	1.0259	1.0259	1	0.9946	0.9946
2	0.9931	1.0160	1.0090	1.0062	1.0275	1.0339
3	0.9122	0.9925	0.9054	1.0302	1.0424	1.0739
4	0.9513	1.0008	0.9521	0.9781	1.0240	1.0015
5	1	1.0303	1.0303	0.9187	0.9074	0.8336
6	0.9315	1.0081	0.9391	1	1.0003	1.0003
7	0.9985	0.9985	0.9970	1	1.0413	1.0413
8	1.0915	1.0052	1.0971	1	0.9978	0.9978
9	1.0708	1.0230	1.0954	0.9892	0.9502	0.9400
10	1	1.0917	1.0917	1	0.9403	0.9403
11	0.92	0.9698	0.8922	1.0434	1.0141	1.0582
12	1	0.9835	0.9835	0.9517	1.0122	0.9634
13	1.0111	1.0642	1.0761	1	1.0007	1.0007
14	1	0.9854	0.9854	1	0.9917	0.9917
15	1.0321	0.9872	1.0188	1.0116	0.9975	1.0092
16	0.9787	0.9798	0.9589	1.0218	0.9952	1.0169
17	1	1.0670	1.0670	1	1.0039	1.0039
18	1	0.9496	0.9496	1	1.0556	1.0556
19	1.0727	0.9625	1.0325	0.9854	1.0134	0.9986
20	1	1.0918	1.0918	1	0.9895	0.9895
21	1	1.0088	1.0088	0.9487	0.9995	0.9482
22	1.0086	0.9678	0.9761	1	1.0916	1.0916
23	0.9572	1.0081	0.9650	0.9852	1.0064	0.9915
24	1.0879	0.9996	1.0874	1	1.0160	1.0160
25	1.1395	1.0197	1.1620	1	1.0031	1.0031
26	1.0553	0.9757	1.0296	1	0.9764	0.9764
27	1.0919	1.0521	1.1487	0.9847	0.9609	0.9462
28	1.0349	1.0389	1.0751	0.9656	1.0132	0.9783
29	0.9866	1.0161	1.0025	1.0216	1.0020	1.0236
30	1	0.9993	0.9993	0.9654	0.9627	0.9294

The real numbers $\eta_1,\ \eta_1',\ \eta_1'',\ \eta_1''',\ \delta_1,\ \delta_1',\ \delta_1''$, are chosen such that all direction vectors are positive and $\mu_{ik}^{t+1} < \mu_{ik}^t$, $\omega_{pk}^{t+1} < \omega_{pk}^t$, $\alpha_{\ell_1 k}^{t+1} < \alpha_{\ell_1 k}^t$, $\sigma_{\ell_2 k}^{t+1} < \sigma_{\ell_2 k}^t$, $\nu_{rk}^{t+1} > \nu_{rk}^t$, $\gamma_{\ell_1 k}^{t+1} > \gamma_{\ell_1 k}^t$, $\rho_{\ell_2 k}^{t+1} > \rho_{\ell_2 k}^t$, to preserve $\overline{D}^{t+1}\big(X^f, Y^f, U^f, C^{df}, C^{uf}\big) > \overline{D}^t\big(X^f, Y^f, U^f, C^{df}, C^{uf}\big),\ f = t,\ t+1$.

The direction vector for DMU_o corresponding to division k applying the production technology of period $t+a$ to the data of period $t+b,\ a, b \in \{0,1\}$, is defined by $(X_{ok}^{t+b} + \mu_k^{t+a},\ Y_{ok}^{t+b} - \nu_k^{t+a},\ U_{ok}^{t+b} + \omega_k^{t+a},\ C_{ok}^{d\,t+b} - \gamma_k^{t+a},\ C_{ok}^{u\,t+b} - \sigma_k^{t+a},\ C_{ok}^{d\,t+b-1} + \alpha_k^{t+a-1},\ C_{ok}^{u\,t+b-1} - \rho_k^{t+a-1})$. For $a, b \in \{0, 1\}$, the dynamic sequential DDFs are obtained by solving the following four linear problems.

$$
(\mathrm{M5})_{a,b} \qquad \overline{D}^{t+a}\big(X^{t+b}, Y^{t+b}, U^{t+b}, C^{d\,t+b}, C^{u\,t+b}\big) = \max\ \beta_o^{a,b}
$$

$$
\text{s.t.} \quad \sum_{\tau=1}^{t+a} \sum_{j=1}^{n} \lambda_{jk}^{\tau}\, y_{rjk}^{\tau} \geq y_{rok}^{t+b} + \beta_o^{a,b}\big(y_{rok}^{t+b} - \nu_{rk}^{t+a}\big), \qquad r = 1, \ldots, s,\ k = 1, \ldots, K, \tag{6.2}
$$

$$
\sum_{\tau=1}^{t+a} \sum_{j=1}^{n} \lambda_{jk}^{\tau}\, x_{ijk}^{\tau} \leq x_{iok}^{t+b} - \beta_o^{a,b}\big(x_{iok}^{t+b} + \mu_{ik}^{t+a}\big), \qquad i = 1, \ldots, m,\ k = 1, \ldots, K, \tag{6.3}
$$

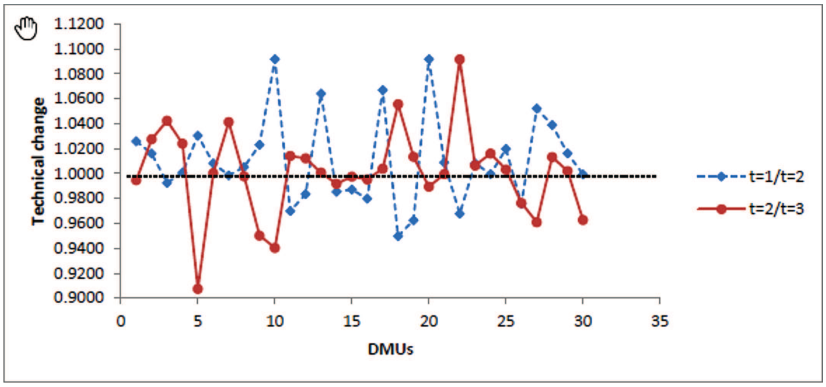


FIGURE 4. The technical change in DMLPI of 30 DMUs in two consecutive periods.

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}\,u_{pjk}^{\tau} \le u_{pok}^{t+b} - \beta_o^{a,b}\Big(u_{pok}^{t+b} + \omega_{pk}^{t+a}\Big), \qquad p=1,\dots,q,\ k=1,\dots,K, \tag{6.4}$$

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1 jk}^{d\tau} \ge c_{\ell_1 ok}^{d\,t+b} + \beta_o^{a,b}\big(c_{\ell_1 ok}^{d\,t+b} - \gamma_{\ell_1 k}^{t+a}\big), \qquad \ell_1=1,\dots,ngood,\ k=1,\dots,K, \tag{6.5}$$

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1 jk}^{d\tau-1} \le c_{\ell_1 ok}^{d\,t+b-1} - \beta_o^{a,b}\big(c_{\ell_1 ok}^{d\,t+b-1} + \alpha_{\ell_1 k}^{t+a-1}\big), \qquad \ell_1=1,\dots,ngood,\ k=1,\dots,K, \tag{6.6}$$

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2 jk}^{u\tau} \le c_{\ell_2 ok}^{u\,t+b} - \beta_o^{a,b}\big(c_{\ell_2 ok}^{u\,t+b} + \sigma_{\ell_2 k}^{t+a}\big), \qquad \ell_2=1,\dots,nbad,\ k=1,\dots,K, \tag{6.7}$$

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2 jk}^{u\tau-1} \ge c_{\ell_2 ok}^{u\,t+b-1} + \beta_o^{a,b}\big(c_{\ell_2 ok}^{u\,t+b-1} - \rho_{\ell_2 k}^{t+a-1}\big), \qquad \ell_2=1,\dots,nbad,\ k=1,\dots,K, \tag{6.8}$$

$$\sum_{j=1}^{n}\lambda_{jk}^{t+a}z_{gjk}^{t+a} = \sum_{j=1}^{n}\lambda_{j,k+1}^{t+a}z_{gjk}^{t+a}, \qquad g=1,\dots,h,\ k=1,\dots,K-1, \tag{6.9}$$

$$\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau} = 1,\ k=1,\dots,K, \tag{6.10}$$

$$\beta_o^{a,b} \in \mathbb{R},\ \lambda_{jk}^{t} \ge 0, \qquad j=1,\dots,n,\ k=1,\dots,K,\ t=1,\dots,T,$$

where $X^{t+1} = \big(X_1^{t+},\dots,X_K^{t+1}\big)'$, $Y^{t+1} = \big(Y_1^{t+1},\dots,Y_K^{t+1}\big)'$, $U^{t+1} = \big(U_1^{t+1},\dots,U_K^{t+1}\big)'$, $C^{d\,t+1} = \big(C_1^{d\,t+1},\dots,C_K^{d\,t+1}\big)'$, $C^{u\,t+1} = \big(C_1^{u\,t+1},\dots,C_K^{u\,t+1}\big)'$, are respectively the input, desirable output, undesirable output, desirable carryover, and undesirable carryover vectors at time $t+1$.

The DSMLPI between periods t and $t+1$ is defined by

$$\mathrm{DSML}^{(t,t+1)} = \left(\frac{1+\overline{D}^{t}\big(X^t,Y^t,U^t,C^{dt},C^{ut}\big)}{1+\overline{D}^{t}(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1})} \times \frac{1+\overline{D}^{t+1}\big(X^t,Y^t,U^t,C^{dt},C^{ut}\big)}{1+\overline{D}^{t+1}(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1})}\right)^{\frac{1}{2}}.$$

Unlike DMLPI, there is no feasibility concern in DSMLPI under both the CRS and VRS technology assumptions in the presence of the undesirable output feature, but it is the negative data values which debar us from using the CRS technology [26], and hence the scale efficiency change factor [22] is not defined. We therefore decompose

DSMLPI into the efficiency change and technical change factors, independent of the scale efficiency change factor, as follows:

$$\begin{aligned}\text{DSML}^{(t,t+1)} &= \left(\frac{1+\overline{D}^{t}\left(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\right)}{1+\overline{D}^{t+1}\left(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1}\right)}\right)\\ &\quad\times\left(\frac{1+\overline{D}^{t+1}\left(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1}\right)}{1+\overline{D}^{t}\left(X^{t+1},Y^{t+1},U^{t+1},C^{d\,t+1},C^{u\,t+1}\right)}\times\frac{1+\overline{D}^{t+1}\left(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\right)}{1+\overline{D}^{t}\left(X^{t},Y^{t},U^{t},C^{dt},C^{ut}\right)}\right)^{\frac{1}{2}}\\ &= \overline{\text{EC}}^{(t,t+1)}\times\overline{\text{TC}}^{(t,t+1)}.\end{aligned}\tag{6.11}$$

Here, $\text{DSML}^{(t,t+1)}=1$ indicates no productivity change over two periods; $\text{DSML}^{(t,t+1)}>1$ depicts productivity growth while $\text{DSML}^{(t,t+1)}<1$ indicates productivity regress from period t to $t+1$.

