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THE IMPLICATIONS OF SUPPLIER ENCROACHMENT *VIA* AN ONLINE PLATFORM

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Abstract. E-commerce provides suppliers with the flexibility to operate an online arm *via* a platform in addition to their pre-existing physical stores. Although such supplier encroachment is becoming increasingly prevalent in e-commerce markets, the literature on supplier encroachment traditionally assumes that suppliers sell products to consumers directly and argues that supplier encroachment can mitigate double marginalisation problems that can secure Pareto improvements. This paper narrows this gap by investigating the implications of the supplier encroachment with an online platform under two scenarios (*i.e.*, the platform owner forgoing or retaining its entry options). A central result obtained is that, unlike supplier-owned direct channels and in addition to the "win–win" outcomes for the supplier and the traditional retailer, supplier encroachment with an online platform may also lead to "win–lose" and "lose–lose" outcomes. Furthermore, when the platform owner retains its entry option, such encroachment is always detrimental for the traditional retailer but beneficial for the supplier.

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1. Introduction

Suppliers can encroach on the retail market in different ways, including supplier outlets, company-owned stores, catalogue sales, telemarketing, and online sales [3]. Over the last decade, with the development of ever-more-sophisticated logistics and the rise of information technology [18], online retail platforms (for example, eBay, Amazon's Marketplace and OLX.in) have become major engines of growth in e-commerce [14]. As a result, a growing number of suppliers have found it attractive to supplement their preexisting retail channels with an online platform channel. For example, many brand-name suppliers, including Apple, Samsung, IBM, and Lenovo, have sold their products on Amazon as well as through stores such as Best Buy and Circuit City. Similarly, among independent sellers on the Alibaba Group's platform (`http://www.taobao.com`; referred to as Taobao for the rest of the paper) can be found many well-known suppliers in a variety of industries, such as Burberry (apparel), HP (electronics and appliances) and Audi (automobiles), who have opened flagship company stores on this online platform.

Keywords. Operations, E-commerce, retail, distribution channels, game theory.

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Supplier encroachment with an online platform occurs in two basic forms. In the first, online retail platforms, such as Taobao, offer suppliers or sales agents direct access to consumers for a fee, while the platform owner forgoes its entry option. In the second, the platform owner retains the option to resell the products from the supplier with an additional allowance for the supplier or sale agent, such as Amazon. In the last decade, these selling formats have become pervasive, facilitated by online retail platforms [27]. For instance, Amazon directly sells 7% of the products in its electronics category, and the remaining 93% of the products are sold by independent sellers [20]. In the fourth quarter of 2016, third-party transactions accounted for 49% of all unit sales on Amazon [35]. This scenario is similar for Taobao. As the world's largest online consumer-to-consumer platform based in China, Taobao has over seven million sellers less than a decade after its establishment in 2003 [14].

Numerous researchers [3,8,11,17,40] have addressed supplier encroachment on the assumption that suppliers sell products through a supplier-owned direct channel, thus ignoring the role played by the online platform and its impact on the traditional retailer, online pricing strategies and profits. Moreover, it has long been recognised that supplier encroachment has strategic consequences on sustainable operations. When the supplier encroaches on the retail market, she has an incentive to reduce the wholesale price to not unduly diminish the traditional retailer's demand. As a result, supplier encroachment can create a "win–win" outcome for the supplier and the traditional retailer [3, 24, 36, 40]. However, the online platform introduces another dimension to the relationship between suppliers and traditional retailers that needs to be addressed. Specifically, if the supplier sells her products through an online platform, the platform owner not only determines its per-unit fee but also ascertains whether to forgo its entry option or not. Furthermore, if the platform owner retains its entry option, then the supplier needs to determine wholesale prices for the traditional retailer and the platform owner, respectively.

Although there has been a rich body of work on platform-based markets, beginning with Rochet and Tirole [32], prior work in this area has primarily focused on platform competition [1,6,21], pricing structure [4,20,25,37] and strategic decisions [5, 14, 19, 31] and less on the interaction between online retail platforms and offline traditional retail channels. This interaction poses several questions that differ from those around supplier-owned channels and traditional retail channels. For example, when facing supplier encroachment with an online platform, a retailer in the traditional channel may be in a more challenging position than when confronting a supplier-owned channel. This is because he is not only essentially competing with the products from the supplier, but also with the products that are resold by the online platform.

In this paper, the focus is on supplier encroachment *via* an online platform. Specifically, a model is developed describing how a supplier encroaches on the retail market by selling products through an online retail platform. We intend to answer the following questions: What are the implications of supplier encroachment with an online platform for the supplier, the traditional retailer and the platform owner? In particular, when the platform owner retains its entry option, should the supplier encroach on the retail market? Finally, how does supplier encroachment with an online platform create strategic issues that differ significantly from those with supplier-owned channels?

These strategic interactions are studied and novel insights into supplier encroachment are provided. The main result is in contrast to those of Arya *et al.* [3]: in addition to "win–win" outcomes for the supplier and traditional retailer, supplier encroachment with an online platform may also lead to "win–lose" and "lose–lose" outcomes. In particular, when the platform owner retains its entry option, supplier encroachment is always beneficial to the supplier but detrimental to the traditional retailer; as a result, "win–lose" is the only possible outcome for the supplier and traditional retailer.

The model developed contributes to the literature in several important ways. First, an aspect of supplier encroachment mostly ignored by the extant research is addressed. Unlike prior studies, which examine scenarios in which the supplier encroaches on the retail market with a supplier-owned direct channel, this study highlights the fact that suppliers can encroach on the retail market by selling products through an online platform. It focuses on the strategic issues created by supplier encroachment with an online platform, which differ significantly from those resulting from a supplier-owned channel. Second, although the question of whether supplier encroachment results in "internet channel conflict" or brings Pareto gains to both parties has been well studied in dual-channel supply chains, aside from the traditional retailer's cost advantage, there is limited knowledge about how the role played by online platforms can affect these results. Third, although there is a rich body of work on platform-based markets, limited attention has been paid to the interaction between online retail platforms and offline traditional retail channels. Therefore, this paper sheds new light on a setting where a supplier encroaches on the retail market by selling products through an online platform – an increasingly prevalent occurrence in e-commerce markets.

The remainder of the paper is organised as follows. Section 2 reviews related literature and explains the contributions of this paper more thoroughly. Section 3 describes the key elements of four models. Section 4 presents our central findings. Section 5 extends the analysis to the simultaneous setting. Section 6 concludes the paper.

2. Related literature

This work lies at the intersection of supplier encroachment and platform-based business models, where the former mainly focuses on the implications of the supplier-owned direct channel on the supplier and traditional retailer, and the latter examines various aspects associated with platform-based markets grounded in the literature on industrial economics and two-sided markets. To clearly delineate the contributions of this paper, each of these literature streams is discussed.

Most research on supplier encroachment has taken one of two familiar approaches. The first emphasises that supplier encroachment can lead to a "win–win" outcome for the supplier and the traditional retailer (see, *e.g.*, [7, 13, 38, 40]). In particular, Arya *et al.* [3] show that supplier encroachment not only creates additional profits for the supplier, but also motivates her to set a lower wholesale price in the encroachment setting than in the no-encroachment setting to offset the advantage that the supplier's retail arm secures. As a result, supplier encroachment should create a "win–win" outcome for the supplier and the traditional retailer because it induces lower wholesale prices to support the wholesale demand of the weakened incumbent retailer. It should be noted that the pioneering literature generally assumes that the supplier encroaches into the retail market through a supplier-owned direct channel. However, platform-based business models have become a major engine of growth in e-commerce [14]. As mentioned earlier, from a research perspective, the forms of these platform-based business models are quite different from those of supplier-owned direct channels. For example, platform owners allow suppliers or sales agents direct access to consumers, charging them only for value-added services. Moreover, with the addition of an allowance for the supplier or sale agent, the online retail platforms may retain the option to resell the products from the supplier. As such, we extend the analysis in Arya *et al.* [3] to address the implications of supplier encroachment *via* an online platform on the traditional retailer, online pricing strategies, and profits. As noted by Arya *et al.* [3], when the supplier encroaches on the retail market by selling products directly, she has an incentive to reduce the wholesale price to avoid unduly diminishing the traditional retailer's demand for the supplier's wholesale products, which can lead to a "win–win" outcome for both parties. Our analysis demonstrates, however, that it is not always true under the supplier encroachment scenario with an online platform. In particular, when the platform owner is endowed with the option of entering the market, the supplier encroachment is always beneficial for the supplier but detrimental to the traditional retailer.

This research stream has recently moved beyond these initial fundamental analyses and considered how the robustness of the "win–win" outcome changes with some assumptions. For example, a few studies have extended supplier encroachment to scenarios with asymmetric information [23], nonlinear pricing [24], and endogenous quality [12, 17], revealing that, under certain conditions, supplier encroachment may always be detrimental to the traditional retailer and may lead to a "lose–lose" situation for the supplier and traditional retailer. Our work complements this stream of research by addressing supplier encroachment with an online platform, leading to results that differ from the current literature. They indicate that there is an alliance between the supplier and the online platform when the platform owner retains its encroachment option.

The second research stream is related to the literature on platform-based markets, which is grounded in the literature on industrial economics and two-sided markets [28, 34]. Although there is a rich body of work on platform economies and two-sided markets – beginning with Rochet and Tirole [32] – prior work on online retail platforms has primarily focused on platform competition [1, 6, 21], pricing structure [4, 20, 25, 37] and strategic decisions by platforms beyond pricing [5, 14, 19, 31]. They have focused less on the interaction between online retail platforms and offline traditional retail channels. The current work complements this stream of research by investigating the implications of supplier encroachment with an online platform under two scenarios (*i.e.*, the platform owner forgoing or retaining its entry options). Specifically, we develop two basic forms of supplier encroachment with an online platform. In the first, the supplier encroaches on the retail market *via* an online platform who forgoes its option to resell the products from the supplier. In the second, the platform owner allows the supplier access to consumers through the online retail platform for a fee, while retaining its entry option.

3. The model

This section introduces the notation and lays out assumptions regarding the four models for the product, supplier, traditional retailer, platform owner, and consumers. We describe each in detail below.

3.1. Formulation

As shown in Figure 1, we set up four models. First, we established Model N and Model EM with the platform owner forgoing its entry option. In Model N (benchmark case), all products are distributed only through the traditional retail channel. In Model EM, the supplier encroaches on the retail market through the online platform. Subsequently, we provided Model ET and Model EB with the platform owner retaining its entry option. In Model ET, the supplier wholesales products to the traditional retailer and platform owner without selling the product directly to consumers. However, in Model EB, both the supplier and platform owner encroach on the traditional retailer's market.

Let π_i^j represent the profit of party i under model j, where $i = s, r, p$ refers to the supplier, traditional retailer, and platform owner, while $j =$ N, EM, ET, EB refers to Model N, Model EM, Model ET and Model EB, respectively. The timeline is as follows (in Fig. 2): the supplier announces the wholesale price to the traditional retailer and platform owner simultaneously (when the platform retains its entry option), then the traditional retailer responds by determining the optimal units. Subsequently, the platform owner selects a per-unit fee for each sale of the supplier, and then it announces the optimal units sold to consumers (when the platform retains its entry option). Meanwhile, the supplier chooses the optimal quantities of products to be sold on the online platform[1].

Like Arya *et al.* [3] and Xiong *et al.* [40], we adopt a single market-clearing price to preclude price discrimination. That is, we assume that consumers have no preference between the online retail platform and traditional retail channel, and their demand for the product is represented by a linear, downward-sloping (inverse) demand, $p = 1 - Q$, where p and $Q = q_r + q_s$ are the price and quantities of the product, respectively.

We let c_p and c_r reflect the differences in selling costs of online and traditional channels, which are uniformly distributed in the interval of $[0, 1]$. Note that, if $c_r < c_p$, the traditional retailer's selling costs may be less compared to the online platform, so it has an advantage in product distribution; otherwise, the opposite is true[2]. Such a difference in selling costs has been widely adopted in the literature in marketing to reflect the level of competition between both channels [3, 9, 17].

In practice, to compete with traditional retailers, many major online platforms, such as Amazon and JD.com, have invested heavily in their marketing efforts (*e.g.*, using novel virtual reality technologies, social and online advertising, and attractive delivery times) that alleviate the consumers' disutility from online transactions.

[1]This consequence of events is consistent with the assumption of supplier encroachment *via* online channel in Arya *et al.* [3]. And this assumption is relaxed in Section 5.

[2]We thank an anonymous reviewer for pointing this out.

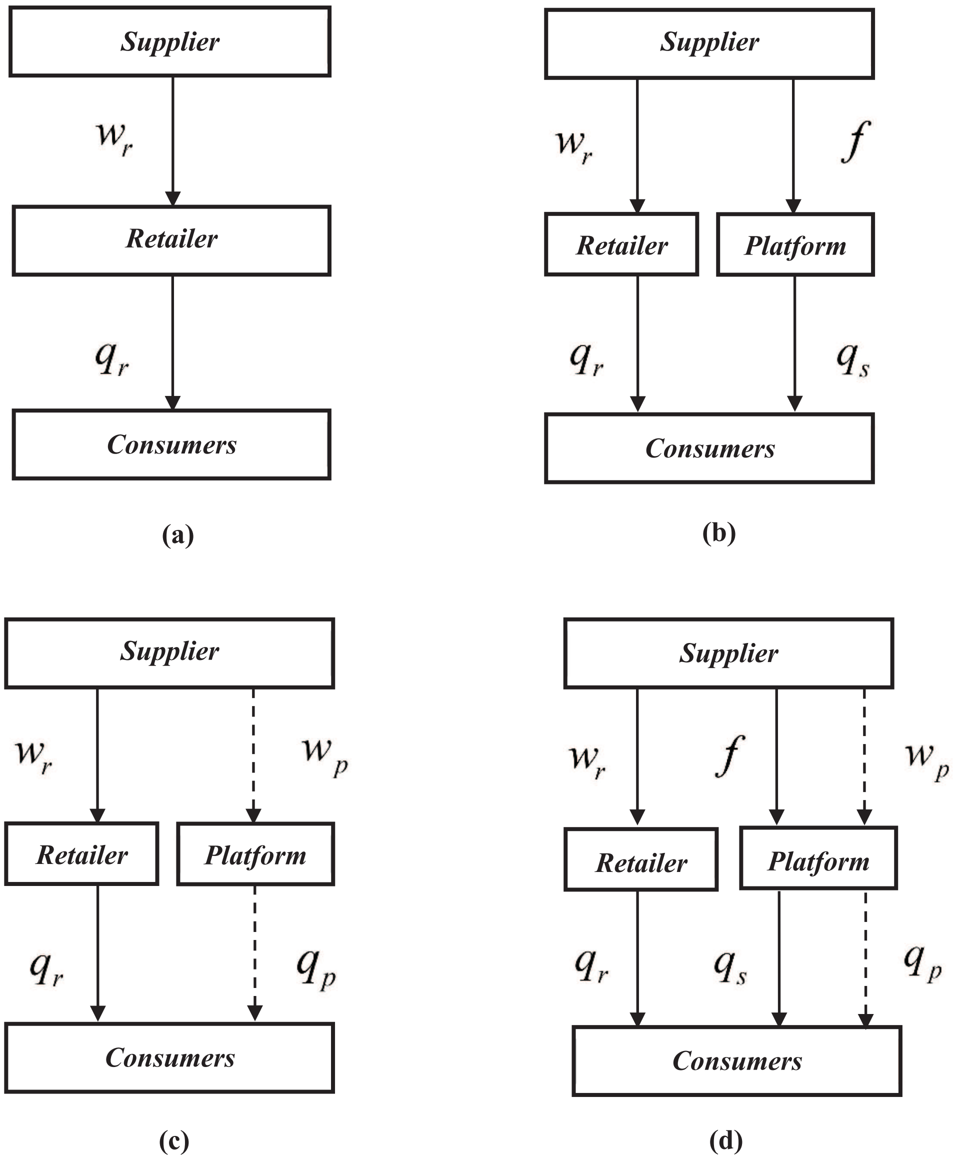


FIGURE 1. Four basic models for supplier encroachment with an online platform. (a) Model N. (b) Model EM. (c) Model ET. (d) Model EB.

Thus, the platform owner's performance is modelled as a function of the demand for products (*i.e.*, q_s) that is influenced by its marketing effort, denoted by I, the investment in marketing efficiency. To characterise the diminishing returns on investment, similar to Savaskan *et al.* [33] and Jiang *et al.* [20], the cost structure $q_s = \sqrt{I/k}$ is used, where k is a scaling parameter that shows the marketing efficiency for the platform owner to give consumers an incentive or enthusiasm for purchasing products in its marketplace. To differentiate from cross-investment to other partners, it is assumed that the platform owner can effectively influence its consumers through a self-investment, that is, $k \in (0, 1)$ [2, 16, 39]. It is natural to expect that, as the scaling parameter k decreases, the online platform is more efficient in marketing a product, as the investment in finding effective

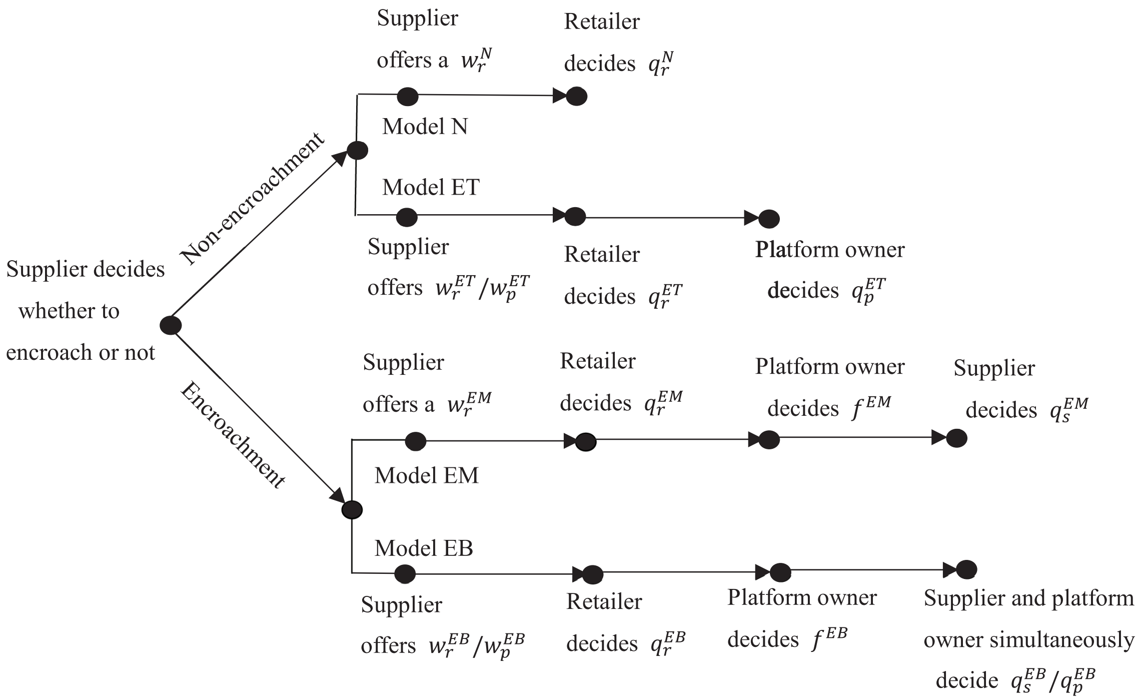


FIGURE 2. Timeline in the sequential setting.

demand or providing a service level is relatively small. Similar forms of response functions have widely been used in salesforce effort response models in the marketing literature [10, 15, 22, 29].

3.2. Equilibrium

3.2.1. Model N (Benchmark without encroachment)

Here, we establish a benchmark-setting with no online retail platform channel, where all the products are distributed only through the traditional retail channel (Model N). From the equation $p = 1 - Q$, the demand with the no encroachment setting is $p = 1 - q_r^{\mathrm{N}}$. In this model, the decisions for the supplier and traditional retailer are as follows.

$$\pi_s^{\mathrm{N}} = w_r^{\mathrm{N}} q_r^{\mathrm{N}} \tag{3.1}$$

$$\pi_r^{\mathrm{N}} = \left(1 - q_r^{\mathrm{N}} - w_r^{\mathrm{N}} - c_r\right) q_r^{\mathrm{N}}. \tag{3.2}$$

We use backward induction to solve this game. Following the timeline of Model N in Figure 1, for any given w_r^{N}, the traditional retailer first chooses the equilibrium output q_r^{N} to maximise its profit $\pi_r^{\mathrm{N}}(w_r^{\mathrm{N}}, q_r^{\mathrm{N}})$:

$$q_r^{\mathrm{N}^*}\left(w_r^{\mathrm{N}}\right) = \frac{1 - w_r^{\mathrm{N}} - c_r}{2}. \tag{3.3}$$

Substituting $q_r^{\mathrm{N}^*}\left(w_r^{\mathrm{N}}\right)$ into equation (3.1), the supplier determines the wholesale price to the traditional retailer w_r^{N} to maximise its profit $\pi_s^{\mathrm{N}}\left(w_r^{\mathrm{N}}, q_r^{\mathrm{N}^*}\left(w_r^{\mathrm{N}}\right)\right)$. Then, we can obtain the equilibrium wholesale prices, demand and profits, as summarised in Lemma 3.1[3].

