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A SINGLE-MANUFACTURER MULTI-RETAILER INTEGRATED INVENTORY MODEL FOR ITEMS WITH IMPERFECT QUALITY, PRICE SENSITIVE DEMAND AND PLANNED BACK ORDERS

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Abstract. In this paper, we develop an integrated two-echelon supply chain inventory model with a single-manufacturer and multi-retailers in which each retailer's demand is dependent on selling price of the product. The manufacturer produces a single product and dispatched the order quantities of the retailers in some equal batches. The production process is imperfect and produces imperfect quality of products with a defective percentage which is random in nature and follows binomial distribution. Inspection process is performed by the retailers to classify the defective items in each lot delivered from the manufacturer. The defective items that were found by the retailer will be returned to the manufacturer at the next delivery. Lead time is random and it follows an exponential distribution. We also assume that shortages are allowed and are completely backlogged at each retailer's end. A closed form solution to maximize the expected average profit for both the centralized and the decentralized scenarios are obtained. The developed models are illustrated with the help of some numerical examples using stochastic search genetic algorithm (GA). It is found that integration of the supply chain players results an impressive increment in the profit of the whole supply chain. Sensitivity analysis is also performed to explore the impacts of key-model parameters on the expected average profit of the supply chain.

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1. Introduction

With the growing fixate on supply chain management over the last two decenniums, firms endeavors to accomplish more preponderant collaborative advantages with their supply chain partners. Subsequently, there is a tremendous growing interest in jointly optimizing the engenderment and inventory quandaries for the supply chain partners. Integrated inventory management has recently received a great deal of attention. Nowadays, due to the globalization of the rialto and expeditiously incrementing competition between sundry opposition organizations, companies are facing sundry obstacles to compete exclusively. So, collaboration between different business units leads to a consequential way to obtain spirited advantage. For better efficiency, the supply chain players are exhibiting great interest in making their decisions jointly. Since 1976 Goyal [15] was the first author

Keywords. Two-echelon supply chain, Single-manufacturer, Multi-retailers, Imperfect quality items, Price dependent demand, Stochastic lead time, Genetic algorithm.

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who introduced integration between supply chain members in inventory model with finite production rate. Later, many researchers (Banerjee [2], Ben-Daya and Hariga [7], Dey *et al.* [10], Jindala and Solanki [21]) presented various integrated inventory models.

The supply chain management involves with the activities for coordinating the raw materials, information, production and financial flow to consummate the customer demand with the aim of maximizing customer value and gaining competitive advantage in the market. In a supply chain, raw materials are distributed to the manufacturer from the supplier, then engendered items are transferred from the manufacturer to the retailer and conclusively distributed to the terminus customers to meet their injunctive authorization. In the present competitive market, the selling price of a product is one of the most consequential factors to the retailers. Generally, higher selling price of a product abbreviates the ordinate dictation, and plausible and low selling price increases the authoritative ordinance of the product. Therefore, price-dependent stochastic demand is very much a practical case in supply chain management. The most paramount factor that affects the consumer demand is the retail price of the product. The customer demand falls with the incrimination in retail prices. So, many companies are now fixating on achieving the best pricing strategy to increment the sales volume. Many researchers like Yang *et al.* [50], Barman and Das [3], Ray and Jewkes [37], Kocabiyikoglu and Popescu [24], Bhowmick and Samanta [8], Mahato *et al.* [28], Pervin *et al.* [34] developed their models considering price sensitive demand. In the classical inventory and supply chain models, the market demand is surmised to be constant. However, in authenticity, there are many items whose demands are merely not constant but depend on the retail price. If the price is low, then the market demand is more; on the other hand, if the price is high then the market demand of the product is less. So, the selling price plays a consequential role in determining the inventory or supply chain strategies. In this paper, we will fixate on an integrated retailer-manufacturer model for imperfect quality of items having price-dependent market demand. Shortages in the retailer's inventory are sanctioned to occur and are planned backlogged. The objective of this study is to determine the optimal decisions which maximize the joint total profit of the whole supply chain.

In today's business world, manufacturer uses several retailers to sell their products rather than rely on a single retailer. The multi-buyer situation is very common in real life. A manufacturer may supply his/her product to different retailers to fulfil their requirements. For example, in health care industries, the manufacturer supplies instruments to different hospitals according to their requirements. They often give much accentuation on controlling distribution lead time in order to efficiently handle a supply chain. Lead time refers to the time interval between order placement and receiving the distribution of items. It may be desultory in nature. It affects the injunctive sanction forecasting and makes the customer probing for alternatives. So, for an efficient management of supply chain and facilitate economic prosperity, it is essential to concentrate on such a sensible factor. Yang and Geunes [51] provided a nice discussion about inventory and lead time planning with lead time sensitive demand. The literary review reflects that numerous research works have been carried out focusing on lead time (Ouyang *et al.* [31], Gholami and Mirzazadeh [11]). In two-level supply chain system, a fascinating strategic quandary arises at the retailer's end in determining optimal authoritatively mandating quantity and the number of shipments decided by the manufacturer to transfer the injunctively sanctioned quantity of the retailer. Keeping in mind the consequentiality of lead time and fixating on multi-retailer scenarios, in our model we develop a single-manufacturer multi-retailer inventory supply chain model postulating lead time as a stochastically arbitrary variable following mundane distribution and the customer demand is influenced by its selling price.

In the classical inventory model, it is often surmised that the production process is impeccable. However, in genuine-life situation, it is not possible that a production process is to be consummately error-free. Due to the imperfect production process of the manufacturer, damage in transit or other unforeseeable circumstances, goods received by the retailer may contain some percentage of defective items. These defective items will affect the inventory level, customer accommodation level and the frequency of orders in the inventory system. Ergo, it is worth studying the effect of defective items on inventory decisions. To resolve the issue, defective items may be repudiated, rehabilitated, reworked or restituted, if those have reached to the retailer. Consequently, for the supply chain system with an imperfect production process, the manufacturer may invest capital on quality amendment, so as to truncate the defective rate of production process. Porteus [36] was the pioneer, who explicitly elaborated the significant relationship between quality imperfection and lot size. Rosenblatt and Lee [39] considered the imperfect production process and controlled the quality of product by inspections. Since the pioneering work by Porteus [36] and Rosenblatt and Lee [39], in order to surmount the common unrealistic assumption of good quality, many researchers have attempted to develop various imperfect-quality inventory models in this important issue.

Salameh and Jaber [40] considered an integrated manufacturer-retailer cooperative inventory model for items with imperfect quality under equal shipment policy, and assumed that the number of defective items follows a given probability density function, and that the manufacturer treats defective items as a single batch at the end of the retailer's 100% screening process at a rate of δ units per unit time with $\delta > D$ (the annual demand). The defective items were sold at a discounted price at the end of the 100% screening process. Khanna *et al.* [23] established inventory and pricing decisions for imperfect quality items with inspection errors. Here, we assume that an arriving order lot to the retailer may contain some imperfect quality of items, and that the number of defective items is a binomial random number. On the arrival of an order, the retailer performs a non-destructive and error-free screening process gradually at a fixed screening rate on receiving a lot before selling, rather than inspecting through a rapid action; and all defective items in each lot are assumed to be discovered. Based on the survey above, we develop a single-manufacturer multi-retailer inventory supply chain model assuming lead time as stochastic and the customer demand is influenced by its selling price.

Virtually, customer demand may be stochastic and thereby sanctioning shortages. When the ordinate dictation is stochastic, lead time becomes a paramount issue. The time that elapses between the placement of an order and the receipt of the order into inventory is called lead time. When shortages are allowable, it can be back authoritatively mandated. Cárdenas-Barrón [9] developed an economic production quantity (EPQ) inventory model with orchestrated back orders for determining the economic production quantity and the size of back orders for a single product, which was made in a single-stage manufacturing process that engendered imperfect quality products and required that all defective products be reworked in the same cycle. He additionally established the range of genuine values of the proportion of defective products for which there is an optimal solution, and the closed form for the total cost of the inventory system. Just like demand is satisfied during the opening time of the firm, shortages are built and eliminated during its non-idle time only. Shortages are sometimes beneficial for the retailer especially when they can be planned backlogged. Thus, the present model is applicable to a variety of retail industries that target ease for their resources, customer satisfaction, and high standards of quality.

The classical economical order quantity (EOQ) model defines isolated inventory problems for retailer and manufacturer in order to minimize their individual costs or maximize their individual profits. This kind of one sided optimal strategy does not provide congruous solution in today's ecumenical market due to ascending cost, shrinking resources, short product life cycle, incrementing competition, *etc.* Ergo, in recent years, researchers pay more attention on integrated vendor-buyer quandaries. Collaboration of the vendor and the buyer is one of the key factors for prosperous management of the supply chain. In two-echelon supply chain system, an intriguing strategic problem arises at the retailer's end in determining optimal authoritatively mandating quantity and the number of shipments decided by the manufacturer to transfer the injunctive authorized quantity of the retailer. Keeping in mind the paramount of multi-retailer in two-echelon supply chain and fixating on lead time and imperfect quality items, in our model we develop a single-manufacturer multi-retailer two-echelon supply chain inventory model postulating lead time as a stochastic desultory variable following exponential distribution and the customer demand is influenced by its selling price.

2. Literature review

The cooperation among the upstream and downstream gammers offers a miles extra gain than a non-collaborative courting. Latest manufacturing era leaps, boom in supply-to-demand ratio, and enterprise competitiveness intensification have brought new challenges to supply chains. A markets got quite aggressive and commercial enterprise surroundings skilled a recession, stock control structures have been a number of the organizational sub-structures that have been affected the most. The time period deliver chain refers to a unified gadget of providers, manufacturers, distributors, wholesalers, and outlets who are dedicated to cooperate to deliver the proper quantity of the right product to satisfy right call for at most beneficial overall performance measures like value, reliability and so on.

Tenaciousness of best pricing strategy and inventory decisions that influence the customer demand has been focused by many researchers and practitioners. In this stream, a seminal research work is done by Whitin [49] in 1955. Several researchers have their great research works assuming demand as constant at the retailer's end, but in many cases it is impossible to forecast demand by an exact value. Also, price of the commodity is a very important factor in deciding end customer demand. In this aspect, during several years, many researchers, like, Sohrabi *et al.* [42] investigated a supplier selection problem in the vendor managed inventory (VMI) scenario under price dependent customer demand. Alfares and Ghaithan [1] developed an inventory model thinking about storage time-dependent conserving cost, quantity cut price and cost-structured call for. Giri and Masanta [13] applied a profit sharing agreement between the store and the producer underneath price touchy market demand in stochastic surroundings. Taghizadeh-Yazdi *et al.* [43] derived greatest retail rate and top-rated time c language for multi-echelon supply chains version considering rate touchy purchaser demand. Paul *et al.* [32] formulate and solve an EOQ model with default risk. They also investigate retailer's optimal replenishment time and credit period for deteriorating items under selling price-dependent demand. Previn *et al.* [33] formulate and solve a deteriorating EPQ inventory model with preservation technology under price- and stock-sensitive demand. Barman *et al.* [6] developed a crisp EPQ model by considering linearly time-dependent demand. Barman and Das [4] discussed about integrated inventory model with variable lead time and ramp-type demand. Mandal and Giri [30] devolved an included stock version and derived surest selections that allows us to optimize the machine income ability for defecting item. Sarkar *et al.* [41] proposed a price bargaining policy where the manufacturer offers a cost bargaining depending on the customer's ordered size in a rate touchy market.

In today's business world, manufacturers use several retailers to sell their products rather than rely on a single retailer. They often give much emphasis on controlling delivery lead time in order to efficiently handle a supply chain. In true sense, lead time is composed of several components and these components can be reduced by adding additional crashing costs. Lead time reduction sometimes appears very advantageous in competitive situation because it can lower safety stock level, reduce stock-out loss and improve the customer service level. Li *et al.* [25] coordinated a two-echelon supply chain by price discount mechanism considering controllable lead time and service-level constraint. Later, Hoque [17] extended this model *via* considering combined identical and unequal batch shipments and confirmed that the version with regular distribution is extra profitable than the model with exponential distribution for lead time. Lin [27] derived manufacturing strategy and investment coverage for reducing the lead time variability for the stochastic lead time. Ritha and Poongodisathya [38] discussed lead time dependent more than one-client and single manufacturer non-stop review stock model with ordering cost discount. Dey *et al.* [10] developed an integrated inventory version, where they confirmed setup cost is decreased with variable protection thing, where demand is promoting fee structured. There are numerous thrilling and relevant papers in stock manage gadget with incorporated dealer-retailer stock model with controllable lead time which includes Giri and Masanta [12] developed a closed-loop deliver chain version considering getting to know effect in the manufacturing in a stochastic lead time scenario underneath fee and fine based call for.

In modern day aggressive business global, it's far difficult to discover that one producer sells a product to a single store. So, it's miles extra practical to awareness at the situations wherein the producer resources the product to numerous stores. Researchers are also currently showing a fantastic hobby in developing such multi-retailer fashions. Taleizadeh *et al.* [46] compared the results of the PSO (Particle Swarm Optimization) approach and GA (Genetic Algorithm) method in a multi-product inventory model comprised of a single manufacturer and multiple retailers where the lead time varies with the lot size. Jha and Shanker [20] considered a single-manufacturer multi-retailer supply chain model under controllable lead time and proposed a Lagrangian multiplier technique to evaluate optimal results under service level constraints. Poorbagheri and Niaki [35] studied a VMI model composed of a single manufacturer and multiple retailers under stochastic demand. Barman *et al.* [5] conceptualized a multi-cycle manufacturer-retailer supply chain production inventory model where they focused on reducing a green-house gas (CO_2) emission to the environment during the production period and shipment of quantity. Uthayakumar and Kumar [47] studied a single-manufacturer multi-retailer integrated model with stochastic demand and controllable lead time.

In conventional supply chain control many researchers have involved that each one the procured gadgets do now not have any disorder, that is unrealistic. consequently, a variety of research has been carried out in previous few a long time doing away with the unrealistic assumption of best items production in classical EOQ model by way of Harris [16] in 1913. The presence of defective items inside the supply chain has the capacity to make the product flow unreliable. consequently, screening of items earlier than selling receives essential, particularly for non-production companies. A number of the preliminary research addressing the production of imperfect quality of items consists of Rosenblatt and Lee citersl. later it is persevered with the aid of a few researchers like Salameh and Jaber [40], *etc.* Later, Lin [26] presented an inventory model considering defective as random variables, where the demand is distribution-free and lead-time is controllable. Further, Wangsa and Wee [48] advanced a manufacturer-retailer stock version in which they take into account a screening manner to look into faulty items in plenty. These days, Kazemi *et al.* [22] investigated an imperfect supply technique to investigate the impact of carbon emission on most beneficial replenishment regulations.

