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PRICING STRATEGY AND PRODUCT SUBSTITUTION OF BULLWHIP EFFECT IN DUAL PARALLEL SUPPLY CHAIN: AGGRAVATION OR MITIGATION?

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Abstract. The bullwhip effect affects not only the revenue of the retailer but also the revenue of the manufacture and weakens the performance of the whole supply chain. In this research, the bullwhip effect influenced by pricing strategy is studied for the first time in two parallel supply chains distributing price-sensitive and substitutable products. The analytical expression of bullwhip effect based on pricing strategy and product substitution is derived for the first time. The effects of pricing strategy and product substitution on the bullwhip effect are studied through simulation. The analytic results and the simulation results show that the bullwhip effect of the two parallel supply chains will be affected by lead time, product substitution rate, and pricing coefficient. The interesting finding is that the bullwhip effect can be efficiently alleviated by adopting a price strategy with many correlations and a small coefficient of autocorrelation.

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1. Introduction

Considering the worldwide high popularity of e-commerce and increasingly fierce commercial competition, it is challenging for a firm to survive as one independent entity. Moreover, companies forming one supply chain are gradually working together to compete with companies in another supply chain. The supply chain management, therefore, becomes extremely significant, it concentrates on all costs-related and consumer-related factors with the ultimate goal of maximizing overall value as well as enhancing efficiency. Among all these elements affecting supply chains, the bullwhip effect (BE) is an important factor which has a profound influence and reflects the supply chain performance. When demands fluctuate, orders are amplified as they travel up the supply chain resulting in excessive overstocking. Especially, in the market of alternative products, the price competition strategies of both sides often lead to frequent demand fluctuations which are likely to induce bullwhip effect.

In the smartphone market, using the pricing strategy to drive sales is very common. The retailers usually take price promotions to attract more consumers during the sales season, such as Double 11 shopping festival and Black Friday. There are plenty of cases, such as Samsung, iPhone and Xiaomi. Hoarding products based

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on price expectations eventually leads to excess inventory throughout the supply chain. In this paper, we seek to explore the BE about the pricing strategy and product substitution. Obviously, in the complicated supply chain net, both the internal coordination of supply chain nodes and the competition between supply chains will inevitably affect the performance of a supply chain. To gain much more profit and market share, competing retailers are always involved in a price war rather than adopting other appropriate measures. The BE, resulting from price variability, however, always leads to unexpected loss. While the BE can be diminished by the demand substitution in most cases Li *et al.* [15], it can also be aggravated by the demand variability caused by price fluctuation. For supply chains with alternative products, the BE is affected not only by product price changes, but also by its replacement products [11].

Recent researches have investigated the impact of the BE on the supply chain and how it could be reduced. However, past researchers scarcely pay attention to the impact of pricing strategies on the BE in supply chain management with alternative products. To make up the gap as well as widen our knowledge about the BE, we construct a model consisting of two parallel supply chains to further probe the impact of the product substitution and price competition BE. As revealed by Ma *et al.* [20], there is a possibility that their interaction can dampen BE when the two parallel supply chains are rivals. Therefore, we mainly investigate how pricing strategy and product substitution affect the BE. Accordingly, we also propose an effective way to reduce BE.

This study examines a model of two parallel price-sensitive supply chains with two substitutable products, where retailers take the first-order vector autoregressive VAR(1) price-setting process. The assumptions of our model conform to many industries. In the mobile phone industry, each enterprise and its downstream distributors form a supply chain, and supply chains share the same demand market. There are interactions BE between their products' pricing process. In other words, take Samsung and iPhone for instance, Samsung retailer makes pricing decisions referring to not only the Samsung phone's previous price but also that of the iPhone. As indicated by these practical examples, it is necessary to quantify BE to analyze its impact, probe its causes, and decrease its barriers to better performance of the supply chains. Generally, BE can cause high inventory and low service levels in the supply chain. What's more, BE also leads to inventory backlog, poor service, low-quality products as well as high manufacturing and transportation costs, which are contrary to supply chain member's demand for low-priced and high-quality products. Therefore, much attention should be paid to investigate how to alleviate the negative impact of BE.

In this paper, we commit to reveal how pricing strategy and product substitution exert an impact on BE, which is seldom involved before. Specially, we explore the way how BE is affected by the pricing strategy and product substitution in two parallel price-sensitive supply chains. Firstly, two supply chains are considered, one of which distributes the focal product and another distributes the substitute product. Secondly, we develop an analytical model to evaluate the impact of price-sensitive demand on BE in two parallel supply chains, and supply chains subject to pricing strategies and product substitution based on the assumptions of two retailers using the order-to-inventory strategy and the MMSE forecasting method. The demand of the supply chain is sensitive to price, while their price-setting processes submit to VAR(1) process.

The structure of the following sections can be summarized as follows: a literature review is shown in Section 2, including the history of BE, demand patterns, and forecasting method, and our models are expanded by comparing the differences between them. Then in Section 3 we develop a basic supply chain model with some setting, which is composed of the inventory policy, forecasting method, demand pattern, and the price-setting decision; in Section 4, we investigate BE in two parallel supply chains with substitute products under VAR(1) price process; a numerical simulation is conducted in Section 5 to explore the impact of the substitution effect and the price strategy on BE; Section 6 draws the main conclusions and present some managerial insights.

2. Literature review

This paper mainly involves the literature of three research fields: bullwhip effect, pricing strategies and complexity in supply chain. The following is mainly to review the literature from these three aspects.

2.1. Bullwhip effect

The BE reveals that the fluctuation of demand is magnified when demand order is delivered step by step from the customer to the supplier. There is a gap between the time it was discovered and the time it was applied to research. Specifically, the phenomenon was firstly discovered in 1961, was formally named BE by Procter & Gamble in the 1990s. And its academic concept was defined in 1997 [14]. Since then, more scholars are actively involved in studying the BE.

The existing work on the BE has concentrated on three aspects, namely the existence of the BE, the measurement of the BE, and the mitigation and control of the BE. Dooley *et al.* [10] studied whether there was a BE in US manufacturing under the economic crisis and further analyzed its impact on the supply chain. Naim *et al.* [25] studied the BE of a complex production and inventory control system by using a single-echelon model, which was a production-distribution system proposed by Forrester [13]. Nagaraja and Mcelroy [24] developed a multi-variate BE expression of products with an order-up-to inventory policy. Chen *et al.* [8] pointed out that the BE measured by the material-flow data was always greater than measured by the information flow. Bray *et al.* [5] established a dynamic discrete selection model of the supply chain including a single supplier and 73 stores to study the BE. According to a known VAR(1) process based on unknown parameters, Pastore *et al.* [27] developed a two-echelon and single-product supply chain considering final demand distributed and analyzed the accuracy of the analytical approximation.

As mentioned before, the demand process has a direct impact on the BE. Another critical issue affecting BE is the forecasting methods and the ordering strategies, arousing intense scholarly interests [31]. Various prediction methods under different ordering strategies, thus, were studied in the field of BE related literature. These prediction techniques are user-friendly so that they have been widely adopted in the industry. Most of the research studying BE merely considered a single chain or a one retailer supply chain. The real economic system, however, is more complex. As an expansion to prior research, Ma *et al* [17] sheds light on the BE under a more complicated supply chain structure. The study gave a comparison of BE in the supply chain with two retailers under MMSE, MA, and ES forecasting techniques, also proposed the measure of BE considering the market competition between two retailers with different VAR.

Numerous papers address demand substitution in the supply chain over the decades. Li *et al.* [15], to our knowledge, is the first one who studied BE and demand substitution. Crucially, Duan *et al.* [11] explored the BE under substitute products *via* empirical research, making it more practical. They found that BE was influenced by its factors as well as its substitute products.

2.2. Pricing strategies related to the BE

Price strategies have also drawn considerable attention from academic fields and practical fields, for the price fluctuation of own products and substitute products are closely related to BE. Specifically, there is a negative correlation between the focal products' price and the BE, and a positive correlation in terms of the price of the substitute products. From the perspective of operation management, BE results from demand forecasting, order lead time, bulk ordering, shortage game, and price fluctuation.

As investigated by Duan *et al.* [11], not only the own product prices but also the replacement product prices will affect BE due to the alternative products in a supply chain. What's more, Wang and Disney [38] hold that future research into the influence of prices on BE should incorporate price-setting and negotiation processes. Since price fluctuation is one of the main reasons for BE, the price-sensitive demand function is generally used as a specification to construct a model for BE research by many scholars [16, 17, 19, 22, 23]. For example, Ma *et al.* [20] proposed an analytical framework that consisted of two parallel supply chains with interacting price-sensitive demands, explored how the interactions exert effects on BE.

Most of the existing literature on BE in two parallel or two retailers supply chains with a substitutable product focus on the demand process or forecasting methods. Research about how fluctuation in prices influences BE, however, is seldom involved. Price fluctuation will lead to demand changes, suggesting that BE will be affected ultimately. Inspired by Ma *et al.* [20] and Duan *et al.* [11], we construct a model, which consists of two parallel supply chains, to explore the relationship between BE and price fluctuation. Differing from the work of Ma *et al.* [20], we assume that the retail price is affected by both its previous price and its substitute product's price. Also, the price-setting process submits to the VAR(1) process. Ozelkan *et al.* [26] investigated the reverse BE with joint replenishment and pricing decisions in supply chains. Ma *et al.* [22] studied the BE in a multi-channel supply chain which included a manufacturer, a dual-channel retailer, and an online retailer.

2.3. Complex supply chain

For better depicting the actual situation, a more complex supply chain, which has more significance in practice, is explored expecting to obtain more practical insights. For example, Vanovermeire and Sörensen [37], working with simple heuristics to solve an integrated collaboration model, integrated the cost allocation approach into operational planning issues in the context of horizontal logical collaboration; and considered the optimization of allocation work and the allocation of costs or benefits as a single decision. Besides, Rezapour *et al.* [30] proposed a method of planning supply chain production, which took the multi-level of the supply process, the unreliability of production equipment and the uncertainty of the market into consideration. In this study, the salient features of uncertainty propagation in the supply chain were introduced and the impact was quantified using test questions from the automotive industry. In the research of Tachizawa and Wong [32] considered the complex interaction between the management governance mechanism of the green supply chain, supply network structure, and environmental performance. Ye and Hu [40], convergence properties for the proposed Nash seeking strategies were developed. They found that network structure and complexity of the supply chain greatly influence GSCM effectiveness, but the effects depend upon the type of governance mechanism.

Besides those models assuming complicated context in the supply chain, other models in various industries, providing some practical and simple researches, are proposed and analyzed as well. The price decision will be affected by the supply chain buyback contracts and government subsidies [2, 39]. Zhu *et al.* [42] pointed out Companies will adopt green technology to reduce environmental pollution. Cui *et al.* [9] studied a two-echelon supply chain where suppliers serve terminals of uncertain final demands. Ponte *et al.* [28] quantified the average financial impact and the variability of production and transportation lead times to multi-level supply chains. Biswal *et al.* [3] explored the impact of RFID on non-profit supply chain scenarios to examine the impact of available ordering rates and shrinkage rates on total warehouse-level costs. Zou *et al.* [43] proved that there is a unique equilibrium in the decision process and that the proposed parallel pricing strategy has a unique Nash equilibrium. Ma *et al.* [22] examined the pricing mechanism with the non-cooperative game and revenue-sharing contracts in the electricity market. Ponte *et al.* [29] proposed four order and inventory variance amplification models with different information transparency and solved the expression of the most profitable rate of return.

