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A NOVEL APPROACH FOR THE SOLUTION OF MULTIOBJECTIVE OPTIMIZATION PROBLEM USING HESITANT FUZZY AGGREGATION OPERATOR

Firoz Ahmad[1,*], Ahmad Yusuf Adhami[3], Boby John[1] and Amit Reza[2]

Abstract. Many decision-making problems can solve successfully by traditional optimization methods with a well-defined configuration. The formulation of such optimization problems depends on crisply objective functions and a specific system of constraints. Nevertheless, in reality, in any decision-making process, it is often observed that due to some doubt or hesitation, it is pretty tricky for decision-maker(s) to specify the precise/crisp value of any parameters and compelled to take opinions from different experts which leads towards a set of conflicting values regarding satisfaction level of decision-maker(s). Therefore the real decision-making problem cannot always be deterministic. Various types of uncertainties in parameters make it fuzzy. This paper presents a practical mathematical framework to reflect the reality involved in any decision-making process. The proposed method has taken advantage of the hesitant fuzzy aggregation operator and presents a particular way to emerge in a decision-making process. For this purpose, we have discussed a couple of different hesitant fuzzy aggregation operators and developed linear and hyperbolic membership functions under hesitant fuzziness, which contains the concept of hesitant degrees for different objectives. Finally, an example based on a multiobjective optimization problem is presented to illustrate the validity and applicability of our proposed models.

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1. Introduction

Many decision-making processes inherently involve different conflicting objectives to be optimized (maximize/minimize) under given circumstances. In the present competitive era, it is indispensable for the decision-maker(s) to obtain better possible outcomes/results when dealing with multiple objectives. Although it is pretty challenging to have an optimal solution that satisfies all the goals efficiently, a compromise solution is possible, which is accepted by the decision-maker(s) up to some extent. Literature reveals various approaches for multi-objective optimization problems, and continuous effort is being made to obtain the best compromise solution. It is often observed that the modeling and formulation of the problem arising in agriculture production planning,

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[1] SQC & OR Unit, Indian Statistical Institute, Bangalore, India.

[2] Indian Institute of Technology Gandhinagar, Gujarat, India.

[3] Department of Statistics and Operations Research, Aligarh Muslim University, Aligarh, India.

[*]Corresponding author: firoz.ahmad02@gmail.com

manufacturing system, transportation problem, supplier selection, etc., take the form of the multiobjective optimization problem, which is realistic. Thus, a multiobjective optimization problem is also a challenging task due to the existence of different Pareto-optimal solutions set, and it is pretty typical to select the best compromise solutions set amongst them.

First [36] introduced the fuzzy programming approach for multiobjective optimization problems, and after that, fuzzy techniques gained popularity among various real applications problems. Li and Lai [22] solved multiobjective transportation problem by using fuzzy compromise programming approach. Zangiabadi and Maleki [33] also solved the multiobjective transportation problem by using fuzzy goal programming techniques with linear, exponential, and hyperbolic membership functions, respectively. Cheng *et al.* [20] solved the fuzzy multiobjective linear programming problem using deviation degree measures and weighted max–min method. Ahmad *et al.* [13] have formulated a pharmaceutical supply chain as a multiobjective programming problem and solved it by using three approaches, namely; *Zimmerman's technique, γ-operator* and *Min. bounded sum operator* with intuitionistic fuzzy parameters. Adhami and Ahmad [1], Adhami *et al.* [6] and Ahmad and John [2] also presented the study on neutrosophic decision set theory and implemented it on real-life applications.

Zhang *et al.* [34] developed a new strategic multi-objective optimization model for designing the supply chain network with multiple distribution channels and solved it by using a modified multi-objective artificial bee colony algorithm. Biswas and Modak [19] also formulated a multi-objective fuzzy chance-constrained model for land allocation in the agricultural sector and solved it by using fuzzy goal programming techniques. Xu *et al.* [31] developed a hesitant fuzzy programming method based on the linear programming technique for multidimensional analysis of preference to solve the hybrid, multiple attribute decision-making problems with incomplete attribute weight information. Wan *et al.* [26] also investigated a new hesitant fuzzy mathematical programming method for hybrid multicriteria group decision making with hesitant fuzzy truth degrees and incomplete criteria weight information. Wan *et al.* [27] suggested a novel solution method named, Pythagorean fuzzy (PF) mathematical programming method to solve multi-attribute group decision-making problems under PF environments. Wan *et al.* [28] also investigated a hesitant fuzzy Preference Ranking Organization Method for Enrichment Evaluations for multicriteria group decision-making and applied it to green supplier selection. Ahmad [3,7] and Ahmad and Smarandache [4] also discussed the multi-objective programming problems under neutrosophic environment.

Most of the existing methods discussed in the literature aggregate multiple objectives problems into a single objective using real-valued and utility functions. The real-valued functions may take different forms, such as weighted sum, max–min or weighted max–min, and the product form. The utility functions methods use the decision-maker(s) preference typically. This preference is translated to mathematical expression by using some utility function. For example, the weighted sum approach cannot guarantee that the achievement levels of fuzzy goals are consistent with desirable relative weights or the decision-maker(s) expectations. Due to the triviality in computation, Zimmermann [36] used the "min" operator in his proposed fuzzy programming approach, which does not guarantee a non-dominated solution. To overcome the above drawback, Li and Lai [22] suggested the weighted root-power mean aggregation operators and used them to evaluate the multiobjective transportation problem globally. Liu *et al.* [23] also used weighted root mean power aggregation operator for the supplier selection problem using modified s-curve membership function.

In real life, hesitancy is the most trivial issue in the decision-making process. It is the situation when the decision-maker(s) is (are) not sure about the exact values of the parameters, although there is some confusion between a few different values. To deal with it, a hesitant fuzzy set (HFS) can be used as an appropriate tool by assigning a set of different membership degrees for an element in the set. Ahmad *et al.* [35] and Zhang *et al.* [8] developed a hesitant mathematical programming technique to solve MCDM problems within the decision environment of hesitant fuzzy elements (HFEs). Bharati [18] presented a hesitant fuzzy computational algorithm for production planning problems. Further, the aggregation operators extended based on hesitant fuzzy sets. Xia *et al.* [30] discussed a new set of hesitant aggregation operators and presented its application in group decision-making problems. Xia and Xu [29] also defined a series of hesitant aggregations operators and illustrated their use in solving decision-making problems. Ye [32] and Ahmadini and Ahmad [15,16] proposed two aggregation operators, namely; a single-valued neutrosophic hesitant fuzzy weighted averaging (SVNHFWA) operator and a single-valued neutrosophic hesitant fuzzy weighted geometric (SVNHFWG) operator based on single-valued neutrosophic hesitant fuzzy and developed a multiple-attribute decision-making method.

Literature reveals that hesitant fuzzy aggregations operators have been widely used in multi-criteria or multi-attribute decision-making problems [4,5,29,30,32]. To the best of our knowledge, there is no such work available for multi-objective optimization problems using a hesitant fuzzy aggregation operator. Therefore, the proposed work fills this gap and provides the future research scope in a multi-objective optimization problem. The summary of the rest of the paper is as follows: In Section 2, the preliminaries regarding hesitant fuzzy set have been discussed while Section 3 represents the proposed model formulation and solution algorithm. In Section 4, an experimental study has been presented to show the applicability and validity of the proposed computational modeling approach. In Section 5, results and discussions have been presented. A comparative study has also been done with other existing approaches. Finally, conclusions and future scope have been discussed based on the present work in Section 6.