Relating to a supply chain with imperfect objects, there may be a sizeable probability that shortages may occur. In this regard, Hsu and Hsu [18] gave an included version in which the production manner is faulty and back-orders are planned. Jaggi *et al.* [19] advanced a replenishment guidelines for imperfect stock gadget beneath herbal idle time. Where they assumes fully backlogged shortages and is solved beneath a profit maximizing framework. Latter, Taleizadeh *et al.* [45] inspected an ordering version wherein shortages are in part again-ordered, incorporating reparation of imperfect products. Then, Taleizadeh [44] studied a limited incorporated imperfect gadget under the assumptions of preventive preservation and partial back-ordering. Barman *et al.* [6] developed an EPQ model for deteriorating items with time-dependent demand and shortages including partially back-ordered under a cloudy fuzzy environment.

"Genetic Algorithm" is an exhaustive search algorithm on the basis of genesis (crossover, mutation, *etc.*) and the mechanics of natural selection (*cf.* Goldberg [14], Maiti and Maiti [29], Barman and Das [4]). It has been successfully applied to many optimization problems because of its generality and several advantages over conventional optimization method.

The comparison of the existing literature with this paper is summarized below in Table 1.

From the above literature overview, we discovered that a single-manufacturer multi-retailer two echelon supply chain inventory model has now not been studied incorporating charge touchy consumer's call for and stochastic lead time, wherein shortages are allowed and are completely back ordered and the manufacturer's manufacturing system is imperfect and produces a certain range of defective products with a known opportunity density feature. We assume that the 100% screening process of the lot is conducted at the retailer's place upon receiving of the lot from the manufacturer. Due to imperfect quality of items, shortages may occur sometimes. In practice, if there are any shortages are allowed then it is often used fully backlogged; for example, companies use planned shortages when the cost of stocking an item exceeds the profit that would come from selling it. Furniture showrooms, for example, do not stock enough furniture to cover all demand, and customers are often asked to wait for their orders to be delivered from suppliers or regional distribution centers. This paper is the first one to incorporate price sensitive customer's demand and stochastic lead time with shortages and imperfect quality items in a two-echelon supply chain inventory model with a single-manufacturer and multi-retailers. The proposed models are developed for both the centralized and the decentralized scenarios. A closed form solution to maximize the expected average profit for both the centralized and the decentralized scenarios are obtained. The developed models are illustrated by numerical examples using stochastic search genetic algorithm (GA). It is found that integration of the supply chain players gives an impressive increment in the profit of the whole supply chain. Sensitivity analysis is also performed to explore the impacts of key-model parameters on the expected average profit of the supply chain.

TABLE 1. Research gap with existing literature.

Author(s)	Model type	Demand type	Imperfect quality	Types of lead time	Allow for shortages
Barman *et al.* [5]	Single-manufacturer Single-retailer	Constant	No	Stochastic	Yes
Gholami and Mirzazadeh [11]	Single period EOQ	Normally distributed	No	Controllable	No
Giri and Masanta [12]	Single-manufacturer Single-retailer	Deterministic	No	Stochastic	No
Hoque [17]	Single-manufacturer Single-retailer	Deterministic	No	Stochastic	Yes
Hsu and Hsu [18]	Single-manufacturer Single-retailer	Constant	Yes	No	Yes (Planned Back-ordered)
Jaggi *et al.* [19]	Single period EOQ	Constant	Yes	No	Yes (Planned back-ordered)
Jha and Shanker [20]	Single-manufacturer Single-retailer	Constant	No	Controllable	Yes
Kazemi *et al.* [22]	Single period EOQ	Constant	Yes	No	No
Khanna *et al.* [23]	Single period EOQ	Price dependent	Yes	No	Yes (Partial Back-ordered)
Lin [26]	Single-manufacturer Single-retailer	Constant	Yes	Stochastic	Yes (Back-ordered)
Mandal and Giri [30]	Single-manufacturer Multi-retailer	Deterministic	Yes	Controllable	No
Ritha and Poongodisathiya [38]	Single-manufacturer Multi-retailer	Constant	No	Stochastic	No
Sohrabi *et al.* [42]	Multi Manufacturer Distributer Multi retailer	Constant	No	No	No
Taleizadeh *et al.* [45]	Single-manufacturer Multi-retailer	Stochastic	No	Stochastic	Yes (Back-ordered)
Wangsa and Wee [48]	Single-manufacturer Single-retailer	Constant	Yes	Stochastic	Yes
This Paper	Single-manufacturer Multi-retailer	Price Dependent	Yes	Stochastic	Yes (Planned Back-ordered)

3. NOTATIONS

The notations used in this paper are as follows:

R	Production rate
S_v	Setup cost for manufacturer/set up
A_i	Ordering cost for ith retailer/order
T_p	Transportation cost per batch shipment
N	Number of retailers
D	Total market demand $[=\sum_{i=1}^{N} D_i]$
D_i	Demand rate on the ith retailer $[R > \sum_{i=1}^{N} D_i]$
α_i	Basic market demand
w_h	Unit wholesale price
β_i	Consumer sensitivity coefficient to retail price
r_i	Reorder point on the ith retailer
L	Lead time, a stochastic random variable
l_i	Lead time variable
T_i	Cycle time
z	The screening rate, $z > D$
z_i	The screening rate on the ith retailer, $z_i > D_i$
δ	The defective percentage in Q.
δ_i	The defective percentage in Q_i on the ith retailer
$f(\delta)$	The probability density function of δ.
h_v	Holding cost for manufacturer/item/unit time $(h_v > h_i)$

h_i	Holding cost for retailer/item/unit time
d_i	Screening cost per unit on the ith retailer
ν	The manufacturer's unit warranty cost per defective item for the retailer
b_i	The back ordering cost per unit per time at the retailer
c_i	The shortage cost per unit per time at the retailer
$f_L(.)$	Probability density function of the lead time
λ_i	Rate parameter of exponential distribution
$E[.]$	Mathematical expectation
$*$	The superscript representing optimal value
Decision variables	
n	Number of batches delivered to each retailer
Q	Total order quantity $[=\sum_{i=1}^{N} Q_i]$
Q_i	Ordering quantity for ith retailer
g_i	Batch size
p_i	Unit retail price

4. Assumptions

The following basic assumptions are used to mathematically formulate the proposed models:

(i) This is a single-manufacturer multi-retailer integrated inventory model in finite time horizon T. Here the manufacturer produces a single product and meets the demand of multiple retailers.

(ii) The ith retailer places his order quantity Q_i. The manufacturer produces the total order quantity $\sum_{i=1}^{N} Q_i$ of all retailers in one set up at a constant production rate R, and then transfers the ordered quantities to the ith retailer, in n equal batches of size g_i. If the total order quantity of N retailers is Q, then we have $ng_i = Q_i$ and $Q = \sum_{i=1}^{N} Q_i$, such that $\frac{Q_i}{D_i} = \frac{Q}{D}$ for all $i = 1, 2, ..., N$.

(iii) Retailer uses a continuous review inventory policy and the order is placed whenever inventory level falls to the reorder point, namely r_i for the ith retailer.

(iv) An arriving lot may contain some defective items. We assume that the number of defective items, δ_i, in an arriving order of size g_i is a random variable which follows a binomial distribution with parameters g_i and γ_i, where $0 < \gamma_i \leq 1$). Upon the arrival of the order, all the items in the lot are inspected with the screening rate z_i by the retailers, and defective items in each lot are discovered and returned to the manufacturer at the time of delivery of the next lot.

(v) To ensure that the manufacturer has enough production capacity to produce the retailer's demand, it is assumed that $\delta < 1 - \frac{D}{R}$. Moreover, to avoid shortages during the screening period, δ is restricted to $\delta < 1 - \frac{D}{z_i}$.

(vi) A 100% screening process of the lot is conducted at the retailer's place upon receipt of the lot. The screening process and demand proceed simultaneously, but the screening rate is greater than the demand rate, $z_i > D_i$. All defective items are returned to the manufacturer at the end of the 100% screening process. A defective item incurs a warranty cost of ν for the manufacturer. The manufacturer will sell the defective items at a reduced price to other retailers.

(vii) The consumer's demand rate depends linearly on the selling price of the product, *i.e.*, the consumer demand rate at the ith retailer is $D_i(p_i) = \alpha_i - \beta_i p_i$, where α_i represents the basic consumer demand and β_i is a positive number such that $\alpha_i > \beta_i p_i$ for all $i = 1, 2, ..., N$.

(viii) The production rate is constant and higher than the collective demand rates of all the retailers, *i.e.*, $R > \sum_{i=1}^{N} D_i$.

(ix) Shortages are allowed and assumed to be completely backlogged at each retailer's end.

(x) The ith retailer places his next order when their inventory stock level reaches a certain reorder point r_i.

(xi) The lead time is stochastic in nature and follows an exponential distribution and the lead time for all shipments are independent of each other.

5. Mathematical formulation of the model

Under the posits, we establish a single-manufacturer multi-retailers included inventory version cognate to shortages, inspections of defective items all through the production run, as expeditiously because the first Q units were engendered, the manufacturer will distribute them to the retailer. The manufacturer will make a delivery to the ith retailer, on an average, every $E[(g_i - \delta_i)/D_i]$ units of time until the inventory level falls to zero, where we have assumed that each lot contains a random number of defective, δ_i. Upon the arrival of order quantity, the retailer inspects all the items at a fixed screening rate, z_i, and all defective items in each lot are discovered and returned to the manufacturer at the time of delivery of the next lot. In order to reduce the production cost, the manufacturer produces Q units at one setup with a finite production rate R when the retailer orders quantity is $Q[= \sum_{i=1}^{N} Q_i]$, and each batch is dispatched to the retailer in n equal lots of size g_i, where $Q_i = ng_i$. Therefore, the expected length of each ordering cycle for the ith retailer is $E[T_i] = E[n(g_i - \delta_i)/D_i] = E[(Q_i - n\delta_i)/D_i]$, and the expected length of each production cycle for the manufacturer is $E[T] = E[(Q - \delta)/D]$, where $E[(Q - \delta)/D] = \sum_{i=1}^{N} E[(Q_i - n\delta_i)/D_i]$.

Let the number of defective items in a lot g_i be a binomial random variable with parameters g_i and γ_i, where γ_i $(0 \leq \gamma_i \leq 1)$ represents the defective rate in a order lot, *i.e.*,

$$P_r(\delta_i) = \begin{pmatrix} g_i \\ \delta_i \end{pmatrix} {\gamma_i}^{\delta_i} (1 - \gamma_i)^{g_i - \delta_i}, \quad \text{for } \delta_i = 0, 1, 2, ..., g_i. \tag{5.1}$$

In this case,

$$E[\delta_i] = g_i \gamma_i \quad \text{and} \quad E[{\delta_i}^2] = g_i^2 {\gamma_i}^2 + g_i \gamma_i (1 - \gamma_i). \tag{5.2}$$

The expected length of cycle time and the expected cycle cost under the lot of size Q_i are

$$E[T_i] = E\left[\frac{ng_i - n\delta_i}{D_i}\right] = \frac{ng_i(1 - \gamma_i)}{D_i}. \tag{5.3}$$

5.1. Mathematical model formulation for retailer

If the producer and the retailer do no longer collaborate in a cooperative manner toward maximizing their mutual benefits, and the retailer makes his personal decisions independently of the manufacturer, then the manufacturer will produce and ship the goods to the retailer on lot-for-lot substratum.

Here, we have developed a stochastic inventory system with shortages under continuous review policies, where the inventory position is continuously monitored. We surmise that the manufacturer transfers the order quantity Q_i to the ith retailer in n equal batches of size g_i, where the total order quantity of N retailers is Q. Thus we have $Q = \sum_{i=1}^{N} Q_i$ and $Q_i = ng_i$. As soon as retailer's inventory position, based on the number of non-defective items, drops to the reorder point r_i and an order is placed. The reorder level is determined so as to arrive the shipment to the retailer's end at or before the time of selling the r_i quantity at a demand rate D_i. The mean lead time is $\frac{r_i}{D_i}$. Due to various reasons, the batches may reach the retailer's end early or late. So, depending on the duration of lead time three cases may arise.

5.1.1. When g_i reaches to the retailer earlier i.e. $0 < l_i < \frac{r_i}{D_i}$

Here batch reaches the retailer's end early. Shortages are not occurred. Upon the arrival of order, the retailer inspects all the items at the fixed screening rate z_i and all defective items in each lot are discovered and returned to the manufacturer at the time of delivery of the next lot. Figure 1 illustrates the behavior of the inventory level over time for the retailer. Here, batch g_i is received at time l_i. Again t, where $t = \frac{g_i}{z_i}$, is the completion time of the 100% screening process, t_1 is the period of time at which demand will be filled from inventory. Among the g_i units, δg_i units are defective and will be removed from inventory at time t.

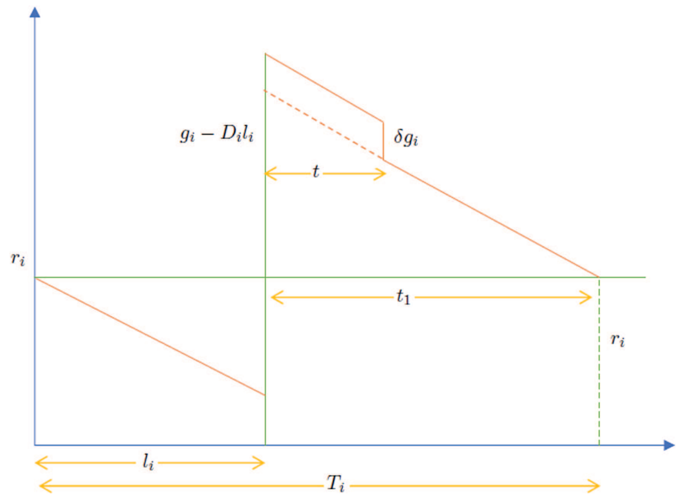

FIGURE 1. Inventory level of retailer when $0 < l_i < \frac{r_i}{D_i}$.