Also, $\overline{\text{EC}}^{(t,t+1)}$ captures efficiency change between periods t and $t+1$; $\overline{\text{EC}}^{(t,t+1)}>1$ means that DMU_o is closer to the efficient frontier in period $t+1$ than in period t, $\overline{\text{EC}}^{(t,t+1)}<1$ depicts that DMU_o is a distance away from the efficient frontier in period $t+1$ than in t, while $\overline{\text{EC}}^{(t,t+1)}=1$ means no efficiency change is observed in DMU_o between the two periods.

The term $\overline{\text{TC}}^{(t,t+1)}$ captures the technical change in form of a shift in the efficient frontier between two periods. Within $\overline{\text{TC}}^{(t,t+1)}$ in (6.11), the first ratio evaluates the frontier shift of the data observed at period $t+1$ while the second ratio evaluates the frontier shift of the data in period t; $\overline{\text{TC}}^{(t,t+1)}>1$ implies technical progress while $\overline{\text{TC}}^{(t,t+1)}=1$ exhibits a neutral state of technical change from period t to $t+1$. Notice that, since $\overline{D}^{t+1}\left(X^{f},Y^{f},U^{f},C^{df},C^{uf}\right)>\overline{D}^{t}\left(X^{f},Y^{f},U^{f},C^{df},C^{uf}\right),\ f=t,\,t+1$, it is not possible to have $\overline{\text{TC}}^{(t,t+1)}<1$. Thus, $\text{DSML}^{(t,t+1)}$ never encounters technical regress.

Further, if $P^{t}\subset P^{t+1},\ t=1,\dots,T-1$, then $\text{DSML}^{(t,t+1)}=\text{DML}^{(t,t+1)}$ [56]. Hence, $\text{DML}^{(t,t+1)}$ can be considered as a special case of $\text{DSML}^{(t,t+1)}$.

Theorem 6.1. *For $a,b\in\{0,1\}$, model $(\text{M5})_{a,b}$ is feasible and bounded, and its optimal value $\beta_o^{*\,a,b}>-1$.*

Proof. If $a=b$, then model is feasible by taking $\beta_o^{a,b}=0,\ \lambda_{ok}^{t+a}=\lambda_{o\,k+1}^{t+a}=1$, and $\lambda_{jk}^{t+a}=\lambda_{j\,k+1}^{t+a}=0,\ j=1,\dots,n,\ j\neq o,\ k=1,\dots,K-1$.

For $a\neq b$, the inequalities (6.2)–(6.8), can be rewritten as follows:

$$\left.\begin{aligned}
\beta_o^{a,b} &\le \frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}y_{rjk}^{\tau}-y_{rok}^{t+b}}{y_{rok}^{t+b}-\nu_{rk}^{t+a}}, && r=1,\dots,s,\\
\beta_o^{a,b} &\le \frac{x_{iok}^{t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}x_{ijk}^{\tau}}{x_{iok}^{t+b}+\mu_{ik}^{t+a}}, && i=1,\dots,m,\\
\beta_o^{a,b} &\le \frac{u_{pok}^{t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}u_{pjk}^{\tau}}{u_{pok}^{t+b}+\omega_{pk}^{t+a}}, && p=1,\dots,q,\\
\beta_o^{a,b} &\le \frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1jk}^{d\tau}-c_{\ell_1ok}^{d\,t+b}}{c_{\ell_1ok}^{d\,t+b}-\gamma_{\ell_1k}^{t+a}}, && \ell_1=1,\dots,ngood,\\
\beta_o^{a,b} &\le \frac{c_{\ell_1ok}^{d\,t+b-1}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1jk}^{d\,\tau-1}}{c_{\ell_1ok}^{d\,t+b-1}+\alpha_{\ell_1k}^{t+a-1}}, && \ell_1=1,\dots,ngood,\\
\beta_o^{a,b} &\le \frac{c_{\ell_2ok}^{u\,t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2jk}^{u\tau}}{c_{\ell_2ok}^{u\,t+b}+\sigma_{\ell_2k}^{t+a}}, && \ell_2=1,\dots,nbad,\\
\beta_o^{a,b} &\le \frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2jk}^{u\,\tau-1}-c_{\ell_2ok}^{u\,t+b-1}}{c_{\ell_2ok}^{u\,t+b-1}-\rho_{\ell_2k}^{t+a-1}}, && \ell_2=1,\dots,nbad.
\end{aligned}\right\}\tag{B}$$

We firstly intend to show that by choosing the six positive constants π, π', π'', ξ, ξ', ξ'', ξ''' appropriately, the seven ratios in block (B) are greater than -1.

From (6.2), for any $\pi > 0$,

$$\frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}y_{rjk}^{\tau}-y_{rok}^{t+b}}{y_{rok}^{t+b}-\nu_{rk}^{t+a}} \geq \frac{\min\limits_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{y_{rjk}^{\tau}\right\}-y_{rok}^{t+b}}{y_{rok}^{t+b}-\nu_{rk}^{t+a}} > -1, \quad \ell=1,\ldots,\zeta,\ k=1,\ldots,K.$$

Similarly, for any $\pi' > 0$ and $\pi'' > 0$,

$$\frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_1 jk}^{d\,t+a}-c_{\ell_1 ok}^{d\,t+b}}{c_{\ell_1 ok}^{d\,t+b}-\gamma_{\ell_1 k}^{t+a}} > -1, \qquad \frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{t+a}c_{\ell_2 jk}^{u\,t+a-1}-c_{\ell_2 ok}^{u\,t+b-1}}{c_{\ell_2 ok}^{u\,t+b-1}-\rho_{\ell_2 k}^{t+a-1}} > -1.$$

Next, consider the ratio in the second inequality in block (A).

We choose

$$\xi > \max\left\{0,\ \max_{1\leq i\leq m,\,1\leq k\leq K}\left\{-\frac{x_{iok}^{t+b}}{\max\limits_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{\left|x_{ijk}^{t+a}\right|\right\}}\right\},\ \max_{1\leq i\leq m,\,1\leq k\leq K}\left\{\frac{\max\limits_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{x_{ijk}^{t+a}\right\}-2x_{iok}^{t+b}}{\max\limits_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{\left|x_{ijk}^{t+a}\right|\right\}}\right\}\right\}.$$

It yields that $\xi > 0$, and

$$\xi\max_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{\left|x_{ijk}^{t+a}\right|\right\} > -x_{iok}^{t+b},$$

$$\xi\max_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{\left|x_{ijk}^{t+a}\right|\right\} > \max_{1\leq j\leq n,\,1\leq\tau\leq t+a}\left\{x_{ijk}^{t+a}\right\}-2x_{iok}^{t+b} \geq \sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{t+a}x_{ijk}^{t+a}-2x_{iok}^{t+b}.$$

Thus, $x_{iok}^{t+b}+\mu_{ik}^{t+a} > 0$, and $\dfrac{x_{iok}^{t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{t+a}x_{ijk}^{t+a}}{x_{iok}^{t+b}+\mu_{ik}^{t+a}} > -1.$

By the similar arguments, we can choose ξ', ξ'', and ξ''' to ensure that the other three ratios in third, fifth, and sixth inequalities in block (B) are greater than -1.

Feasibility of the model

To evaluate the feasible region of model $(\mathrm{M5})_{a,b}$, we are required to find $\lambda_{jk}^{t},\ j=1,\ldots,n,\ k=1,\ldots,K,\ t=1,\ldots,T$, and $\beta_o^{a,b}$ which simultaneously satisfy (6.7), (6.8), and inequalities in the block (B). The equivalent expression of inequalities in block (B) is as follows:

$$\begin{aligned}
-1 < \beta_o^{a,b} \leq \min\Bigg(&\min_{1\leq r\leq s,\,1\leq k\leq K}\left\{\frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}y_{rjk}^{\tau}-y_{rok}^{t+b}}{y_{rok}^{t+b}-\nu_{rk}^{a}}\right\},\min_{1\leq i\leq m,\,1\leq k\leq K}\left\{\frac{x_{iok}^{t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}x_{ijk}^{\tau}}{x_{iok}^{t+b}+\mu_{ik}^{t+a}}\right\},\\
&\min_{1\leq p\leq q,\,1\leq k\leq K}\left\{\frac{u_{pok}^{t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}u_{pjk}^{\tau}}{u_{pok}^{t+b}+\omega_{pk}^{t+a}}\right\},\ \min_{1\leq\ell_1\leq ngood,\,1\leq k\leq K}\left\{\frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1 jk}^{d\tau}-c_{\ell_1 ok}^{d\,t+b}}{c_{\ell_1 ok}^{d\,t+b}-\gamma_{\ell_1 k}^{t+a}}\right\},\\
&\min_{1\leq\ell_1\leq ngood,\,1\leq k\leq K}\left\{\frac{c_{\ell_1 ok}^{d\,t+b-1}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_1 jk}^{d\,\tau-1}}{c_{\ell_1 ok}^{d\,t+b-1}+\alpha_{\ell_1 k}^{t+a-1}}\right\},\\
&\min_{1\leq\ell_2\leq nbad,\,1\leq k\leq K}\left\{\frac{c_{\ell_2 ok}^{u\,t+b}-\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2 jk}^{u\tau}}{c_{\ell_2 ok}^{u\,t+b}+\sigma_{\ell_2 k}^{t+a}}\right\},\\
&\min_{1\leq\ell_2\leq nbad,\,1\leq k\leq K}\left\{\frac{\sum_{\tau=1}^{t+a}\sum_{j=1}^{n}\lambda_{jk}^{\tau}c_{\ell_2 jk}^{u\,\tau-1}-c_{\ell_2 ok}^{u\,t+b-1}}{c_{\ell_2 ok}^{u\,t+b-1}-\rho_{\ell_2 k}^{t+a-1}}\right\}\Bigg).
\end{aligned} \tag{6.12}$$

It follows from (6.12) that the feasible set of $(\mathrm{M5})_{a,b}$ is non-empty and bounded above yielding the finite optimal value of the model and its optimal value $\beta_o^{*\,a,b} > -1$. □

TABLE 7. The DSMLPI and its components on synthetic dataset in Tables 4 and 5 setting parameters $\eta_1 = \eta_1' = \eta_1'' = \eta_1''' = 3$, $\delta_1 = \delta_1' = \delta_1'' = 1$ for period t, and $\eta_1 = \eta_1' = \eta_1'' = \eta_1''' = 1.5$, $\delta_1 = \delta_1' = \delta_1'' = 1$ for period $t+1$, $t = 1, 2$.

