RAIRO-Oper. Res. 56 (2022) 49–75
https://doi.org/10.1051/ro/2021187

RAIRO Operations Research
www.rairo-ro.org

PRICING DECISION OF A DUAL-CHANNEL SUPPLY CHAIN WITH DIFFERENT PAYMENT, CORPORATE SOCIAL RESPONSIBILITY AND SERVICE LEVEL

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Abstract. Strategies such as price, CSR, and service have an important impact on enterprises and supply chains. This paper proposes a two-echelon dual-channel supply chain composed of a manufacturer and a retailer. Considering the product pricing, CSR level, and service level in the supply chain, this paper employs the Stackelberg game to depict supply chain participants' optimal decisions and analyze the influence of explanatory variables on the optimal decision with retailer's payment methods. The results state that market share, service level, CSR, and financing interest rate significantly impact the pricing decision of all participants in the supply chain. In addition, strategies of CSR level and service level are also affected by the discount rate of advance payment, financing interest rate, return on investment, and opportunity cost rate. This paper incorporates CSR and service level into the objective function, considers a variety of retailers' payment methods, enriches the supply chain's pricing model, and is of great value to scientific decision-making of enterprises and sustainable development of supply chains.

Mathematics Subject Classification. 90B06, 91A35.

Received April 8, 2021. Accepted December 16, 2021.

1. Introduction

With the rapid development of Internet technology, the mature e-commerce mode has quietly changed people's consumption habits and strengthened people's consumption desire, in 2019, the total retail sales of China's social consumer goods reached 41.1649 trillion yuan, accounting for 41.5% of GDP in that year. Consumer spending has become the main engine of China's rapid and healthy economic development. In addition, efficient logistics transportation and convenient online payment tools also make more and more consumers tend to diversify the way of shopping, which gives birth to a variety of channels of sales mode, and the competition for market share of enterprises also arises, there is a close relationship between upstream and downstream enterprises in the supply chain, in order to maintain the stability of the whole supply chain, the pricing of enterprises is particularly important; taking social responsibility actively is an important part of corporate culture. This year,

Keywords. Corporate social responsibility, service level, dual-channel supply chain, payment methods, pricing decision, competition.

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affected by the novel coronavirus pneumonia, many farm products are unsalable, major e-commerce platforms have launched assistance measures, Alibaba launched the "love for agriculture" plan, Jingdong (JD) opened the "green channel for fresh agricultural products", Pinduoduo explored the new mode of "live broadcast by city and county heads, and more sales by farmers". Suning cooperated with Suning store, Carrefour and other forces to expand the sales of agricultural products, the "helping agriculture" actions of these enterprises not only guaranteed people's livelihood, stabilized prices, and laid a good social foundation for enterprises, but also innovated and explored the retail industry of China's rural economy. Therefore, the research on the level of CSR is very beneficial to the long-term development of enterprises, and will also promote social harmony and stability to a certain extent. Therefore, it is meaningful for us to study the CSR level for enterprises to formulate its long-term development strategies and improve its comprehensive competitiveness.

With the continuous progress of society and the improvement of people's living standards, people are more and more pursuing the all-weather, all space, and personalized shopping experience, service is an important part of the shopping experience, when shopping online, consumers and sellers can't trade face to face, they can only understand customers' needs and answer customers' questions through customer service, product display is only in the form of pictures or videos, in other words, compared with offline channels, due to the limitations of cost, space and other factors, the service experience provided by online channels is often inferior to offline channels in some aspects, improving the service level of offline channels is very helpful to attract service sensitive customers. Therefore, it is of great significance to study the service level for enterprises to formulate reasonable service strategies and attract more customers.

In this era of big data with a high-speed flow of all kinds of information, the marketing environment is changing rapidly. Enterprises in supply chains need to adjust their strategies in time according to the information they can obtain. In a dual-channel supply chain, various payment methods of retailers will affect the retailer service level, the CSR level, and the objective function of enterprise decision-making. Therefore, studying the impact of different payment methods of retailers on the optimal strategies of all parties in the supply chain is conducive to promote the stability and coordinated development of the whole supply chain.

This paper focuses on constructing Stackelberg game models for a two-level supply chain composed of a manufacturer and a retailer. In addition, this paper incorporates the service level and CSR level into the enterprise's product demand functions and profit functions. At last, this paper analyzes the impact of market share, service level, financing interest rate, and opportunity cost rate on the optimal decision-making of enterprises.

The remainder of this paper is organized as follows. The relevant literature is reviewed in Section 2. Stackelberg game models are depicted in Section 3. The models are analyzed based on various payment methods of the retailer in Section 4. The results is validated by numerical simulations in Section 5. Finally, this paper concludes in Section 6.

2. Literature review

2.1. Distribution channel

With the rapid development of Internet technology and e-commerce platform, more and more enterprises have chosen as many sales channels as possible, for example, they integrate physical stores, online stores, social media, etc. They also ensure the smooth progress of sales through dual-channel supply chain [19], on the basis of traditional retail channels, the operation mode of manufacturer to establish online channels to sell products has become increasingly mature. At the same time, it also changes the sales mode and cooperation mode of the traditional supply chain, so that enterprises in the supply chain not only have cooperative relations, but also have competitive relations, therefore, compared with the traditional single channel supply chain, enterprise decision-making has changed, and the mutual influence is more complex [19, 20]. Therefore, we can learn that it is very valuable to study the coopetition in a dual-channel supply chain.

2.2. Corporate Social Responsibility (CSR)

The literature about CSR at home and abroad mainly focuses on empirical research, and basically only stays at the individual level, and there are not many literatures extending to the supply chain level. In March 2003, the European Union formally put CSR on the agenda of the European Union, which is regarded as one of the strategic tools to make companies more competitive and social cohesion [15]. In practical activities, it also shows that enterprises practicing CSR activities tend to improve the loyalty of investors or customers, reduce the systematic risk of enterprise operation, and strengthen the competitive advantage [1, 5]. Modak et al. framed and pushed forward the debate [17], and studied structure of a supply chain regarding social work donation as a CSR practice [16, 18]. Zhang *et al.* [24] constructed a supply chain operation model composed of manufacturer and retailer through wholesale price contract, to explore the influence of CSR preference on supply chain profitability, the results show that with the increase of CSR preference of manufacturer, product quality and order quantity increase. Fan *et al.* [6, 7] defines the degree of concern of supply chain enterprises to stakeholders as their CSR behavior, and examines the impact of manufacturers' CSR and quality improvement efficiency on supply chain profitability, the research shows that manufacturers' profit changes depend on the relative size of the degree of CSR and quality improvement efficiency. However, little literature involves the coopetition of CSR level in a dual-channel supply chain.

2.3. Enterprise's service level and supply chain pricing

In terms of service level and supply chain pricing, there are relatively few literatures. Based on game theory and optimization theory, Dan *et al.* [5] constructed a decision-making model for manufacturers and sales service integrators, analyzed and compared the service level and product demand under centralized decision-making and decentralized decision-making, the research shows that with the improvement of service level, the product demand increases under the centralized decision-making. Li *et al.* [14] established a supply chain model of two competitive manufacturers and one retailer, introduced channel service level, discussed its impact on the profits of each member of the supply chain, and studied the optimal decisions of each member in different situations, the research shows that with the improvement of channel service level, the profits of each member of the supply chain also increase. For service sensitive customers, He *et al.* [8] analyzed the impact of promotion service level on pricing under decentralized and centralized decision-making, the research results show that with the improvement of service level of retailers, the sales price of traditional retail channels will also increase, while the price of direct sales channels will decrease. Obviously, there are not many researchers analyzed the choice of various payment methods in a dual-channel supply chain.

2.4. The choice of financing methods

The problem of financing difficulty and high cost has been perplexing the development of small and medium-sized enterprises in China. The solution to the problem of financing constraints depends on financial innovation and financial development [10]. The traditional financing methods of enterprises are nothing more than bank lending and supply chain internal financing, the latter belongs to the scope of transaction credit. In the aspect of bank financing, Buzacott *et al.* [3] discussed the operation and financing decision of capital constrained enterprises, when banks are strategic decision makers in the context of inventory management. The research on transaction credit originated from economics, Schwartz *et al.* [21] put forward the Financing Motivation Theory earlier, which emphasizes that if the seller is easier to access the capital market than the buyer, then the seller has the motivation to provide the buyer with cheaper transaction credit. Based on the bilateral capital constrained supply chain, Jin *et al.* [12] established a dynamic game model including manufacturers, retailers and banks, and compared the financing strategies composed of transaction credit and bank loans from the perspective of enterprises and banks, in which transaction credit includes deferred payment and advance payment. It is found that the supply chain's choice of balanced financing strategy depends on the supply chain's capital shortage. From the perspective of financial institutions providing financing services for capital constrained supply chain, Huang *et al.* [9] compared and analyzed the income difference of financial institutions under the strategy of

providing one-way financing services and two-way financing services, and obtained the optimal financing service strategy under the condition of interest rate ceiling. Bing *et al.* [2] studied the supply chain composed of manufacturers and retailers with capital constraints, compared the equilibrium solution of the supply chain under two kinds of loan methods: bank loan and commercial credit, and concluded that under the condition of low marginal cost of products, commercial credit can make the supply chain obtain higher profits than bank loan, and can alleviate the problem of double marginalization. Chen *et al.* [4] compared bank loan and commercial credit, and considered that if commercial credit is selected under wholesale price contract, manufacturers can share the risk of retailers, reduce their marginal cost and increase the order quantity, so commercial credit can make supply chain members get better benefits and achieve the unique financing equilibrium. Kouvelis *et al.* [13] studied the case that both retailers and suppliers are subject to capital constraints, and pointed out that whether retailers prefer trade credit financing or not depends on the specific parameters of trade credit contract. Obviously, there are not many researchers studied the coopetition of service level in a dual-channel supply chain.

2.5. Different payment methods for the retailer

In terms of different payment methods for retailers, Jin *et al.* [11] constructed a supply chain composed of a single manufacturer and a single retailer. Starting from the maximization of enterprise profits, the pricing decision model of the supply chain under different payment methods was derived. The research shows that the selection of different payment methods in the supply chain is based on the financing service interest rate. In another paper of the author, a dynamic game model based on push transaction mode is constructed for three payment methods of retailers, and a wholesale price incentive mechanism is designed under different payment methods, the research shows that retailers will definitely choose to postpone payment without wholesale price incentive, and the selection of retailers and manufacturers under wholesale price incentive is mainly based on wholesale price, risk-free interest rate, production and sales cycle and other parameters, and both sides have similar optimal payment options, which reveals that wholesale price incentive can partially coordinate payment delivery [22]. In addition, based on the manufacturer's wholesale price discount incentive and deferred payment interest rate, the author constructs the supply chain operation model under different payment methods, the research shows that the retailer's optimal payment method is mainly based on the wholesale price discount rate and financing service interest rate [23]. Obviously, there are not many researchers studied the impact of different payment methods of retailers on the optimal strategies of all parties in a dual-channel supply chain.

Based on the existing literature on supply chain pricing decision-making, this paper introduces the service competition between the retailer and the manufacturer, the opportunity cost and investment income derived from capital flow under different payment methods. In addition, the research on CSR at home and abroad mainly focuses on empirical research, by setting variables, this paper brings the service level of the retailer and CSR level of supply chain into the demand functions and profit functions of enterprises, explores the optimal decision of supply chain participants under various payment methods of the retailer, and further analyzes the influence of market share, financing interest rate, opportunity cost rate and other explanatory variables on the optimal decision-making of enterprises.