In this case, the holding inventory area of the ith retailer is determined from Figure 1 as follows:

$$\frac{1}{2}[2r_i - D_i l_i]l_i + t_1 r_i + \frac{1}{2}\frac{[g_i(1-\delta_i) - D_i l_i]^2}{D_i} + g_i \delta t,$$
$$\text{where } t_1 = \frac{g_i - D_i l_i - g_i\delta_i}{D_i} \quad \text{and} \quad r_i = \frac{Q_i D_i}{R}\cdot$$
$$\text{Also, } t_1 r_i = [g_i(1-\delta_i) - D_i l_i]\frac{r_i}{D_i}\cdot$$

The ordered quantity of the ith retailer is given by

$$H_{ai} = n\left[l_i(r_i - \frac{1}{2}D_i l_i) + [g_i(1-\delta_i) - D_i l_i]\frac{r_i}{D_i} + \frac{1}{2}\frac{[g_i(1-\delta_i) - D_i l_i]^2}{D_i} + \frac{g_i^2\delta_i}{z_i}\right]. \tag{5.4}$$

Since δ_i is random and it follows poison distribution, therefore

$$E[H_{ai}] = n\Bigg[l_i(r_i - \frac{1}{2}D_i l_i) + \{g_i(1-E[\delta_i]) - D_i l_i\}\frac{r_i}{D_i} + \frac{g_i^2 E[\delta_i]}{z_i} + \frac{1}{2D_i}\{g_i^2 E[(1-\delta_i)^2] - 2g_i(1-E[\delta_i])D_i l_i + (D_i l_i)^2\}\Bigg]. \tag{5.5}$$

Also, lead time is stochastic random variable and it follows exponential distributions. Hence, expected holding cost for the ordered quantity Q_i of the ith retailer is given by

$$H_{ai}^* = h_i \int_0^{\frac{r_i}{D_i}} E[H_{1i}] f_L(l_i)\mathrm{d}l_i$$
$$= nh_i \int_0^{\frac{r_i}{D_i}} n\Bigg[l_i(r_i - \frac{1}{2}D_i l_i) + \{g_i(1-E[\delta_i]) - D_i l_i\}\frac{r_i}{D_i} + \frac{g_i^2 E[\delta_i]}{z_i} + \frac{1}{2D_i}\{g_i^2 E[(1-\delta_i)^2] - 2g_i(1-E[\delta_i])D_i l_i + (D_i l_i)^2\}\Bigg] f_L(l_i)\mathrm{d}l_i. \tag{5.6}$$

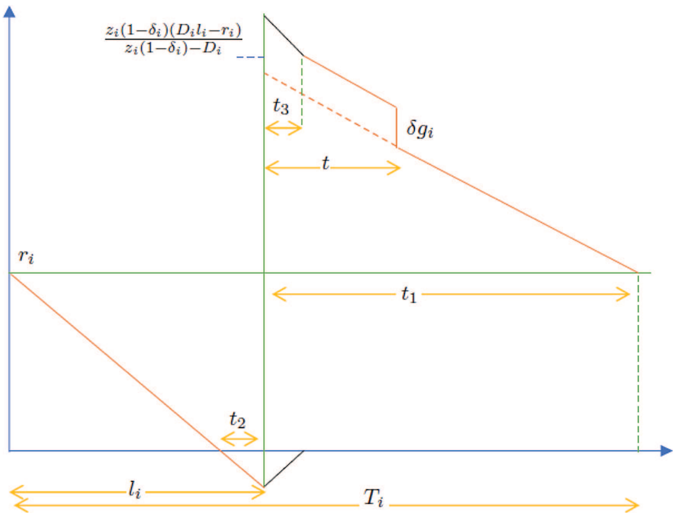

FIGURE 2. Inventory level of retailer when $\frac{r_i}{D_i} \le l_i \le \frac{(r_i+g_i)}{D_i}$.

5.1.2. When g_i reaches late to the ith retailer and the lead time l_i lies in the range $\frac{r_i}{D_i} \le l_i \le \frac{(r_i+g_i)}{D_i}$

Here order quantity reaches late to the ith retailer's end. Shortages are occurred and they are used to satisfy back orders. Upon the arrival of order quantity, the retailer inspects all the items at a fixed screening rate z_i and all defective items in each lot are discovered and returned to the manufacturer at the time of delivery of the next lot. Figure 2 illustrates the behavior of the inventory level over time for the retailer. Here batch g_i is received at time l_i. Again, t, where $t = \frac{g_i}{z_i}$, is the completion time of the 100% screening process, t_1 is the period of time in which demand will be filled from inventory, and t_2 is the period of time in which demand will be back ordered. Among the g_i units, δg_i units are defective and will be removed from inventory at time t.

In this case, the holding inventory area of the ith retailer is determined from Figure 2 and is described as follows: As shortages occur, the amount of shortages will be $(D_il_i - r_i)$. Hence, $(D_il_i - r_i)$ units of items intend to satisfy the back orders in the cycle will be filled at rate of $[z_i(1-\delta_i) - D_i]$. We have

$$t_3 = \frac{D_il_i - r_i}{z_i(1-\delta_i) - D_i},$$
$$\text{and } t_2 = \frac{D_il_i - r_i}{D_i}.$$

Hence back ordering area for ith cycle is given as follows:

$$\frac{1}{2}(D_il_i - r_i)(t_2+t_3) = \frac{1}{2}(D_il_i - r_i)\frac{D_il_i - r_i}{D_i} + \frac{1}{2}(D_il_i - r_i)\frac{D_il_i - r_i}{z_i(1-\delta_i) - D_i}$$
$$= \frac{1}{2}(D_il_i - r_i)^2\left\{\frac{1}{D_i} + \frac{1}{z_i(1-\delta_i - \frac{D_i}{z_i})}\right\}.$$

Thus, back ordered quantity of the ith retailer is given by

$$B = n\left[\frac{1}{2}(D_il_i - r_i)^2\left\{\frac{1}{D_i} + \frac{1}{z_i(1-\delta_i - \frac{D_i}{z_i})}\right\}\right]. \tag{5.7}$$

Now, δ_i is a random variable. Hence,

$$E[B] = n\left[\frac{1}{2}(D_i l_i - r_i)^2\left\{\frac{1}{D_i} + \frac{1}{z_i}E\left[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\right]\right\}\right].$$

Also, lead time is a stochastic random variable and it follows exponential distribution. Hence, expected back ordering cost for the ordered quantity Q_i of the ith retailer is

$$\begin{aligned} B_i^* &= b_i\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} E[B] f_L(l_i)\mathrm{d}l_i \\ &= \frac{nb_i}{2}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \left\{\frac{1}{D_i} + \frac{1}{z_i}E\left[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\right]\right\}(D_i l_i - r_i)^2 f_L(l_i)\mathrm{d}l_i. \end{aligned}$$

After time t_3, the time to fill $(D_i l_i - r_i)$, the maximum shortage level for ith cycle, the inventory level will be reduced by $(D_i l_i - r_i) + t_3 D_i = \frac{z_i(1-\delta_i)(D_i l_i - r_i)}{z_i(1-\delta_i)-D_i}\cdot$ At time t (which is $\frac{g_i}{z_i}$), δg_i defective units will be removed from the inventory. Thus, the holding area for ith cycle is as follows

$$t_1 = \frac{g_i(1-\delta_i) - (D_i l_i - r_i)}{D_i}\cdot$$

Total holding inventory area is determined from Figure 2, which is

$$\begin{aligned} &\frac{r_i^2}{2D_i} + t_1 r_i + \left\{g_i(1-\delta_i) - \frac{1}{2}\frac{z_i(1-\delta_i)(D_i l_i - r_i)}{z_i(1-\delta_i)-D_i}\right\}\frac{D_i l_i - r_i}{z_i(1-\delta_i)-D_i} + \frac{g_i^2\delta_i}{z_i} \\ &+\frac{1}{2}\left\{g_i(1-\delta_i) - \frac{z_i(1-\delta_i)(D_i l_i - r_i)}{z_i(1-\delta_i)-D_i}\right\}\left\{\frac{g_i(1-\delta_i)-(D_i l_i - r_i)}{D_i} - \frac{D_i l_i - r_i}{z_i(1-\delta_i)-D_i}\right\}. \end{aligned}$$

The ordered quantity of the ith retailer Q_i is given by

$$\begin{aligned} H_{bi} = n\Bigg[&\frac{r_i^2}{2D_i} + \frac{g_i^2\delta_i}{z_i} + \frac{r_i}{D_i}\{g_i(1-\delta_i) - (D_i l_i - r_i)\} + \frac{1}{2}\frac{g_i(D_i l_i - r_i)(1-\delta_i)}{z_i(1-\delta_i-\frac{D_i}{z_i})} \\ &+\frac{1}{2}\left\{\frac{g_i^2(1-\delta_i)^2}{D_i} - \frac{g_i(1-\delta_i)^2(D_i l_i - r_i)}{D_i(1-\delta_i-\frac{D_i}{z_i})} - \frac{g_i(1-\delta_i)(D_i l_i - r_i)}{D_i} + \frac{(D_i l_i - r_i)^2(1-\delta_i)}{D_i(1-\delta_i-\frac{D_i}{z_i})}\right\}\Bigg]. \end{aligned} \tag{5.8}$$

Now, δ_i is random and it follows poison distribution. Hence

$$\begin{aligned} E[H_{bi}] = n\Bigg[&\frac{r_i^2}{2D_i} + \frac{g_i^2 E[\delta_i]}{z_i} + \frac{r_i}{D_i}\{g_i(1-E[\delta_i]) - (D_i l_i - r_i)\} + \frac{g_i(D_i l_i - r_i)}{2z_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right] \\ &+\frac{1}{2}\Bigg\{\frac{g_i^2 E[(1-\delta_i)^2]}{D_i} - \frac{g_i(D_i l_i - r_i)}{D_i}E\left[\frac{(1-\delta_i)^2}{(1-\delta_i-\frac{D_i}{z_i})}\right] - \frac{g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i} \\ &+\frac{(D_i l_i - r_i)^2}{D_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right]\Bigg\}\Bigg]. \end{aligned} \tag{5.9}$$

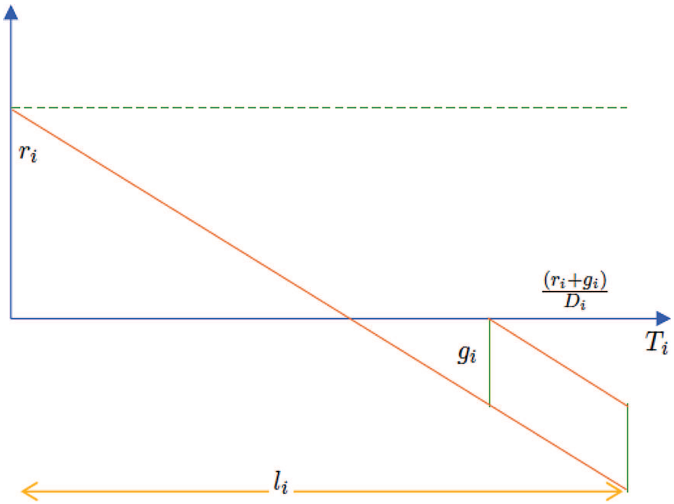

FIGURE 3. Inventory level of retailer when $\frac{(r_i+g_i)}{D_i} \leq l_i \leq \infty$.

Also, lead time is stochastic random variable and it follows exponential distributions. Hence, expected holding cost for the ordered quantity Q_i of the ith retailer is as follows:

$$\begin{aligned} H_{bi}^* &= h_i \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} E[H_{2i}] f_L(l_i) \mathrm{d}l_i \\ &= nh_i \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \left[\frac{r_i^2}{2D_i} + \frac{g_i^2 E[\delta_i]}{z_i} + \frac{r_i}{D_i}\{g_i(1-E[\delta_i]) - (D_i l_i - r_i)\} + \frac{g_i(D_i l_i - r_i)}{2z_i} E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right] \right. \\ &\quad + \frac{1}{2}\left\{ \frac{g_i^2 E[(1-\delta_i)^2]}{D_i} - \frac{g_i(D_i l_i - r_i)}{D_i} E\left[\frac{(1-\delta_i)^2}{(1-\delta_i-\frac{D_i}{z_i})}\right] - \frac{g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i} \right. \\ &\quad \left.\left. + \frac{(D_i l_i - r_i)^2}{D_i} E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right] \right\} \right] f_L(l_i) \mathrm{d}l_i. \end{aligned} \tag{5.10}$$

It is assumed that during this delay period, the batches remain in the manufacturer's stock house. So it causes an extra holding cost to the manufacturer. The extra inventory for this delayed delivery is $\sum_{i=1}^N \frac{g_i(1-\delta_i)(D_i l_i - r_i)}{D_i}$:

$$M_c = \sum_{i=1}^N \frac{g_i(1-\delta_i)(D_i l_i - r_i)}{D_i},$$

$$E[M_c] = \sum_{i=1}^N \frac{g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i}.$$

Therefore, in this scenario, the extra holding cost paid by manufacturer is given by

$$h_v \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \frac{ng_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i} f_L(l_i) \mathrm{d}l_i. \tag{5.11}$$

5.1.3. When g_i reaches late to the ith retailer and the lead time l_i lies in the range $\frac{(r_i+g_i)}{D_i} \leq l_i \leq \infty$

In this case, only shortages occur at the retailer's end. From Figure 3, shortage area of the ith retailer is obtained as follows:

$$H_{ci} = n\left[\frac{g_i^2}{2D_i} + g_i\left\{\frac{(D_i l_i - g_i - r_i)}{D_i}\right\}\right] = n\left\{\frac{g_i}{D_i}(D_i l_i - r_i) - \frac{g_i^2}{2D_i}\right\}. \tag{5.12}$$

Also, lead time is stochastic random variable and it follows exponential distributions. Hence, expected shortage cost for the ordered quantity Q_i of the ith retailer is given by

$$H_{ci}^* = h_i \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} E[H_{3i}] f_L(l_i)\mathrm{d}l_i = nc_i \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} \left\{\frac{g_i}{D_i}(D_i l_i - r_i) - \frac{g_i^2}{2D_i}\right\} f_L(l_i)\mathrm{d}l_i. \tag{5.13}$$

It is assumed that during this delay period, the batches remain in the manufacturer's stock house. So it causes an extra holding cost to the manufacturer. The extra inventory for this delayed delivery is $\sum_{i=1}^{N} \frac{g_i(1-\delta_i)(D_i l_i - r_i)}{D_i}$:

$$M_c = \sum_{i=1}^{N} \frac{g_i(1-\delta_i)(D_i l_i - r_i)}{D_i},$$

$$E[M_c] = \sum_{i=1}^{N} \frac{g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i}.$$

Therefore, in this scenario, the extra holding cost paid by manufacturer is

$$h_v \sum_{i=1}^{N} \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} \frac{n g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i} f_L(l_i)\mathrm{d}l_i. \tag{5.14}$$

5.2. Mathematical model formulation for manufacturer

Once retailers order is placed, the manufacturer start to begin production and produces the total order quantity $\sum_{i=1}^{N} Q_i$ of all retailers in n lots at one set up at a production rate R, and a finite number of units are added to the inventory until the production run has been completed. The manufacturer produces the items in a lot size of Q, where $Q = \sum_{i=1}^{N} Q_i$ and $Q_i = ng_i$ in each production of cycle length $\frac{Q}{R}$ and the retailers will receive the supply in each of size g_i for ith retailer. The first lot of size $\sum_{i=1}^{N} g_i$ is ready for shipment after time $\frac{\sum_{i=1}^{N} g_i}{R}$ just after starting of the production run, and the manufacturer continues making the delivery on average every $E[T] = E[(Q-\delta)/D]$, where $E[(Q-\delta)/D] = \sum_{i=1}^{N} E[(Q_i - n\delta_i)/D_i]$, units of time until the inventory level falls to zero.

