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A BILEVEL GAME MODEL FOR ASCERTAINING COMPETITIVE TARGET PRICES FOR A BUYER IN NEGOTIATION WITH MULTIPLE SUPPLIERS

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Abstract. In this paper, a decision-support is developed for a strategic problem of identifying target prices for the single buyer to negotiate with multiple suppliers to achieve common goal of maintaining sustained business environment. For this purpose, oligopolistic-competitive equilibrium prices of suppliers are suggested to be considered as target prices. The problem of identifying these prices is modeled as a multi-leader-single-follower bilevel programming problem involving linear constraints and bilinear objective functions. Herein, the multiple suppliers are considered leaders competing in a Nash game to maximize individual profits, and the buyer is a follower responding with demand-order allocations to minimize the total procurement-cost. Profit of each supplier is formulated on assessing respective operational cost to fulfill demand-orders by integrating aggregate-production-distribution-planning mechanism into the problem. A genetic-algorithm-based technique is designed in general for solving large-scale instances of the variant of bilevel programming problems with multiple leaders and single follower, and the same is applied to solve the modeled problem. The developed decision support is appropriately demonstrated on the data of a leading FMCG manufacturing firm, which manufactures goods through multiple sourcing.

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1. Introduction

Suppliers compete on prices to attract a competitive demand share from their buyer, in an oligopolistic-monopsony market, for maximizing their profits. On the other side, the buyer intends to minimize the overall procurement cost for receiving a regular supply of required products. To purse this objective, the buyer seeks to exploit the competition between suppliers and therefore negotiates with them to lower down the prices. The status of being a sole buyer of an oligopolistic-monopsony market provides a bargaining power to entice suppliers with a larger share of demand-order in exchange for lowering the prices. In our problem, the buyer's overall demand is less than the combined output capacities of all the suppliers. In this situation, the buyer's bargaining power gets further increased, and therefore suppliers are forced to renegotiate prices for obtaining

Keywords. Competitive target prices, price negotiations, bilevel programming, multi-leader-single-follower bilevel game, aggregate-production-distribution-planning, genetic algorithm.

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demand-orders. In such a scenario, the buyer can adopt an opportunistic discriminatory approach for negotiation with suppliers to reduce the procurement cost to a minimum. Some of the suppliers' prices negotiated in this manner may be well below a point where all the suppliers would have agreed upon due to complete awareness of competition in the discussed business environment. It develops a dissonance among such suppliers, and a tendency of non-cooperation with the buyer starts emerging among suppliers [58]. This antipathy adversely affects the buyer-supplier relationship [40, 68, 78, 84], which, in turn impacts performance of their supply chain detrimentally [93]. On the other hand, if the buyer's price-negotiations with suppliers settle at a point above an equilibrium point of suppliers' oligopolistic competition, the buyer's financial interests are adversely affected, incurring a potentially higher procurement cost. If suppliers resist lowering the prices towards the equilibrium point, it compels the buyer to look for alternative sourcing arrangements as a long-term initiative to intensify the competition among suppliers. This remedial action by the buyer results in reducing the supplier's bargaining power and their demand-shares in the future, resulting in reduced profit in the long run.

The aggrieved stakeholders on either side of the considered market situation have an adverse action space that can thwart from maintaining a sustained and continuous business environment. Accordingly, the outcome of price-negotiations should ensure long term gains through sustained and continuous replenishment of products for the buyer *vis-à-vis* short-term gains through opportunistic discrimination in price negotiation. Similarly, each supplier should prefer getting sustained and continuous replenishment orders from the buyer, *vis-à-vis* creating an adverse action space that may affect their business prospects. Therefore, it is imperative to identify prices for the business deal to achieve common goal while pursuing parallel individual objectives. Consequently, it is appropriate for the stakeholders at both sides to settle at equilibrium prices of suppliers' oligopolistic competition.

Identifying suppliers' oligopolistic competitive equilibrium prices as target prices for negotiation imposes the competition among suppliers up to their individual production-distribution capabilities, and cost-efficiencies. As a result, the negotiated prices leave no space for any opportunistic discrimination on the buyer's part and protect her/his financial interests as well. As these target prices for negotiation are based only on suppliers' competitive capabilities, we appropriately term these as *competitive target prices*. Further, in case the price-negotiations with some supplier(s) settle significantly away from these prices, it indicates that negotiations have not resulted purely due to a fair competition among suppliers.

The oligopolistic-monopsony market ecosystem with the situation described above is common in many sectors. It can emerge because of localization of the market of suppliers due to transportation cost efficiencies. Whenever a manufacturing firm comes up with a new product, which involves a completely new technology, the market situation emerging with its suppliers who associate for providing required components refers to oligopolistic-monopsony market ecosystem. Another example of this market ecosystem is observed in case of sole manufactures of non-substitutable products when they need to procure components or raw material from their suppliers for the execution of manufacturing process. This situation is prevalent in case of specialized products, for example, in case of luxury cars. A particular example for getting more insight can be discussed in context of Indian Railways. Indian Railways is the only organization providing railway travel services within India and for providing these services it manufactures train coaches at its own factories situated at multiple locations within India. For this manufacturing it requires multiple electrical equipments with specific technical requirements, which are fulfilled by a limited number of specialized suppliers. This makes the Indian Railways a single buyer for these specific products, thereby positioning the market system as a monopsony from the buyer's side, and oligopoly from the suppliers' side.

Although, multiple examples of the discussed market ecosystem are observed in many sectors, but to the best of our knowledge, the problem of identifying target prices for negotiations among the buyer and suppliers is not studied in the literature.

In this paper, we develop a decision support for identifying competitive target prices for the buyer's negotiations with multiple suppliers in the oligopolistic-monopsony market. We model the problem of identifying the competitive target prices as a multi-leader-single-follower (MLSF) bilevel programming problem. The model formulated as MLSF bilevel programming problem involves linear constraints and bilinear objective functions.

The model involves integer variables for capturing an actual practice of supply chain where demand orders, production and logistic quantities take integral values. For formulating the action-and-reaction mechanism of price-negotiations in our bilevel programming model, we consider suppliers as leaders and the buyer as a follower. This is based on the chronology of communication among them *viz*, suppliers making the first move by offering the price quotes to the buyer, whereas the buyer responding to those in terms of the demand-order allocation. The price-competition among the group of suppliers for receiving maximum profitable demand shares from the buyer leads to a game situation among these suppliers, thereby making it appropriate to model the problem as an MLSF bilevel programming problem. In the oligopolistic competition among suppliers, the assessment of price offers by the buyer requires an integrated planning, as the total cost of fulfilling demand-orders is related to different operating costs of production and distribution. Therefore, for assessing total operating costs and production-and-logistics capacity constraints of each supplier it is compelling to embed the aggregate-production-distribution-planning (APDP) of suppliers in the formulation of our model. The optimal demand-order allocation problem from the point of view of buyer is woven as the follower's reaction mechanism in the formulation of the model. By considering all the essential factors highlighted above in design of the model, a solution of the problem thus formulated represents competitive target prices.

The structure of modeled problem corresponds to a specific type of MLSF bilevel programming problem which involves linear constraints and bilinear objective functions at both levels. Direct methods existing in the literature have been experienced as incapable of handling the large-scale instance of the modeled problem with practical data taken from industry. In this situation, first a nested GA based solution method is suggested for solving generic problems of the specified type of bilevel programming problems. Then, a modification is further suggested in the solution algorithm to capture a peculiar interdependence of variables of the modeled problem. Finally, the proposed model and the solution method are then illustrated on a data set of an FMCG sector firm concerned about optimality of the procurement cost of ingredients required for manufacturing its products in an environment discussed above.

The paper is organized as follows. Section 2 presents literature review. Section 3 describes the problem, its assumptions and formulation as an MLSF bilevel game model. Section 4 describes the GA-based solution methodology proposed for solving the discussed problem. Section 5 demonstrates an experimental study of a manufacturing firm and analysis of the results obtained through the formulation and algorithm. Managerial implications of adopting the suggested approach and utilizing the proposed decision-support are listed in Section 6. Section 7 concludes the paper with the scope of the research work.

2. Literature review

2.1. Price setting and negotiation pricing problems

From the perspective of a supplier, problems of optimal price-setting (including high and low pricing strategies), flexible pricing (like market segmentation), differential pricing, price skimming, penetration pricing, and revenue management based pricing are studied in literature and reviewed by Dolgui and Proth [32]. The pricing problem, when demand is influenced by prices to maximize the turnover, is modeled as a bilevel programming problem by Labbé *et al.* [57], and implemented on a toll setting problem. Dempe and Zemkoho [27] use nonsmooth analysis to design Karush–Kuhn–Tucker (KKT) type optimality conditions, which preserve essential data in the traffic assignment problem. Some other studies which model the toll price-setting problems include [18, 29, 46, 105]. Kumar *et al.* [55] studied a strategic pricing problem in a non-competitive environment from a small-scale supplier, coherently discerning operational capacities before quoting the prices to the buyer. A monograph by Nagle and Müller [79] is appropriate to refer to the basics of pricing, price competition, and price negotiations. A survey on price setting problems modeled as bilevel programming problems is provided by Labbé and Violin [56].

Negotiation pricing problems attempt to identify the prices where both seller and buyer mutually agree as beneficial price. The research on devising a mechanism for price negotiation seems limited and evolving, with some of research developments in this field to name (*e.g.*, [14, 37, 42, 75, 87, 97, 98]). None of the framework in existing studies match our situation.

2.2. Supplier selection and order allocation

The supplier selection is a crucial strategic decision-making process for the procurement of the required products or the raw material. The primary literature on it begins with the work of Dickson [30]. The next accompanying vital decision is on the order allocation under multiple sourcing. Gaballa [38] and Jayaraman *et al.* [52] developed decision support for the order allocation problem to minimize the total procurement cost. The prevalent use of just-in-time approach for inventory procurement resulted in the recommendation of multiple factors for supplier selection [59, 92, 102]. Several researchers investigated the order allocation in concurrence with supplier selection [10,80,96]. The supplier selection and order allocation issues have been modeled as multi-objective decision-making problems also [4, 25, 90]. One can refer to the recent literature review by Aouadni *et al.* [3] on supplier selection and order allocation techniques.

2.3. Operational planning problems

Planning of various tasks to fulfill the demand of end-users over a short-term horizon is termed as operational planning under the supply chain management [72]. Production planning in regular-time, over-time, and outsourcing (in each plant in each period), shipping volumes from the production facility or stack buffers to warehouses, shipping volumes from warehouses and stack buffers directly to end-users, inventory of finished products in warehouses at each period, is categorized as aggregate production-distribution planning (APDP) [36]. The literature in the field of APDP problems is extensive and classified into seven categories [36]. Some of the significant contributions worth citing in APDP are [35, 64, 65, 85]. Recent research developments in operational planning include [74, 81].

2.4. MLSF bilevel game problem

In certain business planning situations, the decision-makers are often unable to realize their decisions independently but forced to act according to a specific hierarchy. The programming problem formulated to solve this situation is a multilevel programming problem with a case as a bilevel programming problem involving a leader and a follower. A leader is the decision-maker who can take an independent position in analyzing and using the reaction of the dependent decision-maker, and the latter one is the follower. For the theoretical development, algorithms, and applications of bilevel programming, one can refer to excellent texts [6, 16, 26]. One can refer to Sinha *et al.* [91] for a detailed review of classical and evolutionary algorithms and several applications of bilevel optimization. Multilevel decision-making problems, including applications into the supply chain, are reviewed by Lu *et al.* [69]. Some of the issues in supply chain planning and management are studied using the bilevel programming framework (*e.g.*, [5, 88, 99]).

MLSF bilevel games are categorized as a variant of bilevel programming problems, which involve multiple competing leaders and each one needs to incorporate the response of their common follower in their decision making. Some interesting practical problems have been modeled using this bilevel-game framework (*e.g.*, [45, 46, 89]). Theoretical developments are suggested in the literature for solving such problem by reformulating them using stationarity conditions [51, 60]. The suggested solution methodologies in the literature solve the multi-leader-common-follower (MLCF) bilevel game problems in terms of strong-stationary points [60] and strong-stationary equilibrium points [51]. Leyffer and Munson [60] suggested a direct method for obtaining strong-stationary points by solving a nonlinear programming problem derived from strong-stationarity conditions by posing complementarity conditions as the objective function to be minimized to zero. The authors demonstrated the proposed approach through numerical experiments on randomly generated small-scale and medium-scale electricity market problems involving 150 constraints and 160 variables. Hori and Fukushima [51] proposed a Gauss-Seidel method for numerical convergence to a strong-stationary equilibrium point, which involves solving a penalty-based quadratic optimization problem in each of the iterations. Five examples, involving a maximum of 12 variables and less than 10 constraints, are presented to demonstrate the process. An extensive literature review of methodologies for solving MLCF bilevel game problems has indicated that none of the methods have been tested on large-scale problems involving more than 160 variables and 150 constraints. Moreover, no algorithm is developed for solving an MLCF bilevel game problem involving integer variables.

