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BI-LEVEL PRICING AND INVENTORY STRATEGIES FOR PERISHABLE PRODUCTS IN A COMPETITIVE SUPPLY CHAIN

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Abstract. This paper aims to develop a new bi-level game model for joint pricing and inventory decisions in a competitive supply chain consisting of a dominant manufacturer, who produces single perishable product from deteriorating raw materials, and two follower retailers who face nonlinear price-dependent demand and operate under Cournot assumptions. Three levels of warehousing including raw material warehouse, final product warehouse, and retail warehouses with exponential deterioration rates are considered to explore the joint impact of deterioration rate and price elasticity on the equilibrium inventory decisions. A Stackelberg–Nash–Cournot model is developed to seek the equilibrium prices, quantities, and replenishment cycles and is solved through an exact methodology. A numerical example is presented to validate the proposed model and comprehensive sensitivity analyses are carried out to measure the impact of the model's key parameters including the deterioration rate in the producer's and the retailers' warehouses, the retail and competitor price elasticity, and the market scale on the equilibrium.

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1. Introduction

In recent years, supply chain management has been highly considered by researchers. The globalization of trade, the increasing trend of competition, the approaching of product quality to the same level are among the issues persuading the industries and institutes to conduct more studies on more efficient and effective operations through supply chain management. The supply chain management and developed inventory control models aim to understand the operational efficiency and minimize the expected costs of the chain. However, it is possible to control the demand process through the prevailing price. The inventory turnover/production strategy can control the supply amount of each device while pricing policies control the demand amount of a system. The processes of managing and specifying price and inventory complement each other and can reduce the risk of supply-demand mismatch and increase profitability [47].

Keywords. Supply chain coordination, perishable products, inventory decisions, pricing, bi-level programming, Stackelberg–Nash–Cournot equilibrium.

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Furthermore, an increase in the competition in the market would highlight the importance of the customer order management. The order management seeks to manage and control orders in the whole life cycle of the order *i.e.* from customer's request to order delivery. In fact, order management is responsible for creating a primary commitment to the corresponding order. Along with sales, order management defines specific and justifies allocation policies of supplying products to the customers. In this regard, any changes in the supply, capacity, and demand require rescheduling the orders to achieve a new justified commitment.

Today, most producers have concluded that controlling the inventories together with pricing decisions can lead to higher profits [22]. In fact, the production or ordering strategies along with the subsequent pricing policies are the main factors for profitability at different levels of a supply chain. So far, pricing models in the revenue management field have been generally developed based on seasonal products or products with a limited life cycle. The product life cycle and perishability issues makes the inventory and pricing decisions for the products much harder as the prices can be changed during the lifetime in order to manage the demand and to increase the profits [17].

Deterioration means the loss of marginal value of a commodity over time so that its usefulness decreases. Examples of perishable products include medicine, high-tech products, fashion products, food, and agricultural products. Deterioration affects the inventory system in a supply chain so that the manufacturer has to produce more than market demand because some products will be deteriorated and the retailer has to consider the deterioration rate in his inventory level and ordering policies [23]. On the other hand, since pricing can be used as a great tool to control the level of demand, pricing of perishable to match supply and demand is important. Although, there are some studies that simultaneously examine the problem of inventory management and pricing for perishable products [22, 23], to the best knowledge of the authors, such studies have not been presented in competitive market environments. Competition among retailers who sell homogeneous products can be very influential in pricing problem. This study investigates the joint problem of pricing and inventory management for perishable products and deteriorating raw materials in a decentralized supply chain with a dominant producer and two competing retailers.

The main contribution of this research is the analysis of the simultaneous impact of competition among supply chain members and product deterioration on the issue of pricing and inventory decisions in a decentralized supply chain. The focus is on the correlation between perishability of inventories and pricing decisions in a competitive market. For this purpose, a new Stackelberg–Nash–Cournot equilibrium model is presented to determine the best prices, quantities, and replenishment cycles in an equilibrium while the demand is assumed as a nonlinear competitive price-dependent function. This model can be formulated as a bi-level optimization program where the leader manufacturer decides at the upper level taking into account the reaction of the follower retailers in a Nash setting. In this setting, the followers operate under the Cournot assumption and accordingly called Cournot firms. The leader called a Stackelberg firm, which sets production levels in an optimal (profit maximizing) manner by explicitly taking into account the reaction of the Cournot firms to its output variations [34].

The rest of the paper organizes as follows: Section 2 reviews the related literature. Section 3 describes the problem and presents the bi-level mathematical model of the Stackelberg–Nash–Cournot game. A numerical example is examined in Section 4 and sensitivity analyses are carried out. Finally, Section 5 is devoted to concluding remarks and future works.

2. Literature review

The supply chain management coordinates all supply chain decisions and activities. The ultimate goal of supply chain management is to minimize the total cost of the chain, including transport costs, shortages, and maintenance while satisfying the level of service required [35]. In other words, supply chain management presents some ways of configuring a supply chain and collaboration across companies to provide services for customers in a desirable manner [24]. In general, proper supply chain management strategies can improve financial performance throughout the system [13].

So far, a large number of studies have been conducted on coordinated pricing and inventory decisions in supply chains. Inventory control is a flow that guarantees the accessibility of products in an organization by considering factors such as time, location, quantity, quality, and cost. The ultimate goal of any inventory control system is to examine and maintain a level of inventory that minimizes the system costs. Whitin [40] was first explored the theory of pricing and inventory planning simultaneously and formulated the news vendor model by considering the price effects. In this model, demand is assumed dependent on the sale price as a decision variable. Donaldson [9] presented a model in which the increased demand was observed in an increasing inventory period while the reduced demand occurred in a decreasing inventory period. In order to examine the joint product pricing and the inventory control issue for a single product in a limited horizon, Tavana *et al.* [38] proposed a model for coordinating pricing decisions and the order quantity in a supply chain including a producer and a retailer while discussed the advantages and disadvantages of this coordination from different aspects. Zhang *et al.* [47] provided an analytical model to investigate the optimal decisions coordinated with pricing, advertising, and inventory control. In this model, they studied a unit part, a limited horizon, and a periodic review model, in which the demand was affected by the price and promotion to maximize total chain profits. Hsieh and Wu [15] reviewed the process of making coordinated decisions in a decentralized supply chain, including the main equipment manufacturer, a producer, and a distributor with unknown supply and demand. Mo *et al.* [25] studied the inventory and pricing decisions considering a price-dependent demand. Liu *et al.* [20] examined an online dual-channel supply system and its joint decisions on production and pricing under the asymmetry of information. First, the optimal production and pricing strategies were determined according to a centralized system. Then, two types of contracts were designed for the decentralized system to coordinate the system completely and ultimately, the production and pricing decisions were extracted by using an operating principle method for asymmetric information in the traditional channel. In another study, Chen *et al.* [6] investigated the research data by focusing on pricing aspects while integrating the inventory and production decisions. Mokhlesian and Zegordi [26] investigated a bi-level supply chain consisting of a producer and several retailers in a competitive situation with different market powers. In this research, the producer developed various interchangeable products and the demand for each product was a linear function of cost, aiming to coordinate the pricing and inventory control decisions in a multi-dimensional bi-level system. Cardenas-Barron and Sana [4] suggested a bi-level inventory system for the optimal inventory decisions and transport policies based on nonlinear price-dependent demand for a coordinated and non-coordinated supply chain. Kumar *et al.* [18] highlighted the price adjustment decision a retailer who splits orders between an unreliable producer with a cheap price and a reliable producer with a high price while considering a non-linear demand under disruption sourcing risk. Ghashghaei and Mozafari [12] studied the newsboy problem combined with the cooperative advertisement problem in the presence of uncertain demand which depends on retail price as well as local and national advertising expenditures. They determined the equilibrium pricing, ordering, and advertising decisions in a manufacturer-retailer supply chain under three different game scenarios. In a similar study, Mozafari *et al.* [29] developed a possibilistic model to coordinate pricing, ordering and advertising policies in a decentralized supply chain under different power structures.