The main problems to be solved in this paper are as follows:

(1) The optimal decision of each participant in the supply chain when the retailer chooses different payment methods;

(2) The impact of market share, service level, CSR level, discount rate of advance payment, financing interest rate, rate of return on investment and opportunity cost rate on the optimal decision of enterprises.

3. Description and notations

As shown in Figure 1, there is a manufacturer and a retailer in the market. In the decision-making process, the manufacturer is the leader of the Stackelberg game and the retailer is the follower. The consumers are service sensitive customers. The information of each node enterprise in the supply chain is symmetrical.

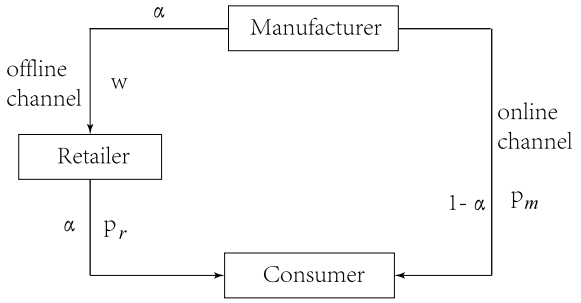


FIGURE 1. Supply chain operation flow chart.

It can be seen from the above literature that service level has the effect of promoting sales. Suppose that the retailer will improve the service level in order to compete for market share, and the service level is set as g, this will bring them additional market share, and at the same time, it will seize part of the customers in the manufacturer's online channel, which is expressed as tg, $t(0 \leq t \leq 1)$ is the service sensitivity coefficient, and the service cost is expressed as $\frac{1}{2}sg^2$, s is the service cost coefficient. As CSR can enhance corporate image, significantly improve the brand awareness of consumers, and thus improve the market demand of products, so the investment of CSR will affect the social demand of enterprises. by_r and by_m are respectively included in the demand functions of the retailer and the manufacturer, b is the marginal impact of CSR level on market demand, that is, the demand elasticity of CSR, y_r and y_m are the levels of CSR of the retailer and manufacturer. Considering that the marginal cost of CSR input per unit product is greater than zero, and the cost of CSR input per unit product is increasing, supposing the retailer's CSR cost input be $\frac{1}{2}c_r y_r^2$, and the manufacturer's CSR cost input is $\frac{1}{2}c_m y_m^2$. c_r and c_m are the cost coefficients of retailer's and manufacturer's CSR. In the above assumptions, $b > 0$, $y_r \geq 0$, $y_m \geq 0$, $c_r \geq 0$, $c_m \geq 0$. The notations are explained in Table 1.

According to the above assumptions, the demand functions of the manufacturer and the retailer are expressed as follows:

$$D_m = a(1-\alpha) - \beta p_m + \eta p_r - tg + by_m, \tag{3.1}$$

$$D_r = a\alpha - \beta p_r + \eta p_m + tg + by_r, \tag{3.2}$$

where β is the price demand elasticity, η is the cross-price elasticity coefficient between the two channels, assuming $\beta > \eta > 0$. In the rest of this paper, the superscript*is added to the relative variables to represent their corresponding optimal values.

4. Model analysis

In order to reveal the pricing decisions of all parties in the supply chain, and the influence of explanatory variables on the optimal decision under different payment methods, this section will model and analyze based on different payment methods of the retailer.

4.1. Model analysis of retailer's punctual payment

When the retailer has no capital constraint, the capital is enough to pay for the goods. The profit functions of the manufacturer and the retailer are as follows:

$$\pi_m = wD_r + p_m D_m - \frac{1}{2}c_m y_m^2, \tag{4.1}$$

$$\pi_r = (p_r - w)D_r - \frac{1}{2}sg^2 - \frac{1}{2}c_r y_r^2. \tag{4.2}$$

TABLE 1. Major notation and explanation.

Notation	Explanation
For the supply chain	
t	Service sensitivity coefficient
b	Demand elasticity of CSR
β	Price elasticity of demand
η	Cross price elasticity coefficient
r_b	The lending rates of bank
For the manufacturer	
α	Market share of offline(retail) channels
$1-\alpha$	Market share of online channels
D_m	Market demand of online channel
p_m	Manufacturer's online sales price
w	Manufacturer's wholesale price
π_m	Manufacturer's profit
y_m	The level of CSR of manufacturer
c_m	Manufacturer's cost coefficient of CSR
r	Discount rate for advance payment
θ	Manufacturer's opportunity cost rate and investment return rate
r_m	Manufacturer's borrowing rate
For the retailer	
α	Market share of retail(offline) channels
D_r	Market demand of retail channel
p_r	Retailer's retail price
B	Initial capital of retailer
π_r	Retailer's profit
g	Service level of retailer
y_r	The level of CSR of retailer
c_r	Retailer's cost coefficient of CSR
r	Discount rate for advance payment
θ	Manufacturer's opportunity cost rate
k	Proportion of advance payment

According to the problem description, when the retailer chooses to pay on time, the manufacturer first determines the sales price p_m and wholesale price w, and then the retailer determines its retail price p_r according to the manufacturer's decision. By using the idea of reverse induction, the following theorems can be obtained.

Theorem 4.1. *When the retailer chooses to pay on time, the optimal decision results of the manufacturer and the retailer are as follows:*

$$p_m^* = \frac{a\beta + \beta(by_m - tg - a\alpha) + \eta(by_r + tg + a\alpha)}{2\beta^2 - 2\eta^2},$$
$$w^* = \frac{a\eta + \beta(by_r + tg + a\alpha) + \eta(by_m - tg - a\alpha)}{2\beta^2 - 2\eta^2},$$
$$p_r^* = \frac{(3\beta^2 - \eta^2)(by_r + tg + a\alpha) + 2\beta\eta(by_m + a - tg - a\alpha)}{4\beta(\beta - \eta)(\beta + \eta)}.$$

Proof. Through calculation, it can be concluded that: $\frac{\partial^2 \pi_r}{\partial (p_r)^2} = -2\beta < 0$, therefore, the retailer's profit function π_r is strictly concave with respect to p_r, by solving $\frac{\partial \pi_r}{\partial p_r} = 0$, we can get the following result:

$p_r(w,p_m)=\frac{gt+by_r+a\alpha+w\beta+p_m\eta}{2\beta}$. We determine whether the optimal solution exists by proving whether $\pi_m(w,p_m)$ is a joint concave function of wholesale price w and sales price p_m. By solving the first and second derivatives of $\pi_m(w,p_m)$ with respect to w and p_m, we can get that $H(p_m,w)=\begin{bmatrix}\frac{\partial^2\pi_m}{\partial(w)^2} & \frac{\partial^2\pi_m}{\partial w\partial p_m}\\ \frac{\partial^2\pi_m}{\partial p_m\partial w} & \frac{\partial^2\pi_m}{\partial(p_m)^2}\end{bmatrix}=\begin{bmatrix}-\beta & \eta\\ \eta & -2\beta+\frac{\eta^2}{\beta}\end{bmatrix}$s, because $\frac{\partial^2\pi_m}{\partial(w)^2}=-\beta<0$ and the 2-order determinant $|H(p_m,w)|=2(\beta^2-\eta^2)>0(\beta>\eta)$, so Hessian matrix is negative definite and $\pi_m(w,p_m)$ is a joint concave function with respect to w and p_m. Furthermore, the optimal solution of manufacturer's sales price p_m and wholesale price w is obtained by combining $\frac{\partial\pi_m}{\partial p_m}=0$ and $\frac{\partial\pi_m}{\partial w}=0$, and substitute the above results into $p_r(w,p_m)$ to obtain the optimal retail price p_r. □

Corollary 4.2. *When the retailer pays on time, with the increase of offline channel market share, p_m^* decreases, w^* and p_r^* increase; with the improvement of service level of the retailer, p_m^* decreases, w^* and p_r^* increase; with the improvement of CSR level, p_m^*, w^* and p_r^* increase.*

Proof. By solving the partial derivatives of p_m^*, w^* and p_r^* with respect to α, g, y_m and y_r respectively, we can obtain the following results: □

$$\begin{aligned}
&\frac{\partial p_m^*}{\partial\alpha}<0,\frac{\partial w^*}{\partial\alpha}>0,\frac{\partial p_r^*}{\partial\alpha}>0,\\
&\frac{\partial p_m^*}{\partial g}<0,\frac{\partial w^*}{\partial g}>0,\frac{\partial p_r^*}{\partial g}>0,\\
&\frac{\partial p_m^*}{\partial y_m}>0,\frac{\partial w^*}{\partial y_m}>0,\frac{\partial p_r^*}{\partial y_m}>0,\\
&\frac{\partial p_m^*}{\partial y_r}>0,\frac{\partial w^*}{\partial y_r}>0,\frac{\partial p_r^*}{\partial y_r}>0.
\end{aligned}$$

The corollary 4.2. shows that:

(1) When the market share of offline channel is large, the manufacturer will reduce the sales price to attract consumers, and increase the wholesale price to obtain more profits from offline channel. Faced with the increase of wholesale price, the retailer will increase the retail price to ensure profits.

(2) With the improvement of service level of the retailer, the product added value will increase, the corresponding cost will also increase, and the retail price will increase. The improvement of service level will bring additional market share to the retailer, in order to compete for market share and maintain its own profit, the manufacturer will correspondingly reduce the online sales price and increase the wholesale price.

(3) With the improvement of the level of CSR, the cost of the manufacturer and the retailer will increase, at this time, they will increase the price to maintain corporate profits.

Theorem 4.3. *Substituting the optimal price into the profit function to obtain the optimal profit of the manufacturer and the retailer, and by combining and solving the first-order condition equations we can obtain the following results:*

$$\begin{aligned}
y_m^*&=\frac{ab\beta(-c_rt^2+s(\alpha-1)(b^2-2c_r\beta)+8c_rs\alpha\eta)}{(b^2s+c_r(t^2-8s\beta))(\beta(b^2-2c_m\beta)+2c_m\eta^2)},\\
y_r^*&=\frac{abs\alpha}{8c_rs\beta-b^2s-c_rt^2},\\
g^*&=\frac{ac_rt\alpha}{8c_rs\beta-b^2s-c_rt^2}.
\end{aligned}$$

Corollary 4.4. *When the retailer pays on time, with the increase of demand elasticity of CSR, the level of CSR determined by the manufacturer and the retailer will increase; with the increase of service sensitivity coefficient, the level of service determined by the retailer will also increase.*

Proof. Solving the partial derivatives of y_m^*, y_r^* with respect to b respectively and the partial derivative of g^* with respect to t, we can obtain the following results: $\frac{\partial y_m^*}{\partial b} > 0$, $\frac{\partial y_r^*}{\partial b} > 0$; $\frac{\partial g^*}{\partial t} > 0$.

The corollary 4.4. shows that:

(1) With the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a greater role in promoting market demand, which encourages them to continuously improve their CSR level.