2. Preliminaries

Hesitant fuzzy set (HFS)

Definition 2.1 ([25])**.** Let there be a fixed set X; a hesitant fuzzy set A on X is defined in terms of a function $h_A(x)$ that when applied to X returns a finite subset of $[0,1]$ and mathematically can be represented as follows:

$$A = \{\langle x, h_A(x)\rangle | x \in X\} \tag{2.1}$$

where $h_A(x)$ is a set of some different values in $[0,1]$, denoting the possible membership degrees of the element $x \in X$ to A. Also, we call $h_A(x)$ a hesitant fuzzy element.

Definition 2.2 ([25])**.** For a given hesitant fuzzy element h, its lower and upper bounds are defined as $h^-(x) = \min h(x)$ and $h^+(x) = \max h(x)$, respectively.

Definition 2.3 ([25])**.** Let h_1 and h_2 be two HFSs in a fixed set X; then their union can be defined as follows:

$$h_1 \cup h_2 = \cup_{\alpha_1 \in h_1, \alpha_2 \in h_2} \ \max\{\alpha_1, \alpha_2\}. \tag{2.2}$$

Definition 2.4 ([25])**.** Let h_1 and h_2 be two HFSs in a fixed set X; then their intersection can be defined as follows:

$$h_1 \cap h_2 = \cup_{\alpha_1 \in h_1, \alpha_2 \in h_2} \ \min\{\alpha_1, \alpha_2\}. \tag{2.3}$$

Definition 2.5 ([25])**.** Let $E = \{h_1, h_2, \dots, h_n\}$ be a set of n HFEs, ψ be a function on E, such that $\psi : [0,1]^N \rightarrow [0,1]$, then $\psi_E = \cup_{\delta \in \{h_1, h_2, \dots, h_n\}} \{\psi(\delta)\}$.

3. Formulation of multiobjective model under different hesitant fuzzy aggregation operator

3.1. Multiobjective optimization problem

Most of the real-life problems are not only confined to the optimization (maximization/minimization) of a single objective, but a set of multiple conflicting and commensurable objectives have to optimize under given circumstances. It is not always possible to have an optimal solution for every single objective at a time, but to some extent, a compromise solution can be obtained that satisfies each objective marginally under certain constraints. Nowadays, the effort is being made to obtain the best compromise solution for the multiobjective optimization problem.

Generally, the formulation of a multiobjective optimization problem with k objectives, m constraints, and q variables is given as follows:

$$\begin{aligned}
&\text{Optimize } (Z_1, Z_2, \ldots, Z_k)\ (k = 1,2,\ldots,K) && \\
&\quad \text{s.t. } g_j(x) \le d_j, && j = 1,2,\ldots,m_1; \\
&\quad\quad g_j(x) \ge d_j, && j = m_1+1, m_1+2,\ldots,m_2; \\
&\quad\quad g_j(x) = d_j, && j = m_2+1, m_2+2,\ldots,m; \\
&\quad\quad x_i \ge 0, && i = 1,2,3,\ldots,q;\quad x_i \in X
\end{aligned} \tag{3.1}$$

where Z_k are a set of k different conflicting objectives, g_j are real valued functions and d_j are real numbers. x_i is q dimensional decision variable vector and X is a feasible solution set.

Definition 3.1 ([9, 12])**.** A feasible solution $x^* = \{x_i^*\} \in X$ is said to be non-dominated (efficient or Pareto-optimal) solution for the multiobjective optimization problem if there does not exist any other feasible solution $x = \{x_i\}$ such that

$$Z_k\{x_i\} \le \ \text{or}\ \ge Z_k^*\{x_i^*\},\ \forall \quad k = 1,2,\ldots,K \tag{3.2}$$

where $\le$ or $\ge$ is used for optimization (minimization or maximization) of different objectives respectively.

Definition 3.2 ([9, 10])**.** An optimal compromise solution of multiobjective optimization problem is a feasible solution $x = \{x_i\} \in X$ at which decision maker's preference value under the different multiple objectives, is optimum (minimum or maximum).

3.2. Development of membership functions under hesitant fuzziness

Until recently, a simple membership function has been defined only in the context of marginal evaluation for every single objective $Z_k(x)$ and without considering the acceptable degree of hesitation by different experts. Hence, by the marginal evaluation for each single objective $Z_k(x)$ under hesitation, we mean a mapping $\mu^{E_n}(Z_k(x)) \rightarrow [0,1]$ assigned by nth expert (E_n) which indicates that up to what extent the decision makers' satisfaction degree is achieved with the involvement of different experts' opinions [8, 11, 14]. Therefore, this kind of membership function would be a handy tool when dealing with some hesitation degree simultaneously.

On Solving each objective functions individually, we have k solutions set, $X^1, X^2, \ldots, X^k$. Then the obtained solutions are substituted into each objective function in order to determine the lower and upper bound for each objective as given below:

$$U_k = \max[Z_k(x)] \ \text{ and } \ L_k = \min[Z_k(x)] \quad \forall k = 1,2,3,\ldots,K. \tag{3.3}$$

1. **Linear membership function:** the linear membership function is the most commonly used membership function due to its simple structure and has gained wide range of applicability under uncertainty. The linear membership function under hesitancy is defined as follows [8]:

$$\mu_L^{E_n}(Z_k(x)) = \begin{cases} 1, & \text{if } \ Z_k < L_k; \\ \delta_n \dfrac{U_k - Z_k}{U_k - L_k}, & \text{if } \ L_k \le Z_k \le U_k; \\ 0, & \text{if } \ Z_k > U_k \end{cases} \tag{3.4}$$

   where $\delta_n \in [0,1]$ is a set of hesitant values assigned by nth expert (E_n). The values of parameters δ_n is suggessted by the different experts based on his/her previous knowledge and experiences.

2. **Hyperbolic membership function:** the hyperbolic membership function shows the flexible characteristic behavior with respect to objective function. It is convex over a part of the objective function values and is concave over the remaining part. When the decision maker is worse off with respect to a goal, the decision maker tends to have a higher marginal rate of satisfaction with respect to that goal. A convex shape part of

the membership function captures that behavior. On the other hand, when decision maker is better off with respect to a goal, the decision maker tends to have a smaller marginal rate of satisfaction. Such behavior is modeled using the concave portion of the membership function [11]. The hyperbolic membership function under hesitancy is defined as follows:

$$\mu_H^{E_n}(Z_k(x)) = \begin{cases} 1, & \text{if } Z_k \leq L_k; \\ \dfrac{\delta_n}{2} + \dfrac{\delta_n}{2}\dfrac{e^{\left\{\frac{U_k+L_k}{2}-Z_k\right\}\alpha_k} - e^{-\left\{\frac{U_k+L_k}{2}-Z_k\right\}\alpha_k}}{e^{\left\{\frac{U_k+L_k}{2}-Z_k\right\}\alpha_k} + e^{-\left\{\frac{U_k+L_k}{2}-Z_k\right\}\alpha_k}}, & \text{if } L_k \leq Z_k < U_k; \\ 0, & \text{if } Z_k \geq U_k \end{cases} \tag{3.5}$$

where $\alpha_k = \frac{6}{U_k - L_k}$ and $\delta_n \in [0,1]$ is a set of hesitant values assigned by nth expert (E_n). The values of parameters δ_n is suggessted by the different experts based on his/her previous knowledge and experiences.

