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ROBUST SUSTAINABLE MULTI-PERIOD HUB LOCATION CONSIDERING UNCERTAIN TIME-DEPENDENT DEMAND

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Abstract. This paper presents a mathematical programming model for designing a sustainable continuous-time multi-period hub network considering time-dependent demand. The present model can be used in situations where the distribution of parameters related to the demand function is unknown, and we only can determine the range of changes of these parameters. To model these conditions, we consider interval uncertainty for the demand function parameters. The proposed model is a non-linear multi-objective model. The objectives of the model cover economic, environmental, and social aspects of sustainability. These objectives include minimizing total costs, minimizing emissions, and maximizing fixed and variable job opportunities. We linearize the model by using some linearization techniques, and then, with the help of Bertsimas and Sim's method, we construct a robust counterpart of the model. We also present some valid inequalities to strengthen the formulation. To solve the proposed model, we use Torabi and Hassini method. From solving the proposed model, network design decisions and the best time to implement decisions during the planning horizon are determined. To validate the model, we solve a sample problem based on the Turkish dataset and compare the designed network in two cases: in the first case, the demand function parameters take nominal values, and in the second case, the value of these parameters can change up to 20% of their nominal values. The results show that in the second case, the total capacity selected for hubs and hub links is greater than the first case. To investigate changes in objective functions to parameters level of conservatism and probability of constraints violation, we perform sensitivity analysis on these parameters in both single-objective and multi-objective optimization cases and report the results.

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1. Introduction

Hub facilities are collection, transmission, and distribution centers utilized in transportation and message transmission networks to reduce transportation links and provide indirect connections rather than direct connections between different origins and destinations. As the number of direct connections in the network reduces, fixed costs of system design decrease. In a hub network, some locations are selected for hubs, and non-hub nodes are allocated to the located hubs. In such networks, direct connections between origins and destinations are

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generally not allowed, and the flows are routed through hubs. In this way, the flows first enter the hub from the origin, are organized at the hub, and finally distributed to the destination. In many transportation systems, a discount factor is considered for inter-hub transshipments. Using this factor, unit transportation costs reduce, and economies of scale are taken into account for inter-hub flows.

The selected locations for hubs directly affect the fixed costs of the system and transportation costs. Various parameters are influential in determining the best locations for hubs in the network, such as transportation demand between different origins and destinations and hub opening costs. One of the assumptions in the classical hub location problem is that the values of the problem parameters are deterministic and will not change in the long run. Therefore, locations selected for hubs and allocation of non-hub nodes to the located hubs remain unchanged for a long time. But in the real world, the parameters change over time, and it is necessary to change the configuration of the transportation network to match changes in the parameters. Therefore, we must consider the time dimension in the hub location problem. To this end, a planning horizon should be defined. The planning horizon is a time frame in which Decision-Maker (DM) plans [50].

The planning horizon can be divided into several periods. Each period is equivalent to a time interval in which the problem parameters are not expected to change over that time interval. In each period, the network configuration in the previous period is updated and completed by the end of the planning horizon. We can make the following decisions at each time period: opening and closing of hubs and hub links (links that connect hubs), allocation of demand nodes to hubs, adjustment of the operational capacity of hubs due to changes in transportation demand (that is more cost-effective than opening new hubs) and routing the flows on the network. In the context of dynamic facility location problem, depending on making the decisions over time, we can define two types of planning horizon. If the timing of decision-making is predefined, the planning horizon is discrete-time. But in a continuous-time planning horizon, decisions can be made at any point in the planning horizon [5]. In the continuous-time planning horizon, the timing of making the decisions is also determined in the decision-making process.

In some transportation systems, such as airlines and message transmission networks, the time required for changes in parameters is much shorter than the length of the planning horizon. In this case, we can assume that parameters are time-dependent. For example, demand in a messaging network may change momentarily, or in airlines, transportation demand may change daily. If the planning horizon is multi-year, in both cases, the time required for changes in demand value is much shorter than the length of the planning horizon, and we assume that this parameter changes continuously. Usually, in such cases, a continuous function can be estimated to indicate changes in the desired parameter. This function forecasts changes in the desired parameter over time. The accuracy of forecasting depends on how the input parameters of the function are estimated. If the input parameters are not estimated correctly, the results may not be consistent with real changes of parameters over time, and the resulting solution may be far from the optimal solution. In real-world conditions, it is difficult to estimate the input parameters of the demand function, and we only can determine the range of changes of the parameters. In such cases, we must consider the uncertainty of the input parameters.

Sustainability is another aspect that we need to consider in the hub location problem. Decisions about the location of hubs are usually strategic and have long-term effects on the economy, environment, and society. The primary dimensions of sustainable development are "safeguarding long-term ecological sustainability, satisfying basic human needs, and promoting inter and intra-generational equity" [36]. Therefore, to move towards sustainable development, we should consider the sustainability aspects in the hub location problem. The following sustainability aspects can be considered in the hub location problem: paying attention to environmental pollution caused by transportations in the hub network, creating fixed and variable job opportunities due to opening hubs, reducing user travel time, regional development, and so on.

Given the importance of dynamic hub network design, uncertainty, and sustainability, in this paper, we present a mathematical modeling framework for a multi-period hub location problem considering a continuous-time planning horizon under uncertainty of time-dependent transportation demand. We also include adjusting operational capacities related to hubs and hub links due to changes in transportation demand over time in the problem modeling. The capacity of hubs and hub links is considered modular, and it is possible to increase their capacity and transfer the capacity between located hubs. For example, terminals in an airport and sorting lines in a postal system can be considered as hub modules [4]. On the other hand, in airlines, the airplanes and pilots moving along conventional airlines can be considered as link modules. Also, in a postal system, the streets used by the post company for delivering the packages together with the drivers moving along the streets can be considered as link modules.

The main contribution of the present study is to consider uncertainty for time-dependent demand in a continuous-time planning horizon and the integration of sustainability aspects in a multi-period hub location problem with a continuous-time planning horizon considering the uncertainty of time-dependent demand. The purpose of this study is to provide a modeling framework for the continuous-time multi-period hub location problem in a situation where demand is time-dependent, and the distribution of parameters related to the demand function is unknown, and we only can determine a range for the values of these parameters. For modelling these conditions, we consider interval uncertainty for these parameters, and with the help of robust optimization tools, we construct a robust counterpart formulation of the problem. The objectives of the problem are sustainability objectives, including minimizing total costs, minimizing emissions, and maximizing fixed and variable job opportunities due to opening hubs throughout the planning horizon. Solving the proposed model provides the best time to implement problem decisions during the planning horizon. These decisions include opening and closing hubs and hub links, allocation of demand nodes to located hubs, increasing the capacity of facilities and transferring the capacity between hubs and routing the flows on the hub network over the planning horizon.

The remainder of the present study is organized as follows: in Section 2, we present the literature review and refer to researches related to the subject of the present study. Then, we will deal with mathematical modeling in Section 3 and explain how to linearize the proposed model. Then, we present a robust optimization approach to construct a robust counterpart formulation. In Section 4, we describe the solution method. In Section 5, we examine the results of applying the robust model on a sample problem and report the sensitivity analysis results. Finally, in Section 6, conclusions and future research are presented.

2. Literature review

In this section, we study the literature of the present research in four research areas: hub location problem, multi-period hub location, hub location under robust uncertainty, and sustainable hub location. In the following, we will review published studies in each of these research areas.

2.1. Hub location problem

At first, reference [35] proposed a network optimization problem for finding the best location of a switching point. Then, reference [60] examined the hub location problem applications in managing airports and airlines. Subsequent researches included the development of a mathematical modeling framework for hub location problem. In the first step, reference [51], proposed the first quadratic model, and reference [52] extended the model considering fixed costs of hubs. References [17]–[41] helped to improve and complete the modeling of hub location problem. To learn more about different types of hub location problem, solution methods, and applications, we refer readers to references [20]–[28].

2.2. Multi-period hub location

In this context, a continuous approximation model was proposed in reference [16] to locate terminals in a fixed region with increasing demand density. In this research, the planning horizon is continuous-time. The location and relocation and the best time to locate and relocate terminals must be determined concerning the changes in demand. In the following, we refer to models in which the planning horizon is discrete-time. Reference [31] presented a model in multi-period hub location problem applications in public transportation. In the proposed model, there is a finite planning horizon that consists of several periods. At the beginning of the planning horizon, there is an initial configuration for the transportation network. Due to parameter changes in different periods, this configuration will be updated in subsequent periods and completed by the end of the planning horizon. Then, reference [21] proposed a quadratic programming model for overcoming demand fluctuations during the planning horizon by finding the optimal location for hubs in each period. Then, reference [58] studied a dynamic virtual hub location problem in which locations for virtual hubs in different periods must be selected to minimize total costs over the planning horizon. Subsequently, reference [32] considered budget constraints for operating costs in a multi-period hub location problem.

Regarding hub capacity management in different periods, in reference [4], authors assumed that the capacity of hubs is modular. At each period, according to changes in demand and other parameters, the capacity of hubs can be increased through modules. The authors proposed models for both single and multiple allocation cases. Also, reference [26] presented a dynamic hub covering location problem considering a flexible coverage radius. In their problem, the coverage radius is a decision variable, and in each period, the coverage radius can be adjusted according to the changes in parameters. Then, reference [10] presented a mobile hub location problem in which, during the planning horizon, it is possible to move located hubs to other locations through rail transport facilities.

In another research, authors studied a reliable, intermodal multi-period freight network expansion problem considering the possibility of disruption in terminals and routes [30]. They divided periods into two sets, namely strategic and routing periods. In the strategic periods, decisions are reopening and increasing the capacity of facilities and repairing disrupted facilities and routes. In the routing periods, the decision is to route the flows on the network. Reference [22] extended dynamic hub location problem with modular capacities and multiple allocations for the case that demand is stochastic. Problem decisions include determining the initial capacity of hubs and increasing the capacity of hubs in different periods due to changes in parameters during the planning horizon. Also, reference [62] provided a reliable model for a multi-modal hierarchical hub location problem considering dynamic disruption (time-dependent and site-dependent). In their proposed model, if a disruption occurs in a period, it affects decisions of subsequent periods. It is necessary to perform activities such as maintenance, temporary closure of facilities, set up, and reopening facilities. Then, a single-allocation hub covering location problem considering the periodic changes of the problem parameters was presented in reference [39]. The authors considered two objectives of maximizing the covered demand and minimizing the costs of opening and closing the hubs during the planning horizon for the problem. Later on, reference [37] studied multi-period single allocation hub location problem considering the life cycle and reconstruction hubs. In their model, each hub, after selecting a contractor to open it, has a certain life cycle and it can be reconstructed or closed at the end of this life cycle. Researchers also considered multiple capacity levels and multiple construction levels in the problem modeling. A bi-objective reliable multi-period hub location problem considering congestion was studied by reference [29]. In their model, the first objective includes minimizing transportation costs, opening and closing costs of hubs, and penalty costs of exceeding hubs capacity from the effective capacity of hubs minus the revenue from closed hub facilities. The second objective is to maximize the minimum reliability of network routes.

2.3. Hub location under robust uncertainty

In this context, we can refer to the following researches. Reference [43] proposed a robust approach to minimize expected costs in a capacitated P-hub location problem. The authors considered uncertainty for demand and processing time. Later on, reference [2] provided a robust optimization approach to an uncapacitated hub location problem for single and multiple allocation cases. They considered uncertainty for the fixed cost of hubs. The objective function was minimizing regret of the worst-case scenario to deal with uncertainty. Also, reference [56] considered an ellipsoidal uncertainty set for demand in an uncapacitated hub location problem and formulated the problem as a mixed-integer conic quadratic programming model with a minimax objective function.

Later on, reference [33] studied a capacitated hub location problem considering stochastic demand and used Bertsimas and Sim's method to solve the problem. Then, reference [15] applied a minimax regret policy for a capacitated hub location problem and considered uncertainty for hubs capacity and fixed cost. Reference [44] studied an uncapacitated multiple allocation P-hub median problem in the case that there are two sets of uncertainty for demand: house and hybrid uncertainty. In the house uncertainty set, the authors assumed that only the upper bound of the total flow adjacent to each demand node is known. In the hybrid uncertainty set, they assumed that in addition to the upper bound of overall flow adjacent to each node, lower and upper bounds of transportation demand for each Origin-Destination (O-D) pair are known. Then, reference [45] extended house uncertainty set for another scenario in which the capacities considered for the hubs affect the feasibility of solutions.

In another work, authors proposed robust counterpart formulations for an uncapacitated multiple allocation hub location problem [65]. They controlled the conservatism level using a budget uncertainty parameter and considered uncertainty for demand and transportation cost in independent and joint cases, assuming that only information about uncertain parameters is the uncertainty interval. The objective function of the problem involves minimizing the total costs of transportation in the worst-case scenario and the fixed costs of hubs. Also, reference [59] provided measures for determining the robustness of a solution of an uncapacitated multiple allocation P-hub median problem under demand uncertainty.

Later on, authors in reference [23] presented a robust counterpart of incomplete multiple allocation hub location problem and assumed that fixed costs of establishing links and hubs and transportation demand have robust uncertainty. They used a budget uncertainty parameter to control the level of conservatism. Then, to extend the previous model, they considered time constraints for each flow of demand in the transportation network and assumed that travel time between demand nodes in the network is subject to uncertainty. The researchers aimed to find solutions with the lowest cost and high probability of feasibility, considering time constraints.

Then, reference [63] proposed a robust approach to a P-hub median location problem and studied the problem under uncertainty of carbon emission parameter. Researchers assumed that the probability distribution of this parameter is only partially available and used a chance constraint-based approach to deal with uncertainty. Also, reference [42] studied an uncapacitated multiple allocation hub location problem under demand uncertainty. The authors considered demand uncertainty as a set of scenarios. They used the absolute deviation of uncertain parameters from their expected value as a measure to determine the robustness of solutions.

2.4. Sustainable hub location

In the sustainable hub location problem, various aspects of sustainability (economic, environmental, and social) have been considered in objectives functions. The models developed in this context are mainly multi-objective optimization models. In the following, we will point out the most important researches in this context. Reference [46] considered the aspects of sustainability in the modeling of a hub covering location problem through two objectives. These objectives are minimizing transportation costs and costs of establishing hubs and minimizing waiting time at hubs. Then, reference [34] considered minimizing total travel time on the network as a social objective and minimizing total costs in a dynamic hub covering location problem. A green hub location-routing problem was studied by reference [54], considering economic and social objectives. Also, in reference [47] minimizing total costs and minimizing the maximum travel time on the network were considered as objectives of a hub covering location problem.

Later on, reference [55] studied a hub covering location problem considering objectives minimization of maximum travel time and minimization of transportation costs and costs of using different transportation modes. Also, reference [48] proposed two environmental objectives minimizing emission costs and minimizing noise pollution at hubs and an economic objective of minimizing total costs.

A multi-modal hub network design problem was studied in reference [49], considering economic, social, and environmental objectives. These objectives are minimizing total travel time on the network, minimizing transportation costs, emission costs, and fuel consumption, and maximizing service reliability. Reference [53] considered minimizing total costs and the maximum travel time on the network as sustainability objectives in a hub covering location problem with different capacity levels of hubs.