Here, the production rate of manufacturer's non-defective items is greater than the retailer's demand rate, so manufacturer's inventory level will increase gradually. When the total required amount Q is fulfilled, the manufacturer stops producing items immediately.

Therefore, the manufacturer's inventory per production cycle can be obtained by subtracting the accumulated retailer inventory level from the accumulated manufacturer inventory level as follows.
Therefore, the total extra holding cost for the manufacturer, from both case (ii) and case (iii), is as follows:

$$h_v \sum_{i=1}^{N} \int_{\frac{r_i}{D_i}}^{\infty} \frac{n g_i E[(1-\delta_i)](D_i l_i - r_i)}{D_i} f_L(l_i)\mathrm{d}l_i. \tag{5.15}$$

In Figure 4, trapezium ABCD represents the joint inventory of the manufacturer-retailer system. The average inventory of the system is obtained as follows:

$$\frac{1}{2}\left[\frac{\sum_{i=1}^{N} g_i}{R} + \left\{\frac{Q-\delta}{D} + \frac{\sum_{i=1}^{N} g_i}{R} - \frac{Q}{R}\right\}\right] Q \frac{D}{Q-\delta} = \frac{DQ}{(Q-\delta)} \frac{\sum_{i=1}^{N} g_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\frac{Q}{(Q-\delta)}\right). \tag{5.16}$$

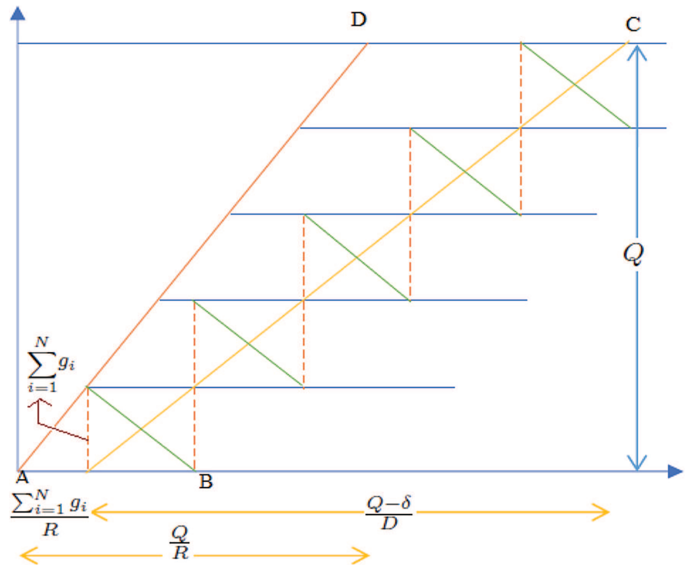

FIGURE 4. Inventory level of retailer when $\frac{r_i}{D_i} \leq l_i \leq \frac{(r_i+Q_i)}{D_i}$.

Average holding area of N retailers is

$$\sum_{i=1}^{N} \frac{g_i^2}{2D_i} \frac{D_i}{n(g_i-\delta_i)} = \sum_{i=1}^{N} \frac{g_i^2}{2n(g_i-\delta_i)}. \tag{5.17}$$

So, average holding area of manufacturer is

$$H_v = \frac{DQ}{(Q-\delta)} \frac{\sum_{i=1}^{N} g_i}{R} + \frac{Q}{2}\left(1 - \frac{D}{R}\frac{Q}{(Q-\delta)}\right) - \sum_{i=1}^{N} \frac{g_i^2}{2n(g_i-\delta_i)}. \tag{5.18}$$

Hence,

$$\begin{aligned}
E[H_v] &= E\left[\frac{DQ}{(Q-\delta)}\right]\frac{\sum_{i=1}^{N} g_i}{R} + \frac{Q}{2}\left\{1 - \frac{D}{R}E\left[\frac{Q}{(Q-\delta)}\right]\right\} - E\left[\sum_{i=1}^{N} \frac{g_i^2}{2n(g_i-\delta_i)}\right] \\
&= E\left[\sum_{i=1}^{N} \frac{Q_i D_i}{Q_i - n\delta_i}\right]\sum_{i=1}^{N} \frac{g_i}{R} + \frac{Q}{2}\left\{1 - \frac{D}{R}E\left[\sum_{i=1}^{N} \frac{Q_i}{Q_i - n\delta_i}\right]\right\} - E\left[\sum_{i=1}^{N} \frac{g_i^2}{2n(g_i-\delta_i)}\right] \\
&= \sum_{i=1}^{N} \frac{Q_i D_i}{n g_i(1-\gamma_i)} \sum_{i=1}^{N} \frac{g_i}{R} + \frac{Q}{2}\left\{1 - \frac{D}{R}\sum_{i=1}^{N} \frac{Q_i}{n g_i(1-\gamma_i)}\right\} - \sum_{i=1}^{N} \frac{g_i^2}{2n g_i(1-\gamma_i)} \\
&= \sum_{i=1}^{N} \frac{g_i}{R} \sum_{i=1}^{N} \frac{D_i}{(1-\gamma_i)} + \frac{Q}{2}\left\{1 - \frac{D}{R}\sum_{i=1}^{N} \frac{1}{(1-\gamma_i)}\right\} - \sum_{i=1}^{N} \frac{g_i}{2n(1-\gamma_i)}.
\end{aligned} \tag{5.19}$$

5.3. Decentralized model

Within the decentralized version situation, the manufacturer and the retailers make their decisions independently with the intention to enhance their own profits. Here we incorporate a Stackelberg gaming structure wherein the retailers act as the leader and the manufacturer as the follower. The manufacturer sets the wide variety of shipments and greening development stage of the product. Then, taking those reaction functions into consideration, the retailers determine optimal retail expenses of the product and batch sizes.

5.3.1. Retailer's profit function

The expected total cost of ith retailer includes ordering cost, screening cost, transportation cost, holding cost, back order cost, shortage cost. Thus Expected total profit for the ith retailer is

$$\begin{aligned}
\mathrm{EAC}_i^T(g_i,p_i) &= p_iQ_i - w_hQ_i - A_i - d_iQ_i - nT_p - H_{ai}^* - H_{bi}^* - B_i^* - H_{ci}^* \\
&= p_iQ_i - w_hQ_i - A_i - dQ_i - nT_p - nh_i\int_0^{\frac{r_i}{D_i}}\Bigg[l_i\left(r_i - \frac{1}{2}D_il_i\right) + \{g_i(1-E[\delta_i]) - D_il_i\}\frac{r_i}{D_i} \\
&\quad + \frac{g_i^2E[\delta_i]}{z_i} + \frac{1}{2D_i}\{g_i^2E[(1-\delta_i)^2] - 2g_i(1-E[\delta_i])D_il_i + (D_il_i)^2\}\Bigg]f_L(l_i)\mathrm{d}l_i \\
&\quad - nh_i\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\Bigg[\frac{r_i^2}{2D_i} + \frac{g_i^2E[\delta_i]}{z_i} + \frac{r_i}{D_i}\{g_i(1-E[\delta_i]) - (D_il_i - r_i)\} + \\
&\quad \frac{g_i(D_il_i-r_i)}{2z_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right] + \frac{1}{2}\Bigg\{\frac{g_i^2E[(1-\delta_i)^2]}{D_i} - \frac{g_i(D_il_i-r_i)}{D_i}E\left[\frac{(1-\delta_i)^2}{(1-\delta_i-\frac{D_i}{z_i})}\right] \\
&\quad - \frac{g_iE[(1-\delta_i)](D_il_i-r_i)}{D_i} + \frac{(D_il_i-r_i)^2}{D_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right]\Bigg\}\Bigg]f_L(l_i)\mathrm{d}l_i \\
&\quad - \frac{nb_i}{2}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left\{\frac{1}{D_i} + \frac{1}{z_i}E\left[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\right]\right\}(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i \\
&\quad - nc_i\int_{\frac{(r_i+g_i)}{D_i}}^{\infty}\left\{\frac{g_i}{D_i}(D_il_i-r_i) - \frac{g_i^2}{2D_i}\right\}f_L(l_i)\mathrm{d}l_i.
\end{aligned}$$

Thus average expected profit of the ith retailer is

$$\begin{aligned}
\mathrm{EAC}_i(g_i,p_i) &= \frac{D_i}{(1-\gamma_i)}(p_i - w_h - d_i) - \frac{D_i}{ng_i(1-\gamma_i)}(A_i + nT_p) \\
&\quad - \frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\Bigg[r_i(1-E[\delta_i]) - \frac{D_il_i}{2g_i} + \frac{D_ig_iE[\delta_i]}{z_i} + \frac{1}{2}\{g_iE[(1-\delta_i)^2] - 2(1-E[\delta_i])D_il_i \\
&\quad + \frac{(D_il_i)^2}{g_i}\}\Bigg]f_L(l_i)\mathrm{d}l_i - \frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\Bigg[\frac{r_i^2}{2g_i} + \frac{D_ig_iE[\delta_i]}{z_i} + r_i\{(1-E[\delta_i]) - \frac{(D_il_i-r_i)}{g_i}\} \\
&\quad + \frac{D_i(D_il_i-r_i)}{2z_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right] + \frac{1}{2}\Bigg\{g_iE[(1-\delta_i)^2] - (D_il_i-r_i)E\left[\frac{(1-\delta_i)^2}{(1-\delta_i-\frac{D_i}{z_i})}\right] \\
&\quad - E[(1-\delta_i)](D_il_i-r_i) + \frac{(D_il_i-r_i)^2}{g_i}E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right]\Bigg\}\Bigg]f_L(l_i)\mathrm{d}l_i \\
&\quad - \frac{b_i}{2g_i(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left\{1 + \frac{D_i}{z_i}E\left[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\right]\right\}(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i \\
&\quad - \frac{c_i}{1-\gamma_i}\int_{\frac{(r_i+g_i)}{D_i}}^{\infty}\left\{(D_il_i-r_i) - \frac{g_i^2}{2}\right\}f_L(l_i)\mathrm{d}l_i \\
&= \frac{D_i}{(1-\gamma_i)}(p_i - w_h - d_i) - \frac{D_i}{ng_i(1-\gamma_i)}(A_i + nT_p)
\end{aligned}$$

$$-\frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[r_i(1-E[\delta_i])-\frac{D_il_i}{2g_i}+\frac{D_ig_iE[\delta_i]}{z_i}+\frac{1}{2}\left\{g_iE[(1-\delta_i)^2]-2(1-E[\delta_i])D_il_i\right.\right.$$
$$\left.\left.+\frac{(D_il_i)^2}{g_i}\right\}\right]f_L(l_i)\mathrm{d}l_i-\frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left[\frac{r_i^2}{2g_i}+\frac{D_ig_iE[\delta_i]}{z_i}+r_i\{(1-E[\delta_i])-\frac{(D_il_i-r_i)}{g_i}\}\right.$$
$$+\frac{D_i(D_il_i-r_i)}{2z_i}A_{1i}+\frac{1}{2}\left\{g_iE[(1-\delta_i)^2]-(D_il_i-r_i)A_{2i}-E[(1-\delta_i)](D_il_i-r_i)\right.$$
$$\left.\left.+\frac{(D_il_i-r_i)^2}{g_i}A_{1i}\right\}\right]f_L(l_i)\mathrm{d}l_i-\frac{b_i}{2g_i(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left\{1+\frac{D_i}{z_i}A_{3i}\right\}(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i$$
$$-\frac{c_i}{1-\gamma_i}\int_{\frac{(r_i+g_i)}{D_i}}^{\infty}\left\{(D_il_i-r_i)-\frac{g_i^2}{2}\right\}f_L(l_i)\mathrm{d}l_i, \tag{5.20}$$

where

$$A_{1i}=E\left[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\right],\quad A_{2i}=E\left[\frac{(1-\delta_i)^2}{(1-\delta_i-\frac{D_i}{z_i})}\right],\quad \text{and } A_{3i}=E\left[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\right].$$

Proposition 5.1. *If $E[\delta_i]<1-\frac{D_i}{z_i}$, then the average expected profit of the ith retailer will be concave in g_i for given p_i, if* $\frac{2D_i(A_i+nT_p)}{n(1-\gamma_i)}+\frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[(D_il_i)^2-D_il_i\right]f_L(l_i)\mathrm{d}l_i+\frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left[2(d_il_i-r_i)^2A_{1i}-2r_i(d_il_i-r_i)^2+r_i^2\right]f_L(l_i)\mathrm{d}l_i+\frac{b_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}A_{3_i}\left(1+\frac{D_i}{z_i}\right)(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i>0.$

Proof. Differentiating profit function with respect to g_i, we obtain

$$\frac{\partial\mathrm{EAC}_i(g_i,p_i)}{\partial g_i}=\frac{D_i(A_i+nT_p)}{ng_i^2(1-\gamma_i)}-\frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[\frac{D_il_i}{2g_i^2}+\frac{D_iE[\delta_i]}{z_i}+\frac{1}{2}E[(1-\delta_i)^2]-\frac{(D_il_i)^2}{2g_i^2}\right]f_L(l_i)\mathrm{d}l_i$$
$$-\frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\left[-\frac{r_i}{2g_i^2}+\frac{D_iE[\delta_i]}{z_i}+\frac{r_i(D_il_i-r_i)}{g_i^2}+\frac{1}{2}(1-E[\delta_i])^2\right.$$
$$\left.-\frac{(D_il_i-r_i)^2}{2g_i^2}A_{1i}\right]f_L(l_i)\mathrm{d}l_i+\frac{b_i}{2g_i^2(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\left(1+\frac{D_i}{z_i}A_{3i}\right)(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i,$$
$$\frac{\partial^2\mathrm{EAC}_i(g_i,p_i)}{\partial g_i^2}=-\frac{2D_i}{ng_i^3(1-\gamma_i)}(A_i+nT_p)-\frac{h_i}{g_i^3(1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[(D_il_i)^2-D_il_i\right]f_L(l_i)\mathrm{d}l_i$$
$$-\frac{h_i}{g_i^3(1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i}{D_i}}\left[2(d_il_i-r_i)^2A_{1i}-2r_i(d_il_i-r_i)^2+r_i^2\right]f_L(l_i)\mathrm{d}l_i$$
$$-\frac{b_i}{g_i^3(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i}{D_i}}A_{3_i}\left(1+\frac{D_i}{z_i}\right)(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i.$$

