RAIRO-Oper. Res. 56 (2022) 165–181
https://doi.org/10.1051/ro/2021191

RAIRO Operations Research
www.rairo-ro.org

ON THE IMPACT OF CORRUPTION ON MANAGERS' AND CONTROLLERS' BEHAVIOR

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Abstract. In this paper, we study the impact of corruption in the context of a game involving a manager and a controller. We propose a model where the controller initiates the bribe demand from the manager. We identify the structure of three potential subgame perfect Nash equilibria, and show their uniqueness. Next, we analyze the influence of the corruption parameters (bribery amount, reciprocity bonus and reputation gain) and the manager's and the controller's bonuses/penalties on the equilibria. Finally, we explain how the manager and the controller may increase, decrease or maintain their performance, when the bribery amount, the reciprocity bonus or the reputation gain index increase.

Mathematics Subject Classification. 91-10, 91-11, 91A68, 91A80.

Received August 7, 2021. Accepted December 23, 2021.

1. Introduction

Corruption and bribery are among the largest impediments to economic and social development [22]. Corruption and Bribery flourish where the criminal justice system and governance are feeble. They proliferate where accountability is absent and where decision-makers are only accessible *via* restricted social networks. They culminate where pay is low and management controls are weak. To investigate the intricacies of bribery and corruption, researchers have analyzed agents' behavior and designed efficient strategies that deter corruption.

Corruption has been tackled through a variety of game theoretic approaches. For instance, Bilokatch [5] studies enterprises' tax evasion through a two-player game involving a businessman and a supervising official. Giamattei [8] extends the Lambsdorff and Frank [11] corruption game experiment to determine the circumstances that favor corrupt decisions. Lianju and Luyan [13] adopt a static non-cooperative game theoretic approach to analyze the behavior of the briber and the bribee. Verma and Sengupta [21] propose an evolutionary game theoretic analysis of the most widespread form of bribery: harassment bribes requested by corrupt officers from citizens in exchange for essential services. Banerjee [3] compares agents' behavior in a harassment bribery game with their behavior in a neutrally framed ultimatum game.

Keywords. Game theory, corruption game, Nash equilibrium, Manager–Controller conflict.

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A special case of corruption is the one-shot game. Spengler [20] constructs a one-shot corruption game with three players: a briber who decides whether to bribe, an official who decides whether to reciprocate, and an inspector who decides whether to inspect. The game has four asymmetrically distributed penalties that punish bribing, bribe-taking, reciprocating, and receiving reciprocation to different degrees. These penalties apply only if corruption is detected. The probability of detection is endogenized, as it depends on inspection. This model differs from other inspection games in that corruption requires the collaboration of two offending players. For this three-player one-shot corruption game, Bone and Spengler [6] study the impact of reporting bribers when this is cumbersome and when it is profitable.

The behavior of players changes when their interaction is not limited to one single game. Dechenaux and Samuel [7] analyze a repeated game where the inspector monitors regulatory compliance of a firm that may offer a bribe to prevent inspection. They consider that corruption is unfeasible in the one-shot game because of the inspector's hold-up, but becomes feasible in an infinitely repeated game. For the repeated game, they characterize the set of bribes that can be sustained as equilibrium paths using a trigger strategy. Their results show that strengthening anti-corruption policies improves compliance only among a subset of firms, despite any increased monitoring effort. Lowen and Samuel [14] present an experimental bribery game where inspectors are hired to find evidence against firm owners who have violated some regulation. Firm owners may bribe the inspector before the inspector starts the search for evidence or after finding inculpating evidence. Inspectors choose costly effort that affects the probability of finding evidence and fine the owner. The results show that inspectors consistently demand bribes below the Nash equilibrium prediction and exert effort below the payoff-maximizing level.

The outcome of a game is obviously affected by the morality of the inspectors and the organization of hierarchical structures. Nikolaev [17] applies a game-theoretical approach that enforces honest behavior of both the audited agents and the auditors. Inspections are carried out by honest inspectors, who always perform honest audits, and by so-called rational inspectors, who take bribes when they find it advantageous. The inspection superintendent has information on the proportion of honest inspectors at each level of audit and uses this information to reduce the cost of enforcing honest behavior.

The outcome of a game, in general, and corruption deterrence, in particular, depend also on whistle-blowing rewards. Abbink and Wu [1] study the impact of whistle-blowing. They test game models involving an importer and an officer, where either one party or both parties may self-report. They infer that allowing only one party to self-report does not significantly deter corruption, while enabling both parties to report does. This effect is more important when parties have no certitude that they could interact with each others in the future. Siggelkow *et al.* [18] analyze the impact of exogenous and endogenous whistle-blowing on the behavior of managers and auditors. They conclude that whistle-blowing pushes a manager to increase her level of professional effort compared to her effort within a basic management conflict game while it lowers the intensity of the controller's effort.

In line with the aforementioned research, our paper studies game agents' behavior in the context of bribery. It considers a particular two-player game, where a controller inspects the manager of a company and prepares a report that she submits to the board of directors. This paper explores a realistic situation where the controller discovering the manager's careless planning asks for a bribe to *cook* a fake report on the manager's work. The manager has the possibility to accept this deal or reject it. We model this conflict as a two-player extensive game involving a manager and a controller. Our model differs from the models of Abbink and Wu [1] and Siggelkow *et al.* [18] because bribery is initiated by the inspector and not the inspectee. Our model also adds a reciprocity bonus, which models the reciprocal trust required, as introduced by Lambsdorff and Nell [12]. Finally, our model uses strategy sequences in its analysis of the ongoing corruption, which allows every player to have a different calibration of her strategic choice at each game information set.

For our game, we show that, under a set of motivated assumptions, the Nash equilibria are of two different types. The first type happens when bribery is not initiated. The second type is a unique Nash equilibrium found when bribery is initiated. We undertake a sensitivity analysis, that explains how the manager and the controller may maintain, decrease or increase their levels of professional effort when the corruption parameters vary, as well as their responsiveness to the bribery proposal.

Section 2 presents the proposed formulation of the Manager–Controller conflict as a two-player extensive form corruption game. Section 3 lists a set of motivated reasonable assumptions that must hold when the players are intelligent and rational. Section 4 identifies the Nash equilibrium structures for the proposed game. Section 5 undertakes a sensitivity analysis of the Nash equilibria. Finally, Section 6 discusses the sensitivity results, extracts some paradoxes and compares the outcomes with the literature.

2. Two-player extensive game with bribery

This paper studies a game model that involves bribery. A potential collusion of interests between a manager $\mathcal{M}$ and a controller $\mathcal{C}$ may rise if $\mathcal{C}$ offers to cover $\mathcal{M}$'s misplanning in exchange of a bribe. This situation defines a two-player extensive form game [10]. An *extensive* form is the most richly-structured way to describe game situations. It is usually represented by a finite decision tree, whose branches correspond to the players' moves. Each player does not opt for a pure strategy but opts for a sequence of moves. Each sequence corresponds to a unique path linking the root node to the leaf nodes of the tree. In addition, each player has perfect recall. That is, each pair of nodes of the tree is uniquely defined by the information set of a player. The paths from the root node to the leaf nodes of the tree have unequal lengths because they may involve different numbers of decisions. The set of all sequences are used to compute the equilibria for the extensive form game.

Figure 1 illustrates a game tree of a two-player extensive corruption game involving the manager $\mathcal{M}$ and the controller $\mathcal{C}$. At her first information set ($M.1$), the manager $\mathcal{M}$ chooses between two pure strategies: m, to plan methodically, with probability x_m, and nm, not to plan methodically, with probability $x_{nm} = 1 - x_m$. The controller $\mathcal{C}$, who controls the work of $\mathcal{M}$, chooses between two pure strategies at her first information set ($C.1$): c, to compile a precise report on the activity of $\mathcal{M}$, with probability x_c, and nc, not to compile a precise report on the activity of $\mathcal{M}$, with probability $x_{nc} = 1 - x_c$. If $\mathcal{C}$ selects c at her first information set and discovers that $\mathcal{M}$ chose nm, she chooses between two pure strategies at her second information set ($C.2$): t, to propose to cover $\mathcal{M}$'s misplanning, with probability x_t against a bribe amount R, and nt, not to propose to cover $\mathcal{M}$'s misplanning, with probability $x_{nt} = 1 - x_t$. If $\mathcal{C}$ selects t at her second information set, $\mathcal{M}$ has to choose between two pure strategies at her second information set ($M.2$): s, to accept $\mathcal{C}$'s proposal, with probability x_s, and ns, and not to accept $\mathcal{C}$'s proposal, with probability x_{ns}. The manager $\mathcal{M}$ receives a payoff $f_{\mathcal{M}}(x_m, x_{nm}, x_s, x_{ns})$, which depends not only on $X_{\mathcal{M}}$, but also on $X_{\mathcal{C}}$. Similarly, $\mathcal{C}$ receives a payoff $f_{\mathcal{C}}(x_c, x_{nc}, x_t, x_{nt})$, which depends on $X_{\mathcal{C}}$ and $X_{\mathcal{M}}$.