Based on the above three research streams, two parallel supply chains distributing two substitutable products with price-sensitive demands are considered. The academic contribution of this paper is to reveal how pricing strategy and product substitution exert an impact on BE, which is seldom involved before. Specially, we explore the way how BE is affected by the pricing strategy and product substitution in two parallel price-sensitive supply chains.

3. The basic model and the analysis of operation mechanism

In this section, we firstly describe the research problem of this paper in detail. Then, the basic game model is constructed and the operation mechanisms about the pricing strategy, order policy and forecasting technique are analyzed for the two parallel supply chain in the following sections.

3.1. Problem description

In two parallel supply chains, the price-sensitive interactions have more complex impact on the bullwhip effect compared to a single supply chain [20].

TABLE 1. A comparative assessment of the previous literatures.

Article	Single channel	Dual (Multi) channel	Pricing	Model & simulation
Chen *et al.* [6]	✓			✓
Vanovermeire *et al.* [37]		✓		✓
Fang *et al.* [12]		✓		✓
Bao *et al.* [43]		✓		✓
Ma *et al.* [20]		✓	✓	✓
Duan *et al.* [11]				
Ma *et al.* [20]		✓		✓
Taleizadeh *et al.* [34]	✓		✓	✓
Taleizadeh *et al.* [33]	✓		✓	✓
Xie *et al.* [39]		✓	✓	✓
Taleizadeh *et al.* [35]		✓		✓
Ma *et al.* [22]		✓	✓	✓
Basban *et al.* [1]				
Ponte *et al.* [29]	✓			✓
Current study		✓	✓	✓

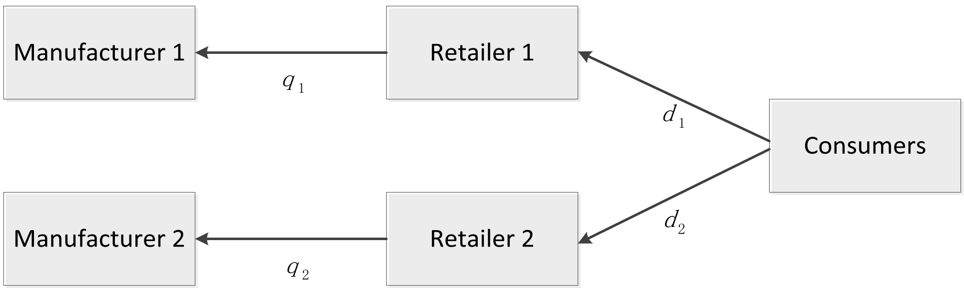


FIGURE 1. Supply chain structure.

3.2. Model construction

Based on [18–20] work, we construct a similar model to explore the different stories. As shown in Figure 1, two supply chains are considered, and each one is comprised of a manufacturer and a retailer, denoted by M_i and R_i $(i = 1, 2)$, respectively. In this paper, $i = 1$ represents the first supply chain, and $i = 2$ represents the second supply chain. The retailers face the same market with price-sensitive demand for two alternative products. They also make and send orders to their manufacturers according to the demand received from consumers. Table 2 reports the variables and parameters related to operation management in the proposed supply chain model.

3.3. Demand pattern

We hold that the demand for each product, as a result of product price sensitivity and substitutability, is affected by not only the price of the product itself but also the demand for its substitute product. Therefore, we assume that the two manufacturers produce the same products and that the products can be replaced with each other. In this case, product demand will be affected by the retail price set by two downstream retailers in two parallel supply chains.

TABLE 2. Notation.

Variables	
d_t^i	The market demand of retailer i $(i = 1, 2)$ at time t
p_t^i	The price of retailer i $(i = 1, 2)$ at time t
P_t	The price vector of retailers at time t
q_t^i	The order quantity of retailer i at time t
S_t^i	The order-up-to point of retailer i at time t
$\hat{D}_t^L$	The estimated vector of the lead-time demand at time t
$\xi_{i,t}$	Disturbance factor for the pricing process of retailer i $(i = 1, 2)$
Parameters	
a_i	The base demand level for the product of the ith supply chain $(i = 1, 2)$
b_{ii}	The price sensitivity coefficients for the ith supply chain $(i = 1, 2)$
b_{ii}	The substitution degree of product j to product i. $(i, j = 1, 2, i \neq j)$
ϕ_{ii}	The autocorrelation coefficient of the ith supply chain's pricing process.
ϕ_{ij}	The mutual-correlation coefficient
E	The stable set of the price decision-making coefficients
L	The lead time
z	The safety factor vector
B	The price-sensitive coefficient matrix
Statistics	
σ_i^2	The variance of normally distributed price shock
δ_i^2	The variance of price process for R_i
δ_{12}	The covariance of price processes for retailers

The demand function models for two alternative products can be described as follows [20, 22]:

$$\begin{cases} d_t^1 = a_1 - b_{11}p_t^1 + b_{12}p_t^2 \\ d_t^2 = a_2 - b_{22}p_t^2 + b_{21}p_t^1 \end{cases} \tag{3.1}$$

where $a_i(a_i > 0, i = 1, 2)$ represents the basic demand of product i, determined by the brand, credit and consumer preference, except their prices, $b_{ii}(b_{ii} > 0, i = 1, 2)$ is the price elastic coefficient for the product of the ith supply chain, $b_{ij}(i = 1, 2, j = 1, 2, i \neq j)$ is the substitution coefficient that measures the degree of substitution of product j to product i. Different from Ma *et al.*'s paper, this paper mainly studies the alternative supply chain, so $b_{ij}(i \neq j)$ is non-negative. p_t^i is the price of retailer $i(i = 1, 2)$ at time t, its variance is $\delta_i^2\left(\mathrm{var}\left(p_t^i\right) = \delta_i^2\right)$, the covariance between two supply chains is $\delta_{12}\left(\mathrm{cov}\left(p_t^1, p_t^2\right) = \delta_{12}\right)$.

For ease of representation and derivation, we use vector and matrix methods to represent the variables and parameters of the two retailers. a is the base demand vector, $a = (a_1, a_2)^T$. d_t is the demand vector and p_t is the price vector for products in period t. $d_t = \left(d_t^1, d_t^2\right)^T$ and $P_t = \left(p_t^1, p_t^2\right)^T$. B is the price-sensitive coefficient matrix for demands, $B = \begin{pmatrix} -b_{11} & b_{12} \\ b_{21} & -b_{22} \end{pmatrix}$.

So that equation (3.1) can be represented in the vector form:

$$d_t = a + BP_t. \tag{3.2}$$

Based on the nature of the time series model, we can infer that the variance of each demand process can be expressed as follows [4]:

$$\begin{cases} \mathrm{var}\left(d_t^1\right) = b_{11}^2\delta_1^2 + b_{12}^2\delta_2^2 - b_{11}b_{12}\delta_{12} \\ \mathrm{var}\left(d_t^2\right) = b_{21}^2\delta_1^2 + b_{22}^2\delta_2^2 - b_{21}b_{22}\delta_{12}. \end{cases} \tag{3.3}$$

3.4. Pricing strategy

The pricing strategy of retailer is that the price of the period t is determined by their own previous price and the price of the substitute product. The price-setting process can be written as:

$$\begin{cases} p_t^1 = \mu_1 + \phi_{11} p_{t-1}^1 + \phi_{12} p_{t-1}^2 + \xi_{1,t} \\ p_t^2 = \mu_2 + \phi_{21} p_{t-1}^1 + \phi_{22} p_{t-1}^2 + \xi_{2,t} \end{cases} \tag{3.4}$$

$\xi_{i,t}(i=1,2)$ is the disturbance factor for the price decision-making process, which meets $E(\xi_{i,t})=0, \operatorname{var}(\xi_{i,t}) = \sigma_i^2 (i=1,2)$, and $\operatorname{cov}(\xi_{1,t}, \xi_{2,t}) = 0$. $\phi_{ii}(i=1,2)$ is the autocorrelation coefficient of the ith supply chain's pricing process. The value of ϕ_{ii} represents the importance of his historical price information in the price-setting process. $\phi_{ij}(i,j=1,2, i\neq j)$ is the mutual-correlation coefficient in the price-setting process. The value of ϕ_{ij} represents the importance of the rival's historical price information in the price-setting process. et $\phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$ and $\xi_t = (\xi_{2,t}, \xi_{1,t})^T$. ϕ is the price decision-making coefficient matrix, and ξ_t is the disturbance vector, then equation (3.3) can be rewritten as the first-order vector autoregressive process (VAR(1)):

$$P_t = \mu + \phi P_{t-1} + \xi_t. \tag{3.5}$$

3.5. Table analysis of the price-setting process

According to the stability judgment basis, the condition for stationarity of the VAR(1) price-setting process is that all eigenvalues of ϕ are less than one in absolute value, while eigenvalues of ϕ are all roots of $\det(\lambda I - \phi) = 0$. Because the eigenvalues of the matrix ϕ are $\lambda_{1,2} = \frac{1}{2}\left(\phi_{11} + \phi_{22} \pm \sqrt{(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21}}\right)$, the price decision-making coefficient matrix should meet the following condition:

$$(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21} < (4 - \phi_{11} - \phi_{22})^2, \tag{3.6}$$

then, based on equation (3.6), we can obtain the stable set of the price decision-making coefficients in equation (3.7):

$$E = \left\{ (\phi_{11}, \phi_{12}, \phi_{21}, \phi_{22}) | (4 - \phi_{11} - \phi_{22})^2 - 4\phi_{12}\phi_{21} - (\phi_{11} - \phi_{22})^2 > 0, \quad \phi_{ij} > 0, i = 1,2, j = 1,2 \right\}. \tag{3.7}$$

Figure 2 shows the stable region of the VAR(1) price-setting process $\phi_{12} = \{0, 0.5, 0.8, 1.0\}$. We find that the increases of the parameters ϕ_{12} and ϕ_{21} narrow the stable region of the supply chain system, and the growth of one parameter at the boundary of the stable region will result in a decrease in the other parameter. It suggests that that when the retailer pays more attention to the price of the opponent, the stable area of the price decision will be reduced.