2.5. Genetic algorithms approaches

Genetic algorithms are bio-inspired artificially intelligent random search algorithms, which are developed due to seminal works of pioneers Holland [50] and Goldberg [43]. Initial development of GA encodes chromosomes as binary strings and thus is called Binary Coded GA (BCGA). The real coded GA (RCGA) [20,70], which encodes chromosomes as strings of real numbers has been found superior to BCGA in terms of efficiency of exploration and against challenges like Hamming Cliff [44, 104]. Contributions towards further development improved the efficiency and acceptance of RCGA [34, 86]. Eventually, several researchers have employed RCGA for solving numerical optimization problems involving continuous variables [21,34,48,49,77,100,104]. Due to the capability of solving majority of complex NP-hard real-world large scale optimization problems, RCGA has been used in almost every area of practical application (*e.g.*, [7, 8, 19, 71, 94, 95]). Later, an Integer Coded GA (ICGA) was developed to solve integer or mixed integer programming problems. Crossover and mutation operators have specifically been developed for ICGA [1,19,22–24,82]. One can refer to Mirjalili [76] for rudimentary prelude and a bird's-eye review on developments of GA. Some standard problems of supply chain planning and management, which are NP-hard, are solved using GA and its variants [35, 41, 67]. GA and its other variants are extensively used in supplier selection problems [28, 31, 66, 106].

Evolutionary approaches for solving the bilevel programming problems are studied separately from those developed for solving general optimization problems due to the special structure of the bilevel problems. Moreover, due to the same reason, evolutionary approaches have been developed in the literature for specific variants of the bilevel programming problems. Pricing problems modeled in the bilevel programming framework have also been solved using GA [33, 103], and a similar artificially intelligent algorithm *viz.*, particle search optimization algorithm [39]. GA based evolutionary algorithms have been used to solve some special classes of bilevel programming problems [9, 11, 15, 47, 54, 55, 61, 63, 101, 107]. Among the evolutionary algorithms developed for solving bilevel programming problems, those based on nested approach are effectively used over those developed for solving general optimization problem. In this approach, only the leader(s) variables are encoded and explored randomly, while the respective values of follower's variables are obtained directly by solving follower(s) problem after passing the values of leader's variables as parameters. Therefore, the nested GA for bilevel programming problems focuses on developing the reaction mechanism for the follower(s) problem in response to the given values of leader's variables and formulating the fitness function. Algorithms developed under this approach for solving some particular cases of bilevel programming problems can be referred in the literature [2, 12, 13, 54, 62, 73, 107, 108]. But none of the studies have considered solving a bilevel programming problem involving integer variables particularly at followers' level.

In this backdrop, we are now ready to present our problem statement, model formulation, and the associated theoretical developments. In subsequent sections we present an algorithm developed for solving the formulated model and its illustration on a real-life case.

3. Bilevel game for ascertaining competitive target prices

3.1. Problem description

The negotiation process among buyer and suppliers is considered in the following setup.

– A buyer has already identified a set of suppliers based on their production and logistic infrastructural capacities and the quality standards of both products and services.

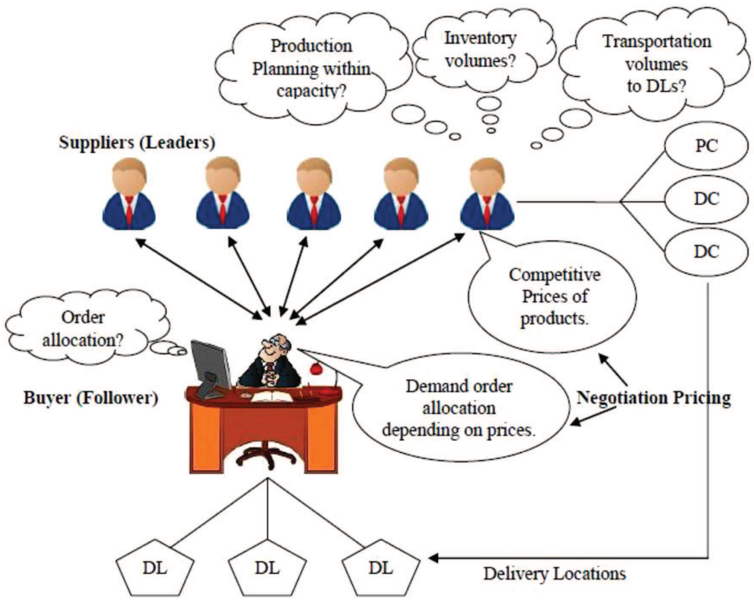

FIGURE 1. Depiction of structure.

- Each of these suppliers can produce some or all the required products and deliver them at various locations of the buyer.
- Considering the supply capacities of all the suppliers the total requirements of the buyer for various products can be fulfilled in each period.
- This setup is considered for a pre-defined planning horizon fixed by suppliers and the buyer together. The planning horizon is further divided into multiple time intervals, called periods[1].

The buyer wishes to negotiate on prices with these suppliers before entering into a new agreement. At this stage, suppliers have opportunity to review and agree on the prices of the goods considering production-and-logistics costs over the planning period and competition from the other suppliers. We consider the business setup where the suppliers bear the cost of on-time delivery at various delivery locations. It compels the suppliers to assess their transportation cost, inventory cost, and production cost in pricing while negotiating with the buyer.

The price-negotiation process of the buyer with each supplier begins with the supplier's offer on the prices of the products. The buyer tries to negotiate with each supplier to lower the prices of the products by offering a greater proportion of the demand-order. The supplier rethinks on the profit while discerning the production-distribution costs and reviews the competition from those suppliers who deal with the buyer in these products particularly. The supplier then agrees for these prices or tries to negotiate further with the buyer. The process of price negotiation gets over once the supplier and the buyer arrive at the final prices. Figure 1 provides a schema for the discussed negotiation process. We develop a model for the buyer to identify competitive target prices for negotiations with a relatively competitive group of suppliers.

[1]The planning horizon is identified as that time duration by which the actual realization of various costs is expected to remain same as per the assessments. The planning horizon can be any duration, for example a month, quarter, or a year and can be discretized into multiple periods, say weeks, fortnights, or months, respectively, depending on replenishment frequencies of various suppliers.

3.2. Basic assumptions of the model

- Any alternative modes or business options will have an adverse impact on the overall business prospects of stakeholders of either side. This consideration for an oligopolistic-monopsony market compels all the stakeholder of the market to discipline themselves for the stability of the market ecosystem.
- The buyer has a knowledge of production and logistic resources and infrastructure of each supplier, to assess an aggregate-production-distribution plan on behalf of the later for any given demand order allocation[2]. This enables the buyer to have an idea of production-distribution capacities and cost efficiencies of suppliers which are required as inputs of our model.
- Each supplier may deal in some or all products among those required by the buyer. This is assumed to consider a more general situation in which different suppliers deal in subsets of the set of all products required by the buyer.
- Each product is homogenous in form, quality, quantity, size, across various suppliers dealing in that product. This is assumed to consider the perfect substitutability of the products of various suppliers which keeps only the price as a decision-variable for competition among suppliers.
- The prices of products are to be negotiated for a fixed time horizon which is discretized into equal subintervals. These subintervals are termed as periods. The demand orders are to be allocated to various suppliers for the delivery of products at each delivery location, during every period of planning horizon.
- The number of periods, into which the planning horizon be discretized, is considered to be the same for all the suppliers. This assumption is considered for reducing the complexity of the model formulation, which would otherwise be involved by considering it to be different for different suppliers. The same can be practically achieved by considering the length of each period as least common multiple of replenishment frequencies of various suppliers.
- Each supplier has a single production center with no capacity to store any inventory over a period. This is an assumption considered to capture into the model the actual practice observed generally about the production-distribution setups of manufacturers in B2B markets.
- All or some part of products manufactured in any period can be transported directly to various delivery locations of the buyer. In general, the products are transported first from the production center to distribution centers for inventory and cross-docking, and from there transported to the different delivery locations. This transportation arrangement of each supplier is depicted in Figure 2. This arrangement of transportation is considered for capturing into the model the most general provisions of transportation arrangements.
- The distribution centers can store each product that the supplier is dealing in. Likewise, all delivery locations can accommodate all types of products.
- Suppliers are aware of the production-distribution capacities of their competitors through their market intelligence. This practical assumption on competitive awareness of suppliers is considered to capture in our model the competency of negotiators from each supplier which enables them to estimate that up to what their counterparts can lower their price offers. Such an assumption is required as a basic consideration from the game theory that players in a game opt only for their best strategy.
- Suppliers' possible cost fluctuations are taken into consideration during the estimation of parameters. Assuming this makes it possible to consider the parameters as fixed numbers any not fuzzy, as fuzziness will further complicate the model to a very high level of complexity.
- The cost (per unit) of the products by the suppliers is estimated by including all direct and indirect expenses, if any. This relieves from the hassle of considering the fixed costs of production or distribution separately in the APDP part of the model responsible for computing the total cost of supply incurred to each supplier.
- No significant changes in technology and business environment are expected during the planning horizon which may impact the costs drastically. This assumption is made for considering the parameter values taken in our model to beat par with their estimates during the planning horizon. As the planning horizon is of short-term this assumption is realized most of the times apart from some exceptional unprecedented incidences.

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[2]Such an assumption is practical in view of the assessment by the buyer during the supplier selection process.

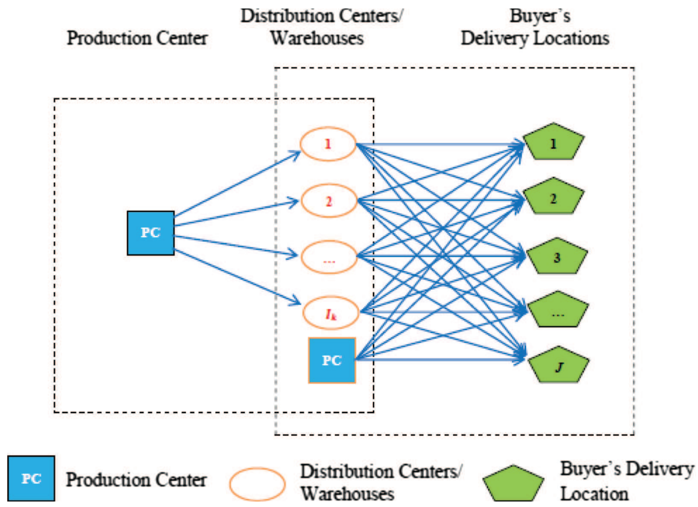

FIGURE 2. Production and distribution structure.

- The variance between the forecasted demand and actual demand for the required products shall be negligible at the buyers end during the planning horizon. Since the planning horizon is short-term, it is reasonable to assume that the buyer can forecast demand encompassing fluctuations in it.
- The buyer knows the price for each supplier below which the supplier would no longer be able to bargain. These prices are identified through break-even prices of suppliers and can be estimated by the buyer during the supplier assessment made during the supplier selection process.
- Discounts or differential pricing are not provisioned in our model. This is assumed for preventing the formulation of our model from including binary integer variables, as they will complicate the model from solution point of view.

3.3. Model formulation

Price quotes from suppliers are a requisition for computing the demand shares by the buyer, which in turn is a requirement for each supplier to assess the profit to be earned as a result. This is a perplexed problem involving action-and-reaction among suppliers and the buyer. The bilevel programming framework is appropriate to model this situation, which terms the action takers as leaders and reaction givers as followers. According to this terminology of bilevel programming, we identify in our problem the suppliers as leaders and the buyer as follower. As there are multiple leaders (suppliers) and single follower (buyer) in the addressed situation, therefore the formulation of model corresponds to a MLSF bilevel programming problem.

Suppliers decide on best price offers, and thereupon prepare the aggregate-production-distribution-plan based on the demand shares received from the buyer in response to the offered prices. Depending on individual resources and surmised prices of other suppliers, each supplier tries to decide on such price offers which can provide appropriate demand allocations from the buyer for achieving the maximum total profit over the planning horizon. This suggests considering the prices and production distribution arrangements of suppliers as leaders variables, whereas the demand allocation decisions as followers variables. Formulating this model of price competition among suppliers thereby requires embedding the APDP problem at leaders' level while posing the demand allocation problem of the supplier at follower's level.

With this basic structure of the model introduced here, the nomenclature of indices, parameters and decision-variables is detailed below, followed by an explanation of components of the model.

3.3.1. Nomenclature

The indices, parameters, and variables used to mathematically formulate our model are listed below. Here, production center of a supplier is termed as "PC". A warehouse/distribution center, which is used for storing the inventory and for cross-docking the in-supply of consignment to out-supply for various delivery locations of the buyer, is termed as "DC". Delivery location of the buyer is denoted as "DL". Prices are discussed in the official currency of the Republic of India, the Indian Rupee, with currency code INR.

