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DESIGNING A SUPPLY CHAIN NETWORK UNDER A DYNAMIC DISCOUNTING-BASED CREDIT PAYMENT PROGRAM

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Abstract. This study examines the effects of dynamic discounting based credit payment on a supply chain network design problem. Dynamic discounting based credit payment is a supply chain finance policy wherein the supplier provides a credit period to a distribution center (DC) with a discount applied if the DC pays the supplier before the end of the credit period. This study also considers the time value of money and applies discounted cash flows to formulate a model that determines the DC's optimal replenishment cycle, selling price, and influence area while maximizing the present value of the total profit. The continuous approximation approach is applied to formulate a mathematical model of the problems, and an algorithm based on non-linear optimization is established to solve the problem. A numerical example and a sensitivity analysis are provided to present the proposed model and solution approach and to illustrate the effect of each cost on the decisions and profit.

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1. Introduction

Supply chain network design is critical in supply chain management that deals with three types of flows, namely, product, information, and financial flows. These flows affect the performance and efficiency of the supply chain. However, it is not easy to optimize all three flows simultaneously. Therefore, a company needs to prioritize when designing a supply chain network. According to Farahani *et al.* [7], financial and material flows are the main parts of supply chain networks. By integrating the supply chain's configuration with financial decisions, the efficiency of a supply chain network can be improved.

In recent years, several studies have tried to integrate product and financial flows. Credit payment is one of the payment schemes that have been used recently in supply chains. In a credit payment, the seller provides the buyer a credit period or additional time to make payment. Credit payment has been applied in the United States, with 55% of the top 100 U.S. contractors in 2008–2009 experiencing difficulties in meeting price margin

Keywords. Supply chain network design, continuous approximation approach, trade credit, dynamic discounting.

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and deciding to extend the payment deadline of suppliers [19]. Early on, Goyal [12] discovered an integration of trade credit and economic order quantity. According to Goyal [12], the idea behind a trade credit (or permissible delay in payments) is for the seller to provide a buyer a credit period or additional time to make payment, and during this credit period, interest is charged and earned by the seller.

Liu and Cruz [18] examined the impact of financial risk on trade credit transactions in supply chain networks. They found that trade credit is affected by financial risk and both play an important role in the decision-making process. Tsao *et al.* [24] used trade credit in designing a distributor center network and considered several scenarios depending on the relationship between the replenishment cycle and the credit period. Giri and Maiti [11] presented a two-level trade credit arrangement in a competitive supply chain environment, wherein two retailers offer two different credit periods to customers and one supplier offers trade credit to both retailers. They found that a two-level trade credit arrangement has a positive effect on the supply chain. Tsao and Linh [23] considered trade credit and advance payment on supply chain network design for deteriorating items and examined shortage and partial backorder. They found that supply chain network cost is higher for advanced payment than trade credit.

Meanwhile, Kim and Sarkar [15] proposed a supply chain model that uses trade credit payment and considered a stochastic lead time and transportation discounts. Different from Kim and Sarkar [15], Li *et al.* [16] examined advance-cash-credit payment for optimizing the selling price and lot size policies for perishable products. Recently, Alavi and Jabbarzadeh [1] provided an optimization model that integrated financial flow (*e.g.*, trade credit), the capital budget, and bank credit in supply chain configuration to determine the location and allocation of a distribution center. They found that trade credit and bank credit have a significant effect on supply chain profit. Zhong *et al.* [30] developed an integrated location and two-echelon inventory network design model and studied the impact of trade credit on network design. They found that integrating trade credit and network design could reduce costs by more than 10%. Similarly, Li *et al.* [16] and Tsao *et al.* [25] used advance-cash-credit payment in supply chain network design. This means that the buyer pays a fraction of the purchasing cost in three stages: before shipping the goods, upon receipt of the goods, and within a credit period. Li *et al.* [17] extended Li *et al.* [16] by considering the time value of money and backorders.

All of the studies above use a one-part trade credit, as the credit consists of a single credit period. Alternatively, there is a two-part trade credit that consists of two different credit periods (*e.g.*, M_1 and M_2), where M_1 is shorter than M_2. The idea of a two-part trade credit is that the buyer can pay the purchasing cost at M_2 at the standard price or get a discount by paying earlier at M_1. Ho *et al.* [14] optimized price-, shipment- and payment-related policies to enhance the supplier's profit. They found that when a seller offers a two-part trade credit to a buyer, this two-part trade credit can increase the profit of the supplier, buyer, and the whole supply chain. Chung and Liao [4] simplified the solution algorithm used in Ho *et al.* [14]. Zhong and Zhou [29] searched for the optimal trade credit policy by comparing a one-part trade credit with a two-part trade credit in a supply chain. They found that the latter is better than the former. Zou and Tian [31] expanded the concept of a two-part trade credit by making it flexible. The idea of a flexible two-part trade credit is that the supplier allows the buyer to pay a fraction of the purchasing cost at M_1 at a discounted price and to pay the rest of the purchasing cost at M_2 at a standard price. The authors found that a flexible two-part trade credit can reduce the total cost of the retailer.

Besides trade credit, another payment scheme is dynamic discounting. According to Gelsomino *et al.* [9,10], dynamic discounting is a supply chain finance policy that extends the two-part trade credit scenario. In such a policy, the seller provides a credit period to the buyer and a discount is applied if the buyer pays the seller before the end of the credit period. The main difference is that dynamic discounting gives buyers more flexibility to choose how and when to pay their suppliers in exchange for a lower price or discount for the goods and services purchased. The "dynamic" component refers to the option to provide discounts based on the dates of payment to suppliers. That means the amount of discount dependents on the length of payment time. The earlier the payment is made, the greater the discount. However, in two-part trade credit, the payment time and discount price are fixed in two different period. The dynamic discounting scheme encourages the buyer to pay before the due date, thereby addressing certain disadvantages of credit payment. In practice, businesses in the UK are increasingly turning to dynamic discounting to solve the growing problem of late payments. Procurement software provider, Wax Digital, asked 100 UK procurement professionals about dynamic discounting, and 27% said they are already using it. Another 30% said that they are planning on implementing the approach in the next 12 months, and 20% said it is a long-term objective. In total, 77% of businesses (more than three quarters) is using or tend to use dynamic discounting in the future [6]. Specially, in the coronavirus era, many businesses have lost significant revenue, many see early pay *via* dynamic discounting as a win–win – help suppliers get cash, and buyers manage cash at much better yields than alternatives [13]. Therefore, more researches of dynamic discounting is necessary to see how to apply it into the real case. Gelsomino *et al.* [9] examined dynamic discounting in a consumer goods retailer in Italy and found that the amount of daily discount has a linear effect on profit. Meanwhile, no studies considered dynamic discounting in supply chain network design problems. To fill this gap, our study considers the dynamic discounting-based credit payment in designing a supply chain network.

There are two kind of models (discrete model and continuous model) in supply chain network design literature. The discrete model is a common method used in formulating the mathematical model [2, 3, 8, 28]. Generally, a discrete model can obtain an optimal solution, but it needs a high level of data and intensive computations, especially if the study tries to illustrate real operational networks [24, 27]. Furthermore, a high level of data leads to a low level of data reliability and model accuracy [24]. Thus, to overcome these disadvantages, some studies use a continuous approximation (CA) approach, which provides a near-closed form solution and reduces the complexity of the problem [5]. The basic idea of the CA approach is to reduce the complexity of the problem by using a continuous function when defining the decision variables [24]. For example, the location of facilities under a CA approach is defined as an influence area, not as an exact location. Dasci and Verter [5] used a CA approach instead of a discrete approach for designing a production-distribution system. Pujari *et al.* [21] presented a CA approach for determining the optimal number and size of shipment in an inventory distribution model. Tsao *et al.* [24] proposed a CA approach for designing a distributor center network. Furthermore, Wang and Ouyang [27] developed a CA approach in a competitive facility location problem that also considered facility disruption risk. Other studies that also used the CA approach for model formulation are Tsao [22], Tsao and Linh [23], and Tsao *et al.* [25].

This study also applies the time value of money concept or discounted cash flow analysis (DCF). Theoretically, DCF assumes that the current price is equal to the cumulative discounted future cash flows at a constant rate [26]. This theory is supported by the fact that both outflow and inflow in a supply chain happen at different times; thus, they have different values. For example, if r is the annual compound interest rate per dollar per year, then \$1 today is equal to $\$e^r$ one year later, and \$1 one year from now is equal to $\$e^{-r}$ today [16]. Therefore, it is important to consider the time value of money and apply discounted cash flow analysis to illustrate real-life situation. Many studies, such as Li *et al.* [16] and Tsao *et al.* [25], have applied DCF analysis.