Table 1. A comparison between the present study and previous studies.

Reference	Time-dependent demand	Planning horizon		Multi-objective	Sustainability	Uncertainty	Solution approach
		Continuous-time	Discrete-time				
[16]	✓	✓	–	–	–	–	Heuristic
[48]	–	–	–	✓	✓	Fuzzy and stochastic	Possibilistic-stochastic programming
[49]	–	–	–	✓	✓	Fuzzy	Inexact rough interval LP
[33]	–	–	–	–	–	Robust	Robust optimization
[44]	–	–	–	–	–	Robust	Robust optimization
[67]	–	–	–	✓	✓	Fuzzy	Differential evolution-ICA
[65]	–	–	–	–	–	Robust	Robust optimization
[62]	–	–	✓	–	–	Stochastic	Two-phase heuristic
[63]	–	–	–	–	✓	Robust	Chance constraint-based approach
This work	✓	✓	–	✓	✓	Robust	Robust optimization

Later on, reference [66] studied the economic and environmental multi-modal hub network design problem. Also, reference [67] integrated social responsibility and responsiveness in a multi-objective hub location problem through the following objectives: minimizing the maximum travel time on the network and maximizing employment and regional development. Later on, a green hub location problem was studied in reference [25]. In this work, the aim is to minimize the emission costs (depending on the vehicles speed and the amount of flow carried by the vehicles on the network) so that the delivery of flows from origin to destination must be done within a predetermined time limit.

Table 1 lists the most relevant studies to the subject of the present study and identifies differences between this study and previous researches.

According to Table 1, we can identify the following research gaps: in the field of multi-period hub location problems, only in one study planning horizon is continuous-time (Ref. [16]). In that study, uncertainty is not considered for parameters, and the problem is single-objective. Also, the authors have not considered sustainability aspects. In sustainable hub location problems, transportation demand is not time-dependent, and the planning horizon is discrete-time. Hub location problems under robust uncertainty are static (single period) and single-objective. Also, in these studies, demand is not time-dependent. The aim of the present study is to cover these gaps.

3. Problem description and mathematical formulation

3.1. Problem statement

The Sustainable Multi-Period Hub Location Problem under Uncertainty of Time-dependent Demand (SMPHLPUTD) is described as follows:

Given $N = \{1, 2, \ldots, n\}$, a set of O-D pairs and matrix $(c_{ij})_{n\times n}$ indicates the cost or distance between each O-D. There is a continuous-time planning horizon with length T, during which decisions can be changed m times, divided into $m+1$ periods. The set of time periods is $S = \{0, 1, 2, \ldots, m+1\}$. Period 0 represents the beginning of the planning horizon. The flow between O-D pairs is time-dependent and represented by a continuous linear function as $\widetilde{w}_{ij}(t) = \widetilde{a}_{ij} + \widetilde{h}_{ij}t, 0 \leq t \leq T$. This function is an adaptation of the function introduced in reference [24]. Parameter $\widetilde{a}_{ij} \geq 0$ is the initial value of demand at the beginning of the planning horizon, and parameter $\widetilde{h}_{ij} \geq \frac{-\widetilde{a}_{ij}}{T}$ is the increasing rate of flow over time. Parameters of the demand function are uncertain.

The set of candidate nodes for opening hubs is given as set $H \subseteq N$. During the planning horizon, new hubs and hub links can be opened, and existing hubs and hub links can be closed. Hubs and hub links can be opened at the beginning of periods, and they can be closed at the end of periods. If a hub is closed, it will not reopen and will not operate in subsequent periods. In each period, P hubs must be opened, and other demand nodes must be allocated to these hubs. Each demand node is allocated to exactly one hub (single allocation rule). A discount factor α is considered for flows on each hub link during the planning horizon.

Costs of opening and closing the hubs are given by matrices $(\mathrm{OC}_k)_{|H|\times 1}$ and $(\mathrm{CC}_k)_{|H|\times 1}$, respectively. The opening and closing costs of the hub links are as matrices $(\mathrm{OCe}_{kl})_{N\times N}$ and $(\mathrm{CCe}_{kl})_{N\times N}$, respectively. The available budget for opening and closing hubs and hub links for the entire planning horizon is limited and denoted by parameter B.

During the planning horizon, the capacity of hubs and hub links can be increased through modules with a certain capacity, and it is possible to transfer the capacity between hubs. In each period, the total number of modules installed on a hub and transferred from other hubs to that hub minus the number of modules from which the hub is transferred to other hubs up to that period determines their capacity. We assumed that transferring the capacity can be done in the second period and subsequent periods. Also, each hub link has a primary capacity, and in each period, additional links can be used to increase the capacity of that hub link, if needed. Each hub link capacity in a certain period is independent of its capacity in the previous periods.

Vehicles traffic on the hub network causes environmental pollution, and also, opening hubs creates fixed and variable job opportunities. Parameters related to environmental pollution and fixed and variable job opportunities are available.

We are looking for a decision about opening and closing hubs and hub links, allocating non-hub nodes to hubs, increasing the capacity of hubs and hub links, transferring the capacity between hubs, routing flows on the network in different periods, and also, the timing of implementing decisions throughout the planning horizon, so that three objectives of sustainability are optimized simultaneously. These objectives are (1) minimizing transportation costs, costs of opening and closing hubs and hub links, costs of increasing and transferring capacity, (2) minimizing emission costs at arcs, and (3) maximizing fixed and variable job opportunities.

Figure 1 helps to clarify the problem. Figure 1 shows a designed hub network with six nodes and three periods. All nodes except node 4 are candidates for hub location. The located hubs are shown in orange, and the hub links are shown in red. The other non-hub nodes are allocated to the hubs. The numbers next to the hubs indicate the modules installed on the hubs, and the numbers on the links indicate the additional links installed on the hub links. We show the capacity transfer between hubs by an arrow. Next to the arrow, the number of transferred modules is specified. The breakpoints of the planning horizon are also marked on the time axis.

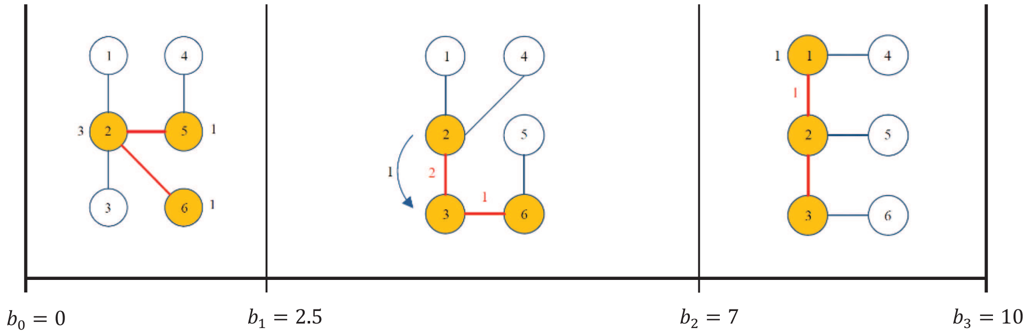


FIGURE 1. A feasible solution with six nodes, five candidate nodes for hubs and three time periods ($T = 10$).

TABLE 2. Sets and indices.

$N = \{1, 2, \ldots, n\}$	Set of demand nodes
$H \subseteq N$	Set of candidate nodes for hubs
$S = \{0, 1, 2, \ldots, m+1\}$	Set of time periods (note that time period 0 denotes the starting point of planning horizon)

3.2. Mathematical modeling

3.2.1. Indices, parameters, and decision variables

The indices, parameters, and decision variables related to the mathematical model are listed in Tables 2–4.

3.2.2. Modeling planning horizon, flows, and cost of emissions

In this subsection, we describe the modelling of continuous-time planning horizon, the calculation of the flow between the demand nodes in a continuous-time planning horizon, and the calculation of the emission costs at arcs.

3.2.2.1. Modeling continuous-time planning horizon. When the planning horizon is continuous-time, the timing of implementing decisions is not predefined and is determined in the decision-making process. Therefore, the length of each period is also unknown, and the timing of implementing decisions determines the starting and ending points of periods. To model these conditions, we use the approach used in references [24,27]. Suppose that we have a finite planning horizon with length T. During the planning horizon, we can change problem decisions m times. Suppose that the timing of implementing decision s is $b_s, \forall s \in S, s \neq \{m+1\}$. Let $b_0 = 0$, $b_{m+1} = T$, and $b_{s-1} < b_s, \forall s \in S \backslash \{0\}$. The first decision is implemented at time $b_0 = 0$, and $b_{m+1} = T$ determines the end of the planning horizon. We call $b_1, b_2, \ldots, b_m$ breakpoints. In this case, period s is determined by time interval $[b_{s-1}, b_s], \forall s \in S \backslash \{0\}$.

3.2.2.2. Calculating demand flow between nodes at each period. In the demand function, parameters $\widetilde{a}_{ij}$ and $\widetilde{h}_{ij}$ are subject to uncertainty, and only the range of their changes is known. These parameters are independent, symmetric, and bounded random variables with an unknown distribution and take values in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$, and $\left[h_{ij} - \hat{h}_{ij}, h_{ij} + \hat{h}_{ij}\right]$, respectively. Parameters a_{ij} and h_{ij} are nominal values, and $\hat{a}_{ij}$ and $\hat{h}_{ij}$ are oscillation ranges of uncertain parameters. Because the demand function values are non-negative, in generating demand function parameters, the following relationship must be considered:

$$\widetilde{a}_{ij} + \widetilde{h}_{ij} t \geq 0 \rightarrow \widetilde{h}_{ij} \geq -\frac{\widetilde{a}_{ij}}{T} \rightarrow h_{ij} - \hat{h}_{ij} \geq -\frac{(a_{ij} - \hat{a}_{ij})}{T}.$$

Table 3. Parameters.

$\widetilde{w}_{ij}(t)$	Demand flow from node $i \in N$ to node $j \in N$ at time $t(t \in [0,T])$
$\widetilde{O}_i(t) = \sum_{j\in N} \widetilde{w}_{ij}(t)$	Total amount of flow originated from node $i \in N$ at time $t(t \in [0,T])$
$\widetilde{D}_i(t) = \sum_{j\in N} \widetilde{w}_{ji}(t)$	Total amount of flow delivered at node $i \in N$ at time $t(t \in [0,T])$
c_{ij}	Unit transportation cost from node $i \in N$ to node $j \in N$ (per unit of distance of one unit of flow)
α	Discount factor for inter-hub connections (per unit of distance of one unit of flow)
P	Number of hubs which must be located
OC_k	Opening cost of hub $k \in H$
OCe_{kl}	Opening cost of hub link (k,l) $(k,l \in H)$
CC_k	Closing cost of hub $k \in H$
CCe_{kl}	Closing cost of hub link (k,l) $(k,l \in H)$
B	Available budget throughout the planning horizon
Q_k	Maximum number of modules that can be installed on hub $k \in H$
Q'_{kl}	Maximum number of additional links that can be installed on hub link (k,l) $(k,l \in H)$
CIM_k	Cost of installing each module in hub $k \in H$
Ctr_{kl}	Cost of transferring each module from hub $k \in H$ to hub $l \in H$
CeAM_{kl}	Cost of installing each additional link on hub link (k,l) $(k,l \in H)$
Δ	Capacity of each hub module
Δ'	Capacity of each additional link
Cape_{kl}	Capacity of link (k,l) $(k,l \in H)$
d	The minimum time interval that must be exist between the breakpoints
Parameters of emissions:	
EW	Empty vehicle weight (kg)
LW	Weight of load carried by the vehicle (kg)
d_{ij}	Distance between nodes $i \in N$ and $j \in N$ (m)
$\mathscr{q}$	Engine friction factor
$\mathscr{N}$	Engine speed
$\mathscr{v}$	Engine displacement
P_t	Total tractive power demand requirement in watts $(\mathrm{kg\,m^2/s^3})$ placed on the wheels
ε	Vehicle drive train efficiency
P_a	Engine power demand associated with running losses of the engine and operation of vehicle accessories such as air conditioning
η	Efficiency parameter for diesel engines
U	Value that depends on some constants including engine speed
M	Mass of the vehicle (kg) (sum of empty and carried load)
vs	Vehicle speed (m/s)
$\mathscr{a}$	Acceleration $(\mathrm{m/s^2})$
$\mathscr{g}$	Gravitational constant (9.81 $\mathrm{m/s^2}$)
θ	Road angle (degree)
AR	Frontal surface area of the vehicle $(\mathrm{m^2})$
ρ	Air density $(\mathrm{kg/m^3})$
C_r	Coefficient of rolling resistance
C_d	Coefficient of aerodynamic drag
Parameters of social objective:	
FJO_k	Fixed job opportunities created by opening hub $k \in H$
VJO_k	Variable job opportunities created by each unit of flow processed in hub $k \in H$

TABLE 4. Decision variables.

Binary variables:	
z_{iks}	If node $i \in N$ is allocated to hub $k \in H$ at time period $s \in S$ is 1, otherwise 0. If $z_{kks} = 1$, then node $k \in H$ is selected as a hub at time period $s \in S$
u_{kqs}	If hub $k \in H$ recieves q modules at time period $s \in S$ is 1, otherwise 0 ($q = 1, \ldots, Q_k$)
trc_{qkls}	If q modules are transferred from hub $k \in H$ to hub $l \in H$ at time period s is 1, otherwise 0 ($q = 1, \ldots, Q_k$)
e_{kls}	If hub link (k, l) $(k, l \in H)$ is opened at time period $s \in S$ is 1, otherwise 0
ec_{kls}	If hub link (k, l) $(k, l \in H)$ is closed at time period $s \in S$ is 1, otherwise 0
$ea_{q'kls}$	If hub link (k, l) $(k, l \in H)$ receives q' additional links at time period $s \in S$ is 1, otherwise 0 ($q' = 1, \ldots, Q'_{kl}$)
zc_{ks}	If hub $k \in H$ is closed at time period $s \in S$, is 1, otherwise 0
Continuous variables:	
Y^i_{kls}	Amount of flow originated from node $i \in N$ and traversed hubs $k \in H$ and $l \in H$ at time period $s \in S$
b_s	Breakpoints ($b_s \in [0, T]$)

Given that demand is time-dependent and the planning horizon is continuous-time, the demand flow between nodes $i \in N$ and $j \in N$ at period s is calculated by integrating the demand function in interval $[b_{s-1}, b_s]$.

$$\int_{b_{s-1}}^{b_s} \widetilde{w}_{ij}(t)\,\mathrm{d}t = \int_{b_{s-1}}^{b_s} \left(\widetilde{a}_{ij} + \widetilde{h}_{ij}t\right)\mathrm{d}t = \widetilde{a}_{ij}(b_s - b_{s-1}) + \widetilde{h}_{ij}\left(\frac{b_s^2 - b_{s-1}^2}{2}\right) \qquad \forall s \in S\backslash\{0\}. \tag{3.1}$$