The game tree has six final leaves; each corresponding to a sequence of strategies of players $\mathcal{M}$ and $\mathcal{C}$, and subsequently having a distinct payoff. The first leaf, corresponding to the path (m, c), stipulates that $\mathcal{M}$ plays strategy m and $\mathcal{C}$ plays c. The respective payoffs are consequently, a bonus $B^c_{\mathcal{M}}$ for $\mathcal{M}$, and a report cost $-K^m_{\mathcal{C}}$ plus a bonus $B^m_{\mathcal{C}}$ for $\mathcal{C}$. The second leaf, corresponding to the path (m, nc), stipulates that $\mathcal{M}$ plays strategy m and $\mathcal{C}$ plays nc. The respective payoffs are consequently, a bonus $B^{nc}_{\mathcal{M}}$ for $\mathcal{M}$, and a penalty $-P^m_{\mathcal{C}}$ for $\mathcal{C}$.

The third leaf, corresponding to the path $(\langle nm, s\rangle, \langle c, t\rangle)$, stipulates that $\mathcal{M}$ plays her sequence $\langle nm, s\rangle$ and $\mathcal{C}$ plays her sequence $\langle c, t\rangle$. The manager $\mathcal{M}$ would receive a bonus $B^c_{\mathcal{M}}$ as if she planned methodically in addition to her leisure gain L. The bribe amount R paid by $\mathcal{M}$ to $\mathcal{C}$ appears with opposite signs in the respective payoffs of $\mathcal{M}$ and $\mathcal{C}$. The controller $\mathcal{C}$ would receive a bonus $B^m_{\mathcal{C}}$ as if she compiled a precise report and $\mathcal{M}$ planned methodically, while the cost of the fake report $K^s_{\mathcal{C}}$ is deducted from her payoff. A marginal reciprocity bonus parameter τ is added to the players' payoffs. These payoffs are consequently, $L + B^c_{\mathcal{M}} - R + \tau$ for $\mathcal{M}$, and $B^m_{\mathcal{C}} - K^s_{\mathcal{C}} + R + \tau$ for $\mathcal{C}$. The reciprocity bonus τ represents a social norm, which makes bribery possible. Corruption requires reciprocal trust. In real life, corruption is immoral and illegal. Therefore, if it were not for fear of reputation loss, manager's retaliation, other forms of retaliation, and social pressure, a corrupt controller who accepts bribes may not reciprocate [12].

The fourth leaf corresponds to the path $(\langle nm, ns\rangle, \langle c, t\rangle)$. It stipulates that $\mathcal{M}$ plays her sequence $\langle nm, ns\rangle$ and $\mathcal{C}$ plays her sequence $\langle c, t\rangle$. The manager $\mathcal{M}$ is penalized as she did not plan methodically, but gets her leisure gain L and a reputation gain ηR as she rejected the offer of $\mathcal{C}$. The controller $\mathcal{C}$ would receive a bonus as she compiled a precise report with cost $K^{nm}_{\mathcal{C}}$, but incurs a reputation loss ηR as she initiated a bribery offer.

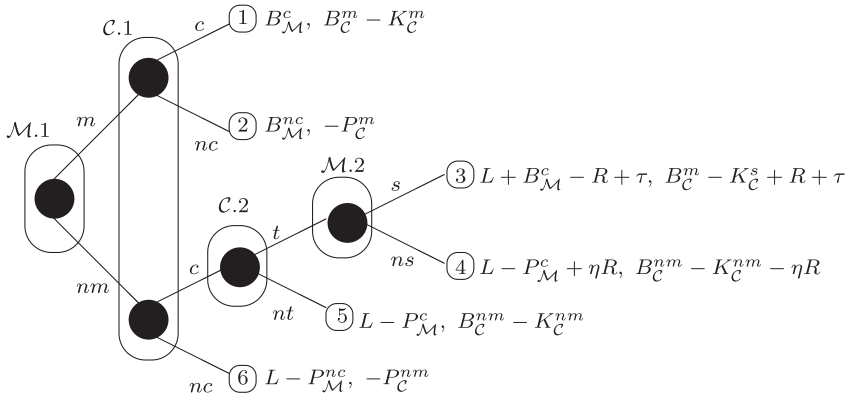

FIGURE 1. Manager–Controller corruption game.

The reputation loss ηR depends on the bribe amount and reflects the risk η taken by the controller. The risk parameter η represents the probability that her case gets revealed, leaked or reported by the manager rejecting the bribery deal. We assume that the whistle-blower gets a reputation gain equivalent to the reputation loss incurred by the inspector. The players' payoffs are consequently $L - P^{c}_{\mathcal{M}} + \eta R$ for $\mathcal{M}$, and $B^{nm}_{\mathcal{C}} - K^{nm}_{\mathcal{C}} - \eta R$ for $\mathcal{C}$. This is in line with the assumptions of Abbink and Wu [1] who study the impact of whistle-blowing rewards on agents' behavior, but differs from their assumption in that they do not consider reciprocation bonuses because their players are uncertain of whether they will interact with each others in the future. In addition, this is in line with the assumptions of Siggelkow *et al.* [18] who also study the impact of whistle-blowing on the agents' behavior and introduce a long term reputation loss occurring at a certain point in time, but do not consider reciprocation bonuses.

The fifth leaf corresponds to the path $(nm, \langle c, nt\rangle)$. It stipulates that $\mathcal{M}$ plays her strategy nm and $\mathcal{C}$ plays her sequence $\langle c, nt\rangle$. The manager $\mathcal{M}$ is penalized as she did not plan methodically but gets her leisure gain L. The controller $\mathcal{C}$ would receive a bonus as she compiled a precise report, with report cost $K^{nm}_{\mathcal{C}}$. The players' payoffs are consequently, $L - P^{c}_{\mathcal{M}}$ for $\mathcal{M}$, and $B^{nm}_{\mathcal{C}} - K^{nm}_{\mathcal{C}}$ for $\mathcal{C}$.

The last leaf corresponds to the path (nm, nc). It indicates that $\mathcal{M}$ plays nm while $\mathcal{C}$ plays nc, with resulting payoffs $L - P^{nc}_{\mathcal{M}}$ and $-P^{nm}_{\mathcal{C}}$, respectively.

All game parameters are exogenous, including the bribery amount R, which is not determined by the controller $\mathcal{C}$.

3. Reasonable assumptions

Even though there is no consensus on the definition of intelligence, it is described by Neisser *et al.* [16] as the ability to perceive information, retain it as knowledge, and translate it to the most appropriate behavior. As differences among individuals may exist, someone's intellectual performance may vary depending on the occasion, judgment criteria and general environment. Similarly, there is no consensus on the definition of rationality. Johnson-Laird and Byrne [9] claim that humans may act erroneously despite their rationality. Simon [19] believes that rationality is bounded by the cognitive limitations of the agents, the lead time the agents have, and the tractability of the problem.

In the following, we show how intelligence and rationality translate to seven assumptions, when applied to our two-player corruption game. These assumptions make our game economically sound.

Motivation 3.1. When $\mathcal{C}$ compiles a precise report, the certitude on $\mathcal{M}$'s actions increases. When the work of $\mathcal{M}$ is non-methodical ($x_m \neq 1$), the penalty imposed on her is larger when $\mathcal{C}$ compiles a report than the penalty imposed on her when $\mathcal{C}$ does not. Differently stated, we should set $P^{c}_{\mathcal{M}} > P^{nc}_{\mathcal{M}}$. On the other hand, when the work of $\mathcal{M}$ is methodical, the bonus rewarded to $\mathcal{M}$ is larger when $\mathcal{C}$ compiles a precise report than when she does not. That is, we should set $B^{c}_{\mathcal{M}} > B^{nc}_{\mathcal{M}}$. The sum of the two inequalities $P^{c}_{\mathcal{M}} > P^{nc}_{\mathcal{M}}$ and $B^{c}_{\mathcal{M}} > B^{nc}_{\mathcal{M}}$ yields $P^{c}_{\mathcal{M}} + B^{c}_{\mathcal{M}} > P^{nc}_{\mathcal{M}} + B^{nc}_{\mathcal{M}}$. Substituting $P^{c}_{\mathcal{M}} + B^{c}_{\mathcal{M}}$ by E and $P^{nc}_{\mathcal{M}} + B^{nc}_{\mathcal{M}}$ by F results in $E > F$. In addition, $P^{c}_{\mathcal{M}} > P^{nc}_{\mathcal{M}}$ is equivalent to $P^{c}_{\mathcal{M}} - P^{nc}_{\mathcal{M}} > 0$ or $J > 0$, where $J = P^{c}_{\mathcal{M}} - P^{nc}_{\mathcal{M}}$. Finally, $B^{c}_{\mathcal{M}} > B^{nc}_{\mathcal{M}}$ can be rewritten as $B^{c}_{\mathcal{M}} - B^{nc}_{\mathcal{M}} > 0$.