In terms of pricing competition, Yang and Zhou [43] proposed a pricing model with one producer as a leader and two retailers as the followers with a competitive demand function, using Stackelberg game theory. They investigated the sale prices and the number of orders independently, as well as the constructional pricing plan. Game theory seeks to use mathematics to evaluate or estimate the behavior in the strategic conditions or in a competition that the success of a player in decision depends on the choice of others. Therefore, this theory is widely used for predicting the behavior of the supply chain members in competitive conditions. Naimi-Sadigh *et al.* [30] addressed the coordination of pricing, inventory, and marketing decisions in a multi-product multi-echelon supply chain composed of multiple suppliers, single manufacturer, and multiple retailers. They assumed that the demand of each product is non-linearly influenced by retailing price and marketing expenditure. Mozafari *et al.* [28] developed a differential Stackelberg–Nash game to find the equilibrium production, price and routing decisions in a system consisting of one dominant producer and multiple oligopolistic transporters over a planning horizon. Chen and Xiao [6] explored the linear competition in a two echelon supply chain with one supplier and two retailers deciding on price and replenishment policies. In another study, Chen *et al.* [6]

analyzed the retailers' decisions by creating a new channel and using the Stackelberg competition theory. They concluded that the two-channel supply chain has higher profitability compared to the single channel supply chain. Considering a supply chain with a dominant manufacturer and a population of retailers, Hafezalkotob *et al.* [14] investigated a Cournot duopoly equilibrium of a pricing problem using the evolutionary game theory approach. Li *et al.* [21] studied the coordination strategies in a competitive supply chain through the Cournot and Stackelberg game methodology while investigating the impact of retailer innovation investment. Naimi-Sadigh *et al.* [31] proposed a Stackelberg–Nash equilibrium model by taking into account the dominant power of the manufacturer and the suppliers' oligopoly competition to coordinate pricing, advertising, and production-inventory decisions in a multi-product three-echelon supply chain.

Perishability plays an important role in inventory control systems because it reduces the inventory level. The term "deterioration" is used for products that lose their inherent value over time. With this definition, perishable products include not only food and medicine, but also a wide range of products such as electronic equipment, clothing, etc. Ghare and Schrader [11] was firs that introduced corruptibility issue into the economic order quantity inventory model assuming fixed deterioration rate. Afterward, the model was expanded in 1966, and the demand-based pricing was added to the model [10]. Covert and Philip [8] extended the inventory control model for perishable products considering a Weibull's two-parameter distribution with the assumption of constant demand rate and not-allowed deficiency. A time-dependent demand is then proposed by Wu [41] to investigate different phases of the product's life cycle in the store. Yang and Wee [42] developed a model for production-inventory integrated planning of corruptible products, aiming to investigate a single-product and a system consisting of a producer and few retailers. Yin and Rajaram [46] considered a periodical review inventory model assuming that pricing and ordering decisions are made at the beginning of each period and all deficiencies are re-entered into the system. The immediate deterioration means that the products start to deteriorate immediately after entering into the system. In contrast, the products that start to deteriorate after a specific time are considered as non-immediate deteriorating products [44]. Soon [36] presented a probabilistic inventory model, in which demand is dependent on price and a random variable. The inventory control and pricing model was developed for perishable products with non-immediate deterioration. A bi-level supply chain model faces a risk-tolerant producer and a retailer who sells a deteriorating item with price-dependent random demand has been presented in Li *et al.* [19]. The producer and retailer would negotiate with each other about the wholesale price, retail price, and the number of orders. In this regard, Taleizadeh and Nematollahi [37] under the conditions of allowed deficiency and backorder suggested an inventory control model and optimal economic order of perishable products. Furthermore, it is possible to pay for the products purchased by the seller later than replenishment. The maximum payment delay time was previously determined and assumed constant by the supplier. In a single-product system, the demand rate is constant and the delivery time is zero. Mokhtari *et al.* [27] proposed an Economic Production Quantity (EPQ) model to determine production-inventory policies for perishable products while the demand rate is stochastic-and stock-dependent. They proposed a simulation-based optimization algorithm by combining a grid search and a simulation model to solve the problem. Otrodi *et al.* [32] addressed a joint pricing and lot-sizing problem in a supply chain where a distributer sells a single perishable product at different prices to various markets and offers a varied credit period to customers. Chernonog [7] investigated a two-echelon supply chain Stackelberg game in which they set the terms of a wholesale price contract for a perishable product. Product demand is assumed dependent on the selling price, the investment in advertising, and the time a unit spends on the shelf before being sold. Salmasnia and Talesh-Kazemi [33] investigated the joint inventory planning, pricing, and maintenance decisions for perishable products in a parallel system including two components. Aiming at optimizing operational performance of a retailer, Wang *et al.* [39] provided a time and price sensitive market strategy for perishable products.

According to the above literature, although many researchers have studied coordination of pricing and inventory decisions, few have considered the perishability of both product and raw materials throughout the supply chain. However, pricing and inventory decisions are highly influenced by product and raw material deterioration. Another gap is the issue of competition and market structure in the joint pricing-inventory problem for deteriorating items. To better model ground truth, this paper adopts the bi-level programming approach to

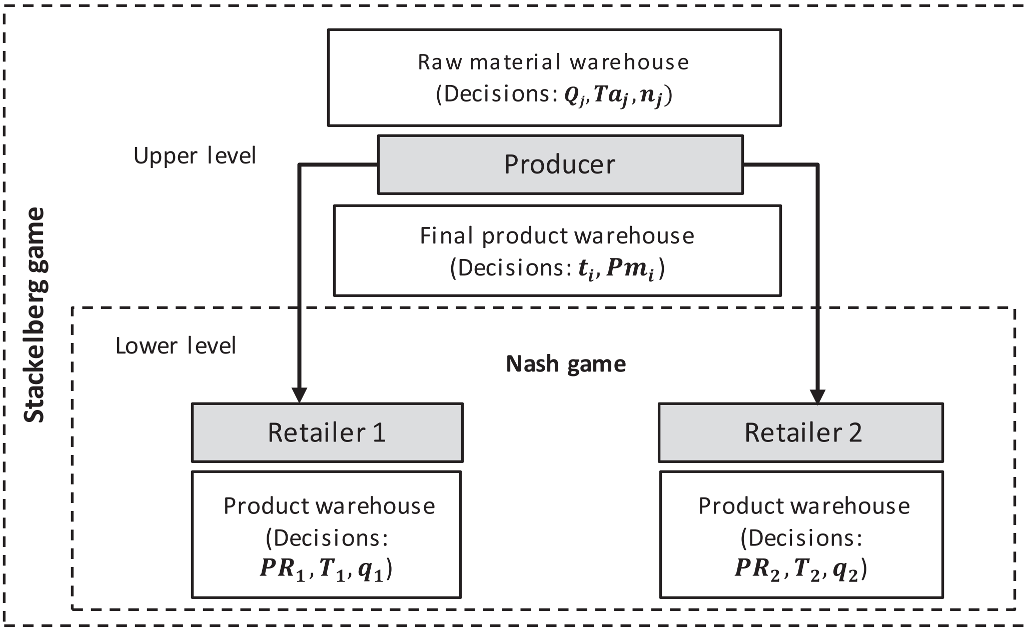

FIGURE 1. The supply chain scheme.

coordinate pricing and inventory decisions for perishable products and raw materials in a two-echelon supply chain including a dominant producer and two Cournot follower retailers with non-linear price-dependent demand and exponential deteriorating inventory.