3.2.2.3. Calculating the cost of emissions at arcs. In this subsection, we present how to calculate the cost of emissions on hub network arcs. These calculations are based on references [8, 9, 48]. Greenhouse gases are emitted as vehicles move on arc (i, j) in the hub network. The emission rate for a greenhouse gas in grams per second $(E(g/s))$ is related to the fuel consumption rate (F) through equation $E = \zeta_1 F + \zeta_2$. In this equation, the parameters ζ_1 and ζ_2 are the specific parameters of the greenhouse gas. Many parameters are influential in calculating F and its calculation is complex. In this research, we use a simplified relation to calculate F as follows.

$$F \approx \left(\mathscr{q}\mathscr{N}\mathscr{v} + \frac{\left(\frac{P_t}{\varepsilon} + P_a\right)}{\eta}\right)U.$$

In the above relation, parameter P_t is calculated as follows:

$$P_t = M\mathscr{a}vs + M\mathscr{g}vs(\sin\theta) + 0.5C_d\mathrm{AR}\rho vs^3 + M\mathscr{g}C_r vs(\cos\theta).$$

It is assumed that all parameters related to P_t are remained unchanged while the vehicle is passing the arc (i, j), except for the vehicle load and speed. Total tractive power demand requirement for arc (i, j) can be estimated from the following equation:

$$P_{ij} \approx P_t\left({}^{d_{ij}}/_{vs_{ij}}\right) \approx \gamma_{ij}(\mathrm{EW} + \mathrm{LW})d_{ij} + \beta vs_{ij}^2 d_{ij}$$

where $\gamma_{ij} = a + g\sin\theta + gC_r\cos\theta_{ij}$, which is the constant of the arc (i, j). The parameter β is also a constant of the vehicle and is equal to $\beta = 0.5C_d\mathrm{AR}\rho$. P_{ij} estimates the energy consumed by the vehicle on the arc (i, j) in joules, which is directly related to fuel consumption and, consequently, greenhouse gas emissions. In our model, each demand node is allocated to exactly one hub. Thus, the entire incoming and outgoing flows of the demand nodes assigned to a hub enter that hub. In period s, this value is $\sum_{i\in N}\int_{b_{s-1}}^{b_s}\left(\widetilde{O}_i(t) + \widetilde{D}_i(t)\right)\mathrm{d}t$. Based on the above descriptions, the emission cost for all vehicles at arcs on the continuous-time multi-period hub network is calculated as follows:

$$(\mathrm{EW}+\mathrm{LW})\left\{\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H}\sum_{i\in N}\gamma_{kl}Y^i_{kls}d_{kl}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{i\in N}d_{ik}z_{iks}\int_{b_{s-1}}^{b_s}\left(\widetilde{O}_i(t)+\widetilde{D}_i(t)\right)\mathrm{d}t\right\}$$
$$+\beta\left\{\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{i\in N}vs^2_{ik}d_{ik}z_{iks}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H,l>k}vs^2_{kl}d_{kl}e_{kls}\right\}.$$

3.2.3. Objective functions

The objective functions of the problem are formulated as equations (3.2)–(3.4).

$$\begin{aligned}\min \mathrm{Obj}_1 = {} & \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}z_{iks}\int_{b_{s-1}}^{b_s}\left(c_{ik}\widetilde{O}_i(t)+c_{ki}\widetilde{D}_i(t)\right)\mathrm{d}t+\sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\sum_{l\in H}\alpha c_{kl}Y^i_{kls}\\ & +\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\mathrm{OC}_k(z_{kks}-z_{kk,s-1}+zc_{ks})+\sum_{s\in S\backslash\{0,1\}}\sum_{k\in H}\mathrm{CC}_k zc_{ks}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{q=1}^{Q_k}\mathrm{CIM}_k qu_{kqs}\\ & +\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H,l>k}\mathrm{OCe}_{kl}(e_{kl,s}-e_{kl,s-1}+ec_{kl,s})+\sum_{s\in S\backslash\{0,1\}}\sum_{k\in H}\sum_{l\in H,l>k}\mathrm{CCe}_{kl}ec_{kl,s}\\ & +\sum_{s\in S\backslash\{0,1\}}\sum_{k\in H}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k}\mathrm{Ctr}_{kl}qtrc_{qkls}\end{aligned}\tag{3.2}$$

$$\begin{aligned}\min \mathrm{Obj}_2 = {} & (\mathrm{EW}+\mathrm{LW})\left\{\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H}\sum_{i\in N}\gamma_{kl}Y^i_{kls}d_{kl}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{i\in N}d_{ik}z_{iks}\int_{b_{s-1}}^{b_s}\left(\widetilde{O}_i(t)+\widetilde{D}_i(t)\right)\mathrm{d}t\right\}\\ & +\beta\left\{\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{i\in N}vs^2_{ik}d_{ik}z_{iks}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H,l>k}vs^2_{kl}d_{kl}e_{kls}\right\}\end{aligned}\tag{3.3}$$

$$\max \mathrm{Obj}_3 = \sum_{s\in S\backslash\{0\}}\sum_{k\in H}z_{kks}\mathrm{FJO}_k+\sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}z_{iks}\mathrm{VJO}_k\int_{b_{s-1}}^{b_s}\left(\widetilde{O}_i(t)+\widetilde{D}_i(t)\right)\mathrm{d}t.\tag{3.4}$$

In the above formulation, the first objective in equation (3.2) minimizes transportation costs, costs of opening and closing hubs and hub links, costs of increasing capacity of hubs and hub links, and costs of transferring the capacity between hubs throughout the planning horizon. The second objective function in equation (3.3) calculates the total emissions caused by vehicles on the hub network throughout the planning horizon and seeks to minimize it. The third objective function in equation (3.4) maximizes fixed and variable job opportunities created by opening hubs throughout the planning horizon.

3.2.4. Constraints

The constraints of the mathematical model are formulated as equations (3.5)–(3.46).

$$\sum_{k\in H}z_{iks}=1 \qquad \forall i\in N, s\in S\backslash\{0\}\tag{3.5}$$

$$\sum_{k\in H}z_{kks}=P \qquad \forall s\in S\backslash\{0\}\tag{3.6}$$

$$z_{iks}\le z_{kks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\}\tag{3.7}$$

$$z_{kks} \leq z_{kk,s+1} + zc_{k,s+1} \qquad \forall k \in H, s \in S\backslash\{0, m+1\} \tag{3.8}$$

$$\sum_{s \in S\backslash\{0\},\{1\}} zc_{ks} \leq 1 \qquad \forall k \in H \tag{3.9}$$

$$zc_{ks} \leq z_{kk,s-1} \qquad \forall k \in H, s \in S\backslash\{0,1\} \tag{3.10}$$

$$zc_{ks} \leq 1 - \frac{1}{m-1} \sum_{s' \in S, s' > s} Z_{kks'} \qquad \forall k \in H, s \in S\backslash\{0,1\} \tag{3.11}$$

$$zc_{ks} \leq 1 - z_{kks} \qquad \forall k \in H, s \in S\backslash\{0,1\} \tag{3.12}$$

$$\sum_{q=1}^{Q_k} u_{kqs} \leq z_{kks} \qquad \forall k \in H, s \in S\backslash\{0\} \tag{3.13}$$

$$e_{kls} \leq z_{kks} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0\} \tag{3.14}$$

$$e_{kls} \leq z_{lls} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0\} \tag{3.15}$$

$$ec_{kls} \leq e_{kl,s-1} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0,1\} \tag{3.16}$$

$$ec_{kls} \leq 1 - e_{kls} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0,1\} \tag{3.17}$$

$$e_{kl,s-1} - e_{kls} \leq ec_{kls} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0,1\} \tag{3.18}$$

$$\sum_{q'=1}^{Q'kl} ea_{q'kls} \leq ec_{kls} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0\} \tag{3.19}$$

$$\begin{aligned} \sum_{l \in H, l \neq k} Y_{kls}^{i} - \sum_{l \in H, l \neq k} Y_{lks}^{i} &= \left(\int_{b_{s-1}}^{b_s} \widetilde{O}_i(t)\,\mathrm{d}t \right) z_{iks} \\ &- \sum_{j \in N} \left(\int_{b_{s-1}}^{b_s} \widetilde{w}_{ij}(t)\,\mathrm{d}t \right) z_{jks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \end{aligned} \tag{3.20}$$

$$\sum_{i \in N} Y_{kls}^{i} + \sum_{i \in N} Y_{lks}^{i} \leq \mathrm{Cape}_{kl}.e_{kls} + \Delta' \sum_{q'=1}^{Q'kl} ea_{q'kls} \qquad \forall k, l \in H, k < l, s \in S\backslash\{0,1\} \tag{3.21}$$

$$Y_{kls}^{i} + Y_{lks}^{i} \leq \left(\int_{b_{s-1}}^{b_s} \widetilde{O}_i(t)\,\mathrm{d}t \right) e_{kls} \qquad \forall i \in N, k, l \in H, k < l, s \in S\backslash\{0\} \tag{3.22}$$

$$\sum_{q=1}^{Q_k} trc_{qkls} \leq z_{kks} \qquad \forall k, l \in H, k \neq l, s \in S\backslash\{0,1\} \tag{3.23}$$

$$\sum_{q=1}^{Q_k} trc_{qkls} \leq z_{lls} \qquad \forall k, l \in H, k \neq l, s \in S\backslash\{0,1\} \tag{3.24}$$

$$\sum_{l \in H, l \neq k} trc_{qkls} \leq 1 \qquad \forall k \in H, q = 1, \ldots, Q_k, s \in S\backslash\{0,1\} \tag{3.25}$$

$$\begin{aligned} \sum_{l \in H, l \neq k} \sum_{q=1}^{Q_k} q \cdot trc_{qkls} &\leq \sum_{q=1}^{Q_k} \sum_{s' \in S\backslash\{0\}, s' < s+1} q u_{kqs'} \\ &+ \sum_{s' \in S\backslash\{0\}, s' < s+1} \sum_{l \in H, l \neq k} \sum_{q=1}^{Q_k} q \cdot trc_{qkls} \qquad \forall k \in H, s \in S\backslash\{0,1\} \end{aligned} \tag{3.26}$$

$$\sum_{s\in S\backslash\{0\}}\sum_{q=1}^{Q_k} q\cdot u_{kqs} - \sum_{s\in S\backslash\{0,1\}}\sum_{l\in H, l\neq k}\sum_{q=1}^{Q_k} q\cdot trc_{qkls}$$

$$+\sum_{s\in S\backslash\{0,1\}}\sum_{l\in H, l\neq k}\sum_{q=1}^{Q_k} q\cdot trc_{qlks} \leq Q_k \qquad \forall k\in H \tag{3.27}$$

$$\sum_{i\in N}\left(\int_{b_{s-1}}^{b_s}\widetilde{O}_i(t)\,\mathrm{d}t\right)z_{iks} + \sum_{l\in H}\sum_{i\in N} Y_{lks}^i$$

$$\leq \Delta\left(\sum_{q=1}^{Q_k} q\left(\sum_{s'\in S\backslash\{0\}, s'<s+1} u_{kqs'} - \sum_{s'\in S\backslash\{0\}, s'<s+1}\right.\right.$$

$$\left.\left.\times\sum_{l\in H, l\neq k} trc_{qkls'} + \sum_{s'\in S\backslash\{0\}, s'<s+1}\sum_{l\in H, l\neq k} trc_{qlks'}\right)\right) \qquad \forall k\in H, s\in S\backslash\{0\} \tag{3.28}$$

$$\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\mathrm{OC}_k(z_{kks} - z_{kk,s-1} + zc_{ks}) + \sum_{s\in S\backslash\{0,1\}}\sum_{k\in H}$$

$$\times\,\mathrm{CC}_k zc_{ks} + \sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H, l>k}\mathrm{OCe}_{kl}(e_{kls} - e_{kl,s-1}$$

$$+\,ec_{kls}) + \sum_{s\in S\backslash\{0,1\}}\sum_{k\in H}\sum_{l\in H, l>k}\mathrm{CCe}_{kl} ec_{kls} \leq B \tag{3.29}$$

$$b_s + d \leq b_{s+1} \qquad \forall s\in S \tag{3.30}$$

$$z_{iks}\in\{0,1\} \qquad \forall i\in N, k\in H, s\in S \tag{3.31}$$

$$u_{kqs}\in\{0,1\} \qquad \forall k\in H, q=1,\ldots,Q_k, s\in S\backslash\{0\} \tag{3.32}$$

$$trc_{qkls}\in\{0,1\} \qquad \forall k,l\in H, k\neq l, q=1,\ldots,Q_k, s\in S\backslash\{0\} \tag{3.33}$$

$$e_{kls}\in\{0,1\} \qquad \forall k,l\in H, k<l, s\in S \tag{3.34}$$

$$ec_{kls}\in\{0,1\} \qquad \forall k,l\in H, k<l, s\in S\backslash\{0\} \tag{3.35}$$

$$ea_{q'kls}\in\{0,1\} \qquad \forall k,l\in H, k<l, q'=1,\ldots,Q'_{kl}, s\in S\backslash\{0\} \tag{3.36}$$

$$zc_{ks}\in\{0,1\} \qquad \forall k\in H, s\in S\backslash\{0\} \tag{3.37}$$

$$Y_{kls}^i \geq 0 \qquad \forall i\in N, k,l\in H, s\in S\backslash\{0\} \tag{3.38}$$

$$b_s\in[0,T] \qquad \forall s\in S \tag{3.39}$$

$$z_{ik,0} = 0 \qquad \forall i\in N, k\in H \tag{3.40}$$

$$zc_{k,1} = 0 \qquad \forall k\in H \tag{3.41}$$

$$e_{kl,0} = 0 \qquad \forall k,l\in H, k<l \tag{3.42}$$

$$ec_{kl,1} = 0 \qquad \forall k,l\in H, k<l \tag{3.43}$$

$$trc_{qkl,1} = 0 \qquad \forall k,l\in H, k\neq l, q=1\ldots,Q_k \tag{3.44}$$

$$b_0 = 0 \tag{3.45}$$

$$b_{m+1} = T. \tag{3.46}$$

In the above formulation, constraint (3.5) states that in each period, each demand node must be allocated to exactly one hub. By constraint (3.6), P hubs must be opened in each period. Constraint (3.7) ensures that in each period, demand nodes are allocated only to opened hubs. Based on constraint (3.8), if a hub is opened in a period, it can continue to operate or be closed in the next period. Constraint (3.9) states that closing each hub can occur at most once during the planning horizon. By constraint (3.10), a hub in a period can be closed if it has been opened in the previous period. If a hub is closed in a certain period, it will remain closed to the end of the planning horizon. This is guaranteed by constraint (3.11). Constraint (3.12) shows the contradiction between opening and closing a hub. If a hub is opened, that hub is not closed. Constraint (3.13) ensures that only opened hubs in a period can receive module(s). Constraints (3.14) and (3.15) state that a hub link can be opened in a period if the beginning and end of it are hub nodes. According to constraint (3.16), a hub link in a period can be closed if it has been opened in the previous period. Constraint (3.17) stipulates that if a hub link is opened, that hub link is not closed. According to constraint (3.18), if a hub is not opened in the current period and has been active in the previous period, that hub is closed. Constraint (3.19) states that only opened hub links in a period can receive the additional links. Constraint (3.20) is related to routing the flows on the network in each period. The capacity constraint of a hub link is determined by equation (3.21). The capacity of each hub link includes the initial capacity and the total capacity of additional links of that hub link. The upper limit of flows on a hub link is also determined by constraint (3.22). Based on constraints (3.23) and (3.24), capacity transferring in each period can only be performed between located hubs. The capacity transferring is done from one hub to exactly one other hub. This is guaranteed by constraint (3.25). Constraint (3.26) states that in each period, the maximum number of modules that can be transferred from one hub to other hubs is equal to the sum of the modules installed on that hub, and the modules transferred from other hubs to that hub, up to that period. Constraint (3.27) sets an upper limit for the total number of modules installed on each hub, and the modules transferred from other hubs to that hub minus the modules transferred from that hub to other hubs. By constraint (3.28), the sum of the input flows to each hub in each period must not exceed that hub capacity. Constraint (3.29) states that the available budget for opening and closing hubs and hub links for the entire planning horizon is limited. Constraint (3.30) ensures that the minimum length of the time interval between the implementation of two consecutive decisions on the planning horizon and between the implementation of the last decision and the end of the planning horizon is d. Constraints (3.31)–(3.39) determine the type of decision variables, and Constraints (3.40)–(3.46) perform a preprocessing on the model.