In the stable region, the price-setting process will be stationary, and the variance and the covariance are fixed.

$$\operatorname{var}(p_t^1) = \operatorname{var}(p_{t-1}^1) = \operatorname{var}(p_{t-2}^1) = \cdots = \delta_1^2 \tag{3.8}$$

$$\operatorname{var}(p_t^2) = \operatorname{var}(p_{t-1}^2) = \operatorname{var}(p_{t-2}^2) = \cdots = \delta_2^2 \tag{3.9}$$

$$\operatorname{cov}(p_t^1, p_t^2) = \operatorname{cov}(p_{t-1}^1, p_{t-1}^2) = \operatorname{cov}(p_{t-2}^1, p_{t-2}^2) = \cdots = \delta_{12}. \tag{3.10}$$

Based on equations (3.8)–(3.10), the variance of the stationary price-setting process can be derived as follows:

$$\operatorname{var}(p_t^1) = \frac{-\sigma_2^2 \phi_{12}^2 (\phi_{11}\phi_{22} - \phi_{12}\phi_{21} + 1) + \sigma_1^2 \left(-\phi_{11}\phi_{22}^3 + \phi_{22}(\phi_{11} + \phi_{22}) + \phi_{12}\phi_{21}\left(\phi_{22}^2 + 1\right) - 1\right)}{(1 - \phi_{11}\phi_{22} + \phi_{12}\phi_{21})(-1 - \phi_{11}\phi_{22} + \phi_{11} + \phi_{12}\phi_{21} + \phi_{22})(1 + \phi_{11}\phi_{22} + \phi_{11} - \phi_{12}\phi_{21} + \phi_{22})} \tag{3.11}$$

$$\operatorname{var}(p_t^2) = \frac{-\sigma_1^2 \phi_{21}^2 (\phi_{11}\phi_{22} - \phi_{12}\phi_{21} + 1) + \left(-\phi_{22}\phi_{11}^3 + \phi_{11}(\phi_{11} + \phi_{22}) + \phi_{12}\phi_{21}\left(\phi_{11}^2 + 1\right) - 1\right)}{(1 - \phi_{11}\phi_{22} + \phi_{12}\phi_{21})(-1 - \phi_{11}\phi_{22} + \phi_{11} + \phi_{12}\phi_{21} + \phi_{22})(1 + \phi_{11}\phi_{22} + \phi_{11} - \phi_{12}\phi_{21} + \phi_{22})} \tag{3.12}$$

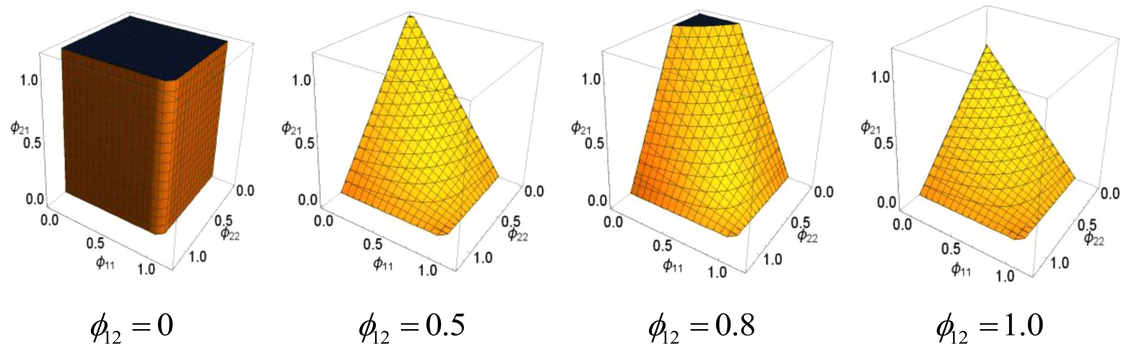


FIGURE 2. Stable region.

and the covariance between two supply chains is expressed as follows:

$$\operatorname{cov}\left(p_t^1, p_t^2\right) = \frac{\sigma_1^2 \phi_{21}\left(\phi_{11}\phi_{22}^2 - \phi_{12}\phi_{21}\phi_{22} - \phi_{11}\right) + \sigma_2^2 \phi_{12}\left(\phi_{22}\phi_{11}^2 - \phi_{11}\phi_{12}\phi_{21} - \phi_{22}\right)}{(1 - \phi_{11}\phi_{22} + \phi_{12}\phi_{21})(-1 - \phi_{11}\phi_{22} + \phi_{11} + \phi_{12}\phi_{21} + \phi_{22})(1 + \phi_{11}\phi_{22} + \phi_{11} - \phi_{12}\phi_{21} + \phi_{22})}. \tag{3.13}$$

The analytic expressions of the variance and the covariance (equations (3.11)–(3.13)) are the important conditions for deriving analytical formula of bullwhip effect.

3.6. Order-up-to inventory policy

As a general ordering algorithm in many ERP systems, the order-up-to inventory policy is for customer service, balancing inventory, and capacity investments. It is supposed that the inventory system is managed and researched at each time, and both retailers adopt an order-up-to inventory strategy. Retailer's demand is a stochastic parameter at period t, and the distribution center is ignorant of that. It is supposed that the periodic review is done at the beginning of every period t. Retailers know exactly their demands of the last period d_{t-1}. Their order-up-to point S_t is, thus, estimated, and the order q_t was sent to their manufacturer. The order vector under an order-up-to policy is given by the following equation:

$$q_t = S_t - S_{t-1} + d_{t-1}. \tag{3.14}$$

In equation (3.14), S_t is the order-up-to point vector of retailers at period t. It should be noted that in this policy, it is assumed that the customer will not accept out of stock during the term or at the end of the period. We postulate that the two retailers both have the same lead time L for orders, so the order-up-to point vector can be written as:

$$S_t = \hat{D}_t^L + z\hat{\sigma}_t^L. \tag{3.15}$$

In equation (3.15), $\hat{D}_t^L = \left(\hat{D}_{1,t}^L \; \hat{D}_{2,t}^L\right)'$ is the estimated vector of the mean demand of retailers during the lead-time which depends upon the demand forecasts of the next period and the lead time. $\hat{\sigma}_t^L = \left(\hat{\sigma}_{1,t}^L \; \hat{\sigma}_{2,t}^L\right)'$ represents the standard deviation of lead-time demand forecast error. $z = (z_1, z_2)'$ represents a constant vector chosen by retailers to meet the expected service level [7].

3.7. Forecasting technique

In this paper, certain forecasting methods are adopted to estimate these factors, which are significant for calculating the mean and variance of demand during delivery. We have proposed a scenario where the supply chain uses MMSE forecasting technology. Since the demand information of retailers is hard to access, we assume that retailers in each supply chain apply an order-up-to inventory policy. Supposing that retailers employ the same inventory strategy and forecasting techniques, we can explore the unique impact of price strategies on BE, ignoring the impact of different inventory strategies and forecasting methods BE between supply chains. Since both retailers use MMSE forecasting technology to predict lead time requirements, we can formulate that:

$$\hat{D}_t^L = \hat{d}_t + \hat{d}_{t+1} + \cdots + \hat{d}_{t+L-1} = \sum\nolimits_{i=0}^{L-1} \hat{d}_{t+i} \tag{3.16}$$

where $\hat{d}_{t+i}$ is the demand forecast of period $t+i$, and can be expressed as equation (3.17):

$$\hat{d}_{t+i} = E[d_{t+i}|d_{t-1}, d_{t-2}, \cdots]. \tag{3.17}$$

Based on the above supply chain model and operation mechanism, supply chain managers can start from price decision to complete demand prediction, to make orders, inventory and other businesses.

4. Quantifying BE in two parallel supply chains

The measure of BE for each supply chain will be derived by the MMSE forecasting method in this section.

4.1. Retailers' ordering decision

As a conventional strategy and for ease of understanding, the safety factor z can be assumed to be equal to 0. This assumption is insufficient to imply a low level of service, and the BE enables the retailer to increase the lead time to achieve the expected service level [6]. Combining equations (3.14) and (3.15), the order vector at the beginning of t period of the retailer can be expressed in the following form:

$$q_t = \hat{D}_t^L - \hat{D}_{t-1}^L + d_{t-1}. \tag{4.1}$$

Substituting equations (3.2) and (3.17) into (3.16), the estimated vector of the lead-time demand can be determined as

$$\hat{D}_t^L = \sum_{j=0}^{L-1} \left(a + B\hat{P}_{t+j}\right), \tag{4.2}$$

where, $\hat{P}_{t+j}$ is the retailers' price vector at period $t+j$, which is determined by the actual prices of retailers for period t. After several iterations of equation (3.5), $\hat{P}_{t+j}$ can be represented as

$$\hat{P}_{t+j} = \sum_{k=0}^{j} \mu\phi^k + \phi^{j+1} P_{t-1} + \sum_{k=0}^{j} \phi^k \xi_{t-k}.$$

Considering $E(\xi_{1,t}) = E(\xi_{2,t}) = 0$, then,

$$\hat{P}_{t+j} = \sum_{k=0}^{j} \mu\phi^k + \phi^{j+1} P_{t-1}. \tag{4.3}$$

Using (4.3) in (4.2), we can derive the demand forecast vector in the lead-time:

$$\hat{D}_t^L = \sum_{j=0}^{L-1} \left(a + B\sum_{k=0}^{j} \mu\phi^k + B\phi^{j+1} P_{t-1}\right). \tag{4.4}$$

Consequently, the retailers' order vector determined by P_{t-1} and P_{t-2} can be derived in equation (4.5).

$$q_t = \sum_{j=0}^{L-1} \hat{d}_{t+j} - \sum_{j=0}^{L-1} \hat{d}_{t+j-1} + d_{t-1} = \sum_{j=0}^{L-1} B\left(\hat{P}_{t+j} - \hat{P}_{t+j-1}\right) + d_{t-1} = \sum_{j=0}^{L-1} B\phi^{j+1}(P_{t-1} - P_{t-2}) + d_{t-1}. \tag{4.5}$$

Proposition 4.1. *In two parallel supply chains with price-sensitive demand, if the retailers use VAR(1) price strategy and order-up-to inventory strategy, the order quantity of period t under the MMSE forecasting method from retailer i $(q_{i,t}, i=1,2)$ can be given as:*

$$q_t^i = h_{i1}\left(p_{t-1}^1 - p_{t-2}^1\right) + h_{i2}\left(p_{t-1}^2 - p_{t-2}^2\right) + d_{t-1}^i, (i=1,2) \tag{4.6}$$

where,

$$(h_{ij})_{2\times 2} = \begin{pmatrix} \frac{-b_{11}(\tau_1(\theta-\phi_{11}+\phi_{22})+\tau_2(\theta+\phi_{11}-\phi_{22}))-2\phi_{21}b_{12}(\tau_1-\tau_2)}{2\theta} & \frac{b_{12}(\tau_1(\theta+\phi_{11}-\phi_{22})+\tau_2(\theta-\phi_{11}+\phi_{22}))+2\phi_{12}b_{11}(\tau_1-\tau_2)}{2\theta} \\ \frac{b_{21}(\tau_1(\theta-\phi_{11}+\phi_{22})+\tau_2(\theta+\phi_{11}-\phi_{22}))+2\phi_{21}b_{22}(\tau_1-\tau_2)}{2\theta} & \frac{b_{12}(\tau_1(\theta+\phi_{11}-\phi_{22})+\tau_2(\theta-\phi_{11}+\phi_{22}))+2\phi_{12}b_{11}(\tau_1-\tau_2)}{2\theta} \end{pmatrix}$$

and $\theta = \sqrt{\phi_{11}^2 + 4\phi_{12}\phi_{21} - 2\phi_{11}\phi_{22} + \phi_{22}^2}$, $\tau_i = \sum_{j=0}^{L-1} \lambda_i^{j+1} = \frac{\lambda_i\left(-1+\lambda_i^L\right)}{-1+\lambda_i}, (i=1,2)$.