[3]For clarity, all proofs are provided in the Appendix A.

Lemma 3.1. *Under Model N, we have:*

(i) *The equilibrium wholesale prices and quantities are:*

$$q_r^{\mathrm{N}^*} = \frac{1-c_r}{4}, w_r^{\mathrm{N}^*} = \frac{1-c_r}{2}\cdot$$

(ii) *The equilibrium profits of the supplier and traditional retailer are:*

$$\pi_s^{\mathrm{N}^*} = \frac{(1-c_r)^2}{8}, \pi_r^{\mathrm{N}^*} = \frac{(1-c_r)^2}{16}\cdot$$

3.2.2. Model EM

In Model EM, the supplier would encroach into the retail market. The demand under this scenario is $p = 1 - q_r^{\mathrm{EM}} - q_s^{\mathrm{EM}}$. Therefore, the profits for the supplier, traditional retailer and platform are as follows:

$$\pi_s^{\mathrm{EM}} = \left(1 - q_r^{\mathrm{EM}} - q_s^{\mathrm{EM}} - f^{\mathrm{EM}}\right)q_s^{\mathrm{EM}} + w_r^{\mathrm{EM}} q_r^{\mathrm{EM}} \tag{3.4}$$

$$\pi_r^{\mathrm{EM}} = \left(1 - q_r^{\mathrm{EM}} - q_s^{\mathrm{EM}} - w_r^{\mathrm{EM}} - c_r\right)q_r^{\mathrm{EM}} \tag{3.5}$$

$$\pi_P^{\mathrm{EM}} = \left(f^{\mathrm{EM}} - c_p\right)q_s^{\mathrm{EM}} - k{q_s^{\mathrm{EM}}}^2. \tag{3.6}$$

We use backward induction to solve this game. Following the timeline of Model EM in Figure 1, for any given w_r^{EM}, q_r^{EM} and f^{EM}, the supplier first chooses the direct sales to consumers q_s^{EM} to maximise its profit $\pi_s^{\mathrm{EM}}(w_r^{\mathrm{EM}}, q_r^{\mathrm{EM}}, f^{\mathrm{EM}}, q_s^{\mathrm{EM}})$:

$$q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right) = \frac{1 - q_r^{\mathrm{EM}} - f^{\mathrm{EM}}}{2}\cdot \tag{3.7}$$

When substituting $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ into equation (3.6), the platform owner determines the commission fee f^{EM} to maximise its profit $\pi_P^{\mathrm{EM}}\left(f^{\mathrm{EM}}, q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)\right)$:

$$f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right) = \frac{c_p + k - q_r^{\mathrm{EM}} - kq_r^{\mathrm{EM}} + 1}{k+2}\cdot \tag{3.8}$$

Then, when substituting $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ and $f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right)$ into equation (3.5), the traditional retailer chooses equilibrium quantities q_r^{EM} to maximise its profit $\pi_r^{\mathrm{EM}}\left(w_r^{\mathrm{EM}}, q_r^{\mathrm{EM}}, q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)\right)$:

$$q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}\right) = \frac{3 + c_p - 4c_r + 2k - 2c_rk - 4w_r^{\mathrm{EM}} - 2kw_r^{\mathrm{EM}}}{2(3+2k)}\cdot \tag{3.9}$$

When substituting $q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}\right)$, $f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right)$, and $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ into Equation (3.4), the supplier determines the wholesale price w_r^{N} to maximise its profit $\pi_s^{\mathrm{EM}}\left(w_r^{\mathrm{EM}}, q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}\right), f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right), q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)\right)$. Then, we can obtain the equilibrium solutions, which are summarised in Lemma 3.2.

Lemma 3.2. *Under Model* EM*:*

(i) *The equilibrium wholesale prices, quantities, and commission fee are:*

$$w_r^{\mathrm{EM}^*} = \frac{5c_p - 44c_r + 68k + 10c_pk - 78c_rk + 4c_pk^2 - 44c_rk^2 - 8c_rk^3 + 40k^2 + 8k^3 + 39}{16k^3 + 88k^2 + 158k + 92}$$

$$q_r^{\mathrm{EM}^*} = \frac{3c_p - 8c_r + 7k + c_pk - 8c_pk - 2c_rk^2 + 2k^2 + 5}{8k^2 + 28k + 23}$$

$$f^{\mathrm{EM}^*} = \frac{10c_p + 4c_r + 15k + 7c_pk + 6c_rk + 2c_rk^2 + 6k^2 + 9}{8k^2 + 28k + 23}$$

$$q_s^{\mathrm{EM}^*} = \frac{4c_r - 13c_p + 6k - 8c_pk + 2c_rk + 9}{2(8k^2 + 28k + 23)}\cdot$$

(ii) *The equilibrium profits of the supplier, traditional retailer, and platform owner are:*

$$\pi_s^{\mathrm{EM}^*} = \frac{\begin{bmatrix} 9c_p^2k + 16c_p^2 - 4c_pc_rk^2 - 20c_pc_rk - 24c_pc_r + 4c_pk^2 + 2c_pk - 8c_p + 4c_r^2k^3 + 24c_r^2k^2 \\ +48c_r^2k + 32c_r^2 - 8c_rk^3 - 44c_rk^2 - 76c_rk - 40c_r + 4k^3 + 20k^2 + 37k + 24 \end{bmatrix}}{4(8k^3 + 44k^2 + 79k + 46)}$$

$$\pi_r^{\mathrm{EM}^*} = \frac{(2k+3)(3c_p - 8c_r + 7k + c_pk - 8c_rk - 2c_rk^2 + 2k^2 + 5)^2}{2(8k^2 + 28k + 23)(8k^3 + 44k^2 + 79k + 46)}$$

$$\pi_p^{\mathrm{EM}^*} = \frac{(k+2)(4c_r - 13c_p + 6k - 8c_pk + 2c_rk + 9)^2}{4(8k^2 + 28k + 23)^2}.$$

3.2.3. Model ET

In Model ET, the supplier wholesales products to the traditional retailer and platform owner without encroaching into the retail market. The demand under this scenario is $p = 1 - q_r^{\mathrm{ET}} - q_P^{\mathrm{ET}}$, where q_P^{ET} is the units sold by the platform owner. Thus, the profits for the supplier, traditional retailer and platform are as follows:

$$\pi_s^{\mathrm{ET}} = w_p^{\mathrm{ET}} q_p^{\mathrm{ET}} + w_r^{\mathrm{ET}} q_r^{\mathrm{ET}} \tag{3.10}$$

$$\pi_r^{\mathrm{ET}} = \left(1 - q_r^{\mathrm{ET}} - q_p^{\mathrm{ET}} - w_r^{\mathrm{ET}} - c_r\right) q_r^{\mathrm{ET}} \tag{3.11}$$

$$\pi_P^{\mathrm{EM}} = \left(1 - q_r^{\mathrm{ET}} - q_p^{\mathrm{ET}} - w_p^{\mathrm{ET}} - c_p\right) q_p^{\mathrm{ET}} - k q_p^{\mathrm{ET}^2}. \tag{3.12}$$

We use backward induction to solve this game. Following the timeline of Model ET in Figure 1, for any given w_r^{ET}, w_P^{ET} and q_r^{ET}, the platform owner first chooses the equilibrium quantities q_p^{ET} to maximise its profit $\pi_p^{\mathrm{ET}}(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}, q_p^{\mathrm{ET}})$:

$$q_p^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}\right) = \frac{1 - c_p - q_r^{\mathrm{ET}} - w_p^{\mathrm{ET}}}{2k+2}. \tag{3.13}$$

Then, when substituting $q_p^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}\right)$ into equation (3.11), the traditional retailer chooses equilibrium quantities q_r^{ET} to maximise its profit $\pi_r^{\mathrm{ET}}\left(w_r^{\mathrm{ET}}, q_r^{\mathrm{ET}}, q_p^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}\right)\right)$:

$$q_r^{\mathrm{ET}^*}\left(w_r^{\mathrm{ET}}, w_p^{\mathrm{ET}}\right) = \frac{c_p - 2c_r + 2k + w_p^{\mathrm{ET}} - 2w_r^{\mathrm{ET}} - 2c_rk - 2kw_r^{\mathrm{ET}} + 1}{4k+2}. \tag{3.14}$$

When substituting $q_p^{\mathrm{ET}^*}(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}})$ and $q_r^{\mathrm{ET}^*}(w_p^{\mathrm{ET}}, w_r^{\mathrm{ET}})$ into equation (3.10), the supplier determines the wholesale price to the traditional retailer w_r^{ET} and the platform owner w_P^{ET} to maximise its profit $\pi_s^{\mathrm{ET}}\left(w_p^{\mathrm{ET}}, w_r^{\mathrm{ET}}, q_p^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}\right), q_r^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, w_r^{\mathrm{ET}}\right)\right)$ simultaneously. Then, we can obtain the equilibrium wholesale prices, demands and profits, as summarised in Lemma 3.3.

Lemma 3.3. *Under Model* ET

(i) *The equilibrium wholesale prices and quantities are as follows:*

$$w_r^{\mathrm{ET}^*} = \frac{1 - c_r}{2}$$

$$w_P^{\mathrm{ET}^*} = \frac{1 - c_p}{2}$$

$$q_r^{\mathrm{ET}^*} = \frac{c_p - 2c_r + 2k - 2c_rk + 1}{8k+4}$$

$$q_p^{\mathrm{ET}^*} = \frac{2c_r - 3c_p + 2k - 4c_pk + 2c_rk + 1}{16k^2 + 24k + 8}.$$

(ii) *The equilibrium profits of the supplier, traditional retailer and platform owner are as follows:*

$$\pi_s^{\mathrm{ET}^*} = \frac{\begin{bmatrix} 4c_p^2k + 3c_p^2 - 4c_pc_rk - 4c_pc_r - 4c_pk - 2c_p + 4c_r^2k^2 + 8c_r^2k + 4cr^2 \\ -8c_rk^2 - 12c_rk - 4c_r + 4k^2 + 8k + 3 \end{bmatrix}}{16(2k^2+3k+1)}$$

$$\pi_r^{\mathrm{ET}^*} = \frac{(c_p - 2c_r + 2k - 2c_rk + 1)^2}{32(2k^2+3k+1)}$$

$$\pi_p^{\mathrm{ET}^*} = \frac{(2c_r - 3c_p + 2k - 4c_pk + 2c_rk + 1)^2}{64(2k+1)^2(k+1)}.$$

3.2.4. Model EB

In Model EB, both the supplier and platform owner encroach into the traditional retailer's market. The inverse demand function for the products is $p = 1 - q_r^{\mathrm{EB}} - q_s^{\mathrm{EB}} - q_p^{\mathrm{EB}}$. Thus, under this scenario, the supplier, traditional retailer and platform owner's decisions are as follows, respectively:

$$\pi_s^{\mathrm{EB}} = \left(1 - q_r^{\mathrm{EB}} - q_s^{\mathrm{EB}} - q_p^{\mathrm{EB}} - f^{\mathrm{EB}}\right)q_s^{\mathrm{EB}} + w_r^{\mathrm{EB}}q_r^{\mathrm{EB}} + w_p^{\mathrm{EB}}q_p^{\mathrm{EB}} \tag{3.15}$$

$$\pi_r^{\mathrm{EB}} = \left(1 - q_r^{\mathrm{EB}} - q_s^{\mathrm{EB}} - q_p^{\mathrm{EB}} - w_r^{\mathrm{EB}} - c_r\right)q_r^{\mathrm{EB}} \tag{3.16}$$

$$\pi_P^{\mathrm{EB}} = \left(1 - q_r^{\mathrm{EB}} - q_s^{\mathrm{EB}} - q_p^{\mathrm{EB}} - w_P^{\mathrm{EB}} - c_p\right)q_P^{\mathrm{EB}} + \left(f^{\mathrm{EB}} - c_p\right)q_s^{\mathrm{EB}} - k\left(q_s^{\mathrm{EB}} + q_P^{\mathrm{EB}}\right)^2. \tag{3.17}$$

We use backward induction to solve this game. Following the timeline of Model EB in Figure 1, for any given w_r^{EB}, w_p^{EB}, q_r^{EB} and f^{EB}, the supplier first chooses the direct sales to consumers q_s^{EB} to maximise its profit $\pi_s^{\mathrm{EB}}\left(w_r^{\mathrm{EB}}, w_p^{\mathrm{EB}}, q_r^{\mathrm{EB}}, f^{\mathrm{EB}}, q_p^{\mathrm{EB}}, q_s^{\mathrm{EB}}\right)$. Further, the platform owner determines the equilibrium quantities q_p^{EB} to maximise its profit $\pi_P^{\mathrm{EB}}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}, q_P^{\mathrm{EB}}, q_s^{\mathrm{EB}}\right)$ simultaneously:

$$q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right) = \frac{1 + c_p - 2f^{\mathrm{EB}} + 2k - q_r^{\mathrm{EB}} + w_P^{\mathrm{EB}} - 2f^{\mathrm{EB}}k - 2kq_r^{\mathrm{EB}}}{2k+3} \tag{3.18}$$

$$q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right) = \frac{1 - 2c_p + f^{\mathrm{EB}} - 2k - q_r^{\mathrm{EB}} - 2w_P^{\mathrm{EB}} + 2f^{\mathrm{EB}}k + 2kq_r^{\mathrm{EB}}}{2k+3}. \tag{3.19}$$

When substituting $q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ and $q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ into equation (3.17), the platform owner determines the commission fee f^{EM} to maximise its profit $\pi_P^{\mathrm{EB}}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}, q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right), q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)\right)$:

$$f^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right) = \frac{5 - 5q_r^{\mathrm{EB}} + 14k + 5c_p - w_P^{\mathrm{EB}} + 4c_pk - 14kq_r^{\mathrm{EB}} - 6kw_P^{\mathrm{EB}} - 8k^2q_r^{\mathrm{EB}} - 4k^2w_P^{\mathrm{EB}} + 8k^2}{8k^2+18k+10}. \tag{3.20}$$

Then, when substituting $q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, $q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ and $f^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ into equation (3.16), the traditional retailer chooses equilibrium quantities q_r^{EB} to maximise its profit $\pi_r^{\mathrm{EB}}\left(w_r^{\mathrm{EB}}, q_r^{\mathrm{EB}}, q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right), q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)\right)$:

$$q_r^{\mathrm{EB}^*}\left(w_r^{\mathrm{EB}}, w_P^{\mathrm{EB}}\right) = \frac{5c_p - 10c_r + 14k + 3w_P^{\mathrm{EB}} - 10w_r^{\mathrm{EB}} + 4c_pk - 18c_rk + 2kw_P^{\mathrm{EB}} - 18kw_r^{\mathrm{EB}} - 8c_rk^2 - 8k^2w_r^{\mathrm{EB}} + 8k^2 + 5}{16k^2+28k+10}. \tag{3.21}$$

When substituting $q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, $q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, $f^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ and $q_r^{\mathrm{EB}^*}\left(w_r^{\mathrm{EB}}, w_P^{\mathrm{EB}}\right)$ into equation (3.15), the supplier determines the wholesale price w_r^{EB}

and $w_p^{\rm EB}$ for the traditional retailer and platform owner to maximise its profit $\pi_s^{\rm EB}\big(w_r^{\rm EB}, w_p^{\rm EB}, q_r^{\rm EB^*}\big(w_r^{\rm EB}, w_P^{\rm EB}\big), f^{\rm EB^*}\big(w_P^{\rm EB}, q_r^{\rm EB}\big), q_p^{\rm EB^*}\big(w_P^{\rm EB}, f^{\rm EB}, q_r^{\rm EB}\big), q_s^{\rm EB^*}\big(w_P^{\rm EB}, f^{\rm EB}, q_r^{\rm EB}\big)\big)$. Then, we can obtain the equilibrium wholesale prices, commission fees, demands and profits, as summarised in Lemma 3.4.

Lemma 3.4. *Under Model* EB*:*

(1) *The equilibrium wholesale prices, quantities, and commission fee are:*

$$w_r^{\rm EB^*} = \frac{\begin{bmatrix} 9c_p - 98c_r + 358k + 56c_pk - 414c_rk + 80c_pk^2 + 32c_pk^3 - 568c_rk^2 \\ -320c_rk^3 - 64c_rk^4 + 488k^2 + 288k^3 + 64k^4 + 89 \end{bmatrix}}{4(32k^4 + 160k^3 + 287k^2 + 214k + 53)}$$

$$w_p^{\rm EB^*} = \frac{\begin{bmatrix} 10c_r - 55c_p + 161k - 189c_pk + 28c_rk - 196c_pk^2 \\ -64c_pk^3 + 26c_rk^2 + 8c_rk^3 + 170k^2 + 56k^3 + 45 \end{bmatrix}}{2(32k^4 + 160k^3 + 287k^2 + 214k + 53)}$$

$$q_r^{\rm EB^*} = \frac{\begin{bmatrix} 32c_p - 54c_r + 99k + 63c_pk - 162c_rk + 40c_pk^2 + 8c_pk^3 \\ -180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 + 22 \end{bmatrix}}{2(32k^4 + 287k^2 + 160k^3 + 53 + 214k)}$$

$$f^{\rm EB^*} = \frac{\begin{bmatrix} 85c_p + 52c_r + 418k + 396c_pk + 254c_rk + 642c_pk^2 + 444c_pk^3 + 112c_pk^4 \\ +470c_rk^2 + 420c_rk^3 + 184c_rk^4 + 32c_rk^5 + 892k^2 + 924k^3 + 472k^4 + 96k^5 + 75 \end{bmatrix}}{4(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)}$$

$$q_s^{\rm EB^*} = \frac{(k+1)\big(2c_r - 11c_p + 25k - 29c_pk + 4c_rk - 16c_pk^2 + 2c_rk^2 + 14k^2 + 9\big)}{32k^4 + 160k^3 + 287k^2 + 214k + 53}$$

$$q_P^{\rm EB^*} = \frac{\begin{bmatrix} 40c_r - 61c_p + 64k - 178c_pk + 114c_rk - 168c_pk^2 - 52c_pk^3 \\ +118c_rk^2 + 52c_rk^3 + 8c_rk^4 + 50k^2 - 8k^4 + 21 \end{bmatrix}}{4(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)}.$$

(2) *The equilibrium profits of the supplier, traditional retailer, and platform owner are:*

$$\pi_s^{\rm EB^*} = \frac{\begin{bmatrix} 72c_p^2k^3 + 236c_p^2k^2 + 252c_p^2k + 87c_p^2 - 32c_pc_rk^4 - 192c_pc_rk^3 - 412c_pc_rk^2 \\ -380c_pc_rk - 128c_pc_r + 32c_pk^4 + 48c_pk^3 - 60c_pk^2 - 124c_pk - 46c_p + 32c_r^2k^5 \\ +208c_r^2k^4 + 536c_r^2k^3 + 684c_r^2k^2 + 432c_r^2k + 108c_r^2 - 64c_rk^5 - 384c_rk^4 - 880c_rk^3 \\ -956c_rk^2 - 484c_rk - 88c_r + 32k^5 + 176k^4 + 416k^3 + 508k^2 + 304k + 67 \end{bmatrix}}{8(k+1)(32k^4 + 287k^2 + 160k^3 + 53 + 214k)},$$

$$\pi_r^{\rm EB^*} = \frac{\begin{bmatrix} (2k+1)\big(32c_p - 54c_r + 99k + 63c_pk - 162c_rk + 40c_pk^2 + 8c_pk^3 \\ -180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 + 22\big)^2 \end{bmatrix}}{4(64k^4 + 320k^3 + 574k^2 + 428k + 106)(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)},$$

$$\pi_P^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l}(2k+3)\big(2048c_p^2k^7 + 15\,872c_p^2k^6 + 51\,872c_p^2k^5 + 92\,712c_p^2k^4 + 98\,000c_p^2k^3 \\ +61\,388c_p^2k^2 + 21\,154c_p^2k + 3103c_p^2 - 1024c_pc_rk^7 - 8320c_pc_rk^6 - 28\,800c_pc_rk^5 \\ -55\,160c_pc_rk^4 - 63\,256c_pc_rk^3 - 43\,508c_pc_rk^2 - 16\,636c_pc_rk - 2728c_pc_r - 3072c_pk^7 \\ -23\,424c_pk^6 - 74\,944c_pk^5 - 130\,264c_pk^4 - 132\,744c_pk^3 - 79\,268c_pk^2 - 25\,672c_pk \\ -3478c_p + 128c_r^2k^7 + 1120c_r^2k^6 + 4216c_r^2k^5 + 8860c_r^2k^4 + 11\,232c_r^2k^3 + 8588c_r^2k^2 \\ +3664c_r^2k + 672c_r^2 + 768c_rk^7 + 6080c_rk^6 + 20\,368c_rk^5 + 37\,440c_rk^4 + 40\,792c_rk^3 \\ +26\,332c_rk^2 + 9308c_rk + 1384c_r + 1152k^7 + 8672k^6 + 27\,288k^5 + 46\,412k^4 \\ +45\,976k^3 + 26\,468k^2 + 8182k + 1047\big)\end{array}\right]}{16(k+1)(32k^4 + 160k^3 + 287k^2 + 214k + 53)^2}.$$

4. Analysis

4.1. Model N *vs.* Model EM

Comparing the outcomes of Model N and EM, we can address the question posed at the beginning of this paper: When the platform owner forgoes its entry option, what implications does supplier encroachment have for the supplier and traditional retailer? In particular, we find that the online platform's selling cost plays a critical role in shaping both parties' profitability. That is,

Proposition 4.1. (i) *Supplier encroachment with an online platform increases the supplier's profit if $c_p < c_{p1}$ or $c_p > c_{p2}$ (i.e., $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$), and also increases the traditional retailer's profit if $c_p > c_{p3}$ (i.e., $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$).*

(ii) *There is a "win–win" outcome if $c_p > c_{p3}$ (i.e., $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$), a "win–lose" outcome if $c_p < c_{p1}$ or $c_{p2} < c_p < c_{p3}$ (i.e., $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$) and a "lose–lose" outcome if $c_{p1} < c_p < c_{p2}$ (i.e., $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$), for the supplier and traditional retailer.*

As Figure 3a shows, Proposition 4.1(i) reveals that when the online platform's selling cost is relatively small or large, supplier encroachment is always beneficial for the supplier. Note that in Model EM, the supplier's profits in this setting come from two sources: selling products through the online platform and wholesaling products to the traditional retailer. When the online platform's selling cost decreases ($i.e.$, $c_p < c_{p1}$), the online platform will charge a lower commission fee (notice from Lem. 3.1 that $\partial f^{\mathrm{EM}^*}/\partial c_p > 0$). As a result, the supplier can gain a higher marginal profit per unit through the online platform. Thus, to gain more profits when $c_p < c_{p1}$, the supplier gives less consideration to the traditional retailer's revenue and encroaches heavily on the retail market. Moreover, when $c_p > c_{p2}$, the major decision faced by the supplier is similar to that explored in Arya *et al.* [3]. In the traditional retail channel, the supplier usually sets a lower wholesale price to avoid unduly reducing the traditional retailer's demand (notice that $w_r^{\mathrm{EM}^*}$ is less than $w_r^{\mathrm{N}^*}$). As a result, although the platform owner will charge a relatively high fee to the supplier when $c_p > c_{p2}$, the supplier can benefit more from the traditional retailer by lowering the wholesale price.