Equations (3.2)–(3.4), (3.20), (3.22), and (3.28), after integration, are converted into equations (3.47)–(3.52), respectively.

$$
\begin{aligned}
\min \mathrm{Obj}_1 = {} & \sum_{s\in S\setminus\{0\}} \sum_{i\in N} \sum_{k\in H} z_{iks} \left(c_{ik} \left(\sum_{j\in N} \widetilde{a}_{ij} \right) (b_s - b_{s-1}) + c_{ik} \left(\sum_{j\in N} \widetilde{h}_{ij} \right) \left(\frac{b_s^2 - b_{s-1}^2}{2} \right) \right) \\
& + \sum_{s\in S\setminus\{0\}} \sum_{i\in N} \sum_{k\in H} z_{iks} \left(c_{ki} \left(\sum_{j\in N} \widetilde{a}_{ji} \right) (b_s - b_{s-1}) + c_{ki} \left(\sum_{j\in N} \widetilde{h}_{ji} \right) \left(\frac{b_s^2 - b_{s-1}^2}{2} \right) \right) \\
& + \sum_{s\in S\setminus\{0\}} \sum_{i\in N} \sum_{k\in H} \sum_{l\in H} \alpha c_{kl} Y_{kls}^{i} + \sum_{s\in S\setminus\{0\}} \sum_{k\in H} \mathrm{OC}_k (z_{kks} - z_{kk,s-1} + zc_{ks}) \\
& + \sum_{s\in S\setminus\{0,1\}} \sum_{k\in H} \mathrm{CC}_k zc_{ks} + \sum_{s\in S\setminus\{0\}} \sum_{k\in H} \sum_{q=1}^{Q_k} \mathrm{CIM}_k q u_{kqs} + \sum_{s\in S\setminus\{0\}} \sum_{k\in H} \sum_{l\in H, l>k} \\
& \times \mathrm{OCe}_{kl} (e_{kl,s} - e_{kl,s-1} + ec_{kl,s}) + \sum_{s\in S\setminus\{0,1\}} \sum_{k\in H} \sum_{l\in H, l>k} \mathrm{CCe}_{kl} ec_{kl,s} \\
& + \sum_{s\in S\setminus\{0\}} \sum_{k\in H} \sum_{l\in H, l>k} \sum_{q'=1}^{Q'_{kl}} \mathrm{CeAM}_{kl} q' ea_{q'kls} + \sum_{s\in S\setminus\{0,1\}} \sum_{k\in H} \sum_{l\in H, l\neq k} \sum_{q=1}^{Q_k} \mathrm{Ctr}_{kl} q trc_{qkls}
\end{aligned} \tag{3.47}
$$

$$\begin{aligned}\min \mathrm{Obj}_2 = (\mathrm{EW}+\mathrm{LW})\Bigg\{ &\sum_{s\in S\setminus\{0\}}\sum_{k\in H}\sum_{l\in H}\sum_{i\in N}\gamma_{kl}Y^i_{kls}d_{kl} + \sum_{s\in S\setminus\{0\}}\sum_{k\in H}\sum_{i\in N} d_{ik}z_{iks}\Bigg[\Bigg(\sum_{j\in N}\widetilde{a}_{ij}\Bigg)(b_s-b_{s-1}) \\ &+\Bigg(\sum_{j\in N}\widetilde{h}_{ij}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)+\Bigg(\sum_{j\in N}\widetilde{a}_{ji}\Bigg)(b_s-b_{s-1})+\Bigg(\sum_{j\in N}\widetilde{h}_{ji}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg]\Bigg\} \\ &+\beta\Bigg\{\sum_{s\in S\setminus\{0\}}\sum_{k\in H}\sum_{i\in N} vs^2_{ik}d_{ik}z_{iks}+\sum_{s\in S\setminus\{0\}}\sum_{k\in H}\sum_{l\in H,l>k} vs^2_{kl}d_{kl}e_{kls}\Bigg\} \end{aligned} \tag{3.48}$$

$$\begin{aligned}\max \mathrm{Obj}_3 = &\sum_{s\in S\setminus\{0\}}\sum_{k\in H} z_{kks}\mathrm{FJO}_k + \sum_{s\in S\setminus\{0\}}\sum_{i\in N}\sum_{k\in H}\mathrm{VJO}_k z_{iks}\Bigg[\Bigg(\sum_{j\in N}\widetilde{a}_{ij}\Bigg)(b_s-b_{s-1}) \\ &+\Bigg(\sum_{j\in N}\widetilde{h}_{ij}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)+\Bigg(\sum_{j\in N}\widetilde{a}_{ji}\Bigg)(b_s-b_{s-1})+\Bigg(\sum_{j\in N}\widetilde{h}_{ji}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg] \end{aligned} \tag{3.49}$$

$$\begin{aligned}&\sum_{l\in H,l\neq k} Y^i_{kls} - \sum_{l\in H,l\neq k} Y^i_{lks} = \Bigg(\Bigg(\sum_{j\in N}\widetilde{a}_{ij}\Bigg)(b_s-b_{s-1})+\Bigg(\sum_{j\in N}\widetilde{h}_{ij}\Bigg) \\ &\times\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg)z_{iks} - \sum_{j\in N}\Bigg(\widetilde{a}_{ij}(b_s-b_{s-1})+\widetilde{h}_{ij}\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg)z_{jks} \qquad \forall i\in N, k\in H, s\in S\setminus\{0\} \end{aligned} \tag{3.50}$$

$$Y^i_{kls}+Y^i_{lks} \le \Bigg(\Bigg(\sum_{j\in N}\widetilde{a}_{ij}\Bigg)(b_s-b_{s-1})+\Bigg(\sum_{j\in N}\widetilde{h}_{ij}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg)e_{kls} \qquad \forall i\in N, k,l\in H, k<l, s\in S\setminus\{0\} \tag{3.51}$$

$$\begin{aligned}&\sum_{i\in N}\Bigg(\Bigg(\sum_{j\in N}\widetilde{a}_{ij}\Bigg)(b_s-b_{s-1})+\Bigg(\sum_{j\in N}\widetilde{h}_{ij}\Bigg)\left(\frac{b_s^2-b_{s-1}^2}{2}\right)\Bigg)z_{iks} \\ &+\sum_{l\in H}\sum_{i\in N} Y^i_{lks} \le \Delta\Bigg(\sum_{q=1}^{Q_k} q\Bigg(\sum_{s'\in S\setminus\{0\},s'<s+1} u_{qks'} - \sum_{s'\in S\setminus\{0\},s'<s+1} \\ &\times\sum_{l\in H,l\neq k} trc_{qkls'} + \sum_{s'\in S\setminus\{0\},s'<s+1}\sum_{l\in H,l\neq k} trc_{qkls'}\Bigg)\Bigg) \qquad \forall k\in H, s\in S\setminus\{0\}. \end{aligned} \tag{3.52}$$

3.2.5. Linearization of the model

The proposed model for SMPHLPUTD, after integration, is nonlinear. The reason for nonlinearity is expressions that consist of multiplication of continuous variables and binary variables and multiplication of quadratic terms and binary variables. It is usually more challenging to solve nonlinear programming models than to solve linear models. In many cases, global optimality cannot be guaranteed for nonlinear models unless specific optimality conditions are met. Therefore, in this section, we use an approach for converting the nonlinear model to an equivalent linear model. At first, we introduce new variables and constraints to linearize the terms, including multiplying continuous variables in binary variables and multiplying quadratic terms in binary variables. Then, we linearize each quadratic term using a linear approximation approach proposed in reference [57].

3.2.5.1. Linearization of multiplicative terms. In the proposed model, the multiplicative terms $b_s z_{iks}$, $b_s^2 z_{iks}$, $b_s e_{kls}$, and $b_s^2 e_{kls}$ are nonlinear. For each of these nonlinear terms, we introduce new variables. Consider that $bz_{siks'} = b_s z_{iks'}$ and $bz'_{siks'} = b_s^2 z_{iks'}$ for $s, s' \in S, i \in N, k \in H$. Similarly, $be_{skls'} = b_s e_{kls'}$ and $be'_{skls'} = b_s^2 e_{kls'}$ for $s, s' \in S, k, l \in H, k < l$. We present the constraints required for linearization along with the linear form of nonlinear constraints.

3.2.5.2. Linear approximation of quadratic terms. In our model, the terms $b_s^2, \forall s \in S$ are quadratic. For linearization, we follow this method. We know that $b_s \in [0, T]$. We divide the range of b_s into $G-1$ subintervals using some grid points as $\varphi_{gs}, \forall \mathrm{GP} = 1, \ldots, G, s = 0, 1, \ldots, m+1$. Consider that $\varphi_{1,s} = 0$ and $\varphi_{Gs} = T$. Therefore, any point in the interval $[0, T]$ can be represented as a convex combination of grid points. Considering coefficients of the convex combination as λ_{gs}, we will have:

$$b_s = \sum_{g \in \mathrm{GP}} \lambda_{gs} \varphi_{gs} \qquad \forall s \in S \tag{3.53}$$

$$b_s^2 = \sum_{g \in \mathrm{GP}} \lambda_{gs} \varphi_{gs}^2 \qquad \forall s \in S \tag{3.54}$$

$$\sum_{g \in \mathrm{GP}} \lambda_{gs} = 1 \qquad \forall s \in S \tag{3.55}$$

$$\lambda_{gs} \geq 0 \qquad \forall s \in S, g \in \mathrm{GP} \tag{3.56}$$

$$\lambda_{gs} \in \mathrm{SOS2} \qquad \forall s \in S, g \in \mathrm{GP}. \tag{3.57}$$

In equations (3.53)–(3.57), the coefficients of convex combination, λ_{gs}, are members of SOS2. This set is a sequential set of non-negative variables that, at most two of them, can take non-zero values. If two variables take non-zero values, these variables must be consecutive in order. Based on the above discussion, the linear form of the objective functions in equations (3.47)–(3.49) is as follows:

$$\begin{aligned}
\min \mathrm{Obj}_1 = & \sum_{s \in S\backslash\{0\}} \sum_{i \in N} \sum_{k \in H} c_{ik} \left(\left(\sum_{j \in N} \widetilde{a}_{ij} \right) (bz_{siks} - bz_{s-1,iks}) + \left(\sum_{j \in N} \widetilde{h}_{ij} \right) \left(\frac{bz'_{siks} - bz'_{s-1,iks}}{2} \right) \right) \\
& + \sum_{s \in S\backslash\{0\}} \sum_{i \in N} \sum_{k \in H} c_{ik} \left(\left(\sum_{j=1}^{n} \widetilde{a}_{ji} \right) (bz_{siks} - bz_{s-1,iks}) + \left(\sum_{j=1}^{n} \widetilde{h}_{ji} \right) \left(\frac{bz'_{siks} - bz'_{s-1,iks}}{2} \right) \right) \\
& + \sum_{s \in S\backslash\{0\}} \sum_{i \in N} \sum_{k \in H} \sum_{l \in H} \alpha c_{kl} Y_{kls}^{i} + \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \mathrm{OC}_k (z_{kks} - z_{kk,s-1} + zc_{ks}) \\
& + \sum_{s \in S\backslash\{0,1\}} \sum_{k \in H} \mathrm{CC}_k zc_{ks} + \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \sum_{q=1}^{Q_k} \mathrm{CIM}_k q u_{kqs} + \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \sum_{l \in H, l>k} \\
& \times \mathrm{OCe}_{kl} (e_{kl,s} - e_{kl,s-1} + ec_{kl,s}) + \sum_{s \in S\backslash\{0,1\}} \sum_{k \in H} \sum_{l \in H, l>k} \mathrm{CCe}_{kl} ec_{kl,s} \\
& + \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \sum_{l \in H, l>k} \sum_{q'=1}^{Q'_{kl}} \mathrm{CeAM}_{kl} q' ea_{q'kls} + \sum_{s \in S\backslash\{0,1\}} \sum_{k \in H} \sum_{l \in H, l \neq k} \sum_{q=1}^{Q_k} \mathrm{Ctr}_{kl} q trc_{qkls}
\end{aligned} \tag{3.58}$$

$$\begin{aligned}
\min \mathrm{Obj}_2 = (\mathrm{EW} + \mathrm{LW}) \Bigg\{ & \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \sum_{l \in H} \sum_{i \in N} \gamma_{kl} Y_{kls}^{i} d_{kl} + \sum_{s \in S\backslash\{0\}} \sum_{k \in H} \sum_{i \in N} d_{ik} \Bigg[\left(\sum_{j \in N} \widetilde{a}_{ij} \right) (bz_{siks} - bz_{s-1,iks}) \\
& + \left(\sum_{j \in N} \widetilde{h}_{ij} \right) \left(\frac{bz'_{siks} - bz'_{s-1,iks}}{2} \right) + \left(\sum_{j \in N} \widetilde{a}_{ji} \right) (bz_{siks} - bz_{s-1,iks}) + \left(\sum_{j \in N} \widetilde{h}_{ji} \right)
\end{aligned}$$

$$\times\left(\frac{bz'_{siks}-bz'_{s-1,iks}}{2}\right)\Bigg]\Bigg\}+\beta\Bigg\{\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{i\in N}vs^2_{ik}d_{ik}z_{iks}+\sum_{s\in S\backslash\{0\}}\sum_{k\in H}\sum_{l\in H,l>k}vs^2_{kl}d_{kl}e_{kls}\Bigg\} \tag{3.59}$$

$$\begin{aligned}\max \mathrm{Obj}_3 = &\sum_{s\in S\backslash\{0\}}\sum_{k\in H}z_{kks}\mathrm{FJO}_k+\sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\mathrm{VJO}_k\left[\left(\sum_{j\in N}\widetilde{a}_{ij}\right)(bz_{siks}-bz_{s-1,iks})+\left(\sum_{j\in N}\widetilde{h}_{ij}\right)\right.\\ &\left.\times\left(\frac{bz'_{siks}-bz'_{s-1,iks}}{2}\right)+\left(\sum_{j\in N}\widetilde{a}_{ji}\right)(bz_{siks}-bz_{s-1,iks})+\left(\sum_{j\in N}\widetilde{h}_{ji}\right)\left(\frac{bz'_{siks}-bz'_{s-1,iks}}{2}\right)\right].\end{aligned} \tag{3.60}$$