Assumption 3.2. *For the Manager–Controller corruption game, the sum of the penalty and bonus of $\mathcal{M}$ should be such that $E > F$ with $J = P^{c}_{\mathcal{M}} - P^{nc}_{\mathcal{M}} > 0$ and $B^{c}_{\mathcal{M}} - B^{nc}_{\mathcal{M}} > 0$.*

Motivation 3.3. When $\mathcal{C}$ does not compile a precise report, the penalty imposed on her is larger when $\mathcal{M}$ does not plan methodically than the penalty imposed when $\mathcal{M}$ plans methodically. Therefore, $P^{nm}_{\mathcal{C}} > P^{m}_{\mathcal{C}}$. On the other hand, when $\mathcal{C}$ compiles a precise report, the bonus rewarded to her is larger when $\mathcal{M}$ does not plan methodically than the bonus rewarded to her when $\mathcal{M}$ plans methodically. Hence, $B^{nm}_{\mathcal{C}} > B^{m}_{\mathcal{C}}$. Adding the two inequalities $P^{m}_{\mathcal{C}} < P^{nm}_{\mathcal{C}}$ and $B^{m}_{\mathcal{C}} < B^{nm}_{\mathcal{C}}$ yields $P^{m}_{\mathcal{C}} + B^{m}_{\mathcal{C}} < P^{nm}_{\mathcal{C}} + B^{nm}_{\mathcal{C}}$. Substituting $P^{m}_{\mathcal{C}} + B^{m}_{\mathcal{C}}$ by G and $P^{nm}_{\mathcal{C}} + B^{nm}_{\mathcal{C}}$ by H yields $G < H$.

Assumption 3.4. *For the Manager–Controller corruption game, the sum of the penalty and bonus of $\mathcal{C}$ should be such that $G < H$ with $B^{m}_{\mathcal{C}} < B^{nm}_{\mathcal{C}}$ and $P^{m}_{\mathcal{C}} < P^{nm}_{\mathcal{C}}$.*

Motivation 3.5. The leisure gain of $\mathcal{M}$ can be estimated in anticipation of the strategy of $\mathcal{C}$. $\mathcal{M}$ will plan methodically if she anticipates $\mathcal{C}$ to compile a precise report and her payoff $B^{c}_{\mathcal{M}}$ to be larger than $L - P^{c}_{\mathcal{M}}$. Were this anticipation to fail, a contradiction would arise causing $\mathcal{M}$ to never plan methodically. Therefore, we should have $L < B^{c}_{\mathcal{M}} + P^{c}_{\mathcal{M}}$. Substituting $B^{c}_{\mathcal{M}} + P^{c}_{\mathcal{M}}$ by E yields $L < E$. On the other hand, $\mathcal{M}$ will not plan methodically if she anticipates $\mathcal{C}$ not to compile a precise report and her payoff $L - P^{nc}_{\mathcal{M}}$ to be larger than $B^{nc}_{\mathcal{M}}$. Were such an anticipation to fail, $\mathcal{M}$ would always plan methodically. Therefore, we should have $B^{nc}_{\mathcal{M}} + P^{nc}_{\mathcal{M}} < L$. Replacing $B^{nc}_{\mathcal{M}} + P^{nc}_{\mathcal{M}}$ by F results in $F < L$. Consequently, $F < L < E$; which guarantees that $\mathcal{M}$ has no strictly dominated strategy at her information set ($\mathcal{M}$.1).

Assumption 3.6. *For the Manager–Controller corruption game, $F < L < E$.*

Motivation 3.7. The payoff of $\mathcal{C}$ can be estimated in anticipation of $\mathcal{M}$. $\mathcal{C}$ will compile a precise report if she anticipates $\mathcal{M}$ not to plan methodically and her payoff to be larger than when she does not compile a precise report. $\mathcal{C}$ should have $B^{nm}_{\mathcal{C}} - K^{nm}_{\mathcal{C}} > -P^{nm}_{\mathcal{C}}$. Therefore, $\mathcal{C}$ should have $B^{nm}_{\mathcal{C}} + P^{nm}_{\mathcal{C}} > K^{nm}_{\mathcal{C}}$. Substituting $B^{nm}_{\mathcal{C}} + P^{nm}_{\mathcal{C}}$ by H yields $H > K^{nm}_{\mathcal{C}}$. On the other hand, $\mathcal{C}$ will not compile a precise report if she anticipates $\mathcal{M}$ to plan methodically and her payoff to be larger than when she compiles a precise report. Therefore, $\mathcal{C}$ should have $-K^{m}_{\mathcal{C}} + B^{m}_{\mathcal{C}} < -P^{m}_{\mathcal{C}}$. Hence, $\mathcal{C}$ should have $P^{m}_{\mathcal{C}} + B^{m}_{\mathcal{C}} < K^{m}_{\mathcal{C}}$. Replacing $P^{m}_{\mathcal{C}} + B^{m}_{\mathcal{C}}$ by G results in $G < K^{m}_{\mathcal{C}}$. In addition, $\mathcal{C}$ will need more time and resources to compile a precise report on the actions of $\mathcal{M}$ when the work of $\mathcal{M}$ is not methodical than when the work of $\mathcal{M}$ is methodical; thus, it is reasonable to assume that $K^{m}_{\mathcal{C}} < K^{nm}_{\mathcal{C}}$.

It follows that $G < K^{m}_{\mathcal{C}} < K^{nm}_{\mathcal{C}} < H$; which guarantees that $\mathcal{C}$ has no strictly dominated strategy at her information set ($\mathcal{C}$.1).

Assumption 3.8. *For the Manager–Controller corruption game, $G < K^{m}_{\mathcal{C}} < K^{nm}_{\mathcal{C}} < H$.*

Motivation 3.9. It is reasonable to think that preparing a fake report on $\mathcal{M}$'s activity to cover each of her misplanning errors is more demanding than preparing a precise report that simply compiles these mistakes if she rejects the bribery deal. Therefore, we may assume that $K^{m}_{\mathcal{C}} < K^{nm}_{\mathcal{C}} < K^{s}_{\mathcal{C}}$.

Assumption 3.10. *For the Manager–Controller corruption game, $K^{m}_{\mathcal{C}} < K^{nm}_{\mathcal{C}} < K^{s}_{\mathcal{C}}$.*

Assumptions 3.2, 3.6 and 3.10 are inline with the assumptions of Belhaiza *et al.* [4]. Assumptions 3.4 and 3.8 are, on the other hand, different due to the presence of a board of directors in their game model.

For the proposed game to be economically interesting, we need to set assumptions that make $\mathcal{C}$ initiate bribery and $\mathcal{M}$ accept it. Therefore, we assume sequential rationality of the game players. A player is sequentially rational if and only if she maximizes her expected payoff at each of her information sets, given her beliefs at this information set. Sequential rationality can be captured through the concept of subgame perfection. A subgame is any part of the game that has a single initial node. That initial node must be the only node in a singleton information set. If a node belongs to the subgame, then so do all its successors. Finally, if a node belongs to the subgame, then all nodes in its information set belong to the subgame. A strategy profile for an extensive-form game is a subgame perfect Nash equilibrium (SPNE) if it specifies a Nash equilibrium in each of its subgames [15]. Under Assumptions 3.2–3.10, an economically interesting SPNE exists only if the bribery amount R satisfies the following assumptions.

Motivation 3.11. For the subgame starting at the information set ($\mathcal{M}$.2), $\mathcal{M}$ would play s if and only if $L + B^c_M - R + \tau \geq L - P^c_M + \eta R$; which is equivalent to $E + \tau \geq (1+\eta)R$. That is $\mathcal{M}$ would play s when her reputation gain is less than the sum of

- her bonus and penalty when she plans methodically and $\mathcal{C}$ compiles a precise report and of
- the reciprocation bonus τ minus R.

For the subgame starting at the information set ($\mathcal{C}$.2), $\mathcal{C}$ would initiate bribery if $B^m_C - K^s_C + R + \tau \geq B^{nm}_C - K^{nm}_C$, which can be written as $R \geq K^s_C - K^{nm}_C + B^{nm}_C - B^m_C - \tau$. $\mathcal{C}$ would initiate bribery if and only if the sum of the bribe R and the reciprocation bonus τ exceeds the sum of

- the difference between the costs of compiling a fake report and of compiling a precise report when $\mathcal{M}$ does not plan methodically and
- the difference between her bonuses B^{nm}_C and B^m_C.

Assumption 3.12. *For the Manager–Controller corruption game,*

$$K^s_C - K^{nm}_C + B^{nm}_C - B^m_C - \tau \leq R \leq \frac{E+\tau}{1+\eta}\cdot$$

Motivation 3.13. For the game to remain economically sound under Assumptions 3.6 and 3.12, $\mathcal{M}$ should prefer to plan methodically when she anticipates $\mathcal{C}$ to compile a precise report and initiate bribery. Otherwise, $\mathcal{M}$ would always prefer not to plan methodically because $L - P^{nc}_M > B^{nc}_M$ already. Therefore, $B^c_M > L + B^c_M - R + \tau$. Hence, $L + \tau < R$.