(2) With the increase of service sensitivity coefficient, consumers are more sensitive to the service level of the retailer, when the retailer improve its service level, it will bring greater market demand, which encourages the retailer to continuously improve its service level. □

4.2. Model analysis of retailer's prepayment

When the retailer signs an order contract with the manufacturer, if the retailer has leisure funds, it can choose to sign an advance payment agreement with the manufacturer to get a payment discount, the proportion of advance payment is $k(0 < k < 1)$, the amount of advance payment is wD_rk, the discount rate of total payment is set as $r(0 < r < 1)$, that is to say, the total amount of payment to be paid by the retailer is $wD_r(1-r)$, the discount rate is set by the manufacturer according to the market situation. The manufacturer uses the payment received in advance to invest, and the rate of return on investment is set as θ, therefore, when the retailer chooses to pay in advance, the profit functions of the manufacturer and the retailer are as follows:

$$\pi_m = wD_r + p_mD_m + wD_r(k\theta - r) - \frac{1}{2}c_m y_m^2, \tag{4.3}$$

$$\pi_r = (p_r - w)D_r + wD_r r - \frac{1}{2}sg^2 - \frac{1}{2}c_r y_r^2. \tag{4.4}$$

According to the problem description, when the retailer chooses to pay in advance, the manufacturer first determines the sales price p_m and wholesale price w, and then the retailer determines its retail price p_r according to the manufacturer's decision. By using the idea of reverse induction, the following theorems can be obtained.

Theorem 4.5. *When the retailer chooses to pay in advance, the optimal decision results of the manufacturer and the retailer are as follows:*

$$\begin{aligned}
p_m^* &= \frac{k\eta\theta(1+k\theta-r)(tg+by_r+a\alpha)}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta(r-1)(\beta-\eta)(\beta+\eta)-k^2\eta^2\theta^2} \\
&\quad + \frac{4(r-1)(1+k\theta-r)((tg-by_m+a\alpha-a)\beta-(tg+by_r+a\alpha)\eta)}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta(r-1)(\beta-\eta)(\beta+\eta)-k^2\eta^2\theta^2}, \\
w^* &= \frac{-4\beta(r-1)(tg(\beta-\eta)+b(\beta y_r+\eta y_m)+a(\alpha\beta+\eta-\alpha\eta))}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta\beta(r-1)(\beta-\eta)(\beta+\eta)-\beta k^2\eta^2\theta^2} \\
&\quad + \frac{k\theta((4\beta^2-\eta^2)(tg+by_r+a\alpha)+2\beta\eta(a-tg+by_m-a\alpha))}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta\beta(r-1)(\beta-\eta)(\beta+\eta)-\beta k^2\eta^2\theta^2}, \\
p_r^* &= \frac{2(r-1)^2((3\beta^2-\eta^2)(tg+by_r+a\alpha)+2\beta\eta(a-tg+by_m-a\alpha))}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta\beta(r-1)(\beta-\eta)(\beta+\eta)-\beta k^2\eta^2\theta^2} \\
&\quad + \frac{(r-1)k\theta((-6\beta^2+2\eta^2)(tg+by_r+a\alpha)+3\beta\eta(tg-by_m-a+a\alpha))}{8(r-1)^2(\beta-\eta)(\beta+\eta)-8k\theta\beta(r-1)(\beta-\eta)(\beta+\eta)-\beta k^2\eta^2\theta^2}.
\end{aligned}$$

Proof. This process is the same as Theorem 4.1. □

Corollary 4.6. *When the retailer pays in advance, with the increase of offline channel market share, p_m^* decreases, w^* and p_r^* increase; with the improvement of service level of the retailer, p_m^* decreases, w^* and*

p_r^* increase; with the improvement of CSR level, p_m^*, w^* and p_r^* increase; with the increase of discount rate of advance payment, p_m^* and w^* increase, the trend of p_r^* with discount rate is influenced by k, θ and α.

Proof. By solving the partial derivatives of p_m^*, w^* and p_r^* with respect to α, g, y_m, y_r and r respectively, we can obtain the following results: □

$$\begin{aligned}
&\frac{\partial p_m^*}{\partial \alpha} < 0, \frac{\partial w^*}{\partial \alpha} > 0, \frac{\partial p_r^*}{\partial \alpha} > 0,\\
&\frac{\partial p_m^*}{\partial g} < 0, \frac{\partial w^*}{\partial g} > 0, \frac{\partial p_r^*}{\partial g} > 0,\\
&\frac{\partial p_m^*}{\partial y_m} > 0, \frac{\partial w^*}{\partial y_m} > 0, \frac{\partial p_r^*}{\partial y_m} > 0,\\
&\frac{\partial p_m^*}{\partial y_r} > 0, \frac{\partial w^*}{\partial y_r} > 0, \frac{\partial p_r^*}{\partial y_r} > 0,\\
&\frac{\partial p_m^*}{\partial r} > 0, \frac{\partial w^*}{\partial r} > 0.
\end{aligned}$$

In addition,the symbol of $\frac{\partial p_r^*}{\partial r}$ is related to k, θ and α.

The corollary 4.6 shows that:

(1) When the market share of offline channel is large, the manufacturer will reduce the sales price to attract consumers, and increase the wholesale price to obtain more profits from offline channel. Faced with the increase of wholesale price, the retailer will increase the retail price to ensure profits.

(2) With the improvement of service level of the retailer, the product added value will increase, the corresponding cost will also increase, and the retail price will increase.The improvement of service level will bring additional market share to the retailer, in order to compete for market share and maintain its own profit, the manufacturer will correspondingly reduce the online sales price and increase the wholesale price.

(3) With the improvement of the level of CSR, the cost of the manufacturer and the retailer will increase, at this time, they will increase the price to maintain corporate profits.

(4) With the increase of the discount rate of advance payment, the manufacturer will receive less payment in the end, so the profit will be maintained by increasing the sales price and wholesale price. The impact of the discount rate on the retailer's pricing is k, θ and α. This part will be introduced in detail in the numerical analysis.

Theorem 4.7. *Substituting the optimal price into the profit functions to obtain the optimal profit of the manufacturer and the retailer, and by combining and solving the first-order condition equations we can obtain the following results:*

$$\begin{aligned}
y_m^* &= \frac{ab\beta(-1+r-k\theta)(2b^2sL_1 + c_r(-2t^2L_2 - sL_3L_4))}{2b^4s\beta(-1+r-k\theta)L_1/(\alpha-1) + c_mc_r(-2t^2L_5 + s\beta L_4^2) + L_9},\\
y_r^* &= \frac{(2absL_1/(\alpha-1))(b^2\alpha\beta(-1+r-k\theta) + c_mL_8)}{2b^4s\beta(-1+r-k\theta)L_1/(\alpha-1) + c_mc_r(-2t^2L_5 + s\beta L_4^2) + L_9},\\
g^* &= \frac{2ac_rtL_2(b^2\alpha\beta(-1+r-k\theta) + c_mL_8)}{2b^4s\beta(-1+r-k\theta)L_1/(\alpha-1) + c_mc_r(-2t^2L_5 + s\beta L_4^2) + L_9},\\
L_1 &= (r-1)^2(\alpha-1)(2\beta^2(-1+r-k\theta) + \eta^2(2-2r+k\theta)),\\
L_2 &= (r-1)^2(\beta-\eta)(2(r-1)(\beta+\eta) - k\theta(2\beta+\eta)),\\
L_3 &= 4(r-1)((\alpha-1)\beta - \alpha\eta) + k\alpha\eta\theta,\\
L_4 &= 8(r-1)^2(\beta-\eta)(\beta+\eta) - 8k\theta(r-1)(\beta-\eta)(\beta+\eta),
\end{aligned}$$

$$
\begin{aligned}
L_5 &= (r-1)^2(\beta-\eta)^2(-2(r-1)(\beta+\eta)+k\theta(2\beta+\eta))^2,\\
L_6 &= 2(r-1)^2(t^2-8s\beta)(\beta-\eta)(\beta+\eta)+2k^2s\beta\eta^2\theta^2,\\
&\quad +k\theta(r-1)(\beta-\eta)(16s\beta(\beta+\eta)-t^2(2\beta+\eta)),\\
L_7 &= (r-1)(2\beta^2(-1+r-k\theta)+\eta^2(2-2r+k\theta))^2\\
L_8 &= -k\beta\eta\theta+\alpha(\beta-\eta)(2(r-1)(\beta+\eta)-k\theta(2\beta+\eta)),\\
L_9 &= 2b^2(r-1)(c_r\beta(-1+r-k\theta)L_6-c_msL_7).
\end{aligned}
$$

Corollary 4.8. *When the retailer pays in advance, with the increase of demand elasticity of CSR, the level of CSR determined by the manufacturer and the retailer will increase; with the increase of service sensitivity coefficient, the level of service determined by the retailer will also increase; with the increase of discount rate, the level of retailer's CSR increases, and the change of service level is affected by market share; with increase of the rate of return on investment, the level of manufacturer's CSR increases.*

Proof. Solving the partial derivatives of y_m^*, y_r^* with respect to b respectively and the partial derivative of g^* with respect to t, we can obtain the following results: $\frac{\partial y_m^*}{\partial b}>0$, $\frac{\partial y_r^*}{\partial b}>0$, $\frac{\partial g^*}{\partial t}>0$; Solving the partial derivatives of y_r^*, g^* with respect to r, we can get $\frac{\partial y_r^*}{\partial r}>0$, and the symbol of $\frac{\partial g^*}{\partial r}$ is related to α; solving the partial derivatives of y_m^* with respect to θ, we can get $\frac{\partial y_m^*}{\partial \theta}>0$.

The corollary 4.8 shows that:

(1) With the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a greater role in promoting market demand, which encourages them to continuously improve their CSR level.

(2) With the increase of service sensitivity coefficient, consumers are more sensitive to the service level of the retailer, when the retailer improve its service level, it will bring greater market demand, which encourages the retailer to continuously improve its service level.

(3) With the increase of discount rate of advance payment, the retailer's purchase cost decreases and the profit increases, and more funds will be used to improve the level of CSR. The trend of service level changing with discount rate is influenced by α, which will be introduced in detail in the numerical analysis.

(4) With the increase of the rate of return on investment, the manufacturer's profit will increase, and the surplus funds will be used to improve the level of CSR. □

4.3. Model analysis of retailer with capital constraints

In the following analysis, it is assumed that the retailer has no initial capital. After financing from the bank or manufacturer, the capital is held in cash from the loan date to the due repayment date, that is, there is no investment behavior of the retailer.

4.3.1. The retailer chooses to borrow from banks

If the retailer with capital constraints chooses bank loans, assuming that the lending rate is r_b, after the end of the sales period, the sum of the principal and interest paid by the retailer is $wD_r(1+r_b)$. The profit functions of the manufacturer and the retailer areas follows:

$$\pi_m = wD_r + p_mD_m - \frac{1}{2}c_my_m^2, \tag{4.5}$$

$$\pi_r = (p_r-w)D_r - wD_rr_b - \frac{1}{2}sg^2 - \frac{1}{2}c_ry_r^2. \tag{4.6}$$

According to the problem description, when the retailer chooses to borrow from banks, the manufacturer first determines the sales price p_m and wholesale price w, and then the retailer determines its retail price p_r according to the manufacturer's decision. By using the idea of reverse induction, the following theorems can be obtained.