DMU	$\overline{\text{EC}}^{(1,2)}$	$\overline{\text{TC}}^{(1,2)}$	$\text{DSML}^{(1,2)}$	$\overline{\text{EC}}^{(2,3)}$	$\overline{\text{TC}}^{(2,3)}$	$\text{DSML}^{(2,3)}$
1	1	1.0371	1.0371	0.9653	1.0059	0.9710
2	0.9740	1.0464	1.0192	1.0164	1.0164	1.0331
3	0.8656	1.0401	0.9003	1.1025	1.0669	1.1763
4	0.9088	1.0512	0.9553	0.9916	1.0273	1.0186
5	1	1.0850	1.0850	0.8571	1.0337	0.8861
6	0.8747	1.0516	0.9198	1.0157	1.0153	1.0313
7	0.9499	1.0371	0.9852	1.0057	1.0415	1.0475
8	1.0915	1.0391	1.1341	0.9749	1.0114	0.9860
9	1.0708	1.0558	1.1305	0.9495	1.0007	0.9501
10	1	1.0994	1.0994	0.9198	1.0081	0.9272
11	0.8630	1.0303	0.8892	1.0623	1.0276	1.0916
12	0.9730	1.0062	0.9790	0.8725	1.0328	0.9011
13	1.0111	1.0800	1.0920	1	1.0289	1.0289
14	0.9845	1.0095	0.9938	1.0128	1.0087	1.0216
15	0.9995	1.0263	1.0258	0.9812	1.0191	0.9999
16	0.9462	1.0271	0.9718	0.9956	1.0211	1.0166
17	1	1.1145	1.1145	0.9787	1.0060	0.9846
18	0.9600	1.0066	0.9663	1.0395	1.0724	1.1148
19	1.0437	1.0167	1.0611	0.9830	1.0207	1.0033
20	1	1.1038	1.1038	1	1.0001	0.9927
21	1	1.0198	1.0198	0.9365	1.0079	0.9439
22	0.9441	1.01099	0.9545	1.0524	1.0842	1.1411
23	0.9185	1.0448	0.9596	0.9528	1.0472	0.9977
24	1.0615	1.0509	1.1155	1.0000	1.0300	1.0300
25	1.1395	1.0667	1.2155	1.0000	1.0119	1.0119
26	1.0033	1.0183	1.0217	1.0000	1.0148	1.0148
27	1.0919	1.0869	1.1868	0.8704	1.0090	0.8782
28	1.0044	1.0837	1.0885	0.8730	1.0322	0.9012
29	0.9405	1.0794	1.0152	0.9699	1.0377	1.0064
30	1	1.0360	1.0360	0.9434	1.0010	0.9444

Consider the synthetic dataset of 30 DMUs in Tables 4 and 5. We apply model $(\text{M5})_{a,b}$ for a, $b \in \{0, 1\}$, and computed DSMLPI. The outcomes are recorded in Table 7.

In Table 7, columns 4 and 7 show the DSML change; 11 units show productivity regress between periods $t = 1\,\&\,2$, whereas this number increases to 14 in $t = 2\,\&\,3$. The values of $\overline{\text{EC}}$ in columns 2 and 5 in Table 7 are equal or lower than those of EC in columns 2 and 5 in Table 6. This indicates that certain DMUs are at a distance away from the efficient frontier in period $t+1$ compared to t, and this gap is more prominent in sequential frontier than contemporaneous frontier. Also, $\overline{\text{TC}}$ values are always one or more than one in columns 3 and 6 in Table 7, while TC values can be less than one as observed in columns 3 and 6 in Table 6, indicating that technical regress is not observed in DSMLPI. The behavior of technical change in DSMLPI can also be observed from Figure 5. As already stated this feature provides a significant advantage to DSMLPI over DMLPI.

7. Empirical study

To demonstrate the practical aspect of the proposed indexes, we consider the annual data for 39 Indian commercial, including both public and private banks from 2008 to 2019. Fukuyama and Weber [29,31], Tavana

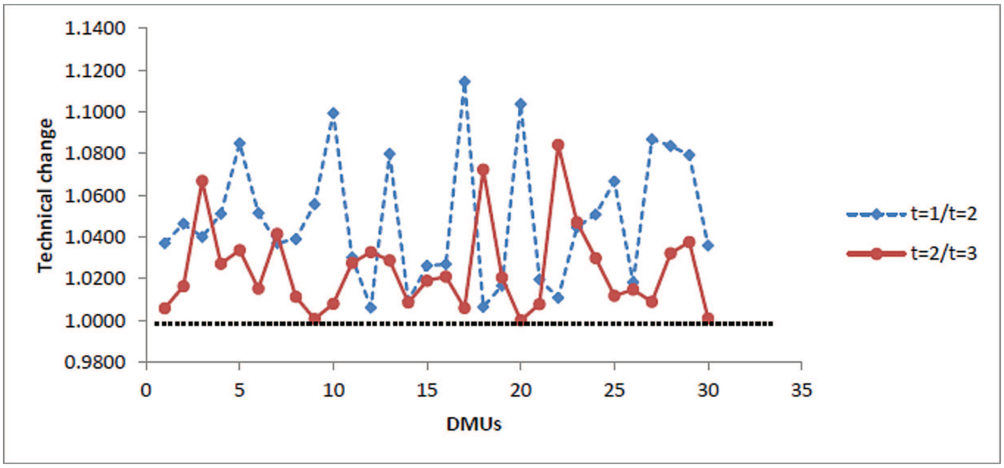


FIGURE 5. The technical change in DSMLPI in two consecutive periods $t = 1 \& 2$ and $t = 2 \& 3$ on the synthetic data of 30 DMUs.

[67], Henriques [34] advocated to apply a two-stage network structure in the banking industry. We follow in their footsteps to consider a two-stage dynamic network production process of an Indian bank. Figure 6 provides the underlying design in which the production process of a bank within each period has two distinct divisions: (i) operations and (ii) profitability. These divisions are linked vertically with intermediate products and horizontally by carryovers that link two distinct periods. Table 8 explains the inputs, outputs, intermediate links, and carryovers variables in the present analysis.

In division 1, we assume that a bank consumes four inputs (labor, fixed assets, borrowings, and operational expenses) to produce four outputs (operating income, deposits, investments, and loans), which serve as an intermediate input for division 2.

Borrowings include interbank borrowings and borrowings from the Reserve bank of India (RBI) and borrowings from outside India. The treatment of deposits as an intermediate product addresses the famous "deposit dilemma" in the banking efficiency literature due to disagreement between the intermediation and production approaches.

Here, we neither treat deposits as an input as proposed in the intermediation approach nor as an output as advocated in the production approach, but treat deposits as an intermediate product in lines suggested by Holod and Lewis [36]. We further refer to [30, 39, 67] for detailed reasons for including these inputs and intermediates in our present study.

In division 2, we assume that a bank utilizes the income generated by division 1 as inputs to maximize returns on assets and net profit and minimize non-performing assets (NPA). To link the network structure of two periods, we follow Fukuyama and Weber [30] and use capital reserves and unused assets of the previous period as the desirable carryovers. The unused assets are measured by the total assets minus the sum of required reserves, capital, loans, and investment.

Some of the features, like return on assets and net profit, can take negative values. We apply the methodologies of Sections 4 and 5 to obtain the DMLPI and DSMLPI and their components efficiency change (EC & $\overline{\text{EC}}$), and technical change (TC & $\overline{\text{TC}}$). Due to the compatibility issue of CRS with negative data, the productivity change discussed here is independent of the scale efficiency change factor.

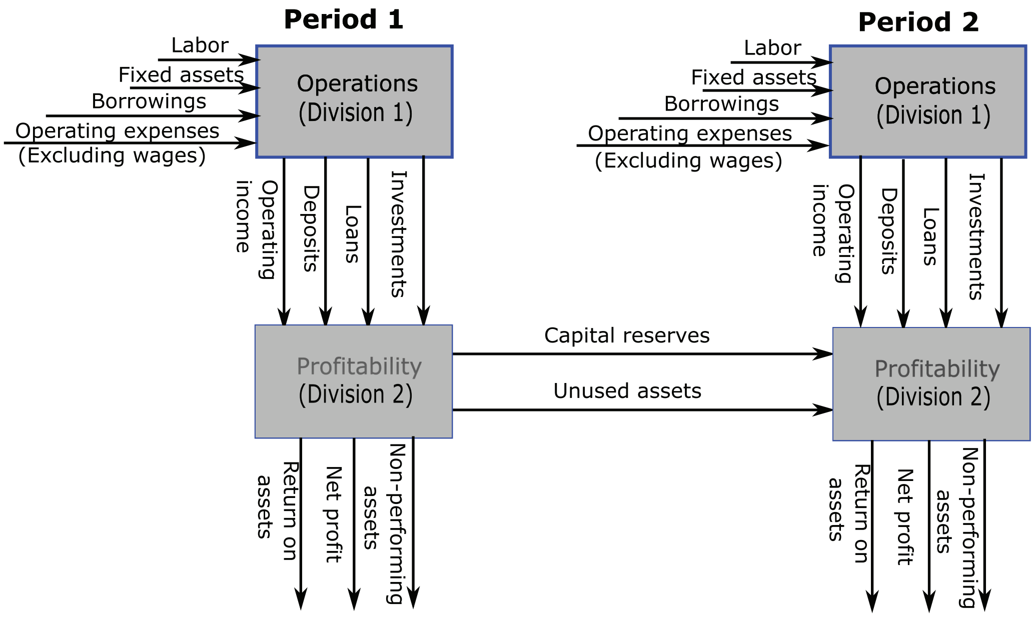


FIGURE 6. A dynamic network structure of a banking unit; the consecutive periods are connected horizontally through desirable and undesirable carryover activities, and the two divisions are connected vertically through intermediate products.

TABLE 8. Inputs, outputs, intermediate products and carryover variables applied in the empirical analysis.

Inputs of division 1	1. Labor 2. Fixed assets 3. Borrowings 4. Operating expenses (excluding wages)
Intermediates between divisions 1 and 2	1. Operating income 2. Deposits 3. Loans 4. Investments
Desirable carryovers in division 2	1. Capital reserves 2. Unused assets
Desirable outputs of division 2	1. Return on assets 2. Net profit
Undesirable outputs of division 2	1. Non-performing assets (NPA)

7.1. Data

We collect the data of 39 Indian commercial public and private sector banks over the period 2008 to 2019 from various issues of Statistical Tables Relating to Banks in India (an annual publication of the RBI). The current price figures of all variables (except labor and return on assets) are deflated by the implicit price deflator of gross domestic product (GDP) at factor cost. Following Denizer *et al.* [17], we normalize the input, output, carryover, and intermediate variables of a bank by the number of bank branches. We assume that all banks

TABLE 9. Summary statistics of variables in the entire period of study; the unit of measurement of all variables (except Labor and Return on assets) is INR in lakhs.