This membership function holds the following systematic properties:

(1) $\mu_H(Z_k(x))$ is strictly monotonically decreasing function with respect to $Z_k(x)$.
(2) $\mu_H(Z_k(x)) = \frac{1}{2} \Leftrightarrow Z_k(x) = \frac{U_k+L_k}{2}$.
(3) $\mu_H(Z_k(x))$ is strictly convex function of $Z_k(x)$ for $Z_k(x) \geq \frac{U_k+L_k}{2}$ and strictly concave function of $Z_k(x)$ for $Z_k(x) \leq \frac{U_k+L_k}{2}$.
(4) $\mu_H(Z_k(x))$ satisfies $0 < \mu_H(Z_k(x)) < 1$ for $L_k < Z_k(x) < U_k$ and approaches asymptotically. $\mu_H(Z_k(x)) = 0$ and $\mu_H(Z_k(x)) = 1$ as $Z_k(x) \to \infty$ and $-\infty$ respectively.

In this sub-section, we have defined linear (Eq. (3.4)) and non-linear (Hyperbolic, Eq. (3.5)) membership functions under a hesitant fuzzy environment. A linear membership function is perhaps the simplest and most common one, as it bounds the upper and lower acceptance levels. In the case of non-linear membership functions, the marginal rate of increase or decrease of membership values as a function of model parameters is not constant. Hence, it reflects the practical situations better than the linear case, and it will be preferable to formulate a non-linear membership function-based model which may reflect the optimal solution.

3.3. Global evaluation of multiple objectives using hesitant fuzzy aggregation operators

Based on the Definition 2.5, Xia and Xu [29] proposed some operations on hesitant fuzzy sets and discussed hesitant fuzzy aggregation operators. Weighted averaging and weighted geometric are two important class of hesitant fuzzy aggregation operator.

Definition 3.3 ([29])**.** For a collection of the hesitant fuzzy elements E_n $(n = 1, 2, \ldots, N)$, a generalized form of hesitant fuzzy weighted averaging (GHFWA) operator family is a mapping GHFWA: $E^n \to E$ such that

$$\text{GHFWA}^{(\lambda)}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \cup_{\alpha_1\in E_1, \alpha_2\in E_2, \ldots, \alpha_n \in E_n}\left(1 - \prod_{n=1}^{N}\left(1-\alpha_n^{\lambda}\right)^{w_n}\right)^{\frac{1}{\lambda}} \tag{3.6}$$

where $(0 < |\lambda| < \infty)$ and $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy elements, with $w_n \in [0,1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set.

Definition 3.4 ([29])**.** Let E_n $(n = 1, 2, \ldots, N)$ be a collection of the hesitant fuzzy elements, then a hesitant fuzzy weighted averaging (HFWA) operator is a mapping $E^n \to E$ such that

$$\text{HFWA}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \cup_{\alpha_1\in E_1, \alpha_2\in E_2, \ldots, \alpha_n \in E_n}\left(1 - \prod_{n=1}^{N}(1-\alpha_n)^{w_n}\right) \tag{3.7}$$

where $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0,1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set. Especially, $\lambda = 1$ transformed a GHFWA to a hesitant fuzzy weighted averaging (HFWA) operator.

Definition 3.5 ([29]). Let E_n $(n = 1, 2, \ldots, N)$ be a collection of the hesitant fuzzy elements, then a hesitant fuzzy quadratic weighted averaging (HFQWA) operator is a mapping $E^n \rightarrow E$ such that

$$\text{HFQWA}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \cup_{\alpha_1 \in E_1, \alpha_2 \in E_2, \ldots, \alpha_n \in E_n} \left(1 - \prod_{n=1}^{N} \left(1 - \alpha_n^2\right)^{w_n}\right)^{\frac{1}{2}} \tag{3.8}$$

where $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0, 1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set. Especially, if we put $\lambda = 2$ in GHFWA then GHFWA reduces to hesitant fuzzy quadratic weighted averaging (HFQWA) operator.

Definition 3.6 ([29]). For a collection of the hesitant fuzzy elements E_n $(n = 1, 2, \ldots, N)$, a generalized form of hesitant fuzzy weighted geometric (GHFWG) operator family is a mapping GHFWG: $E^n \rightarrow E$ such that

$$\text{GHFWG}^{(\lambda)}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \left(1 - \left(1 - \prod_{n=1}^{N} \left(1 - \left(1 - \alpha_n^{\lambda}\right)\right)^{w_n}\right)^{\frac{1}{\lambda}}\right) \tag{3.9}$$

where $(0 < |\lambda| < \infty)$ and $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy elements, with $w_n \in [0, 1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set.

Definition 3.7 ([29]). Let E_n $(n = 1, 2, \ldots, N)$ be a collection of the hesitant fuzzy elements, then a hesitant fuzzy weighted geometric (HFWG) operator is a mapping $E^n \rightarrow E$ such that

$$\text{HFWG}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \cup_{\alpha_1 \in E_1, \alpha_2 \in E_2, \ldots, \alpha_n \in E_n} \left(1 - \left(1 - \prod_{n=1}^{N} \left(1 - \left(1 - \alpha_n\right)\right)^{w_n}\right)\right) \tag{3.10}$$

where $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0, 1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set. Especially, if we put $\lambda = 1$ in GHFWG then GHFWG reduces to hesitant fuzzy weighted geometric (HFWG) operator.

Definition 3.8 ([29]). Let E_n $(n = 1, 2, \ldots, N)$ be a collection of the hesitant fuzzy elements, then a hesitant fuzzy quadratic weighted geometric (HFQWG) operator is a mapping $E^n \rightarrow E$ such that

$$\text{HFQWG}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \cup_{\alpha_1 \in E_1, \alpha_2 \in E_2, \ldots, \alpha_n \in E_n} \left(1 - \left(1 - \prod_{n=1}^{N} \left(1 - \left(1 - \alpha_n^2\right)\right)^{w_n}\right)^{\frac{1}{2}}\right) \tag{3.11}$$

where $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0, 1]$ and $\sum_{n=1}^{N} w_n = 1$ in the set. Especially, if we put $\lambda = 2$ in GHFWG then GHFWG reduces to hesitant fuzzy quadratic weighted geometric (HFQWG) operator.

Based on the above discussed hesitant fuzzy aggregation operator (HFAO), a new general form of achievement or utility function for multiobjective optimization problem may be presented as follows:

$$\text{Max (HFAO)}^{(\lambda)}\left(\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right) = \sum_{k=1}^{K} W_k(\text{HFAO}) \tag{3.12}$$

where $(0 < |\lambda| < \infty)$, HFAO represents the family of hesitant fuzzy aggregation operator (*i.e.*, averaging or geometric) which has been used to maximize the achievement function and W_k $(\forall k = 1, 2, \ldots, K)$, such that $\sum_{k=1}^{K} W_k = 1$ is the weight assigned to each objectives by decision maker(s).