Proof. See Appendix A. □

4.2. BE for each supply chain

According to Chen *et al.* [6], the metric of the BE for each supply chain can be calculated by equation (4.7). When the ratio exceeds 1, there is a BE. In the case of a univariate, the BE metric in equation (4.7) is naturally defined as the ratio of the order variation $\operatorname{Var}\left(q_t^i\right)$ to the future demand variable $\operatorname{Var}\left(d_t^i\right)$.

$$\mathrm{BE}_i = \frac{\operatorname{Var}\left(q_t^i\right)}{\operatorname{Var}\left(d_t^i\right)}, (i=1,2). \tag{4.7}$$

Substituting equations (3.1) and (4.6) into equation (4.7), respectively, the BE for each supply chain can be derived from a series of derivation, its expression is given in Theorem 4.2.

Theorem 4.2. *In two parallel supply chains with price-sensitive demand, if the retailers use VAR(1) price strategy and the order-up-to inventory policy, under the MMSE forecasting method, the BE expression in supply chain $i(i=1,2)$ can be expressed as the following:*

$$\mathrm{BE}_i = 1 + \frac{S_1\delta_1^2 + S_2\delta_2^2 + S_3\delta_{12}}{\operatorname{var}(d_{i,t})} \tag{4.8}$$

here, $S_1 = 2\Big((-1)^i(b_{21}(1-\phi_{11}) + \phi_{21}b_{i2})h_{i1} + (1-\phi_{11})h_{i1}^2 - \phi_{21}h_{i1}h_{i2}\Big)$,

$$S_2 = 2\Big((-1)^{i-1}(\phi_{12}b_{21} + (1-\phi_{22})b_{22})h_{i2} - \phi_{12}h_{i1}h_{i2} + (1-\phi_{22})h_{i2}^2\Big),$$

$$\begin{aligned} S_3 = 2\Big(&(-1)^{i-1}((\phi_{12}b_{i1} + (1-\phi_{22})b_{i2})h_{i1} - ((1-\phi_{11})b_{i1} + \phi_{21}b_{i2})h_{i2}) \\ &+(2-\phi_{11}-\phi_{22})h_{i1}h_{i2} - \phi_{21}h_{i2}^2 - \phi_{12}h_{i1}^2\Big). \end{aligned}$$

Proof. See Appendix B. □

Spreading out the formula of the BE, we can find that the BE_I $(I=1,2)$ is a function of ϕ_{ij} $(i,j=1,2)$, b_{ij} $(i=I, j=1,2)$, σ_i and L. Therefore, we can conclude that the BE is affected by price sensitivity and price setting of both parties, its pricing factors, and the impact of two supply chains' lead time.

5. The impact of the pricing strategy and lead time on BE

After deriving the bullwhip effect for each retailer in the dual supply chain, we are now turning to explore its properties, focusing on the fixed demand model. The following sections analyze the impact of lead time, pricing strategy, and product substitution on the BE by numerical simulation on the basis of equation (4.8). Without loss of generality, we take the first retailer as an example to explore how the pricing strategy, lead time, and product substitution impact the BE. For simplicity, $\sigma_1^2 = 1, \sigma_2^2 = 1$ are set in the following sections, while the relevant literature gives the same value of σ_i^2 $(i=1,2)$ [20].

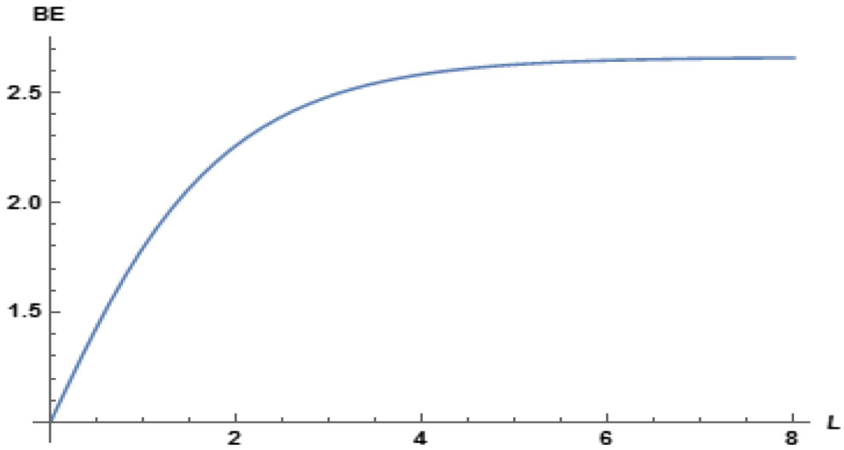


FIGURE 3. BE_1 increases as the growth of the lead time.

5.1. BE affected by the lead time

For a single supply chain, the crucial measure for reducing the BE is to reduce the lead time. The effect of lead time on BE in the dual parallel supply chain will be studied in this section. We set the fixed pricing strategy with parameters $\phi = \begin{pmatrix} 0.2 & 0.3 \\ 0.3 & 0.2 \end{pmatrix}$ in the stable region (According to equation (3.7), $(0.2, 0.3, 0.3, 0.2) \in E$).

Figure 3 shows that the BE_1 increases with the growth of the lead time for a fixed price strategy. As L increases from 0 to 2, the BE varies markedly. When L is larger than 2, the BE grows smoothly. As L grows larger than 3, the BE slightly augments until L takes the value of 8. To further explore the effect of the interaction between lead time and pricing strategy on pricing, we conducted a new set of simulation experiments. We vary the parameters $L \in \{1, 2, 3, 8\}$, as $\{1, 2, 3, 8\}$ are inflection point of L to affect the value of BE, when $\phi_{21} = 0.4$, $\phi_{22} = 0.5$, ϕ_{11} and ϕ_{12} are considered in the stable region (determined by equation (3.7)).

Figure 4 shows the BE_1 affected by the interaction of pricing strategy in different lead time, where $b_{11} = 1$ and $b_{12} = 1.5$ respectively. As depicted in Figure 4, the color surface is the BE of retailer 1 and the green plane is the reference plane which equals to one. The increase of lead time will not only lead to an increase of the BE with the same pricing strategy but also gradually reduce the area without the BE until it disappears, and invalidate the effective pricing strategy. From the perspective of management, shortening the delivery time can provide retailers with a more diversified pricing mechanism and enable retailers to maintain a lower BE or even eliminate the BE.

5.2. BE affected by own pricing strategy and product substitution

First, we consider a basic scenario where the first retailer uses a pricing factor as a variable while his competitors adopt a certain balancing strategy. We assume that the products in the two supply chains can be entirely replaced with each other. Therefore, we set parameters $\phi_{21} = 0.5, \phi_{22} = 0.5, b_{11} = 1, b_{12} = 1, b_{21} = 1, b_{22} = 1, L = 2$, in the following sections, unless otherwise specified. We calculate the BE of retailer1 as ϕ_{11}, varying from 0 to 1, and ϕ_{12}, varying from 0 to 1, which are in the stable region (determined by equation (3.6)). We get to know some properties about the change of the BE_1 affected by our pricing strategy *via* the 3D plot. To determine the existence of the BE, we will plot a green plane where the BE is equal to 1 as a criterion of judgment.

Figure 5(a) shows the BE_1 affected by its pricing factor under the previous condition. We can see that the increase in the weight of the competitor's price factor may reduce or even eliminate the BE of its first supply chain. Note that this finding is limited to a determinative scenario where the price sensitivity coefficients of the

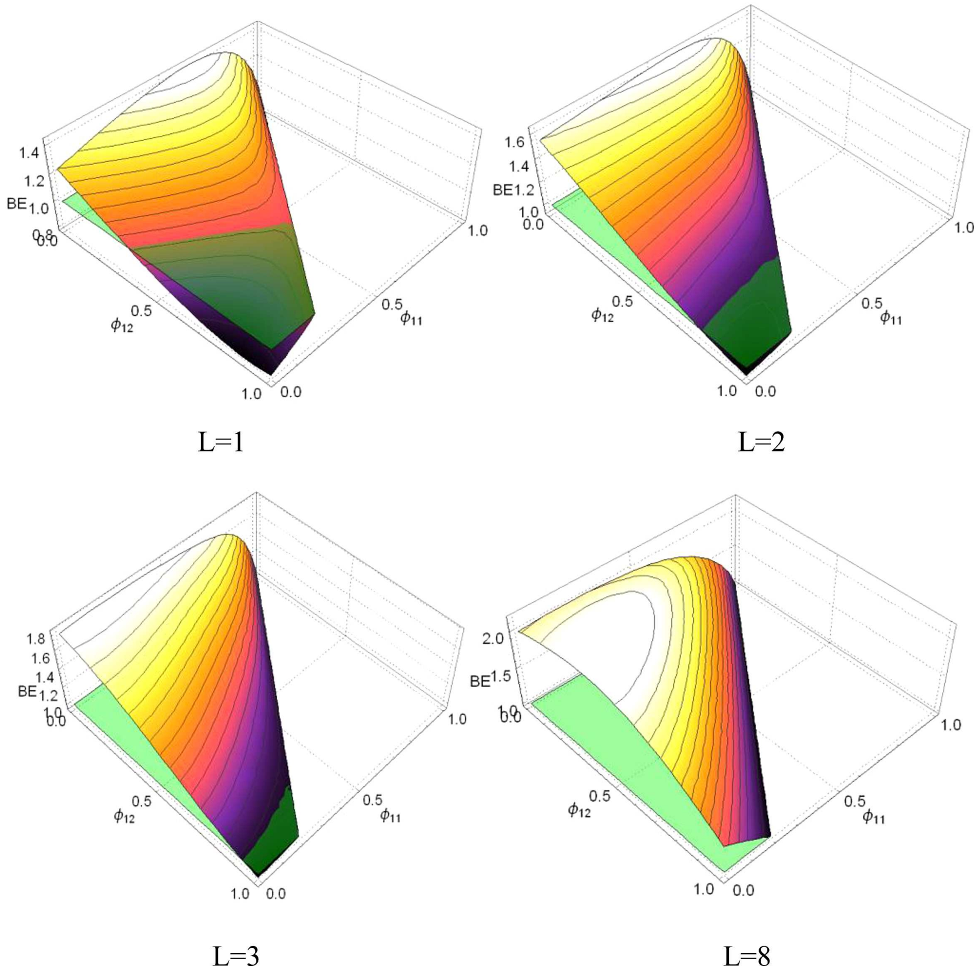


FIGURE 4. 3D color surface figure of BE_1 affected by the interaction of pricing strategy in different lead time.

two products have the same value in the first demand model, indicating that retailers will achieve better supply chain management if they pay more attention to competitors. The performance price is better than the price of the product when the products are consubstantial $b_{12} = b_{21}$.