Theorem 4.9. *When the retailer chooses to borrow from banks, the optimal decision results of the manufacturer and the retailer are as follows:*

$$p_m^* = \frac{4\beta(1+r_b)(tg-by_m+a(\alpha-1))-(4+r_b)(tg+by_r+a\alpha)\eta}{-8\beta^2(1+r_b)+(r_b^2+8r_b+8)\eta^2},$$
$$w^* = \frac{(4\beta^2+r_b\eta^2)(tg+by_r+a\alpha)-2\beta\eta(2+r_b)(tg-by_m+a(\alpha-1))}{8\beta^3(1+r_b)-(r_b^2+8r_b+8)\beta\eta^2},$$
$$p_r^* = \frac{(1+r_b)((6\beta^2-2\eta^2)(tg+by_r+a\alpha)-(4+r_b)\beta\eta(tg-by_m+a(\alpha-1)))}{8\beta^3(1+r_b)-(r_b^2+8r_b+8)\beta\eta^2}.$$

Proof. This process is the same as Theorem 4.1. □

Corollary 4.10. *When the retailer chooses to borrow from banks, with the increase of offline channel market share, p_m^* decreases, w^* and p_r^* increase; with the improvement of service level of the retailer, p_m^* decreases, w^* and p_r^* increase; with the improvement of CSR level, p_m^*, w^* and p_r^* increase; with the increase of lending rate, p_r^* increases, w^* decreases, the changing trend of p_m^* is related to α.*

Proof. By solving the partial derivatives of p_m^*, w^* and p_r^* with respect to α, g, y_m, y_r and r_b respectively, we can obtain the following results: □

$$\frac{\partial p_m^*}{\partial \alpha}<0, \frac{\partial w^*}{\partial \alpha}>0, \frac{\partial p_r^*}{\partial \alpha}>0,$$
$$\frac{\partial p_m^*}{\partial g}<0, \frac{\partial w^*}{\partial g}>0, \frac{\partial p_r^*}{\partial g}>0,$$
$$\frac{\partial p_m^*}{\partial y_m}>0, \frac{\partial w^*}{\partial y_m}>0, \frac{\partial p_r^*}{\partial y_m}>0,$$
$$\frac{\partial p_m^*}{\partial y_r}>0, \frac{\partial w^*}{\partial y_r}>0, \frac{\partial p_r^*}{\partial y_r}>0.$$

In addition, $\frac{\partial p_r^*}{\partial r_b}>0$, $\frac{\partial w^*}{\partial r_b}<0$, the symbol of $\frac{\partial p_m^*}{\partial r_b}$ is related to α.

The corollary 4.10. shows that:

(1) When the market share of offline channel is large, the manufacturer will reduce the sales price to attract consumers, and increase the wholesale price to obtain more profits from offline channel. Faced with the increase of wholesale price, the retailer will increase the retail price to ensure profits.

(2) With the improvement of service level of the retailer, the product added value will increase, the corresponding cost will also increase, and the retail price will increase. The improvement of service level will bring additional market share to the retailer, in order to compete for market share and maintain its own profit, the manufacturer will correspondingly reduce the online sales price and increase the wholesale price.

(3) With the improvement of the level of CSR, the cost of the manufacturer and the retailer will increase, at this time, they will increase the price to maintain corporate profits.

(4) With the increase of lending rate, the retailer's debt service expenditure will increase, and the retailer will maintain the profit by increasing the retail price. In order to maintain the stability of the supply chain, the manufacturer will give the profit to the retailer and reduce the wholesale price accordingly. As for the p_m^*, the influence trend of lending rate on it is affected by α, which will be introduced in detail in the numerical analysis.

Theorem 4.11. *Substituting the optimal price into the profit functions to obtain the optimal profit of the manufacturer and the retailer, and by combining and solving the first-order condition equations we can obtain the following results:*

$$y_m^* = \frac{-ab\beta(N_1 + c_r(N_2 + sN_3N_4))}{N_1b^2\beta/(1-\alpha) + c_mc_r(-N_2N_5 - s\beta N_4^2) + c_mN_1N_6/(1-\alpha) + b^2\beta c_rN_7},$$

$$y_r^* = \frac{N_1a/(b(1-\alpha))(b^2\alpha\beta + c_m(r_b\beta\eta - \alpha N_5))}{N_1b^2\beta/(1-\alpha) + c_mc_r(N_2N_5 - s\beta N_4^2) + c_mN_1N_6/(\alpha-1) + 2b^2\beta c_rN_8},$$

$$g^* = \frac{2atc_r(1+r_b)^2N_5(b^2\alpha\beta + c_m(r_b\beta\eta - \alpha N_5))}{N_1b^2\beta/(\alpha-1) + c_mc_r(N_2N_5 - s\beta N_4^2) + c_mN_1N_6/(\alpha-1) + 2b^2\beta c_rN_8},$$

$$N_1 = 2b^2s(1+r_b)^2(\alpha-1)(-2\beta^2 + (2+r_b)\eta^2),$$

$$N_2 = 2t^2(1+r_b)^2(\beta-\eta)(2\beta + (2+r_b)\eta),$$

$$N_3 = 4\beta(1+r_b)(\alpha-1) - (4+3r_b)\alpha\eta,$$

$$N_4 = 8(1+r_b)^2\beta^2 - (8 + r_b(8+r_b))\eta^2,$$

$$N_5 = (\beta-\eta)(2\beta + (2+r_b)\eta),$$

$$N_6 = -2\beta^2 + (2+r_b)\eta^2,$$

$$N_7 = (N_2 - 4s\beta(1+r_b)N_4),$$

$$N_8 = (1+r_b)(16(1+r_b)s\beta^3 - 2s\beta\eta^2(8 + 8r_b + r_b^2) - (1+r_b)t^2N_5).$$

Corollary 4.12. *When the retailer chooses to borrow from banks, with the increase of demand elasticity of CSR, the level of CSR determined by the manufacturer and the retailer will increase; with the increase of service sensitivity coefficient, the level of service determined by the retailer will also increase; with the increase of lending rate, the retailer will reduce its level of CSR and service.*

Proof. Solving the partial derivatives of y_m^*, y_r^* with respect to b respectively and the partial derivative of g^* with respect to t, we can obtain the following results: $\frac{\partial y_m^*}{\partial b} > 0$, $\frac{\partial y_r^*}{\partial b} > 0$, $\frac{\partial g^*}{\partial t} > 0$; solving the partial derivatives of y_r^*, g^* with respect to r_b, we can get $\frac{\partial y_r^*}{\partial r_b} < 0$ and $\frac{\partial g^*}{\partial r_b} < 0$.

The corollary 4.12 shows that:

(1) With the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a greater role in promoting market demand, which encourages them to continuously improve their CSR level.

(2) With the increase of service sensitivity coefficient, consumers are more sensitive to the service level of the retailer, when the retailer improve its service level, it will bring greater market demand, which encourages the retailer to continuously improve its service level.

(3) With the increase of lending rate, for the retailer, the debt service expenditure increases, and the cost needs to be reduced in time, so the level of CSR and service level will be reduced. □

4.3.2. The retailer chooses to borrow from the manufacturer

If the retailer with capital constraints chooses borrow from the manufacturer, assuming that the borrowing rate is r_m, the opportunity cost rate of manufacturer is θ, after the end of the sales period, the sum of the principal and interest paid by the retailer is $wD_r(1+r_m)$. The profit functions of the manufacturer and the retailer are as follows:

$$\pi_m = wD_r + p_mD_m + wD_r(r_m - \theta) - \frac{1}{2}c_my_m^2, \tag{4.7}$$

$$\pi_r = (p_r - w)D_r + wD_rr_m - \frac{1}{2}sg^2 - \frac{1}{2}c_ry_r^2. \tag{4.8}$$

According to the problem description, when the retailer chooses to borrow from the manufacturer, the manufacturer first determines the sales price p_m and wholesale price w, and then the retailer determines its retail price p_r according to the manufacturer's decision. By using the idea of reverse induction, the following theorems can be obtained.

Theorem 4.13. *When the retailer chooses to borrow from the manufacturer, the optimal decision results of the manufacturer and the retailer are as follows:*

$$\begin{aligned}
p_m^* &= \frac{4(1+r_m-\theta)(1+r_m)(tg(\eta-\beta)+b(y_m\beta+y_r\eta)+a(\beta-\alpha\beta+\alpha\eta))}{8\beta^2(1+r_m)(1+r_m-\theta)-\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)} \\
&\quad - \frac{(1+r_m-\theta)(tg+by_r+a\alpha)\eta\theta}{8\beta^2(1+r_m)(1+r_m-\theta)-\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)}, \\
w^* &= \frac{4\beta(1+r_m)(tg(\beta-\eta)+b(y_r\beta+y_m\eta)+a(\eta+\alpha\beta-\alpha\eta))}{8\beta^3(1+r_m)(1+r_m-\theta)-\beta\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)} \\
&\quad + \frac{((\eta^2-4\beta^2)(tg+by_r+a\alpha)+2\beta\eta(tg-by_m+a(\alpha-1)))\theta}{8\beta^3(1+r_m)(1+r_m-\theta)-\beta\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)}, \\
p_r^* &= \frac{2(1+r_m)^2((3\beta^2-\eta^2)(tg+by_r+a\alpha)+2\beta\eta(a-tg+by_m-a\alpha))}{8\beta^3(1+r_m)(1+r_m-\theta)-\beta\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)} \\
&\quad + \frac{(1+r_m)((2\eta^2-6\beta^2)\theta(tg+by_r+a\alpha)+3\beta\eta\theta(tg-by_m+a(\alpha-1)))}{8\beta^3(1+r_m)(1+r_m-\theta)-\beta\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)}.
\end{aligned}$$

Proof. This process is the same as Theorem 4.1. □

Corollary 4.14. *When the retailer chooses to borrow from the manufacturer, with the increase of offline channel market share, p_m^* decreases, w^* and p_r^* increase; with the improvement of service level of the retailer, p_m^* decreases, w^* and p_r^* increase; with the improvement of CSR level, p_m^*, w^* and p_r^* increase; with the increase of borrowing rate, p_m^* increases, p_r^* and w^* decrease; with the increase of opportunity cost rate, p_r^* and w^* increase, the changing trend of p_m^* is related to α.*

Proof. By solving the partial derivatives of p_m^*, w^* and p_r^* with respect to α, g, y_m, y_r, r_m and θ respectively, we can obtain the following results: □

$$\begin{aligned}
&\frac{\partial p_m^*}{\partial \alpha}<0, \frac{\partial w^*}{\partial \alpha}>0, \frac{\partial p_r^*}{\partial \alpha}>0, \\
&\frac{\partial p_m^*}{\partial g}<0, \frac{\partial w^*}{\partial g}>0, \frac{\partial p_r^*}{\partial g}>0, \\
&\frac{\partial p_m^*}{\partial y_m}>0, \frac{\partial w^*}{\partial y_m}>0, \frac{\partial p_r^*}{\partial y_m}>0, \\
&\frac{\partial p_m^*}{\partial y_r}>0, \frac{\partial w^*}{\partial y_r}>0, \frac{\partial p_r^*}{\partial y_r}>0, \\
&\frac{\partial p_m^*}{\partial r_m}>0, \frac{\partial w^*}{\partial r_m}<0, \frac{\partial p_r^*}{\partial r_m}<0.
\end{aligned}$$

In addition, $\frac{\partial p_r^*}{\partial \theta}>0$, $\frac{\partial w^*}{\partial \theta}>0$, the symbol of $\frac{\partial p_m^*}{\partial \theta}$ is related to α.
The corollary 4.14. shows that:

(1) When the market share of offline channel is large, the manufacturer will reduce the sales price to attract consumers, and increase the wholesale price to obtain more profits from offline channel. Faced with the increase of wholesale price, the retailer will increase the retail price to ensure profits.

(2) With the improvement of service level of the retailer, the product added value will increase, the corresponding cost will also increase, and the retail price will increase. The improvement of service level will bring additional market share to the retailer, in order to compete for market share and maintain its own profit, the manufacturer will correspondingly reduce the online sales price and increase the wholesale price.

(3) With the improvement of the level of CSR, the cost of the manufacturer and the retailer will increase, at this time, they will increase the price to maintain corporate profits.