	Mean	Std. dev.	Kurtosis	Skewness	Median	Max	Min
Labor	11.88	5.76	3.69	2.01	9.82	31.18	6.58
Borrowings	785.51	1230.88	6.69	2.54	373.03	5454.77	2.00
Fixed assets	78.75	63.93	7.63	2.33	61.07	341.95	14.11
Operating expenses	154.13	102.54	2.85	1.88	115.05	484.55	66.54
Deposits	7010.43	3245.41	4.46	1.65	6141.09	18 996.89	2412.79
Investments	2367.14	1387.85	4.21	1.89	1962.49	7401.84	796.03
Loans	4932.59	2941.15	4.19	1.69	4056.17	15 281.93	1215.34
Operating income	776.07	438.59	3.19	1.77	638.38	2266.84	277.12
Unused assets	718.68	561.47	3.85	1.74	507.69	2639.13	102.47
Capital reserves	39.93	37.92	1.13	1.25	23.67	150.49	2.16
Net profit	55.01	84.58	2.52	0.95	44.44	290.08	−135.61
Return on assets	0.59	0.77	1.43	−0.59	0.64	1.89	−1.50
NPA	119.25	93.22	4.58	1.63	110.22	459.81	8.10

under consideration are homogeneous because they all undertake similar activities using the same inputs to produce the same outputs. Table 9 presents the summary statistics of all the variables used in the empirical illustration.

7.2. Results

Model $(\mathrm{M4})_{a,b}$ is solved for $\xi = \xi' = \xi''' = 3,\ \pi = \pi' = 1$, for period t and $t+1$ while model $(\mathrm{M5})_{a,b}$ is solved setting $\eta_1 = \eta_1' = \eta_1''' = 3$, $\delta_1 = \delta_1' = 1$, for period t and $\eta_1 = \eta_1' = \eta_1''' = 1.5$, $\delta_1 = \delta_1' = 1$, for period $t+1$. The results are shown in Tables 10–15, where IFSC codes are used for naming banks. In our analysis, a number greater than one depicts an increase while less than one indicate decrease; value equals 1 depicts no change.

Table 10 presents the efficiency change for each bank and average efficiency change for the period 2008 to 2019, where the average is computed, taking the geometric mean of efficiency changes. UTBI exhibited efficiency change progress in the initial years of study and settled down to no efficiency change in the subsequent years. Four banks (NTBL, RATN, HDFC, ICIC) depict $\mathrm{EC} = 1$ in the entire period indicating these units record no efficiency change. Five banks (IDIB, CSBK, CIUB, UTIB, KKBK) have exhibited $\mathrm{EC} \geq 1$ in ten out of eleven periods of analysis showing more or less a consistent performance in efficiency change by them. A few other banks (BARB, BAID, UCBA, DLXB, TMBL, IBKL, DCBL) have $\mathrm{EC} \geq 1$ on nine out of eleven instances. On the other hand, the largest public sector bank SBIN is inconsistent in efficiency performance with a mix of up and down of EC-values about one. We observe periods of progress as well as periods of regress in efficiency in the other banking units. Six banking units (ALLA, CNRB, IOBA, ORBC, SYNB, JAKA) have shown progress in efficiency change in the recent two periods 2017/18 and 2018/19.

The efficiency change, computed by DSMLPI solving model $(\mathrm{M5})_{a,b}$, is reported in Table 11. One unit (NTBL) shows $\overline{\mathrm{EC}} = 1$ throughout the period indicating its consistency in maintaining efficiency. Another unit (HDFC) registers a consistent performance in terms of efficiency score for almost all, especially in recent times, except for one year, 2008/09. ICIC bank attains $\overline{\mathrm{EC}} \geq 1$ in eight periods out of eleven. Twelve banking units have $\overline{\mathrm{EC}} < 1$ in the whole period of study, including the largest public bank SBIN.

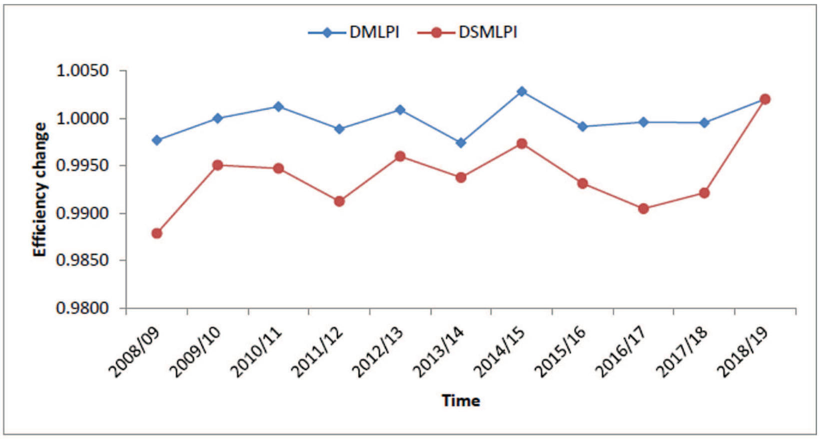
Compared to the number of units with $\mathrm{EC} > 1$, we observe this number is relatively low when it comes to $\overline{\mathrm{EC}}$. The EC and $\overline{\mathrm{EC}}$ measures shift in a DMU towards the efficient frontier composed by the production technology of period t utilizing the data in period $t+1$. The convergence of a DMU towards an efficient frontier happens if either the PPS remains unchanged at $t+1$ or expands but at a relatively lower rate than the rate at which the input-output bundle of a unit at $t+1$ comes more proximate to the efficient frontier at t. If the PPS at

TABLE 10. Efficiency change (EC) of banks over the period 2008–2019 by model $(M4)_{a,b}$.

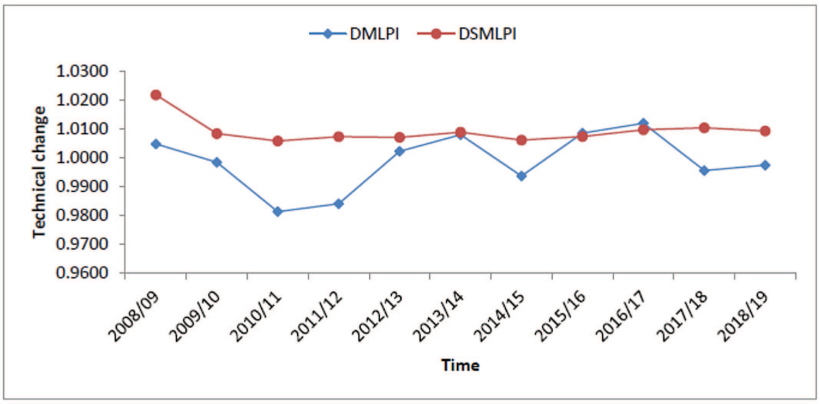
DMU	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	0.9995	1	1	0.9907	1.0016	0.9943	1.0118	1.0019	0.9938	1.0062	0.9981
ALLA	0.9979	1.0019	0.9988	1.0012	0.9930	0.9925	1.0080	1.0066	0.9881	1.0039	1.0063
ANDB	0.9966	1	0.9996	0.9983	0.9999	1.0022	1	1	1	0.9964	1.0009
BARB	0.9985	1.0015	1	1	1	1	1	1	1	0.9728	1.0279
BAID	1	1	0.9946	0.9966	1.0089	1	1	1	1	1	1
MAHB	0.9935	1	0.9962	1.0037	1.0001	0.9948	1.0053	0.9989	1.0011	0.9977	0.9998
CNRB	1	1	1	0.99343	1.0058	1.0008	1	0.9888	0.9966	1.0112	1.0035
CBIN	1	1	0.9926	1	1.0075	0.9834	1.0169	0.9939	1.0061	0.9964	1.0012
CORP	1	1	0.9970	1.0018	0.9945	1.0068	1	0.9923	1.0049	0.9930	1.0021
BKDN	0.9959	0.9970	1.0048	1.0004	1	0.9977	1.0022	0.9903	1.0084	0.9994	1.0012
IDIB	1	1	1	1	0.998	1.002	1	1	1	1	1
IOBA	0.9992	0.9878	0.9995	1.0128	0.9854	0.9997	1.0108	0.9837	0.9999	1.0169	1.0042
ORBC	1.0053	1	0.9934	0.9969	0.9989	0.9954	1.0047	1.0003	0.9919	1.0074	1.0083
PSIB	0.9968	1	1	0.9994	1.0006	1	1	1	1	1	1
PUNB	1.0039	0.9941	0.9993	0.9916	1.0164	0.9689	1.0177	1.0141	0.9840	1.0027	0.9968
SYNB	0.9927	1	0.998	1.0020	0.9963	1.0036	0.9987	1.0015	0.9967	1.0006	1.0015
UCBA	0.9935	0.9973	1.0059	0.9926	1.0033	1.0042	1	1	1	1	1
UBIN	0.9927	0.9944	1.0064	0.9886	1.0066	0.9931	1.0034	1.0038	0.9942	1.0115	0.9972
UTBI	1.0020	1.0008	1	1	1	1	1	1	1	1	1
VIJB	0.9986	0.9933	1.0028	1.0023	1.0014	0.9992	0.9939	0.9984	1.0058	1.0041	0.9951
CSBK	0.9991	1	1	1	1	1	1	1	1	1	1
CIUB	0.9997	1.0002	1	1	1	1	1	1	1	1	1
DLXB	1.0001	0.9986	1.0013	0.9902	1.0091	1.0009	1	1	1	1	1
FDRL	0.9977	0.9911	1.0066	0.9986	1.0038	1	1	0.9969	1.0028	0.9969	1.0027
JAKA	0.9975	0.9964	1.0021	1.0016	1	1	0.9972	0.9925	1.0073	1.0029	1.0001
KARB	1.0012	1	0.9959	0.9998	1.0043	0.9987	1.0013	0.9953	1.0048	0.9993	0.9951
KVBL	0.9999	1	1	0.9995	0.9975	1.0030	0.9950	1.0050	0.9988	0.9943	1.0068
LAVB	0.9982	0.9972	1.0037	0.9922	1.0026	1.0035	0.9992	1.0027	1	1	1
NTBL	1	1	1	1	1	1	1	1	1	1	1
RATN	1	1	1	1	1	1	1	1	1	1	1
SIBL	0.9970	1.0004	1	1	1	1	1	1	0.9997	0.9972	1.0025
TMBL	0.9998	1	1	1	1	0.9993	1.0007	1	1	1	1
UTIB	1	1	1	1	1	1	1	1	1	0.9720	1.0288
HDFC	1	1	1	1	1	1	1	1	1	1	1
ICIC	1	1	1	1	1	1	1	1	1	1	1
INDB	0.9672	1.0143	1.0028	1.0023	1	0.9992	1.0008	1	1	1	0.9997
DCBL	1.0065	0.9718	1.0290	1	1	0.9939	1.0061	1	1	1	1
IBKL	1	0.9817	1.0187	1	1	0.9635	1.0379	1	1	1	1
KKBK	0.9804	1.0166	1	1	1	1	1	1	1	1	1
Average	0.9977	1	1.0012	0.9989	1.0009	0.9974	1.0028	0.9991	0.9996	0.9995	1.0020
Number of units with EC > 1	6	7	11	9	14	10	14	8	8	10	15

TABLE 11. Efficiency change ($\overline{\mathrm{EC}}$) in banks over the period 2008–2019 by model $(\mathrm{M5})_{a,b}$.