Assumption 3.14. *For the Manager–Controller corruption game,*

$$L + \tau < R.$$

Finally, we assume that all the parameters involved in the proposed Manager–Controller extensive corruption game are strictly positive.

4. Sequential equilibrium computation

Herein, we focus on the computational aspects of the proposed corruption game. We show that the proposed game has three extreme sequential Nash equilibria. Two extreme equilibria occur when $\mathcal{C}$ does not initiate bribery while the third occurs when $\mathcal{C}$ initiates bribery. This third equilibrium is more interesting from an economic point of view. The three equilibria are of known structure, and satisfy SPNE conditions.

We define the sequential payoff matrices A_M and A_C, and the sequence matrices E_M and E_C, respectively, for players $\mathcal{M}$ and $\mathcal{C}$ (*cf.* [2] for details on strategy sequences for extensive form games).

$$A_{\mathcal{M}} = \left(\begin{array}{r|ccccc} & \emptyset & \langle c\rangle & \langle nc\rangle & \langle c,t\rangle & \langle c,nt\rangle \\ \hline \emptyset & & & & & \\ \langle m\rangle & & B_M^c & B_M^{nc} & & \\ \langle nm\rangle & & & L-P_M^{nc} & & L-P_M^c \\ \langle nm,s\rangle & & & & L+B_M^c-R+\tau & \\ \langle nm,ns\rangle & & & & L-P_M^c+\eta R & \end{array}\right),$$

$$A_{\mathcal{C}} = \left(\begin{array}{r|ccccc} & \emptyset & \langle c\rangle & \langle nc\rangle & \langle c,t\rangle & \langle c,nt\rangle \\ \hline \emptyset & & & & & \\ \langle m\rangle & & B_C^m-K_C^m & -P_C^m & & \\ \langle nm\rangle & & & -P_C^{nm} & & B_C^{nm}-K_C^{nm} \\ \langle nm,s\rangle & & & & B_C^m-K_C^s+R+\tau & \\ \langle nm,ns\rangle & & & & B_C^{nm}-K_C^{nm}-\eta R & \end{array}\right),$$

$$E_{\mathcal{M}} = \left(\begin{array}{r|ccccc} & \emptyset & \langle m\rangle & \langle nm\rangle & \langle nm,s\rangle & \langle nm,ns\rangle \\ \hline \emptyset & 1 & & & & \\ (\mathcal{M}.1) & -1 & 1 & 1 & & \\ (\mathcal{M}.2) & & & -1 & 1 & 1 \end{array}\right),$$

and

$$E_{\mathcal{C}} = \left(\begin{array}{r|ccccc} & \emptyset & \langle c\rangle & \langle nc\rangle & \langle c,t\rangle & \langle c,nt\rangle \\ \hline \emptyset & 1 & & & & \\ (\mathcal{C}.1) & -1 & 1 & 1 & & \\ (\mathcal{C}.2) & & -1 & & 1 & 1 \end{array}\right).$$

Using the above matrices, we first write the utility maximization programs $(\mathfrak{P}_{\mathcal{M}})$ for $\mathcal{M}$ and $(\mathfrak{P}_{\mathcal{C}})$ for $\mathcal{C}$.

$$\begin{aligned} \max_{x_m,x_s} U_{\mathcal{M}} &= [(E-F)x_c - \eta R x_t + F - L]x_m + [(E+\tau-(1+\eta)R)x_t]x_s + (P_M^{nc}-P_M^c)x_c + \eta R x_t \\ \text{subject to} \quad & 0 \le x_m \le 1, && (4.1) \\ & 0 \le x_s \le 1 - x_m. && (4.2) \\ \max_{x_c,x_t} U_{\mathcal{C}} &= [(K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^m + G - H)x_m + H - K_C^{nm}]x_c \\ &\quad + [(B_{\mathcal{C}}^m - B_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^s + \tau + (1+\eta)R)x_s - \eta R(1-x_m)]x_t \\ &\quad + (P_C^{nm} - P_C^m)x_m - P_{\mathcal{C}}^{nm} \\ \text{subject to} \quad & 0 \le x_c \le 1, && (4.3) \\ & 0 \le x_t \le x_c. && (4.4) \end{aligned}$$

Secondly, we write their respective dual linear programs $(\mathfrak{D}_{\mathcal{M}})$ and $(\mathfrak{D}_{\mathcal{C}})$.

$$\begin{aligned} \min_{\alpha_m,\alpha_s} \quad & \alpha_m + \alpha_s \\ \text{subject to} \quad & \alpha_m + \alpha_s \ge (E-F)x_c - \eta R x_t + F - L, && (4.5) \\ & \alpha_s \ge (E+\tau-(1+\eta)R)x_t, && (4.6) \\ & \alpha_m,\ \alpha_s \ge 0, \\ \min_{\alpha_c,\alpha_t} \quad & \alpha_c \\ \text{subject to} \quad & \alpha_c - \alpha_t \ge (K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^m + G - H)x_m + H - K_C^{nm}, && (4.7) \end{aligned}$$

$$\alpha_t \geq (B_{\mathcal{C}}^m - B_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^s + \tau + (1+\eta)R)x_s - \eta R(1-x_m), \tag{4.8}$$
$$\alpha_c,\ \alpha_t \geq 0.$$

The primal-dual linear complementarity conditions follow.

$$\alpha_m(x_m - 1) = 0, \tag{4.9}$$
$$\alpha_s(x_m + x_s - 1) = 0, \tag{4.10}$$
$$\alpha_c(x_c - 1) = 0, \tag{4.11}$$
$$\alpha_t(-x_c + x_t) = 0, \tag{4.12}$$
$$x_m[\alpha_m + \alpha_s - (E-F)x_c + \eta R x_t - F + L] = 0, \tag{4.13}$$
$$x_s[\alpha_s - (E + \tau - (1+\eta)R)x_t] = 0, \tag{4.14}$$
$$x_c[\alpha_c - \alpha_t - (K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^m + G - H)x_m + H + K_C^{nm}] = 0, \tag{4.15}$$
$$x_t[\alpha_t - (B_{\mathcal{C}}^m - B_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^s + \tau + (1+\eta)R)x_s + \eta R(1-x_m)] = 0. \tag{4.16}$$

Any sequential Nash equilibrium satisfies conditions (4.1)–(4.16), and any solution vector $(x_m, x_s, x_c, x_t, \alpha_m, \alpha_s, \alpha_c, \alpha_t)$ that satisfies these conditions is a sequential Nash equilibrium [2].

4.1. No bribery, two SPNEs

To compute the sequential Nash equilibrium strategies, we approach the two linear programs as unconstrained optimization programs and determine extreme values of $(x_m, x_s, x_c, x_t, \alpha_m, \alpha_s, \alpha_c, \alpha_t)$. We then check whether the Nash equilibrium conditions (4.1)–(4.16) are satisfied.

In the following, we show that two equivalent extreme sequential equilibria are obtained when $\mathcal{C}$ does not initiate bribery. Let $I = H - K_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^m - G$ and $J = B_{\mathcal{C}}^m - B_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^s + \tau + (1+\eta)R$. Based on Assumptions 3.8 and 3.12, we deduce that $I > 0$ and $J > 0$.

Proposition 4.1. *For the Manager–Controller corruption game, two extreme sequential equilibria exist when $\mathcal{C}$ does not initiate bribery. They are SPNE. They are obtained when $x_m^* = \frac{H-K_C^{nm}}{I}$, $x_c^* = \frac{L-F}{E-F}$, $x_t^* = 0$ and x_s^* set to either $\frac{\eta R(1-x_m)}{J}$ for equilibrium (I), or to 0 for equilibrium (II).*

Proof of Proposition 4.1. The first order optimality conditions on $U_{\mathcal{M}}$, $\frac{\partial U_{\mathcal{M}}}{\partial x_m} = 0$, sets $x_t = \frac{(E-F)x_c + F - L}{\eta R}$ as a best response of $\mathcal{C}$. Assumption 3.6 infers that $\frac{L-F}{E-F} \in [0,\ 1]$. When $x_c = \frac{L-F}{E-F}$ and $x_t = 0$, the first order optimality conditions $\frac{\partial U_{\mathcal{M}}}{\partial x_s} = 0$ is satisfied if $E + \tau - (1+\eta)R > 0$. Otherwise, $x_t \leq x_c$; that is, x_t could be non-zero with the extreme value $x_t = x_c$. This case is addressed in Proposition 4.2.

Similarly, the first order optimality conditions on $U_{\mathcal{C}}$, $\frac{\partial U_{\mathcal{C}}}{\partial x_c} = 0$, sets $x_m = (H - K_C^{nm})/I$. Finally, $\frac{\partial U_{\mathcal{C}}}{\partial x_t} = 0$ sets $x_s = \eta R(1-x_m)/J$ as a best response of $\mathcal{M}$. Assumption 3.8 infers that $(H - K_C^{nm})/I \in [0,\ 1]$ and Assumption 3.12 infers that $(\eta R)/J \in [0,\ 1]$. Otherwise, $x_s \leq 1 - x_m$; *i.e.*, x_s could be non-zero with extreme value $x_s = 1 - x_m$. This case is addressed in Proposition 4.2.