In this study, dynamic discounting is used as the payment scheme in a supply chain network design problem. To the best of the author's knowledge, this is the first study that examines the effects of a dynamic discounting scheme on a supply chain network problem. In such a scheme, the supplier provides a credit period to the distribution center (DC) and a discount is applied if the DC pays the supplier before the end of the credit period. This study also considers the time value of money and conducts a DCF analysis. Under a dynamic discounting scheme, the DC determines the replenishment cycle, selling price, and influence area to maximize the present value of the total profit. The problems are formulated using the CA approach, which requires fewer data and computational time. In using this approach, the derived theoretical results, as well as the closed-form solutions, are provided directly. Moreover, this study develops an algorithm based on non-linear optimization to solve the problem. The proposed algorithm looks for the optimum solutions that maximize the present value of the total profit. Lastly, this study provides numerical and sensitivity analyses that provide managerial insights.

The rest of the paper is organized as follows. Section 2 describes the problem, including assumptions and notations. Section 3 discusses the model formulation. Section 4 introduces the theoretical results and the proposed solution approach, as well as the algorithm. Section 5 provides a numerical analysis to gain managerial insights. Section 6 presents the conclusions and avenues for future studies.

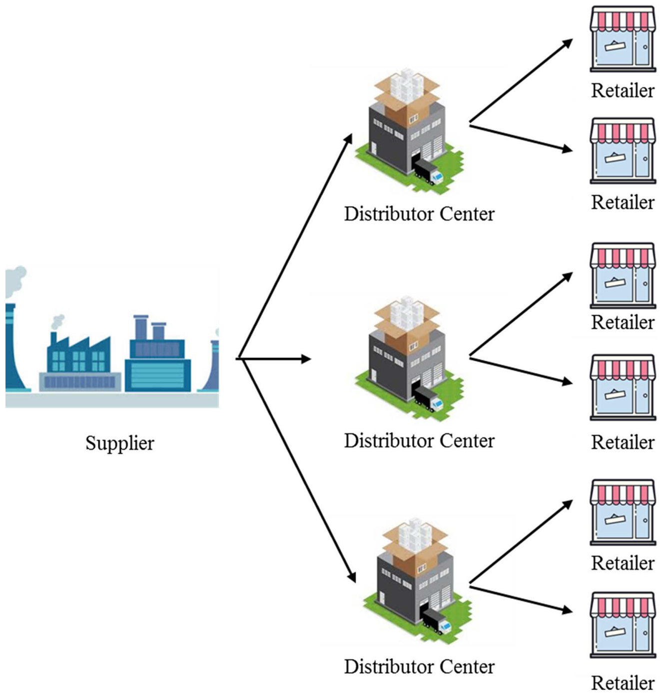

FIGURE 1. Supply chain network.

2. Problem definition

In this study, a network consists of a single supplier, multiple DCs, and multiple retailers, as illustrated in Figure 1. In practice, this kind of network is common, Target, Frito-Lay, and Wal-Mart are examples [22, 24]. Dynamic discounting is provided by the supplier to the DC. Supplier provides a credit period (μ) to the DC and a discount is applied if the DC pays before the end of the credit period. The sooner the payment is settled, the higher the discount is. If the DC does not pay before the credit period, he must pay in the end of credit period. This study develops the model from the viewpoint of the DC. The credit bring benefits for both supplier and distribution centers (DCs). First, the supplier who provides the dynamic discounting based credit payment policy may increase his sale volume because the DCs want to pay before the end of credit time, they need to quickly sell out products (reduce the replenishment cycle time) and order more products to meet customer demand (increase the order quantity). Second, DCs do not need to pay at the ordering time, so they could invest this amount and earn some interests. Because, they can order more than ususal in a replenishment cycle time and do not need to pay immediately. The model formulation is carried out using the CA approach.

The supplier serves a given large service area which is divided into N smaller clusters i, $(i = 1, 2, \ldots, N)$. The area of each cluster is C_i. DCs are responsible for holding, packaging, and delivering to retailer. The number and location of retailers are known and fixed. Let A_{dc_i} be the service area of each DC in the same cluster C_i. Then, the number of DCs should be opened in a cluster, C_i, is $\frac{C_i}{A_{dc_i}}$. The objective of this paper is to determine the service area of each DC, C_i, to know which retailer should be served by which DC. Assume that the DC's service area is roughly circular in shape [24]; irregularly shaped service areas could be roughly circular, hexagonal, or square-shaped, but they have little effect on the optimal solution [5]. Instead of finding the exact location of each DC, we assume that in any case, each DC is located at the center of its service area. Let F_t be the constant that depend on the shape of DC's service area, the outbound distance from a DC to retailers in the same cluster i is $F_t\sqrt{A_{dc_i}}$ [5].

The following assumptions are used in this study.

(1) The demand for each retailer in cluster i is similar, D_{r_i}. The demand function for each retailer in cluster i is a price-dependent demand model with a linear model, as shown in equation (2.1):

$$D_{r_i} = a - bp_i. \tag{2.1}$$

(2) The demand for each DC in cluster i is the total of the demand from retailers within its influence area, as shown in equation (2.2):

$$D_{dc_i} = D_{r_i}\delta_i A_{dc_i}. \tag{2.2}$$

(3) The total demand for DCs in cluster i is shown in equation (2.3):

$$D_i = D_{r_i}\delta_i C_i. \tag{2.3}$$

(4) The order quantity of the DC in cluster i is shown in equation (2.4):

$$Q_{dci} = D_{r_i}\delta_i A_{dc_i} T_i. \tag{2.4}$$

(5) The supplier offers a dynamic discounting program to DC. This study uses the average discount, as shown in equation (2.5):

$$d = \frac{\left(\frac{r_s}{1+((\mu-t_p)r_s)} + \frac{r_b}{1+((\mu-t_p)r_b)}\right)}{2}. \tag{2.5}$$

(6) The buyer is cash-rich and is more trustworthy than the supplier; therefore, $r_b < r_s$ [10].
(7) The invoice is approved after a few days, and the buyer can pay within the credit period [9].
(8) Since this study considers the time value of money, there is an exponential function e^{-rt} [16, 17].

The following notations are used in this study.

Indices

i: Number of clusters $(1, 2, \ldots, N)$
j: Number of cases $(1, 2, 3)$

Decision variables

T_i: DC's replenishment cycle in cluster i, in unit time
p_i: DC's selling price in cluster i, in dollar/unit
A_{dc_i}: DC's influence area in cluster i, in unit area

Parameters

a: Market sizeable factor
b: Price effect parameter
C_f: Fixed cost per inbound shipment, in dollar
C_i: Area of cluster i, in unit area
C_t: Delivery cost, in dollar per unit area per unit
C_v: Unit cost, in dollar per unit
C_{vt}: Variable cost per inbound shipment, in dollar per unit
d: Daily discount for DC from supplier, in percent
D_{dc_i}: Demand for each DC in cluster i, in units
D_i: Total demand for DCs in cluster i, in units
D_{r_i}: Demand for each retailer in cluster i, in units
F_{dc}: Fixed facility cost of each DC, in dollar

F_t: A constant that depends on the distance metric and shape of the DC
h_{dc}: Holding cost of DC, in dollar per unit per unit time
I_c: Interest charged, in percent
I_e: Interest earned, in percent
N_{dci}: Total number of DCs in cluster i, in units
O_C: Order cost per order for DC, in dollar per order
PTP: Present value of the total profit, in dollar
Q_{dci}: Order quantity of each DC in cluster i, in units
r: Annual interest rate, in percent
r_b: Cost of debt of the buyer, in percent per unit time
r_s: Cost of debt of the supplier, in percent per unit time
t_p: Time of payment by DC to the supplier, in unit time
δ_i: Retailer density in cluster i
μ: Credit period, in unit time

3. Model formulation

The objective of the mathematical model is to maximize the present value of the total profit derived from the following elements per replenishment cycle.