3. Problem description and formulation

A leader-follower supply chain is modeled as a bi-level problem with a dominant producer at upper level and two follower retailers under Cournot assumption at lower level. Hence, the relationships among supply chain members form a Stackelberg–Nash–Cournot game in which the demand is assumed a nonlinear price-dependent function and the inventory is deteriorated exponentially with a constant rate. Shortage is not allowed. Three level of warehousing are considered throughout the chain in which materials face deterioration over time; the primary raw material warehouse, the producer's final product warehouse, and the retailers' warehouses; as illustrated in Figure 1.

The producer determines the number of orders for each raw material as well as the wholesale price for each retailer while maximizing his own profit at the upper level given the best responses from the lower level Nash equilibrium. Then, the retailers, as the followers, try simultaneously to maximize their own profits in a horizontal competition by choosing their best replenishment cycles and retail prices. The mathematical notations are illustrated in Table 1.

It is assumed that the process has a production cycle in which the primary materials are purchased as raw materials while the production rate is constant and definite. Figure 2 illustrates the inventory replenishment process related to the three level warehouses during a production cycle.

In each period, it is assumed that retailer i orders q_i unit of the finished product that produced by the producer and sells them during the same period. It is presumed that remaining inventory at the end of each period deteriorates such that the inventory level vanishes. Equation (3.1) represents the inventory level changes

TABLE 1. Notations.

Indices	
i	Retailers
j	Type of raw material
Parameters	
um	Production rate of finished product
θ_i	Deterioration rate of products in the retailer i's warehouse
β	Deterioration rate of products in the producer's warehouse
σ_j	Deterioration rate of raw material type j in the producer's warehouse
$D_i = D_i\left(\mathrm{PR}_i\right)$	Price-dependent demand function for the final product for retailer i
P_j	Purchase price for raw material j
w_j	Proportion of raw material j used in the final product
ch_i	Holding unit cost of products in the retailer i's warehouse
chm	Holding unit cost of products in the producer's warehouse
cha_j	Holding unit cost of products in raw material warehouse for type j
cd_i	Deterioration unit cost of products in the retailer i's warehouse
cdm	Deterioration unit cost of products in the producer's warehouse
cda_j	Deterioration unit cost for raw material type j
C	Production cost per unit of finished product
A_i	Fixed ordering cost for retailer i
Aa_j	Fixed ordering cost for raw material type j
Sm	Setup cost for the producer
γ_i	Self-price sensitivity coefficient for retailer i
α_i	Competitive price sensitivity coefficient
k_i	Market scale for retailer i
Decision variables	
q_i	Quantity of final product ordered by retailer i in each purchase
Q_j	Quantity of raw material j ordered in each purchase
t_i	The producer's lead time to fulfill the retailer i's order
Ta_j	Consumption period of raw material j
T_i	Replenishment cycle for retailer i
n_j	The number of orders for raw material j per period
pm_i	Wholesale price to retailer i
PR_i	Retail price of retailer i
Functions	
πr_i	Retailer i's profit
πm	Producer's profit
$\mathrm{IR}_i\left(t\right)$	Inventory level of retailer i's warehouse
$\mathrm{IM}\left(t\right)$	Inventory level of finished products in the producer's warehouse
$I_j\left(t\right)$	Inventory level of raw material type j

in retailer i's warehouse due to the demand rate as well as the product deterioration rate at time t.

$$\frac{\mathrm{d}\,\mathrm{IR}_i\left(t\right)}{\mathrm{d}t} + \theta_i \cdot \mathrm{IR}_i\left(t\right) = -D_i\left(\mathrm{PR}_i\right) = -D_i, \qquad 0 \leq t \leq T_i. \tag{3.1}$$

The inventory level of retailer i's warehouse at time t can be obtained by solving the differential equation in equation (3.1) with initial condition $\mathrm{IR}_i\left(0\right) = q_i$, as follows:

$$\mathrm{IR}_i\left(t\right) = \frac{-D_i}{\theta_i} + \frac{D_i}{\theta_i}\mathrm{e}^{\theta_i \cdot t}. \tag{3.2}$$

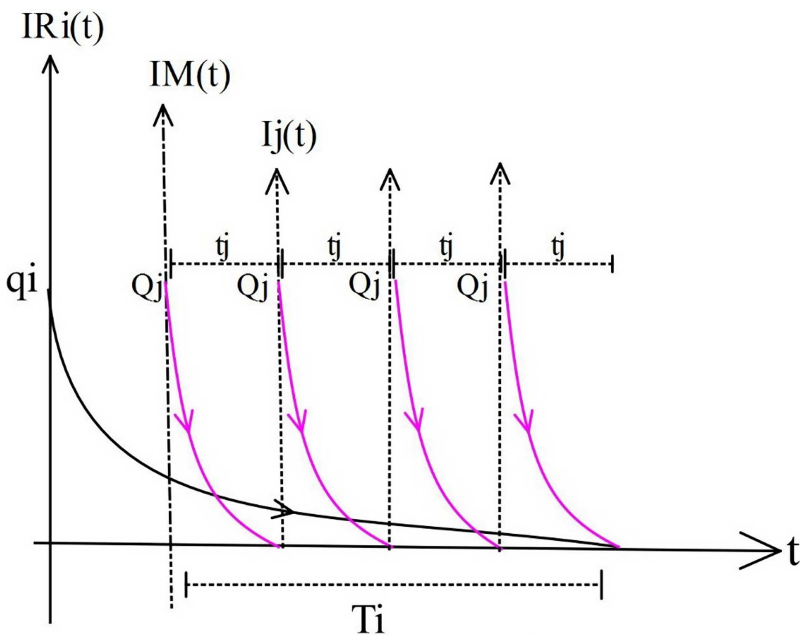

FIGURE 2. Inventory levels at different warehouses.

Since $\mathrm{IR}_i\left(T_i\right)=0$, we have:

$$\mathrm{IR}_i\left(t\right)=\frac{-D_i}{\theta_i}+\frac{D_i}{\theta_i}\mathrm{e}^{\theta_i\left(T_i-t\right)}=\frac{D_i}{\theta_i}\left(\mathrm{e}^{\theta_i\cdot\left(T_i-t\right)}-1\right). \tag{3.3}$$

Retailers compete with each other in the final product market. The demand for each retailer is defined as a price-dependent Cobb Douglas function as follows:

$$D_i\left(\mathrm{PR}_i\right)=k_i\cdot\mathrm{PR}_i^{-\gamma_i}\cdot\mathrm{PR}_{3-i}^{\alpha_i},\qquad i=1,2,\qquad 0<\alpha_i<1,\qquad \gamma_i>1,\qquad k_i>0 \tag{3.4}$$

where k_i denotes the firm i's market scale; γ_i denotes the absolute elasticity of retailer i's demand with respect to its own price; α_i denotes the cross-elasticity with respect to the competitor retailer's price. In this Cobb–Douglas model, the price elasticities γ_i and α_i are both price-independent constants [16]. For some applications and examples, refer to Allon and Federgruen [1] and Bernstein and Federgruen [3].