When $x_t = 0$ both utility maximization programs become single variable optimization programs. The extreme vector (I), with $x_m = (H - K_C^{nm})/I, x_s = \eta R(1-x_m)/J, x_c = \frac{L-F}{E-F}, x_t = 0$ and $\alpha_m = \alpha_s = \alpha_c = \alpha_t = 0$, satisfies conditions (4.1)–(4.16). Switching x_s to 0 provides also a solution satisfying the first order optimization conditions and primal-dual conditions of $\mathfrak{P}_{\mathcal{M}}$. The extreme vector (II), with $x_m = (H - K_C^{nm})/I, x_s = 0, x_c = \frac{L-F}{E-F}, x_t = 0$, and $\alpha_m = \alpha_s = \alpha_c = \alpha_t = 0$, also satisfies conditions (4.1)–(4.16). Particularly, both sides of condition (4.14) are 0. The extreme vectors (I) and (II) are sequential Nash equilibria, and their convex combination is also a sequential equilibrium of the manager controller corruption game. □

Because bribery is not initiated ($x_t^* = 0$), the two sequential equilibria (I) and (II) are not only SPNE, but are also equivalent. They define a unique pair of equilibrium strategies (x_m^*, x_c^*) for $\mathcal{M}$ and $\mathcal{C}$.

4.2. Bribery, unique SPNE

In the following, we investigate the extreme values of x_t and x_s suggested by the feasibility frontiers of $(\mathfrak{P}_{\mathcal{M}})$ and $(\mathfrak{P}_{\mathcal{C}})$, as mentioned in the proof of Proposition 4.1. The economic interpretation of these frontiers assumes that $\mathcal{C}$ would play $x_t = x_c$ as if she were initiating bribery every time she plays x_c with a non-zero probability, and $\mathcal{M}$ would play $x_s = 1 - x_m$, as if she were always accepting the deal. Under these conditions, $(\mathfrak{P}_{\mathcal{M}})$ and $(\mathfrak{P}_{\mathcal{C}})$ would be rewritten as follows.

$$\begin{aligned}\max_{x_m} \quad & U_{\mathcal{M}} = [(R - F - \tau)x_c + F - L]x_m + [B_M^c + P_M^{nc} - R + \tau]x_c + L - P_M^c \\ & \text{subject to} \quad 0 \leq x_m \leq 1,\end{aligned}$$

and

$$\begin{aligned}\max_{x_c} \quad & U_{\mathcal{C}} = [(K_C^s - K_C^m - R - \tau - P_C^{nm} + P_C^m)x_m + K_C^s - R - \tau - P_C^{nm} - B_C^m]x_c \\ & \qquad + (P_C^{nm} - P_C^m)x_m - P_{\mathcal{C}}^{nm} \\ & \text{subject to} \quad 0 \leq x_c \leq 1.\end{aligned}$$

Using $Q = R + \tau + K_{\mathcal{C}}^m + P_{\mathcal{C}}^{nm} - P_{\mathcal{C}}^m - K_{\mathcal{C}}^s$, we derive the resulting unique SPNE.

Proposition 4.2. *For the Manager–Controller corruption game, a unique SPNE equilibrium (III) exits when $\mathcal{C}$ initiates bribery: $\mathcal{M}$ would set*

$$x_m^* = 1 - x_s^* = \frac{R + \tau + B_C^m + P_C^{nm} - K_C^s}{Q}$$

while $\mathcal{C}$ would set

$$x_c^* = x_t^* = \frac{L - F}{R - F - \tau}.$$

Proof of Proposition 4.2. The first order optimality conditions on $U_{\mathcal{M}}$, $\frac{\partial U_{\mathcal{M}}}{\partial x_m} = 0$, sets $x_c = \frac{L-F}{R-F-\tau}$ as a best response of $\mathcal{C}$. Because $L + \tau < R$, $F + \tau < R$ and $\frac{L-F}{R-F-\tau} \in [0,\ 1]$. In the same way, the first order optimality conditions on $U_{\mathcal{C}}$, $\frac{\partial U_{\mathcal{C}}}{\partial x_c} = 0$, sets $x_m = \frac{R+\tau+B_C^m+P_C^{nm}-K_C^s}{R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s}$. Assumption 3.8 infers that $K_C^{nm} < H$; therefore, $K_C^s - P_C^{nm} - \tau < K_C^s - K_C^{nm} + B_C^{nm} - B_C^m - \tau$. Hence, $R + \tau + B_C^m + P_C^{nm} - K_C^s \geq 0$. Similarly, Assumption 3.8 infers that $K_C^m > G$; therefore, $Q = R + \tau + K_C^m + P_C^{nm} - P_C^m - K_C^s > R + \tau + B_C^m + P_C^{nm} - K_C^s$. Hence, $\frac{R+\tau+B_C^m+P_C^{nm}-K_C^s}{Q} \in [0,\ 1]$. The extreme vector (III), with $x_m = \frac{R+\tau+B_C^m+P_C^{nm}-K_C^s}{Q}, x_s = (1 - x_m), x_c = x_t = \frac{L-F}{R-F-\tau}$, dual variables $\alpha_m = \alpha_c = 0$, $\alpha_s = \frac{(L-F)(E+\tau-(1+\eta)R)}{R-F-\tau}$, and $\alpha_t = \frac{(K_C^m-G)(R+\tau+B_C^m-B_C^{nm}+K_C^{nm}-K_C^s)}{Q}$ satisfies conditions (4.1)–(4.16). A unique extreme SPNE is obtained. □

Based on Proposition 4.2, the following corollary, whose proof is straightforward, can be deduced.

Corollary 4.3. *Assumptions 3.2–3.14 guarantee the uniqueness of the sequential Nash equilibrium $(x_m^*, x_s^*, x_c^*, x_t^*)$ of the Manager–Controller corruption game.*

The three sequential Nash equilibria derived from Propositions 4.1 and 4.2 are summarized in Table 1.

5. Sensitivity analysis and economic interpretation

This section analyzes the influence of the variation of the bribe amount and the payoff parameters of $\mathcal{M}$ and $\mathcal{C}$ on the corruption game Nash equilibria. It also highlights the economic interpretations of the findings.

TABLE 1. Sequential Nash equilibria of the manager-controller corruption game.

Eq.	x_m^*	x_s^*	x_c^*	x_t^*
(I)	$\frac{H-K_C^{nm}}{I}$	$\frac{\eta R(1-x_m)}{J}$	$\frac{L-F}{E-F}$	0
(II)	$\frac{H-K_C^{nm}}{I}$	0	$\frac{L-F}{E-F}$	0
(III)	$\frac{R+\tau+B_C^m+P_C^{nm}-K_C^s}{Q}$	$1-x_m$	$\frac{L-E}{R-F-\tau}$	$\frac{L-F}{R-F-\tau}$

Corollary 5.1. *If R increases, x_m^* increases or remains constant, x_s^* increases, decreases or remains constant, while x_c^* and x_t^* remain constant.*

Proof of Corollary 5.1. For equilibria (I) and (II), $\frac{\partial x_m^*}{\partial R}=0$ and $\frac{\partial x_c^*}{\partial R}=\frac{\partial x_t^*}{\partial R}=0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial R}=\frac{\eta(1-x_m)(J-\eta R)}{J^2}$. Under Assumption 3.12, $J-\eta R=R-(K_C^s-K_C^{nm}+B_C^{nm}-B_C^m-\tau)>0$. Therefore, $\frac{\partial x_s^*}{\partial R}>0$. For equilibrium (II), $\frac{\partial x_s^*}{\partial R}=0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial R}=-\frac{\partial x_s^*}{\partial R}=\frac{K_C^m-G}{Q^2}>0$ and $\frac{\partial x_c^*}{\partial R}=\frac{\partial x_t^*}{\partial R}=-\frac{L-F}{(R-F-\tau)^2}$ <0. □

When $\mathcal{C}$ does not initiate bribery, an increase of the bribe amount R does not have any impact on the levels of professional effort produced by $\mathcal{M}$ and $\mathcal{C}$, for equilibria (I) and (II). Meanwhile, for equilibrium (I), $\mathcal{M}$ would be more responsive to the bribing proposal. For equilibrium (III), when $\mathcal{C}$ initiates bribery, an increase of the bribe amount would make $\mathcal{M}$ more careful and less responsive to the bribing proposal, while $\mathcal{C}$ would reduce her levels of professional and bribing effort.