Thus, if $E[\delta_i]<1-\frac{D_i}{z_i}$, then the average expected profit function of the ith retailer is concave in g_i for given p_i, if $\frac{\partial^2\mathrm{EAC}_i(g_i,p_i)}{\partial g_i^2}$ is negative. This implies $\frac{2D_i(A_i+nT_p)}{n(1-\gamma_i)}+\frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[(D_il_i)^2-D_il_i\right]f_L(l_i)\mathrm{d}l_i+\frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left[2(d_il_i-r_i)^2A_{1i}-2r_i(d_il_i-r_i)^2+r_i^2\right]f_L(l_i)\mathrm{d}l_i+\frac{b_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}A_{3_i}\left(1+\frac{D_i}{z_i}\right)(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i>0.$ □

Proposition 5.2. *If* $E[\delta_i] < 1 - \frac{D_i}{z_i}$ *and* $D_i l_i > r_i$, *then the average expected profit of the ith retailer will be concave in* p_i *for given* g_i *if* $\frac{\beta_i}{1-\gamma_i} + \frac{\beta_i^2 h_i}{1-\gamma_i}\left[\int_0^{\frac{r_i}{D_i}} \frac{l_i^2}{g_i} f_L(l_i)\mathrm{d}l_i + \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{1i} l_i (\frac{i}{2z_i} + \frac{1}{g_i}) f_L(l_i)\mathrm{d}l_i\right] + \frac{b_i\beta_i}{2g_i(1-\gamma_i)} J_{11} > 0.$

Proof. Differentiating profit function with respect to g_i, we obtain

$$\frac{\partial \mathrm{EAC}_i(g_i,p_i)}{\partial p_i} = -\frac{\beta_i}{1-\gamma_i}(p_i - w_h - d_i) + \frac{\beta_i(A_i + nT_p)}{ng_i(1-\gamma_i)} - \frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left[\frac{\beta_i}{l_i}2g_i - \frac{\beta_i g_i E[\delta_i]}{z_i}\right.$$
$$\left. -2\beta_i l_i(1-E[\delta_i]) - \frac{2\beta_i D_i l_i^2}{g_i}\right] f_L(l_i)\mathrm{d}l_i - \frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\left[-\frac{\beta_i g_i E[\delta_i]}{z_i} + \frac{\beta_i r_i l_i}{g_i}\right.$$
$$\left. +\beta_i(D_i l_i - r_i)A_{1i}(\frac{1}{2z_i} + \frac{1}{g_i}) + \frac{\beta_i l_i}{2}A_{2i} + \beta_i l_i E[1-\delta_i]\right] f_L(l_i)\mathrm{d}l_i$$
$$-\frac{b_i}{2g_i(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\left[-\beta_i(D_i l_i - r_i) - \frac{\beta_i}{z_i}A_{3i}(D_i l_i - r_i)^2\right] f_L(l_i)\mathrm{d}l_i,$$

$$\frac{\partial^2 \mathrm{EAC}_i(g_i,p_i)}{\partial p_i^2} = -\frac{\beta_i}{1-\gamma_i} - \frac{\beta_i^2 h_i}{1-\gamma_i}\left[\int_0^{\frac{r_i}{D_i}} \frac{l_i^2}{g_i} f_L(l_i)\mathrm{d}l_i + \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{1i} l_i\left(\frac{i}{2z_i} + \frac{1}{g_i}\right) f_L(l_i)\mathrm{d}l_i\right] - \frac{b_i\beta_i^2}{2g_i(1-\gamma_i)} J_{11}.$$

Now $D_i l_i > r_i$, thus $J_{11} = \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \{l_i + \frac{A_{3i}}{z_i}(D_i l_i - r_i)\} f_L(l_i)\mathrm{d}l_i > 0$ as $A_{3i} > 0$. Thus, if $E[\delta_i] < 1 - \frac{D_i}{z_i}$, then, the average expected profit of the ith retailer is concave in p_i for given g_i, if $\frac{\partial^2 \mathrm{EAC}_i(g_i,p_i)}{\partial g_i^2}$ is negative. This implies that

$\frac{\beta_i}{1-\gamma_i} + \frac{\beta_i^2 h_i}{1-\gamma_i}\left[\int_0^{\frac{r_i}{D_i}} \frac{l_i^2}{g_i} f_L(l_i)\mathrm{d}l_i + \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{1i} l_i (\frac{i}{2z_i} + \frac{1}{g_i}) f_L(l_i)\mathrm{d}l_i\right] + \frac{b_i\beta_i}{2g_i(1-\gamma_i)} J_{11} > 0.$ □

5.3.2. Manufacturer's profit function

The expected average cost of the manufacturer includes setup cost, warranty cost for defective items, holding cost along with extra holding cost for manufacturer. Thus Expected average profit for the manufacturer is

$$\mathrm{EAC}_v(n) = w_h \sum_{i=1}^N \frac{D_i}{(1-\gamma_i)} - S_v \sum_{i=1}^N \frac{D_i}{ng_i(1-\gamma_i)} - \nu \sum_{i=1}^N \frac{\gamma_i D_i}{(1-\gamma_i)} - h_v\left[\sum_{i=1}^N \frac{g_i}{R}\sum_{i=1}^N \frac{D_i}{(1-\gamma_i)} + \right.$$
$$\left.\frac{Q}{2}\left\{1 - \frac{D}{R}\sum_{i=1}^N \frac{1}{(1-\gamma_i)}\right\} - \sum_{i=1}^N \frac{g_i}{2n(1-\gamma_i)}\right] - h_v \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} \frac{(1-E[\delta_i])(D_i l_i - r_i)}{(1-\gamma_i)} f_L(l_i)\mathrm{d}l_i.$$

Thus,

$$\mathrm{EAC}_v(n) = w_h \sum_{i=1}^N \frac{D_i}{(1-\gamma_i)} - S_v \sum_{i=1}^N \frac{D_i}{ng_i(1-\gamma_i)} - \nu \sum_{i=1}^N \frac{\gamma_i D_i}{(1-\gamma_i)} - h_v\left[\sum_{i=1}^N \frac{g_i}{R}\sum_{i=1}^N \frac{D_i}{(1-\gamma_i)} + \right.$$
$$\left.\sum_{i=1}^N \frac{ng_i}{2}\left\{1 - \frac{1}{R}\sum_{i=1}^N D_i \sum_{i=1}^N \frac{1}{(1-\gamma_i)}\right\} - \sum_{i=1}^N \frac{g_i}{2n(1-\gamma_i)}\right]$$
$$-h_v \sum_{i=1}^N \int_{\frac{r_i}{D_i}}^{\infty} \frac{(1-E[\delta_i])(D_i l_i - r_i)}{(1-\gamma_i)} f_L(l_i)\mathrm{d}l_i. \tag{5.21}$$

Proposition 5.3. *The average expected profit function of the manufacturer is found to be concave in n if* $2S_v \sum_{i=1}^N \frac{D_i}{g_i(1-\gamma_i)} > h_v \sum_{i=1}^N \frac{g_i}{(1-\gamma_i)}$ *and the optimal number of the batch shipments is obtained as*

$$n^* = \sqrt{\frac{S_v \sum_{i=1}^{N} \frac{D_i}{g_i(1-\gamma_i)} - \frac{h_v}{2} \sum_{i=1}^{N} \frac{g_i}{(1-\gamma_i)}}{h_v \sum_{i=1}^{N} \frac{g_i}{2}\left(1 - \frac{D}{R}\sum_{i=1}^{N} \frac{1}{(1-\gamma_i)}\right)}}.$$

Proof. Differentiating Equation (5.21) with respect to n, we get

$$\frac{\partial \text{EAC}_v(n)}{\partial n} = \frac{S_v}{n^2} \sum_{i=1}^{N} \frac{D_i}{g_i(1-\gamma_i)} - h_v \left[\left(1 - \frac{1}{R} \sum_{i=1}^{N} D_i \sum_{i=1}^{N} \frac{1}{(1-\gamma_i)} \right) \sum_{i=1}^{N} \frac{g_i}{2} + \frac{1}{2n^2} \sum_{i=1}^{N} \frac{g_i}{1-\gamma_i} \right],$$

$$\frac{\partial^2 \text{EAC}_i(n)}{\partial^2 n} = -\frac{1}{n^3} \left[2S_v \sum_{i=1}^{N} \frac{D_i}{g_i(1-\gamma_i)} - h_v \sum_{i=1}^{N} \frac{g_i}{(1-\gamma_i)} \right].$$

So, the manufacturer's average expected profit function $\text{EAC}_v(n)$ will be concave in n if $\frac{\partial^2 \text{EAC}_i(n)}{\partial^2 n} < 0$. This implies that $2S_v \sum_{i=1}^{N} \frac{D_i}{g_i(1-\gamma_i)} > h_v \sum_{i=1}^{N} \frac{g_i}{(1-\gamma_i)}$.
If the above condition holds, then by using the first order necessary optimality condition, *i.e.*, solving $\frac{\partial \text{EAC}_v(n)}{\partial n} = 0$ for n, we get the optimal solution

$$n^* = \sqrt{\frac{S_v \sum_{i=1}^{N} \frac{D_i}{g_i(1-\gamma_i)} - 2h_v \sum_{i=1}^{N} \frac{g_i}{(1-\gamma_i)}}{h_v \sum_{i=1}^{N} \frac{g_i}{2}\left(1 - \frac{D}{R}\sum_{i=1}^{N} \frac{1}{1-\gamma_i}\right)}}.$$

□

5.4. Centralized model

In the centralized structure, the manufacturer and all the retailers act as a single decision maker. They jointly decide the optimal selling prices, number of batch shipments and batch sizes which lead to maximum average expected profit of the entire supply chain. The average expected profit of the supply chain is

$$\begin{aligned}
\text{EAP}(n, g_i, p_i) = {} & \sum_{i=1}^{N} \frac{D_i(p_i - d_i)}{(1-\gamma_i)} - S_v \sum_{i=1}^{N} \frac{D_i}{ng_i(1-\gamma_i)} - \nu \sum_{i=1}^{N} \frac{\gamma_i D_i}{(1-\gamma_i)} - h_v \left[\sum_{i=1}^{N} \frac{g_i}{R} \sum_{i=1}^{N} \frac{D_i}{(1-\gamma_i)} + \right. \\
& \left. \frac{Q}{2}\left\{1 - \frac{D}{R}\sum_{i=1}^{N} \frac{1}{(1-\gamma_i)}\right\} - \sum_{i=1}^{N} \frac{g_i}{2n(1-\gamma_i)} \right] - h_v \sum_{i=1}^{N} \int_{\frac{r_i}{D_i}}^{\infty} \frac{(1 - E[\delta_i])(D_i l_i - r_i)}{(1-\gamma_i)} f_L(l_i) \mathrm{d}l_i \\
& - \sum_{i=1}^{N} \left[\frac{D_i}{ng_i(1-\gamma_i)}(A_i + nT_p) + \frac{h_i}{1-\gamma_i} \int_0^{\frac{r_i}{D_i}} \left[r_i(1 - E[\delta_i]) - \frac{D_i l_i}{2g_i} + \frac{D_i g_i E[\delta_i]}{z_i} \right.\right. \\
& \left. + \frac{1}{2}\left\{ g_i E[(1-\delta_i)^2] - 2(1 - E[\delta_i])D_i l_i + \frac{(D_i l_i)^2}{g_i} \right\} \right] f_L(l_i)\mathrm{d}l_i + \frac{h_i}{1-\gamma_i} \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \left[\frac{r_i^2}{2g_i} \right. \\
& + \frac{D_i g_i E[\delta_i]}{z_i} + r_i \left\{ (1 - E[\delta_i]) - \frac{(D_i l_i - r_i)}{g_i} \right\} + \frac{D_i(D_i l_i - r_i)}{2z_i} A_{1i} + \frac{1}{2}\left\{ g_i E[(1-\delta_i)^2] \right. \\
& \left.\left. - (D_i l_i - r_i)A_{2i} - E[(1-\delta_i)](D_i l_i - r_i) + \frac{(D_i l_i - r_i)^2}{g_i} A_{1i} \right\} \right] f_L(l_i)\mathrm{d}l_i \\
& + \frac{b_i}{2g_i(1-\gamma_i)} \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} \left\{ 1 + \frac{D_i}{z_i} A_{3i} \right\} (D_i l_i - r_i)^2 f_L(l_i)\mathrm{d}l_i \\
& \left. + \frac{c_i}{1-\gamma_i} \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} \left\{ (D_i l_i - r_i) - \frac{g_i^2}{2} \right\} f_L(l_i)\mathrm{d}l_i \right]. \qquad (5.22)
\end{aligned}$$

Proposition 5.4. *If* $E[\delta_i] < 1 - \frac{D_i}{z_i}$, *then the average expected system profit of the ith retailer-manufacturer will be concave in* g_i *for given* p_i *and* n *if*
$\sum_{i=i}^{N} \frac{1}{g_i^3}\left[\frac{2D_i(S_v+A_i+nT_p)}{n(1-\gamma_i)} + \frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\{(D_il_i)^2 - D_il_i\}f_L(l_i)\mathrm{d}l_i + \frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\{2(d_il_i-r_i)^2A_{1i} - 2r_i(d_il_i-r_i)^2 + r_i^2\}f_L(l_i)\mathrm{d}l_i + \frac{b_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{3_i}\left(1+\frac{D_i}{z_i}\right)(D_il_i-r_i)^2 f_L(l_i)\mathrm{d}l_i\right] > 0.$

Proof. Differentiating the profit function (5.22) with respect to g_i, we obtain

$$\begin{aligned}
\frac{\partial \mathrm{EAP}(n,g_i,p_i)}{\partial g_i} &= \sum_{i=i}^{N}\frac{D_iS_v}{ng_i^2(1-\gamma_i)} - h_v\left[\left(\frac{D}{R}-\frac{1}{2n}\right)\sum_{i=i}^{N}\frac{1}{(1-\gamma_i)} + \frac{n}{2}\left(1-\frac{D}{R}\sum_{i=i}^{N}\frac{1}{(1-\gamma_i)}\right)\right] \\
&+\sum_{i=i}^{N}\left[\frac{D_i}{ng_i^2(1-\gamma_i)}(A_i+nT_p) + \frac{h_v\gamma_i}{(1-\gamma_i)}\right. \\
&-\frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\left\{\frac{D_il_i}{2g_i^2} + \frac{D_iE[\delta_i]}{z_i} + \frac{1}{2}E[(1-\delta_i)^2] - \frac{(D_il_i)^2}{2g_i^2}\right\}f_L(l_i)\mathrm{d}l_i \\
&-\frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\left\{-\frac{r_i}{2g_i^2} + \frac{D_iE[\delta_i]}{z_i} + \frac{r_i(D_il_i-r_i)}{g_i^2} + \frac{1}{2}(1-E[\delta_i])^2\right. \\
&\left.\left.-\frac{(D_il_i-r_i)^2}{2g_i^2}A_{1i}\right\}f_L(l_i)\mathrm{d}l_i + \frac{b_i}{2g_i^2(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}(1+\frac{D_i}{z_i}A_{3i})(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i\right].
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial^2 \mathrm{EAP}(n,g_i,p_i)}{\partial g_i^2} &= \sum_{i=i}^{N} -\frac{2D_iS_v}{ng_i^3(1-\gamma_i)} + \sum_{i=i}^{N}\left[-\frac{2D_i}{ng_i^3(1-\gamma_i)}(A_i+nT_p)\right. \\
&-\frac{h_i}{g_i^3(1-\gamma_i)}\int_0^{\frac{r_i}{D_i}}\left\{(D_il_i)^2 - D_il_i\right\}f_L(l_i)\mathrm{d}l_i \\
&-\frac{h_i}{g_i^3(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\left\{2(d_il_i-r_i)^2A_{1i} - 2r_i(d_il_i-r_i)^2 + r_i^2\right\}f_L(l_i)\mathrm{d}l_i \\
&\left.-\frac{b_i}{g_i^3(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{3_i}(1+\frac{D_i}{z_i})(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i\right].
\end{aligned}$$