After that, we consider another scenario in which a competitive product has a higher degree of substitution. We investigate the BE affected by $\phi_{1j}, j = 1, 2$ when $b_{12} = 2$. In Figure 5(b), a larger BE is found regardless of the value of the own price-setting coefficients, and the region without BE disappears in contrast to $b_{12} = 1$. As the parameter ϕ_{1j} grows from 0 to 1, the BE gradually decreases to 1. This means the BE will be eliminated.

Therefore, we can conclude that a higher degree of substitution of competitive products (b_{12} in this case), may lead to greater BE. Also, increasing the price weight of competitors in the decision-making process is an effective method to reduce BE in the alternative supply chain.

5.3. BE affected by the opponent's pricing strategy

Observing the expression of the BE_1, we can find that the BE of the supply chain is not only affected by its pricing strategy but also affected by the opponent's pricing strategy. We set the value of $(\phi_{21}, \phi_{22}) = (0.2, 0.8)$,

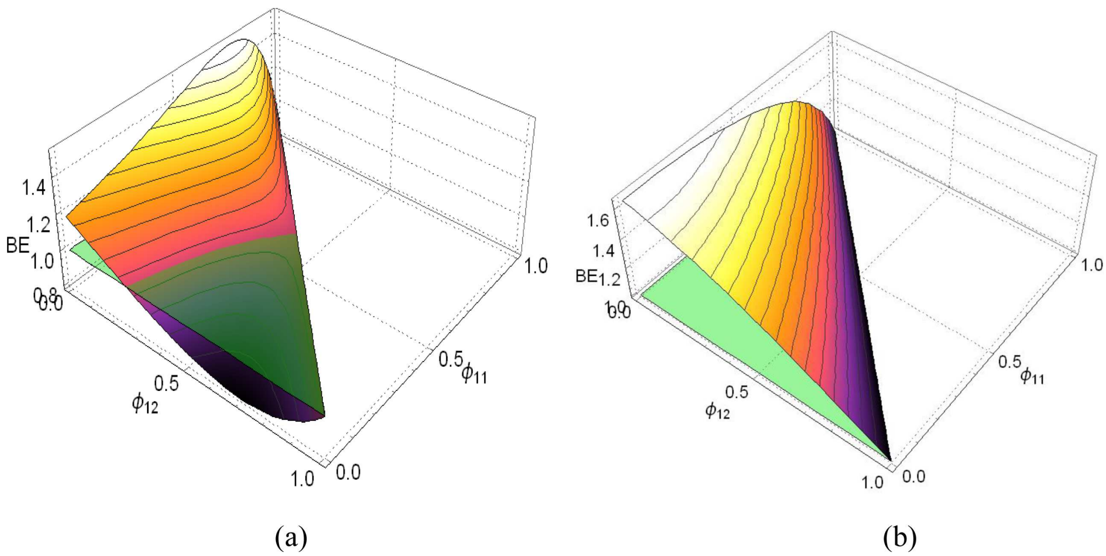


FIGURE 5. The 3D plot of BE_1 affected by their pricing-making coefficients.

(0.8, 0.2), (0.2, 0.2), and (0.8, 0.8) to investigate the impact of opponent's pricing strategy on BE in the stable region (determined by equation (3.6)), as shown in Figure 6.

Figure 6(a) shows the state in which the second retailer severely adopts the autocorrelation price coefficient, but the price coefficient associated with each other is slight. In this state, the first supply chain suffers more BE than the preceding situation illustrated by Figure 5(a). Therefore, a large self-correlation price coefficient and a small mutual-correlation price coefficient will make the opponent face a larger BE.

Figure 6(b) shows a state where the second retailer slightly adopts the self-correlation price coefficient, but the mutual-correlation price coefficient is larger. As shown, in most stable regions, the BE of the first supply chain is less than 1, *i.e.* BE disappears, so a minor self-correlation price coefficient and a larger mutual-correlation price coefficient will reduce or even eliminate competitor BE.

Figure 6(c) shows a scenario where the self-correlation price coefficient and the mutual-correlation price coefficient are both evaluated at a low level by the second retailer. From the graph, we can find that the stable region is expanded compared to Figure 5(a).

Figure 6(d) shows a state where the self-correlation price coefficient and the mutual-correlation price coefficient are both evaluated at a high level by the second retailer. As can be seen from the graph, the stable region reduces compared with Figure 5(a).

In this section, we examine how the competitors' price strategies influence the BE and the price strategy for R_1. Generally, the rival price strategy with a minor self-correlation price coefficient and a large mutual-correlation price coefficient is likely to enable the first supply chain to decrease or even eliminate the BE. Conversely, the focal supply chain may suffer a much more BE. Still, when the supply chain makes price decisions, it can mitigate its BE by increasing the mutual-correlation coefficient.

5.4. The impact of the self-correlation coefficient in the price-setting process

In this part, we investigate how the self-correlation coefficients affect the BE in the price-setting process when the mutual-correlation coefficients are determined.

As illustrated in Figure 7, when the mutual-correlation coefficients keep $\phi_{12} = \phi_{21} = 0.5$ for homogeneous products ($b_{12} = b_{21} = 1$), the simultaneous low self-correlation coefficients of both sides will decrease or even

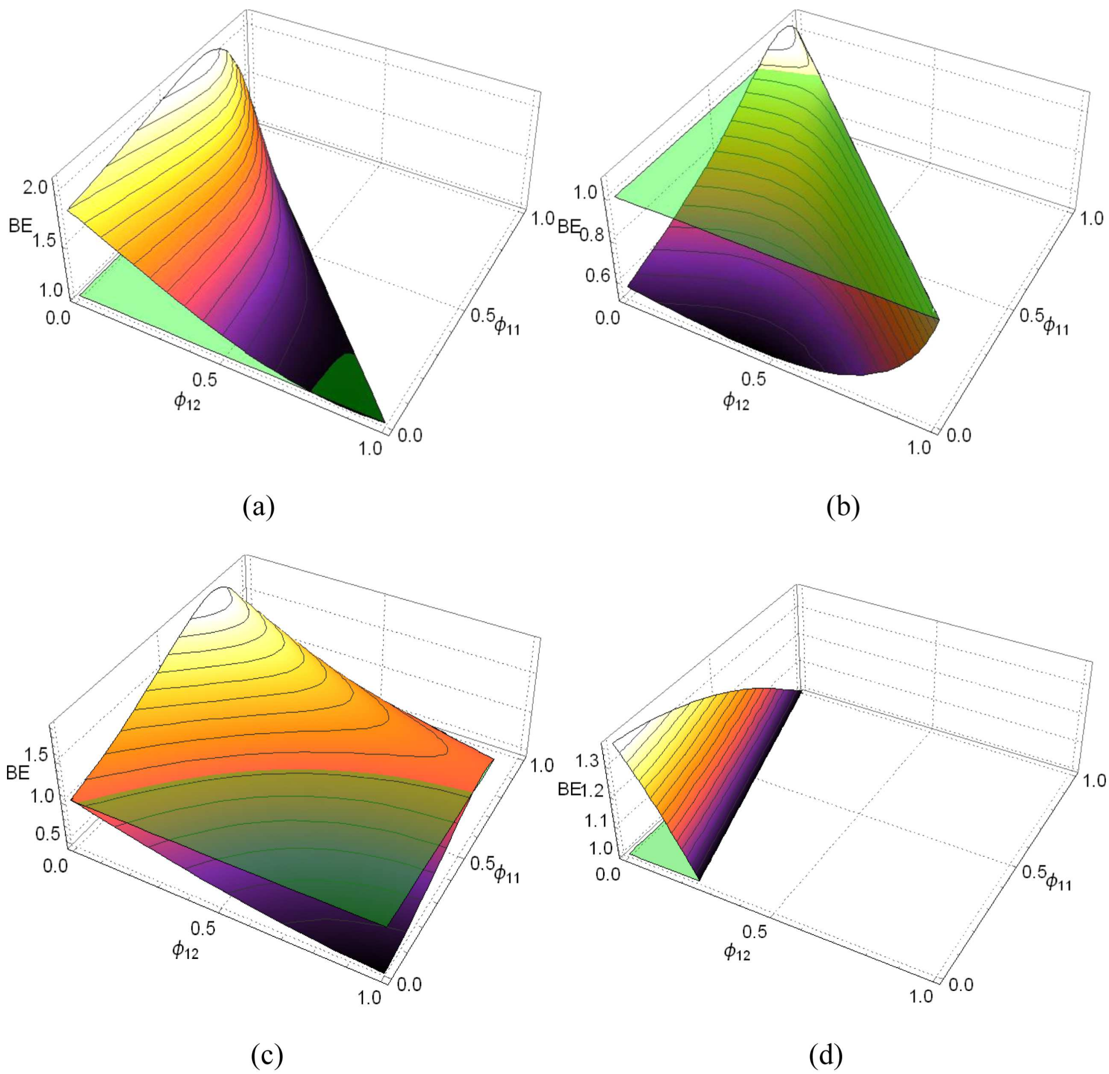


FIGURE 6. The 3D plot of BE_1 affected by the rival pricing-making coefficients. (a) $\phi_{21} = 0.2, \phi_{22} = 0.8$. (b) $\phi_{21} = 0.8, \phi_{22} = 0.2$. (c) $\phi_{21} = 0.2, \phi_{22} = 0.2$. (d) $\phi_{21} = 0.8, \phi_{22} = 0.8$.

eliminate the BE. The following simulations display how the self-correlation coefficient of substitution with different degree affects the BE in the first supply chain in the stable region (determined by equation (3.6)).

We set b_{12} as 2 and 0.5, respectively, in the two different simulations, and the 3D graphs of BE affected by the self-correlation coefficients are shown in Figure 8. As shown in Figure 8(a), the reduction of ϕ_{22} is also effective to prevent the BE of the first supply chain while the variation of ϕ_{11} merely attenuate BE. As a contrast, we can notice, from Figure 8(b), that the decrease in ϕ_{11} can dramatically alleviate BE while the alteration in ϕ_{22} has done little to reduce the BE.

Therefore, when the mutual-correlation coefficients are fixed, the fluctuation of the self-correlation coefficient scarcely impacts the BE; the reduction of the rival self-correlation coefficient can decrease or even eliminate the BE when the rival product is of high substitutability. Contrary to the previous scenario, the change of the rival self-correlation coefficient hardly influences the own BE; the reduction of the own self-correlation coefficient can

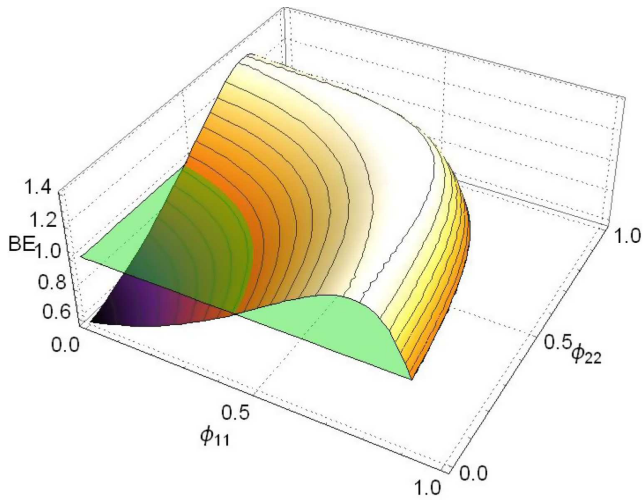


FIGURE 7. The 3D plot of BE affected by the self-correlation coefficient (BE).