Corollary 5.2. *If τ increases, x_m^* increases or remains constant, x_s^* decreases or remains constant, while x_c^* and x_t^* increase.*

Proof of Corollary 5.2. For equilibria (I) and (II), $\frac{\partial x_m^*}{\partial \tau}=0$ and $\frac{\partial x_c^*}{\partial \tau}=\frac{\partial x_t^*}{\partial \tau}=0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial \tau}=-\frac{\eta R(1-x_m)}{J^2}<0$. For equilibrium (II), $\frac{\partial x_s^*}{\partial \tau}=0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial \tau}=-\frac{\partial x_s^*}{\partial \tau}=\frac{K_C^m-G}{\left(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s\right)^2}>0$ and $\frac{\partial x_c^*}{\partial \tau}=\frac{\partial x_t^*}{\partial \tau}=\frac{L-E}{(R-F-\tau)^2}>0$. □

When $\mathcal{C}$ does not initiate bribery, an increase of the reciprocity bonus τ does not have any impact on the level of professional efforts produced by $\mathcal{M}$ and $\mathcal{C}$, for equilibria (I) and (II). Meanwhile, for equilibrium (I), $\mathcal{M}$ would be less responsive to the bribing proposal. For equilibrium (III), when $\mathcal{C}$ initiates bribery, an increase of the reciprocity bonus would make $\mathcal{M}$ more careful and less responsive to the bribing proposal, while $\mathcal{C}$ would increase her levels of professional and bribing effort.

Corollary 5.3. *If η increases, x_m^*, x_c^* and x_t^* remain constant, while x_s^* increases or remains constant.*

Proof of Corollary 5.3. For equilibria (I) and (II), $\frac{\partial x_m^*}{\partial \eta}=0$ and $\frac{\partial x_c^*}{\partial \eta}=\frac{\partial x_t^*}{\partial \eta}=0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial \eta}=\frac{\eta R(1-x_m)(J-\eta R)}{J^2}>0$. For equilibrium (II), $\frac{\partial x_s^*}{\partial \eta}=0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial \eta}=\frac{\partial x_s^*}{\partial \eta}=0$ and $\frac{\partial x_c^*}{\partial \eta}=\frac{\partial x_t^*}{\partial \eta}=0$. □

An increase of the reputation gain index η does not have any direct impact on the behavior of $\mathcal{M}$ and $\mathcal{C}$ for equilibria (II) and (III). For equilibrium (I), $\mathcal{M}$ would be theoretically more responsive to the bribing proposal. However, η regulates the size of bribes: $R<\frac{E}{1+\eta}$, as imposed by Assumption 3.12.

Corollary 5.4. *If E increases, x_m^* and x_s^* remain constant, while x_c^* decreases or remains constant and x_t^* remains constant.*

Proof of Corollary 5.4. For both equilibria (I) and (II), $\frac{\partial x_m^*}{\partial E} = \frac{\partial x_s^*}{\partial E} = \frac{\partial x_t^*}{\partial E} = 0$, and $\frac{\partial x_c^*}{\partial E} = -\frac{L-F}{(E-F)^2} \leq 0$ under Assumption 3.6. For equilibrium (III), $\frac{\partial x_m^*}{\partial E} = \frac{\partial x_s^*}{\partial E} = 0$ and $\frac{\partial x_c^*}{\partial E} = \frac{\partial x_t^*}{\partial E} = 0$. □

For equilibria (I) and (II), when $\mathcal{C}$ does not initiate bribery, an increase of $\mathcal{M}$'s bonus or penalty makes $\mathcal{C}$ further careless. However, this increase has no impact on the performance of $\mathcal{M}$. For equilibrium (III), when $\mathcal{C}$ initiates bribery, an increase of $\mathcal{M}$'s bonus or penalty has no impact on both $\mathcal{M}$ and $\mathcal{C}$.

Corollary 5.5. *If F increases, x_m^* and x_s^* remain constant, while x_c^* decreases and x_t^* decreases or remains constant.*

Proof of Corollary 5.5. For equilibria (I) and (II), $\frac{\partial x_m^*}{\partial F} = \frac{\partial x_s^*}{\partial F} = \frac{\partial x_t^*}{\partial F} = 0$ and $\frac{\partial x_c^*}{\partial F} = \frac{L-E}{(E-F)^2} < 0$ under Assumption 3.6. For equilibrium (III), $\frac{\partial x_m^*}{\partial F} = \frac{\partial x_s^*}{\partial F} = 0$ and $\frac{\partial x_c^*}{\partial F} = \frac{\partial x_t^*}{\partial F} = \frac{L+\tau-R}{(R-F-\tau)^2} < 0$. □

When $\mathcal{C}$ does not initiate bribery, an increase of $\mathcal{M}$'s bonus or penalty would make $\mathcal{C}$ further careless. However, this increase has no impact on the performance of $\mathcal{M}$. Moreover, for equilibrium (III), when $\mathcal{C}$ initiates bribery, an increase of $\mathcal{M}$'s bonus or penalty has no impact on $\mathcal{M}$ but makes $\mathcal{C}$ reduce her levels of both professional and bribing effort.

Corollary 5.6. *If G increases, x_m^* increases, x_s^* decreases, while x_c^* and x_t^* remain constant.*

Proof of Corollary 5.6. For equilibria (I) and (II), under Assumption 3.8, $\frac{\partial x_m^*}{\partial G} = \frac{H-K_C^{nm}}{I^2} > 0$ while $\frac{\partial x_c^*}{\partial G} = 0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial G} = -\frac{\eta R}{Q}\left(\frac{\partial x_m^*}{\partial G} + \frac{1-x_m}{Q}\right) < 0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial B_C^m} = -\frac{\partial x_s^*}{\partial B_C^m} = 1/(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s)^2 > 0$ and $\frac{\partial x_m^*}{\partial P_C^m} = -\frac{\partial x_s^*}{\partial P_C^m} = (1+x_m)/(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s) > 0$. □

For equilibria (I) and (II), when $\mathcal{C}$ does not compile a precise report, an increase of $\mathcal{C}$'s bonus or penalty would make $\mathcal{M}$ more careful. Moreover, for equilibrium (III), an increase of $\mathcal{C}$'s bonus or penalty would also make $\mathcal{M}$ more careful and less responsive to the bribing proposal when $\mathcal{C}$ initiates bribery.

Corollary 5.7. *If H increases, x_m^* increases or decreases or remains constant, x_s^* increases or decreases or remains constant, while x_c^* and x_t^* remain constant.*

Proof of Corollary 5.7. For equilibria (I) and (II), under Assumption 3.8, $\frac{\partial x_m^*}{\partial H} = \frac{K_C^m-G}{I^2} > 0$ while $\frac{\partial x_c^*}{\partial H} = 0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial H} = \frac{(K_C^m-G)(I-J)}{JI^2} \gtrless 0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial B_C^{nm}} = \frac{\partial x_s^*}{\partial B_C^{nm}} = 0$ and $\frac{\partial x_m^*}{\partial P_C^{nm}} = -\frac{\partial x_s^*}{\partial P_C^m} = \frac{K_C^m-G}{\left(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s\right)^2} > 0$. □

For equilibria (I) and (II), when $\mathcal{C}$ compiles a precise report, an increase of $\mathcal{C}$'s bonus or penalty makes $\mathcal{M}$ more careful when $\mathcal{C}$ does not initiate bribery. Meanwhile, for equilibrium (III), an increase of $\mathcal{C}$'s bonus or penalty makes $\mathcal{M}$ indifferent or more careful and less responsive to the bribing proposal when $\mathcal{C}$ initiates bribery.

Corollary 5.8. *If L increases, x_m^* and x_s^* remain constant, while x_c^* and x_t^* increase.*

Proof of Corollary 5.8. For both equilibria (I) and (II), under Assumption 3.2, $\frac{\partial x_m^*}{\partial L} = \frac{\partial x_s^*}{\partial L} = \frac{\partial x_t^*}{\partial L} = 0$ and $\frac{\partial x_c^*}{\partial L} = \frac{1}{E-F} > 0$. For equilibrium (III), under Assumptions 3.6 and 3.14, $\frac{\partial x_m^*}{\partial L} = \frac{\partial x_s^*}{\partial L} = 0$ and $\frac{\partial x_c^*}{\partial L} = \frac{\partial x_t^*}{\partial L} = \frac{1}{R-F-\tau} > 0$. □

When $\mathcal{C}$ initiates bribery, the controller $\mathcal{C}$ should reinforce a larger inspection effort, while $\mathcal{M}$ is more likely to maintain her level of professional effort if her estimated leisure gain increases.

Corollary 5.9. *If K_C^m, K_C^{nm} or K_C^s increase, x_m^* remains constant, x_s^* increases or decreases or remains constant, while x_c^* and x_t^* remain constant.*

Proof of Corollary 5.9. First, for equilibria (I) and (II), under Assumption 3.8, $\frac{\partial x_m^*}{\partial K_C^m} = \frac{K_C^{nm}-H}{\left(H-K_C^{nm}+K_C^m-G\right)^2} < 0$, and $\frac{\partial x_c^*}{\partial K_C^m} = 0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial K_C^m} = -\frac{\eta R}{J}\frac{\partial x_m^*}{\partial K_C^m} > 0$. For equilibrium (III), $\frac{\partial x_c^*}{\partial K_C^m} = \frac{\partial x_t^*}{\partial K_C^m} = 0$ and $\frac{\partial x_m^*}{\partial K_C^m} = -\frac{\partial x_s^*}{\partial K_C^m} = -\frac{R+\tau+B_C^m+P_C^{nm}-K_C^s}{\left(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s\right)^2} < 0$.