As Figure 3b shows, Proposition 4.1(i) further indicates that in the scenario where the platform forgoes its entry option, supplier encroachment may benefit or hurt the traditional retailer depending on the change in the online platform's selling costs. Specifically, when the online platform's selling cost is sufficiently high ($i.e.$, $c_p > c_{p3}$), there is a higher commission charged by the online platform to the supplier, and wholesaling products through the traditional channel is more profitable than selling products through the online platform. Thus, although the supplier encroaches on the traditional retailer's market, she cares greatly about the traditional retailer's profitability and sets a relatively lower wholesale price for the traditional retailer, leading to an increase in the traditional retailer's profitability from the lower wholesale price and outweighing the loss from the supplier encroachment. Then, as illustrated in Figure 3c, Proposition 4.1(ii), a "win–win" outcome arises when $c_p > c_{p3}$. Conversely, when $c_p < c_{p3}$, the latter component dominates, which leads to "win–lose" or "lose–lose" outcomes for the supplier and the traditional retailer.

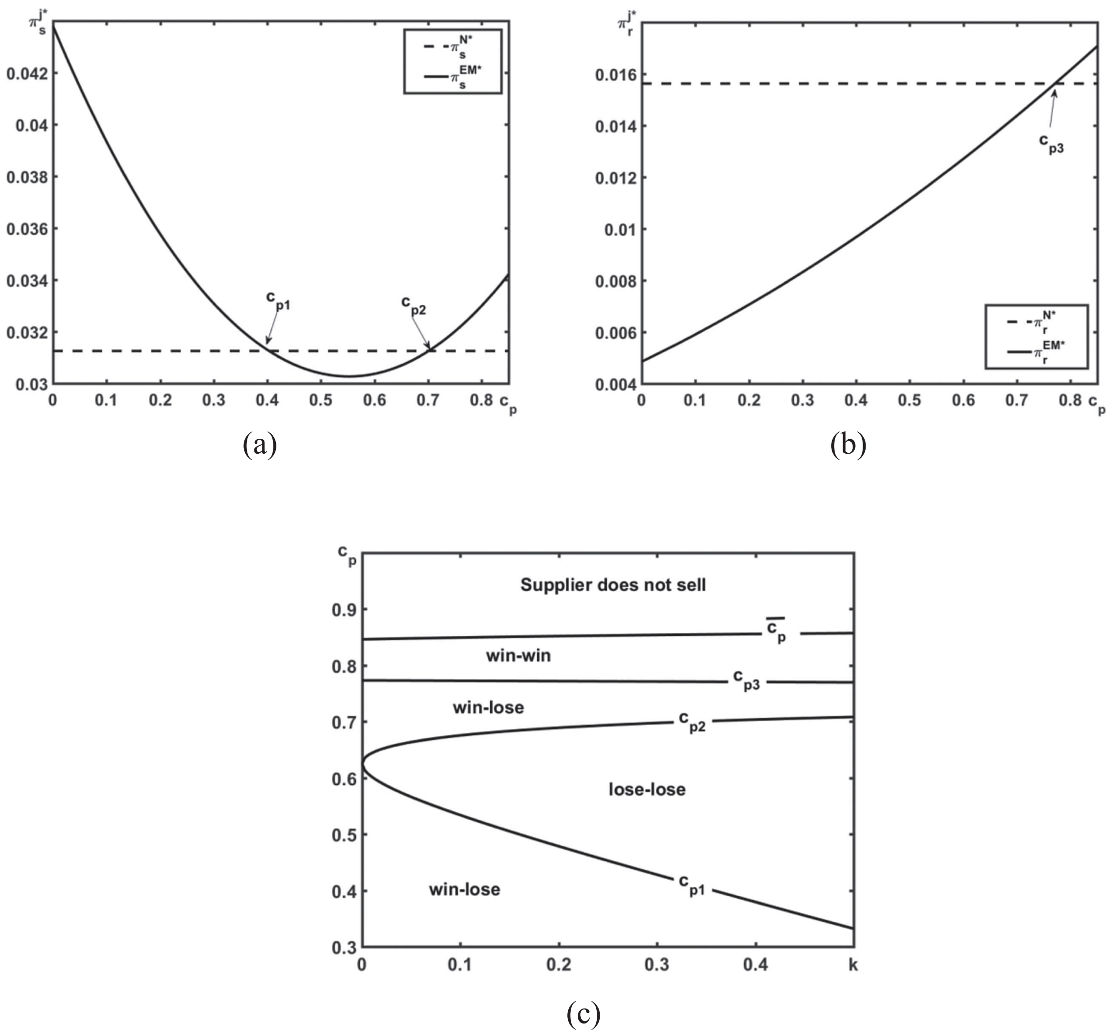
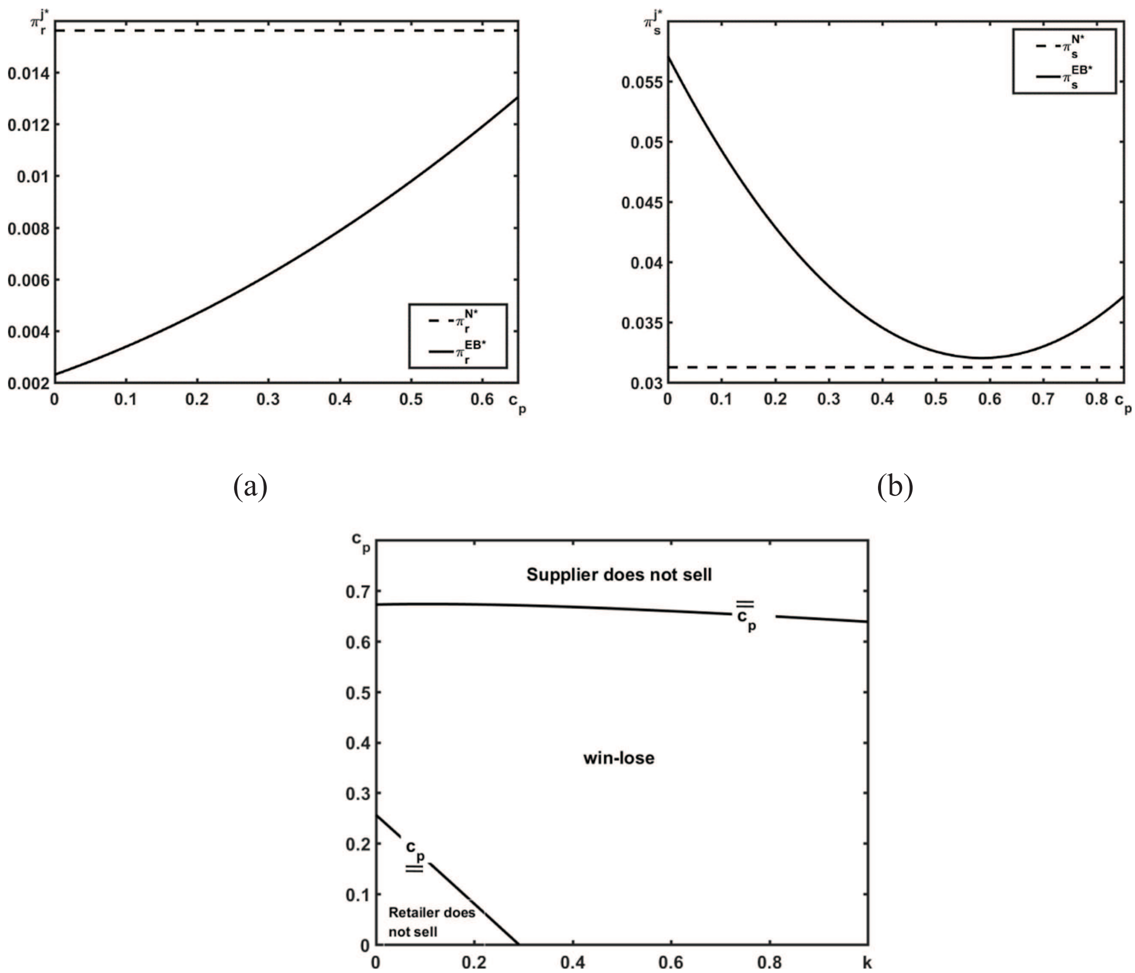


FIGURE 3. The possible outcomes in numerical experiments. (a) Variations of $\pi_s^{\mathrm{EM}^*}$ and $\pi_s^{\mathrm{N}^*}$. (b) Variations of $\pi_r^{\mathrm{EM}^*}$ and $\pi_r^{\mathrm{N}^*}$. (c) The possible outcomes between Model EM and Model N.

In particular, the "lose–lose" outcome would arise when the online platform's selling cost is intermediate (*i.e.*, $c_{p1} < c_p < c_{p2}$), that is, when the retail cost disadvantage of online channels is not profound. The supplier shows no preference between the channels because the marginal profits per unit are similar between wholesaling products in the traditional channel and selling products through the online platform. Consequently, both partners have a close game between both channels. On the one hand, anticipating that the marginal profits per unit for the supplier are similar between the channels, the traditional retailer will vigorously protect his profits if the supplier encroaches into the retail market with an online platform. On the other hand, to deal with the competition from the traditional retailer and the commission fee charged by the platform owner, the supplier must make herself unprofitable by encroaching on the retail market by selling products through the online platform, which leads to a "lose–lose" outcome.

Note, in this example, $c_r = 0.5$, $k = 0.7$.



(c)

FIGURE 4. The possible outcomes in numerical experiments. (a) Variations of $\pi_r^{\mathrm{EB}^*}$ and $\pi_r^{\mathrm{N}^*}$. (b) Variations of $\pi_s^{\mathrm{EB}^*}$ and $\pi_s^{\mathrm{N}^*}$. (c) The possible outcomes between Model EB and Model N.

4.2. Model N *vs.* Model EB

As mentioned earlier, the primary interest of this study is to understand how supplier encroachment with an online platform affects all parties' profitability. Exploring the rationale behind this goal, we now compare the final net change in the supplier's and traditional retailer's profitability from Model N to Model EB. The difference here is that the supplier has encroached on the retail market with an online platform and the platform owner retains its encroachment option. From this perspective, we can conclude our findings in the following proposition:

Proposition 4.2. (i) *Compared to Model N, when the platform retains its encroachment option, supplier encroachment is always beneficial for the supplier but detrimental for the traditional retailer (i.e., $\pi_s^{\mathrm{EB}^*} > \pi_s^{\mathrm{N}^*} \pi_r^{\mathrm{EB}^*} < \pi_r^{\mathrm{N}^*}$), and*

(ii) *There are only "win–lose" outcomes for the supplier and traditional retailer.*

Intuitively, in Model EB, the flexibility of wholesaling products to an online platform allows the supplier to rely less on the traditional retailer, which makes the traditional retailer less likely to benefit from the encroachment; however, Proposition 4.2 shows a stronger result, where the traditional retailer no longer wins. Figure 4a illustrates this result. Before explaining this, it is necessary to examine supplier's profits, which in this setting, come from three sources: selling products through the online platform, wholesaling products to the traditional retailer and wholesaling products to the platform owner. It was found that if the platform retains its encroachment option, the supplier would be better off in Model EB than in Model N, as shown in Figure 4b. In particular, there could be a possible alliance between the supplier and platform owner when the traditional retailer's cost advantage is sufficiently profound (*i.e.*, in exchange for a lower commission fee charged by the platform owner, the supplier would offer a lower wholesale price to the platform owner compared to the traditional retailer). Thus, when the platform retains its encroachment option, the retailer in the traditional channel is in a more challenging position than that confronting a supplier-owned channel: he is not only competing with the products from the supplier but is also dealing with the products that are resold by the online retail platform. Hence, the traditional retailer is always worse off when the platform retains its encroachment option. Put differently, as illustrated in Figure 4c, there are only "win–lose" outcomes for the supplier and the traditional retailer under this scenario.

Note, in this example, $c_r = 0.5$, $k = 0.7$.

4.3. Model ET *vs.* Model EB

Now, we answer the following question: When the platform owner retains its entry option, what implications does supplier encroachment have for the supplier, traditional retailer and platform owner? We compare the supplier, traditional retailer and platform owner's profitability in Model EB to their profitability in Model ET. The difference is that the platform retains its encroachment option, and the supplier decides whether they will encroach on the retail market by selling products directly. Then, we have the following proposition:

Proposition 4.3. (i) *Compared to Model ET, under the scenario of Model EB, supplier encroachment is always beneficial for the supplier ($\pi_s^{\mathrm{EB}^*} > \pi_s^{\mathrm{ET}^*}$) but detrimental for the traditional retailer (i.e., $\pi_r^{\mathrm{EB}^*} < \pi_r^{\mathrm{ET}^*}$).*

(ii) *Compared to Model ET, the platform owner would be better off (i.e., $\pi_P^{\mathrm{EB}^*} > \pi_P^{\mathrm{EM}^*}$) under the scenario of Model EB.*

Proposition 5.1(i) demonstrates that the profits of the supplier are higher when the platform retains its encroachment option, but the traditional retailer's profit is lower in Model EB than in Model ET. As mentioned earlier, in Model EB, supplier encroachment creates an additional distribution channel for the supplier, leading to greater profit for itself. However, the retailer in the traditional channel is in a more challenging position than the supplier; they are not only competing with products resold by the platform owner but may have to deal with products from the supplier. The additional units offered by the supplier through the online platform increase the market competition. Such increased competition reduces the quantities of products in the traditional channel.

Proposition 5.1(ii) further reveals that the platform owner may be pleased to see the supplier encroaching on the retailers' market through its online platform. The reason is that the platform owner's profitability comes from two sources in Model EB: charging a commission fee to the supplier and selling products directly. Although supplier encroachment leads to additional competition from the supplier, who distributes products through the online platform, the platform owner benefits from the flexibility of strategically charging a commission fee to the supplier to limit additional competition. Particularly, more quantities of the product can be sold through the online platform in Model EB than in Model ET, increasing the platform owner's profits. This result explains why more retail platforms, such as Amazon and JD.com, tend to introduce a marketplace that allows the supplier to sell products directly for a commission fee.

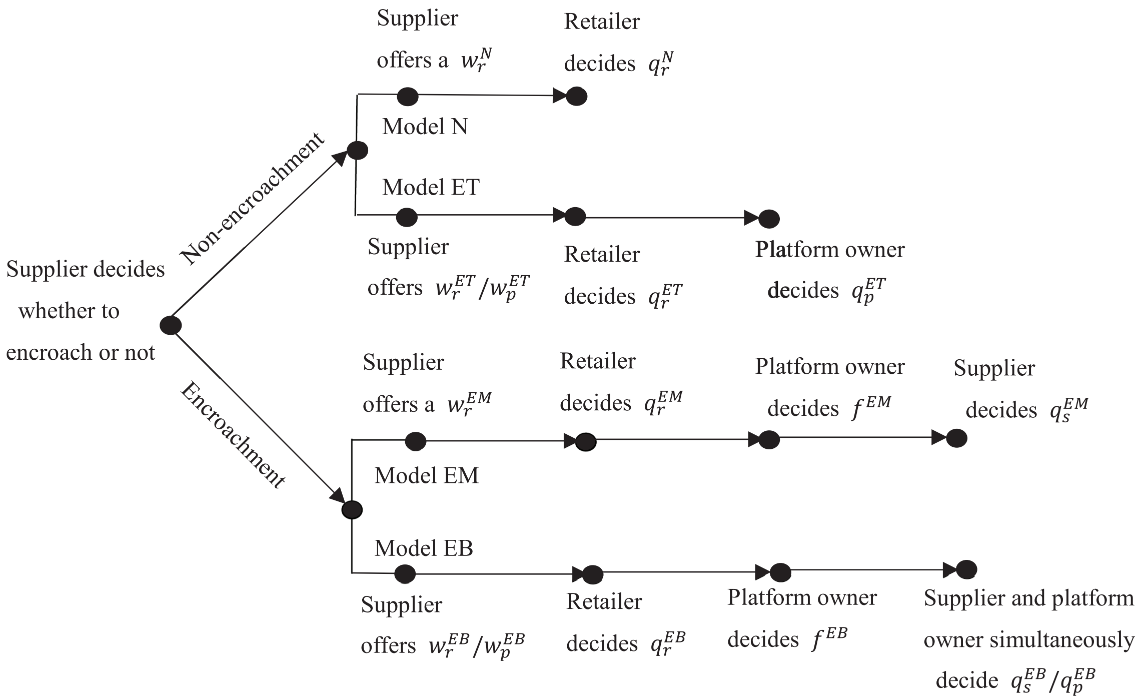


FIGURE 5. Timeline in the simultaneous setting.

5. EXTENSION

The analysis in Section 4 is based on the *sequential setting* in which the supplier and the platform owner choose their sales quantities after the traditional retailer chooses its sales quantities. In this section, as summarised in Figure 5, we now consider simultaneous encroachment settings to further assess the robustness of our key findings[4].

As in the prior section, we can adopt backward induction to obtain the equilibrium solutions in Model EM, Model ET and Model EB under the simultaneous setting[5].

As before, we first address the changes in the profits of the supplier and traditional retailer from Model N to Model EM under the simultaneous setting. We provide the following conclusion:

Proposition 5.1. *Under the simultaneous setting:*

There is still a "win–lose" outcome if $c_p < c_{p4}$(i.e., $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$) and a "lose–lose" outcome if $c_{p4} < c_p < c_{p5}$ (i.e., $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$) for the supplier and traditional retailer. While a new "lose-win" outcome arises if $c_{p5} < c_p$ (i.e., $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$), and the "win–win" outcome disappears.

First, compared to the sequential setting, Proposition 5.1 reveals that the simultaneous decision would have the thresholds $c_{p4} < c_{p1} < c_{p2}$ and $c_{p5} < c_{p3}$ (see Fig. 6). That means the simultaneous decision reduces the supplier's incentive to encroach with an online platform due to its profitable range becoming smaller.