DMU	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	0.9758	0.9894	0.9935	0.9819	0.9853	0.9859	0.9985	0.9811	0.9850	0.9849	0.9544
ALLA	0.9885	0.9921	0.9927	0.9934	0.9973	0.9904	0.9973	0.9923	0.9880	0.9952	0.9917
ANDB	0.9923	0.9991	0.9973	0.9968	0.9992	0.9960	0.9987	0.9974	0.9915	0.9956	0.9907
BARB	0.9971	1.0063	1	1	1	1	1	0.9867	0.9891	0.9678	0.9761
BAID	1	0.9996	0.9951	0.9864	1.0081	1.0005	0.9980	1.0174	0.9701	0.9835	0.9826
MAHB	0.9885	0.9915	0.9939	0.9930	0.9944	0.9877	1.0020	0.9929	0.9935	0.9977	0.9885
CNRB	1	0.9865	0.9985	0.9861	0.9956	1.0078	0.9913	0.9891	0.9816	0.9895	0.9862
CBIN	0.9983	0.9942	0.9870	0.9940	0.9966	0.9879	0.9942	1.0021	0.9955	0.9993	0.9977
CORP	1	1	0.9984	0.9935	0.9936	0.9976	0.9987	0.9873	0.9964	0.9869	0.9955
BKDN	0.9918	0.9953	0.9961	0.9936	0.9926	0.9921	0.9980	0.9887	0.9948	0.9981	0.9983
IDIB	1	0.9994	0.9943	0.9899	1.0000	1.0013	0.9980	0.9960	0.9811	0.9951	0.9935
IOBA	0.9855	0.9758	0.9856	0.9943	0.9925	0.9921	0.9887	0.9768	0.9972	0.9990	0.9979
ORBC	0.9963	1	0.9829	0.9938	0.9904	0.9927	0.9949	0.9909	0.9803	0.9998	0.9855
PSIB	0.9938	1	1	1	1	1	1	0.9977	0.9947	0.9980	0.9990
PUNB	1.0028	0.9780	0.9837	0.9848	1.0143	0.9713	0.9887	0.9753	0.9924	0.9782	0.9948
SYNB	0.9855	0.9891	0.9903	0.9865	0.9905	0.9950	0.9945	0.9937	0.9913	0.9893	0.9897
UCBA	0.9859	0.9869	0.9949	0.9895	0.9948	0.9967	0.9971	0.9895	0.9946	0.9943	0.9951
UBIN	0.9856	0.9866	0.9881	0.9841	0.9938	0.9900	0.9899	0.9861	0.9819	0.9861	0.9832
UTBI	1.0009	1.0000	1	0.9990	1.0000	0.9976	1.0014	0.9979	0.9975	0.9958	0.9988
VIJB	0.9923	0.9930	0.9953	0.9889	0.9922	0.9965	0.9922	0.9907	0.9872	0.9973	0.9998
CSBK	0.9983	0.9991	0.9964	1.0021	0.9974	1.0014	0.9999	0.9998	0.9998	0.9998	0.9998
CIUB	0.9990	0.9992	0.9960	0.9956	0.9965	0.9997	1.0001	0.9998	0.9959	0.9897	1.0031
DLXB	1.0001	0.9969	0.9873	0.9743	1.0089	0.9965	0.9968	0.9989	0.9997	0.9951	0.9998
FDRL	0.9939	0.9888	0.9929	0.9865	0.9953	0.9950	0.9977	0.9960	0.9849	0.9947	0.9919
JAKA	0.9881	0.9993	0.9951	1.0002	1.0006	0.9925	0.9906	0.9944	0.9994	0.9958	0.9920
KARB	1.0012	0.9952	0.9896	0.9945	0.9924	0.9908	1.0006	0.9967	0.9984	0.9981	0.9853
KVBL	0.9995	1.0002	1.0000	0.9782	0.9862	0.9846	0.9923	0.9998	0.9946	0.9932	0.9980
LAVB	0.9966	0.9934	0.9873	0.9967	0.9969	0.9980	0.9976	0.9951	0.9867	0.9822	1.0071
NTBL	1	1	1	1	1	1	1	1	1	1	1
RATN	1	1.0000	0.9995	0.9884	0.9988	0.9877	0.9973	0.9960	0.9803	0.9953	0.98774
SIBL	0.9946	0.9989	0.9990	0.9965	0.9973	0.9970	0.9970	0.9964	0.9924	0.9865	0.9872
TMBL	0.9989	0.9988	0.9997	0.9922	1.0042	0.9964	1.0014	0.9968	0.9985	0.9972	0.9978
UTIB	1	1	1	1	0.9908	0.9859	0.9961	0.9756	0.9528	0.9764	0.9768
HDFC	0.9691	1.0007	1	1	1	1	1	1	1	1	1.0090
ICIC	1	1	1	1	0.9894	1.0065	0.9955	1.0064	1	1	0.9725
INDB	0.9065	0.9998	1.0085	0.9925	0.9897	0.9920	0.9987	0.9920	0.9995	0.9888	0.9751
DCBL	0.9884	0.9636	0.9838	0.9679	0.9819	0.9916	0.9812	0.9912	0.9954	0.9925	0.9914
IBKL	0.9180	0.9845	0.9922	0.9653	0.9937	0.9810	1.0215	0.9873	0.9824	0.9847	0.9852
KKBK	0.9239	1.0286	1	1	0.9931	0.9820	1.0105	0.9819	0.9858	0.9928	0.9935
Average	0.9879	0.9951	0.9947	0.9913	0.9960	0.9938	0.9973	0.9931	0.9905	0.9921	0.9910
Number of units with $\overline{\mathrm{EC}} > 1$	4	4	1	2	7	5	7	3	0	0	4



(a)



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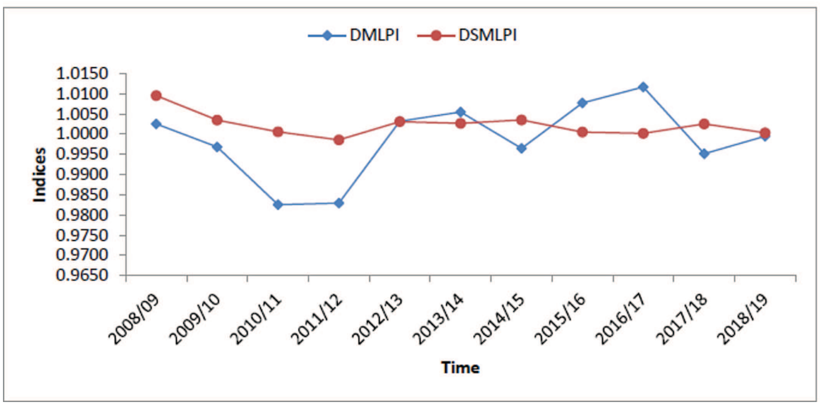

(c)

FIGURE 7. Average DMLPI, DSMLPI and their components for 39 Indian nationalized banks from 2008 to 2019. (a) Efficiency change. (b) Technical change. (c) Productivity indices, liberated from scale change.

Table 12. Technical change (TC) in banks over the period 2008–2019 by model $(M4)_{a,b}$.

Banks	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	1.0016	1.0004	0.9888	0.9950	1.0020	1.0063	1.0021	0.9898	1.0062	0.9988	0.9854
ALLA	0.9993	1.0052	0.9926	0.9985	1.0006	1.0052	0.9935	1.0430	0.9941	0.9937	1.0056
ANDB	1.0010	1.0100	0.9858	1.0020	1.0025	0.9947	1.0038	0.9987	1.0059	0.9915	0.9991
BARB	0.9992	1.0122	0.9947	0.9787	1.0422	1.0815	1.0237	1.0631	1.0235	0.9533	1.0028
BAID	1.0810	0.9821	0.9925	0.9743	1.0118	1.1314	0.9302	1.2150	1.0169	0.9926	1.0113
MAHB	1.0005	1.0018	0.9905	1.0052	0.9955	0.9989	1.0001	0.9880	1.0093	0.9901	0.9998
CNRB	1.0478	0.9725	0.9787	0.9148	1.0071	1.1058	0.8955	0.9943	1.0082	0.9860	1.0043
CBIN	0.9984	1.0057	0.9920	1.0014	1.0019	1.0107	0.9836	1.0202	1.0393	0.9638	1.0014
CORP	1.0026	0.9952	0.9938	1.0024	1.0013	0.9931	1.0053	0.9922	1.0130	0.9902	1.0002
BKDN	1.0005	1.0001	0.9971	1.0011	1.0145	0.9940	1.0075	0.9895	0.9983	0.9986	1.0011
IDIB	1.0364	0.9644	0.9675	0.9804	1.0011	1.0394	0.9586	1.0077	1.0113	0.9886	1.0074
IOBA	1.0338	0.9663	0.9949	0.9885	1.0094	1.0196	0.9781	1.0331	0.9994	0.9847	1.0044
ORBC	1.0020	0.9643	0.9905	0.9951	1.0016	1.0110	0.9853	1.0309	1.0185	0.9849	1.0040
PSIB	1.0032	1.0047	0.9930	0.9815	0.9926	1.1766	0.8900	1.0023	0.9975	0.9981	1.0023
PUNB	1.0033	1.0025	0.9930	1.0005	1.0074	1.0044	0.9923	1.0056	1.0088	0.9765	0.9999
SYNB	1.0008	0.9961	0.9964	1.0046	1.0047	0.9964	1.0033	0.9943	0.9960	0.9988	0.9988
UCBA	1.0009	1.0047	0.9991	0.9825	1.0019	0.9977	0.9995	1.0390	0.9878	1.0043	1.0005
UBIN	1.0016	1.0016	0.9938	1.0052	0.9974	1.0076	0.9959	0.9895	1.0032	0.9903	0.9922
UTBI	1.0004	1.0232	0.9466	0.9297	0.9437	0.9878	0.9980	1.0202	0.9966	0.9827	1.0005
VIJB	1.0010	0.9940	0.9931	1.0017	0.9964	1.0023	1.0022	0.9949	1.0001	0.9893	1.0014
CSBK	1.0001	0.9954	0.9981	1.0030	0.9961	0.9976	1.0003	0.9959	1.0028	0.9958	1.0011
CIUB	1.0002	1.0007	0.9957	1.0022	1.0052	0.9903	1.0039	0.9937	1.0021	1.0030	1.0054
DLXB	1.0000	0.9995	0.9993	1.0035	1.0000	1.0015	1.0010	1.0169	0.9974	0.9868	1.0029
FDRL	1.0024	1.0037	0.9985	0.9989	1.0014	1.0005	0.9994	0.9946	0.9995	0.9984	1.0002
JAKA	1.0022	1.0041	1.0000	1.0224	0.9863	0.9914	0.9812	0.9882	1.0067	0.9975	0.9997
KARB	1.0001	0.9958	0.9963	1.0033	1.0023	0.9979	1.0022	0.9900	1.0001	0.9922	1.0012
KVBL	1.0010	1.0061	0.9945	0.9930	0.9978	0.9949	0.9934	1.0014	0.9968	0.9903	1.0011
LAVB	1.0004	1.0007	0.9964	0.9994	1.0021	0.9984	1.0034	0.9999	1.0029	0.9892	0.9947
NTBL	1.0040	1.0032	0.9831	1.0419	0.9622	1.0002	1.0045	0.9755	1.0196	0.9994	0.9793
RATN	1.0622	0.9754	0.9966	1.0054	0.9894	1.0008	0.9982	0.9902	1.0118	0.9989	1.0037
SIBL	1.0002	1.0018	0.9989	0.9989	1.0027	0.9929	1.0050	0.9908	0.9977	0.9967	0.9986
TMBL	1.0002	1.0005	0.9963	1.0078	1.0204	0.9739	1.0100	0.9949	0.9997	1.0000	1.0024
UTIB	1.0008	1.0178	0.9240	0.9336	1.0313	0.9557	1.0617	0.9675	1.0029	0.9911	0.9899
HDFC	0.9502	1.0225	0.9799	0.9351	1.0106	1.0214	0.9919	1.0057	1.0114	1.0858	0.9704
ICIC	0.9817	0.9710	0.7314	0.8269	1.0500	0.9559	1.0252	1.1438	1.1631	1.0480	1.0068
INDB	0.9789	1.0043	0.9844	0.9819	0.9912	0.9993	1.0367	0.9651	1.0141	1.0002	0.9941
DCBL	1.0129	1.0058	0.9965	0.9858	1.0016	0.9753	0.9913	0.9739	1.0371	0.9686	1.0004
IBKL	0.9775	1.0036	0.9823	0.8739	1.0263	1.0106	1.0157	0.9824	1.0660	1.0422	0.9289
KKBK	1.0078	1.0264	0.9886	1.0527	0.9832	0.9278	0.9992	0.9897	1.0192	0.9968	1.0013
Average	1.0049	0.9985	0.9813	0.9840	1.0023	1.0081	0.9937	1.0086	1.0121	0.9956	0.9975
Number of units with TC > 1	32	26	1	18	27	19	20	15	27	7	25

