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OPTIMIZING A BI-OBJECTIVE LOCATION-ALLOCATION-INVENTORY PROBLEM IN A DUAL-CHANNEL SUPPLY CHAIN NETWORK WITH STOCHASTIC DEMANDS

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Abstract. Integrating strategic and tactical decisions to location-allocation and green inventory planning by considering e-commerce features will pave the way for supply chain managers. Therefore, this study provides an effective framework for making decisions related to different levels of the dual-channel supply chain. We provide a bi-objective location-allocation-inventory optimization model to design a dual-channel, multi-level supply chain network. The main objectives of this study are to minimize total cost and environmental impacts while tactical and strategic decisions are integrated. Demand uncertainty is also addressed using stochastic modeling, and inventory procedure is the periodic review (S, R). We consider many features in inventory modeling that play a very important role, such as lead time, shortage, inflation, and quality of raw materials, to adapt the model to the real conditions. Since a dual-channel supply chain is becoming more important for sustainable economic development and resource recovery, we combine online and traditional sales channels to design a network. We generate five test problems and solve them by using the augmented ε-constraint method. Also, the Grasshopper optimization algorithm was applied to solve the model in a reasonable time for a large size problem. In order to provide managerial insights and investigate the sensitivity of variables and problem objectives with respect to parameters, sensitivity analysis was performed.

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1. Introduction

In today's competitive economic environment, making the right decisions at the strategic and operational levels enables organizations to manage their logistics and supply activities more efficiently. Supply chain management emphasizes the integration of chain members; because to increase a supply chain's productivity, its decisions cannot be optimized separately. Supply chain networks are mainly recognized as an operating basis for various industries. Appropriate design of this chain can reduce operating costs by 60% [29, 51]. Mainly in

Keywords. Inventory-location allocation problem, dual-channel supply chain, periodic review policy, stochastic demands, augmented ε-constraint, grasshopper optimization algorithm.

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the conventional approach, supply chain design is done by considering strategic decisions and the purpose of reducing costs and ignoring operational decisions such as the inventory system [66]. Today, companies must pay close attention to updating their logistics systems and solve the problems of procurement of items, demand uncertainty, warehouses, and retailers to gain significant market share and customer satisfaction. Therefore, one of the key concerns of managers in supply chain management is the integration of operational and strategic decisions [26]. Integrating location-allocation and inventory decisions is one of the approaches that, with proper planning, can play a significant role in creating a competitive advantage and gaining more profit [48].

In addition, the Sustainable vision has encouraged industries and governments to boost environmental sustainability by improving their activities. One of the most important roles in sustainability is controlling greenhouse gas emissions. According to many studies, inventory management activities are the most important issues that release a significant share of greenhouse gases [65]. Numerous research [13, 17, 71] have suggested the integration of inventory systems and environmental requirements because inventory optimization by creating a proper balance in storage and transportation of items reduces greenhouse gas emissions [60]. Therefore, some factors in achieving sustainable supply chains and coping with environmental degradation are the implementation of green inventory management and green supply chain, which specifically focuses on designing an environmentally friendly supply chain [39]. Green inventory management is defined by an economic focus on the costs and attention to the environmental and sustainability considerations [35].

Apart from the environmental aspect, proper response to stochastic demand is necessary in designing an efficient supply chain network [72]. According to Simangunsong *et al.*'s [62] paper, because of various factors and conditions for each organization, demand uncertainty is divided into three categories, including internal uncertainty of the organization, external uncertainty and uncertainty due to the supply chain. Numerous studies have emphasized considering demand uncertainty in order to adapt to real conditions [23,69]. In addition, under conditions of uncertainty and stochastic inventory, the combination of shortages occurs on the part of customers [61, 67]. Some customers wait to receive the product (backorders) and others turn to other alternatives (lost sales) [2]. Therefore, considering a combination of customer behaviors in the face of shortages can practical the results of the problem. Also inventory is considered as an economic asset, which has been invested and affected by inflation, like other assets of the organization. However, in the relevant literature, classic inventory models do not take into account economic considerations such as inflation, while these considerations are important in managerial decisions and affect the amount of economic order [44]. Therefore, expanding inventory models by considering all these attributes and adding practical constraints such as shortage and inflation under uncertain environments brings the model more accurately and closer to real-world conditions.

Moreover, advances in technology and the widespread use of the Internet have led to the use of online sales channels in addition to traditional sales to sell companies' products. Both sales channels (online and traditional) sell the same products at favorable prices so that the manufacturer decides on the wholesale price of products by examining the market share between traditional and online sellers [56]. Thus, considering the numerous benefits of online sales such as better access, faster response, lower distribution costs and a positive effect on environmental sustainability, the approach of combining online and traditional sales channels in supply chain design has been considered [6, 30].

Accordingly, research questions are posed as follows:

(1) What effect will stochastic demand have on tactical and strategic supply chain decisions?
(2) Do considering green and e-commerce issues have a positive effect on location-allocation-inventory decisions?
(3) How can a periodic review policy be modeled and evaluated on a multi-period basis based on economic and green objectives?

Thereby, we provide a bi-objective model to solve the location-allocation-inventory problem (LAIP) in a dual-channel, multi-level supply chain. The main objectives of this model are to minimize total cost and environmental impacts while tactical and strategic decisions are integrated simultaneously. This location-allocation-inventory problem can be formulated as a mixed-integer non-linear programming (MINLP) model. We formulate this model to analyze the expected order quantities and safety stock level for each distribution center.

Based on the above descriptions, we summarize the main contributions of this study that differentiate it from the other relevant research as follows. First, we investigate a location-allocation-inventory problem in a multi-level supply chain under (S, R) inventory policies, with considering stochastic demand and many important Features such as shortage, inflation, quality of raw materials, and positive lead time, which were ignored in most previous research. To the best of our knowledge, this is the first study to provide guidance on how to optimize the location-allocation-inventory problem in a multi-level supply chain network under periodic review inventory policies. Second, we combine online and traditional sales channels to design a dual-channel network. By using online sales channels and e-commerce activities that have been considered in the strategic and tactical decisions, we can develop a dual-channel supply chain and improve its sustainability. Then, we formulate a mixed-integer nonlinear programming model for solving it. Third, for small-sized instances, we solve the models using the augmented ε-constraint method. Then, for large-sized problems, we solve the model in a reasonable time by Grasshopper optimization algorithm.

The rest of the paper is structured in this way. In the next section, the literature review is described. Section 3 presents the development of the model, and Section 4 includes the solution approaches. The computational experiment presents in Section 5. Sections 6 and 7 provide the results and discussion, and sensitivity analysis. Finally, we summarized the study in Section 8 and presented suggestions for future studies.

2. Literature review

Considering the vast literature on inventory management, we review some studies in the concept of various inventory problems. Farahani *et al.* [24] have reviewed the existing literature and its evolution in the field of modeling the location-allocation problems (LAP) during the last three decades. Mousavi *et al.* [45] designed a two-tier network, including distributer and retailer, to investigate the location-inventory problem (LIP) under production constraints. Sadjadi *et al.* [58] considered a lead time and uncertain demand in the LIP model to optimize decisions related to meet the demand. Hajipour *et al.* [28] studied location-allocation decisions considering the three objective functions of minimizing total travel time, cost of launching facilities, and the idle probability of the facilities. Mousavi *et al.* [47] examined the LIP issue in a two-tier network of manufacturers and retailers to reducing purchase and inventory costs. Singha *et al.* [63] proposed a model to minimizing the total cost and finding the optimal reorder point and reorder quantity. Also, inventory management policies, including continuous and periodic reviews, as well as shortage have been considered. Vahdani *et al.* [66] studied the LIP issue to minimize supply chain costs. In this study, shortages are allowed, and periodic review policy has been used to manage inventory level. Puga and Tancrez [53] presented a LIP model to improve the supply chain's economic aspect. A periodic review system and uncertain demand have been considered in this study. Diabat *et al.* [22] used a Markov chain to optimize the LIP model under uncertain demand and lead time to minimize supply chain costs. Rafie-Majd *et al.* [55] studied the inventory-location routing problem (ILRP) in the multi-level network. Araya-Sassi *et al.* [7] studied the periodic review policy in the LIP model to determine warehouse location, reorder point and order size. Braglia *et al.* [14] presented a study to analyzing and optimizing the new inventory models. In this study, inventory policies are periodic or continuous review, and stock out costs, lead time, and lost sales have been considered as well. Behnamian *et al.* [10] discussed multiple cross-dockings where the loads are transferred from origins to destinations through cross-docking facilities. Braglia *et al.* [15] analyzed the (Q, r) policy for an inventory system to minimize the total cost under the conditions of allowable shortages and uncertain demand. Khan and Dey [32] presented a mathematical model to achieve the optimal review policy inventory levels in the review periods. The proposed model minimizes the cost of annual inventory by considering uncertain demand. Araya-Sassi *et al.* [8] proposed the LIP by considering the policies of continuous and periodic review. The developed model by them determines the re-order point, order size and location of warehouses and minimizes system costs simultaneously. Dehghani *et al.* [21] studied the problem of location-inventory by combining mathematical modeling and Markov process technique in a sustainable supply chain. Dehghan *et al.* [20] studied the location-routing problem with the aim of minimizing location and distribution costs under the risk of disruption. Amiri-Aref *et al.* [5] optimized multi-source location and inventory management decisions under stochastic demand and inventory policy (s, S) by formulating a two-stage profit maximization model. Ghasemi and Khalili-Damghani [27] optimized pre-event and post-event preparation decisions by presenting a mathematical model of location-allocation and inventory. They used a combination of mathematical optimization and simulation approaches to solve and estimate earthquake impact scenarios on urban infrastructure. Masoumi *et al.* [36] proposed a mathematical model for multi-objective, multi-period and multi-commodity and inventory routing problems in post-disaster conditions regarding the density of vehicles carrying relief items at the entrance of the border warehouses. Fathi *et al.* [25] studied the problem of warehouse location and inventory management based on stochastic demand and lead time and used the two-phase queuing-stochastic approach. Mokhtarzadeh *et al.* [40] studied the location-allocation problem by minimizing the costs, noise pollution, and inconvenience caused by the establishment, and developed a combination of clustering and meta-heuristic algorithms to solve it. Nayeri *et al.* [50] studied a robust fuzzy stochastic model for the responsive-resilient inventory-location problem.

Moreover, in some studies, the inventory model has been integrated with economic, environmental and e-commerce considerations. Moghadam *et al.* [38] developed a closed-loop supply chain based on an e-commerce platform. Dai *et al.* [18] introduced a LIP model for optimizing supply chain decisions under carbon emission limitations. Wang *et al.* [68] addressed a two-stage stochastic model to control demand uncertainty and solve the LIP of the green supply chain. Mousavi *et al.* [44] to minimize the present value of the inventory system formulated a mathematical model under discount, inflation, and demand certainty conditions. The effects of discount contracts and environmental regulation on pricing in a dual-channel supply network were studied by Xu *et al.* [70]. Rabbani *et al.* [54] developed a joint inventory planning and pricing with adjustment costs under differential inflation. Raza and Govindaluri [56] designed a two-level supply chain for the green product by considering two channels (traditional and online). Amrouche and Yan [6] evaluated the profits from the creation of traditional and online sales channels under discount conditions by designing pricing scenarios. Aziziankohan *et al.* [9] studied a green supply chain management using the queuing theory to control congestion and reduce energy consumption in a supply chain network. Jia and Li [30] studied online and self-run sales policy in the closed-loop supply chain. Naserabadi *et al.* [49] examined the issue of inventory control under inflation and shortage conditions to minimizing the present value of total costs. Bhunia *et al.* [12] addressed an inventory model by proposing two different inventory policies under inflationary conditions and definite demand. Mousavi *et al.* [46] examined the issue of inventory policy for a supply chain under the conditions of shortage and inflation. Alikar *et al.* [3] developed a bi-objective model including minimizing inventory costs and maximizing system reliability by considering inflation rate. Liao *et al.* [33] proposed a multi-objective model to optimize integrated location-inventory-routing decisions in a dual-channel supply chain. Tirkolaee *et al.* [64] studied the green LAIP to formulate a waste management system with the aim of improving the economic aspect. Malekkhouyan *et al.* [34] studied a mixed-integer linear programming for an integrated vehicle routing and mixed-model robotic disassembly sequence scheduling model on an e-waste management system. Kaoud *et al.* [31] evaluated the integration of e-commerce in manufacture and recovery sites in a multi-period closed-loop network. Paul *et al.* [52] addressed the green inventory management model by considering the impact of a carbon tax, variable maintenance costs, change in cost based on green demand level, and maximizing retailer profit. Das *et al.* [19] examined strategic location-routing-inventory decisions with the objectives of minimizing transportation, inventory, and carbon emission costs, and minimizing transportation time.

In order to better show the structure of the problem literature and display the difference between the present study and other researches, previous studies summarized in the gap table (*i.e.*, Tab. 2). We present the codes of this table in Table 1 and some papers categorized based on type of model, General features including inventory policy, Shortage, uncertainty in demand, Economic factors, environmental issue, etc. in Table 2.

According to the research literature, the integration of LAIP issue in the multilevel supply chain network under uncertainty conditions has been less studied by researchers. Also, the integration of features such as green inventory, inflation, e-commerce activities, shortages and the quality of raw materials has not been considered in such research issues simultaneously. Integrating strategic and tactical decisions with considering mentioned characteristics will bring supply chain decisions closer to reality and pave the way for supply chain managers.

TABLE 1. Literature structure codes.

Category	Detail	Code
Type of problem	Inventory problem	IP
	Location-inventory problem	LIP
	Location-allocation-inventory problem	LAIP
General features	Periodic review	Pr
	Continuous review	Cr
	Lost sale	Ls
	Backlogged	Bl
	Deterministic	D
	Stochastic	S
	Economic factors	Ef
	Environmental issue	Ei
	Multi-period	MP
	Multi-product	MPr
	Multi-objective	MO
	Online marketplace	OM
	Dual channel	Dch
Solving methods	Classical and analytical	CA
	Heuristic and Meta-heuristic	HMH

Therefore, we developed a bi-objective model to integrated LAIP in the multilevel supply chain, taking environmental effects, economic factors, inventory shortages, stochastic demand and e-commerce activities into account in a strategy of periodic review.