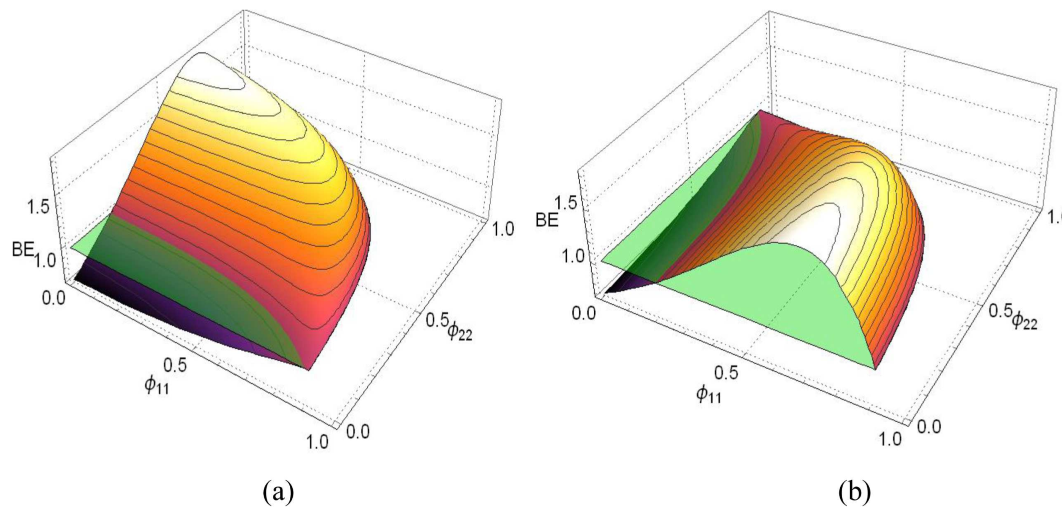


FIGURE 8. The 3D plot of BE affected by the self-correlation coefficient. (a) $b_{12} = 2$. (b) $b_{12} = 0.5$.

decrease or even eliminate the BE when the rival product is of low substitutability. In other words, the degree of product substitutability determines whether the own self-correlation or the rival self-correlation coefficient can reduce the BE.

5.5. The impact of the mutual-correlation coefficient in the price-setting process

In this part, we will probe into the impact of the mutual-correlation coefficients in the price-setting process on the BE when the self-correlation coefficients are determined.

As shown in Figure 9, when the self-correlation coefficients keep $\phi_{11} = \phi_{22} = 0.5$ for consubstantial products ($b_{12} = b_{21} = 1$), any mutual-correlation coefficient of both sides has a negative correlation with the BE. Any supply chain that increases the number of mutual-correlation coefficient in the price-setting process will alleviate BE.

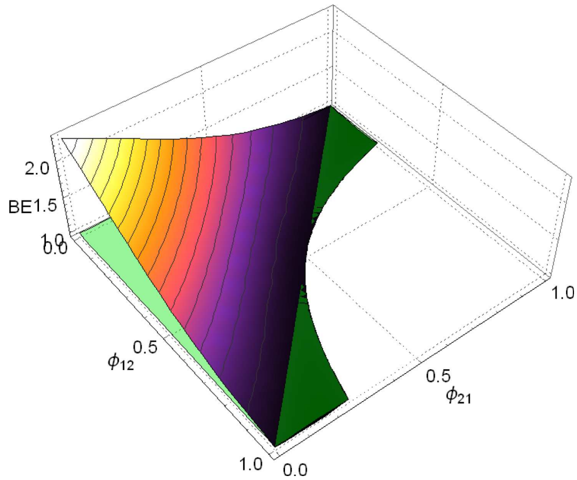
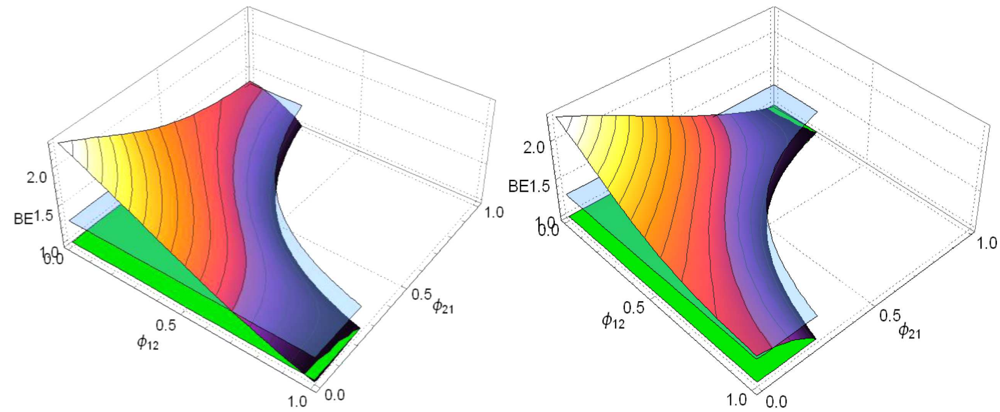


FIGURE 9. The 3D plot of BE affected by the mutual-correlation coefficient.



(a) (b)

FIGURE 10. The 3D plot of BE affected by the mutual-correlation coefficient. (a) $b_{12} = 2$. (b) $b_{12} = 0.5$.

The following simulations illustrate how the BE varies with the variation of mutual-correlation concerning different substitute levels in the stable region (determined by equation (3.6)).

We assign b_{12} to 2 and 0.5 respectively in two different simulations and plot the 3D of the BE affected by the autocorrelation coefficient in Figure 10. A blue plane was plotted where BE is equal to 1.3 as another criterion. As shown by Figure 10(a), compared with ϕ_{21}, the increase of ϕ_{12} has a bigger effect on preserving the BE of the first supply chain. In contrast, Figure 10(b) also shows that the increase in ϕ_{12} scarcely prevents the BE while ϕ_{21} plays an important role.

5.6. Managerial insights

From the above analysis, some managerial insights can be proposed for the supply chain manager. Generally, the rival price strategy with a minor self-correlation price coefficient and a large mutual-correlation price coefficient is likely to enable the first supply chain to decrease or even eliminate the BE. Conversely, the focal supply chain may suffer a much more BE. Still, when the supply chain makes price decisions, it can mitigate its BE by increasing the mutual-correlation coefficient.

6. Conclusions

In this paper, we study the pricing strategy and substitution effect in a competitive environment where two supply chains provide similar products. A two parallel supply chains model was developed to investigate the pricing strategy and substitution effect on the BE. Also, we make it clear that how mutual-correlation and substitution coefficient influences the BE. Consumers' price-sensitive requirement is also considered to determine the characteristic of the BE. Intuitively, each retailer's pricing decision, which we use to characterize price strategy, is affected not only by its previous price but also by that of its rival. We present the analytical metrics of the BE for each supply chain in the VAR(1) price decision process by statistical derivation. From the expression of the BE we induced, we find that differences in price strategy and product substitution may increase or inhibit the BE in the supply chain of different alternative products.

Our results generate several insights into controlling the BE for accomplishing better supply chain performance. Our conclusions are listed as follows:

(1) The lead time somewhat attenuates the positive effect of the pricing strategy. Especially, when supply chain members apply the same pricing strategy, the more lead time will lead to the more BE. What' more, the increase in the lead time will gradually reduce the range without the BE. In other words, the increase in the lead time to some extent makes the price strategy useless. From the perspective of management, shortening the delivery time makes it possible for retailers to reduce or even eliminate the BE by adopting a more diversified pricing mechanism.

(2) As illustrated in our analyses, the BE seems more serious and vital if the own products are more similar to the competitor's products. Thus, more emphasis should be paid to the price-making process when its own products are more easily replaced by others. In terms of price decisions, increasing the weight of competitors' previous prices in the decision-making process is an effective method to reduce the BE in two parallel supply chains with alternative products.

(3) Due to the existence of interaction, the competitor's pricing strategy will also affect the BE of the focus supply chain. The rival's pricing strategy with a minor self-correlation price coefficient and a large mutual-correlation price coefficient is likely to decrease or even eliminate the BE of the focal supply chain. If the rival's pricing strategy is the opposite, the focal supply chain will, however, suffer a much more BE. Nevertheless, the focal supply chain can also mitigate its BE by increasing the mutual-correlation coefficient in its price-setting process.

(4) In a word, adopting a higher mutual-correlation coefficient is an effective means of easing the supply chain BE for focal firms in two parallel supply chains with alternative products. In particular, the mechanism works better for the focal supply chain when competing products are rather similar, or in other words, the degree of substitution is high.

As we select a simple VAR(1) process only, there are some limitations in the current model. In practice, retailers will also adopt some other pricing strategies to ensure the optimal profit rather than choosing a simple VAR(1) process only. Also, we focus on two parallel substitute product supply chains. However, the number of substitute products is more than two in reality and their interaction is more complex. Therefore, future research could be devoted to exploring the impact of a more practical pricing strategy on the BE, as well as the relationship between the BE and the profit of supply chains. Due to a lack of data, empirical analyses were not carried out in our paper. If possible, the combination of empirical analysis and theoretical analysis could be considered in the future.