Thus, considering equations (3.3) and (3.4), the inventory level of retailer i's warehouse at time t can be inferred as follows:

$$\mathrm{IR}_i\left(t\right)=\frac{k_i\cdot\mathrm{PR}_i^{-\gamma_i}\cdot\mathrm{PR}_{3-i}^{\alpha_i}}{\theta_i}\left(\mathrm{e}^{\theta_i\left(T_i-t\right)}-1\right). \tag{3.5}$$

Each retailer aims to maximize his profit during a replenishment cycle that defined as:

$$\begin{aligned}\pi r_i&=\mathrm{PR}_i\int_0^{T_i}D_i\left(\mathrm{PR}_i\right)\mathrm{d}t-ch_i\int_0^{T_i}\mathrm{IR}_i\left(t\right)\mathrm{d}t-A_i-pm_i\cdot q_i-cd_i\int_0^{T_i}\theta_i\cdot\mathrm{IR}_i\left(t\right)\mathrm{d}t\\&=T_i\cdot\mathrm{PR}_i\cdot D_i\left(\mathrm{PR}_i\right)-ch_i\cdot D_i\left(\mathrm{PR}_i\right)\cdot\left(\frac{\mathrm{e}^{\theta_i\cdot T_i}-\theta_i\cdot T_i-1}{\theta_i^2}\right)-A_i-pm_i\cdot\frac{D_i\left(\mathrm{PR}_i\right)}{\theta_i}\left(\mathrm{e}^{\theta_i\cdot T_i}-1\right)\\&\quad-cd_i\frac{D_i\left(\mathrm{e}^{\theta_i\cdot T_i}-\theta_i\cdot T_i-1\right)}{\theta_i},\end{aligned} \tag{3.6}$$

where the first term shows retailer i's revenue as the product of retail prices and realized demand during a replenishment cycle. The second and the third terms indicate holding and ordering costs during a replenishment cycle, respectively. The forth term shows purchasing costs as a function of retailer i's order quantity (q_i) which equals to the initial inventory level at each replenishment cycle $\mathrm{IR}_i\,(0)=\frac{D_i}{\theta_i}\left(\mathrm{e}^{\theta_i\cdot T_i}-1\right)$. The last term depicts the deterioration costs.

In order to reach the best responses of each retailer in the Nash competition at the lower level game, the best values of the retailers' variables (PR_i^* and T_i^*) can be determined by differentiating each retailer's profits regarding to these variables and solving the nonlinear equation system resulted by vanishing the derivatives as follows:

$$\begin{aligned}\frac{\partial \pi r_i}{\partial \mathrm{PR}_i} = &-\Big(k_i\cdot \mathrm{PR}_{3-i}^{\alpha_i}\Big(ch_i\cdot\gamma_i+\theta_i\cdot cd_i\cdot\gamma_i+\theta_i\cdot pm_i\cdot\gamma_i-ch_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}-\theta_i^2\cdot T_i\cdot\mathrm{PR}_i+\theta_i^2\cdot T_i\cdot cd_i\cdot\gamma_i\\ &+\theta_i^2\cdot T_i\cdot\mathrm{PR}_i\cdot\gamma_i+\theta_i\cdot T_i\cdot ch_i\cdot\gamma_i-\theta_i\cdot cd_i\cdot\gamma_i\cdot\mathrm{e}^{(\theta_i\cdot T_i)}-\theta_i\cdot pm_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}\Big)\Big)\Big/\Big(\theta_i^2\cdot\mathrm{PR}_{3-i}^{(\gamma+1)}\Big)=0\end{aligned} \tag{3.7}$$

$$\frac{\partial \pi r_i}{\partial T_i}=\frac{k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\left(ch_i+\theta_i\cdot cd_i+\theta_i\cdot\mathrm{PR}_i-ch_i\cdot\mathrm{e}^{\theta_i\cdot T_i}-\theta_i\cdot cd_i\cdot\mathrm{e}^{\theta_i\cdot T_i}-\theta_i\cdot pm_i\cdot\mathrm{e}^{\theta_i\cdot T_i}\right)}{\theta_i\cdot\mathrm{PR}_i^{\gamma_i}}=0. \tag{3.8}$$

By solving the nonlinear equations system of equations (3.7) and (3.8), we have:

$$\begin{aligned}\mathrm{PR}_i^* = &-\Big(\gamma_i\Big(ch_i+\theta_i\cdot cd_i+\theta_i\cdot pm_i-ch_i\cdot\mathrm{e}^{(\theta_i\cdot T_i)}+\theta_i\cdot T_i\cdot ch_i-\theta_i\cdot cd_i\mathrm{e}^{(\theta_i\cdot T_i)}\\ &-\theta_i\cdot pm_i\mathrm{e}^{(\theta_i\cdot T_i)}+\theta_i^2\cdot T_i\cdot cd_i\Big)\Big)\Big/\left(\theta_i^2\cdot T_i\cdot(\gamma_i-1)\right)\end{aligned} \tag{3.9}$$

$$T_i^* = \ln\left(\frac{ch_i+\theta_i\cdot cd_i+\theta_i\cdot\mathrm{PR}_i}{ch_i+\theta_i\cdot cd_i+\theta_i\cdot pm_i}\right)/\theta_i. \tag{3.10}$$

Proposition 3.1. The lower level Nash game of the rival retailers has a unique equilibrium as (PR_i^*, T_i^*).

Proof. **Definition 3.2.** Considering equations (3.7)–(3.10), the hessian matrix of πr_i is calculated as matrix A.

$$A=\begin{bmatrix}\frac{\partial^2\pi r_i}{\partial\mathrm{PR}_i^2} & \frac{\partial^2\pi r_i}{\partial\mathrm{PR}_i\partial T_i}\\ \frac{\partial^2\pi r_i}{\partial T_i\partial\mathrm{PR}_i} & \frac{\partial^2\pi r_i}{\partial T_i^2}\end{bmatrix}=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} \tag{3.11}$$

where,

$$\begin{aligned}a_{11} =\ &\frac{T_i\cdot k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\cdot\gamma_i\,(\gamma_i+1)}{\mathrm{PR}_i^{\gamma_i+1}}-\frac{2T_i\cdot k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\cdot\gamma_i}{\mathrm{PR}_i^{\gamma_i+1}}-\frac{k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\cdot pm_i\cdot\gamma_i\left(\mathrm{e}^{(\theta_i\cdot T_i)}-1\right)(\gamma_i+1)}{\theta_i\cdot\mathrm{PR}_i^{\gamma_i+2}}\\ &+\frac{cd_i\cdot k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\cdot\gamma_i\,(\gamma_i+1)\left(\theta_i\cdot T_i-\mathrm{e}^{\theta_i\cdot T_i}+1\right)}{\theta_i\cdot\mathrm{PR}_i^{\gamma_i+2}}+\frac{ch_i\cdot k_i\cdot\mathrm{PR}_i^{\alpha_i}\cdot\gamma_i\,(\gamma_i+1)\left(\theta_i\cdot T_i-\mathrm{e}^{\theta_i\cdot T_i}+1\right)}{\theta_i^2\cdot\mathrm{PR}_i^{\gamma_i+2}}\\ a_{12} =\ &\Big(k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\Big(\theta_i\cdot\mathrm{PR}_{3-i}-ch_i\cdot\gamma_i-\theta_i\cdot cd_i\cdot\gamma_i-\theta_i\cdot\mathrm{PR}_{3-i}\cdot\gamma_i+ch_i\cdot\gamma_i\cdot\mathrm{e}^{(\theta_i\cdot T_i)}\\ &+\theta_i\cdot cd_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}+\theta_i\cdot pm_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}\big)\big)\Big/\Big(\theta_i\cdot\mathrm{PR}_i^{\gamma_i+1}\Big)\,\theta_i\\ a_{21} =\ &\big(k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\left(\theta_i\cdot\mathrm{PR}_{3-i}-ch_i\cdot\gamma_i-\theta_i\cdot cd_i\cdot\gamma_i-\theta_i\cdot\mathrm{PR}_{3-i}\cdot\gamma_i+ch_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}\right.\\ &\left.+\ \theta_i\cdot cd_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}+\theta_i\cdot pm_i\cdot\gamma_i\cdot\mathrm{e}^{\theta_i\cdot T_i}\right)\big)\Big/\Big(\theta_i\cdot\mathrm{PR}_i^{\gamma_i+1}\Big)\\ a_{22} =\ &-\frac{\left(k_i\cdot\mathrm{PR}_{3-i}^{\alpha_i}\cdot\mathrm{e}^{(\theta_i\cdot T_i)(ch_i+\theta_i\cdot cd_i+\theta_i\cdot pm_i)}\right)}{\mathrm{PR}_i^{\gamma_i}}.\end{aligned}$$