Indices and sets	
K	Number of suppliers; $k = 1, 2, \ldots, K$
N	Number of different type of products; $n = 1, 2, \ldots, N$
J	Number of Buyer's Delivery Locations (DLs); $j = 1, 2, \ldots, J$
T	Number of periods in the planning horizon; $t = 1, 2, \ldots, T$
N_k	Set of indices of the products that the supplier k deals in ($N_k \subseteq \{1, 2, \ldots, N\}$); $n_k \in N_k$
I_k	Number of Distribution Centers of the supplier k; $i_k = 1, 2, \ldots, I_k$
	$i_k = 0$ stands for the PC of the supplier k, to act as a source to transport products to directly to DLs.
Leaders' parameters and variables (This is practically interpreted as break-even price of the supplier for a product.)	
Parameters	
lp_{kn_kj}	Minimum reservation price of product n_k from supplier k for its demand at buyer's DL j (INR/unit)[3]
Lp_{kn_kj}	Maximum reservation price of product n_k from supplier k for its demand at buyer's DL j (INR/unit)
a_{kn_kt}	Regular time production cost of product n_k for supplier k in period t (INR/unit)
b_{kn_kt}	Overtime production cost of product n_k for supplier k in period t (INR/unit)
r_{kn_kt}	Machine-hours required by supplier k for production of per unit of product n in period t
$tcp_{kn_ki_kt}$	Cost of transportation of product n_k from PC to DC i_k of supplier k in period t (INR/unit)
$tc_{kn_ki_kjt}$	Cost of transportation of product n_k from DC i_k of supplier k to buyer's DL j in period t (INR/unit)
$d_{kn_ki_kt}$	Inventory carrying cost of product n_k at DC i_k of supplier k in period t (INR/unit)
v_n	Space occupied by per unit of product n_k(cu-ft/unit)
MR_{kt}	Maximum regular machine-hours (man-hours) available with supplier k in period t
M_{kt}	Maximum total machine-hours (man-hours) available with supplier k in period t
V_{i_kt}	Maximum space available in DC i_k of supplier k in period t (cu-ft)
Variables	
z_{L_k}	Gross profit of supplier k
p_{kn_kj}	Per unit price of product n_k from supplier k for its demand at DL j (INR/unit)
Q_{kn_kt}	Regular time production volume of product n_k of supplier k in period t (units)
O_{kn_kt}	Overtime production volume of product n_k of supplier k in period t (units)
$\mathrm{SS}_{kn_ki_kt}$	Inventory level (safety stock) of product n_k at DC i_k of supplier k in period t (units)
$I_{kn_ki_kt}$	Consignment volume of product n_k to be sent from PC to DC i_k of the supplier k in period t (units)
$x_{kn_ki_kjt}$	Consignment volume of product n_k from DC i_k of supplier k to buyer's DL j in period t
Follower's parameters and variables	
Parameters	
z_F	Total cost of procurement and holding products at various DLs (INR/unit)
D_{jnt}	Total forecasted demand of product n at buyer's DL j in period t (units)
My_{kn_k}	Maximum purchase volume of product n_k from supplier k in any period
VF_j	Maximum inventory carrying space at DL j of the buyer (cu-ft)
Variables	
y_{kn_kjt}	Number of units of product n_k to be purchased from supplier k for DL j in period t

[3]This is practically interpreted as break-even price of the supplier for a product.

The mathematical model (Price-BLP) for ascertaining competitive target prices using the MLSF bilevel programming framework can be summarized as follows.

3.3.2. Suppliers' optimization problem

Objective function

For each supplier ($k = 1, 2, \ldots, K$), the objective is to maximize the total profit through the decision on values of price variables (p_{kn_kj}) for each product $n_k \in N_k$ and each delivery location $j = 1, 2, \ldots, J$, and thereupon values of variables of production, inventory, and transportation ($Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$) based on the demand shares received as a response to the prices quoted by all the suppliers. The objective function is given as following.

$$\begin{aligned}\text{Max } z_{L_k} = &\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{j=1}^{J} p_{kn_kj}y_{kn_kjt} - \left\{\sum_{t=1}^{T}\sum_{n_k\in N_k}(a_{kn_kt}Q_{kn_kt} + b_{kn_kt}O_{kn_kt})\right.\\ &+\left(\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i_k=1}^{I_k} d_{kn_ki_kt}\mathrm{SS}_{kn_ki_kt} + \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=1}^{I} tcp_{kn_ki_kt}I_{kn_ki_kt}\right.\\ &\left.\left.+\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=0}^{I}\sum_{j=1}^{J} tc_{kn_ki_kjt}x_{kn_ki_kjt}\right)\right\}. \end{aligned} \tag{3.1}$$

Price bounds

The prices are speculated within bounds; lower bounds are the minimum prices acceptable to the supplier emerging due to their cost efficiencies, whereas the upper bounds are enforced through the competition imposed by the other suppliers. Lower bounds on these prices are termed as minimum reservation prices and upper bounds as maximum reservation prices.

$$lp_{kn_kj} \le p_{kn_kj} \le Lp_{kn_kj}, \qquad \forall n_k, \forall j. \tag{3.2}$$

Regular time production hours

The production volumes for various products are restricted by the regular time production hours.

$$\sum_{n_k\in N_k} r_{kn_kt}Q_{kn_kt} \le \mathrm{MR}_{kt}, \qquad \forall t. \tag{3.3}$$

Total production hours

The total production volumes obtainable through the provisions of overtime engagement of labor/machines along with the regular time production are also restricted by the total available production hours.

$$\sum_{n_k\in N_k} r_{kn_kt}(Q_{kn_kt} + O_{kn_kt}) \le M_{kt}, \qquad \forall t. \tag{3.4}$$

Inventory balancing constraints

The demand orders received for each product are fulfilled through the total production volumes together with the available inventory volumes maintained in the previous period while maintaining required inventory volumes for the current period.

$$Q_{kn_kt} + O_{kn_kt} + \sum_{i_k=1}^{I_k}\mathrm{SS}_{kn_ki_k(t-1)} - \sum_{i_k=1}^{I_k}\mathrm{SS}_{kn_ki_kt} = \sum_{j=1}^{J} y_{kn_kjt}, \qquad \forall n_k, \forall t. \tag{3.5}$$

Space constraints at DC

At any period, the consignment volumes of various products to be received at each DC ($I_{kn_k i_k t}$) along with the products already available there as inventory maintained during the previous period ($\text{SS}_{kn_k i_k (t-1)}$) should be capacitated in the available space at the DC.

$$\sum_{n_k \in N_k} v_n \big(I_{kn_k i_k t} + \text{SS}_{kn_k i_k (t-1)} \big) \leq V_{i_k t}, \qquad \forall t, \forall i_k. \tag{3.6}$$

Transport plan for delivery at each DL

For each period, the consignment volumes of various products from PC to DC(s), from DC(s) to DLs, and directly from PC to DLs are to be planned to fulfill the demand of each DL for each product. Following three constraints govern this requirement. First two of the following constraints describe the transportation plan from DC(s) to DLs, whereas the third one describes transportation plan directly from PC to DLs.

$$\sum_{i=0}^{I} x_{kn_k i_k jt} \geq y_{kn_k jt}, \qquad \forall j, \forall n_k, \forall t, \tag{3.7}$$

$$\sum_{j=1}^{J} x_{kn_k i_k jt} \leq I_{kn_k i_k t} + \text{SS}_{kn_k i_k (t-1)} - \text{SS}_{kn_k i_k t}, \qquad \forall i_k \neq 0, \forall n_k, \forall t, \tag{3.8}$$

$$\sum_{j=1}^{J} x_{kn_k 0jt} = Q_{kn_k t} + O_{kn_k t} - \sum_{i_k=1}^{I_k} I_{kn_k i_k t}, \qquad i_k = 0, \forall n_k, \forall t. \tag{3.9}$$

3.3.3. Buyer's optimization problem

Based on the price quotations received from various suppliers, the buyer solves the cost optimal demand allocation problem.

Objective function

The buyer decides on allocating demand shares ($y_{kn_k jt}$) corresponding to the received price quotes ($p_{kn_k j}$) from each supplier for various products for minimum total procurement cost.

$$\text{Min } z_F = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n_k \in N_k} \sum_{t=1}^{T} p_{kn_k j} y_{kn_k jt}. \tag{3.10}$$

Constraint on maximum purchase volumes

The total of purchase volumes of each product over all DLs is restricted by a maximum value in any period by each supplier. Such restrictions are pre-decided by the suppliers due to the potential of their production line or sometimes the supplier managing business with multiple buyers.

$$\sum_{j=1}^{J} y_{kn_k jt} \leq My_{kn_k}, \qquad \forall k, \forall n_k, \forall t. \tag{3.11}$$

Constraint on maximum purchase volumes

The total of purchase volumes of each product for each period from various suppliers must meet the demand of each DL.

$$\sum_{k=1}^{K} y_{kn_k jt} = D_{jnt}, \qquad \forall j, \forall n_k, \forall t. \tag{3.12}$$

Space constraint

The total of purchase volumes of all the products from various suppliers at each DL should be up to maximum inventory carrying capacity of the DL, for each period.

$$\sum_{k=1}^{K}\sum_{n_k\in N_k} v_n y_{kn_kjt} \le \mathrm{VF}_j, \qquad \forall j, \forall t. \tag{3.13}$$

3.3.4. Price-negotiation problem in a MLSF bilevel game framework

The MLSF bilevel game problem for obtaining competitive target prices is described below as (Price-BLP), considering suppliers as leaders and the buyer as follower. The problem formulated to determine values of variables $\{\{p_{kn_kj}\},\{Q_{kn_kt}\},\{O_{kn_kt}\},\{\mathrm{SS}_{kn_ki_kt}\},\{I_{kn_ki_kt}\},\{x_{kn_ki_kjt}\}: n_k, i_k, j, t\}$ for each supplier $(k = 1, 2, \ldots, K)$ and respective demand allocations $\{y_{kn_kjt} : n_k, j, t, k\}$ for the buyer.

(Price-BLP)

$$\begin{aligned}
(\mathrm{LDMP}-k)\ \mathrm{Max}\ z_{L_k} = {} & \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{j=1}^{J} p_{kn_kj}y_{kn_kjt} - \Bigg\{\sum_{t=1}^{T}\sum_{n_k\in N_k}(a_{kn_kt}Q_{kn_kt} + b_{kn_kt}O_{kn_kt}) \\
& + \Bigg(\sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i_k=1}^{I_k} d_{kn_ki_kt}\mathrm{SS}_{kn_ki_kt} + \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=1}^{I} tcp_{kn_ki_kt}I_{kn_ki_kt} \\
& + \sum_{t=1}^{T}\sum_{n_k\in N_k}\sum_{i=0}^{I}\sum_{j=1}^{J} tc_{kn_ki_kjt}x_{kn_ki_kjt}\Bigg)\Bigg\}
\end{aligned}$$

subject to

$$\begin{aligned}
& lp_{kn_kj} \le p_{kn_kj} \le Lp_{kn_kj}, && \forall n_k, \forall j,\\
& \sum_{n_k\in N_k} r_{kn_kt}Q_{kn_kt} \le \mathrm{MR}_{kt}, && \forall t,\\
& \sum_{n_k\in N_k} r_{kn_kt}(Q_{kn_kt} + O_{kn_kt}) \le M_{kt}, && \forall t,\\
& Q_{kn_kt} + O_{kn_kt} + \sum_{i_k=1}^{I_k}\mathrm{SS}_{kn_ki_k(t-1)} - \sum_{i_k=1}^{I_k}\mathrm{SS}_{kn_ki_kt} = \sum_{j=1}^{J} y_{kn_kjt}, && \forall n_k, \forall t,\\
& \sum_{n_k\in N_k} v_n\big(I_{kn_ki_kt} + \mathrm{SS}_{kn_ki_k(t-1)}\big) \le V_{i_kt}, && \forall t, \forall i_k,\\
& \sum_{i=0}^{I} x_{kn_ki_kjt} \ge y_{kn_kjt}, && \forall j, \forall n_k, \forall t,\\
& \sum_{j=1}^{J} x_{kn_ki_kjt} \le I_{kn_ki_kt} + \mathrm{SS}_{kn_ki_k(t-1)} - \mathrm{SS}_{kn_ki_kt}, && \forall i_k \ne 0, \forall n_k, \forall t,\\
& \sum_{j=1}^{J} x_{kn_k0jt} = Q_{kn_kt} + O_{kn_kt} - \sum_{i_k=1}^{I_k} I_{kn_ki_kt}, && i_k = 0, \forall n_k, \forall t,\\
& p_{kn_kj}, Q_{kn_kt}, O_{kn_kt}, I_{kn_ki_kt}, \mathrm{SS}_{kn_ki_kt}, x_{kn_ki_kjt} \ge 0, && \forall n_k, \forall i_k, \forall j, \forall k, \forall t,\\
& Q_{kn_kt}, O_{kn_kt}, I_{kn_ki_kt}, \mathrm{SS}_{kn_ki_kt}, x_{kn_ki_kjt} \text{ are integer variables}, && \forall n_k, \forall i_k, \forall j, \forall k, \forall t;
\end{aligned}$$

where, $\{y_{kn_kjt} : k, n_k, j, t\}$ is rational response obtained from following problem

$$\text{(FDMP) Min } z_F = \sum_{k=1}^{K} \sum_{j=1}^{J} \sum_{n_k \in N_k} \sum_{t=1}^{T} p_{kn_kj} y_{kn_kjt}$$

subject to

$$\sum_{j=1}^{J} y_{kn_kjt} \leq M y_{kn_k}, \qquad \forall k, \forall n_k, \forall t,$$

$$\sum_{k=1}^{K} y_{kn_kjt} = D_{jnt}, \qquad \forall j, \forall n_k, \forall t,$$

$$\sum_{k=1}^{K} \sum_{n_k \in N_k} v_n y_{kn_kjt} \leq \mathrm{VF}_j, \qquad \forall j, \forall t,$$

$$y_{kn_kjt} \geq 0, \qquad \forall j, \forall n_k, \forall k, \forall t,$$

$$y_{kn_kjt} \text{ are integer variables}, \qquad \forall j, \forall n_k, \forall k, \forall t.$$

Here, it is noted that all the constraints of the MLSF bilevel game problem modeled above are linear and objective functions of leaders and follower are bilinear. With this niceness of linearity in constraints and bilinearity of objective functions involved in the model, a challenge is to handle integer variables. In the next section, we concentrate on solving a MLCF bilevel programming problem with the special properties noted above.