Appendix A. Proof of Proposition 4.1

The eigenvalues of the matrix ϕ are $\lambda_1 = \frac{1}{2}\left(\phi_{11} + \phi_{22} - \sqrt{(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21}}\right)$ and $\lambda_2 = \frac{1}{2}\left(\phi_{11} + \phi_{22} + \sqrt{(\phi_{11} - \phi_{22})^2 + 4\phi_{12}\phi_{21}}\right)$, $(\lambda_1 \neq \lambda_2)$. The matrix ϕ is not only invertible, but also can bullwhip effect diagonalizable in the stable region, so there is an invertible matrix T which can meet $\phi = T\Lambda T^{-1}$, here Λ is a diagonalizable matrix $\Lambda = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}$.

$$T = \begin{pmatrix} -\frac{-\phi_{11}+\phi_{22}+\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}}{2\phi_{21}} & -\frac{-\phi_{11}+\phi_{22}-\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}}{2\phi_{21}} \\ 1 & 1 \end{pmatrix} \tag{A.1}$$

$$T^{-1} = \begin{pmatrix} -\frac{\phi_{21}}{\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}} & -\frac{-\phi_{11}+\phi_{22}-\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}}{2\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}} \\ \frac{\phi_{21}}{\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}} & \frac{-\phi_{11}+\phi_{22}+\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}}{2\sqrt{\phi_{11}^2+4\phi_{12}\phi_{21}-2\phi_{11}\phi_{22}+\phi_{22}^2}} \end{pmatrix}$$

therefore,

$$\phi^{j+1} = T(\Lambda)^{j+1}T^{-1} = T\begin{pmatrix} \lambda_1^{j+1} & 0 \\ 0 & \lambda_2^{j+1} \end{pmatrix}T^{-1} \tag{A.2}$$

$$\begin{aligned} q_t &= \int_{j=0}^{L-1} BT\begin{pmatrix} \lambda_1^{j+1} & 0 \\ 0 & \lambda_2^{j+1} \end{pmatrix}T^{-1}(P_{t-1}-P_{t-2}) + d_{t-1} \\ &= BT\left(\int_{j=0}^{L-1}\begin{pmatrix} \lambda_1^{j+1} & 0 \\ 0 & \lambda_2^{j+1} \end{pmatrix}\right)T^{-1}(P_{t-1}-P_{t-2}) + d_{t-1} \\ &= BT\begin{pmatrix} \int_{j=0}^{L-1}\lambda_1^{j+1} & 0 \\ 0 & \int_{j=0}^{L-1}\lambda_2^{j+1} \end{pmatrix}T^{-1}(P_{t-1}-P_{t-2}) + d_{t-1} = H(P_{t-1}-P_{t-2}) + d_{t-1} \end{aligned} \tag{A.3}$$

$$H = \begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix} = BT\begin{pmatrix} \int_{j=0}^{L-1}\lambda_1^{j+1} & 0 \\ 0 & \int_{j=0}^{L-1}\lambda_2^{j+1} \end{pmatrix}T^{-1} = BT\begin{pmatrix} \tau_1 & 0 \\ 0 & \tau_2 \end{pmatrix}T^{-1} \tag{A.4}$$

where $\tau_1 = \int_{j=0}^{L-1}\lambda_1^{j+1} = \frac{\lambda_1(-1+\lambda_1^L)}{-1+\lambda_1}, \tau_2 = \int_{j=0}^{L-1}\lambda_2^{j+1} = \frac{\lambda_2(-1+\lambda_2^L)}{-1+\lambda_2}$,

$$H = \begin{pmatrix} \frac{-b_{11}(\tau_1(\theta-\phi_{11}+\phi_{22})+\tau_2(\theta+\phi_{11}-\phi_{22}))-2\phi_{21}b_{12}(\tau_1-\tau_2)}{2\theta} & \frac{b_{12}(\tau_1(\theta+\phi_{11}-\phi_{22})+\tau_2(\theta-\phi_{11}+\phi_{22}))+2\phi_{12}b_{11}(\tau_1-\tau_2)}{2\theta} \\ \frac{b_{21}(\tau_1(\theta-\phi_{11}+\phi_{22})+\tau_2(\theta+\phi_{11}-\phi_{22}))+2\phi_{21}b_{22}(\tau_1-\tau_2)}{2\theta} & \frac{b_{12}(\tau_1(\theta+\phi_{11}-\phi_{22})+\tau_2(\theta-\phi_{11}+\phi_{22}))+2\phi_{12}b_{11}(\tau_1-\tau_2)}{2\theta} \end{pmatrix} \tag{A.5}$$

here $\theta = \sqrt{\phi_{11}^2 + 4\phi_{12}\phi_{21} - 2\phi_{11}\phi_{22} + \phi_{22}^2}$, so, we can obtain the ordering decision of retailers.

$$\begin{pmatrix} q_t^1 \\ q_t^2 \end{pmatrix} = \begin{pmatrix} h_{11}(p_{t-1}^1 - p_{t-2}^1) + h_{12}(p_{t-1}^2 - p_{t-2}^2) + d_{t-1}^1 \\ h_{21}(p_{t-1}^1 - p_{t-2}^1) + h_{22}(p_{t-1}^2 - p_{t-2}^2) + d_{t-2}^2 \end{pmatrix}. \tag{A.6}$$

The ordering decision of each supply chain can be written as:

$$q_t^i = h_{i1}(p_{t-1}^1 - p_{t-2}^1) + h_{i2}(p_{t-1}^2 - p_{t-2}^2) + d_{t-1}^i, \quad i = 1, 2. \tag{A.7}$$

This completes the proof for Proposition 4.1. □

Appendix B. Proof of Theorem 4.1

According to equation (3.16), we can get the order of retailer i.

$$q_t^i = h_{i1}(p_{t-1}^1 - p_{t-2}^1) + h_{i2}(p_{t-1}^2 - p_{t-2}^2) + d_{t-1}^i, \quad (i = 1, 2). \tag{B.1}$$

Firstly, we give the proof of the expression of BE_1 for the first supply chain. Based on the statistical properties, the order quantity variance of retailer 1 can be written as:

$$\begin{aligned}\operatorname{var}\left(q_t^1\right) = {} & h_{11}^2\operatorname{var}\left(p_{t-1}^1 - p_{t-2}^1\right) + h_{12}^2\operatorname{var}\left(p_{t-1}^2 - p_{t-2}^2\right) + \operatorname{var}\left(d_{t-1}^1\right) + 2h_{11}\operatorname{cov}\left(\left(p_{t-1}^1 - p_{t-2}^1\right), d_{t-1}^1\right) \\ & + 2h_{12}\operatorname{cov}\left(\left(p_{t-1}^2 - p_{t-2}^2\right), d_{t-1}^1\right) + 2h_{11}h_{12}\operatorname{cov}\left(\left(p_{t-1}^1 - p_{t-2}^1\right), \left(p_{t-1}^2 - p_{t-2}^2\right)\right)\end{aligned} \tag{B.2}$$

where

$$\operatorname{var}\left(p_{t-1}^1 - p_{t-2}^1\right) = \operatorname{var}(p_{t-1}^1) + \operatorname{var}(p_{t-2}^1) - 2\operatorname{cov}\left(p_{t-1}^1, p_{t-2}^1\right) = 2\delta_1^2 - 2\operatorname{cov}\left(p_{t-1}^1, p_{t-2}^1\right) \tag{B.3}$$

$$\operatorname{var}\left(p_{t-1}^2 - p_{t-2}^2\right) = \operatorname{var}(p_{t-1}^2) + \operatorname{var}(p_{t-2}^2) - 2\operatorname{cov}\left(p_{t-1}^2, p_{t-2}^2\right) = 2\delta_2^2 - 2\operatorname{cov}\left(p_{t-1}^2, p_{t-2}^2\right). \tag{B.4}$$

Due to equation (3.3),

$$\operatorname{cov}\left(p_{t-1}^1, p_{t-2}^1\right) = \phi_{11}\delta_1^2 + \phi_{12}\operatorname{cov}\left(p_t^1, p_t^2\right) = \phi_{11}\delta_1^2 + \phi_{12}\delta_{12} \tag{B.5}$$

$$\operatorname{cov}\left(p_{t-1}^2, p_{t-2}^2\right) = \phi_{22}\delta_2^2 + \phi_{21}\operatorname{cov}\left(p_t^1, p_t^2\right) = \phi_{22}\delta_2^2 + \phi_{21}\delta_{12}. \tag{B.6}$$

Substituting equations (B.5) and (B.6) into equations (B.3) and (B.4), we obtain the variance terms in (B.2):

$$\operatorname{var}\left(p_{t-1}^1, p_{t-2}^1\right) = 2(1-\phi_{11})\delta_1^2 - 2\phi_{12}\delta_{12} \tag{B.7}$$

$$\operatorname{var}\left(p_{t-1}^2, p_{t-2}^2\right) = 2(1-\phi_{22})\delta_2^2 + 2\phi_{21}\delta_{12}. \tag{B.8}$$

Due to equations (3.1) and (3.3), the covariance terms in (B.2) can be written as:

$$\operatorname{cov}\left(p_{t-2}^1, p_{t-1}^2\right) = \phi_{21}\delta_1^2 + \phi_{22}\delta_{12} \tag{B.9}$$

$$\operatorname{cov}\left(p_{t-1}^1, p_{t-2}^2\right) = \phi_{12}\delta_2^2 + \phi_{11}\delta_{12} \tag{B.10}$$

$$\begin{aligned}\operatorname{cov}\left(p_{t-1}^1 - p_{t-2}^1, p_{t-1}^2 - p_{t-2}^2\right) &= \operatorname{cov}\left(p_{t-1}^1, p_{t-1}^2\right) + \operatorname{cov}\left(p_{t-2}^1, p_{t-2}^2\right) - \operatorname{cov}\left(p_{t-2}^1, p_{t-1}^2\right) - \operatorname{cov}\left(p_{t-1}^1, p_{t-2}^2\right) \\ &= 2\operatorname{cov}\left(p_t^1, p_t^2\right) - \operatorname{cov}\left(p_{t-2}^1, \phi_{21}p_{t-2}^1 + \phi_{22}p_{t-2}^2\right) - \operatorname{cov}\left(\phi_{11}p_{t-2}^1 + \phi_{12}p_{t-2}^2, p_{t-2}^2\right) \\ &= -\left(\phi_{21}\delta_1^2 + \phi_{12}\delta_2^2\right) + (2 - (\phi_{11} + \phi_{22}))\delta_{12}\end{aligned} \tag{B.11}$$

$$\begin{aligned}\operatorname{cov}\left(\left(p_{t-1}^1 - p_{t-2}^1\right), d_{t-1}^1\right) &= \operatorname{cov}\left(\left(p_{t-1}^1 - p_{t-2}^1\right), \left(a - b_{11}p_{t-1}^1 + b_{12}p_{t-1}^2\right)\right) \\ &= -b_{11}\delta_1^2 + b_{11}\operatorname{cov}\left(p_{t-2}^1, p_{t-1}^1\right) + b_{12}\operatorname{cov}\left(p_{t-1}^1, p_{t-1}^2\right) - b_{12}\operatorname{cov}\left(p_{t-2}^1, p_{t-1}^2\right) \\ &= -b_{11}\delta_1^2 + b_{11}\left(\phi_{11}\delta_1^2 + \phi_{12}\operatorname{cov}\left(p_t^1, p_t^2\right)\right) + b_{12}\delta_{12} - b_{12}\left(\phi_{21}\delta_1^2 + \phi_{22}\operatorname{cov}\left(p_t^1, p_t^2\right)\right) \\ &= -(b_{11}(1-\phi_{11}) + b_{12}\phi_{21})\delta_1^2 + (b_{12}(1-\phi_{22}) + b_{11}\phi_{12})\delta_{12}\end{aligned} \tag{B.12}$$

$$\begin{aligned}\operatorname{cov}\left(\left(p_{t-1}^2 - p_{t-2}^2\right), d_{t-1}^1\right) &= \operatorname{cov}\left(\left(p_{t-1}^2 - p_{t-2}^2\right), \left(a - b_{11}p_{t-1}^1 + b_{12}(p_{t-1}^2)\right)\right) \\ &= b_{12}\delta_2^2 - b_{11}\operatorname{cov}\left(p_{t-1}^2, p_{t-1}^1\right) + b_{11}\operatorname{cov}\left(p_{t-1}^1, p_{t-2}^2\right) - b_{12}\operatorname{cov}\left(p_{t-2}^2, p_{t-1}^2\right) \\ &= b_{12}\delta_2^2 - b_{11}\delta_{12} + b_{11}\left(\phi_{12}\delta_2^2 + \phi_{11}\operatorname{cov}\left(p_t^1, p_t^2\right)\right) - b_{12}\left(\phi_{22}\delta_2^2 + \phi_{21}\operatorname{cov}\left(p_t^1, p_t^2\right)\right) \\ &= (b_{12}(1-\phi_{22}) + b_{11}\phi_{12})\delta_2^2 - (b_{11}(1-\phi_{11}) + b_{12}\phi_{21})\delta_{12}\end{aligned} \tag{B.13}$$