(a) *Revenue* is obtained by multiplying the total demand in the DCs in cluster i with the DCs' selling price, as shown in equation (3.1):

$$\mathrm{SR} = \frac{\sum_{i=1}^{N} D_{r_i}\delta_i C_i p_i}{T_i}\int_0^{T_i} e^{-rt}\mathrm{d}t. \tag{3.1}$$

(b) *Facility cost* is the total cost of renting and operating the DC per unit of time [24], and it is derived by multiplying the facility cost for each DC with the number of DCs in cluster i, as shown in equation (3.2):

$$\mathrm{FC} = \sum_{i=1}^{N} F_{dc}\frac{C_i}{A_{dc_i}}\cdot \tag{3.2}$$

(c) *Inbound transportation cost* is the cost of sending shipments from the supplier to the DC [24], and it is derived by multiplying the transportation cost in a single inbound shipment, $C_f + C_{Vt}Q_{dci}$, with the number of shipments, as shown in equation (3.3):

$$\mathrm{TIT} = \sum_{i=1}^{N}(C_f + C_{vt}Q_{dci})\,\frac{D_{r_i}\delta_i C_i}{Q_{dci}}\cdot \tag{3.3}$$

(d) *Outbound transportation cost* is the total cost of shipping goods to retailers located within the DC's influence areas [24], and it is derived by multiplying the delivery cost with the average distance between the DC and the retailer, $F_t\sqrt{A_{dc_i}}$, with the total demand in the DCs in cluster i, as shown in equation (3.4):

$$\mathrm{TOT} = \sum_{i=1}^{N} C_t F_t\sqrt{A_{dc_i}}D_{r_i}\delta_i C_i. \tag{3.4}$$

(e) *Ordering cost* is the total cost every time an order is placed, assuming the ordering time is T_i [24]. It is derived by multiplying the cost per order with the number of DCs and the frequency of orders ${}^1\!/_{T_i}$ as shown in equation (3.5):

$$\mathrm{OC} = \sum_{i=1}^{N}\frac{C_i O_c}{A_{dc_i}T_i}\cdot \tag{3.5}$$

(f) *Purchasing cost* is the total cost of purchasing products from the supplier, and it is derived by multiplying the unit cost C_V with the total demand in the DC in cluster i with the percentage of discount as an effect of early payment as shown in equation (3.6):

$$\mathrm{PC} = \sum_{i=1}^{N} C_v D_{r_i} \delta_i C_i e^{-rt_p} \left(1 - \left(d\left(\mu - t_p\right)\right)\right). \tag{3.6}$$

(g) *Inventory holding cost* is the total cost for storing inventories at the DC [24], and it is derived by multiplying the holding cost with the average amount of inventories as shown in equation (3.7):

$$\mathrm{HC} = \sum_{i=1}^{N} \frac{h_{dc}, T_i, D_{r_i} \delta_i C_i}{2} \int_0^{T_i} e^{-rt} \mathrm{d}t. \tag{3.7}$$

(h) *Interest charged and earned*: the DCs can generate revenues by selling products before paying the supplier, thereby earning interest. However, if the DCs have inventories, they need to pay interest charged to the supplier until all inventories are sold. There are three scenarios as follows:

(1) Case 1: $T_i \le t_p < \mu$.
Interest earned is shown in equation (3.8) and illustrated in Figure 2:

$$\mathrm{IE}_1 = \sum_{i=1}^{N} \frac{I_e p_i D_{r_i} \delta_i C_i}{T_i} \left(\int_0^{T_i} \left(T_i - t\right) e^{-rt} \mathrm{d}t + \int_{T_i}^{t_p} T_i e^{-rt} \mathrm{d}t \right). \tag{3.8}$$

(2) Case 2: $t_p < T_i < \mu$.
Interest earned and interest charged in this scenario are shown in equations (3.9) and (3.10) and further illustrated in Figure 3:

$$\mathrm{IE}_2 = \sum_{i=1}^{N} \frac{I_e p_i D_{r_i} \delta_i C_i}{T_i} \int_0^{t_p} \left(t_p - t\right) e^{-rt} \mathrm{d}t \tag{3.9}$$

$$\mathrm{IC}_1 = \sum_{i=1}^{N} \frac{I_C C_v D_{r_i} \delta_i C_i \left(1 - \left(d\left(\mu - t_p\right)\right)\right)}{T_i} \int_{t_p}^{T_i} \left(T_i - t\right) e^{-rt} \mathrm{d}t. \tag{3.10}$$

(3) Case 3: $t_p < \mu \le T_i$.
Interest charged and interest earned in this scenario are shown in equations (3.11)–(3.13) and further illustrated in Figure 4:

$$\mathrm{IE}_3 = \sum_{i=1}^{N} \frac{I_e p_i D_{r_i} \delta_i C_i}{T_i} \int_0^{t_p} \left(t_p - t\right) e^{-rt} \mathrm{d}t \tag{3.11}$$

$$\mathrm{IC}_2 = \sum_{i=1}^{N} \frac{I_C C_v D_{r_i} \delta_i C_i \left(1 - \left(d\left(\mu - t_p\right)\right)\right)}{T_i} \int_{t_p}^{\mu} \left(\mu - t\right) e^{-rt} \mathrm{d}t \qquad \text{and} \tag{3.12}$$

$$\mathrm{IC}_3 = \sum_{i=1}^{N} \frac{I_C C_v D_{r_i} \delta_i C_i}{T_i} \int_{\mu}^{T_i} \left(T_i - t\right) e^{-rt} \mathrm{d}t. \tag{3.13}$$

Therefore, the profit function, PTP, for each case is as follows:

$$\mathrm{PTP}^* \left(T_i, p_i, A_{dc_i}\right) = \begin{cases} \mathrm{PTP}_1, & \mathrm{SR} - \mathrm{FC} - \mathrm{TIT} - \mathrm{TOT} - \mathrm{OC} - \mathrm{PC} - \mathrm{HC} + \mathrm{IE}_1 \\ \mathrm{PTP}_2, & \mathrm{SR} - \mathrm{FC} - \mathrm{TIT} - \mathrm{TOT} - \mathrm{OC} - \mathrm{PC} - \mathrm{HC} - \mathrm{IC}_1 + \mathrm{IE}_2 \\ \mathrm{PTP}_3, & \mathrm{SR} - \mathrm{FC} - \mathrm{TIT} - \mathrm{TOT} - \mathrm{OC} - \mathrm{PC} - \mathrm{HC} - \mathrm{IC}_2 - \mathrm{IC}_3 + \mathrm{IE}_{3.} \end{cases}$$

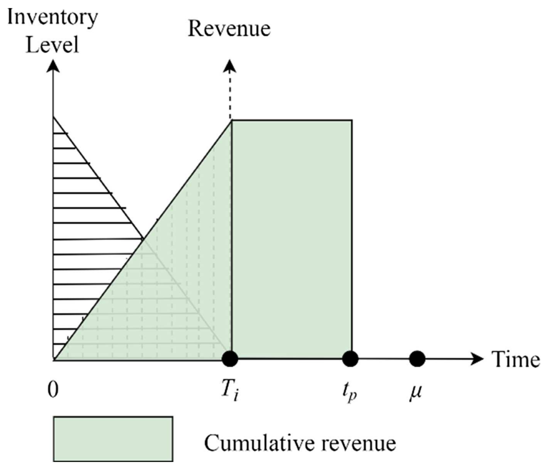

FIGURE 2. Interest earned for case 1.

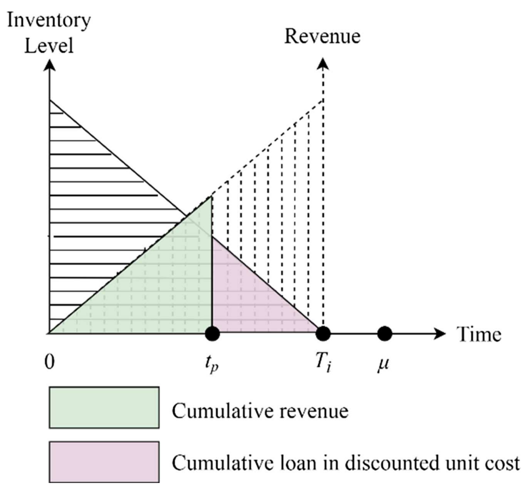

FIGURE 3. Interest charged and interest earned for case 2.

4. Theoretical results and solution approach

4.1. Theoretical results

Theorem 4.1. *For any given T_{ij}, $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ is concave in p_{ij} and $A_{dc_{ij}}$.*

To prove Theorem 4.1, the Hessian Matrix is used for each case as follows:

$$\mathbf{H_j} = \begin{bmatrix} \frac{\partial^2 \mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)}{\partial^2 A_{dc_{ij}}} & \frac{\partial^2 \mathrm{PTP}_{ij}\left(T_{ij} p_{ij} A_{dc_{ij}}\right)}{\partial A_{dc_{ij}} \partial p_{ij}} \\ \frac{\partial^2 \mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)}{\partial A_{dc_{ij}} \partial p_{ij}} & \frac{\partial^2 \mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)}{\partial^2 p_{ij}} \end{bmatrix} = \begin{bmatrix} L & K \\ K & M \end{bmatrix}. \tag{4.1}$$

Based on the sufficient condition of concavity, if $L < 0$, $M < 0$, and $LM - K^2 > 0$, then the Hessian Matrix (H_j) associated with $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ is negative definite, and $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ is concave in p_{ij} and $A_{dc_{ij}}$. First, the second-order derivative of $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ with respect to p_{ij} is carried out. Then, the

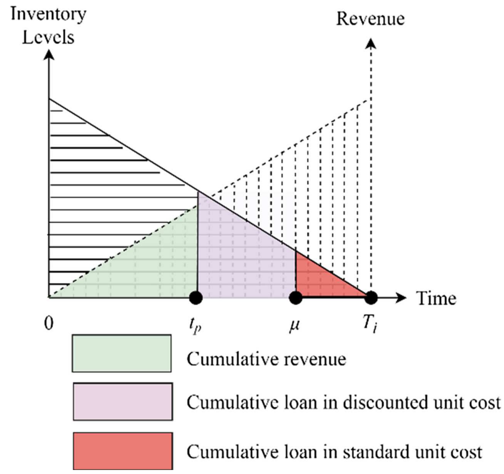

FIGURE 4. Interest charged and interest earned for case 3.

second-order derivative of $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ with respect to $A_{dc_{ij}}$ is carried out. Last, the second partial derivative of $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ with respect to p_{ij} and $A_{dc_{ij}}$ is carried out. The second derivative for each case is presented in Appendix A. According to the result of the second derivation, the sufficient condition of concavity is satisfied for each case. Therefore, *for any given* T_{ij}, $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ *is concave in* p_{ij} and $A_{dc_{ij}}$. Hence, there is an optimal solution or closed form for p_{ij} and $A_{dc_{ij}}$ that is obtained from the first-order derivative of $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ with respect to p_{ij} and $A_{dc_{ij}}$. The closed forms of p^*_{ij} and $A^*_{dc_{ij}}$ are shown in Appendix A.