4. Solution methodology

In the model formulated above, the variables $Q_{kn_kt}, O_{kn_kt}, I_{kn_ki_kt}, \mathrm{SS}_{kn_ki_kt}, x_{kn_ki_kjt}$ and y_{kn_kjt} are considered to have integer values, only for realizing the actual practices of industry. But no direct or evolutionary method is available in the literature for solving a MLSF bilevel programming problem which particularly involves integer variables. In this situation, a possible approach is to relax the integer restriction on these variables and then try to solve the equivalent MLSF bilevel programming problem by applying any of the state-of-the-art approaches by Leyffer and Munson [60] and Hori and Fukushima [51]. We attempted to solve an instance of our practical problem, for which the data was obtained from a manufacturing firm by following this scheme. This instance of the modeled problem involves a total of 1000 variables (760 of leaders' and 240 of the follower). The equivalent nonlinear optimization problems formulated by following the referred approaches could not converge even to a local optimal solution after long hours of computation on computers. From this experience it is evident that direct methods based on the theoretical approaches suggested in literature turn out to be incapable of handling large scale MLSF bilevel game problems. Therefore, it is compelling to adopt some evolutionary search technique for solving a large instance of our problem.

Although, as mentioned in the literature review, evolutionary algorithms are developed specifically for some variants of the bilevel programming problems, even then we made a sincere attempt for solving the considered instance of MLSF bilevel programming problem using the general GA. Also, we attempted to apply integer-coded GA [24] for solving our original problem with integer restriction on variables imposed again. Both the GA approaches also failed to converge to even a feasible solution of the problem. This impels us to develop an algorithm capable of handling large scale MLSF bilevel game problems to obtain strong-stationary points.

In this situation, we propose solution method for solving MLSF bilevel game problems involving integer variables and having the specific structure as involved in our model, *viz.* linear constraints, and bilinear objective functions at both levels. The proposed solution method is based on nested GA approach and uses some theoretical concepts from of Leyffer and Munson approach [60] particularly for devising fitness function. We express below a general MLSF bilevel programming problem with linear constraints and bilinear objective functions at both levels followed by a discussion on appropriately structuring theoretical concepts for developing a nested GA based solution method.

(MLSF-BL-BLP)

$$
\begin{array}{|l|}
\hline
(\mathrm{LDMP}-i)\ \min_{u_i\geq 0}\ f_i(u_i,v) \\
\qquad \text{subject to} \\
\qquad\qquad g_i(u_i,v)\geq 0, \\
\qquad\qquad G_i(u_i,v)=0, \\
\qquad\qquad \text{where}, v \text{ is optimal response of the follower corresponding to the leaders'} \\
\qquad\qquad \text{variables}, u=\{u_i : i=1,2,\ \ldots,k\}, \\
\quad (\mathrm{FDMP})\ \min_{w}\ b(u,\ w) \\
\qquad \text{subject to} \\
\qquad\qquad c(u,w)\geq 0, \\
\qquad\qquad w\geq 0 \\
\qquad\qquad \text{Here, constraints } g_i(u_i,v)\geq 0, G_i(u_i,v)\geq 0, \text{ and } c(u,v)\geq 0 \text{ are} \\
\qquad\qquad \text{considered as linear and objective functions } f_i(u_i,v) \text{ and } b(u,v) \\
\qquad\qquad \text{are bilinear in the sense that functions } f_i \text{ and } b \text{ are linear in the} \\
\qquad\qquad \text{variable } v \text{ for when } u \text{ is considered as parameter and } \textit{vice-versa}. \\
\hline
\end{array}
\tag{4.1}
$$

Here, $(\mathrm{LDMP}-i)$ is optimization problem of leader $i(i = 1, 2, \ldots, k)$, in which the follower's optimization problem (FDMP) is incorporated as a constraint.

A MLSF bilevel programming problem is solved by obtaining strong stationary points [60], or further by obtaining strong stationary Nash-equilibrium points[4], if one such exists [51]. We have developed nested GA based solution method for obtaining strong stationarity points by using some theoretical concepts from the penalty based approach of Leyffer and Munson [60] (explained in Appendix A). The theoretical development in connection with features of the problem (MLSF-BL-BLP) that enables designing the nested GA based solution methodology for it is outlined below.

4.1. Structuring theoretical concepts to design nested GA based solution method for solving (MLSF-BL-BLP)

A nested GA based solution method for any bilevel programming problem principally requires devising the reaction mechanism of follower's (followers') problem, corresponding to a given value of leader's variables, and formulating the fitness function. In the following subsections, we explain devising the reaction mechanism and formulating the fitness function of chromosomes through a typical structuring of the theoretical concepts taken from penalty based approach [60].

4.1.1. Devising the reaction mechanism for the proposed nested GA

For the considered specific structure of the MLSF bilevel programming problem (MLSF-BL-BLP), the follower decision problem is a linear programming problem (LPP) or a mixed integer linear programming problem (MILPP) parameterized in the leader's variables u. We put forward exploring values of leaders' variables u in

[4]If a strong stationary point is obtained, then one can further test whether it is a strong stationary Nash-equilibrium point, following the procedure remarked in Appendix C.

$i = 1$				$i = 2$				...		$i = k$			
$j = 1$	$j = 2$	...	$j = r_1$	$j = 1$	$j = 2$	...	$j = r_2$	...	...	$j = 1$	$j = 2$	...	$j = r_k$
u_{11}	u_{12}	...	u_{1R}	u_{21}	u_{22}	...	u_{2r_2}	...	...	u_{k1}	u_{k2}	...	u_{kr_k}

FIGURE 3. Chromosome structure.

GA, through their encoding as a chromosome, while obtaining the corresponding follower's responses (values of follower's variable v) by directly solving the parameterized LPP/MILPP with the chromosome values supplied as parameter u. Accordingly, for any chromosome representing the value u^* of leaders' variables u, the reaction of the follower, say v^*, can be obtained. This gives (u^*, v^*) as a feasible solution of the MLSF bilevel programming problem with considered specific structure. Devising such a reaction mechanism for nested GA gives an advantage over general GA that every chromosome corresponds to a feasible solution of (MLSF-BL-BLP), thereby the exploration remains in the feasible region throughout all the generations of GA.

Remark 4.1. As we are developing a heuristic search algorithm, it is acceptable to relax the integer condition from follower's variables, if there is any, and then solve the equivalent parameterized LPP instead of MILPP for approximating the follower's response. This privilege of obtaining approximate values of follower's responses is justified because by the heuristic approach anyways obtain an approximate solution of the problem. The approximate real values of the variables (originally integer restricted) can be rounded-off eventually. Relaxing the integer conditions and then rounding-off the approximated real value of the obtained solution is justified by Joseph *et al.* [53] for obtaining near optimal solutions.

4.1.2. Using theoretical developments for formulating the fitness function of nested GA

A feasible solution (u^*, v^*), obtained by the procedure suggested above, can further be tested for strong stationary point by adopting the procedure detailed in Appendix B. For this purpose, we need to solve the LPP (Para – LP) by substituting the values (u^*, v^*) as parameters and check the optimal value. Referring to the result in Appendix B, if the optimal value is zero, then (u^*, v^*) represents a strong stationary point of the considered MLSF bilevel programming problem. Therefore, for a chromosome representing a feasible solution (u^*, v^*) of (MLSF-BL-BLP), we define the fitness value as the objective function value c_{pp} of the LPP (Para – LP) with values of (u^*, v^*) substituted as parameters.

The objective function value of (Para – LP) is always non-negative for any feasible solution. Therefore, for guiding the algorithm towards a chromosome representing strong stationary point (*i.e.*, fitness value 0) it is appropriate to identify a chromosome as better fit than another on the smaller value of fitness function.

With this background, the minute details of the proposed algorithm are provided in the following subsection.

4.2. Nested GA for solving (MLSF-BL-BLP)

GA components used in our algorithm are presented below followed by their assembly into the algorithm presented through a pseudocode.

4.2.1. Chromosome encoding

For leaders' variables $u = \{u_i : i = 1, 2, \ldots, k\}$, where each $u_i = \{u_{i1}, u_{i2}, \ldots, u_{ir_i}\}$, the chromosomes of the population are encoded as an array of length equal to the number of all the leaders' variables indicating the values $\{u_i : i = 1, 2, \ldots, k\}$. A general chromosome structure used in the implementation of the algorithm is shown in Figure 3.

4.2.2. Initialization

The following GA parameters are used in the proposed algorithm: population size *popsize*; number of generations G; current generation g, $(g = 1, 2, \ldots, G)$; crossover rate pc; mutation rate pm; location parameter a and scaling parameter $b > 0$ for Laplace crossover; index of power mutation p.

4.2.3. Incorporating follower's reaction and fitness evaluation

Follower's reaction and inputs for (Para – LP): for each chromosome in population (which corresponds to leaders' variables u), problem (FDMP) is solved. The optimal response v thus obtained is supplied along with u for testing the feasibility of each chromosome (LDMP – k) of (MLSF-BL-BLP). Through this complete information about (u, v), the values of variables s and t_i are calculated using relations referred in (A.7e) and (A.7g).

Fitness evaluation: for each chromosome, with its value $u = \{u_i : i = 1, 2, \ldots, k\}$ and obtained values v, s, $t_i : i = 1, 2, \ldots, k$, the problem (Para – LP) is solved. Corresponding value of the objective function C_{penalty} is considered as the fitness value of that chromosome.

4.2.4. Genetic operators

Selection operator: tournament selection operator is used to choose relatively fit chromosome. A tournament selection mechanism, (with size 2), is adopted for selecting chromosomes with better fitness. For guiding the search of the algorithm towards strong stationary points, a chromosome with smaller fitness value is considered to have better fitness that the other.

Crossover operator: we use a single point crossover for a chromosome, Laplace crossover (LX) operator with the probability pc [24]. The same is explained in [54].

Mutation operator: the mutation is performed on a chromosome with the probability pm using the power mutation (PM) operator [24]. The same is explained in [54].

We note that LX and the PM operators do not disturb feasibility of chromosomes in terms of reservation price bounds in problem (LDMP – k).

4.2.5. Updating the new population

The new population obtained from the parent population P^g is adopted to be a population of the next generation P^{g+1} only if it's maximum fitness value, in comparison to the maximum fitness value of the previous generation, does not decrease. Otherwise, the population P^g is preserved as population P^{g+1} for regenerating the next generation.

4.2.6. Termination criterion

The execution of the algorithm is terminated after the completion of pre-defined maximum number of generations G. The value of G may be tuned by observing stability in the fitness value through various combinations of GA parameters.

Finally, the steps involved in the proposed GA are summarized below through a pseudocode presented in Algorithms 1 and 2. Algorithms 1 and 2, collectively represent a nested type of GA, with Algorithm 2 performing as a subprogram of the main program given by Algorithm 1. Here, Algorithm 2 is responsible for evaluating optimal reaction of the follower corresponding to leaders' potential actions represented by each chromosome of the population (nested action), and thereby determining the fitness values of chromosomes.

Algorithm 1: Nested GA for solving (MLSF-BL-BLP).