(4) With the increase of borrowing rate, for the retailer, its debt service expenses will increase, and it will increase its revenue through low price competition strategy; for the manufacturer, when the retailer face higher loan interest rate, the manufacturer will set a lower wholesale price to give the profit to the retailer, but in order to ensure that its profits will not be affected, and ease the competition with the retailer, the manufacturer will increase the sales price accordingly.

(5) With the increase of opportunity cost rate, the wholesale price set by the manufacturer will increase to make up for the loss of its investment; for the sales price, it is related to the change of market share, which will be introduced in detail in the numerical analysis; for the retailer, when faced with higher borrowing rate and rising wholesale price, the retailer will make up for the loss of its purchase and debt service by setting a higher retail price.

Theorem 4.15. *Substituting the optimal price into the profit functions to obtain the optimal profit of the manufacturer and the retailer, and by combining and solving the first-order condition equations we can obtain the following results:*

$$
\begin{aligned}
y_m^* &= \frac{ab\beta(1+r_m-\theta)(2b^2sM_1+c_r(-2t^2M_2-sM_3M_4))}{2b^4s\beta(1+r_m-\theta)M_1/(\alpha-1)+2b^2(1+r_m)(c_r\beta M_5-c_msM_6)+M_8},\\
y_r^* &= \frac{-((2absM_1/(\alpha-1))(b^2\alpha\beta(1+r_m-\theta)+c_m(\beta\eta\theta-\alpha M_2/(1+r_m)^2)))}{2b^4s\beta(1+r_m-\theta)M_1/(\alpha-1)+2b^2(1+r_m)(c_r\beta M_5-c_msM_6)+M_8},\\
g^* &= \frac{-2ac_rtM_2(b^2\alpha\beta(1+r_m-\theta)+c_m(\beta\eta\theta-\alpha M_2/(1+r_m)^2))}{2b^4s\beta(1+r_m-\theta)M_1/(\alpha-1)+2b^2(1+r_m)(c_r\beta M_5-c_msM_6)+M_8},\\
M_1 &= (1+r_m)^2(\alpha-1)(2\beta^2(1+r_m-\theta)+\eta^2(-2-2r_m+\theta)),\\
M_2 &= (1+r_m)^2(\beta-\eta)(2(1+r_m)(\beta+\eta)-(2\beta+\eta)\theta),\\
M_3 &= (4\beta(1+r_m)(\alpha-1)+\alpha\eta(-4-4r_m+\theta)),\\
M_4 &= (8\beta^2(1+r_m)(1+r_m-\theta)-\eta^2(8(1+r_m)^2-8(1+r_m)\theta+\theta^2)),\\
M_5 &= 2(1+r_m)^2(1+r_m-\theta)(t^2-8s\beta)(\beta^2-\eta^2),\\
&\quad +(1+r_m-\theta)((1+r_m)(\beta-\eta)(16s\beta(\beta+\eta)-t^2(2\beta+\eta))\theta+2s\beta\eta^2\theta^2),\\
M_6 &= (1+r_m)(2\beta^2(1+r_m-\theta)+\eta^2(-2-2r_m+\theta))^2,\\
M_7 &= (1+r_m)^2(\beta-\eta)^2(-2(1+r_m)(\beta+\eta)+(2\beta+\eta)\theta)^2,\\
M_8 &= c_mc_r(-2t^2M_7+s\beta M_4^2).
\end{aligned}
$$

Corollary 4.16. *When the retailer chooses to borrow from the manufacturer, with the increase of demand elasticity of CSR, the level of CSR determined by the manufacturer and the retailer will increase; with the increase of service sensitivity coefficient, the level of service determined by the retailer will also increase; with the increase of borrowing rate, the level of CSR of the manufacturer will also increase.*

Proof. Solving the partial derivatives of y_m^*, y_r^* with respect to b respectively and the partial derivative of g^* with respect to t, we can obtain the following results: $\frac{\partial y_m^*}{\partial b}>0$, $\frac{\partial y_r^*}{\partial b}>0$, $\frac{\partial g^*}{\partial t}>0$; solving the partial derivative of y_m^* with respect to r_m, we can get $\frac{\partial y_m^*}{\partial r_m}>0$.

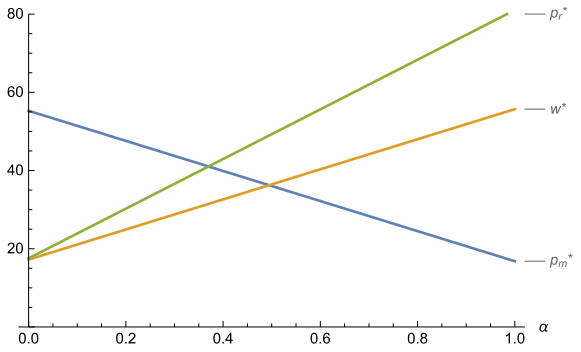


FIGURE 2. The impact of market share on pricing.

The corollary 4.16 shows that:

(1) With the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a greater role in promoting market demand, which encourages them to continuously improve their CSR level.

(2) With the increase of service sensitivity coefficient, consumers are more sensitive to the service level of the retailer, when the retailer improve its service level, it will bring greater market demand, which encourages the retailer to continuously improve its service level.

(3) With the increase of borrowing rate, the income of the manufacturer will increase, and more funds will be used to improve the level of CSR. □

5. Numerical analysis

In this section, we use numerical analysis to explore the impact of market share, financing interest rate, opportunity cost rate and other factors on the decision-making results of supply chain participants. The basic parameters of the example as follows: $a = 100$, $\beta = 1$, $\eta = 0.3$, $t = 0.8$, $s = 2$, $b = 0.5$, $c_m = 20$, $c_r = 25$.

5.1. The retailer chooses to pay on time

5.1.1. The impact of market share on pricing

When the retailer chooses to pay on time, supposing $y_r = 1$, $y_m = 2$, to explore the impact of market share on pricing, the results are as follows.

From Figure 2, we can find that market share has a significant impact on the pricing decisions of all participants in the supply chain. With the expansion of the retailer's market share, the manufacturer will reduce the sales price to attract consumers, and increase the wholesale price to obtain more profits from offline retail channel. Faced with the increase of wholesale price, the retailer will increase the retail price to ensure profits.

5.1.2. The impact of retailer's service level on pricing

When the retailer chooses to pay on time, supposing $\alpha = 0.5$, $y_r = 1$, $y_m = 2$, to explore the impact of retailer's service level on pricing, the results are as follows.

From Figure 3, we can find that with the improvement of service level of the retailer, the product added value will increase, the corresponding cost will also increase, and the retail price will increase. The improvement of service level will bring additional market share to the retailer, in order to compete for market share and maintain its own profit, the manufacturer will correspondingly reduce the online sales price and increase the wholesale price.

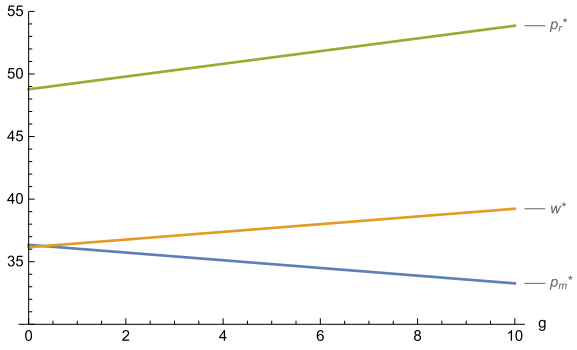


FIGURE 3. The impact of retailer's service level on pricing.

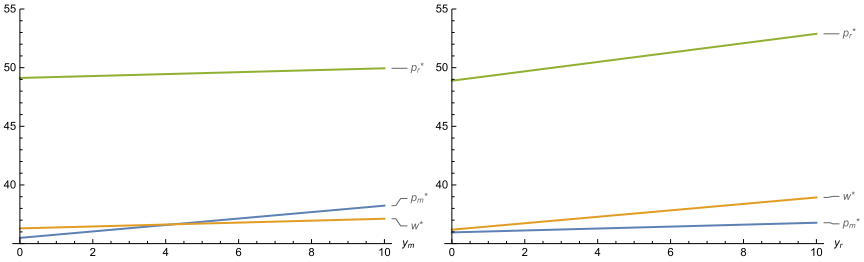


FIGURE 4. The impact of CSR on pricing.

5.1.3. The impact of CSR on pricing

When the retailer chooses to pay on time, supposing $\alpha = 0.5$, $g = 1$, to explore the impact of CSR on pricing, the results are as follows.

From Figure 4, we can find that with the improvement of the level of CSR, the cost of the manufacturer and the retailer will increase, at this time, they will increase the price to maintain corporate profits.

5.1.4. The impact of demand elasticity of CSR on the level of CSR

When the retailer chooses to pay on time, supposing $\alpha = 0.5$, to explore the impact of demand elasticity of CSR on the level of CSR, the results are as follows.

From Figure 5, we can find that with the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a greater role in promoting market demand, which encourages them to continuously improve their CSR level.

5.1.5. The impact of service sensitivity coefficient on service level of the retailer

When the retailer chooses to pay on time, supposing $\alpha = 0.5$, to explore the impact of service sensitivity coefficient on service level of the retailer, the results are as follows.

From Figure 6, we can find that with the increase of service sensitivity coefficient, consumers are more sensitive to the service level of the retailer, when the retailer improve its service level, it will bring greater market demand, which encourages the retailer to continuously improve its service level.

5.2. The retailer chooses to pay in advance

When the retailer pays in advance, the impact of market share, retailer service level and CSR level on the optimal pricing decision, the impact of demand elasticity of CSR on CSR level and the impact of service

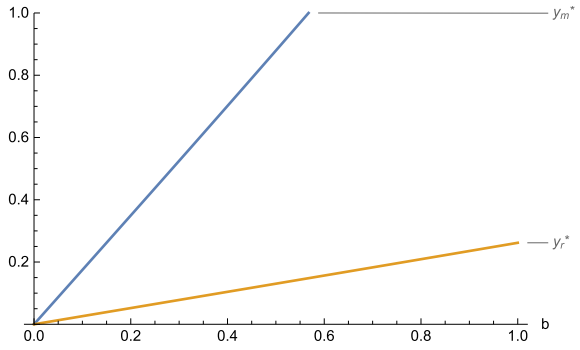


FIGURE 5. The impact of demand elasticity of CSR on the level of CSR.

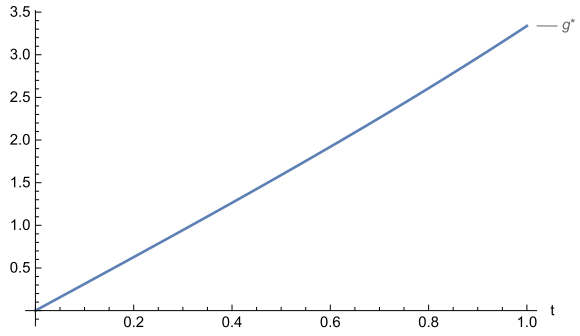


FIGURE 6. The impact of service sensitivity coefficient on service level of the retailer.

sensitivity coefficient on retailer service level are consistent with those when the retailer pays on time, so there is no more analysis here.

5.2.1. The impact of discount rate of advance payment on retail price

In this part, the change of p_m^* and w^* is clear, so we focus on the change of p_r^*. From the analysis, we can see that the trend of p_r^* changing with the discount rate is affected by the comprehensive influence of k, θ and α. We assign different values to k, θ and α, combined with the reality (for the sake of self-interest of manufacturer, the discount rate of advance payment is generally not very large), to explore the impact of the discount rate of advance payment on the retail price. The results are as follows.