3. Problem explanation

This section aims to express the general modeling framework. For this purpose, first, the problem is described, and then the assumptions and features of the model are presented. Finally, by defining the parameters and variables of the problem, a bi-objective model is developed.

3.1. Problem definition

We study a dual-channel, multi-level supply chain to solve location-allocation-inventory problems. The elements of the supply chain consist of suppliers in the first level, plants in the second level, distribution centers (DCs) in the third level and dual channels of sellers in the fourth level. The flow of materials first starts from suppliers and is shipped to plants to produce the products. End products are transported to specific DCs and stored to meet sellers' demand. Finally, the products are sent to traditional and online sellers to trade-off in both physical and e-commerce activities. We intend to integrate strategic and tactical decisions in this dual-channel supply chain network. We develop an inventory model by considering shortages, inflation, and stochastic demand under a periodic review policy (S, R). In this inventory review policy, the stock status checked periodically at equal time intervals (R) and items ordered to achieve the highest inventory level (S). Moreover, the sellers' demand is stochastic, and the shortage occurs as a combination of lost sales and backlogging. The orders' lead times are considered constant and all costs include a fixed annual inflation rate. Thus, the proposed model includes three different decisions: (1) the issue of the location of DCs and traditional sellers, (2) appropriate allocation of sellers to DCs and (3) determining inventory decisions in opened DCs. The goals of our model are to minimize total costs and environmental impacts simultaneously. Figure 1 exhibits the network under consideration.

TABLE 2. Summarized previous studies.

	Type of problem			General features													Solving methods	
				Inventory policy		Shortage		Demand type										
Author	IP	LIP	LAIP	Pr	Cr	Ls	Bl	D	S	Ef	Ei	MP	MPr	MO	OM	Dch	CA	HMH
Amrouche and Yan [6]	✓							✓		✓	✓				✓	✓	✓	
Mousavi *et al.* [46]	✓					✓	✓	✓		✓		✓	✓	✓				✓
Mousavi *et al.* [47]		✓						✓				✓	✓					✓
Singha *et al.* [63]	✓			✓	✓	✓	✓		✓								✓	
Diabat *et al.* [22]		✓			✓	✓			✓									✓
Rafie-Majd *et al.* [55]		✓			✓				✓			✓	✓					✓
Braglia *et al.* [14]	✓			✓	✓	✓	✓		✓									✓
Dai *et al.* [18]		✓						✓			✓	✓						✓
Mousavi *et al.* [48]			✓	✓					✓			✓	✓				✓	✓
Raza and Govindaluri [56]								✓			✓				✓	✓		✓
Wang *et al.* [68]		✓		✓					✓		✓		✓					✓
Araya-Sassi *et al.* [8]		✓		✓	✓			✓										✓
Tirkolaee *et al.* [64]			✓					✓			✓		✓				✓	
Current study			✓	✓		✓	✓		✓	✓	✓	✓		✓	✓	✓	✓	✓

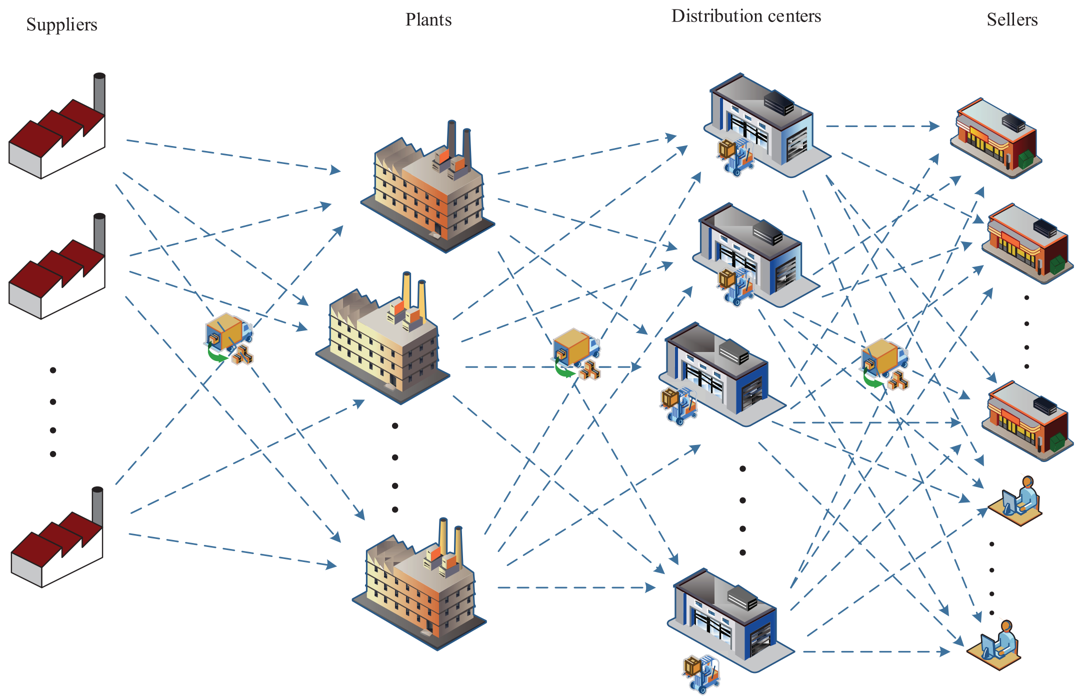


FIGURE 1. Proposed multi-level structure of the supply chain.

3.2. Assumption

The key assumptions of this study are as below:

- A multi-level, multi-period model with one type of transportation vehicle and product is formulated.
- The capacity and number of potential DCs are limited.
- The inflation rate is considered in the model, and inventory shortage is allowed.
- Sellers are considered as traditional and online.
- There are fixed opening costs for DCs and traditional sellers.
- The opening of potential facilities and transportation has environmental impacts.
- Each seller (online or traditional) is assigned to a specific DC.
- Periodic review policy (S, R) is defined as the inventory policy in DCs.
- In order to control and deal with demand fluctuations of sellers, safety stocks have been used in DCs.
- Sellers demand has a normal distribution with a known mean and variance.
- The quality of raw materials provided by suppliers is different and the cost of purchasing from them varies according to their quality.
- There is a fixed shortage cost for both cases of shortage (lost sales and back ordered) [43].
- There is no time-dependent backorder cost [43].
- The backorders occur in very small amounts and the orders received are always enough to meet them [43].
- It is assumed that the related lead times are applied from the beginning of the period.

Figure 2 better illustrates some assumptions related to inventory policy. Figure 2 shows the inventory control policy in the model. According to this figure, each planning horizon includes lead time and review interval so that the lead time of each period is at the beginning of it. The consumption rate in during of review interval is $\mathrm{Dd}_{kt}\,(R_k)$ and during lead time is in accordance with $\mathrm{Dd}_{kt}\,(\mathrm{LTd}_{jkt})$. According to the periodic review policy,

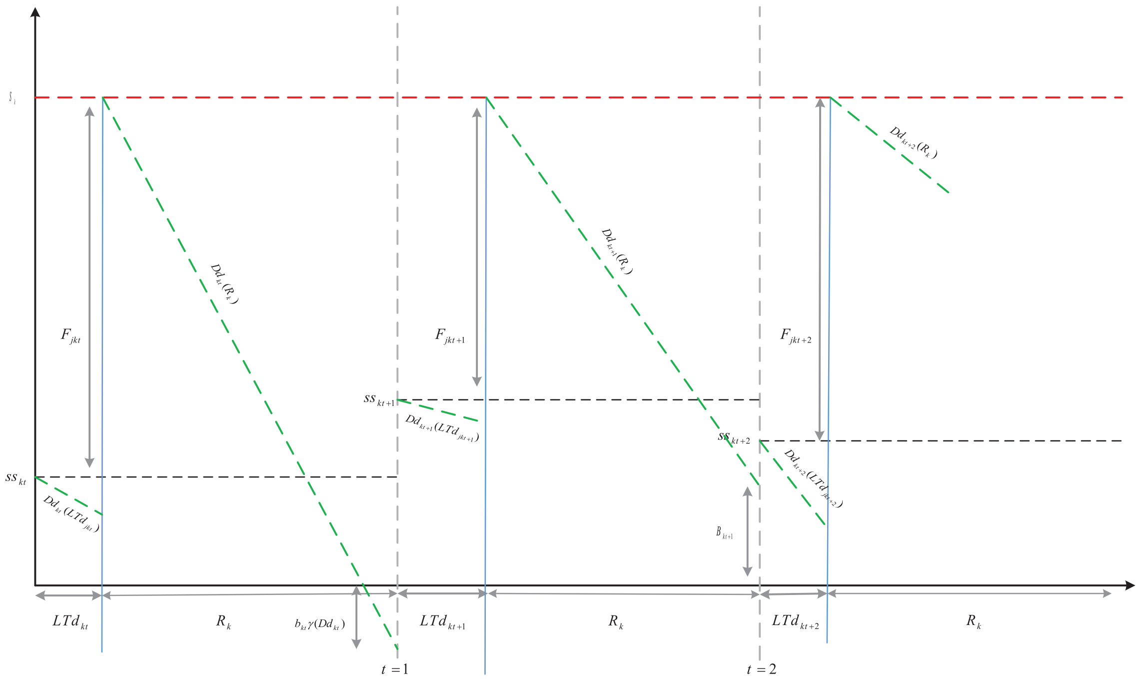


Figure 2. Diagram of inventory control policy in proposed model.

it is necessary to order a quantity of the product in each order so that the inventory level reaches the amount of S.

3.3. Problem formulation

The notations below are used to model the problem if the aforementioned supply chain network.

3.3.1. Notations

Indices

I	Set of suppliers indexed by $i \in I$
S	Set of sellers indexed by $s, l \in S$
S^{tr}	Set of traditional sellers indexed by $s^{\mathrm{tr}} \in S^{\mathrm{tr}}, (S^{\mathrm{tr}} \subset S)$
S^{on}	Set of online sellers indexed by $s^{\mathrm{on}} \in S^{\mathrm{on}}, (S^{\mathrm{on}} \subset S)$
K	Set of potential locations for DCs indexed by $k \in K$
J	Set of plants indexed by $j \in J$
T	Set of time indexed by $t \in T$

Parameters

CS_{it}	Capacity of supplier i in period t
CD_{kt}	Capacity of distribution center k in period t
CP_{jt}	Capacity of plant j in period t
$\mathrm{Cap}_{s^{\mathrm{tr}}t}$	Capacity of traditional seller s^{tr} in period t
$\mathrm{Cap}'_{s^{\mathrm{on}}t}$	Capacity of online seller s^{on} in period t

μ_{st}	Average demand of seller s in period t
σ_{st}^2	Variance in demand of seller s in period t
FCd_{kt}	Fixed cost of opening distribution center k in period t
$\mathrm{FCs}_{s^{\mathrm{tr}}t}$	Fixed cost of opening traditional seller s^{tr} in period t
HCp_{jt}	Inventory holding cost per unit at plant j in period t
HCd_{kt}	Inventory holding cost per unit at distribution center k in period t
$\mathrm{HCst}_{s^{\mathrm{tr}}t}$	Inventory holding cost per unit at traditional seller s^{tr} in period t
$\mathrm{HCso}_{s^{\mathrm{on}}t}$	Inventory holding cost per unit at online seller s^{on} in period t
FC_{jkt}	Fixed cost of a shipment between plant j and distribution center k in period t
TPfs_{kst}	Fixed cost of a shipment between distribution center k and seller s in period t
UC_{ijt}	Purchase cost per unit from supplier i in period t
LT_{jt}	Maximum lead time from suppliers to plant j in period t
LT'_{st}	Maximum lead time from DCs to seller s in period t
LTd_{jkt}	Lead time from plant j to distribution center k in period t
LTp_{ijt}	Lead time from supplier i to plant j in period t
LTs_{kst}	Lead time from distribution center k to seller s in period t
OCp_{ijt}	Fix ordering cost from supplier i to plant j in period t
OCd_{jkt}	Fix ordering cost from plant j to distribution center k in period t
TC_{ijt}	Shipping cost (volume-dependent) per unit raw material per each transmitted unit from supplier i to plant j in period t
TPp_{jkt}	Shipping cost (volume-dependent) per unit from plant j to distribution center k in period t
TPd_{kst}	Shipping cost (volume-dependent) per unit from distribution center k to seller s in period t
$\mathrm{TPd}_{ks^{\mathrm{tr}}t}$	Shipping cost (volume-dependent) per unit from distribution center k to traditional seller s^{tr} in period t
$\mathrm{TPd}'_{ks^{\mathrm{on}}t}$	Shipping cost (volume-dependent) per unit from distribution center k to online seller s^{on} in period t
$Z_k(R_k)$	The accumulated standard normal distribution value such that $P\left(Z \le Z_k\left(R_k\right)\right) = \alpha$ is the service level at distribution center k
π_{kt}	Fixed shortage cost per unit short at distribution center k in period t
π_{0kt}	Unit selling price at distribution center k in period t
R_k	Review interval at distribution center k
ρ_{slt}	Correlation coefficient between demands at seller s and at seller l in period t
EIS_{ijt}	Environmental impacts per each transmitted unit from supplier i to plant j in period t
EIP_{jkt}	Environmental impacts per each transmitted unit from the plant j to distribution center k in period t
$\mathrm{EId}_{ks^{\mathrm{tr}}t}$	Environmental impacts per each transmitted unit from distribution center k to traditional seller s^{tr} in period t
$\mathrm{EIds}_{ks^{\mathrm{on}}t}$	Environmental impacts per each transmitted unit from distribution center k to online seller s^{on} in period t
EIod_{kt}	Environmental impacts related to opening distribution center k in period t
$\mathrm{EIos}_{s^{\mathrm{tr}}t}$	Environmental impacts related to opening traditional seller s^{tr} in period t
θ_i	Defective probability of raw material provided from supplier i
e_t	Inflation rate in period t
Y_t	Number of DCs that can be opened in period t
λ	Conversion coefficient of raw material to product
M	A big number
G	Set of permissible review intervals