Secondly, for equilibria (I) and (II), under Assumption 3.8, $\frac{\partial x_m^*}{\partial K_C^{nm}} = \frac{H+G-K_C^m}{\left(H-K_C^{nm}+K_C^m-G\right)^2} > 0$, and $\frac{\partial x_c^*}{\partial K_C^{nm}} = 0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial K_C^{nm}} = -\frac{\eta R}{J}\left(\frac{\partial x_m^*}{\partial K_C^{nm}} + \frac{1-x_m}{J}\right) < 0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial K_C^{nm}} = \frac{\partial x_s^*}{\partial K_C^{nm}} = 0$ and $\frac{\partial x_c^*}{\partial K_C^{nm}} = \frac{\partial x_t^*}{\partial K_C^{nm}} = 0$.

Finally, for equilibria (I) and (II), $\frac{\partial x_m^*}{\partial K_C^s} = 0$, and $\frac{\partial x_c^*}{\partial K_C^s} = 0$. For equilibrium (I), $\frac{\partial x_s^*}{\partial K_C^s} = \frac{\eta R(1-x_m)}{J^2} > 0$. For equilibrium (III), $\frac{\partial x_m^*}{\partial K_C^s} = -\frac{\partial x_s^*}{\partial K_C^s} = \frac{G-K_C^m}{\left(R+\tau+K_C^m+P_C^{nm}-P_C^m-K_C^s\right)^2} < 0$ and $\frac{\partial x_c^*}{\partial K_C^s} = \frac{\partial x_t^*}{\partial K_C^s} = 0$. □

For equilibria (I) and (II), $\mathcal{M}$ should act more carefully when $\mathcal{C}$ does not initiate bribery and the cost of a precise report on a non-methodical effort of $\mathcal{M}$ increases. However, for equilibrium (III), $\mathcal{M}$ becomes further careless and more responsive to the bribing proposal when $\mathcal{C}$ initiates bribery if either the cost of a precise report on a methodical effort of $\mathcal{M}$ increases or the cost of a fake report increases.

6. Discussion

Herein, we compile the sensitivity analysis results and discuss their economic implications. Using a numerical example, we illustrate these results and analyze the impact of whistle blowing on our current game. We then undertake the same investigation on two alternate scenarios.

Table 2 compiles the sensitivity analysis results. Column "Param." indicates the parameters. Columns (1)–(3) indicate the corresponding equilibria. The symbols $\nearrow$, $\searrow$ and $\rightarrow$ indicate that the variables x_m^*, x_s^*, x_c^* or x_t^* increase, decrease, or remain constant. Table 4 confirms that $\mathcal{C}$ is opportunistic. An increase of the bribery amount leads to a decrease of her level of professional effort. When increasing the bribery amount, $\mathcal{C}$ expects that manager $\mathcal{M}$ lowers her level of professional effort. However, rational manager $\mathcal{M}$ would counter $\mathcal{C}$'s expectation. In fact, in what appears to be a paradoxical behavior, $\mathcal{M}$ increases her level of professional effort. This just confirms that $\mathcal{M}$ is risk averse as any increase of the bribery amount leads to a decrease of her level of responsiveness to the bribery proposal, which in turn leads to a decrease of the bribery effort made by $\mathcal{C}$. Moreover, an increase of the reciprocity bonus leads to a decrease of $\mathcal{M}$'s responsiveness to the bribery proposal, while a rational manager is expected to do the opposite. $\mathcal{M}$ would counter all expectations and increase her level of professional effort as she anticipates an increase of $\mathcal{C}$'s inspection effort. Example 6.1 illustrates theses observations.

Example 6.1. Consider the corruption game whose parameters are listed in Table 3. This game has $E = 260$, $F = 142$, $G = 175$, $H = 550$, $R = 260$, $\tau = 70$ and $\eta = 0.1$. These entities satisfy Assumptions 3.2–3.14.

The three extreme sequential Nash equilibria of this game are listed in Table 4. Figure 2a displays the optimal responses of the two players to an increase of R in equilibrium (III). While the optimal response (blue curve) of $\mathcal{M}$ increases, the optimal response (orange curve) of $\mathcal{C}$ decreases. Figure 2b illustrates the optimal responses of the two players to an increase of τ. The optimal response (orange curve) of $\mathcal{C}$ increases faster than the optimal response of $\mathcal{M}$.

Even though any increase of the reputation gain reduces the size of a potential bribe, it does not have any direct influence on $\mathcal{M}$ or $\mathcal{C}$'s behaviors. This concords with the findings of Abbink and Wu [1] who show that

TABLE 2. Summary of sensitivity analysis results.

	(I)				(II)				(III)			
Param.	x_m^*	x_s^*	x_c^*	x_t^*	x_m^*	x_s^*	x_c^*	x_t^*	x_m^*	x_s^*	x_c^*	x_t^*
$R \nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\searrow$	$\searrow$	$\searrow$
$\tau \nearrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\searrow$	$\nearrow$	$\nearrow$
$\eta \nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$E \nearrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$F \nearrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\searrow$
$G \nearrow$	$\nearrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\searrow$	$\rightarrow$	$\rightarrow$
$H \nearrow$	$\nearrow$	.	$\rightarrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow\rightarrow$	$\rightarrow\searrow$	$\rightarrow$	$\rightarrow$
$L \nearrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\nearrow$
$K_{\mathcal{C}}^m \nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\nearrow$	$\rightarrow$	$\rightarrow$
$K_{\mathcal{C}}^{nm} \nearrow$	$\nearrow$	$\searrow$	$\rightarrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$
$K_{\mathcal{C}}^s \nearrow$	$\rightarrow$	$\nearrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\rightarrow$	$\searrow$	$\nearrow$	$\rightarrow$	$\rightarrow$

TABLE 3. Parameters of $\mathcal{M}$-$\mathcal{C}$ game in Example 6.1.

$\mathcal{M}$	$\mathcal{C}$	Game
$L = 170$	$K_{\mathcal{C}}^m = 250,\ K_{\mathcal{C}}^{nm} = 300,\ K_C^s = 400$	$R = 260$
$B_{\mathcal{M}}^c = 40$	$B_{\mathcal{C}}^m = 75$	$\tau = 70$
$B_{\mathcal{M}}^{nc} = 32$	$B_{\mathcal{C}}^{nm} = 300$	$\eta = 0.1$
$P_{\mathcal{M}}^c = 220$	$P_{\mathcal{C}}^m = 100$	
$P_{\mathcal{M}}^{nc} = 110$	$P_{\mathcal{C}}^{nm} = 250$	

TABLE 4. SPNE of the manager-controller corruption game.

Eq.	x_m^*	x_s^*	x_c^*	x_t^*
(I)	$\frac{10}{13}$	$\frac{6}{31}$	$\frac{14}{59}$	$\frac{14}{59}$
(II)	$\frac{10}{13}$	0	$\frac{14}{59}$	$\frac{14}{59}$
(III)	$\frac{17}{22}$	$\frac{5}{22}$	$\frac{7}{12}$	$\frac{7}{12}$

permitting only one party to self-report does not significantly deter corruption, while enabling both parties to report does.

The alternate scenario, given by Figure 3, confirms our results. In this second scenario, when $\mathcal{M}$ discovers the controller's careful inspection, $\mathcal{M}$ proposes to bribe $\mathcal{C}$ so that $\mathcal{C}$ cooks a fake report. Controller $\mathcal{C}$ has the possibility to either accept or reject this deal. $\mathcal{C}$ may reject this deal when she seeks a reputation gain.