Proposition 5.1 further reveals that when the online platform's sales cost is low, the results in Proposition 4.1 are quite robust; they are labelled as the "win–lose" and "lose–lose" outcomes in Figure 6. Furthermore, unlike Proposition 4.1, Proposition 5.1 indicates that a "lose–win" outcome arises yet a "win–win" outcome no longer

[4] We thank an anonymous reviewer for pointing out such constructive suggestion.

[5] For brevity, all lemmas and their detailed proofs are provided in the Appendix A.

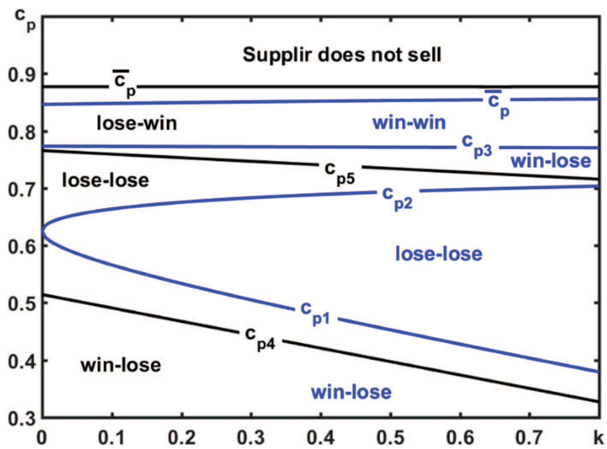


FIGURE 6. The possible outcomes between Model EM and Model N under both settings (The outcomes under the sequential setting are shown in blue and the outcomes under the simultaneous setting are shown in black).

exists when the online platform's sales cost is relatively large ($i.e.$, $c_{p5} < c_p$) in the simultaneous setting. This can be interpreted as follows. When $c_{p5} < c_p$, the supplier lowers wholesale prices to the traditional retailer more substantially in the simultaneous setting than in the sequential setting. The lower wholesale price results in a direct loss for the supplier from the traditional channel. By contrast, the lower wholesale price compensates the traditional retailer's loss due to the supplier encroachment. As such, if $c_{p5} < c_p$, compared to the sequential setting, the supplier can be worse off; the traditional retailer is better off in Model EM than Model N under simultaneous setting, which results in the appearance of a "lose–win" outcome and the disappearance of the "win–win" outcome.

Note, in this example, $c_r = 0.5$.

Next, comparing the outcomes in Model EB and Model N, we can obtain the supplier and the traditional retailer' s profitability, with the platform owner retaining its entry option in the simultaneous setting, and provide the following conclusion:

Proposition 5.2. *Under the simultaneous setting:*

Besides a "win–lose" outcome if $c_p < c_{p6}$ (i.e., $\pi_s^{\mathrm{EB}^*} > \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EB}^*} < \pi_r^{\mathrm{N}^*}$), a new "lose–lose" outcome arises if $c_p > c_{p6}$ (i.e., $\pi_s^{\mathrm{EB}^*} < \pi_s^{\mathrm{N}^*}$, $\pi_r^{\mathrm{EB}^*} < \pi_r^{\mathrm{N}^*}$) for the supplier and traditional retailer.

Consistent with Proposition 4.2, Proposition 5.2 indicates that Model EB would always be detrimental to the traditional retailer when compared to Model N. This result is quite robust and the intuition behind it is not repeated here. Notably, Proposition 4.2 also shows that when the platform retains its entry option, the supplier encroachment is always beneficial for the supplier in the sequential setting.

However, Proposition 5.2 reveals that the supplier's encroachment would hurt her when the online platform's sales cost in the simultaneous setting is relatively large ($i.e.$, $c_p > c_{p6}$). This reveals that – in contrast to Proposition 4.2, under the setting of simultaneous decisions – besides the "win–lose" outcome, there is the additional result of "lose–lose" for the supplier and the traditional retailer (see Fig. 7). This result can be interpreted as follows. First, in Model EB, fewer products are sold through the traditional channel under the simultaneous setting than the sequential setting. Second, in Model EB, when $c_p > c_{p6}$, the supplier may charge a lower wholesale price to traditional retailers under the simultaneous setting than the sequential setting. Thus, when $c_p > c_{p6}$, compared to the sequential setting, the supplier is worse off in Model EB than in Model N under the simultaneous setting.

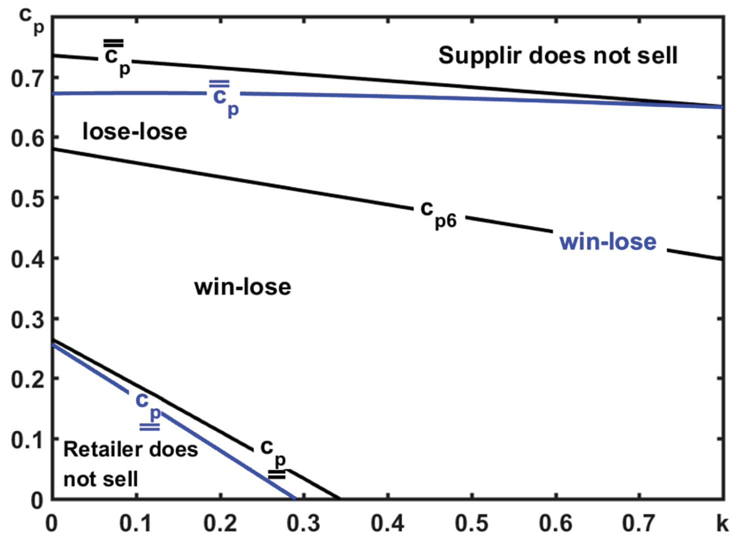


FIGURE 7. The possible outcomes between Model EB and Model N under both settings (The outcomes under the sequential setting are shown in blue and the outcomes under the simultaneous setting are shown in black).

Note, in this example, $c_r = 0.5$.

Finally, we compare the profits of the supplier, traditional retailer and platform owner from Model ET to Model EB under the simultaneous settings. We provide the following conclusion.

Proposition 5.3. *Under the simultaneous setting:*

Compared to Model ET, under Model EB, supplier encroachment still increases the supplier and platform owners' profits but decreases the traditional retailer's profit.

Consistent with Proposition 4.3, Proposition 5.3 shows that when the platform owner retains its entry option to resell the products directly in the simultaneous setting, supplier encroachment would always be beneficial to the supplier and platform owner but detrimental to the traditional retailer. That means timing patterns do not affect the supplier's preference to encroach on the retailer's market, with the platform owner retaining its entry option. This result is quite robust.

6. Conclusion

E-commerce provides suppliers with the potential flexibility to operate an online arm with an online platform in addition to their physical stores. For example, in the electronics industry, many brand-name suppliers, including Apple, IBM, and Lenovo, have sold their products online on Amazon as well as through stores like Best Buy and Circuit City. Although such supplier encroachment is becoming increasingly prevalent in e-commerce markets, the extant studies about supplier encroachment traditionally assume that the supplier sells products directly to consumers through its e-channel and ignores the roles played by online platforms in retail channels.

This research complements this stream of research by investigating the implications of supplier encroachment with an online platform under two scenarios. In the first scenario, the supplier determines whether to encroach on the retail market through an online platform when the platform owner forgoes its option to resell the products from the supplier (Model N and Model EM). In the second scenario, the supplier chooses whether to encroach on the retail market through an online platform when the platform owner retains their entry option (Model ET and Model EB). In four models, we do not facilitate product differentiation or price discrimination[6]. This is to reflect the fact that suppliers are concerned about reputational backlash when customers discover different pricing schemes for different channels, as is noted in Cattani *et al.* [7] and Arya *et al.* [3]. For example, Apple charges the same prices for its Mac, iPhone, and iPad on Amazon as that in retail stores. Particularly, our models extend the work of Cattani *et al.* [7] and Arya *et al.* [3], in which the supplier encroaches on retail market with its own direct channel, to highlight the strategic response to supplier encroachment when the platform owner has the flexibility to forgo and/or retain entry options.

A central result obtained is that, unlike a supplier-owned direct channel and in addition to "win–win" outcomes for the supplier and traditional retailer, supplier encroachment with an online platform may also lead to "win–lose" and "lose–lose" outcomes. When the platform owner retains its entry option, such encroachment is always detrimental for the traditional retailer but beneficial for the supplier. And the platform owner would be better off. Extending all models to the case of both parties making decisions simultaneously, we further find that the simultaneous decision reduces the supplier's incentive to encroach with an online platform but mitigates the traditional retailer's loss from the supplier encroachment.

The model proposed in this study has some limitations as well. First, in all the models, asymmetric information settings have not been addressed. The analysis and insights may be fundamentally different, as revealed in Li *et al.* [23,24]. Second, the supplier is viewed as the Stackelberg leader; that is, she considers the profit-maximising actions of the traditional retailer and simultaneously sets the wholesale price. However, when confronting a dominant traditional retailer, such as Home Depot, the power structure may change. Certainly, it is possible that pricing decisions are made by the traditional retailer and, as a result, the negotiated pricing may be more appropriate in practice. Third, the model assumes that the traditional retailer is a bricks-and-mortar reseller – an assumption that, although common in the literature of e-commerce [3, 17, 38], does not reflect the reality that many traditional retailers have ventured into the online world. Finally, to avoid unnecessary complications, this paper does not focus on the supplier own direct channel, however, in reality, many brand-name suppliers, including Apple, IBM and Lenovo, have opened their own direct channels. This opens up the potential for more realistic models of supplier encroachment.

Appendix A.

Proof of Lemma 3.1. From the equation $p = 1 - Q$, it can be found that the demand with the no-encroachment setting is $p = 1 - q_r^{\mathrm{N}}$. Based on the timeline in the Model N under sequential setting, the equilibrium decisions are solved by backward induction. That is, given wholesale price w_r^{N}, the traditional retailer's problem is $\max_{q_r^{\mathrm{N}}} \pi_r^{\mathrm{N}} = \left(1 - q_r^{\mathrm{N}} - w_r^{\mathrm{N}} - c_r\right) q_r^{\mathrm{N}}$. By applying the first-order condition to π_r^{N} with respect to q_r^{N}, we can obtain $q_r^{\mathrm{N}^*} = \frac{1 - w_r^{\mathrm{N}} - c_r}{2}$. Anticipating the traditional retailer's response, $q_r^{\mathrm{N}^*}\left(w_r^{\mathrm{N}}\right)$, the supplier chooses optimal wholesale price to maximise its profit. In other words, $\max_{w_r^{\mathrm{N}}} \pi_s^{\mathrm{N}} = w_r^{\mathrm{N}}\left(\frac{1 - w_r^{\mathrm{N}} - c_r}{2}\right)$, and this expression yields $w_r^{\mathrm{N}^*} = \frac{1 - c_r}{2}$. Hence, the equilibrium wholesale quantities without encroachment are $q_r^{\mathrm{N}^*} = \frac{1 - c_r}{4}$, and the equilibrium profits are $\pi_s^{\mathrm{N}^*} = \frac{(1 - c_r)^2}{8}$ and $\pi_r^{\mathrm{N}^*} = \frac{(1 - c_r)^2}{16}$. □

Proof of Lemma 3.2. Based on the timeline in the Model EM under sequential setting, solving the first-order condition of the supplier's profit from equation (3.4) with respect to q_s^{EM} yields $q_s^{\mathrm{EM}^*} = \frac{1 - q_r^{\mathrm{EM}} - f^{\mathrm{EM}}}{2}$. After substituting $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ into equation (3.6), the platform owner's profit is given: $\pi_p^{\mathrm{EM}^*} = \left(1 - q_r^{\mathrm{EM}} - f^{\mathrm{EM}}\right)\left(2f^{\mathrm{EM}} - 2c_p - k + f^{\mathrm{EM}}k + kq_r^{\mathrm{EM}}\right)/4$, by applying the first-order condition to it with respect to f^{EM}, we can obtain $f^{\mathrm{EM}^*} = \frac{c_p + k - q_r^{\mathrm{EM}} - kq_r^{\mathrm{EM}} + 1}{k + 2}$. Plugging $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ and $f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right)$ into the traditional retailer's profit, the problem of the traditional retailer is given: $\pi_r^{\mathrm{EM}^*} =$

[6]Liu *et al.* [26], Ofek *et al.* [30] and Yoon *et al.* [41] also provide strong support for the assumption that prices are consistent between different channels.

$\frac{q_r^{\mathrm{EM}}\left(-4c_r+c_p+2k-3q_r^{\mathrm{EM}}-4w_r^{\mathrm{EM}}-2c_rk-2kq_r^{\mathrm{EM}}-2kw_r^{\mathrm{EM}}+3\right)}{2(k+2)}$. Solving first-order condition of the traditional retailer' profit yields $q_r^{\mathrm{EM}^*}=\frac{3+c_p-4c_r+2k-2c_rk-4w_r^{\mathrm{EM}}-2kw_r^{\mathrm{EM}}}{2(3+2k)}$.

Plugging $q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}\right)$, $f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right)$ and $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ into the supplier's profit, we have:

$$\begin{aligned}\pi_s^{\mathrm{EM}^*}=&\frac{\left(4c_r-7c_p+2k+4w_r^{\mathrm{EM}}-4c_pk+2c_rk+2kw_r^{\mathrm{EM}}+3\right)^2}{16(2k^2+7k+6)^2}\\&\times\frac{w_r^{\mathrm{EM}}\left(4c_r-c_p-2k+4w_r^{\mathrm{EM}}+2c_rk+2kw_r^{\mathrm{EM}}-3\right)}{4k+6}.\end{aligned}$$

Therefore, we can derive the optimal w_r^{EM} based on the first-order condition. That is

$$w_r^{\mathrm{EM}^*}=\frac{5c_p-44c_r+68k+10c_pk-78c_rk+4c_pk^2-44c_rk^2-8c_rk^3+40k^2+8k^3+39}{16k^3+88k^2+158k+92}.$$

Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{EM}^*}$ into $q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}\right)$, $f^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}\right)$ and $q_s^{\mathrm{EM}^*}\left(q_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$, we can derive the optimal quantities and commission fee. That is,

$$\begin{aligned}q_r^{\mathrm{EM}^*}&=\frac{3c_p-8c_r+7k+c_pk-8c_pk-2c_rk^2+2k^2+5}{8k^2+28k+23},\\ f^{\mathrm{EM}^*}&=\frac{10c_p+4c_r+15k+7c_pk+6c_rk+2c_rk^2+6k^2+9}{8k^2+28k+23}\quad\text{and}\\ q_s^{\mathrm{EM}^*}&=\frac{4c_r-13c_p+6k-8c_pk+2c_rk+9}{2(8k^2+28k+23)}.\end{aligned}$$

Substituting the optimal $w_r^{\mathrm{EM}^*}$, $q_r^{\mathrm{EM}^*}$, f^{EM^*} and $q_s^{\mathrm{EM}^*}$ into equations (3.4), (3.5) and (3.6), we have the profits of all players as follow:

$$\begin{aligned}\pi_s^{\mathrm{EM}^*}&=\frac{\begin{bmatrix}9c_p^2k+16c_p^2-4c_pc_rk^2-20c_pc_rk-24c_pc_r+4c_pk^2+2c_pk-8c_p+4c_r^2k^3+24c_r^2k^2\\+48c_r^2k+32c_r^2-8c_rk^3-44c_rk^2-76c_rk-40c_r+4k^3+20k^2+37k+24\end{bmatrix}}{4(8k^3+44k^2+79k+46)},\\ \pi_p^{\mathrm{EM}^*}&=\frac{(k+2)(4c_r-13c_p+6k-8c_pk+2c_rk+9)^2}{4(8k^2+28k+23)^2}\quad\text{and}\\ \pi_r^{\mathrm{EM}^*}&=\frac{(2k+3)\left(3c_p-8c_r+7k+c_pk-8c_rk-2c_rk^2+2k^2+5\right)^2}{2(8k^2+28k+23)(8k^3+44k^2+79k+46)}.\end{aligned}$$

□

Proof of Lemma 3.3. Based on the timeline in the Model ET under sequential setting, solving the first-order condition of the platform owner's profit from equation (3.12) with respect to q_p^{ET} yields $q_p^{\mathrm{ET}^*}=\frac{1-c_p-q_r^{\mathrm{ET}}-w_p^{\mathrm{ET}}}{2k+2}$. After substituting $q_p^{\mathrm{ET}^*}(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}})$ into equation (3.11), the traditional retailer's profit is given: $\pi_r^{\mathrm{ET}^*}=\left(q_r^{\mathrm{ET}}\left(-2c_r+c_p+2k-q_r^{\mathrm{ET}}+w_p^{\mathrm{ET}}-2w_r^{\mathrm{ET}}-2c_rk-2kq_r^{\mathrm{ET}}-2kw_r^{\mathrm{ET}}-1\right)\right)/2(k+1)$.

By applying the first-order condition to $\pi_r^{\mathrm{ET}^*}$ with respect to q_r^{ET}, we can obtain $q_r^{\mathrm{ET}^*}=\left(c_p-2c_r+2k+w_p^{\mathrm{ET}}-2w_r^{\mathrm{ET}}-2c_rk-2kw_r^{\mathrm{ET}}+1\right)/(4k+2)$. Anticipating the traditional retailer's response $q_r^{\mathrm{ET}^*}(w_r^{\mathrm{ET}}, w_P^{\mathrm{ET}})$, the supplier chooses optimal wholesale price to maximise its profit. In other words,

$$\max_{w_r^{\mathrm{ET}},w_P^{\mathrm{ET}}}\pi_s^{\mathrm{ET}}=\frac{\begin{bmatrix}c_p+2w_r^{\mathrm{ET}}-4k^2w_r^{\mathrm{ET}^2}-3c_pw_p^{\mathrm{ET}}+2c_pw_r^{\mathrm{ET}}+2c_rw_p^{\mathrm{ET}}-4c_rw_r^{\mathrm{ET}}+2kw_p^{\mathrm{ET}}\\+6kw_r^{\mathrm{ET}}+4w_p^{\mathrm{ET}}w_r^{\mathrm{ET}}-4kw_p^{\mathrm{ET}^2}-8kw_r^{\mathrm{ET}^2}+4k^2w_r^{\mathrm{ET}}-3w_p^{\mathrm{ET}^2}-4w_r^{\mathrm{ET}^2}\\-4c_rk^2w_r^{\mathrm{ET}}-4c_pkw_p^{\mathrm{ET}}+2c_pkw_r^{\mathrm{ET}}+2c_rkw_p^{\mathrm{ET}}-8c_rkw_r^{\mathrm{ET}}+4kw_p^{\mathrm{ET}}w_r^{\mathrm{ET}}\end{bmatrix}}{4(2k^2+3k+1)},$$

and this expression yields $w_r^{\mathrm{ET}^*} = \frac{1-c_r}{2}$ and $w_P^{\mathrm{ET}^*} = \frac{1-c_p}{2}$. Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{ET}^*}$ and $w_P^{\mathrm{ET}^*}$ into $q_r^{\mathrm{ET}^*}\left(w_r^{\mathrm{ET}}, w_P^{\mathrm{ET}}\right)$ and $q_p^{\mathrm{ET}^*}\left(w_p^{\mathrm{ET}}, q_r^{\mathrm{ET}}\right)$, we can derive the optimal quantities. That is,

$$q_r^{\mathrm{ET}^*} = \frac{c_p - 2c_r + 2k - 2c_r k + 1}{8k+4} \quad \text{and} \quad q_p^{\mathrm{ET}^*} = \frac{2c_r - 3c_p + 2k - 4c_p k + 2c_r k + 1}{16k^2 + 24k + 8}.$$

Substituting the optima $w_r^{\mathrm{ET}^*}$, $w_P^{\mathrm{ET}^*}$, $q_p^{\mathrm{ET}^*}$ and $q_r^{\mathrm{ET}^*}$ into equations (3.10), (3.11) and (3.12), we have the profits of all players as follow:

$$\pi_s^{\mathrm{ET}^*} = \frac{\begin{bmatrix} 4c_p^2 k + 3c_p^2 - 4c_p c_r k - 4c_p c_r - 4c_p k - 2c_p + 4c_r^2 k^2 + 8c_r^2 k + 4cr^2 \\ -8c_r k^2 - 12c_r k - 4c_r + 4k^2 + 8k + 3 \end{bmatrix}}{16(2k^2+3k+1)},$$

$$\pi_p^{\mathrm{ET}^*} = \frac{(2c_r - 3c_p + 2k - 4c_p k + 2c_r k + 1)^2}{64(2k+1)^2(k+1)} \quad \text{and} \quad \pi_r^{\mathrm{ET}^*} = \frac{(c_p - 2c_r + 2k - 2c_r k + 1)^2}{32(2k^2+3k+1)}.$$