Auxiliary variables

μp_{jt}	Mean demand allocated to plant j in period t
μd_{kt}	Mean demand allocated to distribution center k in period t
$\sigma^2 p_{jt}$	Variance in demand allocated to plant j in period t
$\sigma^2 d_{kt}$	Variance in demand allocated to distribution center k in period t
I_{jt}	End of period inventory of plant j in period t
B_{kt}	End of period inventory of distribution center k in period t
$H_{s^{\text{tr}}t}$	End of period inventory of traditional seller s^{tr} in period t
$H'_{s^{\text{on}}t}$	End of period inventory of online seller s^{on} in period t
Ld_k	Order lead-time at distribution center k
Lp_j	Order lead-time at plant j
Ls_s	Order lead-time at seller s
ss_{kt}	Level of safety stock at distribution center k in period t

Decision variables

r_{ijt}	The binary variable that specifies the service of supplier i to plant j in period t
z_{jkt}	1 if plant j serves distribution center k in period t; 0 otherwise
w_{kst}	1 if distribution center k serves seller s in period t; 0 otherwise
y_{kt}	1 if distribution center k is open in period t; 0 otherwise
$x_{s^{\text{tr}}t}$	1 if traditional seller s^{tr} open in period t; 0 otherwise
$v_{s^{\text{on}}t}$	1 if online seller s^{on} select in period t; 0 otherwise
b_{kt}	Percentage of demand back-ordered during shortage in period t at distribution center k
β_{ijt}	Number of raw material units developed by supplier i and sent to plant j in period t
F_{jkt}	Number of product that plant j sent to distribution center k in period t
$Q_{ks^{\text{tr}}t}$	Number of product that distribution center k sent to traditional seller s^{tr} in period t
$Q'_{ks^{\text{on}}t}$	Number of product that distribution center k sent to online seller s^{on} in period t

3.3.2. Inventory policy in the distribution center

As we have noted earlier in assumptions, the inventory review policy (S, R) in our study is only regarded for DCs. The purpose of the inventory policy (S, R) is to achieve the optimal values of S and R so that system costs are minimized. According to this strategy, the inventory level is reviewed at fixed intervals (R), and we are obliged to bring the inventory status to its highest level (*i.e.*, S). Therefore, order quantities depend on inventory level check intervals. In this study, we tend to specify the optimal value of S while assuming the value of R is known. The inventory policy (S_k, R_k) shows the inventory system in the distribution center k, so that S_k and R_k represents the order-up-to-level and the review intervals, respectively. The demand in the distribution center k (Dd_{kt}) has a multivariate normal distribution with the mean value of $\mu d_{kt} = \sum_s \mu_{st} w_{kst}$ and the variance of $\sigma^2 d_{kt} = \sum_s \sum_l \rho_{slt}\sigma_{st}\sigma_{lt} w_{kst} w_{klt}$ for each period. Similarly, The demand of the plant j (Dp_{jt}) has a multivariate normal distribution with a mean value of $\mu p_{jt} = \sum_k \mu d_{kt} \cdot z_{jkt}$ and the variance of $\sigma^2 p_{jt}$ for each period. Also, the $\text{Dd}_k(\tau)$ and $\text{Dp}_j(\tau)$, respectively, indicates the demand of the time interval τ with the cumulative distribution function $\text{Fd}_k(.;\tau)$ and $Fp_j(.;\tau)$. The equations $\text{Ld}_{kt} = \sum_j \text{LTd}_{jkt} z_{jkt}$, $\text{Lp}_{jt} = \sum_i \text{LTp}_{ijt} r_{ijt}$ and $\text{Ls}_{st} = \sum_k \text{LTs}_{skt} w_{skt}$ calculated the order lead-time at the distribution center k, the plant j and seller s, respectively for each period. In addition, $\text{Dd}_{kt}(R_k + \text{Ld}_{kt})$ indicates a demand of distribution center k during the lead-time and review interval so that it has normal distribution with average $\mu d_{kt}(R_k + \text{Ld}_{kt})$ and variance $\sigma^2 d_{kt}(R_k + \text{Ld}_{kt})$. Moreover, $\text{Dp}_{jt}(\text{Lp}_{jt})$ indicates the demand of plant j during the lead-time so that has a normal distribution with average $\mu p_{jt}(\text{Lp}_{jt})$ and variance $\sigma^2 p_{jt}(\text{Lp}_{jt})$ [66]. We supposed that the shortage was a combination of backlog and lost sale. Thus, a portion of the shortage is backlogged and the remaining share is lost. In fact, this is derived from the classic model in which for a lost sale $b_{kt} = 0$ and for the backordering, $b_{kt} = 1$ [43].

The costs related to the DC in equation (3.1), is formulated according to the papers of Montgomery *et al.* [43] and Vahdani *et al.* [66].

$$C_k(S_k) = F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt} + \mathrm{FC}_{jkt}}{R_k} + \mathrm{HCd}_{kt}\left[\frac{R_k\mathrm{Dd}_{kt}}{2} + S_k - \mathrm{Dd}_{kt}\left(R_k + \mathrm{LTd}_{jkt}\right)\right]$$
$$+ \left[\mathrm{HCd}_{kt}\left(1 - b_{kt}\right) + \frac{\pi_{kt} + \left(1 - b_{kt}\right)\left[R_k + \pi_{0kt} - \mathrm{TPd}_{kst}\right]}{R_k}\right]\gamma(\mathrm{Dd}_{kt}). \quad (3.1)$$

According to equation (3.1), ordering and shipping costs are presented in the first and second terms. The approximate average cost of inventory and cost of shortages are formulated in the third and fourth terms, respectively. Also S_k is order-up-to-level and $\gamma\left(\mathrm{Dd}_{kt}\right) = E\left[\mathrm{Dd}_{kt}\left(R_k + \mathrm{LTD}_{jkt}\right) - S_k\right]^+$ is the estimated amount of demands short per review period. Then, by considering the unit overage cost $C_{ok}\left(R_k\right) = \mathrm{HCd}_{kt}R_k$ and unit underage cost $C_{uk}\left(R_k\right) = \pi_{kt} + \left(1 - b_{kt}\right)\left(\pi_{okt} - \mathrm{TPd}_{kst}\right) - b_{kt}\mathrm{HCd}_{kt}R_k$, equation (3.1), written as equation (3.2):

$$C_k(S_k) = F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt} + \mathrm{FC}_{jkt}}{R_k} + \mathrm{HCd}_{kt}\left[\frac{R_k\mathrm{Dd}_{kt}}{2} + S_k - \mathrm{Dd}_{kt}\left(R_k + \mathrm{LTd}_{jkt}\right)\right]$$
$$+ \left[\frac{C_{ok}(R_k) + C_{uk}(R_k)}{R_k}\right]E\left[\mathrm{Dd}_{kt}\left(R_k + \mathrm{LTd}_{jkt}\right) - S_k\right]^+. \quad (3.2)$$

According to Vahdani *et al.* [66] the optimum order-up-to-level in the distribution center k is achieved by differentiating $\partial C_k(S_k)/\partial S_k = 0$. Thus, S_k^* and safety stock at distribution center k, denoted by the following equations:

$$S_k^* = \mathrm{Dd}_{kt}\left(R_k + \mathrm{LTd}_{jkt}\right) + Z_k(R_k)\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)} \quad (3.3)$$

$$\mathrm{ss}_{kt} = Z_k(R_k)\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)} = Z_k(R_k)\sqrt{\sum_{s\in S}\sum_{l\in S}\rho_{slt}\sigma_{st}\sigma_{lt}w_{kst}w_{klt}\sum_{j\in J}\left(R_k + \mathrm{LTd}_{jkt}\right)z_{jkt}}. \quad (3.4)$$

Also demand of the distribution center k during the lead-time and review interval *i.e.* $\mathrm{Dd}_{kt}\left(R_k + \mathrm{LTD}_{jkt}\right)$ has a normal distribution with average value $\mu_{kt}\left(R_k + \mathrm{LTD}_{jkt}\right)$ and variance $\sigma_{kt}^2\left(R_k + \mathrm{LTD}_{jkt}\right)$. Therefore, the following equation is achieved [66]:

$$E\left[\mathrm{Dd}_{kt}(R_k + \mathrm{LTD}_{jkt}) - y\right]^+ = \sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)}E(Z - \omega)^+ \quad (3.5)$$

where Z denotes a standard normal random variable and $y = \mu_{kt}\left(R_k + \mathrm{LTD}_{jkt}\right) + \omega\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTD}_{jkt}\right)}$. Also according to Berman *et al.* [11], $E\left(Z - \omega\right)^+ = \varnothing\left(\omega\right) - \omega\left[1 - \varphi\left(\omega\right)\right]$ and $\varphi\left[Z_k\left(R_k\right)\right] = C_{uk}\left(R_k\right)/C_{ok}(R_k) + C_{uk}\left(R_k\right)$. Accordingly, by putting the equation (3.3) at equation (3.2), we can ignore S_k as a decision variable in the proposed model. Therefore, the following equation is gained:

$$C_k(S_k^*) = F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt} + \mathrm{FC}_{jkt}}{R_k} + \mathrm{HCd}_{kt}\left[\frac{R_k\mathrm{Dd}_{kt}}{2} + Z_k(R_k)\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)}\right]$$
$$+ \left[\frac{C_{ok}(R_k) + C_{uk}(R_k)}{R_k}\right]\left[\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)}E\left(Z - Z_k(R_k)\right)^+\right]$$
$$= F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt} + \mathrm{FC}_{jkt}}{R_k} + \frac{\mathrm{HCd}_{kt}R_k\mathrm{Dd}_{kt}}{2}$$
$$+ \left[\mathrm{HCd}_{kt}Z_k(R_k) + \left[\frac{C_{ok}(R_k) + C_{uk}(R_k)}{R_k}\right]\left(\varnothing\left[Z_k(R_k)\right] - Z_k(R_k)\left[1 - \varphi\left[Z_k(R_k)\right]\right]\right)\right]$$
$$\times\sqrt{\sigma_{kt}^2\left(R_k + \mathrm{LTd}_{jkt}\right)}$$

$$= F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt}+\mathrm{FC}_{jkt}}{R_k} + \frac{\mathrm{HCd}_{kt}R_k\mathrm{Dd}_{kt}}{2} + \left[\mathrm{HCd}_{kt}Z_k(R_k) + \left[\frac{C_{ok}(R_k)+C_{uk}(R_k)}{R_k}\right]\right.$$
$$\left.\varnothing\left[Z_k(R_k)\right] - \frac{Z_k(R_k)C_{ok}(R_k)}{R_k}\right] \times \sqrt{\sigma_{kt}^2\left(R_k+\mathrm{LTd}_{jkt}\right)}$$
$$= F_{jkt}\mathrm{TPp}_{jkt} + \frac{\mathrm{OCd}_{jkt}+\mathrm{FC}_{jkt}}{R_k} + \frac{\mathrm{HCd}_{kt}R_k\mathrm{Dd}_{kt}}{2} + \left[\frac{C_{ok}(R_k)+C_{uk}(R_k)}{R_k}\right]$$
$$\varnothing\left[Z_k(R_k)\right]\sqrt{\sigma_{kt}^2\left(R_k+\mathrm{LTd}_{jkt}\right)}. \tag{3.6}$$

Therefore, the costs of distribution center k are represented by equation (3.7).

$$C(X,Y,R) = \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}F_{jkt}\mathrm{TPp}_{jkt} + \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\frac{\mathrm{OCd}_{jkt}+\mathrm{FC}_{jkt}}{R_k} + \sum_{t\in T}\sum_{k\in K}\mathrm{HCd}_{kt}\left(\frac{R_k\mu d_{kt}}{2}\right)$$
$$+\sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\left[\frac{\pi_{kt}+(1-b_{kt})\left[\mathrm{HCd}_{kt}R_k+\pi_{0kt}-\sum_{s\in S}\mathrm{TPd}_{kst}\right]}{R_k}\right]\phi\left[Z_k(R_k)\right]$$
$$\times\sqrt{\sum_{s\in S}\sum_{l\in S}\rho_{slt}\sigma_{st}\sigma_{lt}w_{kst}w_{klt}\sum_{j\in J}\left(R_k+\mathrm{LTd}_{jkt}\right)z_{jkt}}. \tag{3.7}$$

According to Montgomery *et al.* [43], the end of period inventory of distribution center k can be formulated as equation (3.8) that applied in constraint (3.25):

$$B_{kt} = F_{jkt}z_{jkt} + B_{kt-1} + \mathrm{ss}_{kt} - \mathrm{Dd}_{kt}\left(R_k+\mathrm{LTd}_{jkt}\right) + (1-b_{kt})\gamma(\mathrm{Dd}_{kt})$$
$$\rightarrow B_{kt} = \begin{pmatrix} \sum_{j\in J}F_{jkt}z_{jkt} + B_{kt-1} - \mu d_{kt}\left(R_k + \underset{j}{\mathrm{Max}}(\mathrm{LTd}_{jkt})\right) \\ +\left(Z_k(R_k) + (1-b_{kt})\left(\phi\left[Z_k(R_k)\right] - Z_k(R_k)\frac{C_{ok}}{C_{uk}+C_{ok}}\right)\right) \\ \times\sqrt{\sum_{s\in S}\sum_{l\in S}\rho_{slt}\sigma_{st}\sigma_{lt}w_{kst}w_{klt}\sum_{j\in J}\left(R_k+\mathrm{LTd}_{jkt}\right)z_{jkt}} \end{pmatrix} \quad \forall k\in K, t\in T. \tag{3.8}$$