Under this second scenario, Assumption 3.12 must be reformulated as $K_C^s - K_C^{nm} + B_C^{nm} - B_C^m - \tau \leq R \leq \frac{E+\tau}{1-\eta}$. The equilibrium computation results show that the only existing sequential Nash equilibrium is equilibrium (III). Indeed, the pair of strategies played by $\mathcal{M}$ and $\mathcal{C}$ in equilibria (I) and (II), respectively, are no longer best responses to each others' choices. This is mainly due to the fact that condition (4.8) rewritten as $\alpha_t \geq (B_{\mathcal{C}}^m - B_{\mathcal{C}}^{nm} + K_{\mathcal{C}}^{nm} - K_{\mathcal{C}}^s + \tau + (1-\eta)R)x_s + \eta R(1-x_m)$, makes $\alpha_t > 0$ when $x_m = \frac{H-K_{\mathcal{C}}^{nm}}{I}$, as obtained

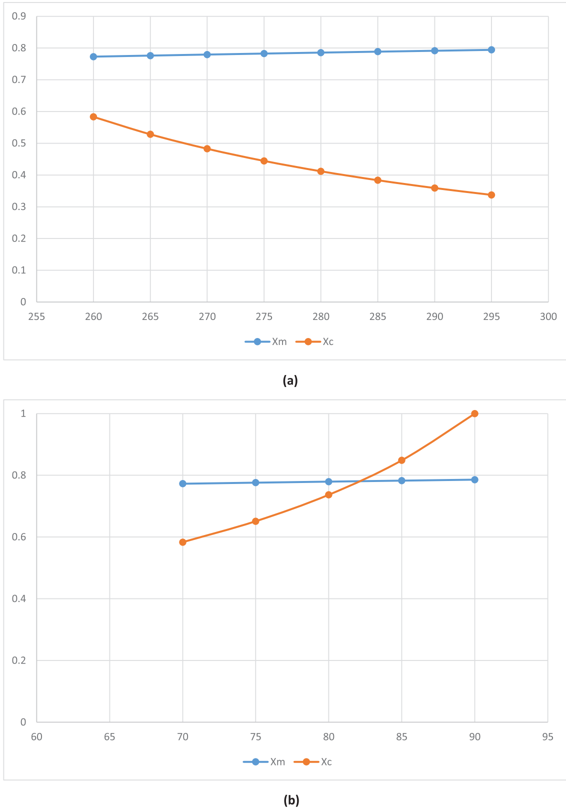

FIGURE 2. Increase of R *vs.* increase of τ in equilibrium (III). (a) Impact of increase of R. (b) Impact of increase of τ.

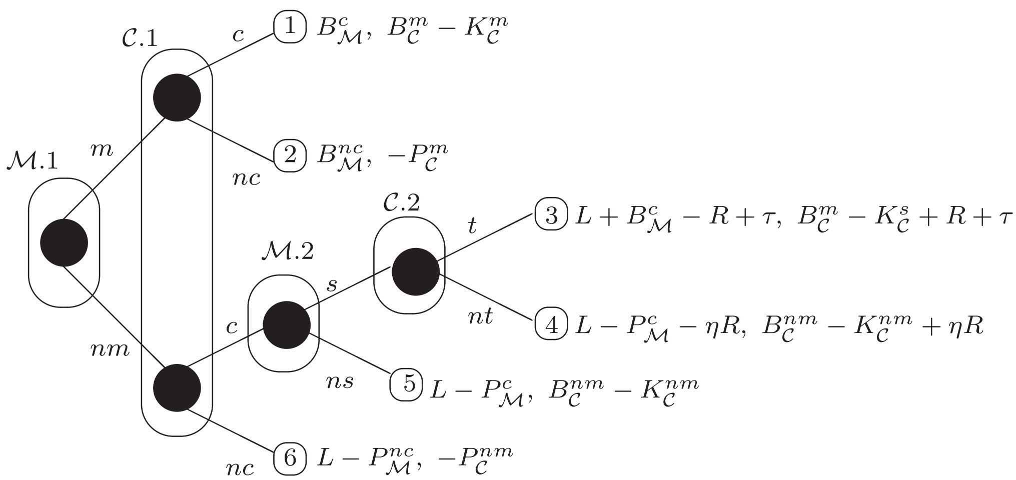

FIGURE 3. Scenario 2: Manager initiates corruption.

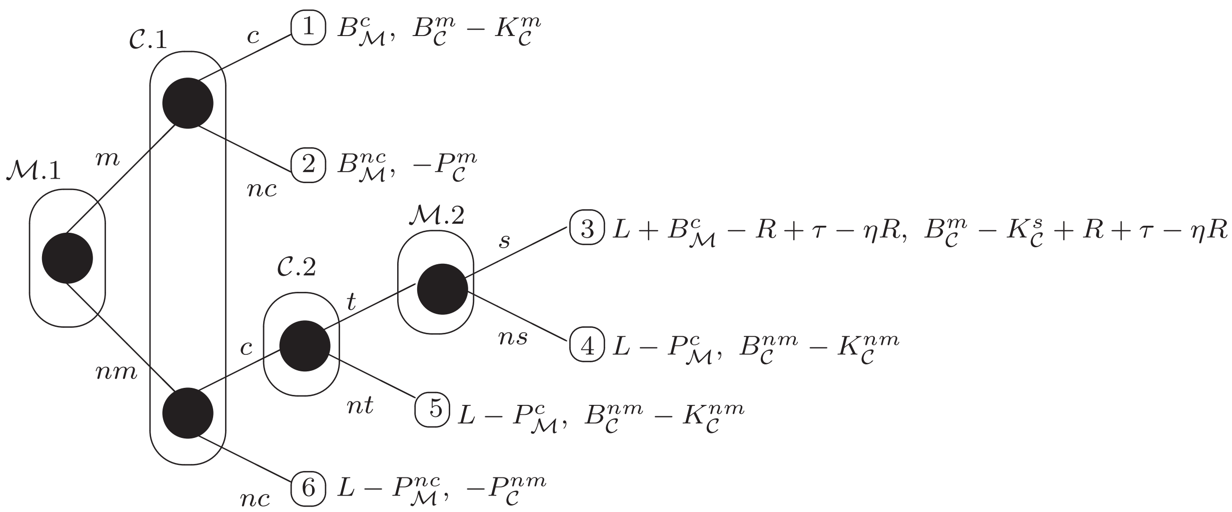

FIGURE 4. Scenario 3: Whistle blowing initiated by an exogenous agent.

in extreme vectors (I) and (II), while condition (4.12) forces $\alpha_t = 0$, when $x_t = 0$. Thus, even though it reduces the size of the bribe amount, endogenous whistle-blowing has no effect on the behavior of both $\mathcal{M}$ and $\mathcal{C}$.

The last alternate scenario, given by Figure 4, yields completely different results. In this third scenario, whistle blowing is initiated by an exogenous agent. Specifically, when $\mathcal{M}$ and $\mathcal{C}$ agree on the corruption terms and bribery is *de facto* consumed (*i.e.*, it is no longer an attempt but has become a fact), an external agent blows the whistle.

Under this third scenario, Assumption 3.12 must be reformulated as

$$\frac{K_{\mathcal{C}}^{s} - K_{\mathcal{C}}^{nm} + B_{\mathcal{C}}^{nm} - B_{\mathcal{C}}^{m} - \tau}{1-\eta} \leq R \leq \frac{E+\tau}{1+\eta},$$

and Assumption 3.14 has to be reformulated as $L + \tau \leq (1+\eta)R$. The equilibrium computation results show the existence of a unique sequential Nash equilibrium with $x_m^* = \frac{P_{\mathcal{C}}^{nm} + B_{\mathcal{C}}^{m} - K_{\mathcal{C}}^{s} + \tau + (1-\eta)R}{P_{\mathcal{C}}^{nm} - P_{\mathcal{C}}^{m} + K_{\mathcal{C}}^{m} - K_{\mathcal{C}}^{s} + \tau + (1-\eta)R}$, $x_s^* = 1 - x_m^*$, and $x_c^* = x_t^* = \frac{L-F}{R-F-\tau+\eta R}$. The first partial derivatives with respect to η are $\frac{\partial x_m^*}{\partial \eta} = -\frac{\partial x_s^*}{\partial \eta} = \frac{G-K_{\mathcal{C}}^{m}}{Q^2} < 0$ and $\frac{\partial x_c^*}{\partial \eta} = \frac{\partial x_t^*}{\partial \eta} = -\frac{L-F}{(R-F-\tau+\eta R)^2} < 0$. Therefore, an increase of the exogenous whistle-blowing reputation loss (or probability) increases the responsiveness of $\mathcal{M}$ to any bribery proposal even though it decreases the level of bribing effort initiated by $\mathcal{C}$ and reduces the levels of professional performance of $\mathcal{M}$ and $\mathcal{C}$. This is paradoxical, but surprisingly quite rational: As $\mathcal{C}$ looses more of her leverage on $\mathcal{M}$'s behavior, she would be less interested in compiling a precise report and detecting any managerial misconduct. $\mathcal{M}$ anticipates $\mathcal{C}$'s reaction and decreases her professional effort. It follows that even though it reduces $\mathcal{C}$'s opportunism, exogenous whistle-blowing may have unhealthy effects on $\mathcal{M}$'s and $\mathcal{C}$'s levels of performance.

7. Conclusion

In this article, we proposed a game theoretical model to analyze the impact of bribery on managers and controllers in the context of an inspection management conflict. Following a number of motivated assumptions, we identified three different subgame perfect sequential Nash equilibria, including a unique equilibrium when bribery is initiated. In addition, we studied the sensitivity of these equilibria to the problem parameters and discussed the effect of whistle-blowing. We showed that whistle blowing is partially effective when it may be initiated by an external agent at some point in time, and may have a negative effect on the company's performance.

A natural extension to our work would introduce a third player representing the board of directors, and study the effects of different corruption, collusion and whistle-blowing scenarios on the game outcome. A further development could be more focused on distinguishing among different levels of controls by internal and external controllers.

Acknowledgements. This work was partly funded by King Fahd University of Petroleum and Minerals research grant POC19103 and the Interdisciplinary Research Center on Smart Mobility and Logistics research grant INML2107. This support is gratefully acknowledged.

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