Here, some new computational models using hesitant fuzzy aggregation operators based on the hesitant fuzzy set have been investigated to solve the multiobjective optimization problem. It allows the decision-maker(s) to express his/her(their) degree of hesitation about the value of parameters and overcome by using the proposed computational model technique efficiently. According to Bellman and Zadeh [17], the fuzzy set includes three concepts, namely, fuzzy decision (D), fuzzy goal (G), and fuzzy constraints (C) and incorporated these concepts in many real-life applications of decision-making under fuzzy environment. So, the fuzzy decision set is defined as follows:

$$D = G \cap C = \cup_{\beta_n \in G, \eta_n \in C} \ \min\{(\beta_n, \eta_n)\}. \tag{3.13}$$

Consequently, the hesitant fuzzy decision set D_h, with hesitant fuzzy objectives (Z) and constraints (C), is defined as follows:

$$\begin{aligned}
D_h = Z \cap C &= \left(\cap_{k=1}^{K} Z_k\right)\left(\cap_{j=1}^{m} C_j\right)\\
&= \{x, \cup \ \min\{(\beta_1,\eta_1),(\beta_2,\eta_2),\ldots,(\beta_n,\eta_n)\}(x) | x \in X\}\\
&= \left\{x, \left\{\mu^{E_1}, \mu^{E_2}, \ldots, \mu^{E_n}\right\}(x) | x \in X\right\}\\
\mu^{E_1} &= \min(\beta_1,\eta_1)\\
\mu^{E_2} &= \min(\beta_2,\eta_2)\\
&\ \vdots\\
\mu^{E_n} &= \min(\beta_n,\eta_n)
\end{aligned}$$

where, μ^{E_n} are a set of degree of acceptance of hesitant fuzzy decision solution under hesitant fuzzy decision set by nth experts.

3.4. Proposed models

We have designed a hesitant fuzzy environment for the multi-objective optimization problem stated in equation (3.1). Since equation (3.1) depicts the multi-objective optimization problem, it can be solved only after converting into a single objective programming problem. For this purpose, we have taken advantage of the HFAO (3.12). Thus, we can transform the objective functions of the optimization problem (Eq. (3.1)) as a maximization on hesitant fuzzy aggregation operator shown in equation (3.12) with the constraints on hesitant membership functions corresponding to each objective function. In equation (3.14), the weighted sum of hesitant fuzzy aggregation operator is summarized and also transformed the multi-objective optimization problem (Eq. (3.1)) into a single objective function (Eq. (3.14)). Therefore, in general, one can reformulate the equation (3.1) as follows (Eq. (3.14)):

$$\text{Max HFAO}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k(\text{HFAO}) \tag{3.14}$$

under the constraints

$$\begin{aligned}
&\mu_A^{E_1}(Z_k(x)) \geq \alpha_{k1},\\
&\mu_A^{E_2}(Z_k(x)) \geq \alpha_{k2},\\
&\ \ldots\\
&\ \ldots\\
&\ \ldots\\
&\mu_A^{E_n}(Z_k(x)) \geq \alpha_{kn},\\
&g_j(x) \leq d_j, \qquad\qquad j = 1, 2, \ldots, m_1,
\end{aligned}$$

$$
\begin{aligned}
&g_j(x) \geq d_j, && j = m_1+1, m_1+2, \ldots, m_2, \\
&g_j(x) = d_j, && j = m_2+1, m_2+2, \ldots, m, \\
&x \geq 0, && 0 \leq \alpha_{k1}, \alpha_{k2}, \ldots, \alpha_{kn} \leq 1, \quad \forall n
\end{aligned} \tag{3.15}
$$

where A represents the type of membership function (*i.e.*, linear or hyperbolic) used by decision maker(s). The decision maker(s) defines the priority among objective functions by assigning crisp weight W_k ($\forall k = 1, 2, \ldots, K$), such that $\sum_{k=1}^{K} W_k = 1$ which takes the maximum value to the most desired objective function. $w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0, 1]$ and $\sum_{n=1}^{N} w_n = 1$. In equation (3.15), each α_{kn} represents the minimum satisfaction degrees corresponding to each hesitant membership function and can be directly obtained while solving the problems; as a result, output. The only restrictions are applied that each α_{kn} can lie between 0 and 1.

$$
\begin{aligned}
&\mu_A^{E_1}(Z_k(x)) : \text{degree of acceptance assigned by 1st expert to the } k\text{th objective} \\
&\mu_A^{E_2}(Z_k(x)) : \text{degree of acceptance assigned by 2nd expert to the } k\text{th objective} \\
&\vdots \\
&\mu_A^{E_n}(Z_k(x)) : \text{degree of acceptance assigned by } n\text{th expert to the } k\text{th objective.}
\end{aligned}
$$

In general, the hesitant fuzzy aggregation operator can be classified into four different categories such as hesitant fuzzy weighted averaging operator, hesitant fuzzy quadratic weighted averaging operator, hesitant fuzzy weighted geometric operator, hesitant fuzzy quadratic weighted geometric operator. Therefore our proposed model can also be classified into four different categories based on these operators. The formulation of proposed computational models for multiobjective optimization problems under hesitant fuzzy environment is given as follows:

Model 1: Based on hesitant fuzzy weighted operator

$$
\text{Max HFWA}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} (1 - \alpha_{kn})^{w_n}\right). \tag{3.16}
$$

Under the constraints provided in equation (3.15).

Model 2: Based on hesitant fuzzy quadratic weighted averaging operator

$$
\text{Max HFQWA}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} \left(1 - \alpha_{kn}^2\right)^{w_n}\right)^{\frac{1}{2}}. \tag{3.17}
$$

Under the constraints provided in equation (3.15).

Model 3: Based on hesitant fuzzy weighted geometric operator

$$
\text{Max HFWG}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k \left(1 - \left(1 - \prod_{n=1}^{N} (1 - (1 - \alpha_{kn}))^{w_n}\right)\right). \tag{3.18}
$$

Under the constraints provided in equation (3.15).

Model 4: Based on hesitant fuzzy quadratic weighted geometric operator

$$
\text{Max HFQWG}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k \left(1 - \left(1 - \prod_{n=1}^{N} \left(1 - \left(1 - \alpha_n^2\right)\right)^{w_n}\right)^{\frac{1}{2}}\right). \tag{3.19}
$$

Under the constraints provided in equation (3.15).

All the above four models differ only concerning different hesitant fuzzy aggregation operators for global evaluation of the multiobjective optimization problem. The aggregation operator used in model 1 is the hesitant fuzzy weighted averaging (HFWA) operator, whereas the hesitant fuzzy quadratic weighted averaging (HFQWA) operator has been used in model 2 to get the best compromise solution. In models 3 and 4, the hesitant fuzzy weighted geometric (HFWG) and hesitant fuzzy quadratic weighted geometric (HFQWG) have been used to obtain the overall best possible solution.

3.5. Solution procedure

To transform the optimization model (3.1) into a hesitant fuzzy model (3.12) (with the hesitant fuzzy environment), one needs to solve each objective function individually and has to determine the bounds U_k and L_k. By using U_k and L_k, construct the preferred membership functions under hesitant fuzzy environment as given in equations (3.4) and (3.5) respectively. The optimal choice of membership function can be varied with practical problems, and one can choose a specific one from a set of well-defined membership functions (Linear, Triangular, Trapezoidal, Exponential, Parabolic, Hyperbolic, etc.). For this study, we have specifically chosen linear and non-linear (hyperbolic membership) functions to demonstrate the efficacy of our proposed method. After considering a specific membership function, the different hesitant values can be assigned based on experts' choices. The expert can provide the value of δ_n defined in equations (3.4) and (3.5). One can select the desired hesitant fuzzy aggregation operator and formulate the computational models under a hesitant fuzzy environment as shown in equations (3.16)–(3.19). The constructed model can be solved using suitable techniques or some optimizing software packages [21] to obtain a compromise solution. The solution process can be shown in the following algorithm form.

Algorithm 1: Hesitant Fuzzy Aggregation operators based solution scheme.