Definition 3.3. A is negative semi-definite in (PR_i^*, T_i^*), if and only if $X^T \cdot A \cdot X \leq 0$ [2].

According to Definition 3.2, we have:

$$H = [\mathrm{PR}_i, T_i] \begin{bmatrix} \frac{\partial^2 \pi r_i}{\partial \mathrm{PR}_i^2} & \frac{\partial^2 \pi r_i}{\partial \mathrm{PR}_i \partial T_i} \\ \frac{\partial^2 \pi r_i}{\partial T_i \partial \mathrm{PR}_i} & \frac{\partial^2 \pi r_i}{\partial T_i^2} \end{bmatrix} \begin{bmatrix} \mathrm{PR}_i \\ T_i \end{bmatrix} \tag{3.12}$$

$$\begin{aligned} &= \frac{T_i \cdot \left(k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \left(\theta_i \cdot \mathrm{PR}_{3-i} - ch_i \cdot \gamma_i - \theta_i \cdot cd_i \cdot \gamma_i - \theta_i \cdot \mathrm{PR}_{3-i} \cdot \gamma_i + ch_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i} + \theta_i \cdot cd_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i} + \theta_i \cdot pm_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i}\right)\right)}{\theta_i \cdot \mathrm{PR}_i^{\gamma_i}} \\ &- \frac{T_i^2 \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot \mathrm{e}^{(\theta_i \cdot T_i)(ch_i + \theta_i \cdot cd_i + \theta_i \cdot pm_i)}}{\mathrm{PR}_i^{\gamma_i}} + \mathrm{PR}_{3-i}^2 \cdot \Bigg(\mathrm{PR}_{3-i} \cdot \Bigg(\frac{T_i \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot \gamma_i (\gamma_i + 1)}{\mathrm{PR}_i^{\gamma_1 + 1}} - \frac{2 T_i \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot \gamma_i}{\mathrm{PR}_i^{\gamma_i + 1}} \\ &- \frac{\left(k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot pm_i \cdot \gamma_i \left(\mathrm{e}^{\theta_i \cdot T_i} - 1\right)(\gamma_i + 1)\right)}{\theta_i \cdot \mathrm{PR}_i^{\gamma_i + 2}} + \frac{\left(cd_i \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot \gamma_i (\gamma_i + 1)\left(\theta_i T_i - \mathrm{e}^{\theta_i \cdot T_i} + 1\right)\right)}{\theta_i \cdot \mathrm{PR}_i^{\gamma_i + 2}} \\ &+ \frac{ch_i \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \cdot \gamma_i (\gamma_i + 1)\left(\theta_i \cdot T_i - \mathrm{e}^{\theta_i \cdot T_i} + 1\right)}{\theta_1^2 \cdot \mathrm{PR}_i^{\gamma_1 + 2}} \Bigg) \\ &+ \frac{T_i \cdot k_i \cdot \mathrm{PR}_{3-i}^{\alpha_i} \left(\theta_i \cdot \mathrm{PR}_{3-i} - ch_i \cdot \gamma_i - \theta_i \cdot cd_i \cdot \gamma_i - \theta_i \cdot \mathrm{PR}_{3-i} \cdot \gamma_i + ch_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i} + \theta_i \cdot cd_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i} + \theta_i \cdot pm_i \cdot \gamma_i \cdot \mathrm{e}^{\theta_i \cdot T_i}\right)}{\theta_i \cdot \mathrm{PR}_i^{\gamma_i + 1}} \Bigg) / \mathrm{PR}_i. \end{aligned} \tag{3.13}$$

Since the non-positivity of the nonlinear equation H cannot be easily proved, we employed the following grid search procedure as a numerical method to analyze the H values inside the feasible region. Since cd_i, γ_i, α_i, k_i, θ_i are all positive constants, we change the values for variables PR_i, T_i, and pm_i, such that $\mathrm{PR}_i \in \left[1, 10^8\right]$, $pm_i \in \left[1, 10^8\right]$, and $T_i \in \left[1, 10^2\right]$.

Step 1. Initialize the open list.
Step 2. Calculate H
Step 3. If $H > 0$ stop, otherwise go to step 4.
Step 4. Update the open list and go to step 2.
Step 5. If all the feasible points on the grid network have been opened and checked, stop the proce.

Given any nonnegative PR_i, pm_i and T_i have shown that H is always non-positive and A is a negative definite matrix. Thus, according to Definition 3.2, (PR_i^*, T_i^*) is the unique Nash equilibrium of the lower level's Nash game and the proof is complete. ☐

On the producer's side, we consider that production operations begin at $t = 0$ with no initial inventories. Given q_i as the product quantity ordered by retailer i in each replenishment cycle, the producer needs t_i unit of time to deliver the orders in a timely manner. Presuming that the finished product inventory level deteriorates at a certain rate β, so we have:

$$\frac{\mathrm{d}\,\mathrm{IM}(t)}{\mathrm{d}t} + \beta \cdot \mathrm{IM}(t) = um, \qquad 0 \leq t \leq t_i. \tag{3.14}$$

By solving the first-order linear differential equation (3.14) with initial condition $\mathrm{IM}(0) = 0$, the inventory level of producer's warehouse at time t is obtained as follows:

$$\mathrm{IM}(t) = \frac{um}{\beta} - \frac{\mathrm{e}^{-\beta \cdot t} \cdot um}{\beta} = \mathrm{e}^{-\beta \cdot t}\left[\frac{um \cdot \mathrm{e}^{\beta \cdot t}}{\beta} - \frac{um}{\beta}\right]. \tag{3.15}$$

The producer's profit function at the upper level can be defined as follows:

$$\pi m = Rm - n \cdot Sm - chm \cdot \sum_{i=1}^{n} \int_0^{t_i} \mathrm{IM}(t)\,\mathrm{d}t - cdm \cdot \int_0^{t_i} \beta \cdot \mathrm{IM}(t)\,\mathrm{d}t - \left(C \cdot um \cdot \sum_{i=1}^{n} t_i\right) - TJ \tag{3.16}$$

$$= \sum_{i=1}^{n} q_i \cdot pm_i - n \cdot Sm - \frac{chm \left(\sum_{i=1}^{n} um \cdot \left(\mathrm{e}^{(-\beta \cdot t_i)} + \beta \cdot t_i - 1\right)\right)}{\beta^2}$$
$$- cdm \cdot \left[\sum_{i=1}^{n} \frac{\left(um \cdot \left(\mathrm{e}^{(-\beta \cdot t_i)} + \beta \cdot t_i - 1\right)\right)}{\beta}\right] - \left(C \cdot um \cdot \sum_{i=1}^{n} t_i\right) - \mathrm{TJ}. \tag{3.17}$$

The first term of equation (3.17) depicts the producer's revenue from which the following costs are deducted respectively: setup costs, holding costs, deterioration costs, variable production costs, and raw material costs.