□

Proof of Lemma 3.4. Based on the timeline in the Model EB under sequential setting, solving the first-order conditions of the supplier's profit from equation (3.15) with respect to q_s^{EB}, and the platform owner's profit from equation (3.17) with respect to q_p^{EB}, we have

$$q_s^{\mathrm{EB}^*} = \frac{c_p - 2f^{\mathrm{EB}} + 2k - q_r^{\mathrm{EB}} + w_p^{\mathrm{EB}} - 2f^{\mathrm{EB}}k - 2kq_r^{\mathrm{EB}} + 1}{2k+3} \quad \text{and}$$
$$q_p^{\mathrm{EB}^*} = \frac{1 - 2c_p + f^{\mathrm{EB}} - 2k - q_r^{\mathrm{EB}} - 2w_p^{\mathrm{EB}} + 2f^{\mathrm{EB}}k + 2kq_r^{\mathrm{EB}}}{2k+3}.$$

After substituting $q_s^{\mathrm{EB}^*}(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}})$ and $q_p^{\mathrm{EB}^*}(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}})$ into equation (3.17), the platform owner's profit is given:

$$\pi_p^{\mathrm{EB}^*} = \frac{\begin{bmatrix} c_p^2 k + c_p^2 + 4c_p f^{\mathrm{EB}} k + 5c_p f^{\mathrm{EB}} + 6c_p k q_r^{\mathrm{EB}} + 4c_p k w_P^{\mathrm{EB}} - 6c_p k + 7c_p q_r^{\mathrm{EB}} + 5c_p w_P^{\mathrm{EB}} \\ -7c_p - 4f^{\mathrm{EB}^2} k^2 - 9f^{\mathrm{EB}^2} k - 5f^{\mathrm{EB}^2} - 8f^{\mathrm{EB}} k^2 q_r^{\mathrm{EB}} - 4f^{\mathrm{EB}} k^2 w_P^{\mathrm{EB}} + 8f^{\mathrm{EB}} k^2 \\ -14f^{\mathrm{EB}} k q_r^{\mathrm{EB}} - 6f^{\mathrm{EB}} k w_P^{\mathrm{EB}} + 14f^{\mathrm{EB}} k - 5f^{\mathrm{EB}} q_r^{\mathrm{EB}} - f w_P^{\mathrm{EB}} + 5f^{\mathrm{EB}^2} - 4k^2 q_r^{\mathrm{EB}^2} \\ -4k^2 q_r^{\mathrm{EB}} w_P^{\mathrm{EB}} + 8k^2 q_r^{\mathrm{EB}} + 4k^2 w_P^{\mathrm{EB}} - 4k^2 - 4k q_r^{\mathrm{EB}^2} + 4w_P^{\mathrm{EB}^2} + 1 - 2k q_r^{\mathrm{EB}} w_P^{\mathrm{EB}} \\ +8k q_r^{\mathrm{EB}} + 3k w_P^{\mathrm{EB}^2} + 2k w_P^{\mathrm{EB}} - 4k + q_r^{\mathrm{EB}^2} + 4q_r^{\mathrm{EB}} w_P^{\mathrm{EB}} - 2q_r^{\mathrm{EB}} - 4w_P^{\mathrm{EB}} \end{bmatrix}}{(2k+3)^2},$$

by applying the first-order condition to it with respect to f^{EB}, we can obtain

$$f^{\mathrm{EB}^*} = \frac{-\left(5q_r^{\mathrm{EB}} - 14k - 5c_p + w_P^{\mathrm{EB}} - 4c_p k + 14kq_r^{\mathrm{EB}} + 6kw_P^{\mathrm{EB}} + 8k^2 q_r^{\mathrm{EB}} + 4k^2 w_P^{\mathrm{EB}} - 8k^2 - 5\right)}{8k^2 + 18k + 10}.$$

Plugging $f^{\mathrm{EB}^*}(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}})$, $q_s^{\mathrm{EB}^*}(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}})$ and $q_p^{\mathrm{EB}^*}(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}})$ into the traditional retailer's profit, the problem of the traditional retailer is given:

$$\pi_r^{\mathrm{EB}^*} = \frac{\begin{bmatrix} q_r\big(-10c_r + 5c_p + 14k - 5q_r^{\mathrm{EB}} + 3w_P^{\mathrm{EB}} - 10w_r^{\mathrm{EB}} + 4c_p k - 18c_r k \\ -14kq_r^{\mathrm{EB}} + 2kw_P^{\mathrm{EB}} - 18kw_r^{\mathrm{EB}} - 8c_r k^2 - 8k^2 q_r^{\mathrm{EB}} - 8k^2 w_r^{\mathrm{EB}} + 8k^2 + 5\big) \end{bmatrix}}{8k^2 + 18k + 10}.$$

Solving the first-order condition of the traditional retailer' profit yields

$$q_r^{\mathrm{EB}^*} = \frac{5c_p - 10c_r + 14k + 3w_P^{\mathrm{EB}} - 10w_r^{\mathrm{EB}} + 4c_pk - 18c_rk + 2kw_P^{\mathrm{EB}} - 18kw_r^{\mathrm{EB}} - 8c_rk^2 - 8k^2w_r^{\mathrm{EB}} + 8k^2 + 5}{16k^2 + 28k + 10}.$$

Plugging $q_r^{\mathrm{EB}^*}\left(w_r^{\mathrm{EB}}, w_P^{\mathrm{EB}}\right)$, $f^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, $q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ and $q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ into the supplier's profit, we have

$$\pi_s^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} -\Big(16c_p^2k^2 + 40c_p^2k + 25c_p^2 - 64c_pc_rk^3 - 224c_pc_rk^2 - 260c_pc_rk - 100c_pc_r + 128c_pk^3w_P^{\mathrm{EB}} \\ -128c_pk^3w_r^{\mathrm{EB}} + 64c_pk^3 + 432c_pk^2w_P^{\mathrm{EB}} - 448c_pk^2w_r^{\mathrm{EB}} + 192c_pk^2 + 484c_pkw_P^{\mathrm{EB}} - 520c_pkw_r^{\mathrm{EB}} \\ +180c_pk + 180c_pw_P^{\mathrm{EB}} - 200c_pw_r^{\mathrm{EB}} + 50c_p + 64c_r^2k^4 + 288c_r^2k^3 + 484c_r^2k^2 + 360c_r^2k + 100c_r^2 \\ +256c_rk^4w_r^{\mathrm{EB}} - 128c_rk^4 - 96c_rk^3w_P^{\mathrm{EB}} + 1152c_rk^3w_r^{\mathrm{EB}} - 512c_rk^3 - 344c_rk^2w_P^{\mathrm{EB}} \\ +1936c_rk^2w_r^{\mathrm{EB}} - 744c_rk^2 - 408c_rkw_P^{\mathrm{EB}} + 1440c_rkw_r^{\mathrm{EB}} - 460c_rk - 160c_rw_P^{\mathrm{EB}} + 400c_rw_r^{\mathrm{EB}} \\ -100c_r + 64k^4w_P^{\mathrm{EB}^2} + 192k^4w_r^{\mathrm{EB}^2} - 256k^4w_r^{\mathrm{EB}} + 64k^4 + 320k^3w_P^{\mathrm{EB}^2} - 128k^3w_P^{\mathrm{EB}}w_r^{\mathrm{EB}} - 32k^3w_P^{\mathrm{EB}} \\ +864k^3w_r^{\mathrm{EB}^2} - 1024k^3w_r^{\mathrm{EB}} + 224k^3 + 596k^2w_P^{\mathrm{EB}^2} - 464k^2w_P^{\mathrm{EB}}w_r^{\mathrm{EB}} - 88k^2w_P^{\mathrm{EB}} + 1452k^2w_r^{\mathrm{EB}^2} \\ -1488k^2w_r^{\mathrm{EB}} + 276k^2 + 488kw_P^{\mathrm{EB}^2} - 556kw_P^{\mathrm{EB}}w_r^{\mathrm{EB}} - 76kw_P^{\mathrm{EB}} + 1080kw_r^{\mathrm{EB}^2} - 920kw_r^{\mathrm{EB}} \\ +140k + 147w_P^{\mathrm{EB}^2} - 220w_P^{\mathrm{EB}}w_r^{\mathrm{EB}} - 20w_P^{\mathrm{EB}} + 300w_r^{\mathrm{EB}^2} - 200w_r^{\mathrm{EB}} + 25\Big) \end{array}\right]}{8(4k+5)^2(2k^2+3k+1)}.$$

Because the objective function is jointly concave in w_r^{EB} and w_P^{EB}, we can easily show that the optimal $w_r^{\mathrm{EB}^*}$ and $w_P^{\mathrm{EB}^*}$ are

$$w_r^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 9c_p - 98c_r + 358k + 56c_pk - 414c_rk + 80c_pk^2 + 32c_pk^3 - 568c_rk^2 \\ -320c_rk^3 - 64c_rk^4 + 488k^2 + 288k^3 + 64k^4 + 89 \end{array}\right]}{4(32k^4 + 160k^3 + 287k^2 + 214k + 53)} \quad \text{and}$$

$$w_p^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 10c_r - 55c_p + 161k - 189c_pk + 28c_rk - 196c_pk^2 \\ -64c_pk^3 + 26c_rk^2 + 8c_rk^3 + 170k^2 + 56k^3 + 45 \end{array}\right]}{2(32k^4 + 160k^3 + 287k^2 + 214k + 53)},$$

respectively. Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{EB}^*}$ and $w_p^{\mathrm{EB}^*}$ into $q_r^{\mathrm{EB}^*}\left(w_r^{\mathrm{EB}}, w_P^{\mathrm{EB}}\right)$, $f^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, $q_s^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$ and $q_p^{\mathrm{EB}^*}\left(w_P^{\mathrm{EB}}, f^{\mathrm{EB}}, q_r^{\mathrm{EB}}\right)$, we can derive the optimal quantities and commission fee. That is,

$$q_r^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 32c_p - 54c_r + 99k + 63c_pk - 162c_rk + 40c_pk^2 + 8c_pk^3 \\ -180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 + 22 \end{array}\right]}{2(32k^4 + 287k^2 + 160k^3 + 53 + 214k)},$$

$$f^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 85c_p + 52c_r + 418k + 396c_pk + 254c_rk + 642c_pk^2 + 444c_pk^3 + 112c_pk^4 \\ +470c_rk^2 + 420c_rk^3 + 184c_rk^4 + 32c_rk^5 + 892k^2 + 924k^3 + 472k^4 + 96k^5 + 75 \end{array}\right]}{4(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)},$$

$$q_s^{\mathrm{EB}^*} = \frac{(k+1)(2c_r - 11c_p + 25k - 29c_pk + 4c_rk - 16c_pk^2 + 2c_rk^2 + 14k^2 + 9)}{32k^4 + 160k^3 + 287k^2 + 214k + 53} \quad \text{and}$$

$$q_P^{\mathrm{EB}^*} = \frac{\begin{bmatrix} 40c_r - 61c_p + 64k - 178c_pk + 114c_rk - 168c_pk^2 - 52c_pk^3 \\ +118c_rk^2 + 52c_rk^3 + 8c_rk^4 + 50k^2 - 8k^4 + 21 \end{bmatrix}}{4(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)}.$$

Substituting the optimal $w_r^{\mathrm{EB}^*}$, $w_p^{\mathrm{EB}^*}$, $q_r^{\mathrm{EB}^*}$, f^{EB^*}, $q_s^{\mathrm{EB}^*}$ and $q_P^{\mathrm{EB}^*}$ into Equation (3.15), (3.16) and (3.17), we have the profits of all players as follow:

$$\pi_s^{\mathrm{EB}^*} = \frac{\begin{bmatrix} 72c_p^2k^3 + 236c_p^2k^2 + 252c_p^2k + 87c_p^2 - 32c_pc_rk^4 - 192c_pc_rk^3 - 412c_pc_rk^2 - 380c_pc_rk - 128c_pc_r \\ +32c_pk^4 + 48c_pk^3 - 60c_pk^2 - 124c_pk - 46c_p + 32c_r^2k^5 + 208c_r^2k^4 + 536c_r^2k^3 + 684c_r^2k^2 + 432c_r^2k \\ +108c_r^2 - 64c_rk^5 - 384c_rk^4 - 880c_rk^3 - 956c_rk^2 - 484c_rk - 88c_r + 32k^5 + 176k^4 + 416k^3 \\ +508k^2 + 304k + 67 \end{bmatrix}}{8(k+1)(32k^4 + 287k^2 + 160k^3 + 53 + 214k)},$$

$$\pi_P^{\mathrm{EB}^*} = \frac{\begin{bmatrix} (2k+3)\big(2048c_p^2k^7 + 15\,872c_p^2k^6 + 51\,872c_p^2k^5 + 92\,712c_p^2k^4 + 98\,000c_p^2k^3 + 61\,388c_p^2k^2 + 21\,154c_p^2k \\ +3103c_p^2 - 1024c_pc_rk^7 - 8320c_pc_rk^6 - 28\,800c_pc_rk^5 - 55\,160c_pc_rk^4 - 63\,256c_pc_rk^3 - 43\,508c_pc_rk^2 \\ -16\,636c_pc_rk - 2728c_pc_r - 3072c_pk^7 - 23\,424c_pk^6 - 74\,944c_pk^5 - 130\,264c_pk^4 - 132\,744c_pk^3 \\ -79\,268c_pk^2 - 25\,672c_pk - 3478c_p + 128c_r^2k^7 + 1120c_r^2k^6 + 4216c_r^2k^5 + 8860c_r^2k^4 + 11\,232c_r^2k^3 \\ +8588c_r^2k^2 + 3664c_r^2k + 672c_r^2 + 768c_rk^7 + 6080c_rk^6 + 20\,368c_rk^5 + 37\,440c_rk^4 + 40\,792c_rk^3 \\ +26\,332c_rk^2 + 9308c_rk + 1384c_r + 1152k^7 + 8672k^6 + 27\,288k^5 + 46\,412k^4 + 45\,976k^3 + 26\,468k^2 \\ +8182k + 1047\big) \end{bmatrix}}{16(k+1)(32k^4 + 160k^3 + 287k^2 + 214k + 53)^2}$$

and

$$\pi_r^{\mathrm{EB}^*} = \frac{\begin{bmatrix} (2k+1)(32c_p - 54c_r + 99k + 63c_pk - 162c_rk + 40c_pk^2 + 8c_pk^3 \\ -180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 + 22)^2 \end{bmatrix}}{4(64k^4 + 320k^3 + 574k^2 + 428k + 106)(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)}.$$

All parameters and variables in this paper must satisfy non-negativity constraints, that is, under sequential setting, we only consider $\frac{2k^2c_r - 2k^2 + 8kc_r - 7k + 8c_r - 5}{k+3} < c_p < \frac{2kc_r + 6k + 4c_r + 9}{8k+13}$ in the scenarios of Model EM (referred to as $\underline{c_p} = \frac{2k^2c_r - 2k^2 + 8kc_r - 7k + 8c_r - 5}{k+3}$, $\overline{c_p} = \frac{2kc_r + 6k + 4c_r + 9}{8k+13}$), and $\frac{\begin{bmatrix} 16c_rk^4 + 88c_rk^3 - 16k^4 + 180c_rk^2 - 80k^3 \\ +162c_rk - 140k^2 + 54c_r - 99k - 22 \end{bmatrix}}{8k^3 + 40k^2 + 63k + 32} < c_p < \frac{\begin{bmatrix} 8c_rk^4 + 52c_rk^3 - 8k^4 + 118c_rk^2 + 114c_rk \\ +50k^2 + 40c_r + 64k + 21 \end{bmatrix}}{52k^3 + 168k^2 + 178k + 61}$ in the scenarios of Model EB (referred to as $\underline{\underline{c_p}} = \frac{\begin{bmatrix} 16c_rk^4 + 88c_rk^3 - 16k^4 + 180c_rk^2 - 80k^3 \\ +162c_rk - 140k^2 + 54c_r - 99k - 22 \end{bmatrix}}{8k^3 + 40k^2 + 63k + 32}$, $\overline{\overline{c_p}} = \frac{\begin{bmatrix} 8c_rk^4 + 52c_rk^3 - 8k^4 + 118c_rk^2 + 114c_rk \\ +50k^2 + 40c_r + 64k + 21 \end{bmatrix}}{52k^3 + 168k^2 + 178k + 61}$). Under these constraints, we compare the outcomes among Model N, Model EM, Model ET and Model EB and draw the Propositions in the sequential setting. □

Proof of Proposition 4.1. Based on the outcomes of Lemmas 3.1 and 3.2, we compare the supplier's profits between Model EM and Model N:

$$\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*} = \frac{\begin{bmatrix} 18c_p^2k + 32c_p^2 - 8c_pc_rk^2 - 40c_pc_rk - 48c_pc_r + 8c_pk^2 + 4c_pk \\ -16c_p + 4c_r^2k^2 + 17c_r^2k + 18c_r^2 + 6c_rk + 12c_r - 4k^2 - 5k + 2 \end{bmatrix}}{8(8k^3 + 44k^2 + 79k + 46)}.$$

$\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*}$ is a quadratic function of c_p, and the coefficient $\frac{16+9k}{4(2+k)(8k^2+28k+23)}$ is positive, which shows that the equation of $\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*}$ is a convex function of c_p. Further, solving $\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*} = 0$, we obtain that there exists two roots (with respect to c_p):

$$c_{p1} = \frac{24c_r - 2k - \left(2k(k+2)\left(8k^2+28k+23\right)\right)^{\frac{1}{2}} + 20c_r k + 4c_r k^2 - 4k^2 + c_r\left(2k(k+2)\left(8k^2+28k+23\right)\right)^{\frac{1}{2}} + 8}{2(18k+32)}$$

and

$$c_{p2} = \frac{24c_r - 2k + \left(2k(k+2)\left(8k^2+28k+23\right)\right)^{\frac{1}{2}} + 20c_r k + 4c_r k^2 - 4k^2 - c_r\left(2k(k+2)\left(8k^2+28k+23\right)\right)^{\frac{1}{2}} + 8}{2(9k+16)}.$$

This, together with the fact that $\underline{c_p} < c_{p1} < c_{p2} < \overline{c_p}$, suggests that $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$ when $c_p < c_{p1}$ or $c_p > c_{p2}$. Otherwise, $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$.

Then, comparing the traditional retailer's profits between Model EM and Model N, we can obtain:

$$\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*} = \frac{\begin{bmatrix} 16c_p^2k^3 + 120c_p^2k^2 + 288c_p^2k + 216c_p^2 - 64c_pc_rk^4 - 544c_pc_rk^3 - 1696c_pc_rk^2 \\ -2304c_pc_rk - 1152c_pc_r + 64c_pk^4 + 512c_pk^3 + 1456c_pk^2 + 1728c_pk + 720c_p \\ +32c_r^2k^4 + 256c_r^2k^3 + 760c_r^2k^2 + 991c_r^2k + 478c_r^2 + 32c_rk^3 + 176c_rk^2 + 322c_rk \\ +196c_r - 32k^4 - 272k^3 - 816k^2 - 1025k - 458 \end{bmatrix}}{16(k+2)(8k^2+28k+23)^2}.$$

$\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*}$ is also a quadratic function of c_p, and the coefficient $\frac{(2k+3)(k+3)^2}{(16k^2+56k+46)(8k^3+44k^2+79k+46)}$ is positive, which shows that the equation of $\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*}$ is a convex function of c_p. There are two roots (with respect to c_p) for the equation $\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*} = 0$:

$$c_{p3} = \frac{-\left(7k - 8c_r - 8c_rk - 2c_rk^2 + 2k^2 + \left(2^{\frac{1}{2}}(c_r-1)(k+2)^{\frac{1}{2}}\left(8k^2+28k+23\right)\right)/\left(4(2k+3)^{\frac{1}{2}}\right) + 5\right)}{k+3}$$

and

$$c_{p3}^* = \frac{8c_r - 7k + 8c_rk + 2c_rk^2 - 2k^2 + \left(2^{\frac{1}{2}}(c_r-1)(k+2)^{\frac{1}{2}}\left(8k^2+28k+23\right)\right)/\left(4(2k+3)^{\frac{1}{2}}\right) - 5}{k+3}.$$

After calculation, we can find $c_{p3}^* < \underline{c_p} < c_{p3} < \overline{c_p}$. Thus, when $c_p > c_{p3}$, the supplier encroachment always increases the traditional retailer's profit, *i.e.*, $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$; Otherwise $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$. Moreover, since $c_{p2} < c_{p3}$, we can easily show that $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$ when $c_p > c_{p3}; \pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$ when $c_p < c_{p1}$ or $c_{p2} < c_p < c_{p3}; \pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$ when $c_{p1} < c_p < c_{p2}$. □

Proof of Proposition 4.2. Based on the outcomes of Lemmas 3.1 and 3.4, we compare the supplier's profits between Model EB and Model N:

$$\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} = \frac{\begin{bmatrix} \left(72k^3 + 236k^2 + 252k + 87\right)c_p^2 - (128c_r + 124k + 380c_rk + 412c_rk^2 + 192c_rk^3 \\ +32c_rk^4 + 60k^2 - 48k^3 - 32k^4 + 46)c_p + 16c_r^2k^4 + 89c_r^2k^3 + 183c_r^2k^2 + 165c_r^2k \\ +55c_r^2 + 14c_rk^3 + 46c_rk^2 + 50c_rk + 18c_r - 16k^4 - 31k^3 + 7k^2 + 37k + 14 \end{bmatrix}}{8(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)}.$$

We find that $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*}$ is a quadratic function of c_p, that is $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} = Ac_p^2 + \mathrm{B}c_p + C$, where $A = \frac{72k^3+236k^2+252k+87}{8(32k^5+192k^4+447k^3+501k^2+267k+53)}$, $B = \frac{\left[\begin{array}{l} -128c_r - 124k - 380c_rk - 412c_rk^2 - 192c_rk^3 \\ -32c_rk^4 - 60k^2 + 48k^3 + 32k^4 - 46 \end{array}\right]}{8(32k^5+192k^4+447k^3+501k^2+267k+53)}$ and $C = \frac{\left[\begin{array}{l} 16c_r^2k^4 + 89c_r^2k^3 + 183c_r^2k^2 + 165c_r^2k + 55c_r^2 + 14c_rk^3 \\ +46c_rk^2 + 50c_rk + 18c_r - 16k^4 - 31k^3 + 7k^2 + 37k + 14 \end{array}\right]}{8(32k^5+192k^4+447k^3+501k^2+267k+53)}$. We obtain that the coefficient A is positive. That means the equation of $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*}$ is a convex function of c_p. Further, after calculation, we find that $B^2 - 4AC < 0$ for any k and c_r, that means there is no root (with respect to c_p) for the equation $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} = 0$. Thus, $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} > 0$ always holds for any c_p.