(1) $k\theta > r$, $\alpha = 0.3$, p_r^* decreases first and then increases with the increase of discount rate.

(2) $k\theta > r$, $\alpha = 0.8$, p_r^* increases with the increase of discount rate.

(3) $k\theta < r$,no matter how α changes, the p_r^* is always on a downward trend.

From Figures 7, 8 and 9, we can find: when $k\theta > r$, if the market share of offline channel is very small, with the increase of discount rate, p_r^* first decreases and then increases, and this inflection point is near $r = 0.9$ (considering the reality, the probability of the situation on the right side of the inflection point is low). The explanation of the situation on the left side of the inflection point is as follows: the retailer avoids risk, does not invest with leisure funds, and pays more opportunity cost than the profit from discount, so the retailer will reduce the price, make small profit but quick turnover, and increase the profit while opening the market; if the market share of offline channel is large, the space for the retailer to reduce the price to expand the market share is not very large, then they will make profits by increasing the price; when $k\theta < r$, it is beneficial to the retailer, which will decrease the retail price to compete for market share and further obtain greater profits.

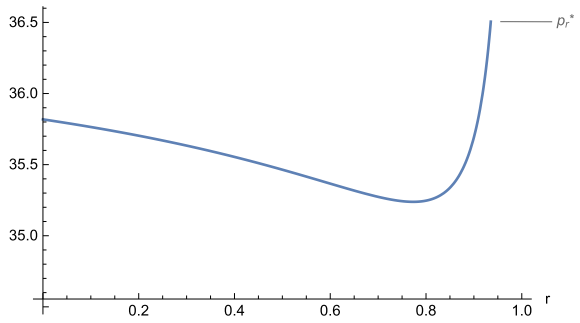


FIGURE 7. The impact of discount rate on retail price($k\theta > r$ and $\alpha = 0.3$).

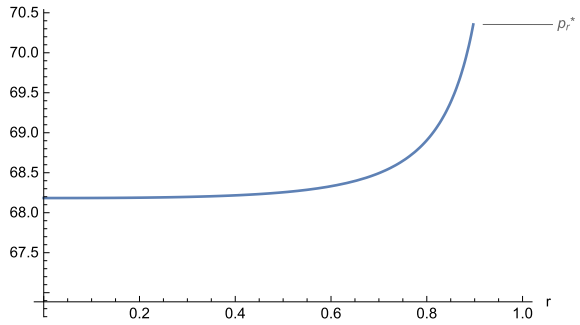


FIGURE 8. The impact of discount rate on retail price($k\theta > r$ and $\alpha = 0.8$).

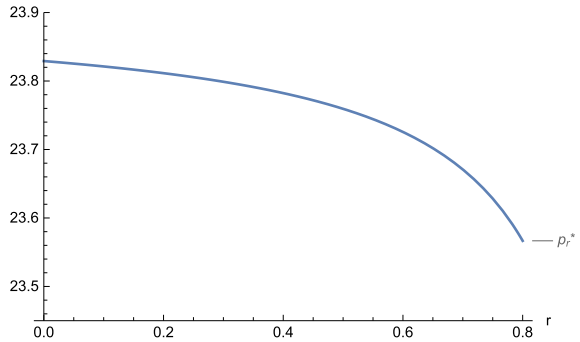


FIGURE 9. The impact of discount rate on retail price($k\theta < r$).

5.2.2. The impact of discount rate on the retailer's CSR and service level

(1) The impact of discount rate of advance payment on the level of retailer's CSR

From Figure 10, we can find that with the increase of discount rate, the retailer's purchase cost decreases and the profit increases, and more funds will be used to improve the level of CSR.

(2) The impact of discount rate on the retailer's service level

From the analysis, we can see that the trend of $g*$ changing with the discount rate is affected by α. Setting $\alpha = 0.3$ and $\alpha = 0.8$ to explore the impact of the discount rate on the retailer's service level, combined with the

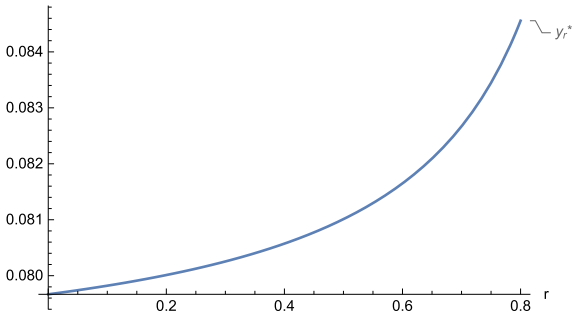
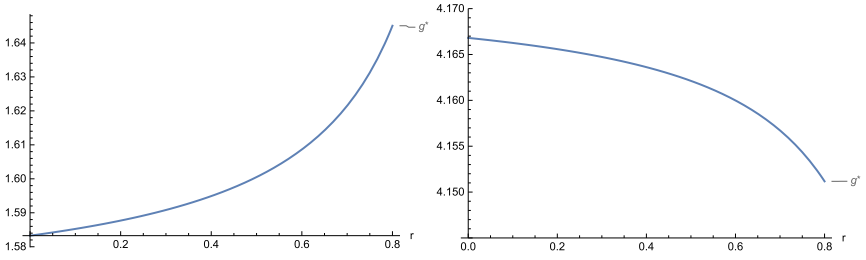


FIGURE 10. The impact of discount rate on the level of the retailer's CSR.



(a) $\alpha = 0.3$ (b) $\alpha = 0.8$

FIGURE 11. The impact of discount rate on the retailer's service level.

actual situation (the discount rate of advance payment is generally not very large for the manufacturer's own interests). The results are as follows.

From Figure 11, we can find that when the market share of offline channel is small, with the increase of discount rate, the retailer's purchase cost will be reduced and the profit will be increased, and the surplus funds will be used to improve the service level of the enterprise, thus expanding the demand and increasing the profit; when the market share of offline channel is large, the space to expand the market share is small, at this time, the retailer will reduce the service level, and then reduce the cost, at the same time, it will ease the competition with the manufacturer for market share, and give the profit to the manufacturer.

5.2.3. The impact of rate of return on investment on the level of manufacturer's CSR

From Figure 12, we can find that with the increase of the rate of return on investment, the manufacturer's profit will increase, and the surplus funds will be used to improve the level of CSR.

5.3. The retailer has capital constraints

5.3.1. The retailer chooses to borrow from banks

When the retailer chooses to borrow from banks, the impact of market share, retailer service level and CSR level on the optimal pricing decision, the impact of demand elasticity of CSR on CSR level and the impact of service sensitivity coefficient on retailer service level are consistent with those when the retailer pays on time, so there is no more analysis here.

(1) The impact of lending rate on pricing

When the retailer chooses to borrow from banks, supposing $\alpha = 0.1$ and $\alpha = 0.9$ respectively, to explore the impact of lending rate on pricing, the results are as follows.

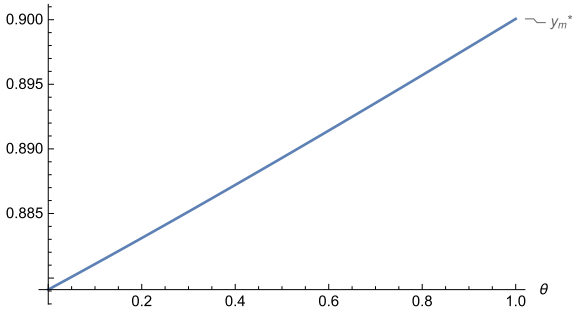


FIGURE 12. The impact of rate of return on the level of manufacturer's CSR.

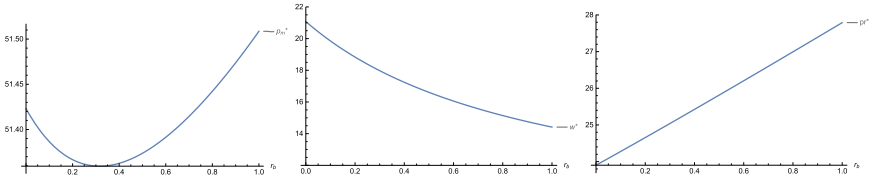


FIGURE 13. The impact of lending rate on pricing($\alpha = 0.1$).

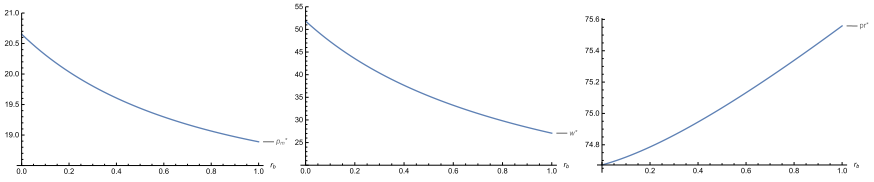


FIGURE 14. The impact of lending rate on pricing($\alpha = 0.9$).

From Figures 13 and 14, we can find that when the retailer chooses bank loans, for the retailer, with the increase of lending rate, the retail price will be increased to make up for its debt repayment expenses; for the manufacturer, when the retailer face higher lending rate, it will set a lower wholesale price to give the profit to the retailer, so as to maintain the stability of the whole supply chain.

As for the sales price, it can be seen from the figure that when the market share of the offline channel is small, the online sales price first decreases and then increases with the increase of lending rate, when the market share of the offline channel exceeds a certain range, the sales price will decrease with the increase of lending rate, this shows that when the manufacturer's online channel market share is dominant and the bank loan interest rate is not very high, the manufacturer will reduce the sales price to attract more customers, however, when the lending rate exceeds a certain limit, the retailer's performance becomes worse, at this time, the manufacturer will increase the sales price to ease the competition with the retailer and cushion the loss caused by customer churn. When the manufacturer's online channel market share is no longer dominant, in order to protect interests, the sales price will decrease with the increase of lending rate to compete with the retailer for market share.

(2) The impact of lending rate on retailer's CSR and service level

When the retailer borrows money from banks, the lending rate will directly affect the retailer's situation and decision-making, supposing $\alpha = 0.5$ to explore the impact of lending rate on retailer's CSR and service level, the results are as follows.

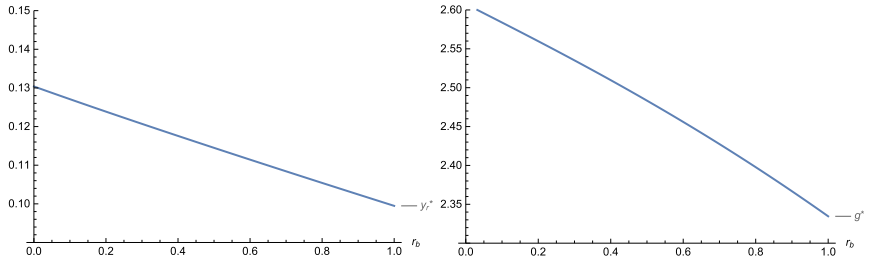


FIGURE 15. The impact of lending rate on retailer's CSR and service level.

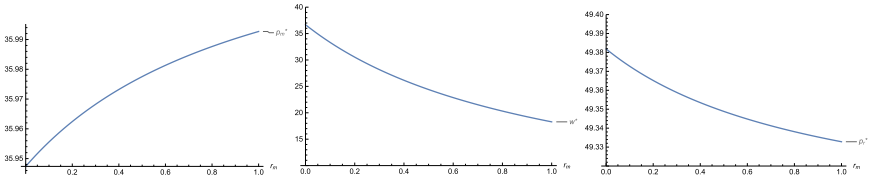


FIGURE 16. The impact of borrowing rate on pricing.