In order to define TJ at the raw material warehousing level, let consider Q_j be the quantity of raw material j received by the producer per purchase and consumed during Ta_j, n_j be the material order frequency. Hence, the changes in the inventory level of raw material j can be calculated as:

$$\frac{\mathrm{d}I_j\left(t\right)}{\mathrm{d}t} + \sigma_j \cdot I_j\left(t\right) = -w_j \cdot um \qquad 0 \leq t \leq Ta_j. \tag{3.18}$$

By solving differential equation (3.18) with the final condition that $I_j\left(Ta_j\right) = 0$, the inventory level of raw material j at time t is derived as follows:

$$I_j\left(t\right) = \mathrm{e}^{-\sigma_j \cdot t} \left[\frac{-w_j \cdot um \cdot \mathrm{e}^{\sigma_j \cdot t}}{\sigma_j} + \frac{w_j \cdot um \cdot \mathrm{e}^{\sigma_j \cdot Ta_j}}{\sigma_j}\right] = \frac{-w_j \cdot um}{\sigma_j} + \frac{w_j \cdot um \cdot \mathrm{e}^{\sigma_j (Ta_j - t)}}{\sigma_j}\cdot \tag{3.19}$$

Then we can introduce TJ as follows:

$$\mathrm{TJ} = \sum_{j=1}^{m} n_j \cdot \left[Aa_j + cha_j \int_0^{Ta_j} I_j\left(t\right)\mathrm{d}t + Q_j \cdot P_j + cda_j \cdot \int_0^{Ta_j} \sigma_j I_j\left(t\right)\mathrm{d}t\right] \tag{3.20}$$

where n_j denotes the number of orders for the raw material j per cycle which is multiplied by the ordering costs, holding costs, purchasing costs, and deterioration costs, respectively. Replacing the levels of holding and deteriorated inventory in equation (3.20), we have:

$$\mathrm{TJ} = \sum_{j=1}^{m} n_j \cdot \left[Aa_j + cha_j \left(\frac{-um \cdot w_j \left(Ta_j \cdot \sigma_j - \mathrm{e}^{Ta_j \cdot \sigma_j} + 1\right)}{\sigma_j^2}\right)\right.$$
$$\left. + Q_j \cdot P_j + cda_j \left(\frac{-um \cdot w_j \left(Ta_j \cdot \sigma_j - \mathrm{e}^{Ta_j \cdot \sigma_j} + 1\right)}{\sigma_j}\right)\right]\cdot \tag{3.21}$$

Since the quantity received by retailer i in each cycle is equal to his initial inventory level, $\mathrm{IR}_i\left(0\right) = q_i$. According to equation (3.3), it is inferred that:

$$q_i = -\frac{D_i}{\theta_i} + \frac{D_i \cdot \mathrm{e}^{\theta_i \cdot T_i}}{\theta_i} = \frac{D_i}{\theta_i}\left(\mathrm{e}^{\theta_i \cdot T_i} - 1\right). \tag{3.22}$$

On the other side, the inventory at the producer's warehouse reaches to the level q_i at time t_i and we have $\mathrm{IM}(t_i) = q_i$. So, according to equation (3.14), q_i can be calculated as:

$$q_i = \frac{um}{\beta} - \frac{um \cdot \mathrm{e}^{-\beta \cdot t_i}}{\beta} = \mathrm{e}^{-\beta \cdot t_i}\left[\frac{um \cdot \mathrm{e}^{\beta \cdot t_i}}{\beta} - \frac{um}{\beta}\right] = \frac{um}{\beta}\left(1 - \mathrm{e}^{-\beta \cdot t_i}\right). \tag{3.23}$$

Putting equations (3.22) and (3.23) equal, the value t_i (in terms of T_i) can be reached as follows:

$$t_i = -\frac{1}{\beta}\ln\left(\frac{D_i \cdot \beta}{\theta_i \cdot um} - \frac{D_i \cdot \mathrm{e}^{\theta_i \cdot T_i} \cdot \beta}{\theta_i \cdot um} + 1\right). \tag{3.24}$$

TABLE 2. Parameter values.

$i = 1,2$	$j = 1,2$	$\theta_1 = 0.04$	$\theta_2 = 0.02$	$\beta = 0.08$
$w_1 = 0.7$	$P_2 = 410$	$P_1 = 405$	$\sigma_1 = 0.04$	$\sigma_2 = 0.02$
$w_2 = 0.8$	$ch_1 = 22$	$ch_2 = 18$	$chm = 40$	$cha_1 = 15$
$cd_1 = 30$	$cd_2 = 28$	$cdm = 30$	$cda_1 = 25$	$cha_2 = 16$
$cda_2 = 23$	$C = 12\,000$	$A_1 = 1000$	$A_2 = 1200$	$Aa_1 = 1000$
$Aa_2 = 2000$	$Sm = 1000$	$\gamma_1 = 1.5$	$\gamma_2 = 1.25$	$\alpha_1 = 0.75$
$um = 900$	$n = 2$	$k_2 = 17\,500$	$k_1 = 17\,000$	$\alpha_2 = 0.5$

At the raw material warehousing level, the initial raw material inventory is $I_j(0) = Q_j$. Thus according to equation (3.19), it is concluded that

$$Q_j = \frac{-w_j \cdot um}{\sigma_j} + \frac{w_j \cdot um \cdot \mathrm{e}^{\sigma_j \cdot Ta_j}}{\sigma_j}. \tag{3.25}$$

Since the average raw material consumption duration is $\frac{\sum_{i=1}^{n} t_i}{n_j}$; by substituting t_i from equation (3.24), we have:

$$Ta_j = \frac{\sum_{i=1}^{n} t_i}{n_j} = \frac{1}{n_j}\left[-\frac{1}{\beta}\sum_{i=1}^{n} \ln\left(\frac{D_i \cdot \beta}{\theta_i \cdot um} - \frac{D_i \cdot \mathrm{e}^{\theta_i \cdot T_i} \cdot \beta}{\theta_i \cdot um} + 1\right)\right]. \tag{3.26}$$

Regarding equations (3.9) and (3.10), the lower level Nash equilibrium is expressed implicitly as a function of the upper level variables. Since the uniqueness of the Nash equilibrium at the lower level is guaranteed *via* Proposition 3.1, we can transform the bi-level Stackelberg–Nash–Cournot game model into an equivalent single level model by adding the Nash equilibrium point of the lower level retailers' competition to the producer's model at the upper level. Furthermore, we replace q_i, t_i, Q_j, and Ta_j with their equivalents according to equations (3.23)–(3.26). Since the equivalent single level model is an unconstrained nonlinear model, we solve it utilizing the Secant algorithm. Therefore, the solution procedure to find the Stackelberg–Nash–Cournot equilibrium is summarized in the following steps:

Step 1. Calculate the lower level Nash equilibrium as a function of the upper level variables according to equations (3.9) and (3.10).
Step 2. Transform the bi-level Stackelberg–Nash–Cournot game model into an equivalent single level model by adding the Nash equilibrium point of the lower level problem to the producer's objective function at the upper level.
Step 3. Replace q_i, t_i, Q_j, and Ta_j with their equivalents according to equations (3.23)–(3.26).
Step 4. Solve the resulting unconstrained nonlinear model using the Secant algorithm.