$t+1$ contracts, a unit can achieve the efficient frontier at t without doing anything; it is one of the drawbacks of DMLPI. As $P_1^t \subseteq \overline{P^t}$, a large number of units can achieve the efficient frontier at t for DMLPI compared to DSMLPI, and hence, $\text{EC} \geq \overline{\text{EC}}$. We can observe the average efficiency changes for the banks over 2008–2019 through Figure 7c, which shows that $\overline{\text{EC}}$ lies below EC.

Tables 12 and 13 present the technical change in the banks from period t to $t+1$. We observe that spurious technical regress is likely to appear in the DMLPI approach. The average values in (Tab. 12) depict five periods with technical advancement and six periods of technical regress. The periods from 2012–14 and 2015–17 emerged as periods when the banking units show technical progress on average. On average, the Indian banking sector achieved a positive technical growth of 1.83% in 2012/13 and 1.5% in 2015/16. Compared to 2016/17, the Indian banking units, on average, suffer a decline of 1.65% in 2017–19 due to technical regress. In 2018/19, the average TC has shown a marginal improvement compared to 2017/18; out of 39 units, almost 25 banks recorded an improvement in their TC values. Moreover, in the most recent three years 2016–19 of the study, only CIUB, and ICIC units recorded TC > 1.

The estimates of technical change obtained by DSMLPI (model $(\mathrm{M5})_{a,b}$) rules out any possibility of technical regress. The same is evident from Table 13 when $\overline{\mathrm{TC}} > 1$ for all 39 banks.

Tables 14 and 15 display productivity changes in banks by DMLPI and DSMLPI approaches, respectively. From Table 14, we observe that the average productivity gains in five periods and loss in six periods. It is noticeable that the periods 2012–14 and 2015–17 showing productivity growth also showed technical progress in Table 12. Compared to year 2011–12, the productivity change records 2.03% increase in 2012–13 while this increase is 1.13% in 2015–16 compared to its previous year. The period 2017–19 has seen a fall in productivity, a phenomena that we observed in technical change also, and the decline on average is 1.424% as compared to 2016–17. At the same time, a marginal improvement is noticed in 2018–19 compared to 2017–18.

The DSMLPI, on average, exhibits progress in productivity in almost all periods. Considering all the banks and all periods together, from Table 14, we found 204 instances of productivity increase and 225 instances of productivity decline showing 48% growth in productivity change. On the other hand, from Table 15, these numbers are respectively 251 and 178, giving 59% increase in productivity by the DSMLPI approach.

Figures 7c, 7b, and 7a illustrate the average efficiency change, technical change, and productivity change, respectively over the period 2008 to 2019 measured by DMLPI and DSMLPI. The following are a few interesting facts noted:

(i) The trends in productivity change and technical change by DMLPI are very similar.
(ii) The movement trends in efficiency change are often opposite of the productivity change by DMLPI. For example, periods 2008–11 and 2013–15 experience an upward increase in efficiency change, while productivity change declines in the same periods.
(iii) The trending behavior of two indexes for efficiency change is similar except in 2010–11 and 2016–17.
(iv) The technical change and productivity change by DMLPI are volatile compared to their respective relatively stable counterparts by DSMLPI.

Table 16 presents the correlation coefficients between the DMLPI, DSMLPI, and their components. Columns (1)–(3) depict that there is a noticeable difference between the two productivity indexes and their components EC and $\overline{\mathrm{EC}}$, and TC and $\overline{\mathrm{TC}}$ in the period of study. Column (4) shows that the correlation between EC and DMLPI is quite weak and almost always less than 0.5 except once. Column (5) establishes a very high positive correlation between TC and DMLPI in all the periods. This trend has also been observed in Figures 7b and 7a. Our observations support our belief that it is the technical change instead of efficiency change that acts as the main driver in computing DMLPI. In other words, technical change plays a vital role in assessing productivity change when measured relative to the two consecutive periods. But, columns (6) and (7) do not give a clear indication as to which among $\overline{\mathrm{EC}}$ or $\overline{\mathrm{TC}}$ contributing more in DSMLPI. The correlation ($\overline{\mathrm{EC}}$, DSMLPI) is higher than that of ($\overline{\mathrm{TC}}$, DSMLPI) on six instances, while the two are almost equal in periods 2009-10 and 2013-14. Thus, both $\overline{\mathrm{EC}}$ and $\overline{\mathrm{TC}}$ are equally participating in DSMLPI. However, it appears that $\overline{\mathrm{EC}}$ is occasionally a primary impact factor in productivity change when measured considering the accumulated information from the past several years.

To explore the possible factors that might influence the productivity change and its components, we apply the panel data regression over 468 observations including 39 (banks) cross-sectional units over a time series length of 12 years (2008–2019) with 13 features. Table 17 reports the estimates of coefficients by the regression model; the

TABLE 13. Technical change ($\overline{\text{TC}}$) of banks in the period 2008–2019 by model $(\text{M5})_{a,b}$.

Banks	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	1.0265	1.0072	1.0064	1.0100	1.0126	1.0140	1.0114	1.0169	1.0144	1.0124	1.0223
ALLA	1.0069	1.0081	1.0050	1.0057	1.0056	1.0081	1.0071	1.0098	1.0116	1.0088	1.0078
ANDB	1.0038	1.0046	1.0030	1.0055	1.0026	1.0024	1.0025	1.0083	1.0080	1.0064	1.0066
BARB	1.0042	1.0097	1.0030	1.0088	1.0104	1.0205	1.0066	1.0150	1.0137	1.0174	1.0225
BAID	1.0853	1.0094	1.0030	1.0084	1.0070	1.0146	1.0009	1.0223	1.0135	1.0172	1.0190
MAHB	1.0032	1.0056	1.0065	1.0066	1.0063	1.0080	1.0064	1.0086	1.0104	1.0074	1.0063
CNRB	1.0556	1.0082	1.0055	1.0075	1.0091	1.0087	1.0027	1.0109	1.0128	1.0137	1.0130
CBIN	1.0016	1.0029	1.0058	1.0074	1.0061	1.0100	1.0095	1.0085	1.0047	1.0037	1.0026
CORP	1.0078	1.0048	1.0004	1.0073	1.0047	1.0044	1.0033	1.0066	1.0090	1.0078	1.0083
BKDN	1.0032	1.0048	1.0046	1.0048	1.0059	1.0070	1.0060	1.0073	1.0075	1.0055	1.0037
IDIB	1.0370	1.0012	1.0024	1.0048	1.0049	1.0025	1.0033	1.0030	1.0074	1.0099	1.0075
IOBA	1.0496	1.0185	1.0119	1.0110	1.0093	1.0103	1.0104	1.0156	1.0147	1.0085	1.0051
ORBC	1.0088	1.0038	1.0043	1.0079	1.0081	1.0093	1.0074	1.0076	1.0110	1.0106	1.0091
PSIB	1.0062	1.0062	1.0033	1.0006	1.0008	1.0366	1.0030	1.0019	1.0055	1.0031	1.0022
PUNB	1.0061	1.0103	1.0132	1.0140	1.0082	1.0079	1.0137	1.0177	1.0172	1.0161	1.0150
SYNB	1.0055	1.0044	1.0073	1.0098	1.0105	1.0088	1.0071	1.0076	1.0062	1.0069	1.0092
UCBA	1.0062	1.0139	1.0072	1.0086	1.0079	1.0062	1.0054	1.0076	1.0081	1.0068	1.0060
UBIN	1.0061	1.0092	1.0109	1.0122	1.0117	1.0119	1.0105	1.0154	1.0173	1.0164	1.0154
UTBI	1.0023	1.0198	1.0024	1.0002	1.0005	1.0014	1.0027	1.0043	1.0033	1.0033	1.0030
VIJB	1.0073	1.0050	1.0054	1.0066	1.0080	1.0071	1.0072	1.0089	1.0118	1.0103	1.0055
CSBK	1.0005	1.0009	1.0013	1.0010	1.0009	1.0014	1.0002	1.0003	1.0002	1.0002	1.0002
CIUB	1.0007	1.0007	1.0013	1.0029	1.0034	1.0026	1.0014	1.0007	1.0014	1.0043	1.0040
DLXB	1.0001	1.0008	1.0043	1.0117	1.0102	1.0040	1.0037	1.0029	1.0018	1.0022	1.0024
FDRL	1.0061	1.0063	1.0074	1.0091	1.0091	1.0070	1.0054	1.0045	1.0069	1.0086	1.0077
JAKA	1.0104	1.0050	1.0077	1.0048	1.0013	1.0022	1.0053	1.0067	1.0049	1.0037	1.0049
KARB	1.0007	1.0012	1.0044	1.0065	1.0064	1.0074	1.0059	1.0037	1.0031	1.0024	1.0053
KVBL	1.0013	1.0030	1.0045	1.0064	1.0115	1.0132	1.0126	1.0084	1.0057	1.0059	1.0052
LAVB	1.0014	1.0037	1.0059	1.0070	1.0051	1.0039	1.0030	1.0034	1.0062	1.0112	1.0081
NTBL	1.0045	1.0021	1.0001	1.0050	1.0024	1.0050	1.0061	1.0011	1.0024	1.0000	1.0000
RATN	1.0624	1.0000	1.0001	1.0034	1.0052	1.0058	1.0067	1.0039	1.0071	1.0078	1.0090
SIBL	1.0016	1.0024	1.0017	1.0020	1.0025	1.0027	1.0029	1.0031	1.0040	1.0074	1.0103
TMBL	1.0008	1.0008	1.0016	1.0026	1.0130	1.0009	1.0010	1.0010	1.0016	1.0019	1.0022
UTIB	1.0315	1.0300	1.0038	1.0015	1.0026	1.0075	1.0102	1.0007	1.0177	1.0305	1.0277
HDFC	1.0112	1.0039	1.0053	1.0043	1.0085	1.0102	1.0023	1.0027	1.0008	1.0119	1.0060
ICIC	1.1393	1.0251	1.0066	1.0052	1.0024	1.0093	1.0028	1.0050	1.0753	1.0848	1.0437
INDB	1.0519	1.0267	1.0226	1.0090	1.0080	1.0099	1.0066	1.0008	1.0010	1.0040	1.0086
DCBL	1.0387	1.0099	1.0126	1.0214	1.0236	1.0286	1.0164	1.0155	1.0090	1.0079	1.0084
IBKL	1.0912	1.0168	1.0130	1.0169	1.0196	1.0225	1.0137	1.0176	1.0197	1.0173	1.0157
KKBK	1.0859	1.0341	1.0138	1.0201	1.0031	1.0069	1.0081	1.0045	1.0074	1.0075	1.0070
Average	1.0219	1.0084	1.0059	1.0074	1.0071	1.0090	1.0062	1.0074	1.0098	1.0105	1.0094