Hence, if $E[\delta_i] < 1 - \frac{D_i}{z_i}$, the average expected profit of the ith retailer-manufacturer is concave in g_i for given p_i if $\frac{\partial^2 \mathrm{EAC}_i(g_i,p_i)}{\partial g_i^2}$ is negative. This implies that $\sum_{i=i}^{N}\frac{1}{g_i^3}\left[\frac{2D_i(S_v+A_i+nT_p)}{n(1-\gamma_i)} + \frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\{(D_il_i)^2 - D_il_i\}f_L(l_i)\mathrm{d}l_i + \frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}\{2(d_il_i-r_i)^2A_{1i}-2r_i(d_il_i-r_i)^2+r_i^2\}f_L(l_i)\mathrm{d}l_i + \frac{b_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{3_i}(1+\frac{D_i}{z_i})(D_il_i-r_i)^2f_L(l_i)\mathrm{d}l_i\right] > 0.$ □

Proposition 5.5. *If* $E[\delta_i] < 1 - \frac{D_i}{z_i}$ *and* $D_il_i > r_i$, *then the average expected profit of the ith retailer-manufacturer will be concave in* p_i *for given* g_i *if*
$\frac{\beta_i}{1-\gamma_i} + \frac{\beta_i^2h_i}{1-\gamma_i}\left[\int_0^{\frac{r_i}{D_i}}\frac{l_i^2}{g_i}f_L(l_i)\mathrm{d}l_i + \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}} A_{1i}l_i(\frac{i}{2z_i}+\frac{1}{g_i})f_L(l_i)\mathrm{d}l_i\right] + \frac{b_i\beta_i}{2g_i(1-\gamma_i)}J_{11} > 0.$

Proof. Differentiating profit function given in Equation (5.22) with respect to g_i, we obtain

$$\frac{\partial \mathrm{EAC}_i(g_i,p_i,n)}{\partial p_i} = \sum_{i=1}^{N}\frac{-\beta_i(p_i-d_i)}{(1-\gamma_i)} - S_v\sum_{i=1}^{N}\frac{-\beta_i}{ng_i(1-\gamma_i)} - \nu\sum_{i=1}^{N}\frac{-\gamma_i\beta_i}{(1-\gamma_i)} - h_v\Bigg\{\sum_{i=1}^{N}\frac{g_i}{R}\sum_{i=1}^{N}\frac{-\beta_i}{(1-\gamma_i)} + $$
$$-\frac{Q}{2}\sum_{i=1}^{N}\frac{-\beta_i}{R}\sum_{i=1}^{N}\frac{1}{(1-\gamma_i)}\Bigg\} - h_v\sum_{i=1}^{N}\int_{\frac{r_i}{D_i}}^{\infty}\frac{-(1-E[\delta_i])\beta_i}{(1-\gamma_i)}f_L(l_i)\mathrm{d}l_i$$
$$-\sum_{i=1}^{N}\Bigg[-\frac{\beta_i(A_i+nT_p)}{ng_i(1-\gamma_i)} + \frac{h_i}{1-\gamma_i}\int_0^{\frac{r_i}{D_i}}\Bigg[\frac{\beta_i}{l_i}2g_i - \frac{\beta_i g_i E[\delta_i]}{z_i}$$
$$-2\beta_i l_i(1-E[\delta_i]) - \frac{2\beta_i D_i l_i^2}{g_i}\Bigg]f_L(l_i)\mathrm{d}l_i + \frac{h_i}{1-\gamma_i}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\Bigg[-\frac{\beta_i g_i E[\delta_i]}{z_i} + \frac{\beta_i r_i l_i}{g_i}$$
$$+\beta_i(D_i l_i - r_i)A_{1i}\Big(\frac{1}{2z_i}+\frac{1}{g_i}\Big) + \frac{\beta_i l_i}{2}A_{2i} + \beta_i l_i E[1-\delta_i]\Bigg]f_L(l_i)\mathrm{d}l_i$$
$$+\frac{b_i}{2g_i(1-\gamma_i)}\int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}}\Bigg[-\beta_i(D_i l_i - r_i) - \frac{\beta_i}{z_i}A_{3i}(D_i l_i - r_i)^2\Bigg]f_L(l_i)\mathrm{d}l_i\Bigg],$$

$$\frac{\partial^2 \mathrm{EAC}_i(g_i,p_i,n)}{\partial p_i^2} = -\sum_{i=1}^{N}\Bigg[\frac{\beta_i}{1-\gamma_i} + \frac{\beta_i^2 h_i}{1-\gamma_i}\Bigg\{\int_0^{\frac{r_i}{D_i}}\frac{l_i^2}{g_i}f_L(l_i)\mathrm{d}l_i + \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}A_{1i}l_i\left(\frac{i}{2z_i}+\frac{1}{g_i}\right)f_L(l_i)\mathrm{d}l_i\Bigg\}$$
$$+\frac{b_i\beta_i^2}{2g_i(1-\gamma_i)}J_{11}\Bigg].$$

Now, $D_i l_i > r_i$ implies that $J_{11} = \int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i}{D_i}}\{l_i + \frac{A_{3_i}}{z_i}(D_i l_i - r_i)\}f_L(l_i)\mathrm{d}l_i > 0$, as $A_{3i} > 0$. Thus, if $E[\delta_i] < 1 - \frac{D_i}{z_i}$, then the average expected profit of the ith retailer-manufacturer is concave in p_i for given g_i if $\frac{\partial^2 \mathrm{EAC}_i(g_i,p_i)}{\partial g_i^2}$ is negative. Which implies

$$\frac{\beta_i}{1-\gamma_i}+\frac{\beta_i^2 h_i}{1-\gamma_i}\Bigg[\int_0^{\frac{r_i}{D_i}}\frac{l_i^2}{g_i}f_L(l_i)\mathrm{d}l_i+\int_{\frac{r_i}{D_i}}^{\frac{(r_i+g_i)}{D_i}}A_{1i}l_i\Big(\frac{i}{2z_i}+\frac{1}{g_i}\Big)f_L(l_i)\mathrm{d}l_i\Bigg]+\frac{b_i\beta_i}{2g_i(1-\gamma_i)}J_{11} > 0. \qquad \square$$

Proposition 5.6. *The average expected system profit function is found to be concave in* n *if* $2\left(S_v\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)} + \sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)}\right) > h_v\sum_{i=1}^{N}\frac{g_i}{(1-\gamma_i)}$ *and the optimal number of the batch shipments is obtained as*

$$n^* = \sqrt{\frac{S_v\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)}+\sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)}-\frac{h_v}{2}\sum_{i=1}^{N}\frac{g_i}{(1-\gamma_i)}}{h_v\sum_{i=1}^{N}\frac{g_i}{2}\left(1-\frac{D}{R}\sum_{i=1}^{N}\frac{1}{(1-\gamma_i)}\right)}}.$$

Proof. Differentiating Equation (5.22) with respect to n, we get

$$\frac{\partial \mathrm{EAP}(n,g_i,p_i)}{\partial n} = \frac{S_v}{n^2}\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)} - h_v\Bigg[\Bigg(1-\frac{D}{R}\sum_{i=1}^{N}\frac{1}{(1-\gamma_i)}\Bigg)\sum_{i=1}^{N}\frac{g_i}{2}+\frac{1}{2n^2}\sum_{i=1}^{N}\frac{g_i}{1-\gamma_i}\Bigg]$$

$$+\frac{1}{n^2}\sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)},$$

$$\frac{\partial^2\mathrm{EAP}(n,g_i,p_i)}{\partial^2 n}=-\frac{1}{n^3}\left[2S_v\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)}+\sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)}-h_v\sum_{i=1}^{N}\frac{g_i}{(1-\gamma_i)}\right].$$

So, the average expected profit function $\mathrm{EAP}(n,g_i,p_i)$ will be concave in n if $\frac{\partial^2\mathrm{EAC}_i(n)}{\partial^2 n}<0$. Which implies $2\left(S_v\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)}++\sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)}\right)>h_v\sum_{i=1}^{N}\frac{g_i}{(1-\gamma_i)}$.

If the above condition holds, then by using the first order optimality condition, *i.e.*, solving $\frac{\partial\mathrm{EAC}_v(n)}{\partial n}=0$, for n we get the optimal solution

$$n^*=\sqrt{\frac{S_v\sum_{i=1}^{N}\frac{D_i}{g_i(1-\gamma_i)}+\sum_{i=1}^{N}\frac{D_iA_i}{g_i(1-\gamma_i)}-\frac{h_v}{2}\sum_{i=1}^{N}\frac{g_i}{(1-\gamma_i)}}{h_v\sum_{i=1}^{N}\frac{g_i}{2}\left(1-\frac{D}{R}\sum_{i=1}^{N}\frac{1}{1-\gamma_i}\right)}}.\qquad\square$$

6. Solution using Genetic Algorithm

It is generally accepted that GA can be habituated to solve decision making problems, which derives its demeanor from an inventory biological metaphor. In simple GA, Goldberg [14] desultorily engendered solution strings, are composed into a population. Strings are decoded and then evaluated according to a fitness/objective function. Following this, individuals are culled to undergo reproduction to engender progeny (individuals for the next generation). The process of engendering progeny consists of two operations. Firstly, culled solution strings are recombined utilizing a recombination operator, *i.e.*, crossover, where two or more parent solution strings provide elements of their string to engender an incipient solution. Secondly, mutation is applied to the progeny following the generation of a consummate population of scion solution strings. The progeny population supersedes the parent population. Each iteration of the process is called a generation. The GA is customarily run for a fine-tuned number of generations or until no amelioration in solution fitness for a number of generations. The Genetic Algorithm procedure is as given below (the details description of the process is presented in Appendix A).

Genetic Algorithm (GA) is a computerized, population predicated, probabilistic search ecumenical optimization technique. GAs are designed to simulate the process of evolution predicated on the Darwin's natural cull principle "Survival of the fittest". It commences with an initial population of individuals representing the possible solutions of a quandary and then endeavors to solve the quandary. Every individual has a concrete characteristic that makes them more or less fittest member of the population. Since in ecosystem, competition among living individuals for environmental issues such as air, water and soil are frequently occurs, the fittest individuals dominates the more impuissant ones. So, the fittest members have more opportunity to compose a mating pool and engendering progenies have desirable characteristics got from the parents than the unfit members. This method is efficient for finding the optimal or near optimal solution of an involute system because it does not impose many inhibitions required by the traditional methods. The concept of GA was first introduced by John Holland and his students and colleagues in the years of 1960–1975. Then it has been extensively used and modified to solve intricate systems in different fields of science and technology. In this model, a Genetic Algorithm with Variable Population (GAVP) in fuzzy age based criteria is used to reproduce a new chromosome at crossover level (*cf.* Maiti [29]). The pseudo codes of GAVP is given in Algorithm 1.

6.1. Convergency of the GA

Following, Goldberg [14] the convergency of GA is predicted to the optimal population $P(t)$ at the number of generations t with population mean at generation t is given by $\overline{f}(t)=nP(t)$ and the variance $\sigma^2(t)=$

Algorithm 1

Step-1	:	**Start**
Step-2	:	Set iteration counter $t = 0$, $Maxsize = 200$, $\varepsilon = 0$ and $p_m(0) = 0.9$.
Step-3	:	Randomly generate **Initial** population $P(t)$.
Step-4	:	**Evaluate** initial population $P(t)$.
Step-5	:	Set Maxfit = Maximum fitness in P(t) and Avgfit = Average fitness of P(t).
Step-6	:	While $(Maxfit - Avgfit \leq \varepsilon)$ **do**
Step-7	:	$t = t + 1$
Step-8	:	Increase age of each chromosome.
Step-9	:	**For** each pair of parents **do**
Step-10	:	Determine probability of crossover p_c for the selected pair of parents.
Step-11	:	Perform crossover with probability p_c.
Step-12	:	**End** for
Step-13	:	**For** each offspring perform mutation with probability p_m **do**
Step-14	:	Store offsprings into offspring set.
Step-15	:	**End** for
Step-16	:	Evaluate P(t).
Step-17	:	Remove from P(t) all individuals with age greater than their lifetime.
Step-18	:	Select a percent of better offsprings from the set and insert into P(t),
	:	such that maximum size of the population is less than Maxsize.
Step-19	:	Remove all offsprings from the offspring set.
Step-20	:	Reduce the value of the probability of mutation p_m.
Step-21	:	**End While**
Step-22	:	Output: Best chromosome of P(t).
Step-23	:	**End** algorithm.

$nP(t)(1 - P(t))$, here $n = pop\_size$.
The increment in population mean fitness can be computed as follows:

$$\overline{f}(t+1) - \overline{f}(t) = \frac{\sigma^2(t)}{\overline{f}(t)} \cdot$$

The increase of the proportion of the population becomes

$$n(P(t+1) - P(t)) = \frac{nP(t)(1 - P(t))}{nP(t)} \cdot$$

Approximating the difference equation with the corresponding differential equation we obtain a simple convergence model expressing the proportion $p(t)$ in function of the number of generations t

$$\frac{\mathrm{d}P(t)}{\mathrm{d}t} = \frac{1 - P(t)}{n} \cdot$$

The above differential equation leads to a convergent solution

$$P(t) = 1 - (1 - P(0))e^{-t/n}.$$

To calculate the convergency speed, we compute the number of population n it takes to let the proportion $p(t)$ come arbitrarily close to 1 or $1 - \epsilon$.

We may also compute the generation at which the global optimum is expected to be found with a given probability. The probability that at least one of the strings in the population consists of all ones is given by Prob(optimality) = $1 - [1 - P^l(t)]^n$, where l = no of generations. For instance, in our example, Maximum no of generations $l = 500$ and pop size $n = 100$ Prob(optimality) = 99%.

The convergency rate of the proportion $P(t)$ for different no generation t yields the following graph for three different pop size (*cf*. Fig. 5).