4. Numerical results

In this paper, the numerical example suggested by Yang *et al.* [45] is used with some modifications to evaluate the proposed bi-level game model and to analyze the sensitivity of the equilibrium to the main parameters variations. Consider a supply chain consisting of two competing retailers, one dominant producer, and two types of raw materials. The parameter values are reported in Table 2.

The bi-level mathematical model of the Stackelberg–Nash–Cournot game between the supply chain's players is addressed by the solution procedure and the equilibrium values of the decisions are listed in Table 3.

Here, we conduct sensitivity analyses on the main parameters of the proposed model. In order to investigate the sensitivity of the equilibrium to the deterioration rate parameter in the retailers' warehouses, we change θ_i value on the interval $[0.5\theta_i, 1.5\theta_i]$ while all the other parameters are fixed. The results are illustrated in Figure 3. As expected, an increase in deterioration rate at retail warehouses reduces the inventory levels and as a result,

Table 3. Equilibrium values.

Raw material warehouse			Final product warehouse	Retail warehouses				Profits		
n_1^*	Q_1^*	Ta_1^*	pm_1^*	t_1^*	T_1^*	PR_1^*	q_1^*	π_m^*	π_{r1}^*	π_{r2}^*
5	36.2419	0.0575	20 986	0.1321	16	48 078	118.29	296 833	1 557 100	1 466 249
n_2^*	Q_2^*	Ta_2^*	pm_2^*	t_2^*	T_2^*	PR_2^*	q_2^*			
5	41.3955	0.0575	28 350	0.1552	21	48 598	138.78			

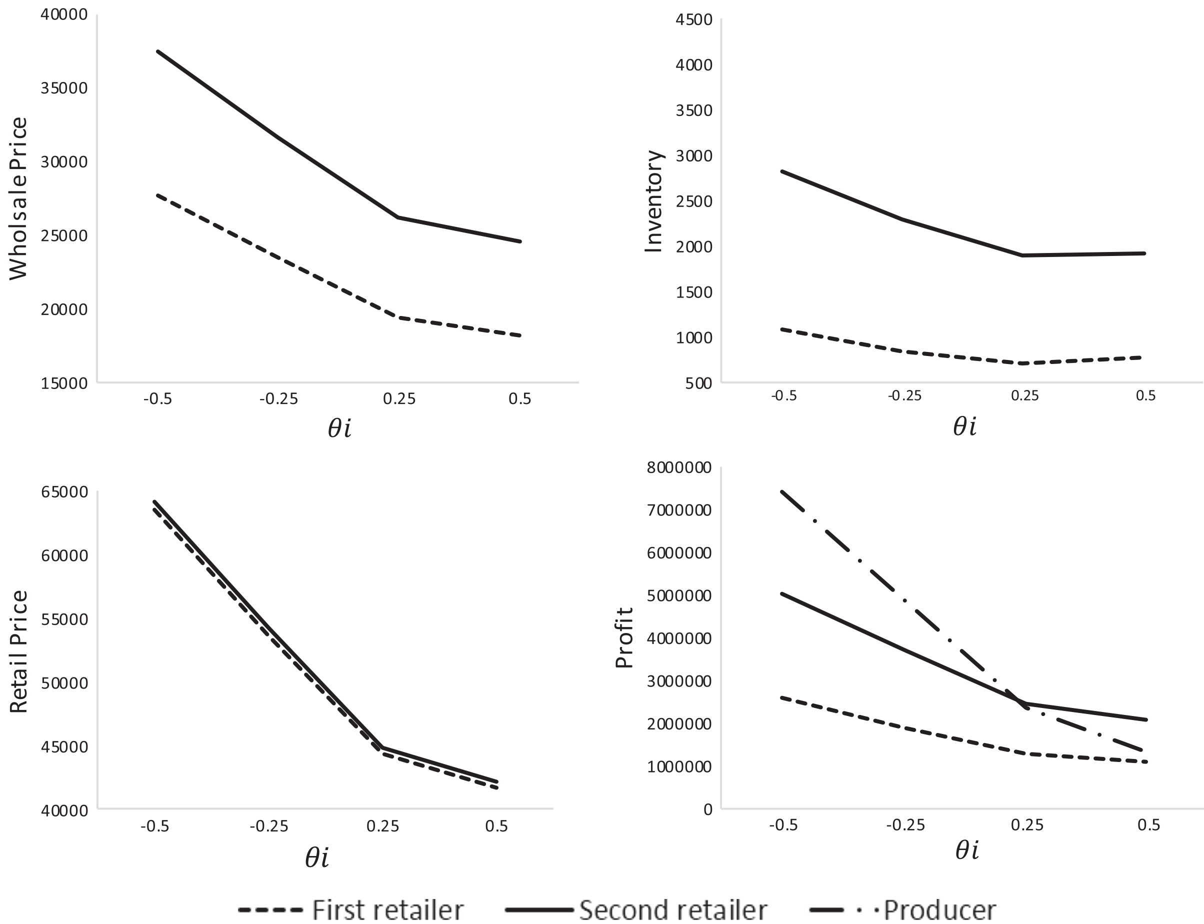

Figure 3. The impact of θ_i.

retailers offer lower prices to receive more demand and prevent the loss of product deterioration. As prices fall and deterioration costs rise, profits decline at the retail tier. Higher levels of demand cause higher levels of production and consequently higher inventory at the producer's warehouse. This increase in costs reduces the profitability of the producer so that the producer's profit turns even negative at $1.5\theta_i$. In Figure 3, note that