Data: Input data and GA parameters
$g \leftarrow 0$;
Initialize population (values of LDMs' variables $u_i : i = 1, 2, \ldots, k$);
Evaluate fitness of population members (along with corresponding response v) (using Algorithm 2);
while $g < G$ **do**
  Tournament selection (retaining best-fit chromosome);
  Generate new individuals through extended Laplace Crossover and Power Mutation.
  Evaluation fitness of population members (along with corresponding response v) (using Algorithm 2);
  Update new population for next generation.
  $g \leftarrow g + 1$;
  Select best-fit chromosome of the new generation (along with corresponding response v);
end
Return best-fit chromosome (along with corresponding response v) over all generations

Algorithm 2: Fitness evaluation of population members – for (MLSF-BL-BLP).

Input: GA population of chromosomes
for $i \leftarrow 1$ **to** Pop_size **do**
  Substitute values of LDM variables $u_i : i = 1, 2, \ldots, k$ (represented by chromosome) in (FDMP) to solve the follower's relaxed LPP for obtaining v;
  Round-off values of v to the closest integers;
  Obtain $s = h(u, v)$ and $t_i = g_i(u_i, v)$;
  Solve (Para – LP) by supplying values of u, v, s, t_i.
  Evaluate the fitness value as objective function value of (Para – LP);
end
Output: Fitness values of all chromosomes of the population (along with corresponding response v)

Going back to solve our model (Price-BLP), a particular problem in the class of (MLSF-BL-BLP) problems, a specific interdependence among leader's variables can be noticed. Some modifications in the proposed algorithm are suggested further in the following subsection to capture this interdependence of variables for solving particularly the modeled problem (Price-BLP).

4.3. Modifications in the proposed algorithm for solving (Price-BLP)

As in the problem (Price-BLP), the values of Leaders' price variables ($\{p_{kn_kj} : k, n_k, j\}$) decide the demand allocations $\{y_{kn_kjt} : k, n_k, j, t\}$ by the follower, and thereafter the values of production-and-distribution planning variables ($Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt} : k, n_k, i_k, j, t$) can be obtained, depending on these demand allocations. This interdependence of one part of leader's variables with other gives a specific structure to the bilevel programming problem (Price-BLP). A problem-specific minor modification is thus presented below for the proposed GA for solving particularly the problem (Price-BLP) for capturing this interdependence and thereby maintaining feasibility during the GA exploration.

We encode the chromosomes as a row vector of values corresponding to leaders' price variables (p_{kn_kj}) only, with each value ranging in the interval $[lp_{kn_kj}, Lp_{kn_kj}]$ for $k = 1, 2, \ldots, K$, $n_k = 1, 2, \ldots, N_k$, $j = 1, 2, \ldots, J$. Values of the rest of leaders' variables are obtained during the fitness evaluation. For the values of leaders' prices $\{p_{kn_kj} : k, n_k, j\}$ generated as a chromosome, we first evaluate the optimal response of the follower $\{y_{kn_kjt} : k, n_k, j, t\}$ by solving (FDMP) as a parameterized LPP in price variables $\{p_{kn_kj} : k, n_k, j\}$. Thereby, $(\mathrm{LDMP} - k)$ is solved to obtain $Q_{kn_kt}, O_{kn_kt}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt} : k, n_k, i_k, j, t$ for each leader k, by solving a LPP parameterized in variables $\{y_{kn_kjt} : k, n_k, j, t\}$.

The concatenated vector values for $\{p_{kn\_kj}, Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}, y_{kn_kjt} : k, n_k, i_k, j, t\}$ thus becomes a feasible solution of (Price-BLP), in concurrence with a feasible solution (u, v) of the general game problem (MLSF-BL-BLP).

Due to the sequential dependence of follower's variables y_{kn_kjt} on leader's prices p_{kn_kj}, and in turn, of other variables of leaders $Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$ on y_{kn_kjt}, these modifications suggested above are appropriate for any chromosome to represent a feasible solution of (Price-BLP) problem. As, on the contrary, if we encode all the variables of leaders $p_{kn\_kj}, Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$ as chromosomes, then the issue of infeasibility of values for $Q_{kn_kj}, O_{kn_kj}, \mathrm{SS}_{kn_ki_kt}, I_{kn_ki_kt}, x_{kn_ki_kjt}$ in connection with the values of p_{kn_kj} (through the responses y_{kn_kjt} calculated as usual by solving (FDMP)) will be of a concern. This manifests the appropriateness of modifications suggested in the proposed GA for the specific bilevel programming problem (Price-BLP) in hand.

The algorithm modified for the problem (Price-BLP) is summarized through the following pseudocode.

Algorithm 3: Modified nested GA for (Price-BLP).

```
Data: Input data and GA parameters
g ← 0;
Initialize population (values of LDMs' variables p_{kn_k j} : k, n_k, j);
Evaluate fitness of population members (along with complete vectors of u and v) (using Algorithm 4);
while g < G do
    Tournament selection (retaining best-fit chromosome);
    Generate new individuals through extended Laplace Crossover and Power Mutation;
    Evaluate fitness of population members (along with complete vectors of u and v) (using Algorithm 4);
    update new population for next generation;
    g ← g + 1;
    Select best-fit chromosome of the new generation (along with complete vector of u and v, as obtained in
    Algorithm 4);
end
Return best-fit chromosome (along with complete vectors of u and v) over all the generations
```

Algorithm 4: Fitness evaluation of population members – for (Price-BLP).

```
Input: GA population of chromosomes
for i ← 1 to Pop_size. do
    Substitute LDM prices p_{kn_k j} (represented by chromosome) in (FDMP) to solve the follower's relaxed LPP for
    obtaining y_{kn_k jt};
    Round-off values of y_{kn_k jt} to the closest integer;
    For each leader k, solve (LDMP − k) to obtain Q_{kn_k t}, O_{kn_k t}, SS_{kn_k i_k t}, I_{kn_k i_k t}, x_{kn_k i_k jt}
    Obtain s and t_i following the rule s = h(u, v) and t_i = g_i(u_i, v) with
    u = {p_{kn_kj}, Q_{kn_k j}, O_{kn_k j}, SS_{kn_k i_k t}, I_{kn_k i_k t}, x_{kn_k i_k jt}, y_{kn_k jt}} and v = {y_{kn_k jt}};
    Solve (Para – LP) with values of u, v, s, t_i supplied as parameters;
    Evaluate the fitness value as objective function value of (Para – LP);
end
Output: Fitness values of all chromosomes of the population (along with complete vectors of u and v)
```

5. An experimental study from a manufacturing firm

5.1. Relevant information about the firm

The formulation of our model is inspired by a scenario of a leading manufacturing firm of the fast-moving consumer goods (FMCG) sector. The firm has 5 production plants in the southern region of the Republic of India. Each of its production plants manufactures certain finished products, which require 5 ingredients of varied

Table 1. Supplier's profile (products and DCs).

	Supplier (k)						
	1	2	3	4	5	6	7
Product sets (N_k)	$\{1\}$	$\{2,3\}$	$\{1\}$	$\{4\}$	$\{2,3\}$	$\{1,4\}$	$\{2,3,4\}$
Number of DC(s) (I_k)	2	1	1	0	1	2	2

quantities over each week, depending upon the production plan. A production plan is prepared for a block of 4 weeks. The requirement arises for procuring different amounts of ingredients for each of the manufacturing plants by suppliers over each week. These ingredients are procured from a total of 8 suppliers who deal with some or all these ingredients. The buyer has already identified these suppliers through various quality and potential parameters. Further, as the buyer and suppliers are at economies of scale in the present environment of localized buying and selling, therefore the ecosystem for purchasing of these ingredients pertains to an oligopolistic-monopsony market. As most of the required components involve frequently fluctuating production costs to vary over each month, the price-quotes must be invited from the suppliers. Based on their price-quotes, the demand allocation is done to these suppliers to supply the ingredients over the planning horizon of 4 weeks.

For its financial planning, the firm is concerned about assessing the total budget allocated for the procurement of ingredients before the invitation of price quotes from suppliers. More importantly, the firm is also keen to observe whether its suppliers are competing for price quotes or having a co-operative game to settle at some higher prices. The latter situation may result in a dominance of its suppliers over the buyer. In such a case, the buyer firm would need to consider more suppliers for selection and demand order allocation to induce more competition to the existing suppliers. A manufacturer is vigilant for performing such analysis in pursuit of minimizing procurement cost, which comes as a significant component of production cost. At the same time, the firm is a production giant with a high brand value in the market of its finished products. Another priority concern of such a firm at this point of time is supplier integration to strengthen its inbound supply chain. In this line of thought, the firm is open to suggesting that any of its suppliers adjust the price quotes to avoid their opportunity losses for the same demand order allocation in case of raised prices. Further, through a prior assessment of prices, the firm would be better prepared to manage its inbound supply chain to be snag-free by allocating demand orders to its suppliers. As the concerns of this manufacturing firm match the theme of our framework therefore a case data from this firm is taken-up our study.

5.2. Data input

Henceforth, the manufacturing firm is the buyer, whereas the ingredients to be procured from its suppliers are products. Out of the five, one product is patent with a specific supplier, and the supplier can supply this product only, so there is no competition. We consider the remaining 4 products to be provided by 7 suppliers. Each of the seven suppliers has a single production center (PC) and a different number of distribution centers/warehouses for inventory storage as well as for cross-docking of shipments of products for transportation from PC to delivery location(s) DL(s). For reference, the products, suppliers, and their DC(s), and DLs numbered with corresponding indices.

The 4th product is a volatile product in a liquid state and is transported only through special containers, so neither its cross-docking nor maintaining inventory is considered practically feasible. This specification of the structural setup can easily be incorporated into the modeling by not defining any variable against the product for inventory and transportation to DC(s) for all suppliers dealing in this product. The details of suppliers dealing in products and other relevant to the problem is tabulated in Tables 1–9.

Table 2. Forecasted demands of products at each DL for each time period.

D_{jnt}	DL (j)																			
	1				2				3				4				5			
Product	Time period (t)				Time period (t)				Time period (t)				Time period (t)				Time period (t)			
(n)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	150	75	100	75	125	100	75	50	75	75	50	50	75	50	75	50	200	100	120	80
2	75	75	75	75	75	75	75	50	75	50	75	25	75	75	50	25	100	100	80	50
3	125	125	100	50	100	75	75	75	100	75	50	50	80	80	100	15	130	130	90	90
4	50	75	75	50	50	75	75	25	75	75	50	0	60	60	80	0	80	80	80	60

Table 3. Reservation prices and maximum purchase volumes.

		Supplier (k)											
		1	2		3	4	5		6		7		
	DL	n_1	n_2		n_3	n_4	n_5		n_6		n_7		
	(j)	1	2	3	1	4	2	3	1	4	2	3	4
lp_{kn_kj}	1	25 700	5560	5550	31 000	89 600	4750	4695	24 250	90 050	5790	6010	92 700
	2	25 800	5660	5700	31 100	89 300	4850	4795	24 100	89 800	6040	6260	92 800
	3	25 950	5810	5450	31 175	89 700	5000	4945	24 200	89 350	6365	6585	92 875
	4	25 950	5560	5050	30 300	91 000	5000	4945	24 350	89 600	6340	6560	92 000
	5	25 600	5160	4850	29 800	90 600	4650	4595	24 250	89 850	5640	5860	91 500
Lp_{kn_kj}	1	28 700	5700	5700	32 900	94 500	5100	4900	27 700	92 500	6250	6350	97 400
	2	28 800	5800	5850	33 000	94 200	5200	5000	27 550	92 250	6500	6600	97 500
	3	28 950	5950	5600	33 075	94 600	5350	5150	27 650	91 800	6825	6925	97 575
	4	28 950	5700	5200	32 200	95 900	5350	5150	27 800	92 050	6800	6900	96 700
	5	28 600	5300	5000	31 700	95 500	5000	4800	27 700	92 300	6100	6200	96 200
My_{kn_k}		175	350	430	225	175	275	350	250	100	300	300	100

Table 4. Production costs – regular-time and overtime (INR per unit), and machine-hours required for production (hours per unit).

	Product	Supplier (k)						
	(n)	1	2	3	4	5	6	7
a_{kn_kt}	1	20 000	–	21 000	–	–	20 500	–
	2	–	3600	–	–	3500	–	3400
	3	–	3500	–	–	3450	–	3600
	4	–	–	–	81 000	–	80 500	83 000
b_{kn_kt}	1	20 200	–	21 200	–	–	20 700	–
	2	–	3650	–	–	3550	–	3450
	3	–	3550	–	–	3500	–	3650
	4	–	–	–	82 000	–	81 500	84 000
r_{kn_kt}	1	0.75	0.5	0.75	–	–	0.75	–
	2	–	0.5	–	–	0.5	–	0.33
	3	–	–	–	–	0.5	–	0.33
	4	–	–	–	1.5	–	1.5	1.25

TABLE 5. Maximum machine-hours available regular-time and total (including overtime).