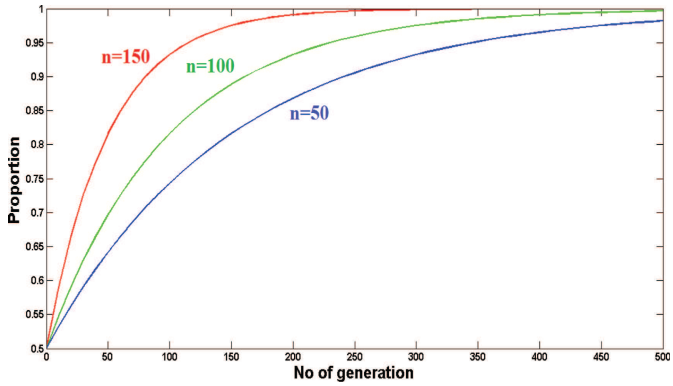

FIGURE 5. Proportion *vs* No. of generation.

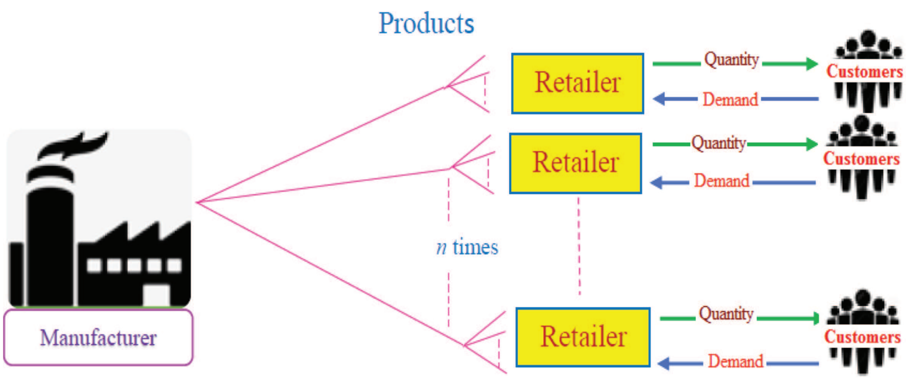

FIGURE 6. The single manufacturer multi retailer supply chain system.

7. NUMERICAL EXAMPLE

To illustrate the proposed model, let us consider an inventory system with the following data:

Example 7.1. Manufacturer set up a production center with set-up cost S_v = \$300/set up, where goods are produces at a production rate $R = 2500$ units/year. The holding cost for manufacturer is (h_v) = \$3 per unit/year. The manufacturer's unit warranty cost per defective item for the retailer is ν = \$3.5/unit, where transportation cost per batch shipment is T_p = \$15.

The input costs of the retailer are used as shown in Table 2.

The problem is to find the optimal batch size (g_i), retail prices (p_i) and number batches delivered to each retailers, which maximizes the average integrated total profit. The results are shown in Table 3.

The p.d.f. of the lead time of ith retailer is assumed as $f_L(l_i) = \lambda_i e^{-\lambda_i l_i}$.

Here we have studied a system consisting of a single manufacturer and two retailers. Concavity of the profit function of the whole supply chain system with respect to its decision variables is observed for the chosen parameter-values. The nature of the profit functions of both the retailers has been found to be concave for different values of n, g_1, g_2, p_1, p_2 in all situations.

Utilizing the prescribed solution algorithm, we find the optimal batch sizes as well as the authoritatively mandating quantity of the retailers, selling price of the product and the number of batch shipments in the decentralized model. Tables 3 and 4 show the optimal results for both the decentralized and centralized models. It can be descried that from Tables 3 and 4 that the average profit in the decentralized model is less than

TABLE 2. Input cost parameters for retailers.

Ordering cost	$A_1 = \$150$/order	$A_2 = \$150$/order
Unit wholesale price	$w_{h1} = \$70$/unit	$w_{h2} = \$80$/unit
Holding cost for retailer/item/unit time	$h_1 = \$8$	$h_2 = \$7$
Screening cost per unit time	$d_1 = \$11$	$d_2 = \$9$
Back ordering cost per unit time	$b_1 = \$4$	$b_2 = \$3$
Shortage cost per unit time	$c_1 = \$3$	$c_2 = \$2$
Basic market demand	$\alpha_1 = 850$ units	$\alpha_2 = 900$ units
Consumer sensitivity	$\beta_1 = 1.5$	$\beta_2 = 1.2$
The screening rate	$z_1 = 20105$	$z_2 = 205241$
The defective rate, a binomial random variable with	$\gamma_1 = 0.01$	$\gamma_2 = 0.012$
Rate parameter of exponential distribution	$\lambda_1 = 0.8$	$\lambda_2 = 0.7$

TABLE 3. Optimal result for decentralize model.

Model	n^*	g_1^*	p_1^*	B_1^*	EAC_1^*	EAC_v	EAP
Decentralized	8	15.94	146.83	0.21	12 048.25	51 427.69	74 003.23
		g_2^*	p_2^*	B_2^*	EAC_2^*		
		17.64	156.16	0.13	10 527.29		

TABLE 4. Optimal result for centralize model.

Model	n^*	g_1^*	g_2^*	p_1^*	p_2^*	B^*	EAP
Centralized	5	12.25	14.73	85.26	100.97	0.22	83834.24

that of centralized model. For the centralized decision making scenario, the manufacturer and all the retailers act as a single business manager and jointly make their optimal decisions in order to achieve highest whole system profit. So, the retailers can provide the product to the customers at a more frugal price than that of the decentralized case. That's why lower priced product increases the consumer demand significantly. Then the retailers order more quantity from the manufacturer. As a result, integration between the manufacturer and the retailers increases the total system profit significantly. We optically canvass that the retail prices of the product decrease for both the retailers. This results in an increase of profit for the entire supply chain and the whole system profit increased by \$83834.24 − \$74003.23 = \$9831.01.

8. Sensitivity analysis

In this section, the impacts of several important parameters on the expected average profit of the whole supply chain system are analyzed. We change one parameter value at a time keeping other parameter values fixed. The numerical results are presented in Table 5–7. Results of Tables 3 and 4 indicate the following observations:

Effects of demand rate on optimal solution:

The impacts of the rudimental market demands α_1 and α_2 on the average expected profit of the system can be realized from the optimal results given in Table 5. Whenever the customer demand increases, the retailers places order for more items from the manufacturer. Additionally, with the increment of the rudimentary market

TABLE 5. Optimal results for different values of α_1, α_2, β_1 and β_2.

Parameter	Value	n	g_1	g_2	p_1	p_2	B^*	EAP
	750	5	11.27	14.24	83.39	99.27	0.16	80 565.75
α_1	850	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	950	5	12.67	15.53	87.12	105.21	0.25	89 686.24
	800	5	11.62	13.54	82.46	98.42	0.14	76 469.53
α_2	900	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	1000	5	12.51	15.53	87.12	101.21	0.23	90 291.21
	1.3	5	11.62	14.33	87.12	102.66	0.18	89 157.35
β_1	1.5	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	1.7	5	12.59	15.13	82.46	99.27	0.24	77 929.35
	1.0	5	11.97	13.54	87.12	102.67	0.17	90 493.19
β_2	1.2	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	1.4	5	12.67	15.43	83.39	100.12	0.23	76 386.18

TABLE 6. Optimal results for different values of defective percentage γ_1 and γ_2.

Defective percentage	Value	n	g_1	g_2	p_1	p_2	B^*	EAP
	0.01	5	12.15	14.73	85.26	100.97	0.22	83 834.24
γ_1	0.02	5	13.81	15.94	87.12	102.66	0.26	76 511.63
	0.03	5	16.27	17.14	88.98	104.36	0.29	57 222.08
	0.012	5	12.15	14.73	85.26	100.97	0.22	83 834.24
γ_2	0.024	5	14.62	16.54	88.05	103.51	0.29	48 411.94
	0.036	5	16.27	17.74	89.91	105.21	0.31	25 308.01

demand, the retailers get the opportunity to raise the retail prices and earn more revenue. This leads the supply chain system to a better profit level.

Effects of consumer sensitivity on optimal solution:

The expected average profit of the entire supply chain is highly sensitive with deference to the parameters β_1 and β_2. It can be descried that, both the retailers truncate their selling prices of the product when the values of β_1 and β_2 increase. As higher values of β_1 and β_2 designate that the customers prefer to purchase more frugal products, so in order to keep the consumer demand intact, the retailers abbreviate their selling prices. Additionally, the inductively authorized quantities of both the retailers decrease as β_1 and β_2 increase. Consequently, the average expected profit of all individual supply chain players as well as the entire supply chain decrease gradually as β_1 and β_2 increase.

Effects on defective percentage:

Table 6 shows the sensitivity analysis on the defective percentage of δ_i (we assume that δ_i are random in nature and $E[\delta_1] = \gamma_1$ and $E[\delta_2] = \gamma_2$ consequently). It is interesting to note that when γ_1 and γ_2 increases, the maximum back ordering quantity as well as batch sizes and retail prices increase. The larger the γ_1, γ_2 values, the smaller the profit of the integrated model.

Table 7. Optimal results for different values of holding costs, h_v, h_1 and h_2.

Holding Costs	Value	n	g_1	g_2	p_1	p_2	B^*	EAP
	1.5	5	11.32	14.13	83.39	99.27	0.19	89 870.67
h_v	3	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	5	5	15.03	17.45	92.71	106.91	0.32	59 680.77
	4	5	11.16	14.01	81.53	96.72	0.24	83 441.28
h_1	8	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	12	5	12.23	14.79	88.52	105.63	0.21	86 105.61
	5	5	11.07	13.95	80.59	95.87	0.25	83 120.25
h_2	7	5	12.15	14.73	85.26	100.97	0.22	83 834.24
	9	5	12.39	14.91	89.15	104.79	0.21	84 768.39

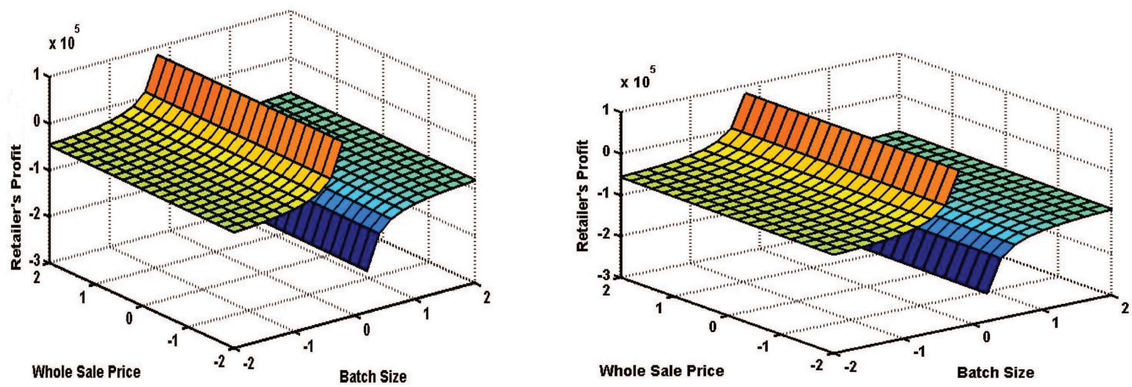

Figure 7. EAC_1 and EAC_2 with respect to g_1, g_2, p_1 and p_2.

Effects on Holding costs:

Table 7 shows that the retailer's holding cost has a reverse impact as the manufacturer's holding cost does on the optimal solution. As the retailer's holding cost increases, the size of shipments per production batch increases, and the cost reduction of the integrated model also increases. Again for manufacturer's holding cost increases, the size of shipments per production batch increases, and the cost reduction of the integrated model also decreases.

Effects on back-order:

The impacts of the rudimental market demands (α_1 and α_2), consumer sensitivity (β_1 and β_2) and defective percentage (γ_1 and γ_2) on back-order can be realized from the above Tables 5 and 6. With the increment of the rudimental market demand, sensitivity and defective percentage in a product it is visually examined that back-order authoritatively mandating quantity increases. Again from Table 7, it is observed that back-order has a reverse impact as the holding costs of manufacturer and retailer do on the optimal solution.

To study the feasibility of the proposed model the sensitivity analysis of EAC_1 and EAC_2 with respect to g_1, g_2, p_1 and p_2 are presented graphically in Figure 7.

9. Discussion

It can be observed from Tables 3 and 4 that the centralized model results more profit level than the decentralized model. In our proposed model, we assume that, the customer demand is linearly dependent on its retail price. In the centralized decision making scenario, the manufacturer and all the retailers acts as a single business manager and jointly make their optimal decisions in order to achieve highest system profit. So, the retailers can provide the product to the customers at a cheaper price than that of the decentralized case. That's why lower priced product increases the consumer demand significantly. Then the retailers order more quantity from the manufacturer. As a result, integration between the manufacturer and all the retailers increase the total system profit significantly. Moreover, the retail prices of the product also decrease for both the retailers. This results in an increase of profit for the entire supply chain system.

10. Practical implementation and future research

This model single manufacturer multi retailer inventory model with imperfect quality of items is useful in real life scenarios in the private, public as well as government sectors. A manufacturer may supply his/her product to different retailers to fulfil their requirements. For example, in health care industries, the vendor supplies instruments to different hospitals according to their requirements. Let a health care industry named Cipla decide to distribute health care instruments like medicine, syringe, *etc.* among unwell people admitted in different hospitals and nursing home in various cities like Tamluk, Contai, Haldia, Minapore, Kharagpur, *etc.* in both East and West Midnapore, West Bengal. So Cipla, a private agency settle different retailers in above said cities to distribute medical commodities in different hospitals and nursing homes of those cities namely Tamluk, Contai, Haldia, Midnapore, Kharagpur, *etc.* of both East and West Midnapore District, West Bengal. Mr. Susmit Saha a retailer in Contai has fair price shopes in different locations at Contai, Ramnagar, Digha. He can distribute medical equipment to different hospitals and nursing homes of those places through various paths. Like S. Saha, other retailers from different cities place the total order quantity Q. Then the manufacturer produces the total order quantity $\sum_{i=1}^{N} Q_i$ of all retailers in one set up at a constant production rate R, and then transfers the ordered quantities to the ith retailer, in n equal batches of size g_i. An arriving lot may contain some defective items. We assume that the number of defective items, δ_i, in an arriving order of size g_i. Upon the arrival of the order, A 100% screening process of the lot is conducted at the retailer's place. Hence defective items in each lot are discovered and returned to the manufacturer at the time of delivery of the next lot. Here, we see that it is a very helpful easier and scientific way that, medical commodities are distributed among right people in the right way. Hence admitted people in the various nursing home and hospitals get essential services and treated well. Here demand is price sensitive. If the price is low, then the market demand is more; on the other hand, if the price is high then the market demand of the product is less. Shortages are allowed and assumed to be completely backlogged at each retailer's end. Thus, the present investigation will be helpful to implement the above real-life phenomena.