1 Solve the objective functions
2 Calculate U_k, L_k. // Use equation (3.3)
3 Construct a preferred membership function. // Use equations (3.4) and (3.5)
4 Choose different hesitant values.
5 Select a desired hesitant fuzzy aggregation operator.
6 Formulate a model under hesitant fuzzy environment.
7 Solve the model numerically.

4. Experimental study

We have adopted the numerical example of manufacturing system presented in Ahmad *et al.* [24] and Singh and Yadav [8]. The decision-maker(s) of the company intends to maximize the total profit incurred over products and minimize the total time required for each product, and also, the decision-maker(s) seeks three experts' opinions in the decision-making process. Therefore, the crisp nonlinear multiobjective programming problem formulation is defined as follows:

$$\text{Max } Z_1(x) = 99.875x_1^{\frac{1}{2}} - 8x_1 + 119.875x_2^{\frac{1}{2}} - 10.125x_2 + 95.125x_3^{\frac{1}{3}} - 8x_3 \tag{4.1}$$

$$\text{Min } Z_2(x) = 3.875x_1 + 5.125x_2 + 5.9375x_3 \tag{4.2}$$

$$\text{s.t. } 2.0625x_1 + 3.875x_2 + 2.9375x_3 \leq 333.125 \tag{4.3}$$

$$3.875x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625 \tag{4.4}$$

$$2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360 \tag{4.5}$$

$$x_1, x_2, x_3 \geq 0. \tag{4.6}$$

One can solve each objective function individually to obtain the individual best solution, along with lower and upper bound for each of the objective function.

$X^1 = (57.82, 13.09, 55.53)$, $X^2 = (62.26, 0, 60.28)$ along with $L_1 = 180.72$, $U_1 = 516.70$, $L_2 = 599.23$ and $U_2 = 620.84$.

Three experts assigned their hesitant values as $\delta_1 = 0.96, \delta_2 = 0.98$ and $\delta_3 = 1$ respectively.

For this multiobjective nonlinear programming problem under hesitant fuzzy environment, one can formulate the following model based on different choice of fuzzy membership function.

1. Using linear membership function

Model 1:

$$\begin{aligned}
&\text{Max HFWA}\left(\mu_L^{E_1}, \mu_L^{E_2}, \ldots, \mu_L^{E_n}\right) = \sum_{k=1}^{2} W_k\left(1 - \prod_{n=1}^{3}(1-\alpha_{kn})^{w_n}\right)\\
&\text{s.t. } 0.96\,\frac{Z_1(x) - 180.72}{516.70 - 180.72} \geq \alpha_{11}\\
&\quad 0.98\,\frac{Z_1(x) - 180.72}{516.70 - 180.72} \geq \alpha_{12}\\
&\quad \frac{Z_1(x) - 180.72}{516.70 - 180.72} \geq \alpha_{13}\\
&\quad 0.96\,\frac{620.84 - Z_2(x)}{620.84 - 599.23} \geq \alpha_{21}\\
&\quad 0.98\,\frac{620.84 - Z_2(x)}{620.84 - 599.23} \geq \alpha_{22}\\
&\quad \frac{620.84 - Z_2(x)}{620.84 - 599.23} \geq \alpha_{23}\\
&\quad 2.0625x_1 + 3.875x_2 + 2.9375x_3 \leq 333.125\\
&\quad 3.875x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625\\
&\quad 2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360\\
&\quad x_1, x_2, x_3 \geq 0, 0 \leq \alpha_{kn} \leq 1 \qquad \forall k = 1,2;\ n = 1,2,3.
\end{aligned} \tag{4.7}$$

Model 2:

$$\text{Max HFQWA}\left(\mu_L^{E_1}, \mu_L^{E_2}, \ldots, \mu_L^{E_n}\right) = \sum_{k=1}^{2} W_k\left(1 - \prod_{n=1}^{3}\left(1-\alpha_{kn}^2\right)^{w_n}\right)^{\frac{1}{2}}$$

s.t. (4.7).

Model 3:

$$\text{Max HFWG}\left(\mu_L^{E_1}, \mu_L^{E_2}, \ldots, \mu_L^{E_n}\right) = \sum_{k=1}^{2} W_k\left(1 - \left(1 - \prod_{n=1}^{3}(1-(1-\alpha_{kn}))^{w_n}\right)\right)$$

s.t. (4.7).

Model 4:

$$\text{Max HFQWG}\left(\mu_L^{E_1}, \mu_L^{E_2}, \ldots, \mu_L^{E_n}\right) = \sum_{k=1}^{2} W_k\left(1 - \left(1 - \prod_{n=1}^{3}\left(1-\left(1-\alpha_n^2\right)\right)^{w_n}\right)^{\frac{1}{2}}\right)$$

s.t. (4.7).

2. Using hyperbolic membership function

Model 1:

$$
\begin{aligned}
&\text{Max HFWA}\left(\mu_H^{E_1}, \mu_H^{E_2}, \ldots, \mu_H^{E_n}\right) = \sum_{k=1}^{2} W_k \left(1 - \prod_{n=1}^{3} (1-\alpha_{kn})^{w_n}\right) \\
&\text{s.t. } \frac{0.96}{2} + \frac{0.96}{2} \frac{e^{\{Z_1-348.71\}0.0178} - e^{-\{Z_1-348.71\}0.0178}}{e^{\{Z_1-348.71\}0.0178} + e^{-\{Z_1-348.71\}0.0178}} \geq \alpha_{11} \\
&\frac{0.98}{2} + \frac{0.98}{2} \frac{e^{\{Z_1-348.71\}0.0178} - e^{-\{Z_1-348.71\}0.0178}}{e^{\{Z_1-348.71\}0.0178} + e^{-\{Z_1-348.71\}0.0178}} \geq \alpha_{12} \\
&\frac{1}{2} + \frac{1}{2} \frac{e^{\{Z_1-348.71\}0.0178} - e^{-\{Z_1-348.71\}0.0178}}{e^{\{Z_1-348.71\}0.0178} + e^{-\{Z_1-348.71\}0.0178}} \geq \alpha_{13} \\
&\frac{0.96}{2} + \frac{0.96}{2} \frac{e^{\{610.035-Z_2\}0.2776} - e^{-\{610.035-Z_2\}0.2776}}{e^{\{610.035-Z_2\}0.2776} + e^{-\{610.035-Z_2\}0.2776}} \geq \alpha_{21} \\
&\frac{0.98}{2} + \frac{0.98}{2} \frac{e^{\{610.035-Z_2\}0.2776} - e^{-\{610.035-Z_2\}0.2776}}{e^{\{610.035-Z_2\}0.2776} + e^{-\{610.035-Z_2\}0.2776}} \geq \alpha_{22} \\
&\frac{1}{2} + \frac{1}{2} \frac{e^{\{610.035-Z_2\}0.2776} - e^{-\{610.035-Z_2\}0.2776}}{e^{\{610.035-Z_2\}0.2776} + e^{-\{610.035-Z_2\}0.2776}} \geq \alpha_{23} \\
&2.0625x_1 + 3.875x_2 + 2.9375x_3 \leq 333.125 \\
&3.875x_1 + 2.0625x_2 + 2.0625x_3 \leq 365.625 \\
&2.9375x_1 + 2.0625x_2 + 2.9375x_3 \geq 360 \\
&x_1, x_2, x_3 \geq 0, 0 \leq \alpha_{kn} \leq 1 \qquad \forall k = 1,2;\ n = 1,2,3.
\end{aligned} \tag{4.8}
$$

Model 2:

$$\text{Max HFQWA}\left(\mu_H^{E_1}, \mu_H^{E_2}, \ldots, \mu_H^{E_n}\right) = \sum_{k=1}^{2} W_k \left(1 - \prod_{n=1}^{3} (1-\alpha_{kn}^2)^{w_n}\right)^{\frac{1}{2}}$$

s.t. (4.8).