	Supplier (k)						
	1	2	3	4	5	6	7
MR_{kt}	168	144	168	168	288	336	456
M_{kt}	168	168	168	168	336	336	504

TABLE 6. Transportation costs from PC to DC(s) and costs of inventory at DC(s) (INR per unit), maximum space available at DC(s).

k	n_k	i_k	$tcp_{kn_ki_kt}$	$d_{kn_ki_kt}$	V_{i_kt}
1	1	1	1200	10	4000
		2	900	10	6000
2	2	1	1000	10	4000
	3	1	1000	10	
3	1	1	6500	10	6000
5	2	1	400	9	4000
	3	1	400	9	
6	1	1	900	10	5000
		2	600	10	5000
7	2	1	850	10	4000
		2	950	10	4000
	3	1	850	10	
		2	950	10	

TABLE 7. Costs of transportation from PC or DC(s) to DLs (As all the suppliers in the problem have DCs not more than 2, due to this fact the costs of transportation have been listed in a single table only. But this doesn't mean that all the suppliers have same DCs. It is simply an enumeration of DCs. They own or hire for their individual warehouses which are located even at different places).

$tc_{kn_ki_kjt}$		$i_k=0$					$i_k=1$					$i_k=2$				
Supplier	Product	DL (j)					DL (j)					DL (j)				
(k)	(n_k)	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
1	1	1000	1100	1250	1000	850	500	600	750	750	400	750	800	900	700	400
2	2	500	600	750	750	400	600	700	850	600	200	–	–	–	–	–
	3	500	600	750	750	400	600	700	850	600	200	–	–	–	–	–
3	1	5500	5000	4800	5500	6500	1400	1500	1575	700	200	–	–	–	–	–
4	4	500	200	600	1900	1500	–	–	–	–	–	–	–	–	–	–
5	2	900	1000	1000	1000	400	500	600	750	750	400	–	–	–	–	–
	3	900	1000	1000	1000	400	500	600	750	750	400	–	–	–	–	–
6	1	1500	1250	800	1050	1300	500	600	750	750	400	1100	950	200	1200	1100
	4	1500	1250	800	1050	1300	–	–	–	–	–	–	–	–	–	–
7	2	1400	1500	1575	700	200	600	800	1150	1550	850	1100	1350	1675	1650	950
	3	1400	1500	1575	700	200	600	800	1150	1550	850	1100	1350	1675	1650	950
	4	1400	1500	1575	700	200	–	–	–	–	–	–	–	–	–	–

TABLE 8. Space occupied per tonne (in sq. ft).

v_n	Product (n)			
	1	2	3	4
	8	8	8	12.5

TABLE 9. Maximum inventory carrying space at DLs of the buyer (This accounts for the capacity to accommodate all the products except product 4, which needs to be stored in a separately installed container (at each DL) having storage capacity which is more sufficient to accommodate the demand for each period.) (sq. ft).

VF_j	DL (j)				
	1	2	3	4	5
	8000	8000	4000	5500	6000

5.3. Implementation of GA

The (Price-BLP) problem modeled as a MLSF game, discussed in context of manufacturing firm with the input data tabulated above is solved using the modified algorithm proposed in the Section 4.3. The program is coded in MATLAB 2019a. The parameters of Laplace crossover and power mutation along with population size (*Pop_size*) are tuned for various combinations of probabilities of crossover, mutation, and tournament selection. The best found are $Pop\_size = 20$, $a = 0$, $b =\ 0.15$, $p = 1$ or 10. We set the maximum number of generations to 1000 after tuning the parameters for attainment of the best fitness value equal to zeros for strong stationary point. For each combination of parameters (Tab. 10), we performed a set of 10 experiments of GA. Table 10 tabulates the relative error of the best solutions obtained for each combination of parameters against the ideal fitness value zero in comparison with the arithmetic mean of the best fitness values of various combinations. Figure 4 shows the variation in the best fitness attained in various generations of a GA run. Among 240 solutions generated (24 combinations with 10 runs each) for (Price-BLP), 22 strong-stationary points (fitness value = 0) were obtained. The arithmetic mean of total procurement cost of the buyer corresponding to these 22 instances of prices (corresponding the strong-stationarity points) is INR 166 844 775, with a standard deviation of 266 396.83, giving coefficient of variation 0.16%. This indicates parity among all the 22 strong-stationarity points from buyer's perspective, in terms of the total procurement cost. The desired fitness value zero (for strong-stationarity points) is attained for the runs of 18 out of 24 combinations of GA parameters, whereas for rest of the combinations there no improvement in fitness value for more than last 300 generations.

5.4. Analysis of the results of computation

The strong-stationarity point with the total procurement cost closest to the arithmetic mean of all the 22 ones obtained from our computation is detailed as following. Table 11 shows the competitive target prices to settle negotiations with each of suppliers for various products they deal in and deliver at each of 5 DLs. As a response to these prices, the consequent demand allocations from the buyer are tabulated in Table 12. The consequent total procurement cost of the buyer in this instance is assessed as INR 166 871 150. The accompanying part of the results comprising of respective aggregate-production-distribution plans for each of 7 suppliers are presented through Tables D.1–D.20 in Appendix D. It is noted that the inventory volumes for all the suppliers at each of their warehouses are obtained as zeros for all products during each period, indicating the availability of a cost-wise efficient production and distribution setup at suppliers ends. It adds an advantage to the suppliers owing to multiple operational challenges of warehousing and inventory management.

TABLE 10. Error analysis for different combinations of parameters in GA; those giving strong-stationary points highlighted in bold.

pt	pc	pm	p	Fitness value	Relative error
0.8	0.7	0.001	1	**0**	0
0.8	0.7	0.001	10	6263	0.920
0.8	0.7	0.005	1	**0**	0
0.8	0.7	0.005	10	**0**	0
0.8	0.8	0.001	1	**0**	0
0.8	0.8	0.001	10	12780	1.877
0.8	0.8	0.005	1	**0**	0
0.8	0.8	0.005	10	**0**	0
0.8	0.9	0.001	1	**0**	0
0.8	0.9	0.001	10	1145	0.168
0.8	0.9	0.005	1	3500	0.514
0.8	0.9	0.005	10	**0**	0
0.9	0.7	0.001	1	**0**	0
0.9	0.7	0.001	10	3500	0. 514
0.9	0.7	0.005	1	**0**	0
0.9	0.7	0.005	10	**0**	0
0.9	0.8	0.001	1	**0**	0
0.9	0.8	0.001	10	**0**	0
0.9	0.8	0.005	1	**0**	0
0.9	0.8	0.005	10	**0**	0
0.9	0.9	0.001	1	197	0.029
0.9	0.9	0.001	10	**0**	0
0.9	0.9	0.005	1	**0**	0
0.9	0.9	0.005	10	**0**	0

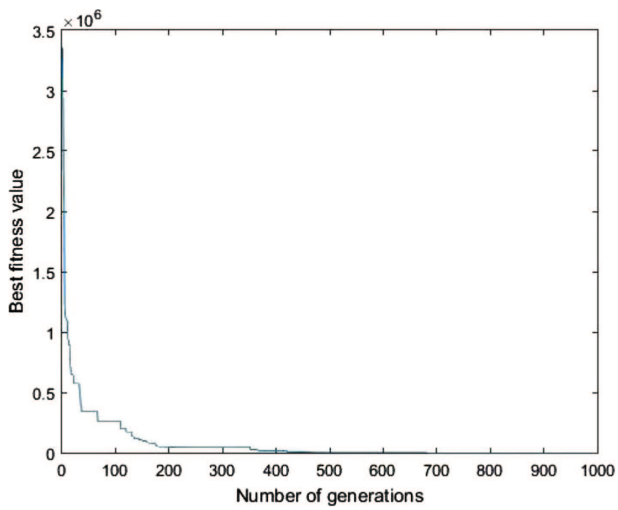

FIGURE 4. Best fitness value *vs*. Generation of a solution.

TABLE 11. Suppliers' prices (competitive target prices) p_{kn_kj} (INR per tonne),

Supplier (k)	Product (n_k)	DL (j) 1	2	3	4	5
1	1	28 700	25 800	25 950	28 950	28 599.92
2	2	5560	5560	5950	5560	5160
	3	5689.914	5850	5450	5050	4850
3	1	32 899.92	33 000	31 175	30 300	29 800
4	4	89 600	94 200	93 504.25	91 000	90 600
5	2	4750	4850	5000	5000	4650
	3	4695	4795	4945	5150	4595
6	1	24 250	27 550	24 200	24 350	24 250
	4	92 499.98	89 800	89 350	92 050	93292.56
7	2	6250	6499.984	6824.942	6800	6100
	3	6350	6599.426	6925	6882.551	6200
	4	92 707.78	92 800	92 875	96 700	91 500

Remark 5.1. Further testing of the 22 strong-stationarity points for strong-stationarity Nash-equilibrium point is not being taken-up for the following reasons.

(1) There is parity among all the obtained strong-stationarity points in terms of total procurement cost of the buyer (the problem being studied from buyer's perspective only).
(2) The decision-makers of the discussed firm (buyer here) confirmed the efficacy of the results obtained so far in the context of total procurement cost assessed corresponding to competitive prices obtained corresponding to all the 22 strong-stationarity points when compared with the actual costs incurred. From the perspective of business management, this indicates that for the buyer a scope of further negotiation on prices was there without any compromise on the cooperative relation with suppliers.
(3) Thus, even if the buyer would have considered the target prices as any one of those obtained corresponding to these strong-stationarity points, and negotiated the same with suppliers, then the objective of strategic pricing discussed in Section 1, would be satisfactorily achieved.
(4) Over that, theoretically, there is no surety of obtaining a strong-stationarity Nash-equilibrium point, and the question of its existence remaining unconfirmed.

The competitive target prices depicted in Table 11 are a set of prices at which the suppliers can settle during negotiations due to a fair competition among them. Further, if the negotiations are settled at these prices, then the demand orders from the buyer is listed in Table 12. Let us try to have an insight of these results in connection with the production and distribution efficiencies of the suppliers. For product 1, the competition is between suppliers 1, 3, and 6, and the demand order allocation for this product seems distributed among them specific to DLs. Demand order for DL 1 is allocated completely to the supplier 6, completely to supplier 1 for DL 2, for DL 3 distributed between suppliers 1 and 6, and for DL 4 and DL 5 completely to supplier 6 and supplier 3, respectively. This allocation of demand orders is clearly reflected to the prices listed in Table 11. This competition of their prices is evidently due to their cost efficiencies. For example, per unit production costs (regular and overtime) for product 1 are lower for supplier 1 and 6 in comparison to supplier 3 (*c.f.*, Tab. 4). Further, it is evident from the reservation prices of these suppliers given in Table 3 that supplier 3 cannot compete with other suppliers except for DL 5, that too up to some extent only. Thereby with its lowest quote it gets almost a complete share of demand order for delivery at DL 5. Similarly from Tables 11 and 12, it can be observed that for the products 2 and 3 the supplier 5, due to its lowest prices, gets more demand orders than suppliers 2 and 7.

TABLE 12. Buyer's Demand allocation y_{kn_kjt} (number of tonnes to be purchased)

		DL (j)																			
		1				2				3				4				5			
Supplier	Product	Time period (t)				Time period (t)				Time period (t)				Time period (t)				Time period (t)			
(k)	(n_k)	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	125	100	75	50	50	0	25	25	0	0	0	0	0	0	0	0
2	2	0	0	0	0	0	0	0	0	0	0	0	0	25	0	0	0	100	100	80	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	80	80	100	15	105	55	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	200	100	120	30
4	4	50	75	75	50	0	0	0	0	0	0	0	0	60	60	80	0	65	40	20	60
5	2	75	75	75	75	75	75	75	50	75	50	75	25	50	75	50	25	0	0	0	50
	3	125	125	100	50	100	75	75	75	100	75	50	50	0	0	0	0	25	75	90	90
6	1	150	75	100	75	0	0	0	0	25	75	25	25	75	50	75	50	0	0	0	50
	4	0	0	0	0	25	25	50	25	75	75	50	0	0	0	0	0	0	0	0	0
7	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	25	50	25	0	0	0	0	0	0	0	0	0	15	40	60	0

Further, even though supplier 7 has better cost competencies of production and distribution for products 2 and 3 (*c.f.*, Tabs. 4 and 7), still the prices for supplier 7 for these products, listed in Table 11, are higher than those of suppliers 2 and 5. Consequently, no demand orders would be allocated to supplier 7 for these products, as reflected in Table 12. Whereas supplier 7 is observed to compete quite well with suppliers 4 and 6 for product 4 to receive a substantial demand share. This indicates that supplier 7 is more interested in dealings product 4, which is costlier in comparison to products 2 and 3. This market insight of supplier 7 seems clear from the reservation prices (given in Tab. 4) as noted here. Minimum reservation prices (lp_{kn_kj}) of products 2 and 3 are higher for supplier 7 in comparison of other suppliers. This restricts the negotiations for products 2 and 3 up to a limit and allows the supplier 7 to negotiate better on product 4.

A significant conclusion comes out from these observations that although the model is being discussed from the buyer's point of view, still the suppliers hold self-control over lowering their prices in the competitive environment. As the considered problem addresses a complex market structure with multiple factors included, accordingly many more similar observations can be made by looking into the results while connecting them with the input data. To summarize, the overall results of computation presented in this section evince the accomplishment of the proposed method to handle a large-scale instance of our problem.