In our proposed model, we have considered that one manufacturer is trading with multiple retailers for a single product. Our model can be further extended considering multiple items. One can also implement any suitable coordination scheme between the manufacturer and all the retailers to improve their profits in the decentralized scenario. Furthermore, our proposed model can be enriched by considering more realistic assumptions, such as combined equal and unequal sized batch shipments, quality dependent demand and multiple manufacturers trading with multiple retailers. Also, The formulate models have great opportunity to extend with breakable items, imperfect production and for another type stochastic distributions. The model can be formulated with fuzzy, fuzzy-rough and fuzzy-random coefficients. Moreover, the model can be solved by using other type of optimisation techniques.

11. Conclusion with managerial implications and insight

In this paper, we have considered a two-level supply chain model consisting of a single manufacturer and multiple retailers under price sensitive customer demand and stochastic lead time. The manufacturer produces a single product at a constant production rate and transfers it to the retailers in some equal batch shipments. An arriving lot may contain some defective items (Which is random in nature) δ_i, in an order of size g_i. On the arrival of order, A 100% screening process of the lot is conducted at the retailer's place. Hence defective items which incur a warranty cost of ν for the manufacturer in each lot are discovered and returned to the manufacturer. The manufacturer will sell the defective items at a reduced price to other retailer or recycle it for delivery of the next lot. Due to the stochastic nature of lead time the batches may reach early or late to the retailers. Depending on the length of lead time duration, all three possible cases are taken into consideration at the retailers end. Decentralized and centralized models are formulated. A solution algorithm is derived to find the optimal solutions in the decentralized model with a Stackelberg gaming structure. It is found that cooperation between the manufacturer and the retailers becomes more profitable for the entire supply chain whenever the consumer demand is price dependent and lead time of delivering the ordered quantity follows a normal distribution. By collaborating with each other, the supply chain players can sell the product at a cheaper price to the end customers and enhance market demand. This leads to a significant increment of the profitability of the whole supply chain.

Supply chain management is an integral part in most of the business system and essential company success through boosting of customer services, reducing of operating costs, improving of financial position. It also ensures sustain of human life, protection of climate extremes, improve the economic growth as well as benefit of customers. In developing countries, like India, Bangladesh, Nepal, China, *etc.*, due to several reasons, many parameters like demand, lead time, quality assurance, *etc.* seemed uncertain in nature. In recent time vast growth of population compel companies to makes huge production to fulfil population demand. So, the selling price plays an important role in determining the inventory or supply chain strategies. For these reasons, if we collect the previous data from any management system belong to these countries and followed by statistical regularity criteria, its probability distribution can be obtained for future correspondence In this paper a focus on the study an integrated single manufacturer multi retailer model for imperfect quality of items having price-dependent market demand determines the optimal decisions which maximize the joint total profit of the whole supply chain.

Appendix A.

Implementing GA
A typical Genetic Algorithm requires the following stages.

(i) Parameters
Firstly, we set the values of different parameters on which this GA depends, like number of generations (MAXGEN), population size (POPSIZE), probability of crossover (PXOVER), and probability of mutation (PMU).

(ii) Chromosome representation
An important issue of a GA is to design appropriate chromosomes for the representation of solutions of the problem together with genetic operators. Since the proposed problem is highly non-linear, to overcome this difficulty the chromosomes are repeated by real-numbers. In this representation, each chromosome X_p is a string of genes G_p, where genes G_p denote the decision variables m and T_1 of the model. Since real-number representation is used here, the value of each chromosome is the actual value of the decision variable.

(iii) Initial population production
We first determine the independent and dependent variables and then their boundaries. All genes corresponding to all the independent variables are generated randomly between its boundaries and dependent variables are

generated by using different conditions.

(iv) Evaluation

The evaluation of function plays the same role in the GA as that which the environment plays in natural evolution. For this problem, the evaluation function is $\text{EVAL}(G_p) = \textit{value of objective function } AC(T_1, m)$.

(v) Selection

The selection scheme of GA determines which chromosomes in the current population are to be selected for next generation. There are several approaches to select chromosomes from the initial population for their mating pool. All these approaches have some merits and demerits over the others. Among these approaches Roulette wheel selection process is very popular. In this study, Roulette wheel selection process is used with the following steps

(a) Find total fitness of the population

$$F = \sum_{p=1}^{\text{POPSIZE}} Z(G_p)$$

(b) Calculate the probability of selection P_{sel} of each solution G_p by the formula

$$P_{\text{sel}}(G_p) = \frac{Z(G_p)}{F}$$

(c) Calculate the cumulative probability $P'_{\text{sel}}(G_p)$ for each solution G_p by the formula

$$P'_{\text{sel}}(G_p) = \sum_{k=1}^{p} P_{\text{sel}}(G_k)$$

(d) Generate a random number r from the range $[0, 1]$.
(e) If $r < P'_{\text{sel}}(1)$ then select G_1 otherwise select $G_p(2 \le p \le \text{POPSIZE})$ if $P'_{\text{sel}}(G_{p-1}) \le r < P'_{\text{sel}}(G_p)$.
(f) Repeat steps (iv) and (v) POPSIZE times to select the chromosomes for mating pool. These chromosomes are denoted by GN_j, $j = 1, 2, \ldots.., \text{POPSIZE}$. Clearly one chromosome may be selected more than once and so some members of this set may be identical.
(g) Selected chromosomes set is denoted by $T = GN_1, GN_2, \ldots., GN_{\text{POPSIZE}}$ in the proposed GA algorithm.

(vi) Crossover

Crossover is a key operator in the GA and is used to exchange the main characteristics of parent individuals and pass them to the children. it consists of two steps:

(a) Selection for crossover: For each solution of $P^1(T)$ generate a random number r from the range $[0, 1]$. If $r < p_c$ then the solution is taken for the crossover. Where p_c is the probability of the crossover.
(b) Crossover process: Crossover taken place on the selected solutions. for pair coupled solution M_1, M_2 a random number c is generated from the range $[0, 1]$ and M_1, M_2 are replaced by their offsprings G_1, G_2 respectively. where $G_1 = c_1 M_1 + (1-c)M_2$,$G_2 = (1-c)M_1 + c_1 M_2$, provided G_1, G_2 satisfied the constraints of the problem.

(vii) Mutation

As in the conventional GA scheme, a parameter P_m of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

(viii) Implementation

With the above function and values the algorithm is implemented using C-programming language.

(ix) Termination

If number of iterations is less than to MAXGEN then the process continues, otherwise it terminates. Therefore, in this study at the end of last generation, each individual in the last generation is studied to check there is any local optimal in their neighborhoods. To do this, all the neighbor solutions to each individual are searched and evaluated successfully.

Appendix B.

The profit function can be computed as:

$$\begin{aligned}
\text{EAP}(n, g_i, p_i) = & \sum_{i=1}^{N} \frac{D_i(p_i - d_i)}{(1-\gamma_i)} - S_v \sum_{i=1}^{N} \frac{D_i}{ng_i(1-\gamma_i)} - \nu \sum_{i=1}^{N} \frac{\gamma_i D_i}{(1-\gamma_i)} - h_v \Bigg[\sum_{i=1}^{N} \frac{g_i}{R} \sum_{i=1}^{N} \frac{D_i}{(1-\gamma_i)} \\
& + \frac{Q}{2} \Bigg\{ 1 - \frac{D}{R} \sum_{i=1}^{N} \frac{1}{(1-\gamma_i)} \Bigg\} - \sum_{i=1}^{N} \frac{g_i}{2n(1-\gamma_i)} \Bigg] - h_v \sum_{i=1}^{N} \frac{1 - g_i\gamma_i}{1-\gamma_i} \Bigg\{ D_i I_6 \\
& - r_i(1 - F(\frac{r_i}{D_i})) \Bigg\} - \sum_{i=1}^{N} \Bigg[\frac{D_i}{ng_i(1-\gamma_i)} (A_i + nT_p) + \frac{h_i}{1-\gamma_i} \Bigg\{ r_i(1 - g_i\gamma_i) \\
& + \frac{D_i g_i^2 \gamma_i}{z_i} + \frac{g_i}{2}(1 - g_i\gamma_i - g_i\gamma_i^2 + g_i^2\gamma_i^2) \Bigg\} F(\frac{r_i}{D_i}) - \frac{h_i}{1-\gamma_i} \Bigg\{ \frac{D_i}{2g_i} + (1 - g_i\gamma_i) D_i \Bigg\} I_1 \\
& + \frac{h_i}{1-\gamma_i} \frac{D_i^2}{2g_i} I_2 + \frac{h_i}{1-\gamma_i} \Bigg\{ \frac{r_i^2}{3g_i} + \frac{D_i g_i^2 \gamma_i}{z_i} + r_i(1 - g_i\gamma_i) - \frac{D_i r_i}{2z_i} A_{1i} \\
& + \frac{1}{2}(g_i(1 - g_i\gamma_i - g_i\gamma_i^2 + g_i^2\gamma_i^2) + r_i A_{2i}) + r_i(1 - g_i\gamma_i) + \frac{r_i^2}{g_i} A_{1i} \Bigg\} I_3 - \frac{h_i}{1-\gamma_i} \Bigg\{ \frac{D_i}{g_i} \\
& - \frac{D_i^2}{2z_i} A_{1i} + D_i A_{2i} + D_i(1 - g_i\gamma_i) + \frac{2D_i r_i}{g_i} A_{1i} \Bigg\} I_4 + \frac{h_i D_i}{g_i(1-\gamma_i)} I_5 \\
& + \frac{b_i}{2g_i(1-\gamma_i)} \Bigg\{ 1 + \frac{D_i}{z_i} A_{3i} \Bigg\} \Bigg(D_i^2 I_5 - 2D_i r_i I_4 + r_i^2 I_3 \Bigg) \\
& + \frac{c_i}{1-\gamma_i} \{ D_i I_7 - (r_i + \frac{g_i^2}{2})(1 - F(\frac{(r_i + g_i)}{D_i})) \} \Bigg],
\end{aligned}$$

where

$$A_{1i} = E\Bigg[\frac{(1-\delta_i)}{(1-\delta_i-\frac{D_i}{z_i})}\Bigg] = \frac{(1-g_i\gamma_i)}{(1-g_i\gamma_i-\frac{D_i}{z_i})},$$

$$A_{2i} = E\Bigg[\frac{(1-\delta_i^2)}{(1-\delta_i-\frac{D_i}{z_i})}\Bigg] = \frac{(1-g_i\gamma_i-g_i\gamma_i^2+g_i^2\gamma_i^2)}{(1-g_i\gamma_i-\frac{D_i}{z_i})},$$

$$A_{3i} = E\Bigg[\frac{1}{(1-\delta_i-\frac{D_i}{z_i})}\Bigg] = \frac{1}{(1-g_i\gamma_i-\frac{D_i}{z_i})}.$$

Appendix C.

When l_i is exponentially distributed with $p.d.f.$ $f(l_i) = \lambda_i e^{-\lambda_i l_i}$, the expression of $F(\frac{r_i}{D_i})$ as well as $1-F(\frac{r_i}{D_i})$, $F(\frac{r_i+g_i}{D_i})$, $1-F(\frac{r_i+g_i}{D_i})$ I_1, I_2, I_3, I_4, I_5, I_6, I_7 are the following:

$$F(\frac{r_i}{D_i}) = \int_0^{\frac{r_i}{D_i}} f(l_i)\mathrm{d}l_i = \int_0^{\frac{r_i}{D_i}} \lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = 1 - e^{\frac{-\lambda_i r_i}{D_i}},$$

and $$F(\frac{r_i+g_i}{D_i}) = \int_0^{\frac{r_i+g_i}{D_i}} f(l_i)\mathrm{d}l_i = \int_0^{\frac{r_i+g_i}{D_i}} \lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = 1 - e^{\frac{-\lambda_i(r_i+g_i)}{D_i}}.$$

Hence, $$1 - F(\frac{r_i}{D_i}) = e^{\frac{-\lambda_i r_i}{D_i}},$$

$$1 - F(\frac{r_i+g_i}{D_i}) = e^{\frac{-\lambda_i(r_i+g_i)}{D_i}}.$$

Similarly,

$$I_1 = \int_0^{\frac{r_i}{D_i}} l_i f(l_i)\mathrm{d}l_i = \int_0^{\frac{r_i}{D_i}} l_i\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = \frac{1}{\lambda_i} - (\frac{1}{\lambda_i} + \frac{r_i}{D_i})e^{\frac{-\lambda_i r_i}{D_i}},$$

$$I_2 = \int_0^{\frac{r_i}{D_i}} l_i^2 f(l_i)\mathrm{d}l_i = \int_0^{\frac{r_i}{D_i}} l_i^2\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = \frac{2}{\lambda_i^2} - \{\frac{1}{\lambda_i^2} + (\frac{1}{\lambda_i} + \frac{r_i}{D_i})^2\}e^{\frac{-\lambda_i r_i}{D_i}},$$

$$I_3 = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} f(l_i)\mathrm{d}l_i = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} \lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = e^{\frac{-\lambda_i r_i}{D_i}} - e^{\frac{-\lambda_i(r_i+g_i)}{D_i}},$$

$$I_4 = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} l_i f(l_i)\mathrm{d}l_i = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} l_i\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = (\frac{1}{\lambda_i} + \frac{r_i}{D_i})e^{\frac{-\lambda_i r_i}{D_i}} - (\frac{1}{\lambda_i} + \frac{(r_i+g_i)}{D_i})e^{\frac{-\lambda_i(r_i+g_i)}{D_i}},$$

$$I_5 = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} l_i^2 f(l_i)\mathrm{d}l_i = \int_{\frac{r_i}{D_i}}^{\frac{r_i+g_i}{D_i}} l_i^2\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i$$

$$= \frac{1}{\lambda_i^2}\{1 + \frac{1}{D_i^2}(D_i + \lambda_i r_i)^2\}e^{\frac{-\lambda_i r_i}{D_i}} - \frac{1}{\lambda_i^2}[1 + \frac{1}{D_i^2}\{D_i + \lambda_i(r_i+g_i)\}^2]e^{\frac{-\lambda_i(r_i+g_i)}{D_i}},$$

$$I_6 = \int_{\frac{r_i}{D_i}}^{\infty} l_i f(l_i)\mathrm{d}l_i = \int_{\frac{r_i}{D_i}}^{\infty} l_i\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = (\frac{1}{\lambda_i} + \frac{r_i}{D_i})e^{\frac{-\lambda_i r_i}{D_i}},$$

$$I_7 = \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} l_i f(l_i)\mathrm{d}l_i = \int_{\frac{(r_i+g_i)}{D_i}}^{\infty} l_i\lambda_i e^{-\lambda_i l_i}\mathrm{d}l_i = \{\frac{1}{\lambda_i} + \frac{(r_i+g_i)}{D_i}\}e^{\frac{-\lambda_i r_i}{D_i}}.$$

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