3.3.3. Objective function

In terms of the above notation and equation (3.7) the model presented as:

$$\mathrm{Min}\ Z_1 = \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\mathrm{TC}_{ijt}\beta_{ijt}(1+e_t) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\mathrm{OCp}_{ijt}r_{ijt}(1+e_t) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\mathrm{UC}_{ijt}\beta_{ijt}(1-\theta_i)(1+e_t)$$
$$+\sum_{t\in T}\sum_{k\in K}\mathrm{FCd}_{kt}y_{kt}(1+e_t) + \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\frac{\mathrm{OCd}_{jkt}+\mathrm{FC}_{jkt}}{R_k}z_{jkt}(1+e_t)$$
$$+\sum_{t\in T}\sum_{k\in K}\sum_{j\in J}F_{jkt}\mathrm{TPp}_{jkt}z_{jkt}(1+e_t) + \sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\mathrm{HCp}_{jt}I_{jt}z_{jkt}(1+e_t)$$
$$+\sum_{t\in T}\sum_{k\in K}\mathrm{HCd}_{kt}\left(\frac{R_k\mu d_{kt}}{2}\right)(1+e_t) + \sum_{t\in T}\sum_{k\in K}\mathrm{HCd}_{kt}Z_k(R_k)$$
$$\times\sqrt{\sum_{s\in S}\sum_{l\in S}\rho_{slt}\sigma_{st}\sigma_{lt}w_{kst}w_{klt}\sum_{j\in J}\left(R_k+\mathrm{LTd}_{jkt}\right)z_{jkt}}(1+e_t)$$
$$+\sum_{t\in T}\sum_{k\in K}\sum_{j\in J}\left[\frac{\pi_{kt}+(1-b_{kt})\left[\mathrm{HCd}_{kt}R_k+\pi_{0kt}-\sum_{s\in S}\mathrm{TPd}_{kst}\right]}{R_k}\right]\phi\left[Z_k(R_k)\right]$$
$$\times\sqrt{\sum_{s\in S}\sum_{l\in S}\rho_{slt}\sigma_{st}\sigma_{lt}w_{kst}w_{klt}\sum_{j\in J}\left(R_k+\mathrm{LTd}_{jkt}\right)z_{jkt}}(1+e_t) + \sum_{t\in T}\sum_{s^{\mathrm{tr}}\in S^{\mathrm{tr}}}\mathrm{FCs}_{s^{\mathrm{tr}}t}x_{s^{\mathrm{tr}}t}(1+e_t)$$

$$+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm tr}\in S^{\rm tr}}\mathrm{TPd}_{ks^{\rm tr}t}Q_{ks^{\rm tr}t}(1+e_t)+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm on}\in S^{\rm on}}\mathrm{TPd}'_{ks^{\rm on}t}Q'_{ks^{\rm on}t}(1+e_t)$$
$$+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm tr}\in S^{\rm tr}}\mathrm{HCst}_{s^{\rm tr}t}H_{s^{\rm tr}t}w_{kst}(1+e_t)+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm on}\in S^{\rm on}}\mathrm{HCso}_{s^{\rm on}t}H'_{s^{\rm on}t}w_{kst}(1+e_t). \quad (3.9)$$

The first objective function developed in equation (3.9) minimizes total costs. The first term is transportation costs from each supplier to each plant. The second and third terms refer to the fixed ordering cost from each supplier to each factory and the purchase cost, respectively. Fixed costs of establishment of DCs are presented in the fourth term. Variable ordering costs and transportation costs from each plant to DCs are shown in the fifth and sixth terms, respectively. Holding costs and safety stocks are presented in the seventh to ninth terms. The cost of inventory shortages is formulated in the tenth term. The eleventh term is related to the fixed costs of opening traditional seller. The twelfth and thirteenth terms are the cost of transportation from each DC to traditional and online sellers. The final terms refer to the holding costs in each seller.

The second objective function is presented in (3.10).

$$\mathrm{Min}\ Z_2=\sum_{t\in T}\sum_{i\in I}\sum_{j\in J}\mathrm{EIS}_{ijt}\beta_{ijt}+\sum_{t\in T}\sum_{j\in J}\sum_{k\in K}\mathrm{EIP}_{jkt}F_{jkt}+\sum_{t\in T}\sum_{k\in K}\mathrm{EIod}_{kt}y_{kt}$$
$$+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm tr}\in S^{\rm tr}}\mathrm{EId}_{ks^{\rm tr}t}Q_{ks^{\rm tr}t}+\sum_{t\in T}\sum_{k\in K}\sum_{s^{\rm on}\in S^{\rm on}}\mathrm{EIds}_{ks^{\rm on}t}Q'_{ks^{\rm on}t}+\sum_{t\in T}\sum_{k\in K}\mathrm{EIos}_{s^{\rm tr}t}x_{s^{\rm tr}t}. \quad (3.10)$$

The environmental impacts of the network are minimized according to equation (3.10). The first term denotes the environmental effects of the transfer of raw materials purchased from suppliers. The second and third terms are related to the environmental impacts resulting from the transfer from plants to DCs and the opening of DCs, respectively. The environmental effects of transferring items from DCs to each seller and opening the traditional sellers are presented in the rest of the terms, respectively. The entire model is subject to constraints (3.11)–(3.35).

3.3.4. Constraints

The constraints are presented below.

$$\sum_{k\in K}w_{kst}=1 \qquad \forall s\in S,t\in T \quad (3.11)$$

$$\sum_{k\in K}w_{ks^{\rm tr}t}\le x_{s^{\rm tr}t} \qquad \forall s^{\rm tr}\in S^{\rm tr},t\in T \quad (3.12)$$

$$\sum_{k\in K}w_{ks^{\rm on}t}\le v_{s^{\rm on}t} \qquad \forall s^{\rm on}\in S^{\rm on},t\in T \quad (3.13)$$

$$H_{s^{\rm tr}t}=\sum_{k\in K}Q_{ks^{\rm tr}t}+H_{s^{\rm tr}t-1}-\mu_{s^{\rm tr}t} \qquad \forall s^{\rm tr}\in S^{\rm tr},t\in T \quad (3.14)$$

$$H'_{s^{\rm on}t}=\sum_{k\in K}Q'_{ks^{\rm on}t}+H'_{s^{\rm on}t-1}-\mu_{s^{\rm on}t} \qquad \forall s^{\rm on}\in S^{\rm on},t\in T \quad (3.15)$$

$$\sum_{k\in K}Q_{ks^{\rm tr}t}+H_{s^{\rm tr}t-1}\le \mathrm{Cap}_{s^{\rm tr}t}x_{s^{\rm tr}t} \qquad \forall s^{\rm tr}\in S^{\rm tr},t\in T \quad (3.16)$$

$$\sum_{k\in K}Q'_{ks^{\rm on}t}+H'_{s^{\rm on}t-1}\le \mathrm{Cap}'_{s^{\rm on}t}v_{s^{\rm on}t} \qquad \forall s^{\rm on}\in S^{\rm on},t\in T \quad (3.17)$$

$$\sum_{s\in S}\mu_{st}w_{kst}=\mu d_{kt} \qquad \forall k\in K,t\in T \quad (3.18)$$

$$\sum_{s\in S}\sigma^2_{st}w_{kst}=\sigma^2 d_{kt} \qquad \forall k\in K,t\in T \quad (3.19)$$

$$\sum_{k \in K} \mu d_{kt} z_{jkt} = \mu p_{jt} \qquad \forall j \in J, t \in T \tag{3.20}$$

$$\sum_{k \in K} \sigma^2 d_{kt} z_{jkt} = \sigma^2 p_{jt} \qquad \forall j \in J, t \in T \tag{3.21}$$

$$\sum_{s \in S} w_{kst} \leq \sum_{j \in J} z_{jkt} \leq y_{kt} \qquad \forall k \in K, t \in T \tag{3.22}$$

$$\sum_{k \in K} y_{kt} \leq Y_t \qquad \forall t \in T \tag{3.23}$$

$$I_{jt} = \sum_{i \in I} \beta_{ijt}(1-\theta_i)\lambda + I_{jt-1} - \mu p_{jt} \qquad \forall j \in J, t \in T \tag{3.24}$$

$$B_{kt} = \left(\begin{array}{l} \sum_{j \in J} F_{jkt} z_{jkt} + B_{kt-1} - \mu d_{kt} \left(R_k + \operatorname*{Max}_j (\mathrm{LTd}_{jkt}) \right) \\ + \left(Z_k(R_k) + (1 - b_{kt}) \left(\phi\left[Z_k(R_k)\right] - Z_k(R_k) \frac{C_{ok}}{C_{uk} + C_{ok}} \right) \right) \\ \times \sqrt{\sum_{s \in S} \sum_{l \in S} \rho_{slt} \sigma_{st} \sigma_{lt} w_{kst} w_{klt} \sum_{j \in J} \left(R_k + \mathrm{LTd}_{jkt}\right) z_{jkt}} \end{array} \right) \qquad \forall k \in K, t \in T \tag{3.25}$$

$$\sum_{i \in I} \beta_{ijt}(1-\theta_i)\lambda + I_{jt-1} \leq \mathrm{CP}_{jt} \qquad \forall j \in J, t \in T \tag{3.26}$$

$$\sum_{j \in J} \beta_{ijt} \leq \mathrm{CS}_{it} \qquad \forall i \in I, t \in T \tag{3.27}$$

$$\beta_{ijt} \leq r_{ijt} \times M \qquad \forall i \in I, j \in J, t \in T \tag{3.28}$$

$$\sum_{i \in I} \mathrm{LTp}_{ijt} r_{ijt} \leq \mathrm{LT}_{jt} \qquad \forall j \in J, t \in T \tag{3.29}$$

$$\sum_{k \in K} \left(\sum_{j \in J} \mathrm{LTd}_{jkt} z_{jkt} + \mathrm{LTs}_{kst} \right) w_{kst} \leq \mathrm{LT}'_{st} \qquad \forall s \in S, t \in T \tag{3.30}$$

$$Z_k(R_k) \sqrt{\sum_{s \in S} \sum_{l \in S} \rho_{slt} \sigma_{st} \sigma_{lt} w_{kst} w_{klt} \sum_{j \in J} \left(R_k + \mathrm{LTd}_{jkt}\right) z_{jkt}} + \sum_{j \in J} F_{jkt} z_{jkt} + B_{kt-1} \leq \mathrm{CD}_{kt} y_{kt} \qquad \forall k \in K, t \in T \tag{3.31}$$

$$\beta_{ijt}, \mu p_{jt}, \mu d_{kt}, \sigma^2 d_{kt}, \sigma^2 p_{jt}, F_{jkt}, Q_{ks^{\mathrm{tr}}t}, Q'_{ks^{\mathrm{on}}t}, I_{jt}, B_{kt} \in Z^+ \qquad \forall i \in I, j \in J, k \in K, s \in S, t \in T \tag{3.32}$$

$$r_{ijt}, y_{kt}, z_{jkt}, w_{kst}, x_{s^{\mathrm{tr}}t}, v_{s^{\mathrm{on}}t} \in \{0,1\} \qquad \forall i \in I, j \in J, k \in K, s \in S, t \in T \tag{3.33}$$

$$0 \leq b_{kt} \leq 1 \qquad \forall k \in K, t \in T \tag{3.34}$$

$$R_k \in G \qquad \forall k \in K. \tag{3.35}$$

Constraint (3.11) is the sourcing strategy assumption for each seller which means each seller is allocated to one DC. Constraints (3.12) and (3.13) confirm that DCs are allocated to sellers when they are selected and established. Constraints (3.14) and (3.15) formulated the end-of-period inventory of each seller. Constraints (3.16) and (3.17) show the capacity of traditional and online sellers, respectively. The mean and variance of the demand of the assigned sellers to the DCs are presented in equations (3.18) and (3.19) and similarly, the mean and variance of the demand of the DCs assigned to each plant are shown in equations (3.20) and (3.21) respectively. Constraint (3.22) indicates that allocation to plants and sellers will be possible if a DC is established. Constraint (3.23) shows the upper bound of the number of established DCs. Constraints (3.24) and

(3.25) formulated each plant's end of period inventory and distribution center, respectively. The constraint for capacity of plants is given by (3.26). Constraint (3.27) ensures that the total raw materials do not surpass the capacity of the supplier. Constraint (3.28) ensures that raw materials are shipped to plants when a supplier is selected. Constraints (3.29) and (3.30) calculate the maximum lead time for plants and sellers, respectively. Constraint (3.31) controls the capacity in each DC. Finally, the nature of the problem variables is presented in constraints (3.32)–(3.34). Constraint (3.35) indicates the set of allowable intervals for inventory review at each DC.

4. Solution method

In this study, we delineate a medium-sized problem and solve this by the augmented ε-constraint method. The model is calculated in the GAMS software 25.1.3 using the BARON solver.Moreover, Grasshopper optimization Algorithm (GOA) method is employed for solving our model in large size.

4.1. Multi-objective methods

Different approaches developed to solve multi-objective decision making (MODM), including the Weighted Sum Approach (WSA), Epsilon Constraint (EC), Augmented Epsilon Constraint (AEC) Lexicography (Lex), Goal Programming (GP), $\mathrm{L_p}$-metrics methods and more. In comparison to other multi-objective problem solving methods, the EC method has undeniable advantages.

The advanced version of the EC method is the AEC method, which improves the weaknesses of the classical EC method and modifies the generation process of the Pareto front and ensures its optimality. In the AEC method, the first objective is considered as the main objective, and the other objectives are constrained as the upper bound of epsilon [16], and the following model is replaced in which s_i are the nonnegative variables for the slack, and the parameter $\varphi_i = R\left(f_1\right)/R\left(f_i\right)$ is considered for normalizing the first objective function relative to the objective i. In order to better implement the AEC method, the acceptable range of epsilons can be obtained using the lexicographic method [37].

$$
\begin{aligned}
&\text{Min } f_1(x) - \sum_{i=2}^{n} \phi_i s_i \\
&\text{s.t.} \\
&\quad f_i(x) + s_i = \varepsilon_i^l \qquad\qquad \forall i = 2,3,\ldots,n \\
&\quad x \in X \\
&\quad s_i \geq 0.
\end{aligned}
\tag{4.1}
$$

In the offered model the objective function of cost is expressed as the function $f_1\left(x\right) = \mathrm{obj}_1$ and environmental function is demonstrated as constraint under the equation $\mathrm{obj}_2 + s_2 = \varepsilon_2^l$ [37].