The following constraints in equations (3.61)–(3.66) are used to linearization and must be added.

$$bz_{siks'}\le T\cdot z_{iks'} \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.61}$$

$$bz_{siks'}\le \sum_{g\in \mathrm{GP}}\lambda_{gs}\varphi_{gs} \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.62}$$

$$bz_{siks'}\ge \sum_{g\in \mathrm{GP}}\lambda_{gs}\varphi_{gs}-T\cdot(1-z_{iks'}) \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.63}$$

$$bz'_{siks'}\le T^2\cdot z_{iks'} \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.64}$$

$$bz'_{siks'}\le \sum_{g\in \mathrm{GP}}\lambda_{gs}\varphi^2_{gs} \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.65}$$

$$bz'_{siks'}\ge \sum_{g\in \mathrm{GP}}\lambda_{gs}\varphi^2_{gs}-T^2\cdot(1-z_{iks'}) \qquad \forall i\in N,k\in H,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1. \tag{3.66}$$

Finally, the SMPHLPUTD is formulated as a mixed-integer linear programming problem as follows.
Equations (3.58)–(3.60)
Subject to.

$$\begin{aligned}&\sum_{l\in H,l\ne k}Y^i_{kls}-\sum_{l\in H,l\ne k}Y^i_{lks}=\left(\left(\sum_{j\in N}\widetilde{a}_{ij}\right)(bz_{siks}-bz_{s-1,iks})\right.\\ &+\left.\left(\sum_{j\in N}\widetilde{h}_{ij}\right)\left(\frac{bz'_{siks}-bz'_{s-1,iks}}{2}\right)\right)-\sum_{j\in N}\left(\widetilde{a}_{ij}(bz_{sjks}-bz_{s-1,jks})\right.\\ &+\left.\widetilde{h}_{ij}\left(\frac{bz'_{sjks}-bz'_{s-1,jks}}{2}\right)\right) \qquad \forall i\in N,k\in H,s\in S\backslash\{0\}\end{aligned} \tag{3.67}$$

$$\begin{aligned}&Y^i_{kls}+Y^i_{lks}\le\left(\sum_{j\in N}\widetilde{a}_{ij}\right)(be_{skls}-be_{s-1,kls})+\left(\sum_{j\in N}\widetilde{h}_{ij}\right)\\ &\times\left(\frac{be'_{sjks}-be'_{s-1,jks}}{2}\right) \qquad \forall i\in N,k,l\in H,k<l,s\in S\backslash\{0\}\end{aligned} \tag{3.68}$$

$$be_{skls'}\le T\cdot e_{kls'} \qquad \forall k,l\in H,k<l,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.69}$$

$$be_{skls'}\le \sum_{g\in \mathrm{GP}}\lambda_{gs}\varphi_{gs} \qquad \forall k,l\in H,k<l,s,s'\in S\backslash\{0\},s'=s\ \text{ or }\ s'=s+1 \tag{3.70}$$

$$be_{skls'} \geq \sum_{g \in \mathrm{GP}} \lambda_{gs}\varphi_{gs} - T \cdot (1 - e_{kls'}) \qquad \forall k,l \in H, k<l, s,s' \in S\backslash\{0\}, s'=s \text{ or } s'=s+1 \tag{3.71}$$

$$be'_{skls'} \leq T^2 \cdot e_{kls'} \qquad \forall k,l \in H, k<l, s,s' \in S\backslash\{0\}, s'=s \text{ or } s'=s+1 \tag{3.72}$$

$$be'_{skls'} \leq \sum_{g \in \mathrm{GP}} \lambda_{gs}\varphi_{gs}^2 \qquad \forall k,l \in H, k<l, s,s' \in S\backslash\{0\}, s'=s \text{ or } s'=s+1 \tag{3.73}$$

$$be'_{skls'} \geq \sum_{g \in \mathrm{GP}} \lambda_{gs}\varphi_{gs}^2 - T^2 \cdot (1 - e_{kls'}) \qquad \forall k,l \in H, k<l, s,s' \in S\backslash\{0\}, s'=s \text{ or } s'=s+1 \tag{3.74}$$

$$\begin{aligned}&\sum_{i \in N}\left(\left(\sum_{j \in N}\widetilde{a}_{ij}\right)(bz_{siks} - bz_{s-1,iks}) + \left(\sum_{j \in N}\widetilde{h}_{ij}\right)\left(\frac{bz'_{siks} - bz'_{s-1,iks}}{2}\right)\right)\\ &+ \sum_{l \in H}\sum_{i \in N} Y^i_{lks} \leq \Gamma\left(\sum_{q=1}^{Q_k} q\left(\sum_{s' \in S\backslash\{0\}, s'<s+1} u_{kqs'} - \sum_{s' \in S\backslash\{0\}, s'<s+1}\right.\right.\\ &\left.\left.\times \sum_{l \in H, l \neq k} trc_{qkls'} + \sum_{s' \in S\backslash\{0\}, s'<s+1}\ \sum_{l \in H, l \neq k} trc_{qlks'}\right)\right) \qquad \forall k \in H, s \in S\backslash\{0\}\end{aligned} \tag{3.75}$$

$$\sum_{g \in \mathrm{GP}} \lambda_{gs}\varphi_{gs} + d \leq \sum_{g \in \mathrm{GP}} \lambda_{g,s+1}\varphi_{g,s+1} \qquad \forall s \in S\backslash\{m+1\} \tag{3.76}$$

$$bz_{siks'}, bz'_{siks'} \geq 0 \tag{3.77}$$

$$be_{skls'}, be'_{skls'} \geq 0 \tag{3.78}$$

$$bz_{0,ik,1} = 0 \tag{3.79}$$

$$bz'_{0,ik,1} = 0 \tag{3.80}$$

$$be_{0,kl,1} = 0 \tag{3.81}$$

$$be'_{0,kl,1} = 0 \tag{3.82}$$

and equations (3.5)–(3.19), (3.21), (3.23)–(3.27), (3.29), (3.31)–(3.46), (3.53)–(3.57), (3.61)–(3.66).

3.2.6. Valid inequalities

In this subsection, we present some valid inequalities to improve the formulation. Valid inequality in equation (3.83) states that in each period, the graph of hubs must be a connected graph. The other valid inequality in equation (3.84) stipulates that in the first period, exactly P hubs receive modules. The third valid inequality in equation (3.85) ensures that at least one module must be available in each opened hub to process flows in each period.

$$\sum_{k \in H}\sum_{l \in H, l>k} e_{kls} \geq P - 1 \qquad \forall s \in S\backslash\{0\} \tag{3.83}$$

$$\sum_{k \in H}\sum_{q=1}^{Q_k} u_{kq,1} = P \tag{3.84}$$

$$\begin{aligned}z_{kks} \leq &\sum_{q=1}^{Q_k}\ \sum_{s' \in S\backslash\{0\}, s'<s+1} q u_{kqs'} - \sum_{s' \in S\backslash\{0\}, s'<s+1}\ \sum_{l \in H, l \neq k}\sum_{q=1}^{Q_k} q \cdot trc_{qkls'}\\ &+ \sum_{s' \in S\backslash\{0\}, s'<s+1}\ \sum_{l \in H, l \neq k}\sum_{q=1}^{Q_k} q \cdot trc_{qlks'} \qquad \forall k \in H, s \in S\backslash\{0\}.\end{aligned} \tag{3.85}$$

In the following, we present another valid inequality based on reference [22]. Consider constraint (3.27). We can enhance this constraint through equation (3.86):

$$\sum_{s\in S\setminus\{0\}}\sum_{q=1}^{Q_k} q\cdot u_{kqs} - \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} q\cdot trc_{qkls} + \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} q\cdot trc_{qlks} \leq Q_k\sum_{s\in S} z_{kks} \quad \forall k\in H. \tag{3.86}$$

In equation (3.86), each term can be divided by an integer value $\eta \geq 1$, and we can get the following relation:

$$\sum_{s\in S\setminus\{0\}}\sum_{q=1}^{Q_k} \frac{q}{\eta}\cdot u_{kqs} - \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \frac{q}{\eta}\cdot trc_{qkls} + \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \frac{q}{\eta}\cdot trc_{qlks} \leq \frac{Q_k}{\eta}\sum_{s\in S} z_{kks} \qquad \forall k\in H, \eta=1,\dots,Q_k.$$

Then, we can round down coefficients on the left-hand side and get to the following relation:

$$\sum_{s\in S\setminus\{0\}}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot u_{kqs} - \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot trc_{qkls} + \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot trc_{qlks} \leq \frac{Q_k}{\eta}\sum_{s\in S} z_{kks} \qquad \forall k\in H, \eta=1,\dots,Q_k.$$

In each feasible solution, the left-hand side is an integer value. Therefore, equation (3.87) is valid.

$$\sum_{s\in S\setminus\{0\}}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot u_{kqs} - \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot trc_{qkls} + \sum_{s\in S\setminus\{0,1\}}\sum_{l\in H,l\neq k}\sum_{q=1}^{Q_k} \left\lfloor\frac{q}{\eta}\right\rfloor\cdot trc_{qlks} \leq \left\lfloor\frac{Q_k}{\eta}\right\rfloor\sum_{s\in S} z_{kks} \qquad \forall k\in H, \eta=1,\dots,Q_k. \tag{3.87}$$

Valid inequalities in equations (3.83)–(3.85) and (3.87) are added to the problem model to strengthen the formulation.

3.2.7. Robust counterpart formulation

In this subsection, we present a robust counterpart formulation of SMPHLPUTD (after linearization). To convert the uncertain model to a robust counterpart model, we use the approach proposed in reference [13] and its extension for discrete optimization problems in reference [14]. This approach that is based on budget uncertainty, leads to a linear robust counterpart formulation and provides a balance between robustness and level of conservatism. To use this approach, suppose that c, l, and u are column vectors with n elements, A is a $m\times n$ matrix, and b is a column vector with n elements. Consider the following Mixed-Integer Programming (MIP) problem with nominal values and n variables. Of these variables, the first k variables are integer variables.

$$\begin{aligned}&\max\ c'x\\ &\text{s.t.}\\ &\qquad Ax\leq b\end{aligned}$$

$$
\begin{aligned}
& l \leq x \leq u \\
& x_i \in \mathbb{Z}, \forall i = 1, \dots, k.
\end{aligned} \tag{3.88}
$$

Without loss of generality, we suppose that in problem (3.88), only the elements of matrix A are subject to uncertainty. The coefficients of the objective function and right-hand side values are deterministic. Each element of the coefficient matrix is modeled as an independent, symmetric, and bounded random variable with an unknown distribution. This random variable is represented by $\widetilde{\mu}_{ij}, \forall i = 1, \dots, m, j = 1, \dots, n$ and takes value in interval $[\mu_{ij} - \hat{\mu}_{ij}, \mu_{ij} + \hat{\mu}_{ij}]$. For each constraint i, parameters $\Gamma_i, \forall i = 1, \dots, m$, is defined that takes value in $[0, |J_i|]$ where $J_i = \{j | \hat{\mu}_{ij} > 0\}$. This parameter is called the uncertainty budget for constraint i and is not necessarily integer. This parameter adjusts robustness against the conservatism level. If $\Gamma_i = 0$, the effect of changes in elements of the coefficients matrix is ignored, and if $\Gamma_i = |J_i|$, all possible changes in elements of the coefficients matrix are considered, which is equivalent to the highest level of conservatism. But if $\Gamma_i \in (0, |J_i|)$, the DM determines what level of changes must be considered to the elements of the coefficients matrix. Consider constraint i of integer programming problem with nominal values as $\mu_i' x \leq b_i$. In this constraint, J_i is related to a set of μ_{ij} coefficients that are subject to uncertainty. The uncertain elements of this constraint, $\widetilde{\mu}_{ij}, j \in J_i$, take values independently and according to an unknown and symmetric distribution with the mean nominal values μ_{ij} in $[\mu_{ij} - \hat{\mu}_{ij}, \mu_{ij} + \hat{\mu}_{ij}]$. It is unlikely that all the uncertain elements of this constraint will take their worst values simultaneously. Therefore, the aim is controlling $\lfloor \Gamma_i \rfloor$ of elements that take the worst values and an element μ_{it} whose range of changes is equal to $(\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{\mu}_{it}$. Thus, the robust counterpart of problem (3.88) is written as follows.

$$
\begin{aligned}
& \max\ c' x \\
& \text{ s.t.} \\
& \qquad \sum_j \mu_{ij} x_j + \max_{\{S_i \bigcup \{t_i\} | S_i \subseteq J_i, |S_i| \leq \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{\mu}_{ij} |x_j| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{\mu}_{it_i} |x_{t_i}| \right\} \leq b_i, \quad \forall i \\
& \qquad l \leq x \leq u \\
& \qquad x_i \in \mathbb{Z}, \qquad \forall i = 1, \dots, k.
\end{aligned} \tag{3.89}
$$

In problem (3.89), S_i shows the subset containing Γ_i uncertain parameters in constraint i. Also, t_i explains the additional uncertain parameter. This constraint states that Γ_i uncertain parameters can take their worst values simultaneously. Authors in reference [13] showed that the following problem is equivalent to problem (3.89):

$$
\begin{aligned}
& \max\ c' x \\
& \text{ s.t.} \\
& \qquad \sum_j \mu_{ij} x_j + z_i \Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i && \forall i \\
& \qquad z_i + p_{ij} \geq \hat{\mu}_{ij} y_j && \forall i, j \in J_i \\
& \qquad -y_j \leq x_j \leq y_j, && \forall j \\
& \qquad l_j \leq x_j \leq u_j, && \forall j \\
& \qquad y_j \geq 0 && \forall j \\
& \qquad z_i \geq 0 && \forall i \\
& \qquad p_{ij} \geq 0 && \forall i, j \in J_i \\
& \qquad x_i \in \mathbb{Z}, && \forall i = 1, \dots, k.
\end{aligned} \tag{3.90}
$$

In problem (3.90), if all variables are non-negative, there is no need to use variables y. Reference [13] presented bounds to determine the maximum probability of constraints violation. They showed that the following bound is better than the other bounds.

$$pr\left(\sum_j \widetilde{a}_{ij}x_j^* > b_i\right) \leq \frac{1}{2^n}\left\{(1-\mu)\sum_{l=\lfloor v\rfloor}^{n}\binom{n}{l} + \mu\sum_{l=\lfloor v\rfloor+1}^{n}\binom{n}{l}\right\} \tag{3.91}$$

where $n = |J_i|$, $v = \frac{\Gamma_i+n}{2}$ and $\mu = v - \lfloor v\rfloor$. In this study, objective functions in equations (3.47)–(3.49) and constraints (3.50)–(3.52) are subject to uncertainty. We present the robust counterparts of these relations in Appendix A.