Similarly, we compare the traditional retailer's profits between Model EB and Model N:

$$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} = \frac{\left[\begin{array}{l} (2k+1)(32c_p - 54c_r + 99k + 63c_pk - 162c_rk + 40c_pk^2 + 8c_pk^3 \\ -180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 + 22)^2 \end{array}\right]}{8(32k^4 + 160k^3 + 287k^2 + 214k + 53)(32k^5 + 192k^4 + 447k^3 + 501k^2 + 267k + 53)} - \frac{(c_r - 1)^2}{16}.$$

$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*}$ is a quadratic function of c_p, and the coefficient $\frac{(2k+1)(8k^3+40k^2+63k+32)^2}{8(k+1)(32k^4+287k^2+160k^3+53+214k)^2}$ is positive. Then, there are two roots (with respect to c_p) for the equation $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} = 0$:

$$c_p^* = \frac{\left[\begin{array}{l} 54c_r - 99k + 162c_rk + 180c_rk^2 + 88c_rk^3 + 16c_rk^4 - 140k^2 - 80k^3 - 16k^4 \\ +\left(2^{\frac{1}{2}}(c_r - 1)(k+1)^{\frac{1}{2}}\left(32k^4 + 160k^3 + 287k^2 + 214k + 53\right)\right)/\left(2(2k+1)^{\frac{1}{2}}\right) - 22 \end{array}\right]}{32 + 63k + 40k^2 + 8k^3}$$

and

$$c_p^{**} = \frac{\left[\begin{array}{l} -\Big(99k - 54c_r - 162c_rk - 180c_rk^2 - 88c_rk^3 - 16c_rk^4 + 140k^2 + 80k^3 + 16k^4 \\ +\left(2^{\frac{1}{2}}(c_r - 1)(k+1)^{\frac{1}{2}}\left(32k^4 + 160k^3 + 287k^2 + 214k + 53\right)\right)/\left(2(2k+1)^{\frac{1}{2}}\right) + 22\Big) \end{array}\right]}{32 + 63k + 40k^2 + 8k^3}.$$

This, together with the fact that $c_p^{**} < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^*$, suggests that, for any c_p, $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} < 0$ always holds. □

Proof of Proposition 4.3. Based on the outcomes of Lemmas 3.3 and 3.4, we compare the supplier's profits between Model EB and Model ET:

$$\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} = \frac{\left[\begin{array}{l} -128c_p^2k^4 - 320c_p^2k^3 - 220c_p^2k^2 - 17c_p^2k - 15c_p^2 - 64c_pc_rk^3 - 180c_pc_rk^2 - 160c_pc_rk \\ -44c_pc_r + 256c_pk^4 + 704c_pk^3 + 620c_pk^2 + 194c_pk + 14c_p + 4c_r^2k^3 + 12c_r^2k^2 + 12c_r^2k \\ +4cr^2 + 56c_rk^3 + 156c_rk^2 + 136c_rk + 36c_r - 128k^4 - 380k^3 - 388k^2 - 165k - 25 \end{array}\right]}{16(64k^5 + 352k^4 + 734k^3 + 715k^2 + 320k + 53)}.$$

$\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient $\frac{-\left(256k^4+640k^3+440k^2+34k+30\right)}{1024k^5+5632k^4+11\,744k^3+11\,440k^2+5120k+848}$ is negative, which shows that the equation of $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*}$ is a concave function of c_p. Further, solving $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_p^{**} = -\frac{\begin{bmatrix} 97k - 22c_r - 80c_r k - 90c_r k^2 - 32c_r k^3 + 310k^2 + 352k^3 + 128k^4 + 2\sqrt{2}\big(64k^5 + 352k^4 + 734k^3 \\ +715k^2 + 320k + 53\big)^{\frac{1}{2}} - 2\sqrt{2}c_r\big(64k^5 + 352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} + 2\sqrt{2}k\big(64k^5 \\ +352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} - 2\sqrt{2}c_r k\big(64k^5 + 352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} \\ +7 \end{bmatrix}}{128k^4 + 320k^3 + 220k^2 + 17k + 1}$$

and

$$c_p^{*} = -\frac{\begin{bmatrix} 97k - 22c_r - 80c_r k - 90c_r k^2 - 32c_r k^3 + 310k^2 + 352k^3 + 128k^4 - 2\sqrt{2}\big(64k^5 + 352k^4 + 734k^3 \\ +715k^2 + 320k + 53\big)^{\frac{1}{2}} + 2\sqrt{2}c_r\big(64k^5 + 352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} - 2\sqrt{2}k\big(64k^5 \\ +352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} + 2\sqrt{2}c_r k\big(64k^5 + 352k^4 + 734k^3 + 715k^2 + 320k + 53\big)^{\frac{1}{2}} \\ +7 \end{bmatrix}}{128k^4 + 320k^3 + 220k^2 + 17k + 1}.$$

This, together with the fact that $c_p^* < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^{**}$, shows that, for any c_p, $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} > 0$ always holds. Next, we compare the traditional retailer's profits between Model EB and Model ET:

$$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} = \frac{\begin{bmatrix} 1024c_p^2k^6 + 7232c_p^2k^5 + 20\,352c_p^2k^4 + 29\,135c_p^2k^3 + 22\,253c_p^2k^2 + 8541c_p^2 * k + 1287c_p^2 - 2048c_pc_rk^7 \\ -16\,128c_pc_rk^6 - 53\,248c_pc_rk^5 - 95\,228c_pc_rk^4 - 99\,248c_pc_rk^3 - 60\,024c_pc_rk^2 - 19\,424c_pc_rk - 2588c_pc_r \\ +2048c_pk^7 + 14\,080c_pk^6 + 38\,784c_pk^5 + 54\,524c_pk^4 + 40\,978c_pk^3 + 15\,518c_pk^2 + 2342c_pk + 14c_p \\ +256c_r^2k^7 + 2048c_r^2k^6 + 6908c_r^2k^5 + 12\,716c_r^2k^4 + 13\,768c_r^2k^3 + 8744c_r^2k^2 + 3004c_r^2k + 428c_r^2 + 1536c_rk^7 \\ +12\,032c_rk^6 + 39\,432c_rk^5 + 69\,796c_rk^4 + 71\,712c_rk^3 + 42\,536c_rk^2 + 13\,416c_rk + 1732c_r - 1792k^7 \\ -13\,056k^6 - 39\,108k^5 - 62\,160k^4 - 56\,345k^3 - 29\,027k^2 - 7879k - 873 \end{bmatrix}}{32(2k+1)(32k^4 + 160k^3 + 287k^2 + 214k + 53)^2}.$$

$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient $\frac{1024k^6+7232k^5+20\,352k^4+29\,135k^3+22\,253k^2+8541k+1287}{16(2k+1)(32k^4+160k^3+287k^2+214*+53)^2}$ is positive, which shows that the equation of $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*}$ is a convex function of c_p. Further, solving $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_p^{**} = \frac{2c_r + 25k + 4c_rk + 2c_rk^2 + 14k^2 + 9}{16k^2 + 29k + 11}$$

and

$$c_p^{*} = \frac{\begin{bmatrix} 214c_r - 606k + 1074c_rk + 2010c_rk^2 + 1790c_rk^3 + 768c_rk^4 \\ +128c_rk^5 - 1391k^2 - 1454k^3 - 704k^4 - 128k^5 - 97 \end{bmatrix}}{64k^4 + 336k^3 + 619k^2 + 468k + 117}.$$

This, together with the fact that $c_p^* < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^{**}$, shows that, for any c_p, $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} < 0$ always holds.

Additionally, we compare the platform owner's profits between Model EB and Model ET:

$$\pi_P^{\mathrm{EB}^*} - \pi_P^{\mathrm{ET}^*} = \frac{\begin{bmatrix} 49\,152c_p^2k^9 + 434\,176c_p^2k^8 + 1\,651\,712c_p^2k^7 + 3\,542\,272c_p^2k^6 + 4\,709\,552c_p^2k^5 + 4\,020\,856c_p^2k^4 + 2\,204\,895c_p^2k^3 \\ +750\,109c_p^2k^2 + 144\,089c_p^2k + 11\,955c_p^2 - 16\,384c_pc_rk^9 - 139\,264c_pc_rk^8 - 502\,784c_pc_rk^7 - 1\,001\,984c_pc_rk^6 \\ -1\,197\,168c_pc_rk^5 - 868\,212c_pc_rk^4 - 364\,368c_pc_rk^3 - 74\,728c_pc_rk^2 - 2512c_pc_rk + 972c_pc_r - 81\,920c_pk^9 \\ -729\,088c_pk^8 - 2\,800\,640c_pk^7 - 6\,082\,560c_pk^6 - 8\,221\,936c_pk^5 - 7\,173\,500c_pk^4 - 4\,045\,422c_pk^3 \\ -1\,425\,490c_pk^2 - 285\,666c_pk - 24\,882c_p - 3072c_r^2k^8 - 27\,136c_r^2k^7 - 102\,656c_r^2k^6 - 216\,708c_r^2k^5 \\ -278\,420c_r^2k^4 - 222\,184c_r^2k^3 - 107\,144c_r^2k^2 - 28\,436c_r^2k - 3172c_r^2 + 16\,384c_rk^9 + 145\,408c_rk^8 \\ +557\,056c_rk^7 + 1\,207\,296c_rk^6 + 1\,630\,584c_rk^5 + 1\,425\,052c_rk^4 + 808\,736c_rk^3 + 289\,016c_rk^2 + 59\,384c_rk \\ +5372c_r + 32\,768k^9 + 291\,840k^8 + 1\,121\,792k^7 + 2\,437\,632k^6 + 3\,295\,676k^5 + 2\,874\,224k^4 + 1\,618\,343k^3 \\ +568\,237k^2 + 113\,141k + 9755 \end{bmatrix}}{64(64k^5+352k^4+734k^3+715k^2+320k+53)^2}.$$

$\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient $\frac{\begin{bmatrix} 49\,152k^9 + 43\,4176k^8 + 1\,651\,712k^7 + 3\,542\,272k^6 + 4\,709\,552k^5 \\ +4\,020\,856k^4 + 2\,204\,895k^3 + 750\,109k^2 + 144\,089k + 11\,955 \end{bmatrix}}{32(64k^5+352k^4+734k^3+715k^2+320k+53)^2}$ is positive. That means the equation of $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*}$ is a convex function of c_p. Further, solving $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*} = 0$, we obtain that there exists two roots (with respect to c_p):

$$c_p^* = \frac{\begin{bmatrix} \Big(142\,833k - 486c_r + 1256c_rk + 1696c_r\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} - 11\,936k\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} + 37\,364c_rk^2 + 182\,184c_rk^3 + 434\,106c_rk^4 + 598\,584c_rk^5 \\ +500\,992c_rk^6 + 251\,392c_rk^7 + 69\,632c_rk^8 + 8192c_rk^9 - 1696\big(\big((2k+3)\big(512k^5 + 2432k^4 \\ +4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 712\,745k^2 + 2\,022\,711k^3 + 3\,586\,750k^4 \\ +4\,110\,968k^5 + 3\,041\,280k^6 + 1\,400\,320k^7 + 364\,544k^8 + 40\,960k^9 - 33\,120k^2\big(\big((2k+3)\big(512k^5 \\ +2432k^4 + 4512k^3 + 4048k^2 + 1730^*k + 283\big)\big)/64\big)^{(1/2)} - 46\,368k^3\big(\big((2k+3)\big(512k^5 + 2432k^4 \\ +4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} - 34\,752k^4\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 \\ +4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} - 13\,312k^5\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} - 2048k^6\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k \\ +283\big)\big)/64\big)^{(1/2)} + 33\,120c_rk^2\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big) \\ /64\big)^{(1/2)} + 46\,368c_rk^3\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ +34\,752c_rk^4\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ +13\,312c_rk^5\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ +2048c_rk^6\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ +11\,936c_rk\big(\big((2k+3)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 12\,441\Big) \end{bmatrix}}{\begin{bmatrix} 49\,152k^9 + 434\,176k^8 + 1\,651\,712k^7 + 3\,542\,272k^6 + 4\,709\,552k^5 \\ +4\,020\,856k^4 + 2\,204\,895k^3 + 750\,109k^2 + 144\,089k + 11\,955 \end{bmatrix}}$$

and

$$c_p^{**} = \frac{\left[\begin{array}{l}\big(142\,833k - 486c_r + 1256c_r k - 1696c_r\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} + 11\,936k\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} + 37\,364c_r k^2 + 182\,184c_r k^3 + 434\,106c_r k^4 + 598\,584c_r k^5 \\ +500\,992c_r k^6 + 251\,392c_r k^7 + 69\,632c_r k^8 + 8192c_r k^9 + 1696\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 \\ +4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 712\,745k^2 + 2\,022\,711k^3 + 3\,586\,750k^4 \\ +411\,0968k^5 + 3\,041\,280k^6 + 1\,400\,320k^7 + 364\,544k^8 + 40\,960k^9 - 33\,120k^2\big(\big(\big(2k+3\big)\big(512k^5 \\ +2432k^4 + 4512k^3 + 4048k^2 + 1730^*k + 283\big)\big)/64\big)^{(1/2)} + 46\,368k^3\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 \\ +4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 34\,752k^4\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 \\ +4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 13\,312k^5\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 \\ +1730k + 283\big)\big)/64\big)^{(1/2)} + 2048k^6\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k \\ +283\big)\big)/64\big)^{(1/2)} - 33\,120c_r k^2\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big) \\ /64\big)^{(1/2)} - 46\,368c_r k^3\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ -34\,752c_r k^4\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ -13\,312c_r k^5\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ -2048c_r k^6\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} \\ -11\,936c_r k\big(\big(\big(2k+3\big)\big(512k^5 + 2432k^4 + 4512k^3 + 4048k^2 + 1730k + 283\big)\big)/64\big)^{(1/2)} + 12\,441\big)\end{array}\right]}{\left[\begin{array}{l}49\,152k^9 + 434\,176k^8 + 1\,651\,712k^7 + 3\,542\,272k^6 + 4\,709\,552k^5 \\ +4\,020\,856k^4 + 2\,204\,895k^3 + 750\,109k^2 + 144\,089k + 11\,955\end{array}\right]}.$$

To simplify the comparison, we can obtain that, $\underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^* < c_p^{**}$. Thus, we can conclude that, for any c_p, $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*} > 0$ always holds. □

Lemma A.1. *In Model EM under simultaneous setting, we have:*

(i) *The equilibrium wholesale prices, quantities, and commission fee are:*

$$\begin{aligned} w_r^{\mathrm{EM}^*} &= \frac{-2c_p - 17c_r + 12k - 12c_r k + 19}{28k + 41} \\ f_p^{\mathrm{EM}^*} &= \frac{20c_p + 6c_r + 20k + 8c_r k + 15}{28k + 41} \\ q_s^{\mathrm{EM}^*} &= \frac{4c_r - 14c_p + 10}{28k + 41} \\ q_r^{\mathrm{EM}^*} &= \frac{8c_p - 14c_r + 8k - 8c_r k + 6}{28k + 41}. \end{aligned}$$

(ii) *The equilibrium profits of the supplier, traditional retailer, and platform owner are:*

$$\pi_s^{\mathrm{EM}^*} = \frac{\left[\begin{array}{l}180c_p^2 - 80c_p c_r k - 220c_p c_r + 80c_p k - 140c_p + 96c_r^2 k^2 + 304c_r^2 k \\ +254c_r^2 - 192c_r k^2 - 528c_r k - 288c_r + 96k^2 + 224k + 214\end{array}\right]}{(28k + 41)^2}$$

$$\pi_r^{\mathrm{EM}^*} = \frac{4(4c_p - 7c_r + 4k - 4c_r k + 3)^2}{(28k+41)^2}$$
$$\pi_p^{\mathrm{EM}^*} = \frac{2(2k+3)(2c_r - 7c_p + 5)^2}{(28k+41)^2}.$$

Proof of Lemma A.1. Based on the timeline in the Model EM under simultaneous setting, solving the first-order conditions of the supplier's profit from equation (3.4) with respect to q_s^{EM}, and the traditional retailer's profit from equation (3.5) with respect to q_r^{EM} yield $q_s^{\mathrm{EM}^*} = \frac{c_r - 2f^{\mathrm{EM}} + w_r^{\mathrm{EM}} + 1}{3}$ and $q_r^{\mathrm{EM}^*} = \frac{f^{\mathrm{EM}} - 2c_r - 2w_r^{\mathrm{EM}} + 1}{3}$.

After substituting $q_s^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ and $q_r^{\mathrm{EM}^*}\left(w_r^{\mathrm{EM}}, f^{\mathrm{EM}}\right)$ into equations (3.4) and (3.6), the supplier and platform owner's profits are given respectively: $\pi_s^{\mathrm{EM}^*} = (2c_r - 4f^{\mathrm{EM}} + 5w_r^{\mathrm{EM}} - 4c_r f^{\mathrm{EM}} - 4c_r w_r^{\mathrm{EM}} - f^{\mathrm{EM}} w_r^{\mathrm{EM}} + c_r^2 + 4f^{\mathrm{EM}^2} - 5w_r^{\mathrm{EM}^2} + 1)/9$ and $\pi_p^{\mathrm{EM}^*} = (-c_r + 2f^{\mathrm{EM}} - w_r^{\mathrm{EM}} - 1)(3c_p - 3f^{\mathrm{EM}} + k + c_r k - 2f^{\mathrm{EM}} k + k w_r^{\mathrm{EM}})/9$. Then, solving the first-order conditions of the supplier's profit with respect to w_r^{EM}, and the platform owner's profit with respect to f^{EM} yield $w_r^{\mathrm{EM}^*} = \frac{-2c_p - 17c_r + 12k - 12c_r k + 19}{28k+41}$ and $f^{\mathrm{EM}^*} = \frac{20c_p + 6c_r + 20k + 8c_r k + 15}{28k+41}$.

Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{EM}^*}$ and f^{EM^*} into $q_s^{\mathrm{EM}^*}(w_r^{\mathrm{EM}}, f^{\mathrm{EM}})$, and $q_r^{\mathrm{EM}^*}(w_r^{\mathrm{EM}}, f^{\mathrm{EM}})$, we derive the optimal quantities. That is, $q_s^{\mathrm{EM}^*} = \frac{4c_r - 14c_p + 10}{28k+41}$ and $q_r^{\mathrm{EM}^*} = \frac{8c_p - 14c_r + 8k - 8c_r k + 6}{28k+41}$.