Model 3:

$$\text{Max HFWG}\left(\mu_H^{E_1}, \mu_H^{E_2}, \ldots, \mu_H^{E_n}\right) = \sum_{k=1}^{2} W_k \left(1 - \left(1 - \prod_{n=1}^{3} (1-(1-\alpha_{kn}))^{w_n}\right)\right)$$

s.t. (4.8).

Model 4:

$$\text{Max HFQWG}\left(\mu_H^{E_1}, \mu_H^{E_2}, \ldots, \mu_H^{E_n}\right) = \sum_{k=1}^{2} W_k \left(1 - \left(1 - \prod_{n=1}^{3} (1-(1-\alpha_n^2))^{w_n}\right)^{\frac{1}{2}}\right)$$

s.t. (4.8).

$w_n = (w_1, w_2, \ldots, w_n)$ be the weight vector assigned to each hesitant fuzzy element, with $w_n \in [0,1]$ and $\sum_{n=1}^{N} w_n = 1$.

At $w_1 = w_2 = w_3 = \frac{1}{3}$ and by tuning the different pairs of weight $W_1, W_2 = (1 - W_1)$ assigned to each objective functions, the optimal solutions set are obtained and summarized in Tables 1 and 2.

All the proposed models have been written in AMPL language and solved using available solvers on NEOS server on-line facility provided by Wisconsin Institutes for Discovery at the University of Wisconsin in Madison for solving Optimization problems, see Dolan [21].

TABLE 1. The optimal compromise objective values using linear membership function.

Weight (W_1, W_2)	Optimal objective values: (Z_1, Z_2) Model 1	Model 2	Model 3	Model 4
$(0.1, 0.9)$	$(184.56, 599.47)$	$(184.69, 599.22)$	$(210.56, 599.73)$	$(215.53, 599.86)$
$(0.8, 0.2)$	$(236.65, 599.43)$	$(236.74, 599.34)$	$(243.45, 599.37)$	$(247.87, 599.65)$
$(0.3, 0.7)$	$(279.87, 600.54)$	$(279.36, 600.75)$	$(281.34, 600.34)$	$(283.40, 600.29)$
$(0.4, 0.6)$	$(323.34, 602.95)$	$(323.74, 602.91)$	$(324.64, 602.84)$	$(324.32, 602.76)$
$(0.5, 0.5)$	$(372.39, 604.76)$	$(372.49, 604.89)$	$(373.45, 604.59)$	$(373.28, 604.69)$
$(0.6, 0.4)$	$(428.37, 609.69)$	$(428.43, 609.22)$	$(428.32, 609.74)$	$(428.93, 609.75)$
$(0.7, 0.3)$	$(516.50, 620.27)$	$(516.81, 620.26)$	$(490.45, 616.23)$	$(490.18, 616.35)$
$(0.8, 0.2)$	$(516.64, 620.24)$	$(516.48, 620.67)$	$(516.87, 620.29)$	$(516.46, 620.74)$
$(0.9, 0.1)$	$(516.29, 620.65)$	$(516.49, 620.64)$	$(516.28, 620.76)$	$(516.35, 620.56)$

TABLE 2. The optimal compromise objective values using hyperbolic membership function.

Weight (W_1, W_2)	Optimal objective values: (Z_1, Z_2) Model 1	Model 2	Model 3	Model 4
$(0.1, 0.9)$	$(358.46, 603.56)$	$(358.87, 603.34)$	$(359.63, 603.24)$	$(360.34, 603.21)$
$(0.8, 0.2)$	$(375.32, 604.34)$	$(375.34, 604.34)$	$(375.23, 604.24)$	$(375.23, 604.98)$
$(0.3, 0.7)$	$(384.24, 605.34)$	$(384.67, 605.23)$	$(384.34, 605.65)$	$(385.38, 605.45)$
$(0.4, 0.6)$	$(391.74, 605.73)$	$(391.28, 605.87)$	$(391.84, 605.97)$	$(392.45, 605.87)$
$(0.5, 0.5)$	$(398.56, 606.37)$	$(397.52, 606.62)$	$(398.82, 606.12)$	$(398.32, 606.92)$
$(0.6, 0.4)$	$(404.43, 606.29)$	$(404.68, 606.45)$	$(404.28, 606.93)$	$(404.45, 606.46)$
$(0.7, 0.3)$	$(411.78, 607.45)$	$(411.18, 607.98)$	$(410.48, 607.87)$	$(410.47, 607.45)$
$(0.8, 0.2)$	$(419.27, 608.86)$	$(419.45, 608.56)$	$(419.48, 608.29)$	$(419.57, 608.76)$
$(0.9, 0.1)$	$(435.56, 609.39)$	$(435.45, 609.48)$	$(434.69, 609.45)$	$(434.39, 609.65)$

5. Results and discussions

The compromise solution has been obtained using linear and hyperbolic membership functions under four models with different crisp weights, as shown in Figures 1 and 2, respectively. The weights given to the first objective are chosen randomly between 0 to 1 and assigned by the decision-maker(s) according to his/her(their) satisfaction level, whereas weights given to the second objective complement the weight given to the first objective function, respectively. The sensitivity analysis has also been performed by tuning the weight parameter, and the obtained results can be revealed in the following three aspects:

1. Based on assigned individual weight to each objective function

The solution results obtained by using the linear membership function has been interpreted as follows:

In model 1; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(184.47, 599.23)$, while at $(0.5, 0.5)$, the values are $(372.92, 604.62)$ and at $(0.9, 0.1)$, the objective values reaches to $(516.37, 620.84)$ respectively.

As the assigned weight is increasing for the first objective, the value of the first objective is increasing and reaches almost its best solution while at the same time the assigned weight is decreasing for the second objective and the value of the second objective is also increasing and reached to almost its worst solution.

In model 2; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(184.50, 599.23)$, while at $(0.5, 0.5)$, the values are $(372.92, 604.62)$ and at $(0.9, 0.1)$, the objective values reaches to $(516.37, 620.84)$ which is almost individual best and worst solution of each objectives respectively.

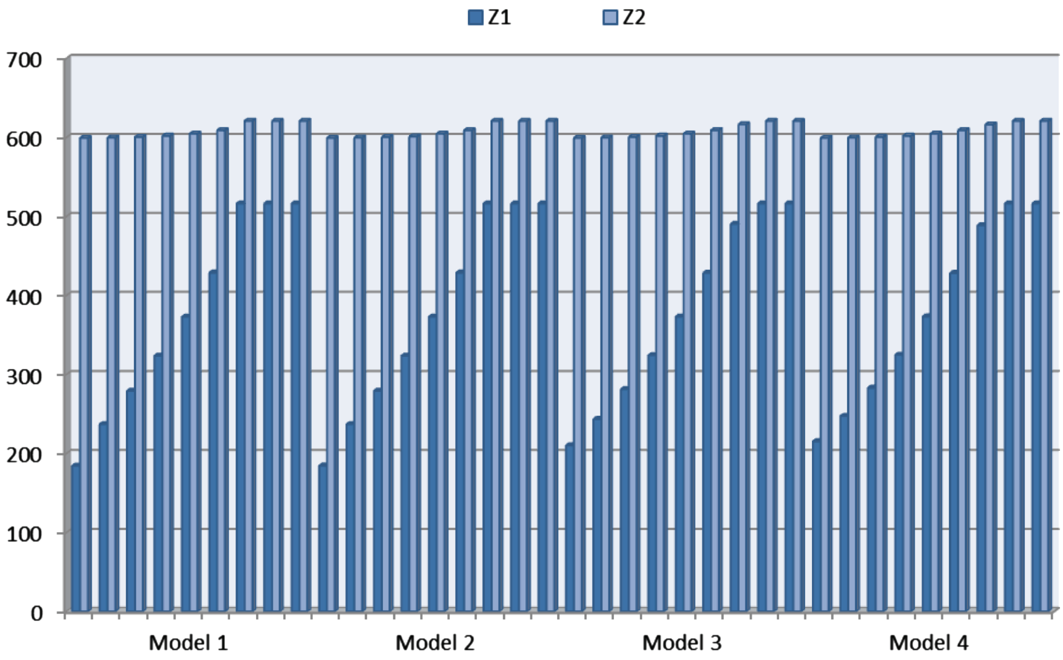

Figure 1. Solution results using linear membership function with different weights.