6. Managerial implications

Negotiations handled transparently using this decision support system will portray the buyer's intention of creating constructive competition and non-indulgence into opportunistic discrimination with the motive of minimizing procurement cost. Similarly it will ensure that the suppliers arrive at a settlement price for supply which have adequate margin of safety with regard to their profitability. Thus, the use of developed decision support will ensure that there is no opportunistic discrimination from the buyer by exploiting the bargaining power as a single buyer. On the contrary, with the negotiations settled at prices suggested by the decision support the suppliers too cannot reap enormous surplus profits defying the competition. Hence, this model helps to arrive at that satisficing point of demand and supply which will be logically beneficial for both buyers and sellers in the discussed market situation.

In all, the decision support system developed for our study will help in creating healthy entrepreneurial platform in which the business interests of the buyer and sellers are protected. Eventually, such a business relationship with the suppliers inculcates a support behavior in them which proves to be helpful in combating sudden unforeseen contingency situations in the future. The use of this decision support system can help even in reducing the lengthy negotiation and communication time span, as the target prices ascertained by the model can be used as consensus base prices for fixing the deal.

The proposed model can be appropriately used by any buyer firm which is concerned for minimizing the total procurement costs by creating a healthy competition among its suppliers through a non-cooperative game for their price quotes. The scope of application of our model is listed below.

(1) The model can be used by government authorities for assessment of competitive prices for the tenders invited for the purchase of required material. An indication can be obtained about the existing suppliers for any possible cartels, as these practices are prevalently found among those who supply products to government organizations.
(2) In case of the setup of strategic business units by the suppliers dealing in multiple products, our model is capable of catering such a scenario due to consideration of transportation cost as product dependent.
(3) This model can be used by non-trading and non-manufacturing organizations like Universities, hospitals which procure a sizable proportion of purchases locally.
(4) The situation of suppliers having multiple production centers can also be catered through simple extension of our model by merely introducing additional index for the production centers.

Major challenges of study

Exact assessment of suppliers' operational parameters is a major challenge. Technical expertise at managerial level is a major requirement for an effective use of this model.

7. Conclusions

This work is first of its kind to mathematically address the problem of ascertaining competitive target prices for the buyer of an oligopolistic-monopsony market to negotiate with suppliers in line with a balanced approach of safeguarding the financial interests of each supply-chain partner. The problem is formulated as an MLSF bilinear bilevel game model, featuring suppliers as leaders and the buyer as a follower, as it fits naturally to the problem. Annexing suppliers' operational planning with the bilevel game problem of price-setting enables the buyer to assess the capacities of suppliers for fulfilling the demand-orders. Faced with lack of solution methodologies to handle large-scale instances, a GA-based method is proposed to solve a general MLSF bilevel game problem with bilinear objectives and linear constraints. Further, a modification to the proposed algorithm is suggested to solve the bilevel game problem having a specific dependence structure among variables at leaders' level, as present in our proposed model. The decision support developed is finally illustrated by a study on an FMCG manufacturing firm's procurement setup having similar concerns.

An interesting future research direction is to specifically design an algorithm for handling MLCF bilevel programming problems with integer variables in a large-scale situation. Another challenging potential problem can be addressed for the case of differential pricing. Similar problems may be studied for other market setups, for example, where multiple buyers also compete to fulfill their demands-orders.

Appendix A. Penalty approach [60]

Theoretical developments of the penalty approach [60] for solving the general MLSF bilevel programming problem (MLSF-BLP) given in (4.1) is explained below.

(MLSF-BLP)

$$
\begin{array}{|l|}
\hline
(\mathrm{LDMP}-i)\ \min\limits_{u_i\geq 0}\ f_i(u_i,v) \\
\quad\quad \text{subject to} \\
\quad\quad\quad\quad g_i(u_i,v)\geq 0, \\
\quad\quad\quad\quad G_i(u_i,v)=0, \\
\quad\quad\quad\quad \text{where, } v \text{ is optimal response of the follower corresponding} \\
\quad\quad\quad\quad \text{to the leaders' variables, } u=\{u_i: i=1,2,\ldots,k\}, \\
\quad (\mathrm{FDMP})\ \min\limits_{w}\ b(u,w) \\
\quad\quad \text{subject to} \\
\quad\quad\quad\quad c(u,w)\geq 0, \\
\quad\quad\quad\quad w\geq 0. \\
\hline
\end{array}
\tag{A.1}
$$

Observation: The MLSF bilevel programming problem with linear constraints and bilinear objective functions (MLSF-BL-BLP) is a particular case of the general problem (MLSF-BLP), with all the notations taken appropriately as same. Therefore, the penalty approach [60] explained here for the general problem can directly be referred in the discussion in Section 4 with notations $f_i, g_i, G_i, b, c, w, u, v$ symbolizing same functions and variables at both places.

Definition A.1. If, for any given vector $u = \{u_i : i = 1, 2, \ldots, k\}$, y is an optimal solution of (FDMP) in (MLSF-BLP), such that the vector (u, v) satisfy constraints $g_i(u_i, v) \geq 0$ and $G_i(u_i, v) = 0, (i = 1, 2, \ldots, k)$, then (u, v) is a *feasible solution* of (MLSF-BLP).

For the (MLSF-BLP) problem described above, the penalty approach is described in following the steps.

Step 1. The KKT equivalent of (FDMP) (in w)

$$0 \leq w \perp \nabla_w b(u, w) - \nabla_w c(u, w) z \geq 0 \tag{A.2a}$$

$$0 \leq z \perp c(u, w) \geq 0. \tag{A.2b}$$

Step 2: (a) Redefining y as $v = (w, z)$, and defining

$$h(u, v) = \begin{bmatrix} \nabla_w b(u, w) - \nabla_w c(u, w) z \\ c(u, w) \end{bmatrix}. \tag{A.3}$$

(b) Introducing slack variables s, the conditions become

$$h(u, v) - s = 0 \tag{A.4a}$$

$$0 \leq v \perp s \geq 0. \tag{A.4b}$$

Step 3. Incorporating the conditions reduces (MLSF-BLP) to the following Equilibrium Problem with Equilibrium Constraints (EPEC).
For each leader $i = 1, 2, \ldots, k$

$$
\begin{aligned}
&\min_{u_i \geq 0}\ f_i(u_i, v) \\
&\text{subject to} \\
&\quad g_i(u_i, v) \geq 0, \\
&\quad G_i(u_i, v) = 0,
\end{aligned}
$$

$$h(u,v) - s = 0,$$
$$0 \le v \perp s \ge 0.$$

The above EPEC is rewritten as following.

$$\left.\begin{array}{ll}(\text{SLNG})\min & f_i(u_i,v)\\ \text{subject to} & \\ & -g_i(u_i,v) \le 0,\\ & G_i(u_i,v) = 0,\\ & s - h(u,v) = 0,\\ & -u_i \le 0,\\ & -v \le 0,\\ & -s \le 0,\\ & Vs \le 0.\end{array}\right\} \tag{A.5}$$

(Here, $V = \text{diag}(v_1, v_2, \ldots, v_r)$.)

Step 4. A desired solution to (SLNG) is a solution of the following strong-stationarity conditions

$$\nabla_{x_i} f_i(u_i,v) - \lambda_i' \nabla_{x_i} g_i(u_i,v) - \nu_i' \nabla_{x_i} G_i(u_i,v) - \mu_i' \nabla_{x_i} h(u_i,v) - \chi_i = 0 \tag{A.6a}$$
$$\nabla_y f_i(u_i,v) - \lambda_i' \nabla_y g_i(u_i,v) - \nu_i' \nabla_y G_i(u_i,v) - \mu_i' \nabla_y h(u_i,v) - \psi_i + S\xi_i = 0 \tag{A.6b}$$
$$\mu_i' \nabla_s (s - h(u,v)) - \sigma_i' \nabla_s(s) - \xi_i' \nabla_s(Vs) = 0 \tag{A.6c}$$
$$0 \le g_i(u_i,v) \perp \lambda_i \ge 0 \tag{A.6d}$$
$$G_i(u_i,v) = 0 \tag{A.6e}$$
$$h(u,v) - s = 0 \tag{A.6f}$$
$$0 \le u_i \perp \chi_i \ge 0 \tag{A.6g}$$
$$0 \le v \perp \psi_i \ge 0 \tag{A.6h}$$
$$0 \le s \perp \sigma_i \ge 0 \tag{A.6i}$$
$$0 \le -Vs \perp \xi_i \ge 0. \tag{A.6j}$$

(Here, $S = \text{diag}(s_1, s_2, \ldots, s_r)$.)

Definition A.2. A feasible solution (u,v) of (MLSF-BLP) is called a *strong-stationarity point* if there exist multipliers λ_i, χ_i, ψ_i, σ_i, ξ_i, ν_i, μ_i which satisfy strong-stationarity conditions (A.6).

Step 5. Among the strong-stationarity conditions written above, as (A.6d), (A.6g)–(A.6j) are complementarity conditions, therefore the process of solving the system of conditions (A.6) can be eased out by solving instead the following nonlinear programming problem. If an optimal solution of (A.7) gives the objective function value zero *i.e.*, $C_{\text{penalty}} = 0$, then that optimal solution satisfies the strong-stationarity conditions (A.6).

$$(\text{Pen-NLP})\min\ C_{\text{penalty}} = \sum_{i=1}^{k} (u_i'\chi_i + t_i'\lambda_i + v'\psi_i + s'\sigma_i) + v's \tag{A.7a}$$

subject to

$$\nabla_{u_i} f_i(u_i,v) - \lambda_i' \nabla_{u_i} g_i(u_i,v) - \nu_i' \nabla_{u_i} G_i(u_i,v) - \mu_i' \nabla_{u_i} h(u_i,v) - \chi_i = 0, \quad \forall i = 1,2,\ldots,k, \tag{A.7b}$$
$$\nabla_v f_i(u_i,v) - \lambda_i' \nabla_v g_i(u_i,v) - \nu_i' \nabla_v G_i(u_i,v) - \mu_i' \nabla_v h(u_i,v) - \psi_i + S\xi_i = 0, \quad \forall i = 1,2,\ldots,k, \tag{A.7c}$$
$$\mu_i - \sigma_i + \text{V}\xi_i = 0, \quad \forall i = 1,2,\ldots,k, \tag{A.7d}$$

$$- g_i(u_i, v) + t_i = 0, \qquad \forall i = 1, 2, \ldots, k, \tag{A.7e}$$

$$G_i(u_i, v) = 0, \qquad \forall i = 1, 2, \ldots, k, \tag{A.7f}$$

$$h(u, v) - s = 0, \tag{A.7g}$$

$$u_i \geq 0, v \geq 0, s \geq 0, t_i \geq 0, \lambda_i \geq 0, \chi_i \geq 0, \psi_i \geq 0, \sigma_i \geq 0, \xi_i \geq 0, \tag{A.7h}$$

$$\nu_i, \mu_i \text{ unrestricted in sign.} \tag{A.7i}$$

(Here, $V = \mathrm{diag}(v_1, v_2, \ldots, v_r)$.)

Result. If $(u^*, v^*, s^*, t_i^*, \lambda_i^*, \chi_i^*, \psi_i^*, \sigma_i^*, \xi_i^*, \nu_i^*, \mu_i^*)$ is a local solution of (A.7) with $C_{\text{penalty}} = 0$, then (u^*, v^*) is a strong-stationary point of (MLSF-BLP).

Thus for testing a feasible solution (u^*, v^*) of (MLSF-BLP)to be a strong-stationary point, it is sufficient to solve the (Pen-NLP) parameterized in $(u, v) = (u^*, v^*)$ for optimal values of s, t_i, λ_i, χ_i, ψ_i, σ_i, ξ_i, ν_i, μ_i and check whether the objective function value $C_{\text{penalty}} = 0$.

Appendix B. A special case of (MLSF-BLP)

We now discuss a special case of (MLSF-BLP) and observe a practical method to test the strong-stationarity for a feasible solution of this special case.

If in (MLSF-BLP) the upper level constraint functions g_i, G_i are linear in (u, v), lower level constraints c are linear in (u, w); and upper level objective functions f_i are bilinear in (u_i, v), lower level objective is bilinear in (u, w), then for a feasible solution (u^*, v^*) of (MLSF-BLP) the optimization problem (A.7) becomes a linear programming problem in variables λ_i, $\chi_i, \psi_i, \sigma_i, \xi_i, \nu_i, \mu_i$, and parameterized in corresponding values u, v, s, t_i.

Reason: because, for a given feasible solution (u^*, v^*), the values of s can be obtained using (A.7g), say it as s^*. Also, then the values of S and V become known as $S = \mathrm{diag}(s_1, s_2, \ldots, s_r)$ and $V = \mathrm{diag}(v_1, v_2, \ldots, v_r)$. Further, the non-negativity of t_i can be tested using the condition (A.7e), say it as t_i^*.