In order to calculate the vector ε_i^l, the lowest and highest values of the kth objective achieved using the payoff table and are displayed as $f_k^{\min}$ and $f_k^{\max}$. Then, the distance between the values of the kth objective and the value of ε_k^l can be obtained as follows:

$$
r_k = f_k^{\max} - f_k^{\min}; \quad \varepsilon_k^l = f_k^{\max} - \frac{r_k}{q_k} \times l; \quad \forall k \neq i, l = 0,\ldots,q_k. \tag{4.2}
$$

Based on the explanations provided in the proposed model, the first objective function, *i.e.* cost minimization, is considered as the main objective function and the second objective function, *i.e.* environmental effects, is limited to the amount of epsilon and placed in the constraints (see Eq. (4.3)). The amount of epsilon changes to six values based on the distance between the changes of the worst and the best value and the decision maker's opinion, and the result is the creation of six points on the Pareto front.

$$\begin{aligned}
&\text{Min } Z_1 - \phi_2 s_2 \\
&\text{s.t.} \\
&\qquad Z_2 + s_2 = \varepsilon_2^l \\
&\qquad x \in X \\
&\qquad s_2 \geq 0.
\end{aligned} \tag{4.3}$$

4.2. Grasshopper optimization algorithm (GOA)

The Grasshopper optimization algorithm (GOA) was addressed by Saremi *et al.* [59] by mimicking the mass interactions of grasshoppers in nature. Three characteristics of gravity (G_i), wind advection (A_i) and social interaction (S_i) affect the flight route of grasshopper in a crowd, so that the component of social interaction as the main search mechanism is calculated by equation (4.4):

$$S_i = \sum_{j=1, j\neq i}^{N} s(d_{ij})\hat{d}_{ij}. \tag{4.4}$$

In this equation, d_{ij} calculates the distance between grasshoppers by equation $d_{ij} = |x_j - x_i|$. Also, s is a function to determining the power of social forces and $\hat{d}_{ij} = x_j - x_i / d_{ij}$. In this equation, function s is considered as the main part of the social interaction function, which defines the direction of movement of the grasshopper in the group as follows:

$$s(r) = f e^{\frac{-r}{l}} - e^{-r}. \tag{4.5}$$

In the above equation, the power of attraction with parameter f and the attractive distance scale with parameter l are shown. This function leads to the creation of attraction and repulsion forces between the grasshoppers, so that changing the parameters of function s significantly affects the swarm behavior. Therefore, Saremi *et al.* [59] adjusted the following model to design an optimization algorithm:

$$X_i^d = c\left(\sum_{j=1, j\neq i}^{N} c\frac{ub_d - lb_d}{s} s\left(\left|x_j^d - x_i^d\right|\right)\frac{x_j - x_i}{d_{ij}}\right) + \hat{T}_d \tag{4.6}$$

ub_d and lb_d are the upper and lower bound of the d-th dimension, respectively. The parameters $\hat{T}_d$ and c are considered as controller parameters in achieving to the goal. Grasshoppers' interactions and goal pursuit lead to update the best solution, and parameter c is the main controller parameter, which is calculated by equation (4.7):

$$c = c_{\max} - l\frac{c_{\max} - c_{\min}}{L} \tag{4.7}$$

L and l show the maximum number of iteration and the current iteration respectively. The value of $c_{\max}$ is 1 and $c_{\min}$ is 0.00001.

This algorithm continuously generates initial solutions to generate discrete variables and then changes them to discrete [1, 41, 59]. Figure 3 shows how to create a binary variable y_{kt} for 6 potential DCs. This variable is used as a discrete variable to determine the number of potential DCs, and the algorithm for generating it first creates a continuous vector between zero and one. After creating this vector, it arranges its elements in ascending order and thus creates a new vector. Since the maximum number of DCs allowed to be established is known, another random number is created between one and the number of allowable DCs, and the distribution centers are randomly selected in each iteration.

Despite its simplicity, this algorithm can solve complex models and effectively finds optimal solutions for complex problems [4, 57, 59]. The number of algorithm parameters for adjustment is low and due to the use of

A random continuous vector

DCs	1	2	3	4	5	6
UNIF(0,1)	0.2	0.7	0.5	0.3	0.4	0.6

↓

Sorted vector

DCs	1	4	5	3	6	2
UNIF(0,1)	0.2	0.3	0.4	0.5	0.6	0.7

↓

A random permutation of DCs

1	4	5	3	6	2

FIGURE 3. Solution representation for the GOA [1].

TABLE 3. Test problems' sizes.

Test problems No.	No. of suppliers (I)	No. of sellers (S)	No. of traditional sellers (S^{tr})	No. of online sellers (S^{on})	No. of potential DCs (K)	No. of plants (J)	No. of time period (T)
1	2	3	2	1	2	2	2
2	2	4	2	2	3	3	2
3	3	4	2	2	4	3	3
4	3	5	3	2	4	4	3
5	3	5	3	3	4	4	3
6	3	6	4	3	4	4	4
7	4	6	4	4	5	4	5

vectors in this algorithm, the dimensions can be extended to any dimension. Therefore, it has been used in the present study. In order to solve the model using this algorithm, the payoff table for objectives is created and by changing the values to the right of the second constrained objective function, the Pareto front is obtained.

5. Numerical examples

5.1. Computational experiment

In this section, the test problems are produced to examine the application of the model. The values of each test problem are presented in Table 3. The range of model parameters is also shown in Table 4. The results of model solving based on each of the test problems are presented in the results section.

5.2. Parameters tuning

Taguchi technique is a suitable approach for designing experiments to parameters tuning of meta-heuristic methods. This technique works by designing a number of experiments based on the algorithm's parameters and determines the appropriate value for each parameter. The effective parameters in GOA, such iteration and npop (the number of grasshoppers) are presented in Table 5.

Based on the number of parameters (factors) and their status, 9 experiments are designed in Minitab software. These experiments and their results are calculated and reviewed by the software, and the results are illustrated in Figure 4. According to this figure, the appropriate parameters are iteration = 100 and npop = 50.

5.3. Model verification

We examined the achievement of logical results to validate the mathematical model of the supply chain, including 3 suppliers, 5 sellers (3 traditional sellers and 2 online sellers), 4 DCs, 4 plants and three time periods

TABLE 4. The ranges of parameters of the test problems.

Parameters	Values	Parameters	Values	Parameters	Values
CS_{it}	$U(20,22)$	TPfs_{kst}	$U(2,5)$	λ	$U(0.2,0.5)$
CD_{kt}	$U(80,225)$	UC_{ijt}	$U(1,2)$	π_{kt}	$U(5,6)$
CP_{jt}	$U(80,225)$	LT_{jt}	$U(8,15)$	π_{0kt}	$U(2,3)$
$\mathrm{Cap}_{s^{\mathrm{tr}}t}$	$U(22,25)$	LT'_{st}	$U(8,15)$	R_k	$U(1,2)$
$\mathrm{Cap}'_{s^{\mathrm{on}}t}$	$U(22,25)$	LTd_{jkt}	$U(1,2)$	ρ_{slt}	$U(0,1)$
μ_{st}	$U(1,6)$	LTp_{ijt}	$U(1,2)$	EIS_{ijt}	$U(0.002,0.005)$
σ^2_{st}	$U(1,2)$	LTs_{kst}	$U(1,2)$	EIP_{jkt}	$U(0.4,0.5)$
FCd_{kt}	$U(2,5)$	OCp_{ijt}	$U(1,2)$	$\mathrm{EId}_{ks^{\mathrm{tr}}t}$	$U(0.4,0.5)$
$\mathrm{FCs}_{s^{\mathrm{tr}}t}$	$U(2,5)$	OCd_{jkt}	$U(2,5)$	$\mathrm{EIds}_{ks^{\mathrm{on}}t}$	$U(0.4,0.5)$
HCp_{jt}	$U(3,4.5)$	TC_{ijt}	$U(0.05,0.1)$	EIod_{kt}	$U(0.4,0.5)$
HCd_{kt}	$U(3,4.5)$	TPp_{jkt}	$U(0.05,1)$	$\mathrm{EIos}_{s^{\mathrm{tr}}t}$	$U(0.4,0.5)$
$\mathrm{HCst}_{s^{\mathrm{tr}}t}$	$U(3,4.5)$	TPd_{kst}	$U(0.05,1)$	θ_i	$U(0.1,0.5)$
$\mathrm{HCso}_{s^{\mathrm{on}}t}$	$U(3,5)$	$\mathrm{TPd}_{ks^{\mathrm{tr}}t}$	$U(0.05,1)$	e_t	$U(0.1,0.5)$
FC_{jkt}	$U(2,5)$	$\mathrm{TPd}'_{ks^{\mathrm{on}}t}$	$U(0.05,1)$	Y_t	$U(1,3)$

TABLE 5. State table and Taguchi analyze parameters.

	Iteration	npop
State 1	75	30
State 2	100	40
State 3	125	50

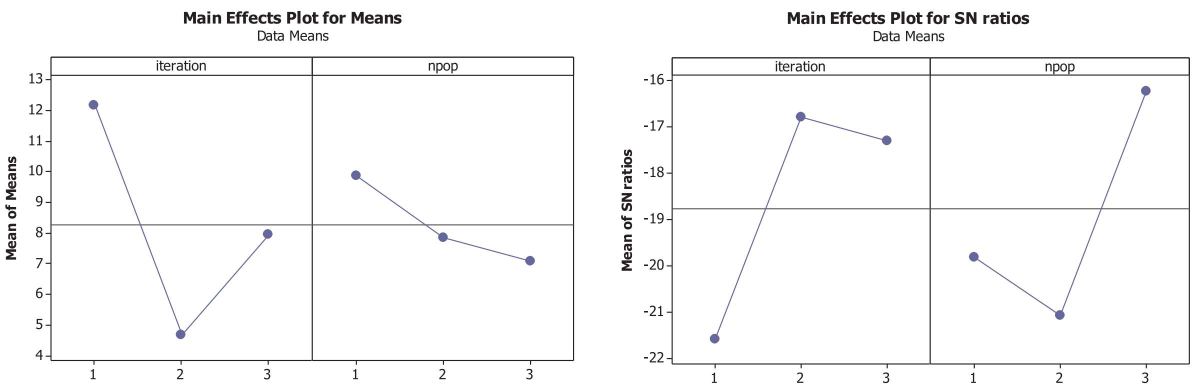


FIGURE 4. Taguchi's result.

(fourth test problem). The results of running AEC method code in GAMS for each test problem are presented in Figure 5, which displays the contrast between two objectives. Moreover, according to Figure 6, the performance of the objective functions is balanced by considering different weights and the set of Pareto optimal solutions is achieved. According to the policy and preferences proposed by decisionmakers, the best answer can be chosen from the values of the results. Thus, we presented the achievement of logical results according to the different values of the objective functions.

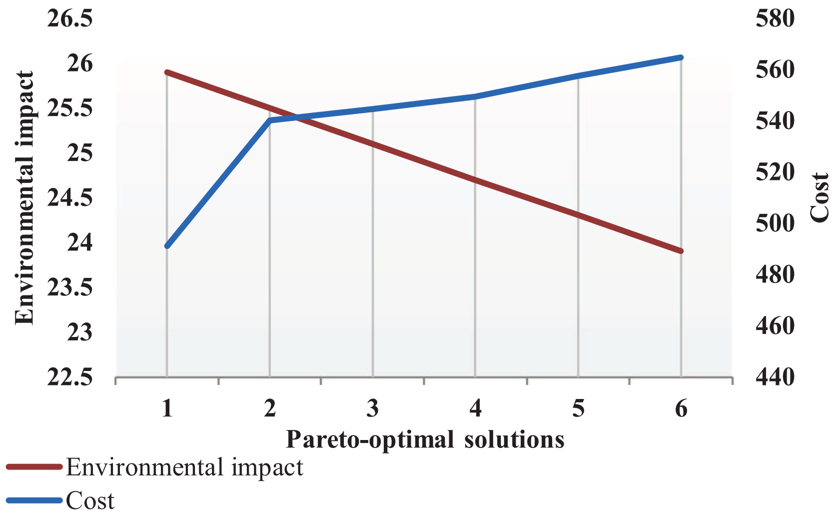


FIGURE 5. Pareto's Objective functions obtained through the AEC method.

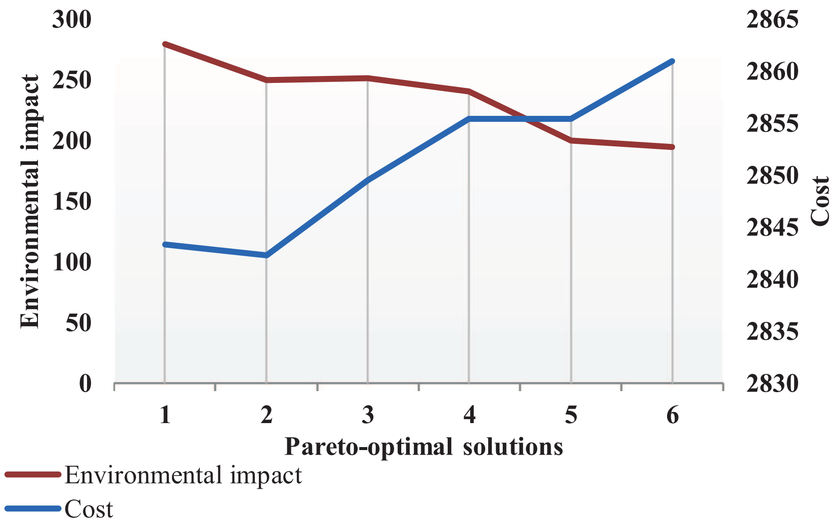


FIGURE 6. Trade-off between two objective functions.

6. RESULT AND DISCUSSION

6.1. Results

The results of Pareto-optimal solutions for each test problem by using AEC method shown in Table 6. Also, computational time for each test problem shows in the second column of Table 6.

In the following, the Payoff table of the fourth test problem is presented in Table 7.

The results of Pareto-optimal solutions of fourth test problem by using the AEC method are presented in Figure 7. Figure 7a shows the second Pareto solution (grid point) of the fourth test problem. The second supplier is selected in the first period, and it is served all the factories. Also, in this period, the second factory hasn't shipped any product to any DCs, and the third DC is served the fifth seller (an online seller). In the second

TABLE 6. The results for each test problem AEC method.

Test problems No.	Time (s)	Obj. function values	Pareto-optimal solutions					
			1	2	3	4	5	6
1	4.44	Cost	122.146	128.19	128.308	128.642	141.307	157.188
		EI	13.166	12.304	11.86	11.129	10.565	9.769
2	22.54	Cost	180.567	182.845	195.312	202.899	218.937	231.019
		EI	15.17	14.707	14.243	13.78	13.316	12.852
3	34.71	Cost	409.543	413.131	435.371	444.555	448.854	455.306
		EI	22.286	21.829	21.371	20.913	20.456	19.998
4	73.52	Cost	525.967	539.998	544.548	549.307	557.281	564.611
		EI	25.581	25.496	25.097	24.699	24.3	23.901
5	113.12	Cost	599.604	606.315	613.485	678.015	680.474	682.416
		EI	32.733	32.033	31.333	30.633	29.933	29.232
6	286.95	Cost	701.16	712.54	720.43	743.12	748.55	756.28
		EI	35.12	34.85	34.202	33.56	33.018	32.73
7	371.28	Cost	782.65	793.57	800.35	811.37	840.22	849.44
		EI	40.83	40.011	39.29	39.85	38.79	38.26

TABLE 7. Payoff table of the fourth test problem.