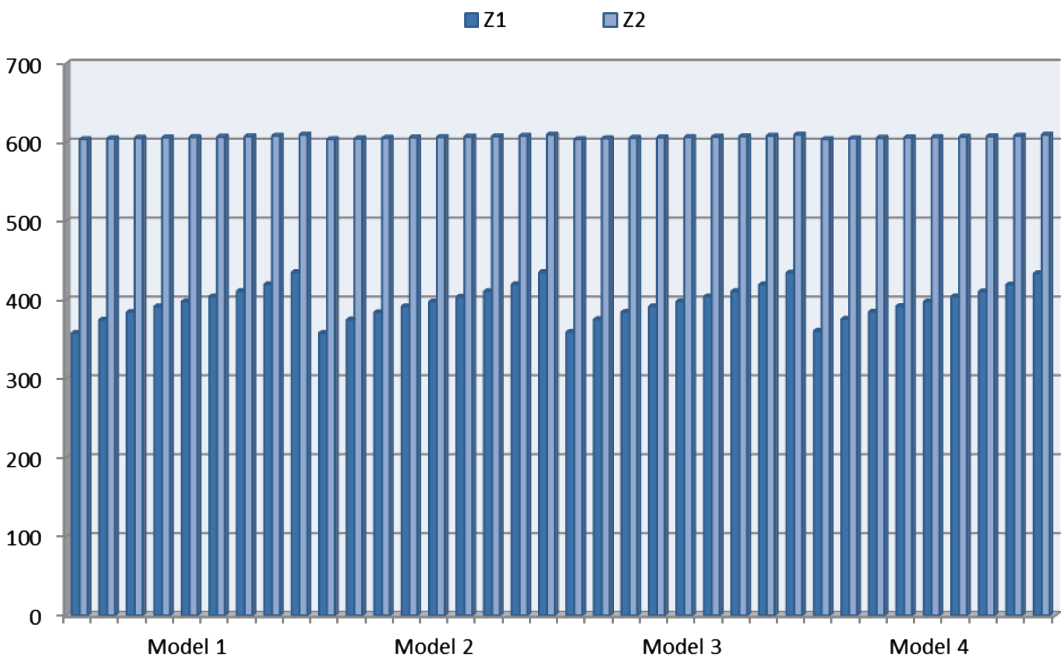

Figure 2. Solution results using hyperbolic membership function with different weights.

It is found that with the increase(decrease) in the weights of the first(second) objective function, the values are approaching towards its best(worst) solution.

In model 3; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(210.27, 599.33)$, while at $(0.5, 0.5)$, the values are $(373.09, 604.64)$ and at $(0.9, 0.1)$, the objective values reaches to $(516.37, 620.84)$ which is almost individual best and worst solution of each objectives respectively.

It is noted that with the increase(decrease) in the weights of the first(second) objective function, the values tend towards their best(worst) solution.

In model 4; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(215.35, 599.33)$, while at $(0.5, 0.5)$, the values are $(373.24, 604.64)$ and at $(0.9, 0.1)$, the values reaches to $(516.37, 620.84)$ which is almost individual best and worst solution of each objectives respectively.

It is concluded that with the increase(decrease) in the weights of the first(second) objective function, the values are reaching their best(worst) solution.

The solution results obtained by using the hyperbolic membership function has been interpreted as follows:

In model 1; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(358.03, 603.72)$, while at $(0.5, 0.5)$, the values are $(398.01, 606.38)$ and at $(0.9, 0.1)$, the objective values reaches to $(435.97, 609.68)$ respectively.

As the assigned weight is increasing for the first objective, the value of the first objective is also increasing and reaching its best solution while at the same time the assigned weight is decreasing for the second objective, the value of the second objective is also increasing and reaching to its worst solution.

In model 2; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(358.03, 603.72)$, while at $(0.5, 0.5)$, the values are $(397.88, 606.39)$ and at $(0.9, 0.1)$, the objective values reaches to $(435.15, 609.70)$. It is noted that with the increase(decrease) in the weights of first(second) objective function, the values are approaching towards its best(worst) solution.

In model 3; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(359.41, 603.80)$, while at $(0.5, 0.5)$, the values are $(398.09, 606.40)$ and at $(0.9, 0.1)$, the objective values reaches to $(434.42, 609.62)$. It is concluded that with the increase(decrease) in the weights of first(second) objective function, the values are reaching towards its best(worst) solution.

In model 4; at weights $(0.1, 0.9)$ assigned to first and second objective function, the value of each objectives are $(360.72, 603.80)$, while at $(0.5, 0.5)$, the values are $(398.94, 606.40)$ and at $(0.9, 0.1)$, the values reaches to $(434.69, 609.55)$. It is observed that with the increase(decrease) in the weights of first(second) objective function, the values are tending towards its best(worst) solution.

2. Based on the performance of hesitant fuzzy aggregation operators

To develop our computational scheme, we have used two distinct hesitant fuzzy aggregation operator families, namely; (a) weighted averaging and (b) weighted geometric. One can use any values of λ, but we have demonstrated results by considering $\lambda = 1$ and $\lambda = 2$ for both the families.

In the case of the linear membership function, the compromise solution obtained by using HFQWA is better than HFWA, *i.e.*, (HFQWA $\geq$ HFWA) and likewise, the performance of HFQWG is better than HFWG, *i.e.*, (HFQWG $\geq$ HFWG) for all different combination of weight assigned to both objectives. Whereas in the case of the hyperbolic membership function, the compromise solution obtained by using HFWA is better than HFQWA, *i.e.*, (HFWA $\geq$ HFQWA) and similarly, the performance of HFQWG is better than HFWG, *i.e.*, (HFQWG $\geq$ HFWG) for all different values of weight assigned to both objectives. Therefore, the best selection of aggregation operator depends on the decision-maker(s) choice because each aggregation operator is well enough to provide an efficient solution based on different preference criteria.

3. Comparison between the performance of linear and hyperbolic membership functions

Under hesitancy, the linear and hyperbolic membership function concept is a benefit because it allows the decision-maker(s) to seek opinions from different experts in any decision-making process. In the case of the linear membership function, after combining all four models, it can be observed that the best and worst solution value of the first objective is 516.37 and 184.47, whereas the best and worst solution value of the second objective is 599.23 and 620.84. Therefore, it provides a wide range of optimal solutions set by tuning the weight parameter assigned to each objective function.