Thus, for testing a feasible solution (u^*, v^*) of this special case of (MLSF-BLP) to be a stationary point, it reduces to:

(1) obtain values of t_i^* and s^* using (A.7e) and (A.7g), respectively, such that $s^* \perp v^*$, test check for non-negativity conditions, then
(2) solve the linear programming problem (Para-LP) given in (B.1) parameterized in (u^*, v^*, s^*, t_i^*), for $\lambda_i, \chi_i, \psi_i, \sigma_i, \xi_i, \nu_i, \mu_i$, and then
(3) check the objective function value $C_{pp} = 0$ for an optimal solution.

$$\text{(Para-LP)} \min_{\lambda_i, \chi_i, \psi_i, \sigma_i, \xi_i, \nu_i, \mu_i} \; C_{pp} = \sum_{i=1}^{k} (u_i^{*\prime} \chi_i + t_i^{*\prime} \lambda_i + v^{*\prime} \psi_i + s^{*\prime} \sigma_i)$$

subject to

$$\begin{aligned}
&\nabla_{u_i} f_i(u_i^*, v^*) - \lambda_i' \nabla_{u_i} g_i(u_i^*, v^*) - \nu_i' \nabla_{u_i} G_i(u_i^*, v^*) - \mu_i' \nabla_{u_i} h(u_i^*, v^*) - \chi_i = 0, && \forall i = 1, 2, \ldots, k,\\
&\nabla_v f_i(u_i^*, v^*) - \lambda_i' \nabla_v g_i(u_i^*, v^*) - \nu_i' \nabla_v G_i(u_i^*, v^*) - \mu_i' \nabla_v h(u_i^*, v^*) - \psi_i + S^* \xi_i = 0, && \forall i = 1, 2, \ldots, k,\\
&\mu_i - \sigma_i + V^* \xi_i = 0, && \forall i = 1, 2, \ldots, k,\\
&\lambda_i \geq 0, \chi_i \geq 0, \psi_i \geq 0, \sigma_i \geq 0, \xi_i \geq 0,\\
&\nu_i, \mu_i \text{ unrestricted in sign.}
\end{aligned} \tag{B.1}$$

Appendix C. Theoretical development for solving special case of (MLSF-BLP)

For the special case of (MLSF-BLP) discussed above, if a vector values for the variable u is chosen randomly, then the lower-level problem (FDMP) in (4.1) becomes a linear programming problem (LPP) in variables w, parametrized in the values of u. This LPP can be easily solved for obtaining an optimal solution v. And therefore, the corresponding vector of values for (u, v) would satisfy the KKT conditions (A.2). Accordingly, the vector of values for the variable s can be obtained using (A.7g) (same as (A.4)). Further, the values of each variable t_i can be obtained using (A.7e) and tested for non-negativity. If, through all these computations a vector of values for (u, v) is obtained which satisfies the testing criteria just discussed, then it is a feasible solution of (MLSF-BLP). Then for the obtained values of (u, v, s, t_i) the (Para-LP) can be solved for an optimal solution. If the objective function value $C_{pp} = 0$, then we get (u, v) as a strong stationary point of (MLSP-BLP).

Through this discussion it is learnt that if we randomly generate a vector of values for leaders' variables u, we can obtain follower's reaction v and test for strong-stationary point of this special case of (MLSP-BLP). Thus, if we use a real-coded GA by coding the chromosomes as vectors of values for u, obtain corresponding values for v, and take the fitness value of each chromosome as objective function value C_{pp} to be computed by the above procedure, then we can achieve a stationary-point in some generation obtained through reproduction operators of GA. A GA-based approach is proposed in Section 4 for solving the special case of (MLSF-BLP) which involves linear constraints and bilinear objective functions.

Based on above discussion, a GA based approach, for solving a MLSF game involving linear constraints and bilinear objective functions, is proposed in Section 4.

Remark: a strong-stationarity point (u^*, v^*) for $u^* = \{u_i^* : i = 1, 2, \ldots, k\}$, thus obtained can further be tested for a strong-stationary Nash-equilibrium point of the (MLSF-BLP) by repeatedly solving single-leader-single-follower bilevel programming problems for each leader given by:

$$\boxed{\begin{aligned}
&\left(\text{LDMP} - i^{-1}\right) \min_{u_i \geq 0}\ f_i(u_i, v) \\
&\qquad \text{subject to} \\
&\qquad\qquad g_i(u_i, v) \geq 0, \\
&\qquad\qquad G_i(u_i, v) = 0, \\
&\qquad\qquad \text{where, } v \text{ is optimal response of the follower corresponding} \\
&\qquad\qquad \text{to above values of } u_i \text{ and keeping values of other leaders'} \\
&\qquad\qquad \text{variables fixed as } u_{\bar{i}} = u_{\bar{i}}^* : \bar{i} = 1, 2, \ldots, k, \bar{i} \neq i, \\
&\quad (\text{FDMP}) \min_{w}\ b(u, w) \\
&\qquad \text{subject to} \\
&\qquad\qquad c(u, w) \geq 0 \\
&\qquad\qquad w \geq 0.
\end{aligned}} \tag{C.1}$$

For solving such a problem having large number of variables, it further requires a heuristic algorithm. One such algorithm is proposed by Kumar *et al.* [55].

Appendix D. Complementary results

Table D.1. Production volumes (regular-time and over-time): Supplier 1.

		Time period (t)			
		1	2	3	4
Q_{kn_kt}	$n_k = 1$	175	100	100	75
O_{kn_kt}	$n_k = 1$	0	0	0	0

TABLE D.2. Consignment volumes from PC to DC(s): Supplier 1.

$I_{kn_ki_kt}$	Time period (t)			
DC (i_k)	1	2	3	4
1	0	0	0	0
2	0	0	0	0

TABLE D.3. Transportation volumes from PC to DC(s): Supplier 1 ($n_k = 1$).

$x_{kn_ki_kjt}$	DC (i_k)											
	0				1				2			
DL	Time period (t)				Time period (t)				Time period (t)			
(j)	1	2	3	4	1	2	3	4	1	2	3	4
1	0	0	0	0	0	0	0	0	0	0	0	0
2	125	100	75	50	0	0	0	0	0	0	0	0
3	50	0	25	25	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0

TABLE D.4. Production volumes (regular-time and over-time): Supplier 2.

		Time period (t)			
	Product (n_k)	1	2	3	4
Q_{kn_kt}	1	103	100	80	0
	2	185	135	100	15
O_{kn_kt}	1	22	0	0	0
	2	0	0	0	0

TABLE D.5. Consignment volumes from PC to DC(s): Supplier 2 (DC: $i_k = 1$).

$I_{kn_ki_kt}$	Time period (t)			
Product (n_k)	1	2	3	4
2	0	0	0	0
3	0	0	0	0

TABLE D.6. Transportation volumes from PC to DC(s): Supplier 2.

$x_{kn_ki_kjt}$		$i_k = 0$				$i_k = 1$			
Product	DL	Time period (t)				Time period (t)			
(n_k)	(j)	1	2	3	4	1	2	3	4
2	1	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
	4	25	0	0	0	0	0	0	0
	5	100	100	80	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0
	4	80	80	100	15	0	0	0	0
	5	105	55	0	0	0	0	0	0

Table D.7. Production volumes (regular-time and over-time): Supplier 3.

	Product (n_k)	Time period (t) 1	2	3	4
Q_{kn_kt}	1	200	100	120	30
O_{kn_kt}	1	0	0	0	0

Table D.8. Consignment volumes from PC to DC(s) ($I_{kn_ki_kt}$; $n_3 = 1,\ i_3 = 1$): Supplier 3.

Time period (t)			
1	2	3	4
0	0	0	0

Table D.9. Transportation volumes from PC to DC(s) ($n_3 = 1$): Supplier 3.

$x_{kn_ki_kjt}$	$i_k = 0$				$i_k = 1$			
DL	Time period (t)				Time period (t)			
(j)	1	2	3	4	1	2	3	4
1	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	0	0
5	200	100	120	30	0	0	0	0

Table D.10. Production volumes (regular-time and over-time) ($n_k = 5$): Supplier 4.

	Time period (t) 1	2	3	4
Q_{kn_kt}	175	175	175	110
O_{kn_kt}	0	0	0	0

Table D.11. Transportation volumes from PC to DC(s) ($i_k = 0,\ n_k = 5$): Supplier 4.

DL (j)	Time period (t) 1	2	3	4
1	50	75	75	50
2	0	0	0	0
3	0	0	0	0
4	60	60	80	0
5	65	40	20	60

TABLE D.12. Production volumes (regular-time and over-time): Supplier 5.

		Time period (t)			
		1	2	3	4
Q_{kn_kt}	$n_k = 2$	226	226	261	225
	$n_k = 3$	350	350	315	265
O_{kn_kt}	$n_k = 2$	49	49	14	0
	$n_k = 3$	0	0	0	0

TABLE D.13. Consignment volumes from PC to DC(s): Supplier 5.

	Time period (t)			
	1	2	3	4
$n_k = 2, i_k = 1$	0	0	0	0
$n_k = 3, i_k = 1$	0	0	0	0

TABLE D.14. Transportation volumes from PC to DC(s): Supplier 5.

$x_{kn_ki_kjt}$		$i_k = 0$				$i_k = 1$			
	DL	Time period (t)				Time period (t)			
	(j)	1	2	3	4	1	2	3	4
$n_k = 2$	1	75	75	75	75	0	0	0	0
	2	75	75	75	50	0	0	0	0
	3	75	50	75	25	0	0	0	0
	4	50	75	50	25	0	0	0	0
	5	0	0	0	50	0	0	0	0
$n_k = 3$	1	125	125	100	50	0	0	0	0
	2	100	75	75	75	0	0	0	0
	3	100	75	50	50	0	0	0	0
	4	0	0	0	0	0	0	0	0
	5	25	75	90	90	0	0	0	0

TABLE D.15. Production volumes (regular-time and over-time): Supplier 6.

	Product	Time period (t)			
	(n_k)	1	2	3	4
Q_{kn_kt}	1	250	200	200	200
	5	100	100	100	25
O_{kn_kt}	1	0	0	0	0
	5	0	0	0	0

Table D.16. Consignment volumes from PC to DC(s): Supplier 6.

$I_{kn_ki_kt}$	Time period (t) 1	2	3	4
$n_k = 2, i_k = 1$	150	75	100	75
$n_k = 3, i_k = 1$	0	0	0	0

Table D.17. Transportation volumes from PC to DC(s): Supplier 6

$x_{kn_ki_kjt}$		DC (i_k) 0				1				2			
Product (n_k)	DL (j)	Time period (t) 1	2	3	4	Time period (t) 1	2	3	4	Time period (t) 1	2	3	4
1	1	0	0	0	0	150	75	100	75	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	25	75	25	25	0	0	0	0	0	0	0	0
	4	75	50	75	50	0	0	0	0	0	0	0	0
	5	0	0	0	50	0	0	0	0	0	0	0	0
5	1	0	0	0	0	–	–	–	–	–	–	–	–
	2	25	25	50	25	–	–	–	–	–	–	–	–
	3	75	75	50	0	–	–	–	–	–	–	–	–
	4	0	0	0	0	–	–	–	–	–	–	–	–
	5	0	0	0	0	–	–	–	–	–	–	–	–

Table D.18. Production volumes (regular-time and over-time): Supplier 7.

	Product (n_k)	Time period (t) 1	2	3	4
Q_{kn_kt}	1	0	0	0	0
	3	0	0	0	0
	5	40	90	85	0
O_{kn_kt}	1	0	0	0	0
	3	0	0	0	0
	5	0	0	0	0

Table D.19. Consignment volumes from PC to DC(s) ($I_{kn_ki_kt}$): Supplier 7.

Product (n_k)	DC (i_k)	Time period (t) 1	2	3	4
2	1	0	0	0	0
	2	0	0	0	0
3	1	0	0	0	0
	2	0	0	0	0

Table D.20. Transportation volumes from PC to DC(s): Supplier 7.

$x_{kn_ki_kjt}$		DC (i_k)											
		0				1				2			
Product	DL	Time period (t)				Time period (t)				Time period (t)			
(n_k)	(j)	1	2	3	4	1	2	3	4	1	2	3	4
1	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0
3	1	0	0	0	0	0	0	0	0	0	0	0	0
	2	0	0	0	0	0	0	0	0	0	0	0	0
	3	0	0	0	0	0	0	0	0	0	0	0	0
	4	0	0	0	0	0	0	0	0	0	0	0	0
	5	0	0	0	0	0	0	0	0	0	0	0	0
5	1	0	0	0	0	–	–	–	–	–	–	–	–
	2	25	50	25	0	–	–	–	–	–	–	–	–
	3	0	0	0	0	–	–	–	–	–	–	–	–
	4	0	0	0	0	–	–	–	–	–	–	–	–
	5	15	40	60	0	–	–	–	–	–	–	–	–

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