	Z_1	Z_2
Min Z_1	518.915	25.683
Min Z_2	755.311	23.104

period, the second supplier is served the second factory, and it's served the third DC, and it's served all the sellers. Also, in the third period, the first supplier is served the third factory, and it's served the fourth DC, and from it, the products are sent to all the sellers. Similarly, Figure 7b, which is the fifth Pareto's solution from the fourth test problem, can be interpreted.

6.2. Comparison of exact and metaheuristic methods

We solved the generated test problems using the GOA method and compared the results with GAMS to confirm its validity for large size applications. As shown in Table 8, due to the acceptable difference between the GOA method and GAMS results, the relevant algorithm can be applied to solve the model on a large scale.

The present model in small and medium sizes can be used in some areas of a zone, while by increasing the dimensions of the model, the practical application of the model can be considered in covering the whole of the zone. Thus, increasing the dimensions of the model can be used operationally in location-allocation-inventory decisions on a greater level. We increased the dimensions of the model and listed the results of GOA in Table 8. Also, the convergence diagram of the metaheuristic method is presented in Figure 8.

Moreover, the statistical analysis on the equality of the value of objective functions obtained by AEC and the GOA algorithm is investigated. A medium-sized test problem has been run 25 times by the GOA and the normality of this data is tested with Minitab software and is plotted in Figures 9 and 10 for the first and second objective functions, respectively. Therefore, the assumption of a normal distribution for the data obtained by the metaheuristic algorithm is logical.

According to Montgomery [42], the following assumptions are made for the ANOVA test:

$$H_0 : \mu_{Z_{\text{GOA}}} = \mu_{Z_{\text{AEC}}}$$

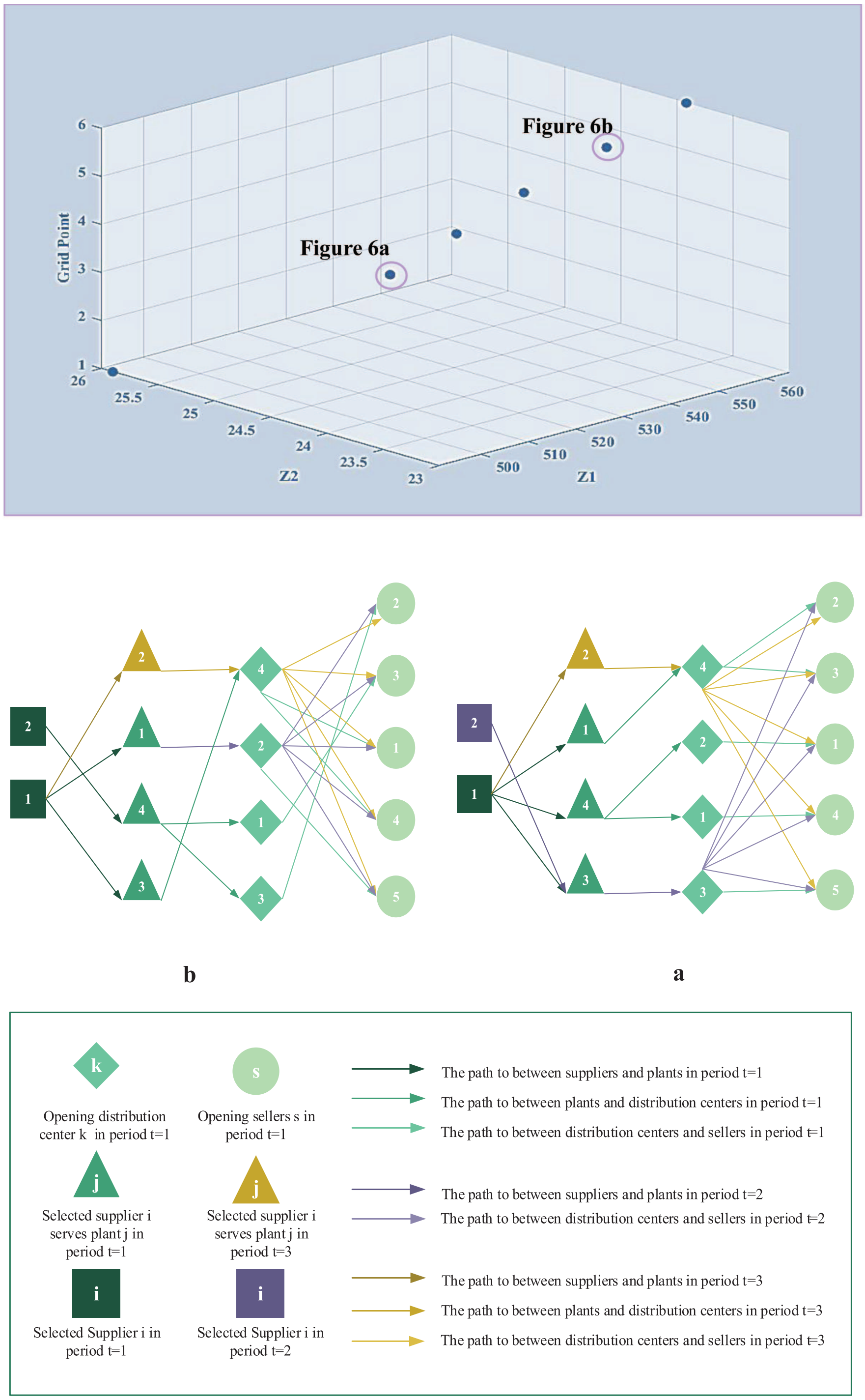


FIGURE 7. Results of Pareto frontier and feasible solutions based on AEC method. (a) 1st structure of the feasible result. (b) 2nd structure of the feasible result.

TABLE 8. Solutions obtained by GOA and GAMS.

Test problem	AEC		GOA		Gap (%)	
	Z_1	Z_2	Z_1	Z_2	Z_1	Z_2
1	128.642	11.129	129.094	11.238	0.351	0.979
2	218.937	13.316	220.013	13.518	0.491	1.516
3	444.555	20.913	448.109	21.321	0.799	1.95
4	549.307	24.699	553.582	24.988	0.778	1.17
5	613.485	31.333	619.233	31.764	0.937	1.375
6	743.12	33.56	749.68	34.01	0.883	1.34
7	800.35	39.29	808.71	39.98	1.044	1.756
8	–	–	1075.3059	52.622	–	–
9	–	–	1628.5187	77.190	–	–

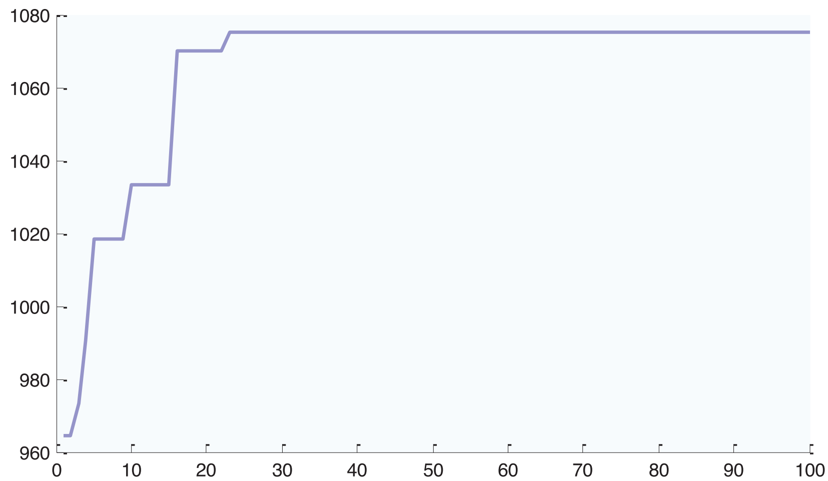


FIGURE 8. Convergence diagram of the model in large size.

$$H_1 : \mu_{Z_{\mathrm{GOA}}} \neq \mu_{Z_{\mathrm{AREC}}}.$$

The null hypothesis indicates the equality of the mean of the results of the two approaches and the opposite hypothesis indicates their inequality. The results of ANOVA statistical analysis are presented in Table 9.

Based on the results shown in Table 9, at a significance level of 0.05, all sample statistics are in the acceptance area and null hypothesis cannot be rejected. Therefore, there is no significant difference between the results AEC and GOA methods.

7. Sensitivity analysis

To evaluate the proposed model's efficiency, the sensitivity analysis is presented. We consider different values for the main parameters and assess the behavior of the objective functions and the value of decision variables.

7.1. The impact of seller's demand and storage capacity of suppliers

We accomplished the sensitivity analysis in four categories. In the first category, we've increased the storage capacity of suppliers by keeping the distribution functions of the seller's demand constant. In the second category, we examined the effect of reducing the storage capacity of suppliers while demand is constant. In the third

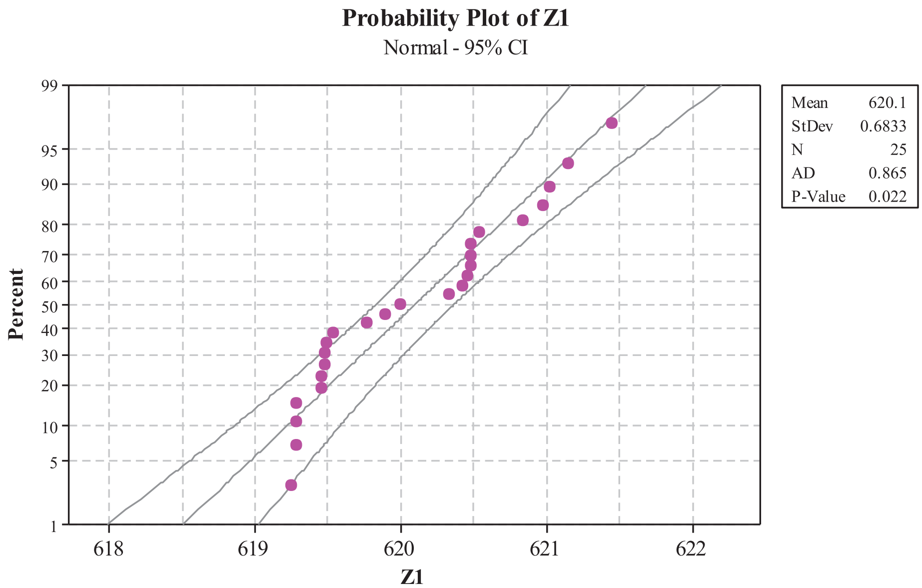


Figure 9. Probability plot of first objective function.

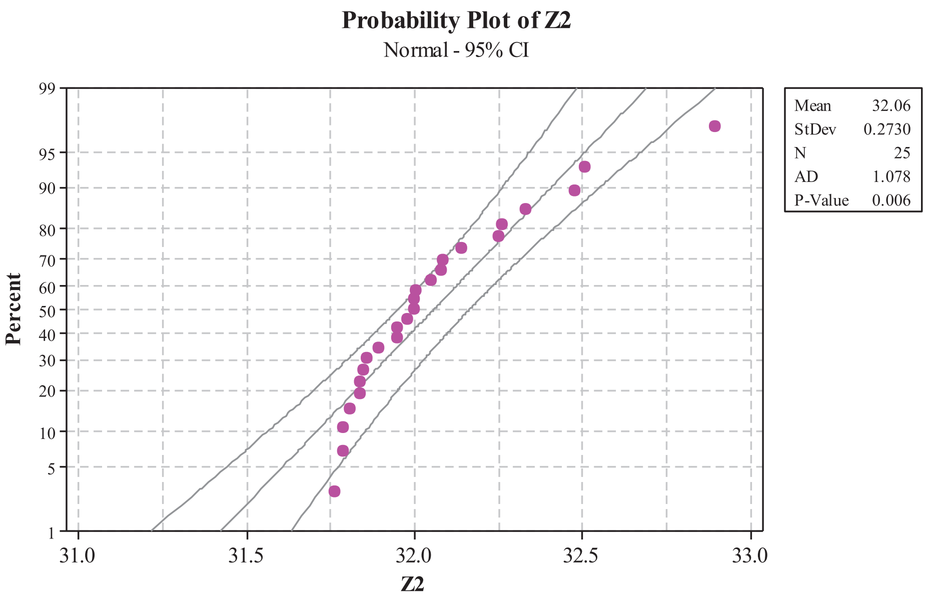


Figure 10. Probability plot of the second objective function.

category, by keeping the supplier's storage capacity stable, we've increased the demand for the sellers, and finally, in the fourth category, we've examined the combined effect of increasing demand for sellers and increasing the storage capacity of suppliers. The results of the first and second categories are shown in Figures 11a and 11b, and the results from the third and fourth categories are depicted in Figures 12a and 12b.

Looking horizontally at Figure 8a, we find that with increasing the supplier's storage capacity, the value of Z_1 and Z_2 in Pareto solutions behaves inversely. Increasing cost leads to reduced environmental impact and

Table 9. ANOVA test output results.

Source of variation		DF	Sum of squares	Mean square	F	P-value
Z_1	Response	1	545.757	545.757	2337.73	0.984
	Error	48	11.206	0.233		
	Total	49	556.963			
Z_2	Response	1	6.5403	6.5403	175.51	0.957
	Error	48	1.7887	0.0373		
	Total	49	8.3289			

vice versa. The cause of these ups and downs can be explained by the amount that the Beta variable takes on. Similarly, Figure 8b can be expressed in the case of decreasing the minimum capacity of the suppliers.

In Figure 11a, it is clear that as the demand of sellers increases, Z_1 and Z_2 at the Pareto solutions is gradually increasing. According to Figure 11a, if the capacity of suppliers remains constant and the average demand increases, the total costs and environmental impacts will increase. Increasing the shortage costs and also the transportation of products in the network leads to an increase in both objective functions.

According to Figure 11b, it is clear that as the storage capacity of suppliers and the average seller's demand increases simultaneously, the objective functions behave is reverse, so that the increase of the first objective function leads to a decrease in environmental impact and *vice versa*. According to Figure 12a, an increase in demand leads to an increase in cost and environmental impact, while according to Figure 12b, an appropriate adjustment can be made to the cost and environmental impact by increasing capacity.