Substituting (B.7), (B.8), and (B.11)–(B.13) into (B.2), the order quantity variance of retailer 1 can be derived.

$$\begin{aligned}\operatorname{var}\left(q_t^1\right) = {} & \operatorname{var}\left(d_t^1\right) + 2((\phi_{12}b_{11} + (1-\phi_{22})b_{12})h_{11} - ((1-\phi_{11})b_{11} + \phi_{21}b_{12})h_{12} \\ & + (2 - \phi_{11} - \phi_{22})h_{11}h_{12} - \phi_{12}h_{11}^2 - \phi_{21}h_{12}^2\big)\delta_{12} \\ & + 2\big((-(1-\phi_{11})b_{11} - \phi_{21}b_{12})h_{11} + (1-\phi_{11})h_{11}^2 - \phi_{21}h_{11}h_{12}\big)\delta_1^2 \\ & + 2\big((\phi_{12}b_{11} + (1-\phi_{22})b_{12})h_{12} - \phi_{12}h_{11}h_{12} + (1-\phi_{22})h_{12}^2\big)\delta_2^2\end{aligned} \tag{B.14}$$

$$\begin{aligned}\mathrm{BE}_1 &= \frac{\operatorname{var}(q_t^1)}{\operatorname{var}(d_t^1)} \\ &= 1 + (2((\phi_{12}b_{11} + (1-\phi_{22})b_{12})h_{11} - ((1-\phi_{11})b_{11} + \phi_{21}b_{12})h_{12}\end{aligned}$$

$$+ (2-\phi_{11}-\phi_{22})h_{11}h_{12} - \phi_{12}h_{11}^2 - \phi_{21}h_{12}^2\big)\delta_{12}$$
$$+ 2\big((-(1-\phi_{11})b_{11} - \phi_{21}b_{12})h_{11} + (1-\phi_{11})h_{11}^2 - \phi_{21}h_{11}h_{12}\big)\delta_1^2$$
$$+ 2\big((\phi_{12}b_{11} + (1-\phi_{22})b_{12})h_{12} - \phi_{12}h_{11}h_{12} + (1-\phi_{22})h_{12}^2\big)\delta_2^2\big)/\mathrm{var}\big(d_t^1\big). \tag{B.15}$$

The expression of BE_1 for the first supply chain can be derived by dividing the two sides of equation (B.14) by $\mathrm{var}(d_t^1)$.

Secondly, we deduce the expression of BE_2 for the second supply chain. Similarly, we can get the order quantity variance of retailer 2:

$$\begin{aligned}\mathrm{var}\big(q_t^2\big) = {} & h_{21}^2\mathrm{var}\big(p_{t-1}^1 - p_{t-2}^1\big) + h_{22}^2\mathrm{var}\big(p_{t-1}^2 - p_{t-2}^2\big)\\ & + \mathrm{var}(d_{2,t}) + 2h_{21}\mathrm{cov}\big(p_{t-1}^1 - p_{t-2}^1), d_{t-1}^2\big)\\ & + 2h_{22}\mathrm{cov}\big((p_{t-1}^2 - p_{t-2}^2), d_{t-1}^2\big) + 2h_{21}h_{22}\mathrm{cov}\big((p_{t-1}^1 - p_{t-2}^1), (p_{t-1}^2 - p_{t-2}^2)\big).\end{aligned} \tag{B.16}$$

Due to equations (3.1) and (3.3), we can get the following covariance equations:

$$\mathrm{cov}\big(p_{t-2}^1, p_{t-1}^1\big) = \phi_{11}\delta_1^2 + \phi_{12}\delta_{12} \tag{B.17}$$
$$\mathrm{cov}\big(p_{t-2}^2, p_{t-1}^1\big) = \phi_{12}\delta_2^2 + \phi_{11}\delta_{12} \tag{B.18}$$
$$\mathrm{cov}\big(p_{t-2}^2, p_{t-1}^2\big) = \phi_{22}\delta_2^2 + \phi_{21}\delta_{12}. \tag{B.19}$$

Using equations (B.9) and (B.16)–(B.18), we can obtain the following covariance equations.

$$\begin{aligned}\mathrm{cov}\big((p_{t-1}^1 - p_{t-2}^1), d_{t-1}^2\big) &= \mathrm{cov}\big((p_{t-1}^1 - p_{t-2}^1), \big(a + b_{21}p_{t-1}^1 - b_{22}p_{t-1}^2\big)\big)\\ &= b_{21}\mathrm{cov}\big(p_{t-1}^1, p_{t-1}^1\big) - b_{21}\mathrm{cov}\big(p_{t-2}^1, p_{t-1}^1\big) - b_{22}\mathrm{cov}\big(p_{t-1}^1, p_{t-1}^2\big) + b_{22}\mathrm{cov}\big(p_{t-2}^1, p_{t-1}^2\big)\\ &= b_{21}\delta_1^2 - b_{21}\big(\phi_{11}\delta_1^2 + \phi_{12}\delta_{12}\big) - b_{22}\delta_{12} + b_{22}\big(\phi_{21}\delta_1^2 + \phi_{22}\delta_{12}\big)\\ &= (b_{21}(1-\phi_{11}) + b_{22}\phi_{21})\delta_1^2 - (b_{22}(1-\phi_{22}) + b_{21}\phi_{12})\delta_{12}\end{aligned} \tag{B.20}$$

$$\begin{aligned}\mathrm{cov}\big((p_{t-1}^2 - p_{t-2}^2), d_{t-1}^2\big) &= \mathrm{cov}\big((p_{t-1}^2 - p_{t-2}^2), \big(a + b_{21}p_{t-1}^1 - b_{22}p_{t-1}^2\big)\big)\\ &= b_{21}\mathrm{cov}\big(p_{t-1}^2, p_{t-1}^1\big) - b_{21}\mathrm{cov}\big(p_{t-2}^2, p_{t-1}^1\big) - b_{22}\mathrm{cov}\big(p_{t-1}^2, p_{t-1}^2\big) + b_{22}\mathrm{cov}\big(p_{t-2}^2, p_{t-1}^2\big)\\ &= b_{21}\delta_{12} - b_{21}\big(\phi_{12}\delta_2^2 + \phi_{11}\delta_{12}\big) - b_{22}\delta_2^2 + b_{22}\big(\phi_{22}\delta_2^2 + \phi_{21}\delta_{12}\big)\\ &= (-b_{22}(1-\phi_{22}) - b_{21}\phi_{12})\delta_2^2 + (b_{21}(1-\phi_{11}) + b_{22}\phi_{21})\delta_{12}.\end{aligned} \tag{B.21}$$

Substituting (B.7), (B.8), (B.11), (B.19) and (B.20) into (B.15), the order quantity variance of the second supply chain can be derived.

$$\begin{aligned}\mathrm{var}\big(q_t^2\big) = {} & 2\big((b_{21}(1-\phi_{11}) + \phi_{21}b_{22})h_{21} + (1-\phi_{11})h_{21}^2 - \phi_{21}h_{21}h_{22}\big)\delta_1^2\\ & - 2\big((\phi_{12}b_{21} + (1-\phi_{22})b_{22})h_{22} + \phi_{12}h_{21}h_{22} - (1-\phi_{22})h_{22}^2\big)\delta_2^2\mathrm{var}(d_t^2)\\ & + 2\delta_{12}((-\phi_{12}b_{21} + (-1+\phi_{22})b_{22})h_{21} + ((1-\phi_{11})b_{21} + \phi_{21}b_{22})h_{22}\\ & + (2-\phi_{11}-\phi_{22})h_{21}h_{22} - \phi_{21}h_{22}^2 - h_{21}^2\phi_{12}\big).\end{aligned} \tag{B.22}$$

We can get the expression of BE for the second supply chain BE_2 by dividing the two sides of equation (B.21) by $\mathrm{var}(d_t^2)$.

$$\begin{aligned}\mathrm{BE}_2 &= \frac{\mathrm{var}(q_t^2)}{\mathrm{var}(d_t^2)}\\ &= 1 + (2((-\phi_{12}b_{21} + (-1+\phi_{22})b_{22})h_{21} + ((1-\phi_{11})b_{21} + \phi_{21}b_{22})h_{22}\\ &\quad + (2-\phi_{11}-\phi_{22})h_{21}h_{22} - \phi_{21}h_{22}^2 - h_{21}^2\phi_{12}\big)\delta_{12}\\ &\quad + 2\big((b_{21}(1-\phi_{11}) + \phi_{21}b_{22})h_{21} + (1-\phi_{11})h_{21}^2 - \phi_{21}h_{21}h_{22}\big)\delta_1^2\\ &\quad - 2\big((\phi_{12}b_{21} + (1-\phi_{22})b_{22})h_{22} + \phi_{12}h_{21}h_{22} - (1-\phi_{22})h_{22}^2\big)\delta_2^2\big)/\mathrm{var}\big(d_t^2\big).\end{aligned} \tag{B.23}$$

Equations (B.15) and (B.23) can BE expressed as one formula.

$$\begin{aligned}\mathrm{BE}_i = 1 + \Big(2\Big((-1)^{i-1}((\phi_{12}b_{i1} + (1-\phi_{22})b_{i2})h_{i1} - ((1-\phi_{11})b_{i1} + \phi_{21}b_{i2})h_{i2}) \\ + (2-\phi_{11}-\phi_{22})h_{i1}h_{i2} - \phi_{21}h_{i2}^2 - \phi_{12}h_{i1}^2\Big)\delta_{12} \\ + 2\big((-1)^i(b_{21}(1-\phi_{11}) + \phi_{21}b_{i2})h_{i1} + (1-\phi_{11})h_{i1}^2 - \phi_{21}h_{i1}h_{i2}\big)\delta_1^2 \\ + 2\big((-1)^{i-1}(\phi_{12}b_{21} + (1-\phi_{22})b_{22})h_{i2} - \phi_{12}h_{i1}h_{i2} + (1-\phi_{22})h_{i2}^2\big)\delta_2^2\Big)/\mathrm{var}\big(d_t^i\big).\end{aligned} \tag{B.24}$$

The above content completes the proof of Theorem 4.2. □

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