Theorem 4.2. *For any given* $A_{dc_{ij}}$ *and* p_{ij}, $PTP\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ *is strictly pseudo-concave in* T_{ij}.

To prove Theorem 4.2, first, we set $\mathrm{PTP}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right) = \frac{f_j(T_{ij})}{g_j(T_{ij})}$ where $g_j\left(T_{ij}\right) = T_{ij}$. Then the second-order derivative of $f_j\left(T_{ij}\right)$ with respect to T_{ij} is carried out. The result is presented in Appendix B. According to the result of the second derivation, the sufficient condition of concavity is satisfied for each case. Therefore, *for any given* p_{ij} and $A_{dc_{ij}}$, $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ *is strictly pseudo-concave in* T_{ij}. Hence, there is an optimal solution or closed form for T_{ij} that maximizes $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$. However, due to the high level of complexity, the closed form of T^*_{ij} is not found.

4.2. Solution approach

To solve this problem, a non-linear optimal solution approach is applied. We first find the optimal closed-form of all decision variables based on the theoretical results in Section 4.1. Two closed forms of p^*_{ij} and $A^*_{dc_{ij}}$ for all the cases ($j = 1, 2, 3$) are found. However, both closed forms are dependent on each other. Therefore, we cannot directly substitute both into $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$. We first substitute $A_{dc_{ij}}\left(p_{ij}, T_{ij}\right)$ into $p_{ij}\left(A_{dc_{ij}}, T_{ij}\right)$ to get $p_{ij}\left(T_{ij}\right)$, and substitute $p_{ij}\left(T_{ij}\right)$ into $A_{dc_{ij}}\left(p_{ij}, T_{ij}\right)$ to get $A_{dc_{ij}}\left(T_{ij}\right)$. Then, we substitute $p_{ij}\left(T_{ij}\right)$ and $A_{dc_{ij}}\left(T_{ij}\right)$ into $\mathrm{PTP}_{ij}\left(T_{ij}, p_{ij}, A_{dc_{ij}}\right)$ to reduce the number of decision variables as $N * 3$ decision variables as follows:

$$\mathrm{PTP}_{ij}\left(T_{ij}\right) = \begin{cases} \mathrm{PTP}_{i1}, & \text{when } T_{i1} \leq t_p < \mu \\ \mathrm{PTP}_{i2}, & \text{when } t_p < T_{i2} < \mu \\ \mathrm{PTP}_{i3}, & \text{when } t_p < \mu \leq T_{i3}. \end{cases} \tag{4.2}$$

Due to the complicated derivative, the closed form of T^*_{ij} cannot be found from the first derivation of $\mathrm{PTP}_{ij}\left(T_{ij}\right)$ with respect to T_{ij}. Thus, a searching algorithm is developed to find the DC's optimal replenishment cycle, $\left(T^*_{ij}\right)$, as follow

Algorithm

Step 1. For all the cases ($j = 1, 2, 3$):

1.1. Let $T_{i=1,2,3\ldots,N,j=1,2,3,x=1} = C$, where C is the minimum time required to approve an invoice, and x is the number of iteration (*i.e.*, $x = 1, 2, \ldots, m$).

1.2. Solve $A_{dc_{ijx}}(p_{ijx}, T_{ijx})$ and $p_{ijx}\left(A_{dc_{ijx}}, T_{ijx}\right)$ to get $p_{ijx}(T_{ijx})$ and $A_{dc_{ijx}}(T_{ijx})$ given T_{ijx}.

1.3. Calculate $\mathrm{PTP}_{ijx}\left(T_{ijx}, p_{ijx}, A_{dc_{ijx}}\right)$.

Step 2. For $j = 1$ (Case 1):

2.1. IF $T_{i,1,x} \in T_{i,1,x} < t_p$,

2.1.1. Let $T_{i,1,x+1} = T_{i,1,x} + \varepsilon$, where ε is any positive small value.

2.1.2. IF $T_{i,1,x+1} \in T_{i,1,x+1} \le t_p$, go to step 2.1.3,
ELSE go to step 3.

2.1.3. Solve $A_{dc_{i,1,x+1}}(p_{i,1,x+1}, T_{i,1,x+1})$ and $p_{i,1,x+1}\left(A_{dc_{i,1,x+1}}, T_{i,1,x+1}\right)$ to get $p_{i,1,x+1}(T_{i,1,x+1})$ and $A_{dc_{i,1,x+1}}(T_{i,1,x+1})$ given $T_{i,1,x+1}$.

2.1.4. Calculate $\mathrm{PTP}_{i,1,x+1}\left(T_{i,1,x+1}, p_{i,1,x+1}, A_{dc_{i,1,x+1}}\right)$.

2.1.5. IF $\mathrm{PTP}_{i,1,x}\left(T_{i,1,x}, p_{i,1,x}, A_{dc_{i,1,x}}\right) < \mathrm{PTP}_{i,1,x+1}\left(T_{i,1,x+1}, p_{i,1,x+1}, A_{dc_{i,1,x+1}}\right)$,
let $T_{i,1,x} = T_{i,1,x+1}$, and go to step 2.1.1;
ELSE, let $T^*_{i,1} = T_{i,1,x}, p^*_{i,1} = p_{i,1,x}, A^*_{dc_{i,1}} = A_{dc_{i,1,x}}$.

Step 3. For $j = 2$ (Case 2):

3.1. IF $T_{i,2,x} \in T_{i,2,x} > t_p$,

3.1.1. Let $T_{i,2,x+1} = T_{i,2,x} + \varepsilon$, where ε is any positive small value.

3.1.2. IF $T_{i,2,x+1} \in T_{i,2,x+1} < \mu$, go to step 3.1.3;
ELSE, go to step 4.

3.1.3. Solve $A_{dc_{i,2,x+1}}(p_{i,2,x+1}, T_{i,2,x+1})$ and $p_{i,2,x+1}\left(A_{dc_{i,2,}}, T_{i,2,x+1}\right)$ to get $p_{i,2,x+1}(T_{i,2,x+1})$ and $A_{dc_{i,2,x+1}}(T_{i,2,x+1})$ given $T_{i,2,x+1}$.

3.1.4. Calculate $\mathrm{PTP}_{i,2,x+1}\left(T_{i,2,x+1}, p_{i,2,x+1}, A_{dc_{i,2,x+1}}\right)$.

3.1.5. IF $\mathrm{PTP}_{i,2,x}\left(T_{i,2,x}, p_{i,2,x}, A_{dc_{i,2,x}}\right) < \mathrm{PTP}_{i,2,x+1}\left(T_{i,2,x+1}, p_{i,2,x+1}, A_{dc_{i,2,x+1}}\right)$,
let $T_{i,2,x} = T_{i,2,x+1}$, and go to step 3.1.1;
ELSE, let $T^*_{i,2} = T_{i,2,x}, p^*_{i,2} = p_{i,2,x}, A^*_{dc_{i,2}} = A_{dc_{i,2,x}}$.

Step 4. For $j = 3$ (Case 3):

4.1. IF $T_{i,3,x} \in T_{i,3,x} \ge \mu$,

4.1.1. Let $T_{i,3,x+1} = T_{i,3,x} + \varepsilon$, where ε is any positive small value.

4.1.2. Solve $A_{dc_{i,3,x+1}}(p_{i,3,x+1}, T_{i,3,x+1})$ and $p_{i,3,x+1}\left(A_{dc_{i,3,}}, T_{i,3,x+1}\right)$ to get $p_{i,3,x+1}(T_{i,3,x+1})$ and $A_{dc_{i,3,x+1}}(T_{i,3,x+1})$ given $T_{i,3,x+1}$.