7.2. The impact of inflation on lack of inventory and objective functions

The results of the sensitivity analysis of changes in the inflation rate are shown in Table 10. The results show that as the inflation rate grows, the cost function increases due to the increase in the cost of back-ordered demand and inventory shortages. This means that the rate of production throughout the supply chain is declining, resulting in reduced environmental impact.

As shown in Figure 13, a rising inflation rate will lead to increased system costs and shortages, and consequently, the cost of the whole supply chain will increase. Moreover, as the inflation rate and shortage increase, the quantities produced and transmitted decrease, resulting in reduced environmental impacts. Moreover, the effect of the inflation rate on the lack of inventory is presented in Figure 14. This figure illustrates that increasing the inflation rate leads to increasing costs and decreasing the volume of orders and consequently, shortages increase.

7.3. The impact of defective probability on lack of inventory, objective functions, and safety stock

The results of the changes in the defective probability of the product are shown in Table 11 and Figures 15 and 16. Based on these results, we find that the more defective products are produced, the cost performance increases due to the increase in shortages and the preservation of higher safety stocks (see Fig. 16). In addition, as defective probability increases, transfers at different levels of the chain to meet demand increase and accordingly increased environmental impact.

7.4. The impact of fixed cost of opening DCs on the objective functions

According to the results of Table 12, it is clear that the higher the fixed costs of opening DCs, the less DCs will be ready to serve, which will lead to a reduction in environmental impacts resulting from the opening of these centers. This is clearly shown in Figure 17. Moreover, increasing the fixed cost of opening DCs leads to

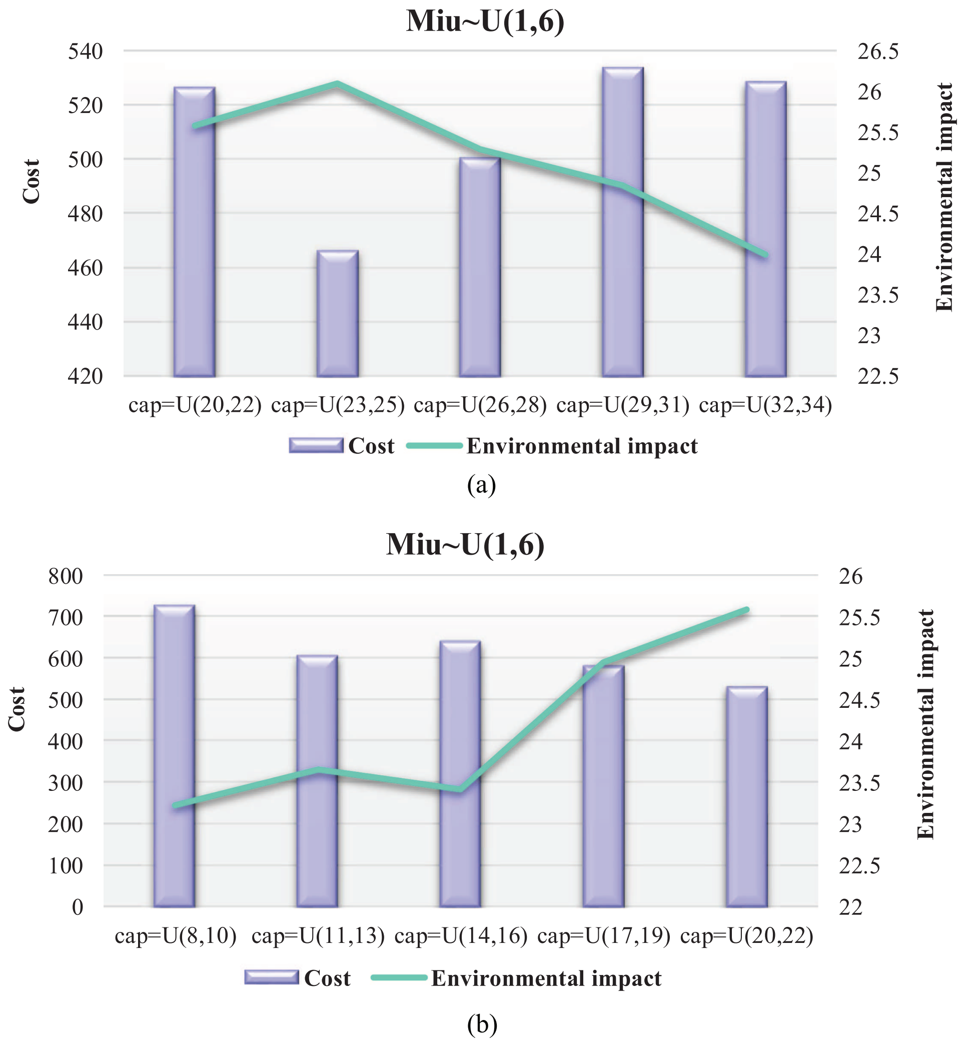


FIGURE 11. (a) The results of the first category. (b) The results of the second category.

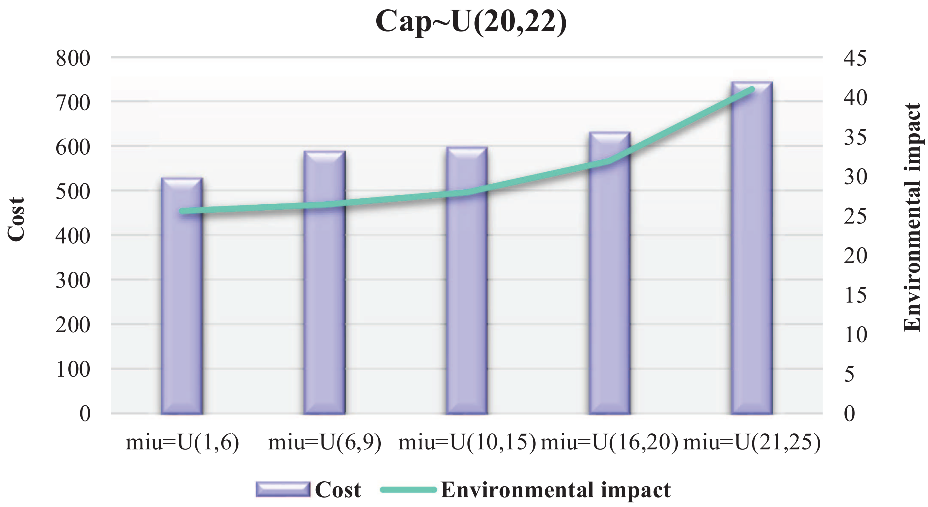
an increase in the total cost. In this way, managers can have a good balance between environmental impact and cost function by choosing the optimal number of DCs.

7.5. The impact of maximum lead time from DCs to sellers on the objective functions

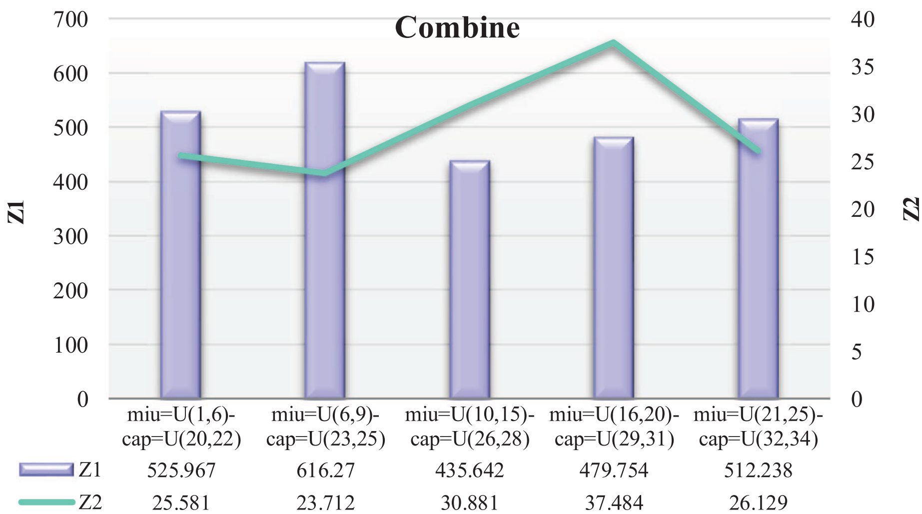
The results of changes in the maximum lead time from DCs to sellers parameter in Figure 18 and Table 13 demonstrate that the longer the lead time, the lesser the value of both objective functions. This means that transportation-related costs are reduced, and as a result, the total costs and environmental effects on the entire network are reduced.

7.6. The impact of sellers and DCs capacity on the environmental impact

The effect of variations in the capacity of DCs and sellers (traditional and online) on the second objective function, namely environmental impact, is investigated in Figures 19–21, respectively. As shown in Figure 19,



(a)



(b)

FIGURE 12. (a) The results from the third category. (b) The results from the fourth category.

TABLE 10. Result of the first Pareto-solution of fourth test problem.

Inflation rate	Objective function		Total lack of inventory
	Cost	Environmental impact	
$U(0.1, 0.2)$	472.822	29.911	1.659
$U(0.2, 0.3)$	518.219	26.885	3.027
$U(0.3, 0.4)$	590.018	24.697	5.513
$U(0.4, 0.5)$	664.236	25.011	7.324

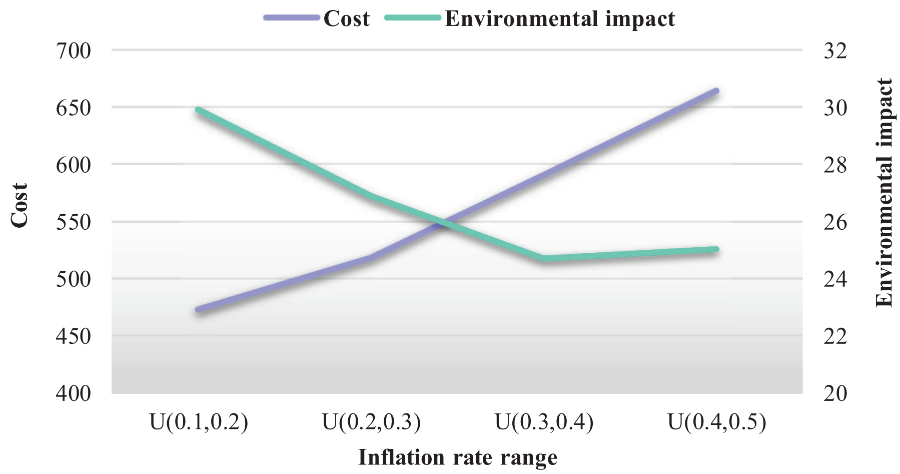


FIGURE 13. The effect of Inflation rate on the objective functions.

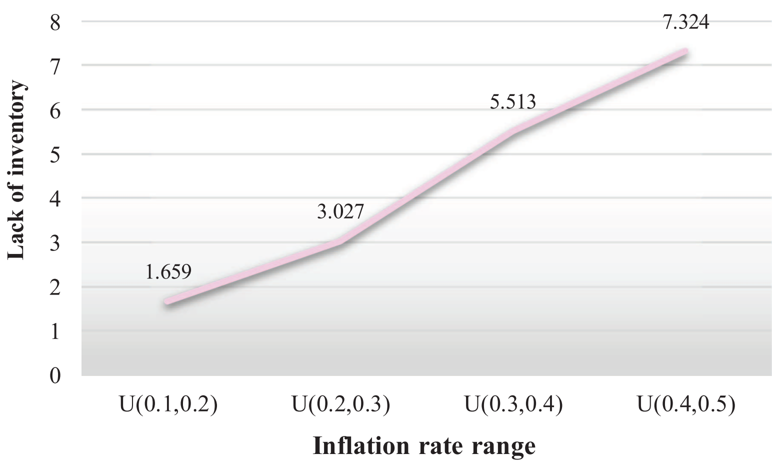


FIGURE 14. The effect of the Inflation rate on the lack of inventory.

TABLE 11. Result of the first Pareto-solution of fourth test problem.

Defective probability	Objective function		Safety stock	Total lack of inventory
	Cost	Environmental impact		
$U(0.1, 0.2)$	52.642	21.699	8.984	6.444
$U(0.2, 0.3)$	519.387	26.882	8.796	9.754
$U(0.3, 0.4)$	543.234	29.464	9.315	10.368
$U(0.4, 0.5)$	581.159	33.878	1.157	12.292

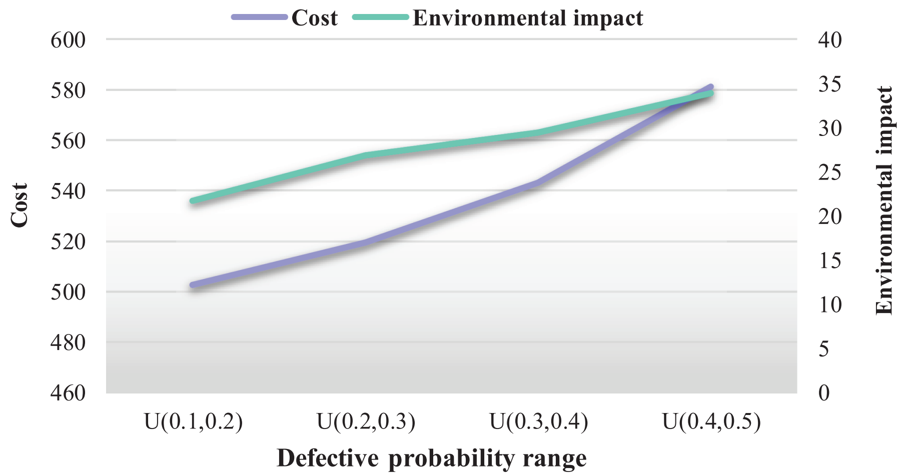


FIGURE 15. The effect of defective probability on the objective functions.