Substituting the optimal $w_r^{\mathrm{EM}^*}$, f^{EM^*}, $q_r^{\mathrm{EM}^*}$ and $q_s^{\mathrm{EM}^*}$ into equations (3.4), (3.5) and (3.6), we have the profits of all players as follow:

$$\pi_s^{\mathrm{EM}^*} = \frac{\begin{bmatrix} 180c_p^2 - 80c_p c_r k - 220c_p c_r + 80c_p k - 140c_p + 96c_r^2 k^2 + 304c_r^2 k \\ +254c_r^2 - 192c_r k^2 - 528c_r k - 288c_r + 96k^2 + 224k + 214 \end{bmatrix}}{(28k+41)^2},$$
$$\pi_p^{\mathrm{EM}^*} = \frac{2(2k+3)(2c_r - 7c_p + 5)^2}{(28k+41)^2} \quad \text{and} \quad \pi_r^{\mathrm{EM}^*} = \frac{4(4c_p - 7c_r + 4k - 4c_r k + 3)^2}{(28k+41)^2}.$$

□

Lemma A.2. *In Model* ET *under simultaneous setting, we have:*

(i) *The equilibrium wholesale prices and quantities are:*

$$w_r^{\mathrm{ET}^*} = \frac{1 - c_r}{2}$$
$$w_P^{\mathrm{ET}^*} = \frac{1 - c_p}{2}$$
$$q_r^{\mathrm{ET}^*} = \frac{c_p - 2c_r + 2k - 2c_r k + 1}{8k+6}$$
$$q_p^{\mathrm{ET}^*} = \frac{c_r - 2c_p + 1}{8k+6}.$$

(ii) *The equilibrium profits of the supplier, traditional retailer, and platform owner are:*

$$\pi_s^{\mathrm{ET}^*} = \frac{k - c_r - c_p - c_p c_r - 2c_r k + c_r^2 k + c_p^2 + c_r^2 + 1}{8k+6}$$
$$\pi_r^{\mathrm{ET}^*} = \frac{(c_p - 2c_r + 2k - 2c_r k + 1)^2}{(8k+6)^2}$$
$$\pi_p^{\mathrm{ET}^*} = \frac{(k+1)(c_r - 2c_p + 1)^2}{4(4k+3)^2}.$$

Proof of Lemma A.2. Based on the timeline in the Model ET under simultaneous setting, solving the first-order conditions of the traditional retailer's profit from equation (3.11) with respect to q_r^{ET}, and the platform owner's profit from equation (3.12) with respect to q_p^{ET} yield $q_r^{\mathrm{ET}^*} = (c_p - 2c_r + 2k + w_p^{\mathrm{ET}} - 2w_r^{\mathrm{ET}} - 2c_r k - 2kw_r^{\mathrm{ET}} + 1)/(4k+3)$ and $q_p^{\mathrm{ET}^*} = (c_r - 2c_p - 2w_p^{\mathrm{ET}} + w_r^{\mathrm{ET}} + 1)/(4k+3)$.

After substituting $q_r^{\mathrm{ET}^*}(w_r^{\mathrm{ET}}, w_p^{\mathrm{ET}})$ and $q_p^{\mathrm{ET}^*}(w_r^{\mathrm{ET}}, w_p^{\mathrm{ET}})$ into equation (3.10), the supplier's profits are given:

$$\pi_s^{\mathrm{ET}^*} = \frac{\begin{bmatrix} w_p^{\mathrm{ET}} + w_r^{\mathrm{ET}} - 2c_p w_p^{\mathrm{ET}} + c_p w_r^{\mathrm{ET}} + c_r w_p^{\mathrm{ET}} - 2c_r w_r^{\mathrm{ET}} + 2kw_r^{\mathrm{ET}} \\ +2w_p^{\mathrm{ET}} w_r^{\mathrm{ET}} - 2kw_r^{\mathrm{ET}^*} - 2w_p^{\mathrm{ET}^*} - 2w_r^{\mathrm{ET}^*} - 2c_r k w_r^{\mathrm{ET}} \end{bmatrix}}{4k+3}.$$

Then, solving the first-order conditions of the supplier's profit with respect to w_r^{ET} and w_p^{ET} yield: $w_r^{\mathrm{ET}^*} = \frac{1-c_r}{2}$ and $w_p^{\mathrm{ET}^*} = \frac{1-c_p}{2}$. Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{ET}^*}$ and $w_P^{\mathrm{ET}^*}$ into $q_p^{\mathrm{ET}^*}(w_r^{\mathrm{ET}}, w_p^{\mathrm{ET}})$ and $q_r^{\mathrm{ET}^*}(w_r^{\mathrm{ET}}, w_p^{\mathrm{ET}})$, we can derive the optimal quantities. That is, $q_p^{\mathrm{ET}^*} = (c_r - 2c_p + 1)/(8k+6)$ and $q_r^{\mathrm{ET}^*} = (c_p - 2c_r + 2k - 2c_r k + 1)/(8k+6)$. Substituting the optimal $w_r^{\mathrm{ET}^*}$, $w_P^{\mathrm{ET}^*}$, $q_p^{\mathrm{ET}^*}$ and $q_r^{\mathrm{ET}^*}$ into equations (3.10), (3.11) and (3.12), we have the profits of all players as follow:

$$\pi_s^{\mathrm{ET}^*} = (k - c_r - c_p - c_p c_r - 2c_r k + c_r^2 k + c_p^2 + c_r^2 + 1)/(8k+6),$$
$$\pi_r^{\mathrm{ET}^*} = (c_p - 2c_r + 2k - 2c_r k + 1)^2/(8k+6)^2 \quad \text{and} \quad \pi_p^{\mathrm{ET}^*} = (k+1)(c_r - 2c_p + 1)^2/(4(4k+3)^2).$$

□

Lemma A.3. *In Model* EB *under simultaneous setting, we have:*

(i) *The equilibrium wholesale prices, quantities, and commission fee are:*

$$w_r^{\mathrm{EB}^*} = \frac{75k - 30c_r + 8c_p k - 83c_r k + 8c_p k^2 - 68c_r k^2 - 16c_r k^3 + 60k^2 + 16k^3 + 30}{2(16k^3 + 70k^2 + 89k + 34)}$$
$$w_p^{\mathrm{EB}^*} = \frac{4c_r - 34c_p + 61k - 64c_p k + 3c_r k - 32c_p k^2 + 32k^2 + 30}{2(16k^3 + 70k^2 + 89k + 34)}$$
$$f_p^{\mathrm{EB}^*} = \frac{17c_p + 5c_r + 41k + 29c_p k + 19c_r k + 14c_p k^2 + 17c_r k^2 + 4c_r k^3 + 39k^2 + 12k^3 + 12}{2(16k^3 + 70k^2 + 89k + 34)}$$
$$q_s^{\mathrm{EB}^*} = \frac{3c_r - 17c_p + 28k - 30c_p k + 2c_r k - 16c_p k^2 + 16k^2 + 14}{32k^3 + 140k^2 + 178k + 68}$$
$$q_P^{\mathrm{EB}^*} = \frac{9c_r - 17c_p + 5k - 18c_p k + 13c_r k + 4c_r k^2 - 4k^2 + 8}{32k^3 + 140k^2 + 178k + 68}$$
$$q_r^{\mathrm{EB}^*} = \frac{17c_p - 25c_r + 35k + 20c_p k - 55c_r k + 4c_p k^2 - 38c_r k^2 - 8c_r k^3 + 34k^2 + 8k^3 + 8}{32k^3 + 140k^2 + 178k + 68}.$$

(ii) *The equilibrium profits of the supplier, traditional retailer, and platform owner are:*

$$l\pi_s^{\mathrm{EB}^*} = \frac{\begin{bmatrix} 288c_p^2 k^4 + 1728c_p^2 k^3 + 3436c_p^2 k^2 + 2856c_p^2 k + 867c_p^2 - 128c_p c_r k^5 - 1088c_p c_r k^4 \\ -3444c_p c_r k^3 - 5102c_p c_r k^2 - 3600c_p c_r k - 986c_p c_r + 128c_p k^5 + 512c_p k^4 \\ -12c_p k^3 - 1770c_p k^2 - 2112c_p k - 748c_p + 128c_r^2 k^6 + 1152c_r^2 k^5 + 4128c_r^2 k^4 \\ +7546c_r^2 k^3 + 7464c_r^2 k^2 + 3816c_r^2 k + 795c_r^2 - 256c_r k^6 - 2176c_r k^5 - 7168c_r k^4 \\ -11\,648c_r k^3 - 9826c_r k^2 - 4032c_r k - 604c_r + 128k^6 + 1024k^5 + 3328k^4 \\ +5830k^3 + 5798k^2 + 3072k + 676 \end{bmatrix}}{4(16k^3 + 70k^2 + 89k + 34)^2}$$
$$\pi_r^{\mathrm{EB}^*} = \frac{(17c_p - 25c_r + 35k + 20c_p k - 55c_r k + 4c_p k^2 - 38c_r k^2 - 8c_r k^3 + 34k^2 + 8k^3 + 8)^2}{4(16k^3 + 70k^2 + 89k + 34)^2}$$

$$\pi_p^{\mathrm{EB}^*} = \frac{\begin{bmatrix} 256c_p^2k^5 + 1792c_p^2k^4 + 4704c_p^2k^3 + 5964c_p^2k^2 + 3672c_p^2k + 867c_p^2 - 128c_pc_rk^5 \\ -928c_pc_rk^4 - 2616c_pc_rk^3 - 3576c_pc_rk^2 - 2344c_pc_rk - 578c_pc_r - 384c_pk^5 \\ -2656c_pk^4 - 6792c_pk^3 - 8352c_pk^2 - 5000c_pk - 1156c_p + 16c_r^2k^5 + 136c_r^2k^4 \\ +433c_r^2k^3 + 641c_r^2k^2 + 440c_r^2k + 111c_r^2 + 96c_rk^5 + 656c_rk^4 + 1750c_rk^3 + 400 \\ +2294c_rk^2 + 1464c_rk + 356c_r + 144k^5 + 1000k^4 + 2521k^3 + 3029k^2 + 1768k \end{bmatrix}}{4(16k^3 + 70k^2 + 89k + 34)^2}.$$

Proof of Lemma A.3. Based on the timeline in the Model EB under simultaneous setting, solving the first-order conditions of the supplier's profit from equation (3.15) with respect to q_s^{EB}, the platform owner's profit from equation (3.16) with respect to q_r^{EB} and the traditional retailer's profit from equation (3.17) with respect to q_p^{EB} yield $q_s^{\mathrm{EB}^*} = \frac{c_p+c_r-3f^{\mathrm{EB}}+2k+w_p^{\mathrm{EB}}+w_r^{\mathrm{EB}}+2c_rk-4f^{\mathrm{EB}}k+2kw_r^{\mathrm{EB}}+1}{4k+4}$, $q_r^{\mathrm{EB}^*} = \frac{c_p-3c_r+f^{\mathrm{EB}}+2k+w_p^{\mathrm{EB}}-3w_r^{\mathrm{EB}}-2c_rk-2kw_r^{\mathrm{EB}}+1}{4k+4}$ and $q_P^{\mathrm{EB}^*} = \frac{c_r-3c_p+f^{\mathrm{EB}}-2k-3w_p^{\mathrm{EB}}+\ w_r^{\mathrm{EB}}-2c_rk+\ 4f^{\mathrm{EB}}k-2kw_r^{\mathrm{EB}}+\ 1}{4k+4}$.

After substituting $q_s^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$, $q_P^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$ and $q_r^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$ into equations (3.15) and (3.16), the supplier and platform owner's profits are given respectively:

$$\pi_s^{\mathrm{EB}^*} = \frac{\begin{bmatrix} c_p^2 + 4c_pc_rk + 2c_pc_r - 8c_pf^{\mathrm{EB}}k - 6c_pf^{\mathrm{EB}} - 12c_pkw_p^{\mathrm{EB}} + 8c_pkw_r^{\mathrm{EB}} + 4c_pk - 10c_pw_p^{\mathrm{EB}} + 6c_pw_r^{\mathrm{EB}} \\ +2c_p + 4c_r^2k^2 + 4c_r^2k + c_r^2 - 16c_rf^{\mathrm{EB}}k^2 - 20c_rf^{\mathrm{EB}}k - 6c_rf^{\mathrm{EB}} - 8c_rk^2w_p^{\mathrm{EB}}8c_rk^2 - 12c_rkw_r^{\mathrm{EB}} \\ +8c_rk + 6c_rw_p^{\mathrm{EB}} - 10c_rw_r^{\mathrm{EB}} + 2c_r + 16f^{\mathrm{EB}^2}k^2 + 24f^{\mathrm{EB}^2}k + 9f^{\mathrm{EB}^2} + 16f^{\mathrm{EB}}k^2w_p^{\mathrm{EB}} - 16f^{\mathrm{EB}}k^2w_r^{\mathrm{EB}} \\ -16f^{\mathrm{EB}}k^2 + 12f^{\mathrm{EB}}kw_p^{\mathrm{EB}} - 16f^{\mathrm{EB}}kw_r^{\mathrm{EB}} - 20f^{\mathrm{EB}}k - 2f^{\mathrm{EB}}w_p^{\mathrm{EB}} - 2f^{\mathrm{EB}}w_r^{\mathrm{EB}} - 6f^{\mathrm{EB}} - 8k^2w_p^{\mathrm{EB}}w_r^{\mathrm{EB}} \\ -8k^2w_p^{\mathrm{EB}} - 4k^2w_r^{\mathrm{EB}^2} + 16k^2w_r^{\mathrm{EB}} + 4k^2 - 12kw_p^2 + 4kw_p^{\mathrm{EB}}w_r^{\mathrm{EB}} - 16kw_r^{\mathrm{EB}^2} + 20kw_r^{\mathrm{EB}} \\ +4k - 11w_p^{\mathrm{EB}^2} + 10w_p^{\mathrm{EB}}w_r^{\mathrm{EB}} + 6w_p^{\mathrm{EB}} - 11w_r^{\mathrm{EB}^2} + 6w_r^{\mathrm{EB}} + 1 \end{bmatrix}}{16(k+1)^2}$$

and

$$\pi_p^{\mathrm{EB}^*} = \frac{\begin{bmatrix} 4c_p^2k + 5c_p^2 - 8c_pc_rk - 10c_pc_r + 8c_pf^{\mathrm{EB}}k + 10c_pf^{\mathrm{EB}} + 12c_pkw_p^{\mathrm{EB}} - 8c_pkw_r^{\mathrm{EB}} - 8c_pk + 14c_pw_p^{\mathrm{EB}} \\ -10c_pw_r^{\mathrm{EB}} - 10c_p - 4c_r^2k^2 - 4c_r^2k + c_r^2 + 16c_rf^{\mathrm{EB}}k^2 + 24c_rf^{\mathrm{EB}}k + 6c_rf^{\mathrm{EB}} + 8c_rk^2w_p^{\mathrm{EB}} - 8c_rk^2w_r^{\mathrm{EB}} \\ -8c_rk^2 + 4c_rkw_p^{\mathrm{EB}} - 8c_rkw_r^{\mathrm{EB}} - 8c_rk - 6c_rw_p^{\mathrm{EB}} + 2c_rw_r^{\mathrm{EB}} + 2c_r - 16f^{\mathrm{EB}^2}k^2 - 28f^{\mathrm{EB}^2}k - 11f^{\mathrm{EB}^2} \\ -16f^{\mathrm{EB}}k^2w_p^{\mathrm{EB}} + 16f^{\mathrm{EB}}k^2w_r^{\mathrm{EB}} + 16f^{\mathrm{EB}}k^2 - 20f^{\mathrm{EB}}kw_p^{\mathrm{EB}} + 24f^{\mathrm{EB}}kw_r^{\mathrm{EB}} + 24f^{\mathrm{EB}}k - 2f^{\mathrm{EB}}w_p^{\mathrm{EB}} \\ +6f^{\mathrm{EB}}w_r^{\mathrm{EB}} + 6f^{\mathrm{EB}} + 8k^2w_p^{\mathrm{EB}}w_r^{\mathrm{EB}} + 8k^2w_p^{\mathrm{EB}} - 4k^2w_r^{\mathrm{EB}^2} - 8k^2w_r^{\mathrm{EB}} - 4k^2 + 8kw_p^2 + 4kw_p^{\mathrm{EB}}w_r^{\mathrm{EB}} \\ +4kw_p^{\mathrm{EB}} - 4kw_r^{\mathrm{EB}^2} - 8kw_r^{\mathrm{EB}} - 4k + 9w_p^{\mathrm{EB}^2} - 6w_p^{\mathrm{EB}}w_r^{\mathrm{EB}} - 6w_p^{\mathrm{EB}} + w_r^{\mathrm{EB}^2} + 2w_r^{\mathrm{EB}} + 1 \end{bmatrix}}{16(k+1)^2}.$$

Then, solving the first-order conditions of the supplier's profit with respect to w_r^{EB} and w_p^{EB}, and the platform owner's profit with respect to f^{EB} yield:

$$w_r^{\mathrm{EB}^*} = \frac{75k - 30c_r + 8c_pk - 83c_rk + 8c_pk^2 - 68c_rk^2 - 16c_rk^3 + 60k^2 + 16k^3 + 30}{2(16k^3 + 70k^2 + 89k + 34)},$$

$$w_p^{\mathrm{EB}^*} = \frac{4c_r - 34c_p + 61k - 64c_pk + 3c_rk - 32c_pk^2 + 32k^2 + 30}{2(16k^3 + 70k^2 + 89k + 34)} \quad \text{and}$$

$$f_p^{\mathrm{EB}^*} = \frac{17c_p + 5c_r + 41k + 29c_pk + 19c_rk + 14c_pk^2 + 17c_rk^2 + 4c_rk^3 + 39k^2 + 12k^3 + 12}{2(16k^3 + 70k^2 + 89k + 34)}.$$

Furthermore, substituting the optimal wholesale prices $w_r^{\mathrm{EB}^*}$, $w_P^{\mathrm{EB}^*}$ and f^{EB^*} into $q_s^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$, $q_P^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$ and $q_r^{\mathrm{EB}^*}(w_p^{\mathrm{EB}}, w_r^{\mathrm{EB}}, f^{\mathrm{EB}})$, we can derive the optimal quantities. That is,

$$q_s^{\mathrm{EB}^*} = \frac{3c_r - 17c_p + 28k - 30c_pk + 2c_rk - 16c_pk^2 + 16k^2 + 14}{32k^3 + 140k^2 + 178k + 68},$$

$$q_P^{\mathrm{EB}^*} = \frac{9c_r - 17c_p + 5k - 18c_pk + 13c_rk + 4c_rk^2 - 4k^2 + 8}{32k^3 + 140k^2 + 178k + 68} \quad \text{and}$$
$$q_r^{\mathrm{EB}^*} = \frac{17c_p - 25c_r + 35k + 20c_pk - 55c_rk + 4c_pk^2 - 38c_rk^2 - 8c_rk^3 + 34k^2 + 8k^3 + 8}{32k^3 + 140k^2 + 178k + 68}.$$

Substituting the optimal $w_r^{\mathrm{EB}^*}$, $w_P^{\mathrm{EB}^*}$, f^{EB^*}, $q_r^{\mathrm{EB}^*}$, $q_P^{\mathrm{EB}^*}$ and $q_s^{\mathrm{EB}^*}$ into equations (3.15), (3.16) and (3.17), we have the profits of all players as follow:

$$\pi_s^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 288c_p^2k^4 + 1728c_p^2k^3 + 3436c_p^2k^2 + 2856c_p^2k + 867c_p^2 - 128c_pc_rk^5 - 1088c_pc_rk^4 - 3444c_pc_rk^3 \\ -5102c_pc_rk^2 - 3600c_pc_rk - 986c_pc_r + 128c_pk^5 + 512c_pk^4 - 12c_pk^3 - 1770c_pk^2 - 2112c_pk \\ -748c_p + 128c_r^2k^6 + 1152c_r^2k^5 + 4128c_r^2k^4 + 7546c_r^2k^3 + 7464c_r^2k^2 + 3816c_r^2k + 795c_r^2 \\ -256c_rk^6 - 2176c_rk^5 - 7168c_rk^4 - 11\,648c_rk^3 - 9826c_rk^2 - 4032c_rk - 604c_r + 128k^6 \\ +1024k^5 + 3328k^4 + 5830k^3 + 5798k^2 + 3072k + 676 \end{array}\right]}{4(16k^3 + 70k^2 + 89k + 34)^2},$$

$$\pi_p^{\mathrm{EB}^*} = \frac{\left[\begin{array}{l} 256c_p^2k^5 + 1792c_p^2k^4 + 4704c_p^2k^3 + 5964c_p^2k^2 + 3672c_p^2k + 867c_p^2 - 128c_pc_rk^5 - 928c_pc_rk^4 \\ -2616c_pc_rk^3 - 3576c_pc_rk^2 - 2344c_pc_rk - 578c_pc_r - 384c_pk^5 - 2656c_pk^4 - 6792c_pk^3 \\ -8352c_pk^2 - 5000c_pk - 1156c_p + 16c_r^2k^5 + 136c_r^2k^4 + 433c_r^2k^3 + 641c_r^2k^2 + 440c_r^2k + 111c_r^2 \\ +96c_rk^5 + 656c_rk^4 + 1750c_rk^3 + 400 + 2294c_rk^2 + 1464c_rk + 356c_r + 144k^5 + 1000k^4 \\ +2521k^3 + 3029k^2 + 1768k \end{array}\right]}{4(16k^3 + 70k^2 + 89k + 34)^2} \quad \text{and}$$

$$\pi_r^{\mathrm{EB}^*} = \frac{(17c_p - 25c_r + 35k + 20c_pk - 55c_rk + 4c_pk^2 - 38c_rk^2 - 8c_rk^3 + 34k^2 + 8k^3 + 8)^2}{4(16k^3 + 70k^2 + 89k + 34)^2}.$$