associated coefficient estimates the change in the mean response per unit increase in the factor when all other predictors are held constant. It is observed that the two factors *viz.* deposits and investments significantly affect TC, DMLPI, $\overline{\text{TC}}$, and DSMLPI at 5% significance level. Since the coefficients of deposits are positive while that of investment are negative for all four dependent indexes, the technical change and productivity change are favourably impacted by the deposits and see adverse affect by the investments during the period 2008 to 2019. The unused assets and capital reserves depict unfavorable impact on TC and DMLPI. Our findings report a positive influence of the operating income on TC and DSMLPI and operating expenses on $\overline{\text{TC}}$ and DSMLPI.

Table 14. DMLPI of banks showing productivity change in the period 2008–2019.

Banks	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	1.0012	1.0004	0.9888	0.9857	1.0036	1.0005	1.0139	0.9917	1.0000	1.0050	0.9835
ALLA	0.9971	1.0072	0.9913	0.9997	0.9937	0.9977	1.0014	1.0499	0.9823	0.9976	1.0120
ANDB	0.9976	1.0100	0.9854	1.0003	1.0023	0.9969	1.0038	0.9987	1.0059	0.9880	0.9999
BARB	0.9977	1.0137	0.9947	0.9787	1.0422	1.0815	1.0237	1.0631	1.0235	0.9274	1.0308
BAID	1.0810	0.9821	0.9871	0.9710	1.0208	1.1314	0.9302	1.2150	1.0169	0.9926	1.0113
MAHB	0.9940	1.0018	0.9867	1.0090	0.9956	0.9937	1.0054	0.9869	1.0104	0.9878	0.9996
CNRB	1.0478	0.9725	0.9787	0.9088	1.0129	1.1067	0.8955	0.9831	1.0048	0.9970	1.0078
CBIN	0.9984	1.0057	0.9846	1.0014	1.0095	0.9939	1.0003	1.0140	1.0457	0.9604	1.0026
CORP	1.0026	0.9952	0.9908	1.0042	0.9958	0.9998	1.0053	0.9845	1.0179	0.9832	1.0023
BKDN	0.9964	0.9972	1.0019	1.0014	1.0145	0.9918	1.0097	0.9799	1.0067	0.9980	1.0023
IDIB	1.0364	0.9644	0.9675	0.9804	0.9991	1.0415	0.9586	1.0077	1.0113	0.9886	1.0074
IOBA	1.0330	0.9545	0.9944	1.0012	0.9947	1.0193	0.9886	1.0162	0.9992	1.0013	1.0086
ORBC	1.0073	0.9643	0.9840	0.9921	1.0005	1.0063	0.9899	1.0313	1.0103	0.9921	1.0123
PSIB	1.0000	1.0047	0.9930	0.9808	0.9932	1.1766	0.8900	1.0023	0.9975	0.9981	1.0023
PUNB	1.0072	0.9965	0.9922	0.9921	1.0240	0.9731	1.0099	1.0198	0.9926	0.9792	0.9967
SYNB	0.9935	0.9961	0.9944	1.0066	1.0009	1.0000	1.0020	0.9957	0.9928	0.9994	1.0003
UCBA	0.9944	1.0020	1.0050	0.9752	1.0052	1.0018	0.9995	1.0390	0.9878	1.0043	1.0005
UBIN	0.9943	0.9960	1.0002	0.9938	1.0040	1.0007	0.9993	0.9932	0.9974	1.0016	0.9894
UTBI	1.0023	1.0240	0.9466	0.9297	0.9437	0.9878	0.9980	1.0202	0.9966	0.9827	1.0005
VIJB	0.9996	0.9873	0.9959	1.0040	0.9978	1.0015	0.9961	0.9933	1.0059	0.9933	0.9965
CSBK	0.9991	0.9954	0.9981	1.0030	0.9961	0.9976	1.0003	0.9959	1.0028	0.9958	1.0011
CIUB	0.9999	1.0009	0.9957	1.0022	1.0052	0.9903	1.0039	0.9937	1.0021	1.0030	1.0054
DLXB	1.0001	0.9981	1.0006	0.9936	1.0091	1.0024	1.0010	1.0169	0.9974	0.9868	1.0029
FDRL	1.0002	0.9948	1.0051	0.9975	1.0052	1.0005	0.9994	0.9915	1.0024	0.9953	1.0030
JAKA	0.9997	1.0005	1.0021	1.0240	0.9863	0.9914	0.9785	0.9808	1.0140	1.0004	0.9998
KARB	1.0013	0.9958	0.9923	1.0031	1.0065	0.9966	1.0035	0.9853	1.0049	0.9915	0.9963
KVBL	1.0009	1.0061	0.9945	0.9925	0.9953	0.9979	0.9884	1.0064	0.9956	0.9847	1.0079
LAVB	0.9986	0.9979	1.0001	0.9916	1.0047	1.0019	1.0026	1.0026	1.0029	0.9892	0.9947
NTBL	1.0040	1.0032	0.9831	1.0419	0.9622	1.0002	1.0045	0.9755	1.0196	0.9994	0.9793
RATN	1.0622	0.9754	0.9966	1.0054	0.9894	1.0008	0.9982	0.9902	1.0118	0.9989	1.0037
SIBL	0.9972	1.0022	0.9989	0.9989	1.0027	0.9929	1.0050	0.9908	0.9974	0.9939	1.0012
TMBL	1.0000	1.0005	0.9963	1.0078	1.0204	0.9733	1.0107	0.9949	0.9997	1.0000	1.0024
UTIB	1.0008	1.0178	0.9240	0.9336	1.0313	0.9557	1.0617	0.9675	1.0029	0.9634	1.0184
HDFC	0.9502	1.0225	0.9799	0.9351	1.0106	1.0214	0.9919	1.0057	1.0114	1.0858	0.9704
ICIC	0.9817	0.9710	0.7314	0.8269	1.0500	0.9559	1.0252	1.1438	1.1631	1.0480	1.0068
INDB	0.9468	1.0187	0.9872	0.9842	0.9912	0.9984	1.0375	0.9651	1.0141	1.0002	0.9938
DCBL	1.0195	0.9774	1.0254	0.9858	1.0016	0.9694	0.9973	0.9739	1.0371	0.9686	1.0004
IBKL	0.9775	0.9852	1.0007	0.8739	1.0263	0.9738	1.0541	0.9824	1.0660	1.0422	0.9289
KKBK	0.9881	1.0435	0.9886	1.0527	0.9832	0.9278	0.9992	0.9897	1.0192	0.9968	1.0013
Average	1.0025	0.9968	0.9825	0.9829	1.0032	1.0055	0.9965	1.0077	1.0117	0.9951	0.9995

The feature of fixed assets show a negative impact on EC and $\overline{\text{EC}}$ and a positive impact on $\overline{\text{TC}}$. The factors like labor, net profit, return on assets, and NPA are not playing a significant role on the two indexes and their components.

8. Conclusions and future directions of research

In this study, we introduce an enhanced ML productivity index (MLPI) to address the problem of infeasibility when evaluating cross-period DDFs in the presence of negative data. In the dynamic network framework,

TABLE 15. DSMLPI of banks over the period 2008 to 2019.