For the hyperbolic membership function, the best and worst solution value (combining all four different models) of the first objective is 435.97 and 358.03, whereas the second objective's best and worst solution value is 603.72 and 609.70. It comparatively (comparison to linear membership) a narrow range of the optimal solutions set for both the objective functions by tuning the weight parameters. Therefore, the use of linear membership function under hesitancy provides more flexibility to generate the different optimal solutions and covers a wide range of decision-making solution spectrum, which offers an opportunity to the decision-maker(s) to select the most desired set compared to hyperbolic membership function.

Comparative study with other existing approaches

The plausible solution of the experimental study presented in Section 4 can also obtain using three other criteria, namely; *Zimmerman's technique,* γ*-operator* and *Min. bounded sum operator* (see [24]). The optimal compromise solution using these three methods are $(409.70, 607.28)$, $(288.86, 599.64)$, and $(416.58, 607.88)$, respectively. All these three solutions set also lie in the same coverage spectrum of solution set obtained by our proposed computational scheme with linear membership function with a combination of different values of weights. A variation on the weight parameters helps achieve a better solution compared to the other existing methods. Therefore, one can conclude that the proposed computational modeling scheme provides a generalized framework for getting different optimal solutions set by tuning the weight parameters. Apart from this specific contribution, the following points reveal the superiority of the proposed computational modeling approach over the method used by Singh and Yadav [24].

(1) To the best of our knowledge, the hesitant fuzzy aggregation operator has not been used for the global evaluation of the multiobjective optimization problem under a hesitant fuzzy environment. Therefore, this work filled this gap and laid down the new base of future research scope in the hesitant fuzzy domain.
(2) The representation of linear and hyperbolic membership function under hesitancy for marginal evaluation of different objective functions provides an opportunity for the decision-maker(s) to express his/her(their) degree of hesitation effectively. It also provides a more general framework for the involvement of different experts' opinions in the decision-making process. The method used in [24] does not consider these vantages.
(3) The proposed computational models offer an opportunity of selecting the most desired/preferred optimal solutions set by tuning the weight parameter, whereas the method used in Singh and Yadav [24] has no such facility.

6. Conclusions

This study investigated a new optimal computational scheme for solving a multiobjective optimization problem in a hesitant fuzzy environment. The proposed scheme comprises different hesitant fuzzy aggregation operators for obtaining the global solution of the multiobjective optimization problem. Under hesitancy, the utilization of linear and hyperbolic membership functions provides a practical framework and computes the marginal evaluation of decision-maker(s) satisfaction and the different experts' opinions. It also enables the decision-maker(s) to execute his/her strategy to generate the most desired solutions. The sensitivity analysis for the optimal solution has been performed by adjusting different weight parameters, resulting in conflicting optimal solutions. Therefore, the proposed scheme would be helpful as it allows the active involvement of different experts' opinions, especially when the decision-maker(s) is(are) not sure about the values of parameters and having some confusion in implementing his/her strategy directly in any decision-making process.

6.1. Supplementary material

Remark 6.1.

$$\text{HFWA}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) = \sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} (1-\alpha_{kn})^{w_n}\right). \tag{6.1}$$

The above equation (6.1) can be lower bounded as follows:

$$\text{HFWA}\left(\mu_A^{E_1}, \mu_A^{E_2}, \ldots, \mu_A^{E_n}\right) \geq \sum_{k=1}^{K} W_k (1 - (1 + CM_{k_0})^{w_N}) \tag{6.2}$$

where, $C = \sum_{i=1}^{N} c_i$ and $c_1 = \sum_{n=1}^{N} (-1)^n \delta_n$, $c_2 = \sum_{i,j=1, i\neq j}^{N} \delta_i \delta_j$, …, $c_N = (-)^n \delta_1 \delta_2 \ldots \delta_N$.

Proof. Considering the fact that

$$\mu_A^{E_n}(Z_k(x)) \geq \alpha_{kn}$$

and For a fixed $k = k_0$ one can rewrite the above relation as:

$$\mu_A^{E_n}(Z_{k_0}(x)) \geq \alpha_{k_0 n}.$$

It is clearly shown that

$$-\alpha_{k_0 n} \geq -\mu_A^{E_n}(Z_{k_0}(x)).$$

Therefore,

$$\prod_{n=1}^{N} (1-\alpha_{k_0 n})^{w_n} \geq \prod_{n=1}^{N} \left(1 - \mu_A^{E_n}(Z_{k_0}(x))\right)^{w_n}. \tag{6.3}$$

The above equations (6.3) can be generalized as

$$\sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} (1-\alpha_{k_0 n})^{w_n}\right) \geq \sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} \left(1 - \mu_A^{E_n}(Z_{k_0}(x))\right)^{w_n}\right). \tag{6.4}$$

Now opening the each product term from the right hand side of the equation (6.4), one can write as follows:

$$\begin{aligned}\prod_{n=1}^{N} \left(1 - \mu_A^{E_n}(Z_{k_0}(x))\right)^{w_n} &= \left(1 - \mu_A^{E_1}(Z_{k_0}(x))\right)^{w_1} \left(1 - \mu_A^{E_2}(Z_{k_0}(x))\right)^{w_2} + \ldots + \left(1 - \mu_A^{E_N}(Z_{k_0}(x))\right)^{w_N} \\ &= \left(\left(1 - \mu_A^{E_1}(Z_{k_0}(x))\right) \left(1 - \mu_A^{E_2}(Z_{k_0}(x))\right) + \ldots + \left(1 - \mu_A^{E_N}(Z_{k_0}(x))\right)\right)^{w_N}.\end{aligned}$$

Here we have considered, equal weights *i.e.*, $w_1 = w_2 = \ldots = w_N$

The above equation (6.5) finally simplified as

$$\begin{aligned}\prod_{n=1}^{N} \left(1 - \mu_A^{E_n}(Z_{k_0}(x))\right)^{w_n} &= ((1 + (-\delta_1 M_{k_0}))\ (1 + (-\delta_2 M_{k_0})) \ldots (1 + (-\delta_N M_{k_0})))^{w_N} \\ &= \left(1 + \sum_{n=1}^{N} (-1)^n (\delta_i M_{k_0}) + \sum_{n=1}^{N} (-1)^{2n} (\delta_i \delta_j M_{k_0}) + \sum_{n=1}^{N} (-1)^{2n+1} (\delta_i \delta_j \delta_k M_{k_0}) + \ldots \right. \\ &\quad \left. + \sum_{n=1}^{N} (-1)^n (\delta_i \delta_j \delta_k \ldots \delta_N M_{k_0})\right)^{w_N} \\ &= (1 + (c_1 + c_1 + \ldots + c_N) M_{k_0})^{w_N} = (1 + C\ M_{k_0})^{w_N}.\end{aligned}$$

Here we have defined $C = c_1 + c_2 + \ldots + c_N$ and $\delta_i = \delta_1\delta_2 \ldots \delta_N$. Finally,

$$\sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} \left(1 - \mu_A^{E_n}(Z_{k_0}(x))\right)\right)^{w_n} = \sum_{k=1}^{K} W_k(1 - (1 + C\ M_{k_0})^{w_N}). \tag{6.5}$$

Combining equations (6.4) and (6.5), we can state that,

$$\sum_{k=1}^{K} W_k \left(1 - \prod_{n=1}^{N} (1 - \alpha_{k_0 n})^{w_n}\right) \geq \sum_{k=1}^{K} W_k(1 - (1 + CM_{k_0})^{w_N}).$$

□

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