4. Solution method

The robust counterpart formulation of SMPHLPUTD is a multi-objective MIP problem. To solve multi-objective problems, various methods have been proposed in the literature. Among these proposed methods, fuzzy interactive methods that can determine the degree of satisfaction of each objective function according to the DM preferences are increasingly applied. One of these methods is the two-phase Torabi–Hassini (TH) method [61]. Despite the classical methods such as the weighted sum method, this method can identify only efficient Pareto solutions to multi-objective problems according to the DM preferences. In this study, we use the TH method to solve SMPHLPUTD. In the first phase of this method, an uncertain problem is converted into an equivalent problem without uncertainty. In phase two, using an aggregation objective function, the problem becomes a single-objective parametric problem. By selecting different values for the parameters in this problem, we get efficient Pareto solutions. The steps of the TH method for the present model is as follows:

Step 1. Formulate SMPHLPUTD as a multi-objective MIP problem.
Step 2. Construct the robust counterpart of SMPHLPUTD.
Step 3. Determine Γ-Positive Ideal Solution (Γ-PIS) and Γ-Negative Ideal Solution (Γ-NIS) for each objective function.
Step 4. Determine the linear membership function for each objective function.
Step 5. Convert the multi-objective problem into a single-objective problem using the aggregation function of the TH method.
Step 6. Select the values of parameters of the single-objective problem.
Step 7. Solve the single-objective problem to get a Pareto optimal solution.
Step 8. If the DM is satisfied, stop. Otherwise, go to step 6 and change the value of the parameters to get another Pareto optimal solution.

So far, we have taken the first and second steps. In step 3, to determine Γ-PIS, *i.e.*, $\left(\mathrm{Obj}_1^{\Gamma\text{-PIS}}, x_1^{\Gamma\text{-PIS}}\right)$, $\left(\mathrm{Obj}_2^{\Gamma\text{-PIS}}, x_2^{\Gamma\text{-PIS}}\right)$, $\left(\mathrm{Obj}_3^{\Gamma\text{-PIS}}, x_3^{\Gamma\text{-PIS}}\right)$, each objective function must be optimized separately under the problem constraints. Γ-NIS for each objective function is derived from equations (4.1) and (4.2).

$$\mathrm{Obj}_f^{\Gamma\text{-NIS}} = \max \mathrm{Obj}_f; \qquad \forall f = 1,2 \tag{4.1}$$

$$\mathrm{Obj}_3^{\Gamma\text{-NIS}} = \min \mathrm{Obj}_3. \tag{4.2}$$

Equations (4.3) and (4.4) determine the degree of satisfaction for each objective function.

$$\mu_f(x) = \begin{cases} 1 & \mathrm{Obj}_f < \mathrm{Obj}_f^{\Gamma\text{-PIS}} \\ \frac{\mathrm{Obj}_f^{\Gamma\text{-NIS}} - \mathrm{Obj}_f}{\mathrm{Obj}_f^{\Gamma\text{-NIS}} - \mathrm{Obj}_f^{\Gamma\text{-PIS}}} & \mathrm{Obj}_f^{\Gamma\text{-PIS}} \leq \mathrm{Obj}_f \leq \mathrm{Obj}_f^{\Gamma\text{-NIS}} \\ 0 & \mathrm{Obj}_f > \mathrm{Obj}_f^{\Gamma\text{-NIS}} \end{cases} \qquad \forall f = 1,2 \tag{4.3}$$

FIGURE 2. Turkish network.

$$\mu_3(x) = \begin{cases} 1 & \mathrm{Obj}_3 > \mathrm{Obj}_3^{\Gamma\text{-PIS}} \\ \frac{\mathrm{Obj}_3 - \mathrm{Obj}_3^{\Gamma\text{-NIS}}}{\mathrm{Obj}_3^{\Gamma\text{-PIS}} - \mathrm{Obj}_3^{\Gamma\text{-NIS}}} & \mathrm{Obj}_3^{\Gamma\text{-NIS}} \leq \mathrm{Obj}_3 \leq \mathrm{Obj}_3^{\Gamma\text{-PIS}} \\ 0 & \mathrm{Obj}_3 < \mathrm{Obj}_3^{\Gamma\text{-NIS}}. \end{cases} \tag{4.4}$$

Then, using the aggregation function of the TH method, the multi-objective problem becomes a single-objective problem as follows:

$$\max \omega(x) = \psi\omega_0 + (1-\psi)\sum_{f=1}^{3} \pi_f \mu_f(x) \tag{4.5}$$

$$\text{s.t. } \omega_0 \leq \mu_f(x), \quad f = 1,2,3 \tag{4.6}$$

$$x \in F,\ \omega_0 \in [0,1]. \tag{4.7}$$

In which parameter $\psi \in [0,1]$ is called the coefficient of compensation. The importance of each objective is determined by parameter $\pi_f \in [0,1]$ and $\sum_{f=1}^{3} \pi_f = 1, \pi_f > 0$. This formulation seeks a compromise solution between the minimum and weighted sum operators. By selecting different values for parameters ψ and π_f, we get efficient Pareto solutions.

5. COMPUTATIONAL EXPERIMENTS

5.1. Numerical example

In this section, we use Turkish network data (TR dataset) for validating the proposed model. Demand nodes are considered as $N = \{1,2,3,4,5,6,7,16\}$ and the candidate nodes for opening hubs are considered as $H = \{1,3,5,6,7,16\}$. Figure 2 shows the demand nodes and candidate nodes for hubs. Note that transportation demand (the flows between demand nodes) are normalized so that the total transportation demand for these 8 points is equal to one. We assume that the amount of change in the demand function parameters can be up to 20% of the nominal values.

There are three periods, and the length of the planning horizon is $T = 5$. In each period, $P = 3$ hubs must be located. For parameters used in the TH method, we assume that $\psi = 0.5$ and the importance of the objective functions are considered the same. That is, $\pi_f = \frac{1}{3}$, $\forall f = 1,2,3$. The level of conservatism in Bertsimas and Sim's method is 50%. It means that in any constraint on the problem, at most half of the uncertain coefficients can simultaneously select their worst value. Calculations are performed on a laptop with the following characteristics: processor Intel(R) Core (TM) i5-5200U CPU @ 2.20 GHz and 6 GB of RAM on

TABLE 5. Details of data generation for the test problem.

Symbol	Value	Symbol	Value	Symbol	Value
N	$\{1,2,3,4,5,6,7,16\}$	OC_k	$\mathrm{FixedCostHub}_k^{\mathrm{TR}}$ [11]	Cape_{kl}	Uniform $(0.5,1)$
H	$\{1,3,5,6,7,16\}$ [3]	OCe_{kl}	$\mathrm{FixedCostHubLink}_{kl}^{\mathrm{TR}}$ [11]	Δ'	0.75
P	3	CC_k	$0.4\times\mathrm{OC}_k$	d	0.25
S	$\{0,1,2,3\}$	CCe_{kl}	$0.4\times\mathrm{OCe}_{kl}$	vs	$\left[\frac{1000*\mathrm{distance}_{ij}^{\mathrm{TR}}}{60*\mathrm{TravelTime}_{ij}^{\mathrm{TR}}}\right]$
T	5	B	4000	EW	4000
GP	$\{1,2,\ldots,T+1\}$	Q_k	5	LW	100
a_{ij}	w_{ij}^{TR} [11]	Q'_{kl}	5	d_{ij}	$\mathrm{distance}_{ij}^{\mathrm{TR}}\times 1000$
h_{ij}	$\mathrm{Uniform}\left(\frac{-a_{ij}}{T},a_{ij}\right)$ [64]	CIM_k	$0.2\times\mathrm{OC}_k$	γ_{ij}	0.098 [12]
$\hat{a}_{ij}$	$0.2\times\lvert a_{ij}\rvert$	Ctr_{kl}	$10\times\mathrm{distance}_{ij}^{\mathrm{TR}}$	β	2.107 [12]
$\hat{h}_{ij}$	$0.2\times\lvert h_{ij}\rvert$	CeAM_{kl}	$0.4\times\mathrm{OCe}_{kl}$	FJO_k	RandomInteger $(10,100)$ [67]
c_{ij}	$\mathrm{distance}_{ij}^{\mathrm{TR}}$ [11]	Δ	0.75	VJO_k	Uniform $(10,100)$
α	0.9				

windows eight (64-bit). The TH method is coded in GAMS software version 27.3.0, and the CPLEX solver is used for solving the instance. Details of generating other parameters are as Table 5.

Figure 3 shows the results of solving the test problem in two cases. The first case is when all the parameters of the demand function take their nominal values. The second case is when the parameters are subject to the robust uncertainty, and half of the uncertain coefficients of each constraint can simultaneously take their worst values. The numbers next to the hubs displayed in blue indicate the selected capacity for a hub in a given period, which is equal to available modules of that hub up to that period (installed and transferred modules from other hubs to that hub, minus the modules transferred from that hub to other hubs). The numbers next to the hub links displayed in purple indicate the number of additional links installed on that hub link.

According to Figure 3, when the parameters take the nominal values, in the first period, hubs 1, 6, and 7 and hub links $(1,6)$ and $(6,7)$ are opened. Nodes 3, 5, and 16 are allocated to hub 6, and nodes 2 and 4 are allocated to hub 1. The total number of modules received by the hubs is nine, and no additional links are installed on hub links. In the robust case, in the first period, hubs 3, 6, and 16 and hub links $(3,16)$ and $(6,16)$ are opened. Nodes 2, 4, and 7 are allocated to hub 3. Node 1 is allocated to hub 6, and node 5 is allocated to hub 16. In total, opened hubs receive fourteen modules, and ten additional links are installed on hub links. In periods 2 and 3, the network configuration in the robust case differs from the case nominal values. In the robust case, the total selected capacity for hubs and hub links is greater than the case nominal values. In the second period, the hubs receive eleven modules, and three additional links are installed on hub links in the case nominal values. While in the robust case, the hubs receive seventeen modules, and nine additional links are installed on hub links. In the third period, the total number of modules installed on the hubs in the case nominal values and robust is eleven and fourteen modules, respectively. Also, the total number of additional links installed on hub links, in the case nominal values and robust, is three and fifteen links, respectively. Given the differences of solutions in the two cases, it can be concluded that if the uncertainty in the problem parameters is not taken into account, the resulting solution may be non-optimal or even infeasible. Therefore, it is necessary to consider the uncertainty of the parameters. In the robust case, in each period, total capacities selected for hubs and hub links increase significantly. This makes the designed network robust to real changes in transportation demand during the planning horizon and allows the flows to be routed on the network.

In the case of nominal values, the optimal values of breakpoints are as follows: $b_0^*=0$, $b_1^*=1.036$, $b_2^*=3.323$, and $b_3^*=5$. Thus, the first, second, and third periods, in this case, are as intervals $[0,1.036]$, $[1.036,3.323]$, and $[3.323,5]$, respectively. Also, the optimal values of breakpoints for the robust case are: $b_0^*=0$, $b_1^*=1.027$, $b_2^*=3.923$, and $b_3^*=5$. The first, second, and third time periods in this case are as intervals $[0,1.027]$, $[1.027,3.923]$, and $[3.923,5]$, respectively. The optimal objective function value of the TH method for nominal

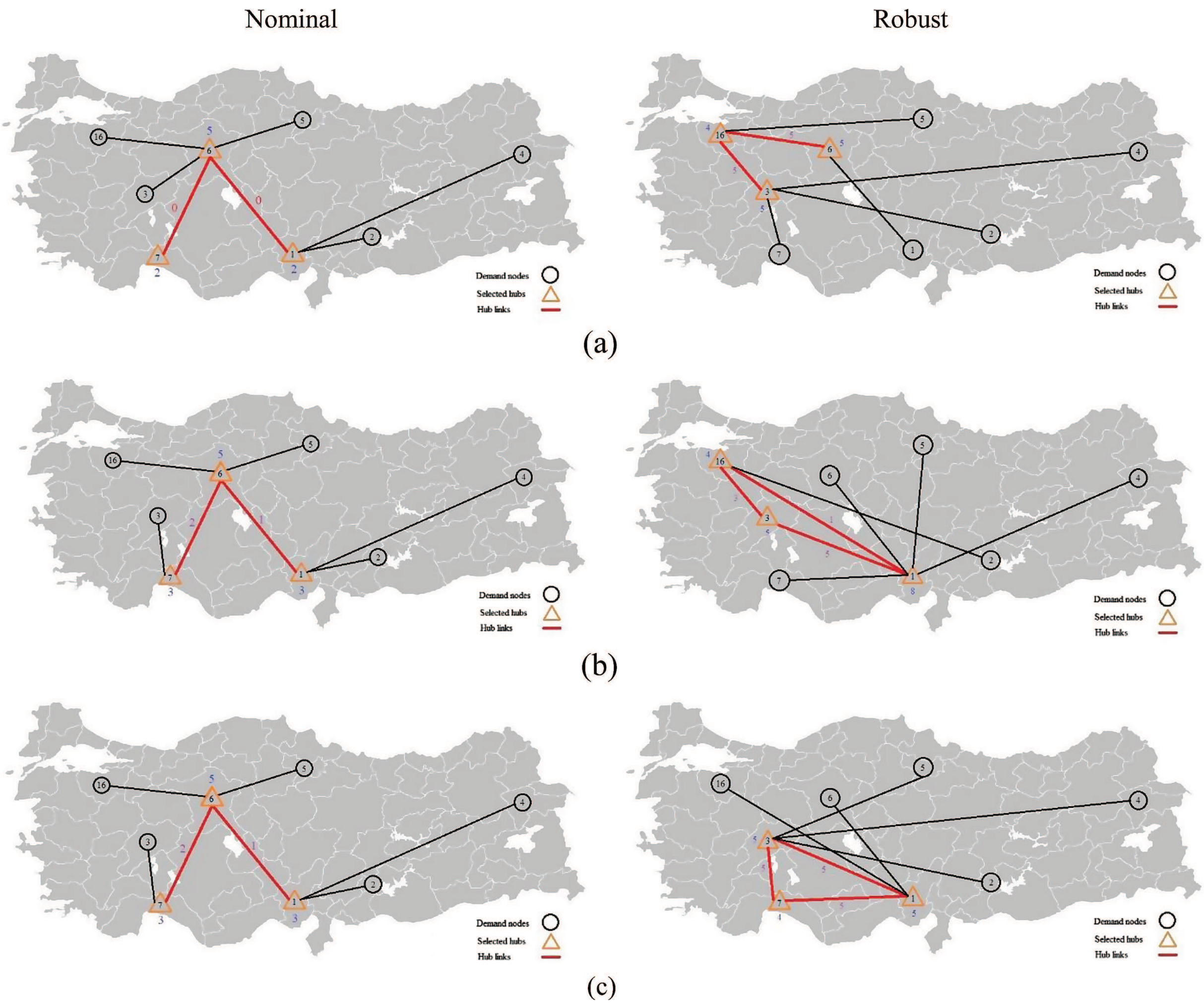


FIGURE 3. Hub network configuration for Turkish network with eight nodes in (a) first period, (b) second period and (c) third period.