From Figure 15, we can find that with the increase of lending rate, for the retailer, the debt service expenditure increases, and the cost needs to be reduced in time, so the level of CSR and service level will be reduced.

5.3.2. The retailer chooses to borrow from the manufacturer

When the retailer chooses to borrow from the manufacturer, the influence of market share, retailer service level and CSR level on the optimal pricing decision, the influence of demand elasticity of CSR on CSR level and the influence of service sensitivity coefficient on retailer service level are consistent with those when the retailer pays on time, so there is no more analysis here.

(1) The impact of borrowing rate on pricing

When the retailer chooses to borrow from the manufacturer, supposing $\alpha = 0.5$, $g = 1$, $y_r = 1$ and $y_m = 2$, to explore the impact of borrowing rate on pricing, the results are as follows.

From Figure 16, we can find that when the retailer chooses to borrow from the manufacturer, with the increase of borrowing rate, for the retailer, its debt service expenses will increase, and it will increase its revenue through low price competition strategy; for the manufacturer, when the retailer face higher borrowing rate, the manufacturer will set a lower wholesale price to give the profit to the retailer, but in order to ensure that its profits will not be affected, and ease the competition with the retailer, the manufacturer will increase the sales price accordingly.

(2) The impact of manufacturer's opportunity cost rate on pricing

When the retailer chooses to borrow from the manufacturer, supposing $\alpha = 0.2$ and $\alpha = 0.6$ respectively, according to the six-month loan interest rate of commercial banks, supposing r_m =4.35% to explore the impact of manufacturer's opportunity cost rate on pricing, the results are as follows.

From Figures 17 and 18, we can find that when the retailer chooses to borrow from the manufacturer, with the increase of opportunity cost rate of the manufacturer, the wholesale price set by the manufacturer will increase to make up for the loss of its investment; the sales price will first decrease and then increase with the increase of the opportunity cost rate, this is because when the opportunity cost rate is controlled in a small range, the manufacturer's loss is not very large, it will compete for market share by reducing the sales price and make up for the loss by sales volume. However, when the opportunity cost rate exceeds a certain limit, the manufacturer will increase the sales price and make up for the loss by increasing the unit sales price.

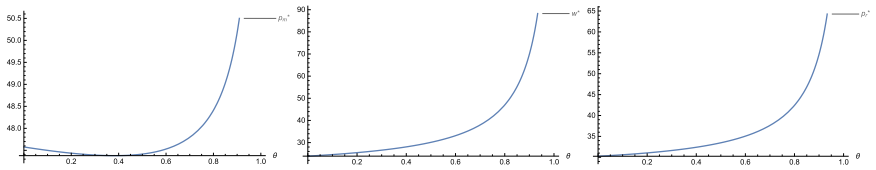


FIGURE 17. The impact of manufacturer's opportunity cost rate on pricing($\alpha = 0.2$).

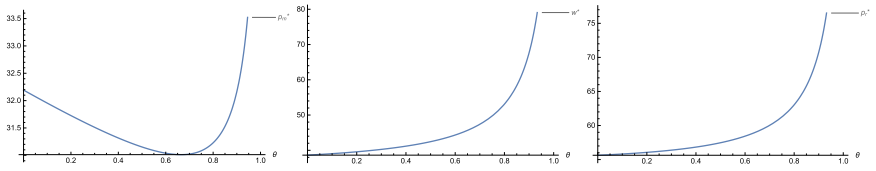


FIGURE 18. The impact of manufacturer's opportunity cost rate on pricing($\alpha = 0.6$).

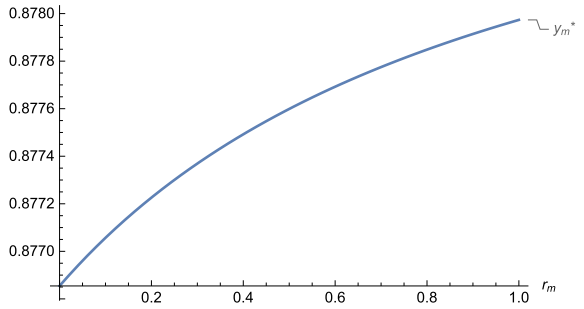


FIGURE 19. The impact of borrowing rate on the level of manufacturer's CSR.

For the retailer, when faced with higher borrowing rate and rising wholesale price, the retailer will make up for the loss of its purchase and debt service by setting a higher retail price.

We can also see a phenomenon from the figure, that is, with the decrease of α, the inflection point of the curve will move to the left, which indicates that with the decrease of α, the tolerance of the manufacturer to the increase of opportunity cost rate will decrease, this is because when the market share of the offline channel is small, that is, the market share of the manufacturer's online channel is already large, there is little room for the manufacturer to adopt the low price strategy to attract consumers. The manufacturer will adjust the price strategy in time according to the increase of the opportunity cost rate, and obtain higher profits by increasing the sales price.

(3) The impact of borrowing rate on the level of manufacturer's CSR

When the retailer chooses to borrow from the manufacturer, supposing $\alpha = 0.5$, according to the six-month loan interest rate of commercial banks, supposing $\theta = 4.35\%$ to explore the impact of borrowing rate on the level of manufacturer's CSR, the results are as follows.

From Figure 19, we can find that with the increase of borrowing rate, the income of the manufacturer will increase, and more funds will be used to improve the level of CSR.

6. Conclusion and future extensions

6.1. The conclusion

Constructing Stackelberg game in a two-echelon dual-channel supply chain, this paper considers the product pricing, CSR level, and service level to analyze the supply chain participants' optimal decisions with retailer's payment methods. The following conclusion is drawn.

6.1.1. The market share, service level, and CSR level have impact on optimal pricing decisions

No matter what kind of payment method the retailer chooses, market share, service level, and CSR level have the same impact on the pricing decisions of all participants in the supply chain.

When the market share of the offline channel is large, the manufacturer should reduce its online sales price and raise its wholesale price to attract consumers and make more profits in retail channels. In the face of the increase in wholesale price, the retailer must increase retail price to ensure profitability.

With the improvement of the retailer's service level, the added value of products will increase, the corresponding cost will also increase, and the retailer's sales price will increase. The improvement of service level will bring additional market share to the retailer. The manufacturer must correspondingly reduce online sales price and raise the wholesale price to compete for market share and maintain its profit.

With the improvement of the level of CSR, the cost paid by the manufacturer and the retailer for taking responsibility will increase. At this time, the manufacturer and the retailer will raise their prices to maintain enterprise profits.

6.1.2. The discount rate of advance payment has an impact on optimal pricing decisions

With the increase of the discount rate of advance payment, the manufacturer will eventually receive less pay for goods, so they must maintain profits by increasing the sales and wholesale prices. The impact of discount rates on retailer pricing is more complex.

When the retailer does not invest with leisure funds, if the opportunity cost is greater than the profit from discounts and the market share of the offline retail channel is very small, the retailer will reduce the sales price and adopt the low-margin strategy of quick turnover. However, when the market share of the offline retail channel is large, there is not much room for retailers to reduce the sales price to expand the market share. At this time, it will make a profit by increasing the sales price. If the opportunity cost is less than the profit from the discount, it is beneficial to the retailer. It will reduce the sales price to compete for market share and further obtain more significant profits.

6.1.3. The interest rate has an impact on optimal pricing decisions

When the retailer chooses bank loans with the lending rate increase, its retail price will be raised to offset its debt repayment expenses. When the retailer faces a higher lending rate, the manufacturer will set a lower wholesale price to give some profit to maintain the stability of the whole supply chain. As for the sales price, it can be seen from the figure that when the market share of the offline channel is small, the online sales price first decreases and then increases with the increase of lending rate, when the market share of the offline channel exceeds a specific range, the sales price will reduce with the rise of lending rate, this shows that when the manufacturer's online channel market share is dominant, and the bank loan interest rate is not very high, the manufacturer will reduce the sales price to attract more customers, however, when the lending rate exceeds a specific limit, the retailer's performance becomes worse, At this time, the manufacturer will increase the sales price to ease the competition with the retailer and cushion the loss caused by customer churn. When the manufacturer's online channel market share is no longer dominant, to protect interests, the sales price will decrease with the increase of lending rate to compete with the retailer for market share.

When the retailer chooses to borrow from the manufacturer, with the increase of borrowing rate, for the retailer, its debt service expenses will increase, and it will increase its revenue through low price competition strategy; for the manufacturer, when the retailer faces higher borrowing rate, the manufacturer will set a lower

wholesale price to give the profit to the retailer, but to ensure that its profits will not be affected, and ease the competition with the retailer, the manufacturer will increase the sales price accordingly.

6.1.4. The manufacturer's opportunity cost rate has an impact on optimal pricing decisions

When the retailer chooses to finance from the manufacturer, increasing the opportunity cost rate of the manufacturer, the wholesale price set by the manufacturer will increase to make up for the loss of its investment. The sales price will first decrease and then increase with the opportunity cost rate because when the opportunity cost rate is controlled in a small range, the manufacturer's loss is not very large. The retailer will compete for market share by reducing the sales price and make up for the loss by sales volume. However, when the opportunity cost rate exceeds a particular limitation, the manufacturer will increase the sales price and compensate for the loss by increasing the unit sales price.

When faced with a higher borrowing rate and rising wholesale price, the retailer will make up for the loss of its purchase and debt service by setting a higher retail price. With the decrease of the offline channel's market share, the manufacturer's tolerance to the increase of opportunity cost rate will decrease with the decline in market share of the offline channel. When the market share of the offline channel is small, there is little room for the manufacturer to adopt the low price strategy to attract consumers. The manufacturer will adjust the price strategy in time according to the increase of the opportunity cost rate and obtain higher profits by increasing the sales price.

6.1.5. The demand elasticity of CSR has an impact on the CSR level

With the increase of demand elasticity of CSR, the manufacturer and the retailer's improvement of CSR level will have a more significant role in promoting market demand, encouraging them to improve their CSR level continuously.

6.1.6. The service sensitivity coefficient has an impact on the retailer service level

With the increase of service sensitivity coefficient, consumers are more sensitive to the retailer's service level. When the retailer improves its service level, it will bring greater market demand, encouraging the retailer to improve its service level continuously.

6.1.7. The discount rate of advance payment and lending rate have impact on retailers CSR level and service level

With the increase of discount rate, the retail purchase cost decreases and the profit increases, and more funds will be used to improve CSR level.

When the market share of the offline channel is small, with the increase of discount rate, the retail purchase cost will reduce. The profit will be increased, and the surplus funds will be used to improve the service level of the enterprise, thus expanding the demand and increasing the profit. When the market share of the offline channel is large, the space to expand the market share is small. At this time, the retailer will reduce the service level and then reduce the cost. At the same time, it will ease the competition with the manufacturer for market share and give the profit to the manufacturer.

With the increase of the lending rate, the debt service expenditure increases, and the retailer's cost needs to be reduced in time, so the CSR and service levels should be reduced.

6.1.8. The investment return rate and borrowing rate have impact on the manufacturer's CSR level

With the increase of the rate of return on investment and borrowing rate, the manufacturer's profit will increase, and the surplus funds will be used to improve the CSR level.

In summary, the market share, service level, and other factors will directly impact the optimal decision-making of all participants in the supply chain. The market share will make the enterprises more sensitive to the changes in the discount rate of advance payment, financing interest rate, and opportunity cost rate, affecting pricing decision-making. To better adapt to the changes of the market while timely adjusting their own decisions,

enterprises should always pay attention to the response and decision changes of upstream and downstream affiliated enterprises and pay attention to the rationality of capital allocation to have more choices in the future.