4.1.3. Calculate $\mathrm{PTP}_{i,3,x+1}\left(T_{i,3,x+1}, p_{i,3,x+1}, A_{dc_{i,3,x+1}}\right)$.

4.1.4. IF $\mathrm{PTP}_{i,3,x}\left(T_{i,3,x}, p_{i,3,x}, A_{dc_{i,3,x}}\right) < \mathrm{PTP}_{i,3,x+1}\left(T_{i,3,x+1}, p_{i,3,x+1}, A_{dc_{i,3,x+1}}\right)$,
let $T_{i,3,x} = T_{i,3,x+1}$, and go to step 4.1.1;
ELSE, $T^*_{i,3} = T_{i,3,x}, p^*_{i,3} = p_{i,3,x}, A^*_{dc_{i,3}} = A_{dc_{i,3,x}}$.

Step 5. Determine the global optimum solution as follows:

$$\mathrm{PTP}^*\left(T^*_i, p^*_i, A^*_{dc_i}\right) = \mathrm{Max}\begin{pmatrix} \mathrm{PTP}^*_{i,1}\left(T^*_{i,1}, p^*_{i,1}, A^*_{dc_{i,1}}\right), \mathrm{PTP}^*_{i,2}\left(T^*_{i,2},, p^*_{i,2}, A^*_{dc_{i,2}}\right), \\ \mathrm{PTP}^*_{i,3}\left(T^*_{i,3}, p^*_{i,3}, A^*_{dc_{i,3}}\right) \end{pmatrix}.$$

5. Numerical analysis

5.1. Numerical example

This section is to show if the proposal model and solution approach could be applicable in the real data case. All used parameters are referred from company data and Indonesia market according to model's required

TABLE 1. Optimum solution with dynamic discounting.

	Cluster 1	Cluster 2	Cluster 3
T_i^* (year)	0.042041	0.03992	0.03928
p_i^* (\$)	117.471	117.227	117.156
$A_{dc_i}^*$ (km^2)	4265.83	3512.16	3306.66
PTP (\$)		2 766 136	

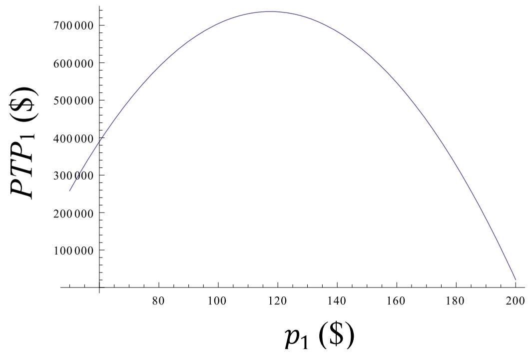

FIGURE 5. Graphic of optimal DC's selling price in cluster 1.

parameters. The Indonesian batik business is Batik Butimo, located in Yogyakarta, Indonesia is considered. Batik Butimo produces batik *tulis* with natural colors and sells it to several retailers. Each retailer's demand is represented by the function $D_{r_i} = 405 - 2p_i$. Batik Butimo plans to expand its business by building DCs in Central Java. Based on historical data, there are three clusters of demand in Central Java, represented as follows: $C_1 = 11\,200\,\mathrm{km}^2$; $C_2 = 9220\,\mathrm{km}^2$; and $C_3 = 12\,380\,\mathrm{km}^2$. Batik Butimo buys its materials from a single supplier, which uses a dynamic discounting program. Payment can be made on the same day the invoice is approved until the end of the credit period. Generally, it takes five days or 0.014 year to approve an invoice. In this example, Batik Butimo agrees to pay the supplier on the same day the invoice is approved, that is, at $t_p = 0.014$ year. Suppose that the following parameters are used: $C_f = \$5$; $C_t = \$8$; $C_v = \$26.32$/unit; $C_{vt} = \$1$/unit; $F_{dc} = \$8194$; $F_t = \$0.01$; $h_{dc} = \$0.8$; $I_c = 0.05$; $I_e = 0.04$; $O_c = \$25$/order; $r = 0.07$; $r_b = 0.02$; $r_s = 0.1$; $\mu = 0.1$ year; $\delta_1 = 0.0047$; $\delta_2 = 0.0063$; and $\delta_3 = 0.0069$. First, a case with dynamic discounting is considered. The optimum solution for this problem is Case 2, as shown in Table 1. The optimum solution for each decision variable is illustrated in Figures 5–13.

Second, Table 2 presents the results of the case without dynamic discounting. Based on the results of Table 2, we can see that the replenishment cycle time for three clusters in the case with dynamic accounting is shorter that in the case without dynamic accounting. The reason is that the DCs want to reduce the cycle time to get the benefits from early payment when dynamic discounting scheme is applied. The optimal price does not change so much when applying dynamic discounting. That means the dynamic discounting scheme affects the price slightly. However, the service areas of DCs become smaller in the case of a non-dynamic discounting application. The reason is that the DCs tend to reduce the service area when they cannot get the benefits from dynamic discounting. Finally, according to the results in Tables 1 and 2, dynamic discounting can help to increase the total profit by 0.15%. Thus, dynamic discounting could be considered in the supply chain network to increase profit.

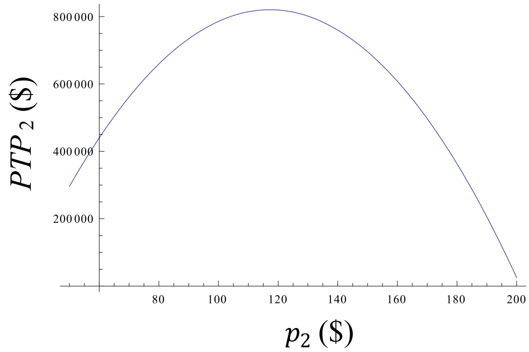

Figure 6. Graphic of optimal DC's selling price in cluster 2.

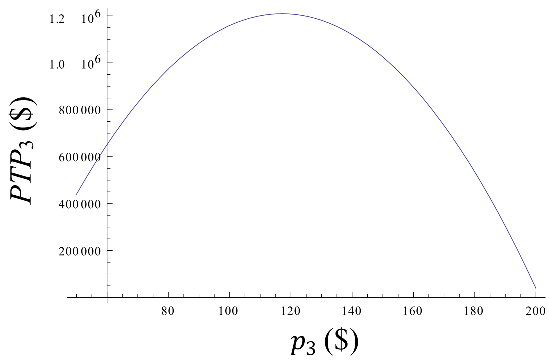

Figure 7. Graphic of optimal DC's selling price in cluster 3.

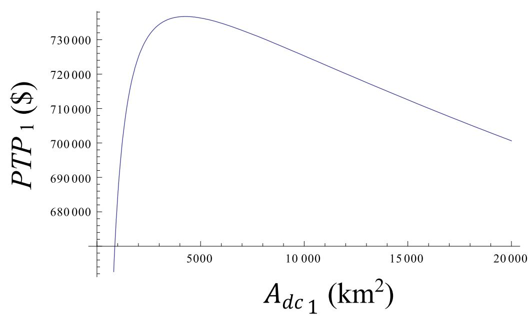

Figure 8. Graphic of optimal DC's influence area in cluster 1.

5.2. Sensitivity analysis

A sensitivity analysis is carried out to analyze the effect of changing the cost parameter from –40%, –20%, +20%, and +40% to the present value of the total profit, as well as the optimal decision variables. These calculations are conducted to help the decision makers make the decisions when the market condition changes. The results of the sensitivity analysis are presented in Table 3.

The sensitivity analysis reveals the following.

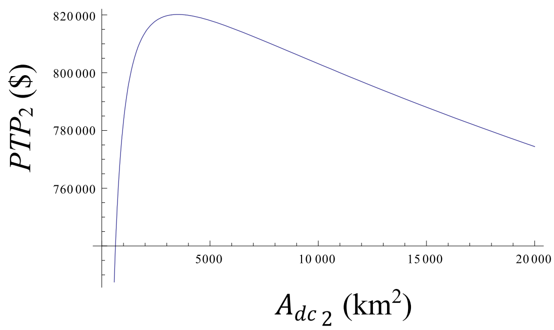

FIGURE 9. Graphic of optimal DC's influence area in cluster 2.

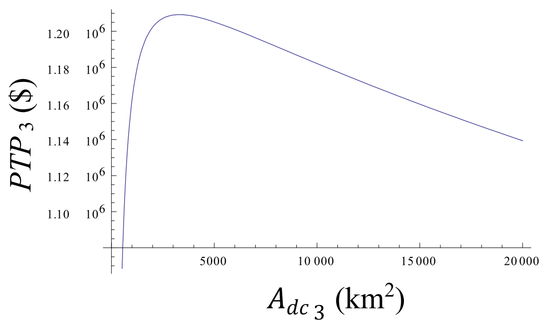

FIGURE 10. Graphic of optimal DC's influence area in cluster 3.

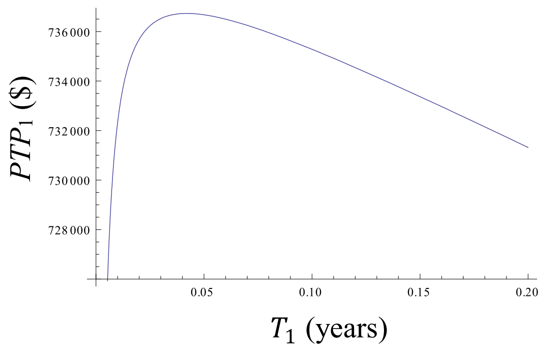

FIGURE 11. Graphic of optimal DC's replenishment cycle in cluster 1.