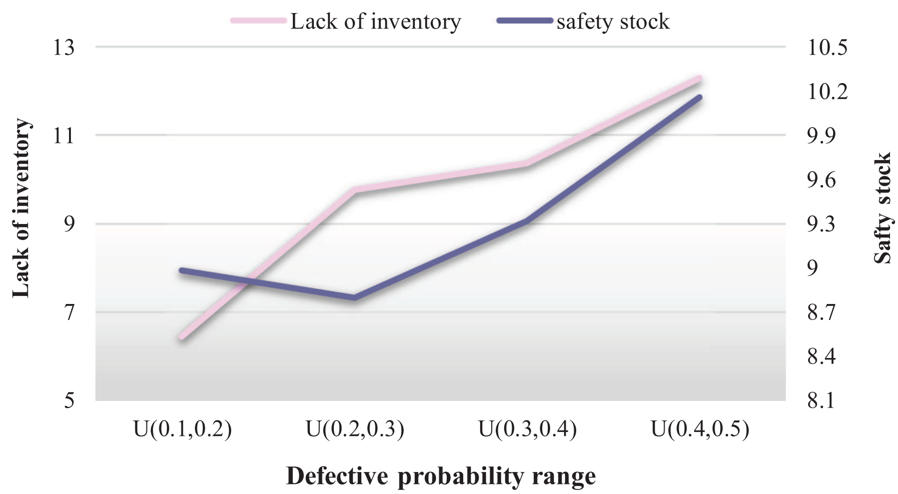


FIGURE 16. The effect of defective probability on the lack of inventory and safety stock.

TABLE 12. Result of the first Pareto-solution of fourth test problem

Fixed cost of opening DCs	Cost	Environmental impact
$U(2,5)$	525.967	25.581
$U(5,8)$	563.028	24.965
$U(8,11)$	594.336	24.322
$U(11,14)$	667.219	23.784

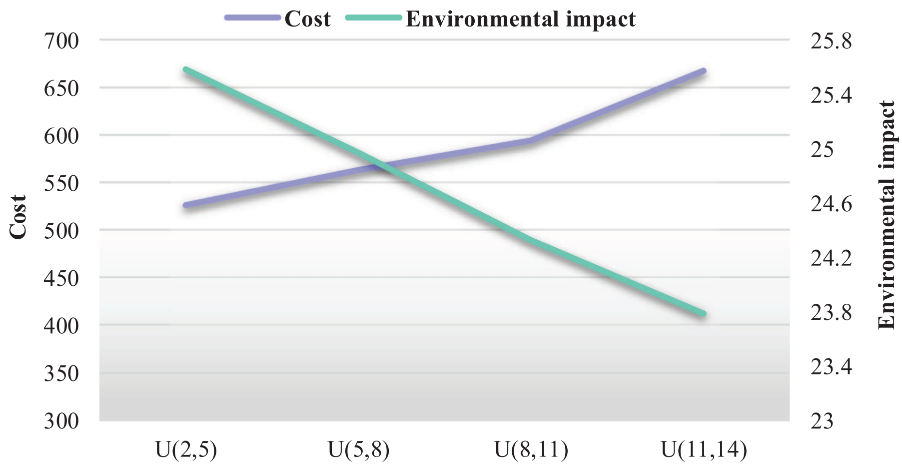


FIGURE 17. The effect of fixed cost of opening DCs on the objective functions.

TABLE 13. Result of the first Pareto-solution of fourth test problem.

Lead time	Objective function	
	Cost	Environmental impact
$U\,(8,10)$	573.902	25.429
$U\,(10,15)$	537.038	25.878
$U\,(15,20)$	511.739	24.502
$U\,(20,25)$	483.586	24.760

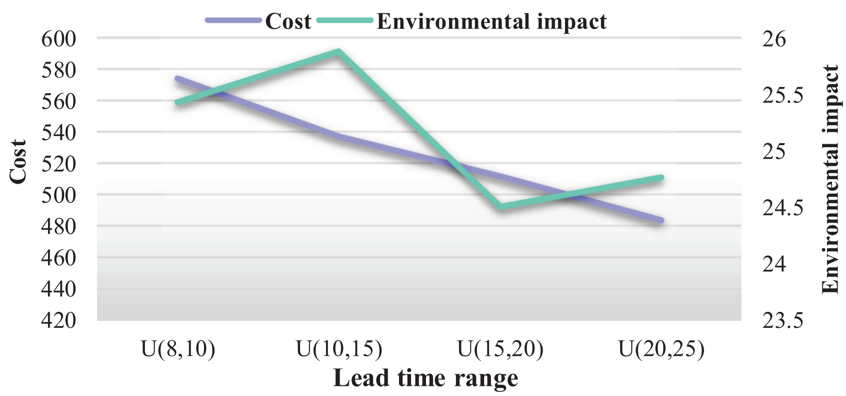


FIGURE 18. The impact of maximum lead time from DCs to sellers on the objective functions.

increasing the capacity of DCs leads to a decreasing trend in the behavior of Z_2. This can be explained by reducing the number of established DCs and changing the amounts sent to them, which leads to a reduction in environmental impact.

Figures 20 and 21 indicate a rather descending trend in the performance of Z_2 with increasing sellers (traditional and online) capacity. These figures indicate that the second objective function is sensitive to fluctuations in sellers' capacity, and by determining the appropriate seller capacity; the environmental impact can be improved.

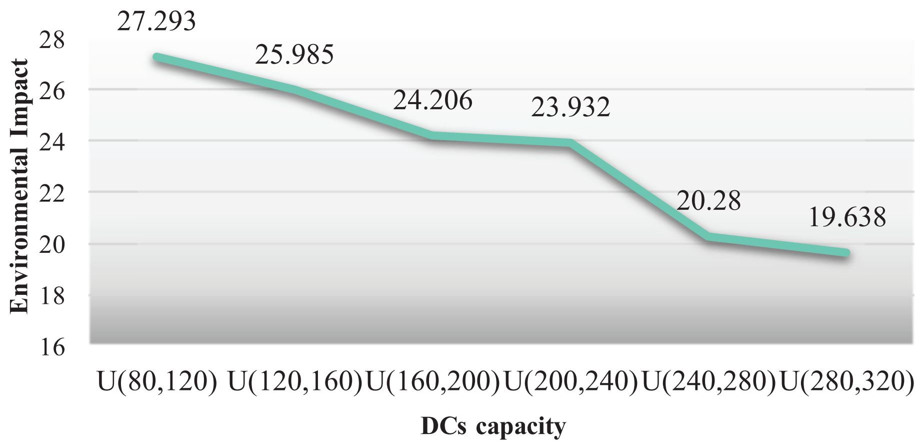


FIGURE 19. The impact of DCs capacity on the second objective function.

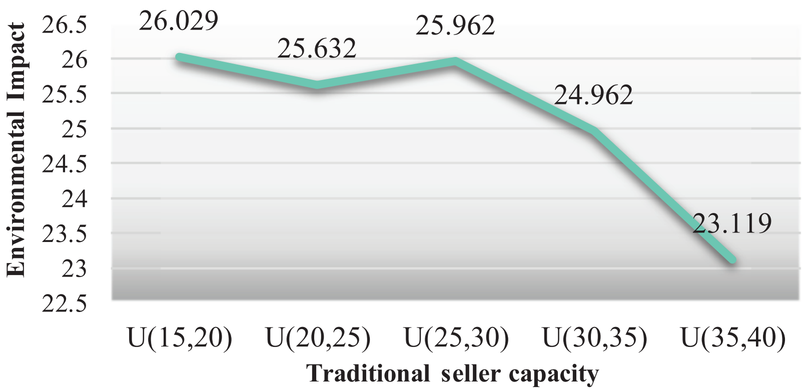


FIGURE 20. The impact of traditional seller capacity on the second objective function.

7.7. Discussion and managerial insights

In this research, we studied the integrated optimization of LAIP by developing a bi-objective model under uncertainty conditions. In this model, the characteristics of green inventory, inflation and quality of raw materials are considered simultaneously. We also added e-commerce activities by introducing two traditional and online channels for selling products. Moreover, we examined the inventory policy of multi-period periodic reviews under a combination of shortages. To the best of our knowledge, previous studies are not included these features. Vahdani *et al.* [66] studied the LIP issue in a three-level supply network to minimize supply chain costs. In this study, shortages are allowed, and periodic review policy has been used to manage inventory level. Raza and Govindaluri [56] examined a two-tier supply chain for the green product by considering two channels (traditional and online). Mousavi *et al.* [48] studied LAIP issue in the two-tier supply network considering the continuous periodic review policy and stochastic demand. Braglia *et al.* [15] analyzed the (Q, r) policy for an inventory system to minimize the total cost under the conditions of allowable shortages and uncertain demand. Wang *et al.* [68] addressed a stochastic model to control demand uncertainty and solve the LIP issue of the green supply chain. Obviously, the researchers did not provide a model that considered the characteristics of this study.

We considered five test problems in different dimensions. Test problems 1–3 are considered as small-sized problems, and test problems 4–7 are considered as medium-sized problems. The main results of the sensitivity

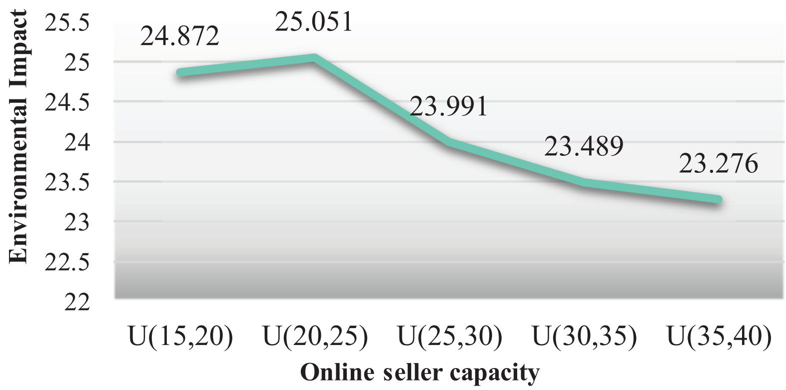


FIGURE 21. The impact of online seller capacity on the second objective function.

analysis are provided in the before sections. According to the results of sensitivity analysis, important management insights are classified into two general categories as follows:

(1) *Environmental sustainability*

The capacity of suppliers and the end levels of the chain play a key role in environmental impact. On the one hand, allocating adequate capacity to suppliers enables the supply chain to increase the sustainability of environmental issues in addition to the economic aspect; on the other hand, increasing the capacity of DCs has a considerable effect on reducing environmental impact as well as increasing the capacity of the dual-channel, but what leads to its further improvement is the capacity of DCs which is one of the important characteristics that affect supply chain outcomes. These features become especially important during the time of increasing demand. According to the results of sensitivity analysis, when only demand increases, it leads to a rising trend in costs and environmental impacts. In fact, the simultaneous increase in demand and capacity controls their rising trend. So, managers can make appropriate adjustments to costs and environmental effects in the face of increasing demand by capacity increases.

Based on the results, the design of a dual-channel supply chain and the increase of defective probability of products have positive and negative effects on the environment, respectively. As we can see in Figures 20 and 21, delivering products through the sharing of supply between traditional and online channels has a significant role in controlling environmental impacts. According to Figures 15 and 16, when the defective products are increasing, the shortage increases. Therefore, managers are advised in addition to using e-commerce activities, to have more comprehensive quality control over products and the status of production equipment with proper maintenance, keep their performance at a high level, and minimize the production of defective products.

(2) *Economic sustainability*

Increasing lead time and the inflation rate are the parameters that reduce economic sustainability. As stated in the sensitivity analysis results, an increase in the inflation rate leads to an increase in shortage and, consequently, to a decline in economic sustainability. Contrary to the changes in the inflation rate, which affects supply chain decisions and goals, the timely delivery of items and creating an online sales channel can improve supply chain costs. Thus, managers are advised to special attention to increasing the level of service and reinforcing the flexibility of supply chain members as a way to compensate for the shortage under conditions of the rising inflation rate. Also, DCs are advised to the expansion of e-commerce activities and increase the share of online sales by advertising and promotion. These subjects lead to a decrease in the total cost and consequently increase the total income and economic sustainability.

8. Conclusion

In the present study, a bi-objective model was formulated for the optimization of LAIP. The network under study is integrated by suppliers, plants, DCs, and dual channels of sellers. The main aim of this study is to improve the economic and environmental aspects of the supply network simultaneously while tactical and strategic decisions are integrated. Demand uncertainty is also addressed using stochastic modeling. Features such as lead time, shortage, inflation, quality of raw materials and associated demand in a periodic review policy joined the model to adapt it to the real conditions. Moreover, the combination of online and traditional sales channels to design a dual-channel network is studied in this research. This network effort to help managers find suitable providers, proper DCs locations, and the optimal number of products shipped so that they are in a favorable economic and environmental situation. We delineated medium-sized problems and solved them by using the AEC approach. Also, the GOA method has been used to solve the model for large size problems. The results of solving the model show that considering features such as inflation, shortage, dual-channel sales and supplier quality of raw materials play a key role in location, inventory, and managers' decisions in real situations. Also, integrating stochastic demand with a mathematical model brings the results closer to the real world. Moreover, considering the environmental aspects not only help companies improve the economic aspect, but also avoid penalties by environmental officials and make them popular from the people's perspective.

With the development and increase of competition between different manufacturing industries, updating the logistics system and solving purchasing problems are the concerns of supply chain managers. Moreover, the importance of sustainable development has encouraged industry and governments to increase environmental sustainability by improving their activities. Appropriate and integrated decision-making at the strategic and tactical levels of the supply chain will make a major contribution to reducing costs, increasing competitive advantage, and moving towards sustainable development. Moreover, with the advancement of technology and the expansion of the use of the Internet and e-commerce, the creation of online sales channels enables the supply chain to reduce share of carbon emissions from transportation and the establishment of facilities. Therefore, this research helps different industries to reduce costs and move towards sustainable development by optimizing appropriate investment and proper planning at different levels of the supply chain. In addition, paying attention to demand uncertainty in the proposed model not only provides a better decision for the decision-maker, but also increases the speed of response to changes in demand. Industries of apparel and home textile, kitchen and home appliances are industries that, due to the adaptation of their decisions to the framework of the present research, can use the results for planning at tactical and strategic levels.

This study also had some limitations; for example, we rely on numerical examples to solve the proposed model. Moreover, the lack of implementation in the real example due to the unavailability of a suitable case study at the time of research is another limitation that can affect the performance of the results. The application of some facilitator assumptions can also be considered as research limitations. In order to expand the present study, it is suggested to future researchers to extend the model in a multi-product, multi-vehicle supply chain with stochastic lead time. In addition, they can consider reverse logistics and add discount conditions, the concept of resilience, and the time value of money in an infinite planning horizon.

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