Next, we provide the proofs of propositions under simultaneous setting, similarly, all parameters and variables must satisfy non-negativity constraints. Hence, in the simultaneous setting, we only consider $c_rk + \frac{7}{4}c_r - k - \frac{3}{4} < c_p < c_r\frac{2}{7} + \frac{5}{7}$ in the scenarios of Model EM (referred to as $\underline{c_p} = c_rk + \frac{7}{4}c_r - k - \frac{3}{4}$, $\overline{c_p} = c_r\frac{2}{7} + \frac{5}{7}$), and $\frac{8k^3c_r - 8k^3 + 38k^2c_r - 34k^2 + 55kc_r - 35k + 25c_r - 8}{4k^2 + 20k + 17} < c_p < \frac{4k^2c_r - 4k^2 + 13kc_r + 5k + 9c_r + 8}{18k + 17}$ in the scenarios of Model EB (referred to as $\underline{\underline{c_p}} = \frac{8k^3c_r - 8k^3 + 38k^2c_r - 34k^2 + 55kc_r - 35k + 25c_r - 8}{4k^2 + 20k + 17}$, $\overline{\overline{c_p}} = \frac{4k^2c_r - 4k^2 + 13kc_r + 5k + 9c_r + 8}{18k + 17}$). Under these constraints, we compare the outcomes among Model N, Model EM, Model ET and Model EB and draw the propositions in the simultaneous setting. □

Proof of Proposition 5.1. Based on the outcomes of Lemmas 3.1 and A.1, we compare the supplier's profits between Model EM and Model N:

$$\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*} = \frac{\left[\begin{array}{l} 1440c_p^2 - 640c_pc_rk - 1760c_pc_r + 640c_pk - 1120c_p - 16c_r^2k^2 + 136c_r^2k \\ +351c_r^2 + 32c_rk^2 + 368c_rk + 1058c_r - 16k^2 - 504k + 31 \end{array}\right]}{8(28k + 41)^2}.$$

$\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*}$ is a quadratic function of c_p, and the coefficient $\frac{360}{(28k+41)^2}$ is positive, which shows that the equation of $\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*}$ is a convex function of c_p. Further, solving $\pi_s^{\mathrm{EM}^*} - \pi_s^{\mathrm{N}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_{p4} = (11c_r)/18 - (2k)/9 + \left(41\sqrt{10}c_r\right)/360 - \left(7\sqrt{10}k\right)/90 + (2c_rk)/9 - \left(41\sqrt{10}\right)/360 + \left(7\sqrt{10}c_rk\right)/90 + 7/18$$

and

$$c_{p4}^* = (11c_r)/18 - (2k)/9 - \left(41\sqrt{10}c_r\right)/360 + \left(7\sqrt{10}k\right)/90 + (2c_rk)/9 + \left(41\sqrt{10}\right)/360 - \left(7\sqrt{10}c_rk\right)/90 + 7/18.$$

This, together with the fact that $\underline{c_p} < c_{p4} < \overline{c_p} < c^*_{p4}$, suggests that $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$ when $c_p < c_{p4}$. Otherwise, $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$.

Similarly, comparing the traditional retailer's profits between Model EM and Model N, we can obtain:

$$\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*} = \frac{\begin{bmatrix} 1024c_p^2 - 2048c_pc_pk - 3584c_pc_p + 2048c_pk + 1536c_p + 240c_r^2k^2 + 1288c_r^2k \\ +1455c_r^2 - 480c_pk^2 - 528c_pk + 674c_p + 240k^2 - 760k - 1105 \end{bmatrix}}{16(28k+41)^2}.$$

$\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*}$ is also a quadratic function of c_p, and the coefficient $\frac{128}{(28k+41)^2}$ is positive, which shows that the equation of $\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*}$ is a convex function of c_p. There are two roots (with respect to c_p) for the equation $\pi_r^{\mathrm{EM}^*} - \pi_r^{\mathrm{N}^*} = 0$: $c_{p5} = \frac{15c_r}{32} - \frac{k}{8} + \frac{c_rk}{8} + \frac{17}{32}$ and $c^*_{p5} = \frac{97c_r}{32} - \frac{15k}{8} + \frac{15c_rk}{8} - \frac{65}{32}$.

After calculation, we can find $c^*_{p5} < \underline{c_p} < c_{p5} < \overline{c_p}$. Thus, when $c_p > c_{p5}$, we have $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$; Otherwise $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$. Furthermore, since $c_{p4} < c_{p5}$, we can show that $\pi_s^{\mathrm{EM}^*} > \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$ when $c_p < c_{p4}$; $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} < \pi_r^{\mathrm{N}^*}$ when $c_{p4} < c_p < c_{p5}$; $\pi_s^{\mathrm{EM}^*} < \pi_s^{\mathrm{N}^*}$ and $\pi_r^{\mathrm{EM}^*} > \pi_r^{\mathrm{N}^*}$ when $c_{p5} < c_p$. □

Proof of Proposition 5.2. Based on the outcomes of Lemmas 3.1 and A.3, we compare the supplier's profits between Model EB and Model N:

$$\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} = \frac{\begin{bmatrix} 576c_p^2k^4 + 3456c_p^2k^3 + 6872c_p^2k^2 + 5712c_p^2k + 1734c_p^2 - 256c_pc_rk^5 - 2176c_pc_rk^4 \\ -6888c_pc_rk^3 - 10204c_pc_rk^2 - 7200c_pc_rk - 1972c_pc_r + 256c_pk^5 + 1024c_pk^4 \\ -24c_pk^3 - 3540c_pk^2 - 4224c_pk - 1496c_p + 64c_r^2k^5 + 508c_r^2k^4 + 1544c_r^2k^3 \\ +2247c_r^2k^2 + 1580c_r^2k + 434c_r^2 + 128c_rk^5 + 1160c_rk^4 + 3800c_rk^3 + 5710c_rk^2 \\ +4040c_rk + 1104c_r - 192k^5 - 1092k^4 - 1888k^3 - 1085k^2 + 92k + 196 \end{bmatrix}}{8(16k^3+70k^2+89k+34)^2}.$$

We find that $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*}$ is a quadratic function of c_p, and the coefficient $\frac{576k^4+3456k^3+6872k^2+5712k+1734}{8(16k^3+70k^2+89k+34)^2}$ is positive, which shows that the equation of $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*}$ is a convex function of c_p. Further, solving $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{N}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_{p6} = \frac{\begin{bmatrix} 986c_r + 2112k - 34\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 3600c_rk + 5102c_rk^2 + 3444c_rk^3 + 1088c_rk^4 \\ +128c_rk^5 + 1770k^2 + 12k^3 - 512k^4 - 128k^5 + 34c_r\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} - 89k\big(2(4k \\ +5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} - 70k^2\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} - 16k^3\big(2(4k+5)(8k^3 \\ +38k^2+48k+19)\big)^{\frac{1}{2}} + 70c_rk^2\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 16c_rk^3\big(2(4k+5)(8k^3+38k^2 \\ +48k+19)\big)^{\frac{1}{2}} + 89c_rk\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 748 \end{bmatrix}}{2(288k^4+1728k^3+3436k^2+2856k+867)}$$

and

$$c^*_{p6} = \frac{\begin{bmatrix} 986c_r + 2112k - 34\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 3600c_rk + 5102c_rk^2 + 3444c_rk^3 + 1088c_rk^4 \\ +128c_rk^5 + 1770k^2 + 12k^3 - 512k^4 - 128k^5 - 34c_r\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 89k\big(2(4k \\ +5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 70k^2\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 16k^3\big(2(4k+5)(8k^3 \\ +38k^2+48k+19)\big)^{\frac{1}{2}} - 70c_rk^2\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} - 16c_rk^3\big(2(4k+5)(8k^3+38k^2 \\ +48k+19)\big)^{\frac{1}{2}} - 89c_rk\big(2(4k+5)(8k^3+38k^2+48k+19)\big)^{\frac{1}{2}} + 748 \end{bmatrix}}{2(288k^4+1728k^3+3436k^2+2856k+867)}.$$

This, together with the fact that $\underline{c_p} < c_{p6} < \overline{c_p} < c_{p6}^*$, suggests that $\pi_s^{\mathrm{EB}^*} > \pi_s^{\mathrm{N}^*}$ when $c_p < c_{p6}$. Otherwise, $\pi_s^{\mathrm{EB}^*} < \pi_s^{\mathrm{N}^*}$.

Then, we compare the traditional retailer's profits between Model EB and Model N:

$$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} = \frac{\begin{bmatrix} 64c_p^2k^4 + 640c_p^2k^3 + 2144c_p^2k^2 + 2720c_p^2k + 1156c_p^2 - 256c_pc_rk^5 - 2496c_pc_rk^4 \\ -8928c_pc_rk^3 - 14\,768c_pc_rk^2 - 11\,480c_pc_rk - 3400c_pc_r + 256c_pk^5 + 2368c_pk^4 \\ +7648c_pk^3 + 10\,480c_pk^2 + 6040c_pk + 1088c_p + 192c_r^2k^5 + 1548c_r^2k^4 + 4772c_r^2k^3 \\ +7019c_r^2k^2 + 4948c_r^2k + 1344c_r^2 - 128c_rk^5 - 600c_rk^4 - 616c_rk^3 + 730c_rk^2 \\ +1584c_rk + 712c_r - 64k^5 - 884k^4 - 3516k^3 - 5605k^2 - 3812k - 900 \end{bmatrix}}{16(16k^3 + 70k^2 + 89k + 34)^2}.$$

$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*}$ is a quadratic function of c_p, and the coefficient $\frac{(4k^2+20k+17)^2}{2(16k^3+70k^2+89k+34)^2}$ is positive, which shows that the equation of $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*}$ is a convex function of c_p. Then, there are two roots (with respect to c_p) for the equation $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} = 0$: $c_p^* = \frac{16c_r+19k+21c_rk+6c_rk^2+2k^2+18}{8k^2+40k+34}$ and $c_p^{**} = \frac{84c_r-159k+199c_rk+146c_rk^2+32c_rk^3-138k^2-32k^3-50}{8k^2+40k+34}$. This, together with the fact that $c_p^{**} < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^*$, suggests that, for any c_p, $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{N}^*} < 0$ always holds. □

Proof of Proposition 5.3. Based on the outcomes of Lemmas A.2 and A.3, we compare the supplier's profits between Model EB and Model ET:

$$\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} = \frac{\begin{bmatrix} -\Big(512c_p^2k^6 + 3328c_p^2k^5 + 7720c_p^2k^4 + 8168c_p^2k^3 + 3630c_p^2k^2 + 68c_p^2k + 289c_p^2 + 256c_pc_rk^5 \\ +1544c_pc_rk^4 + 3644c_pc_rk^3 + 4344c_pc_rk^2 + 2640c_pc_rk + 646c_pc_r - 1024c_pk^6 - 6912c_pk^5 \\ -16\,984c_pk^4 - 19\,980c_pk^3 - 11\,604c_pk^2 - 2776c_pk - 68c_p + 8cc_r^2k^5 + 24c_r^2k^4 - 36c_r^2k^3 \\ -190c_r^2k^2 - 212c_r^2k - 73c_r^2 - 272c_rk^5 - 1592c_rk^4 - 3572c_rk^3 - 3964c_rk^2 - 2216c_rk - 500c_r \\ +512k^6 + 3592k^5 + 9288k^4 + 11776k^3 + 7784k^2 + 2496k + 284\Big) \end{bmatrix}}{4(4k+3)(16k^3 + 70k^2 + 89k + 34)^2}.$$

We find that $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient $\frac{-(1024k^6+6656k^5+15\,440k^4+16\,336k^3+7260k^2+136k+578)}{4(4k+3)(16k^3+70k^2+89k+34)^2}$ is negative, which shows that the equation of $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*}$ is a concave function of c_p. Further, solving $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_p^{**} = \frac{\begin{bmatrix} 256k - 86c_r + 2((k+1)^3(1024k^7 + 7552k^6 + 22976k^5 + 37\,108k^4 + 34\,037k^3 + 17\,525k^2 + 4629k \\ +477))^{\frac{1}{2}} - 292c_rk - 376c_rk^2 - 218c_rk^3 - 48c_rk^4 + 664k^2 + 846k^3 + 528k^4 + 128k^5 - 2c_r((k+1)^3 \\ \big(1024k^7 + 7552k^6 + 22\,976k^5 + 37\,108k^4 + 34\,037k^3 + 17\,525k^2 + 4629k + 477)\big)^{\frac{1}{2}} + 37 \end{bmatrix}}{128k^5 + 480k^4 + 628k^3 + 288k^2 - 36k + 5}$$

and

$$c_p^{*} = \frac{\begin{bmatrix} 256k - 86c_r - 2((k+1)^3(1024k^7 + 7552k^6 + 22\,976k^5 + 37\,108k^4 + 34\,037k^3 + 17\,525k^2 + 4629k \\ +477))^{\frac{1}{2}} - 292c_rk - 376c_rk^2 - 218c_rk^3 - 48c_rk^4 + 664k^2 + 846k^3 + 528k^4 + 128k^5 + 2c_r((k+1)^3 \\ \big(1024k^7 + 7552k^6 + 22\,976k^5 + 37\,108k^4 + 34\,037k^3 + 17\,525k^2 + 4629k + 477)\big)^{\frac{1}{2}} + 37 \end{bmatrix}}{128k^5 + 480k^4 + 628k^3 + 288k^2 - 36k + 5}.$$

This, together with the fact that $c_p^* < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^{**}$, suggests that, for any c_p, $\pi_s^{\mathrm{EB}^*} - \pi_s^{\mathrm{ET}^*} > 0$ always holds.

Next, we compare the traditional retailer's profits between Model EB and Model ET:

$$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} = \frac{\left[\begin{array}{l}(k+1)\big(704c_p^2k^4 + 4108c_p^2k^3 + 7528c_p^2k^2 + 5559c_p^2k + 1445c_p^2 - 1536c_pc_rk^5 - 9776c_pc_rk^4 \\ -23\,296c_pc_rk^3 - 26\,404c_pc_rk^2 - 14\,372c_pc_rk - 3026c_pc_r + 1536c_pk^5 + 8368c_pk^4 + 15\,080c_pk^3 \\ +11\,348c_pk^2 + 3254c_pk + 136c_p + 256c_r^2k^6 + 2160c_r^2k^5 + 7232c_r^2k^4 + 12\,304c_r^2k^3 + 11\,268c_r^2k^2 \\ +5293c_r^2k + 1001c_r^2 - 512c_rk^6 - 2784c_rk^5 - 4688c_rk^4 - 1312c_rk^3 + 3868c_rk^2 + 3786c_rk \\ +1024c_r + 256k^6 + 624k^5 - 1840k^4 - 6884k^3 - 7608k^2 - 3520k - 580\big)\end{array}\right]}{4(64k^4 + 328k^3 + 566k^2 + 403k + 102)^2}.$$

$\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient of the quadratic function is $\frac{704k^5+4812k^4+11\,636k^3+13\,087k^2+7004k+1445}{2(64k^4+328k^3+566k^2+403k+102)^2}$. Further, solving $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_p^{**} = \frac{7c_r + 10k + 12c_rk + 4c_rk^2 - 4k^2 + 10}{22k + 17} \quad \text{and} \quad c_p^{*} = \frac{\left[\begin{array}{l}143c_r - 294k + 511c_rk + 652c_rk^2 + 348c_rk^3 \\ +64c_rk^4 - 490k^2 - 316k^3 - 64k^4 - 58\end{array}\right]}{32k^3 + 162k^2 + 217k + 85}.$$

After calculation, we can find that, $c_p^* < \underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^{**}$. Hence, for any c_p, $\pi_r^{\mathrm{EB}^*} - \pi_r^{\mathrm{ET}^*} < 0$ always holds.

Additionally, we compare the platform owner's profits between Model EB and Model ET:

$$\pi_P^{\mathrm{EB}^*} - \pi_P^{\mathrm{ET}^*} = \frac{\left[\begin{array}{l}3072c_p^2k^7 + 24\,832c_p^2k^6 + 80\,624c_p^2k^5 + 139\,264c_p^2k^4 + 139\,308c_p^2k^3 + 80\,744c_p^2k^2 + 25\,024c_p^2k \\ +3179c_p^2 - 1024c_pc_rk^7 - 7936c_pc_rk^6 - 25\,328c_pc_rk^5 - 43\,168c_pc_rk^4 - 41\,956c_pc_rk^3 - 22\,756c_pc_rk^2 \\ -6136c_pc_rk - 578c_pc_r - 5120c_pk^7 - 41\,728c_pk^6 - 135\,920c_pk^5 - 235\,360c_pk^4 - 236\,660c_pk^3 \\ -138\,732c_pk^2 - 43\,912c_pk - 5780c_p + 64c_r^2k^6 + 348c_r^2k^5 + 576c_r^2k^4 + 92c_r^2k^3 - 628c_r^2k^2 - 584c_r^2k \\ -157c_r^2 + 1024c_rk^7 + 7808c_rk^6 + 24\,632c_rk^5 + 42\,016c_rk^4 + 41\,772c_rk^3 + 24\,012c_rk^2 + 7304c_rk \\ +892c_r + 2048k^7 + 16\,960k^6 + 55\,644k^5 + 96\,672k^4 + 97\,444k^3 + 57\,360k^2 + 18\,304k + 2444\end{array}\right]}{4(64k^4 + 328k^3 + 566k^2 + 403k + 102)^2}.$$

$\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*}$ is a quadratic function of c_p, and the coefficient $\frac{\left[\begin{array}{l}6144k^7 + 49\,664k^6 + 161\,248k^5 + 278\,528k^4 \\ +278\,616k^3 + 161\,488k^2 + 50\,048k + 6358\end{array}\right]}{4(64k^4+328k^3+566k^2+403k+102)^2}$ is positive. That means the equation of $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*}$ is a convex function of c_p. Further, solving $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*} = 0$, we obtain that there exist two roots (with respect to c_p):

$$c_p^{*} = \frac{\left[\begin{array}{l}289c_r + 21\,956k + 204c_r\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} + 3068c_rk - 806k\big((k+1)^3 \\ \big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} - 204\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} + 11\,378c_rk^2 \\ +20\,978c_rk^3 + 21\,584c_rk^4 + 12\,664c_rk^5 + 3968c_rk^6 + 512c_rk^7 - 1132k^2\big((k+1)^3\big(16k^3 + 24k^2 \\ +29k + 14\big)\big)^{(1/2)} - 656k^3\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} - 128k^4\big((k+1)^3\big(16k^3 + 24k^2 \\ +29k + 14\big)\big)^{(1/2)} + 69\,366k^2 + 118\,330k^3 + 117\,680k^4 + 67\,960k^5 + 20\,864k^6 + 2560k^7 + 2890 \\ +806c_rk\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} + 1132c_rk^2\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} \\ +656c_rk^3\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)} + 128c_rk^4\big((k+1)^3\big(16k^3 + 24k^2 + 29k + 14\big)\big)^{(1/2)}\end{array}\right]}{3072k^7 + 24\,832k^6 + 80\,624k^5 + 139\,264k^4 + 139\,308k^3 + 80\,744k^2 + 25\,024k + 3179}$$

and

$$c_p^{**} = \frac{\left[\begin{array}{l} 289c_r + 21\,956k - 204c_r\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} + 3068c_rk + 806k\big((k+1)^3 \\ \big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} - 204\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} + 11\,378c_rk^2 \\ +20\,978c_rk^3 + 21\,584c_rk^4 + 12\,664c_rk^5 + 3968c_rk^6 + 512c_rk^7 + 1132k^2\big((k+1)^3\big(16k^3+24k^2 \\ +29k+14\big)\big)^{(1/2)} + 656k^3\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} + 128k^4\big((k+1)^3\big(16k^3+24k^2 \\ +29k+14\big)\big)^{(1/2)} + 69\,366k^2 + 118\,330k^3 + 117\,680k^4 + 67\,960k^5 + 20\,864k^6 + 2560k^7 + 2890 \\ -806c_rk\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} - 1132c_rk^2\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} \\ -656c_rk^3\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} - 128c_rk^4\big((k+1)^3\big(16k^3+24k^2+29k+14\big)\big)^{(1/2)} \end{array}\right]}{3072k^7 + 24\,832k^6 + 80\,624k^5 + 139\,264k^4 + 139\,308k^3 + 80\,744k^2 + 25\,024k + 3179}.$$

To simplify the comparison, we can obtain that, $\underline{\underline{c_p}} < \overline{\overline{c_p}} < c_p^* < c_p^{**}$. Thus, we can conclude that, for any c_p, $\pi_p^{\mathrm{EB}^*} - \pi_p^{\mathrm{ET}^*} > 0$ always holds. □

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