(1) As the price effect parameter, b, increases, the DC's replenishment cycle and influence area increase, but the selling price and present value of the total profit decrease. This is understandable because a decrease in the present value of the total profit implies an increase in the present value of the total cost. Therefore, the DC's influence area and replenishment cycle are increased to reduce the number of DCs that operate and reduce the frequency of orders and the volume of inventories.
(2) As the fixed cost per inbound shipment, C_f, increases, the selling price and the DC's replenishment cycle and influence area increase, but the present value of the total profit decreases. The selling price is increased to maintain the present value of the total profit, and the demand decreases as an effect of increasing the

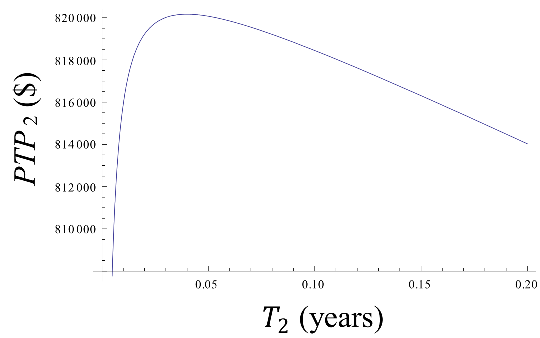

FIGURE 12. Graphic of optimal DC's replenishment cycle in cluster 2.

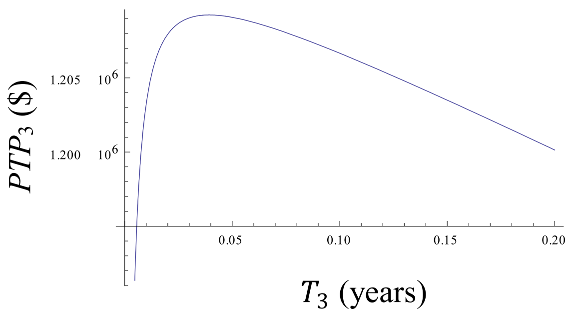

FIGURE 13. Graphic of optimal DC's replenishment cycle in cluster 3.

TABLE 2. Optimum solution for case without dynamic discounting.

	Cluster 1	Cluster 2	Cluster 3
T_i^* (year)	0.04912	0.0401	0.0411
p_i^* ($)	117.46	117.217	117.147
$A_{dc_i}^*$ (km^2)	4132.38	3392.7	3191.31
PTP ($)		2 761 914	

selling price. A decrease in demand means that there are not many customers to serve. Therefore the DC's influence area is increased to reduce the number of DCs that operate. Besides, as the inbound shipment cost increases, the DC's replenishment cycle is increased to reduce the frequency of product deliveries from the supplier.

(3) As the delivery cost, C_t, increases, the DC's selling price and replenishment cycle increase as well, but the DC's influence area and the present value of the total profit decrease. The selling price is increased to maintain the present value of the total profit and the replenishment cycle is increased to reduce the number of products for delivery. Moreover, the DC's influence area is decreased to increase the number of DCs that operate and to reduce the distance covered for delivery.

(4) As the unit cost, C_v, increases, the DC's influence area and selling price increase, but the DC's replenishment cycle and the present value of the total profit decrease. The selling price is increased to maintain the present value of the total profit, and the demand decreases as a result of increasing the selling price. A decrease in demand means that there are not many customers to serve. Therefore, the DC's influence area is increased

TABLE 3. Sensitivity analysis.

	Decision variables			PTP_i	Percentage of deviation to the average PTP				
	T_i	p_i	A_{dc_i}		−40%	−20%	0	+20%	+40%
b	+	−	+	−	72.145%	19.461%	−11.887%	−32.568%	−47.152%
C_f	+	+	+	−	0.0166%	0.0082%	−0.0001%	−0.0079%	−0.0157%
C_t	+	+	−	−	2.442%	1.121%	−0.086%	−1.209%	−2.268%
C_v	−	+	+	−	12.828%	6.212%	−0.196%	−6.408%	−12.436%
C_{vt}	−	+	+	−	0.287%	0.045%	−0.196%	−0.437%	−0.678%
F_{dc}	−	+	+	−	1.169%	0.500%	−0.071%	−0.573%	−1.024%
h_{dc}	−	No changes	+	−	0.0005%	0.0000%	−0.0001%	−0.0002%	−0.0003%
I_c	−	+	+	−	0.006%	0.003%	0.000%	−0.003%	−0.005%
I_e	−	−	+	+	−0.006%	−0.003%	0.000%	0.003%	0.006%
O_c	+	+	+	−	0.086%	0.039%	−0.004%	−0.042%	−0.078%
r	−	+	+	−	0.072%	0.032%	−0.004%	−0.036%	−0.065%
r_b	+	−	−	+	−0.011%	−0.005%	0.000%	0.006%	0.011%
r_s	+	−	−	+	−0.054%	−0.027%	0.000%	0.027%	0.054%

to reduce the number of DCs that operate. Moreover, as the unit cost increases, the ordered quantities also decrease; therefore, the DC's replenishment cycle is decreased.

(5) As the variable cost per inbound shipment, C_{vt}, increases, the DC's influence area and selling price increase, but the DC's replenishment cycle and the present value of the total profit decrease. The selling price is increased to maintain the present value of the total profit, and the demand decreases as a result of increasing the selling price. A decrease in demand means that there are not many customers to serve. Therefore, the DC's influence area is increased to reduce the number of DCs that operate. Moreover, as the variable cost per unit of shipment increases, the ordered quantities decrease; therefore, the DC's replenishment cycle is decreased.

(6) As the facility cost, F_{dc}, increases, the DC's selling price and influence area increase, but the DC's replenishment cycle and the present value of the total profit decrease. The selling price is increased to maintain the present value of the total profit, and the DC's influence area is increased to reduce the number of DCs that operate.

(7) As the holding cost, h_{dc}, increases, the present value of the total profit and the DC's replenishment cycle decrease. However, the selling price is slightly increased to maintain the present value of the total profit, and the DC's influence area is increased to reduce the number of DCs that operate. Meanwhile, as the holding cost increases, the DC's replenishment cycle is decreased to reduce the ordered quantities and thereby reduce the inventory cost.

(8) As the interest charged, I_c, increases, the selling price and the DC's influence area increase, but the present value of the total profit and the DC's replenishment cycle decrease. The selling price is increased to maintain the present value of the total profit, and the demand decreases as a result of increasing the selling price. A decrease in demand means that there are not many customers to serve. Therefore, the DC's influence area is increased to reduce the number of DCs that operate. Furthermore, when the interest charged increases, it is reasonable to decrease the DC's replenishment cycle to reduce the number of inventories so that the interest charged can be reduced.

(9) As the interest earned, I_e, increases, the selling price and the DC's replenishment cycle decrease, but the DC's influence area and the present value of the total profit increase. The selling price is decreased to stimulate and increase the demand when the interest earned is high, thereby increasing the present value of the total profit.

(10) As the ordering cost, O_c, increases, the DC's selling price, influence area, and replenishment cycle increase, but the present value of the total profit decreases. The selling price is increased to maintain the present value of the total profit, and the demand decreases as the selling price increases. A decrease in demand means that there are not many customers to serve. Therefore, the DC's influence area is increased to reduce the number of DCs that operate. Meanwhile, when the ordering cost increases, it is reasonable to increase the DC's replenishment cycle to reduce the replenishment frequency so that the ordering cost can be reduced.

(11) As the annual interest rate, r, increases, the DC's selling price and influence area increase, but the DC's replenishment cycle and the present value of the total profit decrease. This is reasonable because the higher the annual interest rate, the smaller the present value of money. Moreover, when the annual interest rate increases, the selling price is also increased to maintain the profit, and the demand decreases as a result of increasing the selling price. A decrease in demand and the present value of the total profit means that there are not so many customers to serve and that the total cost has increased. Therefore, the DC's influence area is increased to reduce the number of DCs that operate and to reduce the number of ordered quantities and the volume of inventories.

As the cost of debt of the buyer, r_b, or the supplier, r_s, increases, the DC's influence area and selling price decrease, but the DC's replenishment cycle and the present value of the total profit increase. Here, it is reasonable to decrease the selling price because the buyer's cost of debt in a dynamic discounting scheme is used as a basis to determine the amount of discount. Therefore, the total cost that should be covered by revenues decreases because of the higher discount. Moreover, the demand increases as the selling price decreases, and when the demand increases, the DC's influence area is decreased to increase the number of DCs that operate. Moreover, the DC's replenishment cycle is increased to increase the ordered quantities so that the purchasing cost can be reduced while optimizing the discount.