6.2. The future extension

The following extensions are of interest for future research:

(1) The Stackelberg game is deterministic and static, but the real world is stochastic and dynamical. Further development is to consider the stochastic Stackelberg game.

(2) We use some linear functions, which can be extended to nonlinear functions.

(3) We can conduct more in-depth research combined with the agricultural product supply chain, concretize the research subject, and study the impact of their specific behaviors on the decision-making of the supply chain.

Appendix A.

A.1. Proof of Theorem 4.1

Firstly, deriving the first-order condition of π_r with respect to p_r, and $p_r(p_m, w) = \frac{gt+by_r+a\alpha+\beta w+\eta p_m}{2\beta}$ is obtained, then substituting $p_r(p_m, w)$ into π_m to get $\pi_m(p_m, w)$, and solving the first-order condition of $\pi_m(p_m, w)$ with respect to p_m and w to obtain:$p_m* = \frac{a\beta-tg\beta+by_m\beta-a\alpha\beta+gt\eta+by_r\eta+a\alpha\eta}{2\beta^2-2\eta^2}$ and $w* = \frac{tg\beta+by_r\beta+a\alpha\beta+a\eta-gt\eta+by_m\eta-a\alpha\eta}{2\beta^2-2\eta^2}$, by simplifying the equations,
$p_m^* = \frac{a\beta+\beta(by_m-tg-a\alpha)+\eta(by_r+tg+a\alpha)}{2\beta^2-2\eta^2}$ and $w^* = \frac{\beta(by_r+tg+a\alpha)+\eta(a+by_m-tg-a\alpha)}{2\beta^2-2\eta^2}$ can be obtained. Finally, by substituting p_m^* and $w*$ into $p_r(p_m, w)$,
$p_r^* = \frac{(3\beta^2-\eta^2)(by_r+tg+a\alpha)+2\beta\eta(a+by_m-tg-a\alpha)}{4\beta(\beta-\eta)(\beta+\eta)}$ can be obtained.

A.2. Proof of Corollary 4.1

Because β is the price elasticity of demand and η is the cross-price elasticity coefficient, thus $\beta > \eta$.Then we can get the following results:

$$
\begin{aligned}
\frac{\partial p_m*}{\partial \alpha} &= \frac{a(\eta-\beta)}{2\beta^2-2\eta^2} = -\frac{a}{2(\beta+\eta)} < 0,\\
\frac{\partial w*}{\partial \alpha} &= \frac{a(\beta-\eta)}{2\beta^2-2\eta^2} = \frac{a}{2(\beta+\eta)} > 0,\\
\frac{\partial p_r*}{\partial \alpha} &= \frac{(3\beta^2-\eta^2)a-2\beta\eta a}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{(3\beta^2-\eta^2-2\beta\eta)a}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{(2\beta^2-2\beta\eta+\beta^2-\eta^2)a}{4\beta(\beta-\eta)(\beta+\eta)}\\
&= \frac{a(2\beta(\beta-\eta)+(\beta+\eta)(\beta-\eta))}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{a((3\beta+\eta)(\beta-\eta))}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{a(3\beta+\eta)}{4\beta(\beta+\eta)} > 0,\\
\frac{\partial p_m*}{\partial g} &= \frac{t(\eta-\beta)}{2\beta^2-2\eta^2} = -\frac{t}{2(\beta+\eta)} < 0,\\
\frac{\partial w*}{\partial g} &= \frac{t(\beta-\eta)}{2\beta^2-2\eta^2} = \frac{t}{2(\beta+\eta)} > 0,\\
\frac{\partial p_r*}{\partial g} &= \frac{(3\beta^2-\eta^2)t-2\beta\eta t}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{(3\beta^2-\eta^2-2\beta\eta)t}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{(2\beta^2-2\beta\eta+\beta^2-\eta^2)t}{4\beta(\beta-\eta)(\beta+\eta)}\\
&= \frac{t(2\beta(\beta-\eta)+(\beta+\eta)(\beta-\eta))}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{t((3\beta+\eta)(\beta-\eta))}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{t(3\beta+\eta)}{4\beta(\beta+\eta)} > 0,\\
\frac{\partial p_m*}{\partial y_m} &= \frac{b\beta}{2\beta^2-2\eta^2} = \frac{b\beta}{2(\beta+\eta)(\beta-\eta)} > 0,
\end{aligned}
$$

$$\frac{\partial w*}{\partial y_m} = \frac{b\eta}{2\beta^2 - 2\eta^2} = \frac{b\eta}{2(\beta+\eta)(\beta-\eta)} > 0,$$
$$\frac{\partial p_r*}{\partial y_m} = \frac{2\beta\eta b}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{b\eta}{2(\beta-\eta)(\beta+\eta)} > 0,$$
$$\frac{\partial p_m*}{\partial y_r} = \frac{b\eta}{2\beta^2 - 2\eta^2} = \frac{b\eta}{2(\beta+\eta)(\beta-\eta)} > 0,$$
$$\frac{\partial w*}{\partial y_r} = \frac{b\beta}{2\beta^2 - 2\eta^2} = \frac{b\beta}{2(\beta+\eta)(\beta-\eta)} > 0,$$
$$\frac{\partial p_r*}{\partial y_r} = \frac{b(3\beta^2 - \eta^2)}{4\beta(\beta-\eta)(\beta+\eta)} = \frac{b\beta}{2(\beta-\eta)(\beta+\eta)} + \frac{b}{4\beta} > 0.$$

A.3. Proof of Theorem 4.2

Firstly, substitute p_r^*, p_m^* and w^* into π_r^* and π_m^*, by combining and solving the first-order condition equations: $\frac{\partial \pi_r^*}{\partial g} = 0$, $\frac{\partial \pi_m^*}{\partial y_m} = 0$ and $\frac{\partial \pi_r^*}{\partial y_r} = 0$, we can obtain the following results:

$$y_m^* = \frac{ab\beta(-c_r t^2 + s(\alpha-1)(b^2 - 2c_r\beta) + 8c_r s\alpha\eta)}{(b^2 s + c_r(t^2 - 8s\beta))(\beta(b^2 - 2c_m\beta) + 2c_m\eta^2)},$$
$$y_r^* = \frac{abs\alpha}{8c_r s\beta - b^2 s - c_r t^2},$$
$$g^* = \frac{ac_r t\alpha}{8c_r s\beta - b^2 s - c_r t^2}.$$

A.4. Proof of Corollary 4.2

This proof process is complex, and readers can understand it in combination with numerical analysis. (See Figs. 5 and 6).

The detailed proof process of other theorems and corollaries is similar to the above proof, and will not be repeated here.

Acknowledgements. This work is supported partly by National Office for Philosophy and Social Sciences of China (Grant No. 21BJY206).

References

[1] Y. Aan, C. Kuzey, M.F. Acar and A. Akgz, The relationships between corporate social responsibility,environmental supplier development, and firm performance. *J. Clean. Prod.* **112** (2016) 1872–1881.
[2] J. Bing and A. Seidmann, Finance souring in a supply chain. Decis. *Support. Syst.* **58** (2014) 15–20.
[3] J.A. Buzacott and R.Q. Zhang, Inventory management with asset-based financing. *Manag. Sci.* **50** (2004) 1274–1292.
[4] X. Chen, A model of trade credit in a capital-constrained distribution channel. *Int. J. Prod. Econ.* **159** (2015) 347–357.
[5] B. Dan, G. Xu and C. Liu, Pricing policies in a dual-channel supply chain with retail services. *Int. J. Prod. Econ.* **139** (2012) 312–320.
[6] J.C. Fan, D.B. Ni and X.W. Tang, Product quality choice in two-echelon supply chains under post-sale liability:insights from wholesale price contracts. *Int. J. Prod. Res.* **55** (2017) 2556–2574.
[7] J.C. Fan, D. B. Ni and X. Fang, Liability cost sharing, product quality choice, and coordination in two-echelon supply chains. *Eur. J. Oper. Res.* **284** (2020) 514–537.
[8] N. He, Z.-Z. Jiang, J. Wang, M. Sun and G. Xie, Maintenance optimisation and coordination with fairness concerns for the service-oriented manufacturing supply chain. *Enterp. Inf. Syst.-UK* **15** (2021) 694–724.
[9] S. Huang, Z.-P. Fan and X. Wang, The impact of transportation fee on the performance of capital-constrained supply chain under 3PL financing service. *Comput. Ind. Eng.* **130** (2019) 358–369.
[10] J. Huang, W. Yang and Y. Tu, Financing mode decision in a supply chain with financial constraint. *Int. J. Prod.Econ.* **220** (2020) 107441.

[11] W. Jin, J. Luo and Q. Zhang, Optimal ordering and financing decisions under advance selling and delayed payment for a capital-constrained supply chain. *J. Oper. Res. Soc.* **69** (2018) 1978–1993.
[12] W. Jin, Q. Zhang and J. Luo, Non-collaborative and collaborative financing in a bilateral supply chain with capital constraints. *OMEGA-Int. J. Manage. Sci.* **88** (2019) 210–222.
[13] P. Kouvelis and W. Zhao, Financing the newsvendor: Supplier vs. Bank, and the structure of optimal trade credit contracts. *Oper. Res.* **60** (2012) 566–580.
[14] Q.H. Li and B. Li, Dual-channel supply chain equilibrium problems regarding retail services and fairness concerns. *Appl. Math. Model.* **40** (2016) 7349–7367.
[15] Y. Liu, B.-T. Quan, Q. Xu and J.Y.-L. Forrest, Corporate social responsibility and decision analysis in a supply chain through government subsidy. *J. Clean. Prod.* **208** (2019) 436–447.
[16] N.M. Modak, N. Kazemi and L.E. Cardenas-Barron, Investigating structure of a two-echelon closed-loop supply chain using social work donation as a Corporate Social Responsibility practice. *Int. J. Prod. Econ.* **207** (2019) 19–33.
[17] N.M. Modak, S. Sinha, A. Raj, S. Panda, J.M. Merigo and A. Jabbour, Corporate social responsibility and supply chain management: Framing and pushing forward the debate. *J. Clean. Prod.* **273** (2020) 122981.
[18] N.M. Modak and P. Kelle, Using social work donation as a tool of corporate social responsibility in a closed-loop supply chain considering carbon emissions tax and demand uncertainty. *J. Oper. Res. Soc.* **72** (2021) 61–77.
[19] S. Panda, N.M. Modak, S.S. Sana and M. Basu, Pricing and replenishment policies in dual-channel supply chain under continuous unit cost decrease. *Appl. Math. Comput.* **256** (2015) 913–929.
[20] S. Panda, N.M. Modak and L.E. Cardenas-Barron, Does extended warranty depict competitive advantage to a retailer in a retail-e-tail channel supply chain. *Comput. Ind. Eng.* **149** (2020) 106770.
[21] R.A. Schwartz, An economic model of trade credit. *J. Financial Quant. Anal.* **99** (1974) 643–657.
[22] A.J. Vakharia and L. Wang, Uniform vs. Retailer-Specific Pricing: Incentive Alignment to Enhance Supply Chain Efficiency. *Prod. Oper. Manag.* **23** (2014) 1176–1182.
[23] H. Yang, W. Zhuo, Y. Zha and H. Wan, Two-period supply chain with flexible trade credit contract. *Expert Syst. Appl.* **66** (2016) 95–105.
[24] C.-H. Zhang, P. Xing and J. Li, Optimal Strategy of Social Responsibility and Quality Effort in Service Supply Chain with Quality Preference. *Asia-Pacific J. Oper. Res.* **35** (2018) 1850018.

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