Banks	2008/09	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17	2017/18	2018/19
SBIN	1.0017	0.9966	0.9999	0.9917	0.9977	0.9998	1.0099	0.9977	0.9992	0.9970	0.9757
ALLA	0.9954	1.0001	0.9976	0.9991	1.0029	0.9985	1.0044	1.0020	0.9994	1.0039	0.9994
ANDB	0.9961	1.0037	1.0003	1.0022	1.0018	0.9983	1.0012	1.0056	0.9994	1.0020	0.9972
BARB	1.0014	1.0160	1.0030	1.0088	1.0104	1.0205	1.0066	1.0015	1.0026	0.9847	0.9981
BAID	1.0853	1.0089	0.9981	0.9947	1.0152	1.0151	0.9989	1.0401	0.9833	1.0005	1.0013
MAHB	0.9916	0.9971	1.0003	0.9995	1.0007	0.9956	1.0084	1.0014	1.0038	1.0051	0.9947
CNRB	1.0556	0.9946	1.0040	0.9935	1.0047	1.0165	0.9940	0.9998	0.9942	1.0031	0.9991
CBIN	1.0000	0.9971	0.9927	1.0013	1.0026	0.9978	1.0036	1.0107	1.0002	1.0030	1.0003
CORP	1.0078	1.0048	0.9988	1.0008	0.9983	1.0020	1.0021	0.9939	1.0054	0.9946	1.0038
BKDN	0.9950	1.0001	1.0006	0.9984	0.9984	0.9991	1.0040	0.9959	1.0022	1.0037	1.0020
IDIB	1.0370	1.0006	0.9967	0.9947	1.0050	1.0038	1.0013	0.9990	0.9883	1.0049	1.0010
IOBA	1.0344	0.9939	0.9974	1.0052	1.0018	1.0023	0.9989	0.9921	1.0118	1.0075	1.0030
ORBC	1.0051	1.0038	0.9871	1.0017	0.9984	1.0019	1.0023	0.9984	0.9910	1.0103	0.9944
PSIB	1.0000	1.0062	1.0033	1.0006	1.0008	1.0366	1.0030	0.9996	1.0002	1.0011	1.0012
PUNB	1.0089	0.9880	0.9967	0.9986	1.0226	0.9790	1.0023	0.9926	1.0095	0.9940	1.0097
SYNB	0.9909	0.9935	0.9975	0.9961	1.0009	1.0038	1.0016	1.0012	0.9975	0.9962	0.9989
UCBA	0.9921	1.0005	1.0021	0.9980	1.0027	1.0028	1.0026	0.9970	1.0027	1.0011	1.0011
UBIN	0.9917	0.9957	0.9989	0.9961	1.0054	1.0018	1.0003	1.0013	0.9990	1.0023	0.9984
UTBI	1.0032	1.0198	1.0024	0.9993	1.0005	0.9989	1.0042	1.0022	1.0008	0.9991	1.0018
VIJB	0.9996	0.9980	1.0007	0.9954	1.0002	1.0036	0.9993	0.9995	0.9989	1.0076	1.0053
CSBK	0.9988	1.0001	0.9977	1.0032	0.9983	1.0027	1.0000	1.0001	1.0000	1.0000	1.0000
CIUB	0.9998	0.9999	0.9974	0.9985	0.9999	1.0024	1.0015	1.0005	0.9973	0.9939	1.0071
DLXB	1.0001	0.9977	0.9915	0.9857	1.0192	1.0004	1.0005	1.0018	1.0016	0.9973	1.0022
FDRL	0.9999	0.9950	1.0002	0.9955	1.0044	1.0020	1.0030	1.0004	0.9918	1.0032	0.9995
JAKA	0.9984	1.0042	1.0028	1.0050	1.0020	0.9947	0.9958	1.0010	1.0043	0.9994	0.9968
KARB	1.0019	0.9964	0.9940	1.0009	0.9988	0.9982	1.0065	1.0004	1.0015	1.0005	0.9906
KVBL	1.0007	1.0032	1.0045	0.9845	0.9976	0.9976	1.0048	1.0082	1.0002	0.9991	1.0032
LAVB	0.9980	0.9971	0.9931	1.0037	1.0020	1.0019	1.0006	0.9985	0.9928	0.9932	1.0153
NTBL	1.0045	1.0021	1.0001	1.0050	1.0024	1.0050	1.0061	1.0011	1.0024	1.0000	1.0000
RATN	1.0624	1.0000	0.9997	0.9917	1.0040	0.9934	1.0040	1.0000	0.9872	1.0031	0.9967
SIBL	0.9962	1.0013	1.0007	0.9984	0.9999	0.9997	0.9999	0.9995	0.9963	0.9939	0.9974
TMBL	0.9997	0.9995	1.0013	0.9948	1.0173	0.9973	1.0024	0.9978	1.0002	0.9991	1.0000
UTIB	1.0315	1.0300	1.0038	1.0015	0.9935	0.9933	1.0063	0.9763	0.9697	1.0062	1.0039
HDFC	0.9800	1.0046	1.0053	1.0043	1.0085	1.0102	1.0023	1.0027	1.0008	1.0119	1.0151
ICIC	1.1393	1.0251	1.0066	1.0052	0.9918	1.0159	0.9983	1.0115	1.0753	1.0848	1.0150
INDB	0.9535	1.0266	1.0313	1.0015	0.9976	1.0018	1.0054	0.9928	1.0004	0.9927	0.9835
DCBL	1.0267	0.9732	0.9961	0.9886	1.0050	1.0200	0.9973	1.0066	1.0044	1.0004	0.9997
IBKL	1.0017	1.0011	1.0052	0.9816	1.0132	1.0031	1.0355	1.0047	1.0018	1.0018	1.0007
KKBK	1.0033	1.0636	1.0138	1.0201	0.9962	0.9887	1.0186	0.9864	0.9931	1.0003	1.0004
Average	1.0095	1.0035	1.0006	0.9986	1.0031	1.0027	1.0035	1.0005	1.0002	1.0025	1.0003

where multiple divisions are connected through intermediate products and multiple periods are linked through carryovers, the paper's significant contribution is an extension of MLPI to the dynamic MLPI (DMLPI) and dynamic sequential MLPI (DSMLPI) involving undesirable outputs and real data values. The proposed indexes capture productivity change subject to efficiency change and technical change independent of scale change. The sequential technique considers the past information of the data and eliminates the chance of detecting technical regress and inconsistency. To demonstrate the usefulness of the suggested indexes, we compare the findings from DMLPI and DSMLPI using data from 39 Indian commercial banks from 2008 to 2019. We discovered that the DSMLPI is a superior indicator for detecting efficiency changes than the DMLPI. The

Table 16. Correlation coefficients between indexes and their components over the period 2008 to 2019.

	(EC, $\overline{\text{EC}}$) (1)	(TC, $\overline{\text{TC}}$) (2)	(DMLPI, DSMLPI) (3)	(EC, DMLPI) (4)	(TC, DMLPI) (5)	($\overline{\text{EC}}$, DSMLPI) (6)	($\overline{\text{TC}}$, DSMLPI) (7)
2008/09	0.72	0.26	0.58	0.44	0.96	0.34	0.72
2009/10	0.82	0.23	0.54	0.53	0.92	0.80	0.71
2010/11	−0.17	0.00	−0.18	0.19	0.99	0.75	0.53
2011/12	0.31	0.07	0.24	0.10	0.99	0.85	−0.12
2012/13	0.49	0.30	0.26	0.27	0.96	0.73	0.43
2013/14	0.64	0.54	0.71	0.20	0.98	0.66	0.67
2014/15	0.33	0.23	0.44	0.29	0.97	0.82	0.38
2015/16	0.03	0.39	0.71	0.15	0.99	0.78	0.41
2016/17	0.21	0.84	0.74	0.21	0.99	0.57	0.74
2017/18	0.50	0.36	0.48	0.45	0.95	0.38	0.85
2018/19	−0.27	−0.08	0.02	0.47	0.90	0.62	0.08

Table 17. Coefficients (p-value) of the panel data regression.

	Dependent variables					
Predictors	EC	TC	DMLPI	$\overline{\text{EC}}$	$\overline{\text{TC}}$	DSMLPI
Labor	−7.37 (−0.1977)	33.52 (−0.1794)	26.02 (−0.3043)	−8.75 (−0.2707)	14.77 (−0.0871)	5.08 (−0.6253)
Borrowings	0.037 (−0.1391)	−0.073 (−0.5063)	−0.036 (−0.744)	−0.093 (−0.0082)	−0.012 (−0.7587)	−0.114 (−0.0134)
Fixed assets	−0.982 (−0.0059)	1.9 (−0.2201)	0.928 (−0.5551)	−2.537 (0)	3.361 (0)	0.63 (−0.3292)
Operating expenses	1.102 (−0.0565)	1.086 (−0.6661)	2.199 (−0.3896)	0.157 (−0.8442)	2.314 (−0.0081)	2.432 (−0.021)
Deposits	0.0002 (−0.9784)	0.189 (0)	0.19 (0)	0.005 (−0.6387)	0.04 (−0.0011)	0.046 (−0.0018)
Investments	0.062 (−0.0614)	−0.859 (0)	−0.798 (0)	0.028 (−0.5446)	−0.297 (0)	−0.264 (0)
Loans	0.014 (−0.3182)	−0.007 (−0.9091)	0.007 (−0.9153)	0.015 (−0.4528)	−0.031 (−0.144)	−0.016 (−0.5248)
Operating income	−0.363 (−0.0521)	1.717 (−0.0353)	1.355 (−0.1014)	0.21 (−0.4183)	0.503 (−0.074)	0.737 (−0.0303)
Unused assets	−0.179 (−0.0006)	−1.198 (0)	−1.376 (0)	−0.189 (−0.009)	0.03 (−0.7063)	−0.165 (−0.082)
Capital reserves	−0.504 (−0.4258)	−19.273 (0)	−19.77 (0)	0.043 (−0.9612)	−1.36 (−0.1545)	−1.231 (−0.2853)
Net profit	−0.007 (−0.9929)	−4.707 (−0.1419)	−4.711 (−0.1477)	0.765 (−0.4529)	0.005 (−0.9966)	0.762 (−0.5683)
Return on assets	−35.82 (−0.462)	206.3 (−0.3319)	170.99 (−0.4282)	−70.12 (−0.3002)	103.03 (−0.1611)	30.9 (−0.7274)
NPA	0.304 (−0.1686)	1.128 (−0.2409)	1.429 (−0.1435)	−0.034 (−0.9114)	0.42 (−0.2064)	0.376 (−0.3481)

discrepancies in estimations are attributable to the use of different postulates when constructing these indices. The empirical findings clearly reveal that productivity change measured by DMLPI is predominantly driven by technical change, however when measured by DSMLPI, both efficiency and technical change contributed to the productivity development of Indian banks. Our method is general in that it can handle network dynamic structures in series including any real data with both desirable and undesirable characteristics. There is currently no approach that can be utilised to compare the findings of our proposed method described in this research, to the best of our knowledge.

Extending the current research to a parallel network structure or more complicated network structures with a feedback mechanism has the potential to be fruitful in the future. Lin and Liu [47] supplied direction vector characterizations, resulting in a plausible super-efficiency DDF model. Lozano and Soltani [50] suggested utilising a multi-directional distance function approach to analyse efficiency. The two research papers provide motivation for developing new direction vectors for computing MLPI. The new directions could shorten the projections of DMUs on the efficient frontier and possibly project them to the strong efficient frontier. For measuring MLPI, Arabi *et al.* [5] suggested a slacks-based measure (SBM) technique. Integrating the SBM and DDF techniques to compute MLPI in a dynamic DEA network system may yield some interesting results.

Acknowledgements. We would like to thank the editor and the anonymous referees for their valuable suggestions which helped us to improve the quality of the paper substantially. The authors are responsible for any remaining errors.

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