Aside from the results discussed above, we have several key findings as follows.

(1) The cost elements that have the most significant effect on the present value of the total profit are the price effect parameter and the unit cost. Therefore, under a dynamic discounting scheme, the manager should pay more attention to both of these cost elements before agreeing to pay at a certain time and to use dynamic discounting as the payment scheme so as to maximize the total profit.

(2) The cost element that has the least significant effect on the present value of the total profit is the holding cost. Thus, under a dynamic discounting scheme, the manager can prioritize other costs in the decision-making process.

6. Conclusions

This study presented a supply chain network design that consists of a single supplier, multiple DCs, and multiple retailers. It also considered the time value of money by conducting a DCF analysis. Specifically, the analysis involved a supplier that provides a dynamic discounting program for a DC. A mathematical model was built under three cases with three decision variables, namely, the DC's replenishment cycle, selling price, and influence area so that the present value of the total profit is maximized. To solve the problem, an algorithm based on non-linear optimization is developed. A numerical example was provided to illustrate the proposed model and solution approaches. The numerical example illustrated that the proposed model can be applied to a real case problem to obtain the optimal solution. A sensitivity analysis was also conducted to demonstrate the effect of changing the cost parameter in relation to the total profit, as well as the optimal decision variables. The sensitivity analysis revealed that the price effect parameter and the unit cost have the most significant effect on the present value of the total profit. Meanwhile, the holding cost has the least significant effect on the present value of the total profit. Future studies can use a network that consists of multiple suppliers, where all suppliers provide dynamic discounting and a trade credit payment is provided to the downstream of the network (*i.e.*, from the DC to the retailers). Furthermore, future studies can consider a perishable product and a shortage of products because both scenarios are common in real life.

Appendix A.

Case 1. $T_{ij} \le t_p < \mu$ where $j = 1$.

$$\frac{\partial^2 \mathrm{PTP}_{i1}\left(T_{i1}, p_{i1}, A_{dc_{i1}}\right)}{\partial^2 p_{i1}} = -\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i \mathrm{Log}\left[e^{-r}\right]} - \frac{2bI_e\left(\frac{T_i\left(-\left(e^{-r}\right)^{T_i}+\left(e^{-r}\right)^{t_p}\right)}{\mathrm{Log}\left[e^{-r}\right]}+\frac{-1+\left(e^{-r}\right)^{T_i}-T_i\mathrm{Log}\left[e^{-r}\right]}{\mathrm{Log}\left[e^{-r}\right]^2}\right)C_i\delta_i}{T_i} \tag{A.1}$$

$$\frac{\partial^2 \mathrm{PTP}_{i1}\left(T_{i1}, p_{i1}, A_{dc_{i1}}\right)}{\partial^2 A_{dc_{i1}}} = -\frac{2F_{dc}C_i}{A_{dc_i}^3} - \frac{2O_C C_i}{T_i A_{dc_i}^3} + \frac{2C_{vt}C_i\left(a-bp_i\right)\delta_i}{A_{dc_i}^2} + \frac{C_t F_t C_i\left(a-bp_i\right)\delta_i}{4A_{dc_i}^{3/2}} - \frac{2C_i\left(C_f + C_{vt}T_i A_{dc_i}\left(a-bp_i\right)\delta_i\right)}{T_i A_{dc_i}^3} \tag{A.2}$$

$$\frac{\partial^2 \mathrm{PTP}_{i1}\left(T_{i1}, p_{i1}, A_{dc_{i1}}\right)}{\partial A_{dc_{i1}}\partial p_{i1}} = \frac{bC_t F_t C_i \delta_i}{2\sqrt{A_{dc_i}}} \tag{A.3}$$

$$A_{dc_{i1}}^* = \frac{\left(4F_{dc}^2 + \frac{4C_f^2}{T_i^2} + \frac{8C_f O_C}{T_i^2} + \frac{4O_C^2}{T_i^2} + \frac{8C_f F_{dc}}{T_i} + \frac{8F_{dc}O_C}{T_i}\right)^{1/3}}{C_t^{2/3}F_t^{2/3}\left(a^2 - 2abp_i + b^2p_i^2\right)^{1/3}\delta_i^{2/3}} \tag{A.4}$$

$$p_{i1}^* = \frac{-bC_{vt}C_i\delta_i - bC_v\left(\mathrm{e}^{-r}\right)^{t_p}\left(1-\frac{1}{2}\left(-t_p+\mu\right)\left(\frac{r_s}{1+((\mu-t_p)r_s)}+\frac{r_b}{1+((\mu-t_p)r_b)}\right)\right)C_i\delta_i + A}{-\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]} - \frac{2bI_e\left(\frac{T_i\left(-\left(e^{-r}\right)^{T_i}+\left(e^{-r}\right)^{t_p}\right)}{\mathrm{Log}\left[e^{-r}\right]}+\frac{-1+\left(e^{-r}\right)^{T_i}-T_i\mathrm{Log}\left[e^{-r}\right]}{\mathrm{Log}\left[e^{-r}\right]^2}\right)C_i\delta_i}{T_i}} \tag{A.5}$$

$$A = -\frac{a\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]} - \frac{b\left(-1+\left(e^{-r}\right)^{T_i}\right)h_{dc}T_iC_i\delta_i}{2\mathrm{Log}\left[e^{-r}\right]} - \frac{aI_e\left(\frac{T_i\left(\left(e^{-r}\right)^{T_{dc}}+\left(e^{-r}\right)^{t_p}\right)}{\mathrm{Log}\left[e^{-r}\right]}+\left(\frac{-1+\left(e^{-r}\right)^{T_i}-T_i\mathrm{Log}\left[e^{-r}\right]}{\mathrm{Log}\left[e^{-r}\right]^2}\right)\right)C_i\delta_i}{T_i} - bC_tF_t\sqrt{A_{dc_i}}C_i\delta_i. \tag{A.6}$$

Case 2. $t_p < T_{ij} < \mu$ where $j = 2$.

$$\frac{\partial^2 \mathrm{PTP}_{i2}\left(T_{i2}, p_{i2}, A_{dc_{i2}}\right)}{\partial^2 p_{i2}} = -\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]} - \frac{2bI_e\left(-1+\left(e^{-r}\right)^{t_p}-t_p\mathrm{Log}\left[e^{-r}\right]\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]^2} \tag{A.7}$$

$$\frac{\partial^2 \mathrm{PTP}_{i2}\left(T_{i2}, p_{i2}, A_{dc_{i2}}\right)}{\partial^2 A_{dc_{i2}}} = -\frac{2F_{dc}C_i}{A_{dc_i}^3} - \frac{2O_C C_i}{T_i A_{dc_i}^3} + \frac{2C_{vt}C_i\left(a-bp_i\right)\delta_i}{A_{dc_i}^2} + \frac{C_t F_t C_i\left(a-bp_i\right)\delta_i}{4A_{dc_i}^{3/2}} - \frac{2C_i\left(C_f + C_{vt}T_i A_{dc_i}\left(a-bp_i\right)\delta_i\right)}{T_i A_{dc_i}^3} \tag{A.8}$$

$$\frac{\partial^2 \mathrm{PTP}_{i2}\left(T_{i2}, p_{i2}, A_{dc_{i2}}\right)}{\partial A_{dc_{i2}}\partial p_{i2}} = \frac{bC_t F_t C_i \delta_i}{2\sqrt{A_{dc_i}}} \tag{A.9}$$

$$A_{dc_{i2}}^* = \frac{\left(4F_{dc}^2 + \frac{4C_f^2}{T_i^2} + \frac{8C_f O_C}{T_i^2} + \frac{4O_C^2}{T_i^2} + \frac{8C_f F_{dc}}{T_i} + \frac{8F_{dc}O_C}{T_i}\right)^{1/3}}{C_t^{2/3}F_t^{2/3}\left(a^2 - 2abp_i + b^2p_i^2\right)^{1/3}\delta_i^{2/3}} \tag{A.10}$$

$$p_{i2}^{*}=\frac{-bC_{vt}C_i\delta_i-bC_v\left(e^{-r}\right)^{t_p}\left(1-\frac{1}{2}\left(-t_p+\mu\right)\left(\frac{r_s}{1+((\mu-t_p)r_s)}+\frac{r_b}{1+((\mu-t_p)r_b)}\right)\right)C_i\delta_i+B}{-\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}[e^{-r}]}-\frac{2bI_e\left(-1+\left(e^{-r}\right)^{t_p}-t_p\mathrm{Log}[e^{-r}]\right)C_i\delta_i}{T_i\mathrm{Log}[e^{-r}]^2}} \tag{A.11}$$

$$\begin{aligned}B=&-\frac{a\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]}-\frac{b\left(-1+\left(e^{-r}\right)^{T_i}\right)h_{dc}T_iC_i\delta_i}{2\mathrm{Log}\left[e^{-r}\right]}\\&-\frac{aI_e\left(-rt_p+\frac{r^2t_p^2}{2}-t_p\mathrm{Log}\left[e^{-r}\right]\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]^2}-\frac{1}{T_i\mathrm{Log}\left[e^{-r}\right]^2}bC_vI_c\\&\times\left(1-\frac{1}{2}\left(-t_p+\mu\right)\left(\frac{r_b}{1+r_b\left(-t_p+\mu\right)}+\frac{r_s}{1+r_s\left(-t_p+\mu\right)}\right)\right)\\&\times\left(\left(e^{-r}\right)^{T_i}+\left(e^{-r}\right)^{t_p}\left(-1+\left(-T_i+t_p\right)\mathrm{Log}\left[e^{-r}\right]\right)\right)C_i\delta_i-bC_tF_t\sqrt{A_{dc_i}}C_i\delta_i.\end{aligned} \tag{A.12}$$