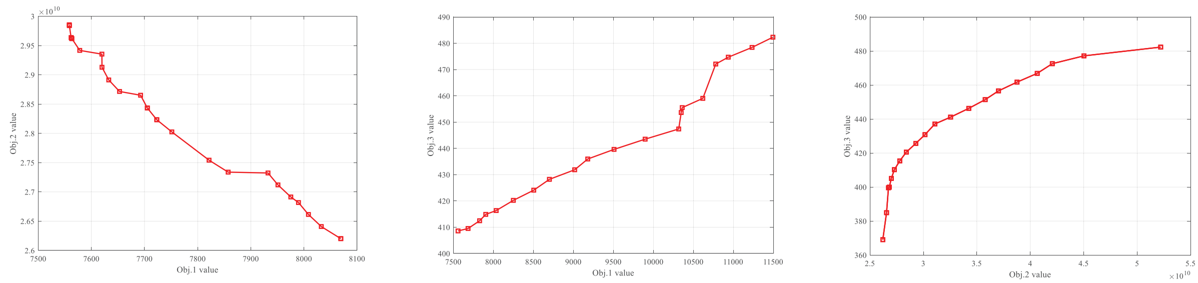
values is 0.897, and for the robust case is 0.574. The solution time of the model with nominal values is 4619.58 s, and the solution time of the robust model is 7757.02 s. This solution time for each case is the sum of solution time to find Γ-PIS and Γ-NIS and the solution time of the single-objective problem of the TH method.

5.2. Sensitivity analysis

In this section, we make a sensitivity analysis on the parameters of the Bertsimas and Sim's method. To this end, we first examine the conflict between the objectives of the problem. Because the study of the conflict between the objectives helps to understand the sensitivity of the TH objective function to the parameters of the Bertsimas and Sim's method. The payoff table for $\Gamma = 0$ is reported in Table 6. According to Table 6, when the first objective function has its best value, the second and third objective functions do not have their best value. When the value of the second objective function is at its lowest, the third objective function has the worst value, and the first objective function is not in the best condition. The best value for the third objective is achieved when the first and second objectives are at their worst. Therefore, it can be concluded that the

TABLE 6. Payoff table ($\Gamma = 0$).

	Obj.1	Obj.2	Obj.3
Min Obj.1	7558.36	29 855 194 293.50	408.58
Min Obj.2	8069.32	26 201 482 585.42	369.05
Max Obj.3	11 492.79	55 586 485 685.65	482.31



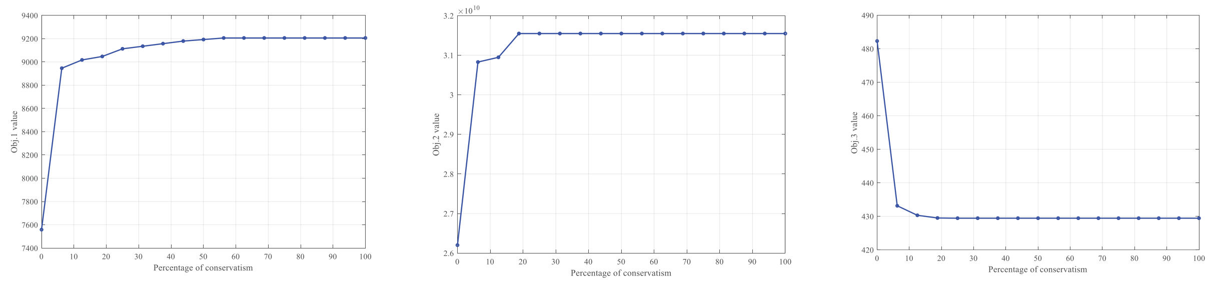
(a) (b) (c)

FIGURE 4. Optimal Pareto frontier for (a) first and second objectives, (b) first and third objectives and (c) second and third objectives ($\Gamma = 0$).

objectives are in conflict with each other. To examine the conflict between the objectives, we also perform a bi-objective optimization for each pair of objectives. The optimal Pareto front for each pair of objectives is shown in Figure 4. According to Figure 4a, as the total costs in the first objective function increase, emissions decrease. In Figure 4b, to increase fixed and variable job opportunities in the third objective function, the total costs in the first objective function must be increased. According to Figure 4c, the objectives of minimizing emission costs and maximizing fixed and variable job opportunities are in conflict. That is, increasing fixed and variable job opportunities increases emission costs (note that the first and second objective functions are minimization and the third objective function is maximization).

In the following, we look at changes in optimal objective function value to the percentage of conservatism. The percentage of conservatism represents a percentage of the coefficients of all uncertain constraints that can simultaneously take their worst value. This percentage is considered simultaneously for all uncertain constraints. For example, if the percentage of conservatism is 50%, half of the uncertainty coefficients in all uncertain constraints can simultaneously take their worst value. To investigate the sensitivity of the values of the objective functions to the level of conservatism, we first perform a single-objective optimization for each objective. The results of are shown in Figure 5. According to Figure 5a, as the level of conservatism increases to 56.25%, the first objective value increases. After that, the first objective value remains constant at 9205.91. In Figure 5b, as the level of conservatism increases to 18.75%, the second objective value increases. After that, the second objective value remains constant at 31 548 025 840.18. According to Figure 5c, as the level of conservatism increases to 25%, the third objective value decreases. After that, the third objective value remains constant at 429.44.

To investigate the sensitivity of the objectives to the level of conservatism in multi-objective optimization, we optimize the aggregation function of the TH method. The results are shown in Figure 6. According to Figure 6, four return points are observed. The highest value of the TH objective function is obtained for the lowest level of conservatism. This value is equal to 0.897. As the level of conservatism increases to 18.75%, the curve shows a decreasing trend, and the value TH objective value decreases to 0.359 (the first return point of the curve). Then, by increasing the level of conservatism to 31.25%, the curve shows an increasing trend, and the TH objective value reaches 0.553. Then, as the level of conservatism increases to 37.5%, a decreasing trend



(a) (b) (c)

FIGURE 5. Changes in optimal value of the objective functions to the percentage of conservatism for (a) first objective, (b) second objective and (c) third objective.

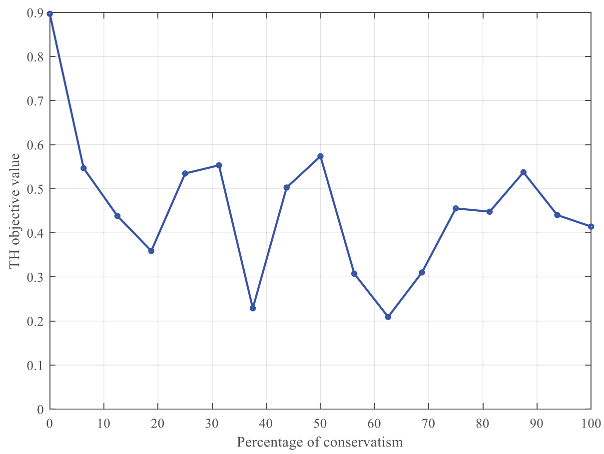
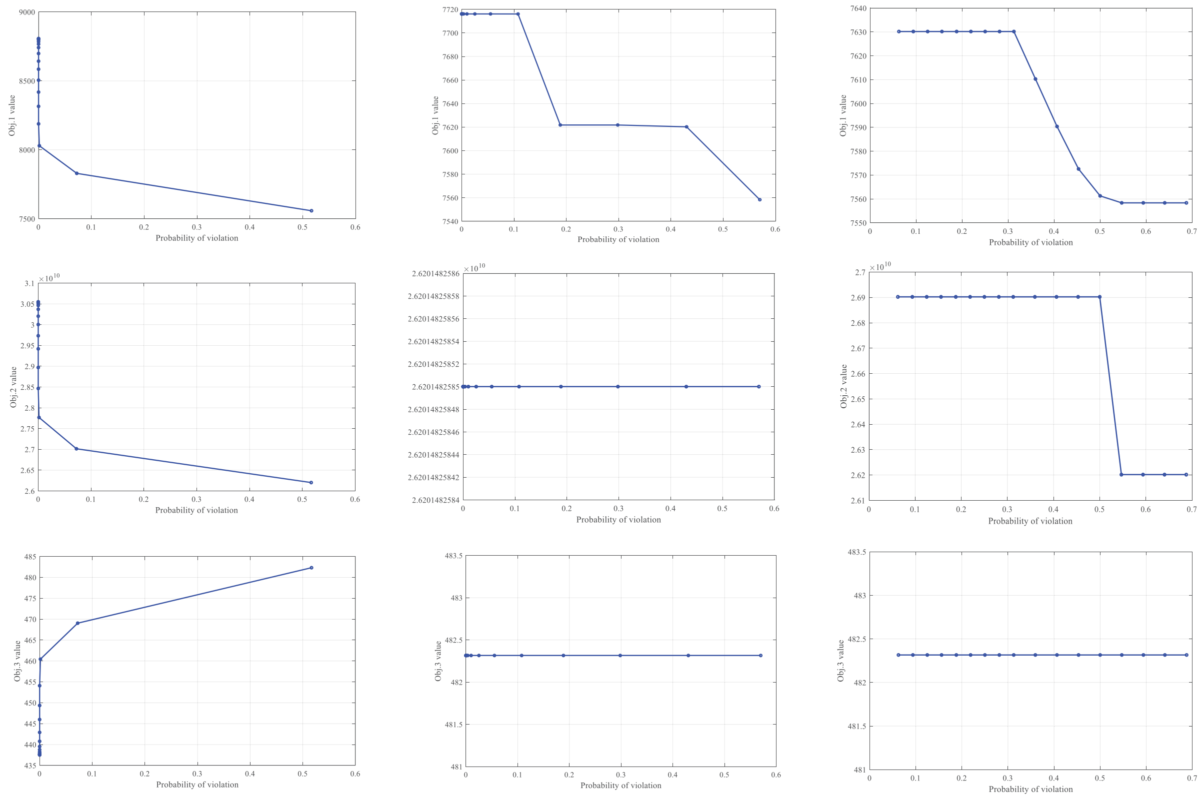


FIGURE 6. Sensitivity analysis on the percentage of conservatism for the TH objective function.

is observed, and the TH objective value reaches 0.229 (the second return point of the curve). Then, as the level of conservatism increases to 50%, the curve is increasing, and the TH objective value reaches 0.574. After that, until the conservatism level of 62.50% is reached, the curve is decreasing, and the TH objective value reaches the minimum value of 0.209 (the third return point of the curve). Then, reaching the conservatism level of 75%, the TH objective value reaches 0.455. Reaching the conservatism level of 81.25%, the TH objective value reaches 0.448 (the fourth return point of the curve). Reaching the conservatism level of 87.50%, the TH objective value reaches 0.537. After that, the curve is decreasing, and at the highest level of conservatism, the TH objective value is 0.414. The existence of return points in the curve can be analyzed in such a way that due to the conflict between the objectives, in multi-objective optimization performed by the TH method, by changing the value of an objective (improvement or deterioration of that objective) due to the increase in the level of conservatism, The other two objectives do not necessarily change in the same direction of that objective. The result of these changes is reflected in the aggregation objective function of the TH method. Thus, the trend of changes in the TH objective function can be increasing or decreasing. If the aggregated value decreases, the curve will have a decreasing trend, and if the aggregated value increases, the curve will have an increasing trend.

To examine the changes in the values of the objective functions relative to the changes in the probability of constraints violation, we first perform a single-objective optimization for each objective for different values of violation probability. The results are shown in Figure 7. According to Figure 7a, the trend of changes in the

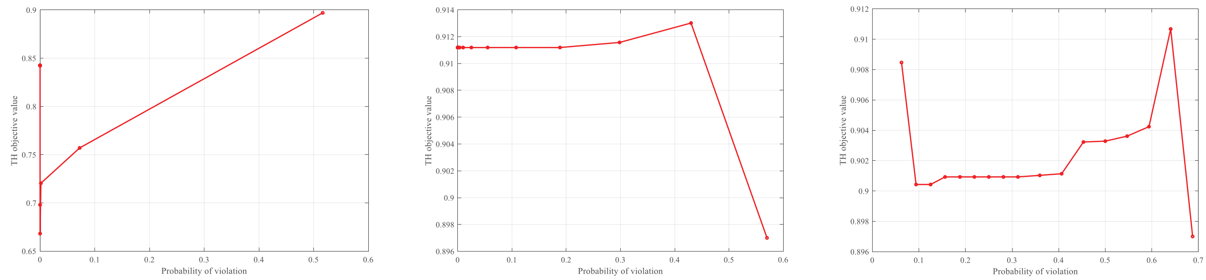


(a) (b) (c)

FIGURE 7. Changes in optimal value of the objective functions to the probability of violation for (a) constraints of objective function, (b) flow balance and hub capacity constraints and (c) constraints of flow on hub links.

first and second objective functions is decreasing, and the trend of changes in the third objective function is increasing. Therefore, as the probability of constraints violation increases, the values of the first, second, and third objective functions improve. Figure 7b shows a non-increasing trend in the first objective function as the probability of violation increases (the curve from probability 0 to 0.1077 shows a steady trend). In contrast, the second and third objectives remain at their optimal level. According to Figure 7c, as the probability of violation increases, a non-increasing trend is observed in the first and second objective functions (value of the first objective decreases between the probability of 0.3125 and 0.5469, and it is fixed in the rest points). The second objective value decreases between the probabilities of 0.5000 and 0.5469, and it is fixed in the rest points. In contrast, the third objective function remains constant at its optimal level.

To examine the changes in the objective functions value to the changes in the probability of constraints violation in multi-objective optimization, we optimize the aggregation function of the TH method for different violation probability values. The results are shown in Figure 8. According to Figure 8a, for probabilities less than 0.0015, the TH objective value is between 0.668 and 0.842 (when the probability of violation is lower than or equal to 0.0015, the level of conservatism is between 12.5% and 100%). The best value for the TH objective function is obtained when the probability of violation for these constraints is the highest possible value (*i.e.*, 0.516). In Figure 8b, by increasing the probability of violation from 0.0251 to 0.430, an increasing trend in the curve is observed. The best objective function value (0.913) is obtained when the probability of



(a) (b) (c)

FIGURE 8. Sensitivity analysis on probability of violation for in (a) constraints of the TH method, (b) flow balance and hub capacity constraints and (c) constraints of flow on hub links.

violation is 0.430. As the probability of violation increases to 0.570, the TH objective value is reduced from 0.913 to 0.897. According to Figure 8c, as the probability of violation increases from 0.0625 to 0.0938, the TH objective value is reduced from 0.908 to 0.900. When the probability of violation is between 0.0938 and 0.6404, an increasing trend is observed in the curve. Also, as the probability of violation increases from 0.6404 to 0.6875, the TH objective value decreases from 0.910 (the highest TH objective value) to 0.897 (the lowest TH objective value). The existence of the return points in the curves in Figure 8 can be explained by the existing conflict between the objectives. By increasing the probability of constraints violation, the changes in the objectives (their improvement or worsening) are not necessarily consistent. The result of changing the objectives is reflected in the aggregation objective function of the TH method. If the aggregated value increases, the curve will have an increasing trend, and if the aggregated value decreases, the curve will have a decreasing trend.