Case 3. $t_p<\mu\leq T_{ij}$ where $j=3$.

$$\frac{\partial^2\mathrm{PTP}_{i3}\left(T_{i3},p_{i3},A_{dc_{i3}}\right)}{\partial^2p_{i3}}=-\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]}-\frac{2bI_e\left(-1+\left(e^{-r}\right)^{t_p}-t_p\mathrm{Log}\left[e^{-r}\right]\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]^2} \tag{A.13}$$

$$\begin{aligned}\frac{\partial^2\mathrm{PTP}_{i3}\left(T_{i3},p_{i3},A_{dc_{i3}}\right)}{\partial^2A_{dc_{i3}}}=&-\frac{2F_{dc}C_i}{A_{dc_i}^3}-\frac{2O_CC_i}{T_iA_{dc_i}^3}+\frac{2C_{vt}C_i\left(a-bp_i\right)\delta_i}{A_{dc_i}^2}+\frac{C_tF_tC_i\left(a-bp_i\right)\delta_i}{4A_{dc_i}^{3/2}}\\&-\frac{2C_i\left(C_f+C_{vt}T_iA_{dc_i}\left(a-bp_i\right)\delta_i\right)}{T_iA_{dc_i}^3}\end{aligned} \tag{A.14}$$

$$\frac{\partial^2\mathrm{PTP}_{i3}\left(T_{i3},p_{i3},A_{dc_{i3}}\right)}{\partial A_{dc_{i3}}\partial p_{i3}}=\frac{bC_tF_tC_i\delta_i}{2\sqrt{A_{dc_i}}} \tag{A.15}$$

$$A_{dc_{i3}}^{*}=\frac{\left(4F_{dc}^2+\frac{4C_f^2}{T_i^2}+\frac{8C_fO_C}{T_i^2}+\frac{4O_C^2}{T_i^2}+\frac{8C_fF_{dc}}{T_i}+\frac{8F_{dc}O_C}{T_i}\right)^{1/3}}{C_t^{2/3}F_t^{2/3}\left(a^2-2abp_i+b^2p_i^2\right)^{1/3}\delta_i^{2/3}} \tag{A.16}$$

$$p_{i3}^{*}=\frac{-bC_{vt}C_i\delta_i-bC_v\left(e^{-r}\right)^{t_p}\left(1-\frac{1}{2}\left(-t_p+\mu\right)\left(\frac{r_s}{1+((\mu-t_p)r_s)}+\frac{r_b}{1+((\mu-t_p)r_b)}\right)\right)C_i\delta_i+C}{-\frac{2b\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}[e^{-r}]}-\frac{2bI_e\left(-1+\left(e^{-r}\right)^{t_p}-t_p\mathrm{Log}[e^{-r}]\right)C_i\delta_i}{T_i\mathrm{Log}[e^{-r}]^2}} \tag{A.17}$$

$$\begin{aligned}C=&-\frac{a\left(-1+\left(e^{-r}\right)^{T_i}\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]}-\frac{b\left(-1+\left(e^{-r}\right)^{T_i}\right)h_{dc}T_iC_i\delta_i}{2\mathrm{Log}\left[e^{-r}\right]}\\&-\frac{aI_e\left(-rt_p+\frac{r^2t_p^2}{2}-t_p\mathrm{Log}\left[e^{-r}\right]\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]^2}-\frac{1}{T_i\mathrm{Log}\left[e^{-r}\right]^2}bC_vI_c\\&\times\left(1-\frac{1}{2}\left(-t_p+\mu\right)\left(\frac{r_b}{1+r_b\left(-t_p+\mu\right)}+\frac{r_s}{1+r_s\left(-t_p+\mu\right)}\right)\right)\\&\times\left(\left(e^{-r}\right)^{T_i}+\left(e^{-r}\right)^{t_p}\left(-1+\left(-T_i+t_p\right)\mathrm{Log}\left[e^{-r}\right]\right)\right)C_i\delta_i\\&-\frac{bC_vI_c\left(\left(e^{-r}\right)^{T_i}+\left(e^{-r}\right)^{\mu}\left(-1+\left(-T_i+\mu\right)\mathrm{Log}\left[e^{-r}\right]\right)\right)C_i\delta_i}{T_i\mathrm{Log}\left[e^{-r}\right]^2}\end{aligned}$$

$$- bC_tF_t\sqrt{A_{dc_i}}C_i\delta_i. \tag{A.18}$$

Appendix B.

Case 1. $T_{ij} \le t_p < \mu$ where $j = 1$.

$$\mathrm{PTP}_{i1}\left(T_{i1}, p_{i1}, A_{dc_{i1}}\right) = \frac{f_1\left(T_{i1}\right)}{g_1\left(T_{i1}\right)} \tag{B.1}$$

$$f_1\left(T_{i1}\right) = \mathrm{PTP}_{i1}\left(T_{i1}, p_{i1}, A_{dc_{i1}}\right)T_{i1} \tag{B.2}$$

$$g_1\left(T_{i1}\right) = T_{i1} > 0 \tag{B.3}$$

$$\begin{aligned}\frac{\partial^2 f_1\left(T_{i1}\right)}{\partial^2 T_{i1}} = \frac{C_i\left(a - bp_i\right)}{-2r}&\Big(2h_{dc} - \left(e^{-r}\right)^{T_i}\left(h_{dc}\left(2 - rT_i\left(4 - rT_i\right)\right)\right.\\ &\left. - 2r\left(I_e + \left(-1 + I_eT_i\right)(-r)\right)p_i\right)\Big)\delta_i < 0.\end{aligned} \tag{B.4}$$

Case 2. $t_p < T_{ij} < \mu$ where $j = 2$.

$$\mathrm{PTP}_{i2}\left(T_{i2}, p_{i2}, A_{dc_{i2}}\right) = \frac{f_2\left(T_{i2}\right)}{g_2\left(T_{i2}\right)} \tag{B.5}$$

$$f_2\left(T_{i2}\right) = \mathrm{PTP}_{i2}\left(T_{i2}, p_{i2}, A_{dc_{i2}}\right)T_{i2} \tag{B.6}$$

$$g_2\left(T_{i2}\right) = T_{i2} > 0 \tag{B.7}$$

$$\begin{aligned}\frac{\partial^2 f_2\left(T_{i2}\right)}{\partial^2 T_{i2}} = \frac{C_i\left(a - bp_i\right)}{2}&\left(\frac{C_v\left(e^{-r}\right)^{T_i}I_c\left(-2 + \left(r_b + r_s\right)\left(t_p - \mu\right)\right)}{\left(-1 + r_b\left(t_p - \mu\right)\right)\left(-1 + r_s\left(t_p - \mu\right)\right)}\right.\\ &\left. - \frac{h_{dc}\left(2 - \left(e^{-r}\right)^{T_i}\left(2 - rT_i\left(4 - rT_i\right)\right)\right)}{r} - 2r\left(e^{-r}\right)^{T_i}p_i\right)\delta_i < 0.\end{aligned} \tag{B.8}$$

Case 3. $t_p < \mu \le T_{ij}$ where $j = 3$.

$$\mathrm{PTP}_{i3}\left(T_{i3}, p_{i3}, A_{dc_{i3}}\right) = \frac{f_3\left(T_{i3}\right)}{g_3\left(T_{i3}\right)} \tag{B.9}$$

$$f_3\left(T_{i3}\right) = \mathrm{PTP}_{i3}\left(T_{i3}, p_{i3}, A_{dc_{i3}}\right)T_{i3} \tag{B.10}$$

$$g_3\left(T_{i3}\right) = T_{i3} > 0 \tag{B.11}$$

$$\frac{\partial^2 f_3\left(T_{i3}\right)}{\partial^2 T_{i3}} = \frac{C_i\left(a - bp_i\right)}{-2r}\left(2h_{dc} - \left(e^{-r}\right)^{T_i}\left(2h_{dc} - r\left(-2C_vI_c + 4h_{dc}T_i - h_{dc}T_i^2r\right) - 2r^2p_i\right)\right)\delta_i < 0. \tag{B.12}$$

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