6. Conclusions and future research

In this study, we presented a mathematical programming model for a sustainable multi-period hub location problem when transportation demand is time-dependent and uncertain and the planning horizon is continuous-time. Also, the capacity of hubs and hub links is modular. We formulated the problem as a multi-objective mixed-integer nonlinear model under the robust uncertainty of time-dependent demand. Using some linearization techniques, we converted the problem into a multi-objective mixed-integer linear programming problem under uncertainty. To convert the model to a robust counterpart, we used Bertsimas and Sim's method. With the help of the TH method aggregation function, we converted the problem into a single-objective parametric problem. Also, we proposed some valid inequalities to improve the formulation of the problem. We used Turkish network data for validating the proposed model and examined the results of solving a test problem in two cases. In the first case, the parameters of the demand function take their nominal values, and in the second case, parameters can change within a specific range. The results showed that in the robust case, the selected capacity for hubs and hub links is higher than the case of nominal values. Also, the network configuration at different periods, in two cases, is different. Then, we performed a sensitivity analysis on the parameters related to robust optimization. Changes in the optimal objective functions value to the level of conservatism and the probability of constraints violation in two cases (single-objective optimization and multi-objective optimization) were examined, and the results were reported. For future research, we can mention the following research gaps: presenting alternative formulation or tighter formulation for the problem, using different robust optimization techniques to deal with uncertainty, developing exact or heuristic methods to reduce the solution time of robust model, and considering the model for other types of hub location problem.

Appendix A. Robust counterpart formulation of equations (3.47)–(3.49) and (3.50)–(3.52)

(a) Robust counterpart of equation (3.47)

$$\max -\omega_1 \tag{A.1}$$

$$-\omega_1 + \mathrm{Obj}_1 + r^0\Gamma^0 + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{s-1,iks}$$

$$+\sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{s-1,iks} \leq 0 \tag{A.2}$$

$$r^0 + \rho^0_{siks} \geq \left[\left(c_{ik}\left(\sum_{j\in N}\hat{a}_{ij}\right) + c_{ki}\left(\sum_{j\in N}\hat{a}_{ji}\right)\right)\right] bz_{siks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.3}$$

$$r^0 + \rho^0_{s-1,iks} \geq -\left[\left(c_{ik}\left(\sum_{j\in N}\hat{a}_{ij}\right) + c_{ki}\left(\sum_{j\in N}\hat{a}_{ji}\right)\right)\right] bz_{s-1,iks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.4}$$

$$r^0 + \tau^0_{siks} \geq \frac{1}{2}\left[\left(c_{ik}\left(\sum_{j\in N}\hat{h}_{ij}\right) + c_{ki}\left(\sum_{j\in N}\hat{h}_{ji}\right)\right)\right] bz'_{siks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.5}$$

$$r^0 + \tau^0_{s-1,iks} \geq -\frac{1}{2}\left[\left(c_{ik}\left(\sum_{j\in N}\hat{h}_{ij}\right) + c_{ki}\left(\sum_{j\in N}\hat{h}_{ji}\right)\right)\right] bz'_{s-1,iks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.6}$$

$$r^0 \geq 0 \tag{A.7}$$

$$\rho^0_{siks'}, \tau^0_{siks'} \geq 0 \qquad \forall s\in S, s'\in S\backslash\{0\}, s=s' \text{ or } s=s'-1, i\in N, k\in H. \tag{A.8}$$

(b) Robust counterpart of equation (3.48)

$$\max -\omega_2 \tag{A.9}$$

$$-\omega_2 + \mathrm{Obj}_2 + r^0\Gamma^0 + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{s-1,iks}$$

$$+\sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{s-1,iks} \leq 0 \tag{A.10}$$

$$r^0 + \rho^0_{siks} \geq \left[(\mathrm{EW}+\mathrm{LW})d_{ik}\left(\sum_{j\in N}\hat{a}_{ij} + \sum_{j\in N}\hat{a}_{ji}\right)\right] bz_{siks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.11}$$

$$r^0 + \rho^0_{s-1,iks} \geq -\left[(\mathrm{EW}+\mathrm{LW})d_{ik}\left(\sum_{j\in N}\hat{a}_{ij} + \sum_{j\in N}\hat{a}_{ji}\right)\right] bz_{s-1,iks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.12}$$

$$r^0 + \tau^0_{siks} \geq \frac{1}{2}\left[(\mathrm{EW}+\mathrm{LW})d_{ik}\left(\sum_{j\in N}\hat{h}_{ij} + \sum_{j\in N}\hat{h}_{ji}\right)\right] bz'_{siks} \qquad \forall i\in N, k\in H, s\in S\backslash\{0\} \tag{A.13}$$

$$r^0 + \tau^0_{s-1,iks} \geq -\frac{1}{2}\left[(\mathrm{EW}+\mathrm{LW})d_{ik}\left(\sum_{j\in N}\hat{h}_{ij} + \sum_{j\in N}\hat{h}_{ji}\right)\right] bz'_{s-1,iks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \tag{A.14}$$

and (A.7) and (A.8).

(c) Robust counterpart of equation (3.49)

$$\max \omega_3 \tag{A.15}$$

$$\begin{aligned}
&\omega_3 - \mathrm{Obj}_3 + r^0\Gamma^0 + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\rho^0_{s-1,iks}\\
&+ \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{siks} + \sum_{s\in S\backslash\{0\}}\sum_{i\in N}\sum_{k\in H}\tau^0_{s-1,iks} \leq 0
\end{aligned} \tag{A.16}$$

$$r^0 + \rho^0_{siks} \geq -\left[\mathrm{VJO}_k\left(\sum_{j\in N}\hat{a}_{ij} + \sum_{j\in N}\hat{a}_{ji}\right)\right] bz_{siks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \tag{A.17}$$

$$r^0 + \rho^0_{s-1,iks} \geq \left[\mathrm{VJO}_k\left(\sum_{j\in N}\hat{a}_{ij} + \sum_{j\in N}\hat{a}_{ji}\right)\right] bz_{s-1,iks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \tag{A.18}$$

$$r^0 + \tau^0_{siks} \geq -\frac{1}{2}\left[\mathrm{VJO}_k\left(\sum_{j\in N}\hat{h}_{ij} + \sum_{j\in N}\hat{h}_{ji}\right)\right] bz'_{siks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \tag{A.19}$$

$$r^0 + \tau^0_{s-1,iks} \geq \frac{1}{2}\left[\mathrm{VJO}_k\left(\sum_{j\in N}\hat{h}_{ij} + \sum_{j\in N}\hat{h}_{ji}\right)\right] bz'_{s-1,iks} \qquad \forall i \in N, k \in H, s \in S\backslash\{0\} \tag{A.20}$$

and (A.7), (A.8).

In the above relations, Obj_1, Obj_2, and Obj_3 are expressions of the first, second, and third objective functions with nominal values of uncertain parameters. In the following, we present the robust counterpart formulation of constraints (3.50)–(3.52). Note that because the flow balance constraint (3.50) is tight in optimality, we modify this constraint as inequality to conform to the general form of Bertsimas and Sim's method.

(d) Robust counterpart of equation (3.50)

$$\begin{aligned}
&-\sum_{l\in H, l\neq k} Y^i_{kls} - \sum_{l\in H, l\neq k} Y^i_{lks} = \left(\left(\sum_{j\in N} a_{ij}\right)(bz_{siks} - bz_{s-1,iks})\right.\\
&\left. + \left(\sum_{j\in N} h_{ij}\right)\left(\frac{bz'_{siks} - bz'_{s-1,iks}}{2}\right)\right) - \sum_{j\in N}\left(a_{ij}(bz_{sjks} - bz_{s-1,jks})\right.\\
&\left. + h_{ij}\left(\frac{bz'_{sjks} - bz'_{s-1,jks}}{2}\right)\right) + r^4_{iks}\Gamma^4_{iks} + \sum_{j\in N}\rho^4_{sjks} + \sum_{j\in N}\rho^4_{s-1,jks}\\
&+ \sum_{j\in N}\tau^4_{sjks} + \sum_{j\in N}\tau^4_{s-1,jks} \leq 0 \qquad \forall i \in N, k \in H, s \in S\backslash\{0\}
\end{aligned} \tag{A.21}$$

$$r^4_{iks} + \rho^4_{siks} \geq \left[-\left(\sum_{j' \in N} \hat{a}_{ij}\right) + \hat{a}_{ii}\right] bz_{siks} \qquad \forall i \in N, k \in H, s \in S \backslash \{0\} \tag{A.22}$$

$$r^4_{iks} + \rho^4_{sjks} \geq [\hat{a}_{ij}] bz_{sjks} \qquad \forall i, j \in N, i \neq j, k \in H, s \in S \backslash \{0\} \tag{A.23}$$

$$r^4_{iks} + \rho^4_{s-1,iks} \geq -\left[-\left(\sum_{j' \in N} \hat{a}_{ij}\right) + \hat{a}_{ii}\right] bz_{s-1,iks} \qquad \forall i \in N, k \in H, s \in S \backslash \{0\} \tag{A.24}$$

$$r^4_{iks} + \rho^4_{s-1,jks} \geq -[\hat{a}_{ij}] bz_{s-1,jks} \qquad \forall i, j \in N, i \neq j, k \in H, s \in S \backslash \{0\} \tag{A.25}$$

$$r^4_{iks} + \tau^4_{siks} \geq \frac{1}{2}\left[-\left(\sum_{j' \in N} \hat{h}_{ij}\right) + \hat{h}_{ii}\right] bz'_{siks} \qquad \forall i \in N, k \in H, s \in S \backslash \{0\} \tag{A.26}$$

$$r^4_{iks} + \tau^4_{sjks} \geq \frac{1}{2}\left[\hat{h}_{ij}\right] bz'_{sjks} \qquad \forall i, j \in N, i \neq j, k \in H, s \in S \backslash \{0\} \tag{A.27}$$

$$r^4_{iks} + \tau^4_{s-1,iks} \geq -\frac{1}{2}\left[-\left(\sum_{j' \in N} \hat{h}_{ij}\right) + \hat{h}_{ii}\right] bz'_{s-1,iks} \qquad \forall i \in N, k \in H, s \in S \backslash \{0\} \tag{A.28}$$

$$r^4_{iks} + \tau^4_{s-1,jks} \geq -\frac{1}{2}\left[\hat{h}_{ij}\right] bz'_{s-1,jks} \qquad \forall i, j \in N, i \neq j, k \in H, s \in S \backslash \{0\} \tag{A.29}$$

$$r^4_{iks} \geq 0 \qquad \forall i \in N, k \in H, s \in S \backslash \{0\} \tag{A.30}$$

$$\rho^4_{siks'}, \tau^4_{siks'} \geq 0 \qquad \forall s \in S, s' \in S \backslash \{0\}, s = s' \text{ or } s = s' - 1, i \in N, k \in H. \tag{A.31}$$

(e) Robust counterpart of equation (3.51)

$$\begin{aligned} &Y^i_{kls} + Y^i_{lks} - \left(\sum_{j \in N} a_{ij}\right)(be_{skls} - be_{s-1,kls}) - \left(\sum_{j \in N} h_{ij}\right) \\ &\times \left(\frac{be'_{skls} - be'_{s-1,kls}}{2}\right) + r^5_{ikls}\Gamma^5_{ikls} + \rho^5_{sikls} + \rho^5_{s-1,ikls} \\ &+ \tau^5_{sikls} + \tau^5_{s-1,ikls} \leq 0 \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \end{aligned} \tag{A.32}$$

$$r^5_{ikls} + \rho^5_{sikls} \geq -\left(\sum_{j \in N} \hat{a}_{ij}\right) be_{skls} \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \tag{A.33}$$

$$r^5_{ikls} + \rho^5_{s-1,ikls} \geq \left(\sum_{j \in N} \hat{a}_{ij}\right) be_{s-1,kls} \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \tag{A.34}$$

$$r^5_{ikls} + \tau^5_{sikls} \geq -\frac{1}{2}\left(\sum_{j \in N} \hat{h}_{ij}\right) be'_{sikls} \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \tag{A.35}$$

$$r^5_{ikls} + \tau^5_{s-1,ikls} \geq +\frac{1}{2}\left(\sum_{j \in N} \hat{h}_{ij}\right) be'_{s-1,ikls} \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \tag{A.36}$$

$$r^5_{ikls} \geq 0 \qquad \forall i \in N, k, l \in H, k < l, s \in S \backslash \{0\} \tag{A.37}$$

$$\rho^5_{sikls'}, \tau^5_{sikls'} \geq 0 \qquad \forall s \in S, s' \in S \backslash \{0\}, s = s' \text{ or } s = s' - 1, i \in N, k, l \in H, k < l. \tag{A.38}$$

(f) Robust counterpart of equation (3.52)

$$\sum_{i\in N}\left(\left(\sum_{j\in N}a_{ij}\right)(bz_{siks}-bz_{s-1,iks})+\left(\sum_{j\in N}h_{ij}\right)\left(\frac{bz'_{siks}-bz'_{s-1,iks}}{2}\right)\right)$$
$$+\sum_{l\in H}\sum_{i\in N}Y^i_{lks}-\Gamma\left(\sum_{q=1}^{Q_k}q\left(\sum_{s'\in S\setminus\{0\},s'<s+1}u_{qks'}-\sum_{s'\in S\setminus\{0\},s'<s+1}\right.\right.$$
$$\left.\left.\times\sum_{l\in H,l\neq k}trc_{qkls'}+\sum_{s'\in S\setminus\{0\},s'<s+1}\sum_{l\in H,l\neq k}trc_{qkls'}\right)\right)+\eta^6_{ks}\Gamma^6_{ks}$$
$$+\sum_{i\in N}\rho^6_{siks}+\sum_{i\in N}\rho^6_{s-1,iks}+\sum_{i\in N}\tau^6_{siks}+\sum_{i\in N}\tau^6_{s-1,iks}\leq 0 \qquad \forall k\in H, s\in S\setminus\{0\} \tag{A.39}$$

$$r^6_{ks}+\rho^6_{siks}\geq\sum_{i'\in N}\left(\sum_{j\in N}\hat{a}_{ij}\right)\cdot bz_{siks} \qquad \forall i\in N,k\in H,s\in S\setminus\{0\} \tag{A.40}$$

$$r^6_{ks}+\rho^6_{s-1,iks}\geq-\sum_{i'\in N}\left(\sum_{j\in N}\hat{a}_{ij}\right)\cdot bz_{s-1,iks} \qquad \forall i\in N,k\in H,s\in S\setminus\{0\} \tag{A.41}$$

$$r^6_{ks}+\tau^6_{siks}\geq\frac{1}{2}\sum_{i'\in N}\left(\sum_{j\in N}h_{ij}\right)\cdot bz'_{siks} \qquad \forall i\in N,k\in H,k<l,s\in S\setminus\{0\} \tag{A.42}$$

$$r^6_{ks}+\tau^6_{s-1,iks}\geq-\frac{1}{2}\sum_{i'\in N}\left(\sum_{j\in N}h_{ij}\right)\cdot bz'_{s-1,iks} \qquad \forall i\in N,k\in H,s\in S\setminus\{0\} \tag{A.43}$$

$$r^6_{ks}\geq 0 \qquad \forall k\in H,s\in S\setminus\{0\} \tag{A.44}$$

$$\rho^6_{siks},\tau^6_{siks}\geq 0 \qquad \forall s\in S,s'\in S\setminus\{0\},s=s' \text{ or } s=s'-1,i\in N,k\in H. \tag{A.45}$$

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