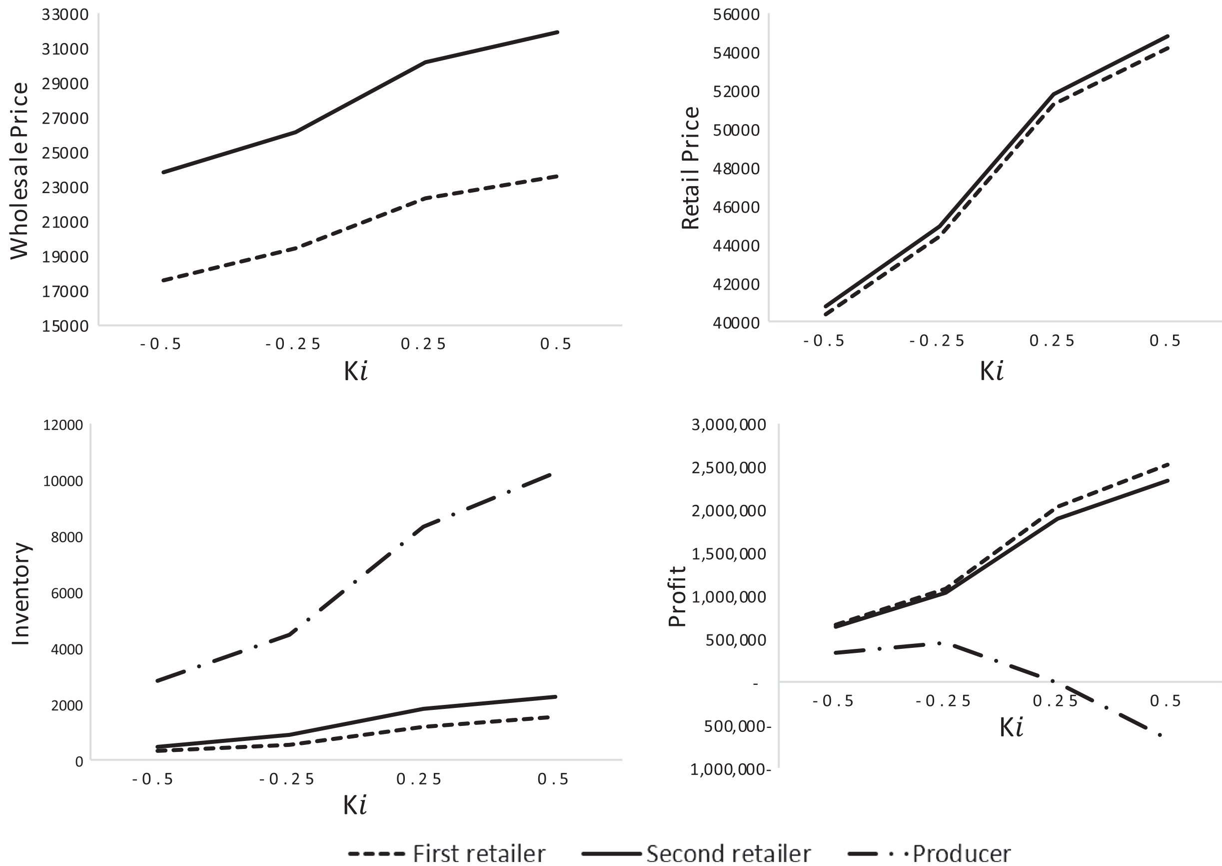

FIGURE 4. The impact of k_i.

the number -0.5 on the horizontal axis indicates a 50% decrease and the number $+0.5$ indicates 50% increase in θ_i's value, respectively.

In a similar way, to investigate the sensitivity of the Stackelberg–Nash–Cournot equilibrium to the market scale factor, we change k_i on the interval $[0.5k_i, 1.5k_i]$ while all the other parameters are fixed. As it is observed in Figure 4, both the retailers offer higher prices, achieve more demand, and consequently more benefit when they have larger market scale. Although the producer increases wholesale prices to the retailers, his benefit declines due to the higher level of increase in production, purchasing and inventory costs at the producer's echelon.

Here, we examine how any changes in each retailer's self-price elasticity can influence the equilibrium point of the game. We alter γ_i on the interval $[1.15\gamma_i, 1.75\gamma_i]$. As can be observed in Figure 5, a growth in parameter γ_i implies a reduction in the pricing power of retailer i. Hence, the lower pricing power causes lower retail prices and consequently lower profits for the whole chain.

In general, an increase in competitor price sensitivity coefficient (α_i) increases the amount of demand and inventory levels of each retail warehouse, resulting in increasing the profitability of the retailer due to increased demand and retail prices, while it is only at specific times. The reason for the lack of increase in the producer profit is the costs increased more than the revenue, as the producer has two warehouses of raw materials and the final product holding and production costs. The costs of the aforementioned factors will reduce profit. Figure 6 shows the sensitivity analysis of the competitor's price sensitivity coefficient (α_i).

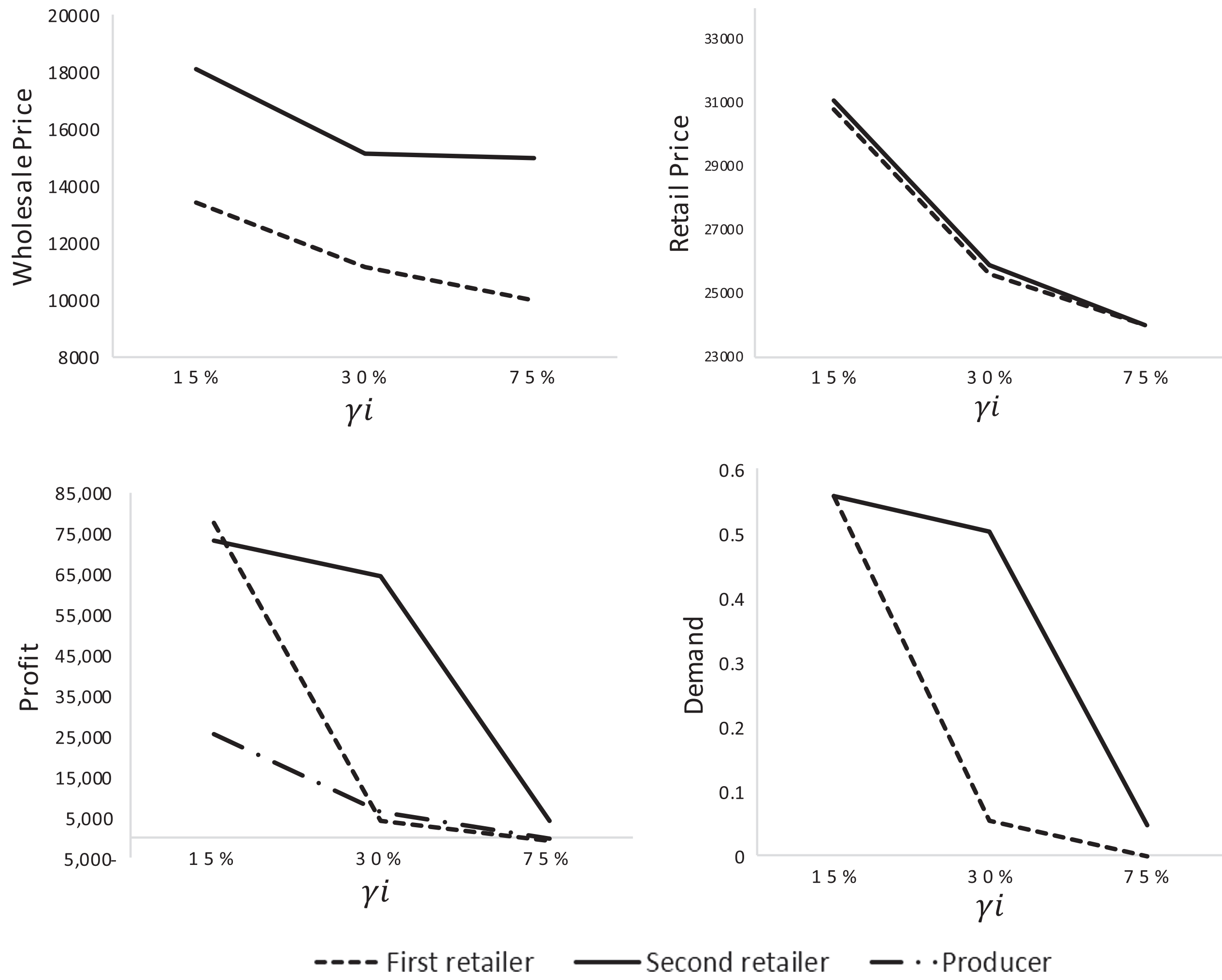

FIGURE 5. The impact of γ_i.

By increasing the product deterioration rate in the producers' warehouse, the inventory level of the producer is increased due to the faster deterioration rate of products in the warehouse. Therefore, the producer and the retailers reduce prices to increase sales and raise consumer's demand, resulting in increasing demand. The decrease in price will reduce retailers' profit, although the producers' profit will be further decreased by increasing production costs and reducing producer revenue. Figure 7 depicts the sensitivity analysis of the products deterioration rate for producer (β).

5. Conclusion

In this study, a bi-level programming model was presented to coordinate pricing and inventory decisions in a competitive supply chain consisting of a dominant producer and two Cournot follower retailers with nonlinear price-dependent demand. The supply chain produces perishable products from perishable raw materials that decay at a certain rate inside the warehouses. Three levels of warehousing including raw material warehouse, final

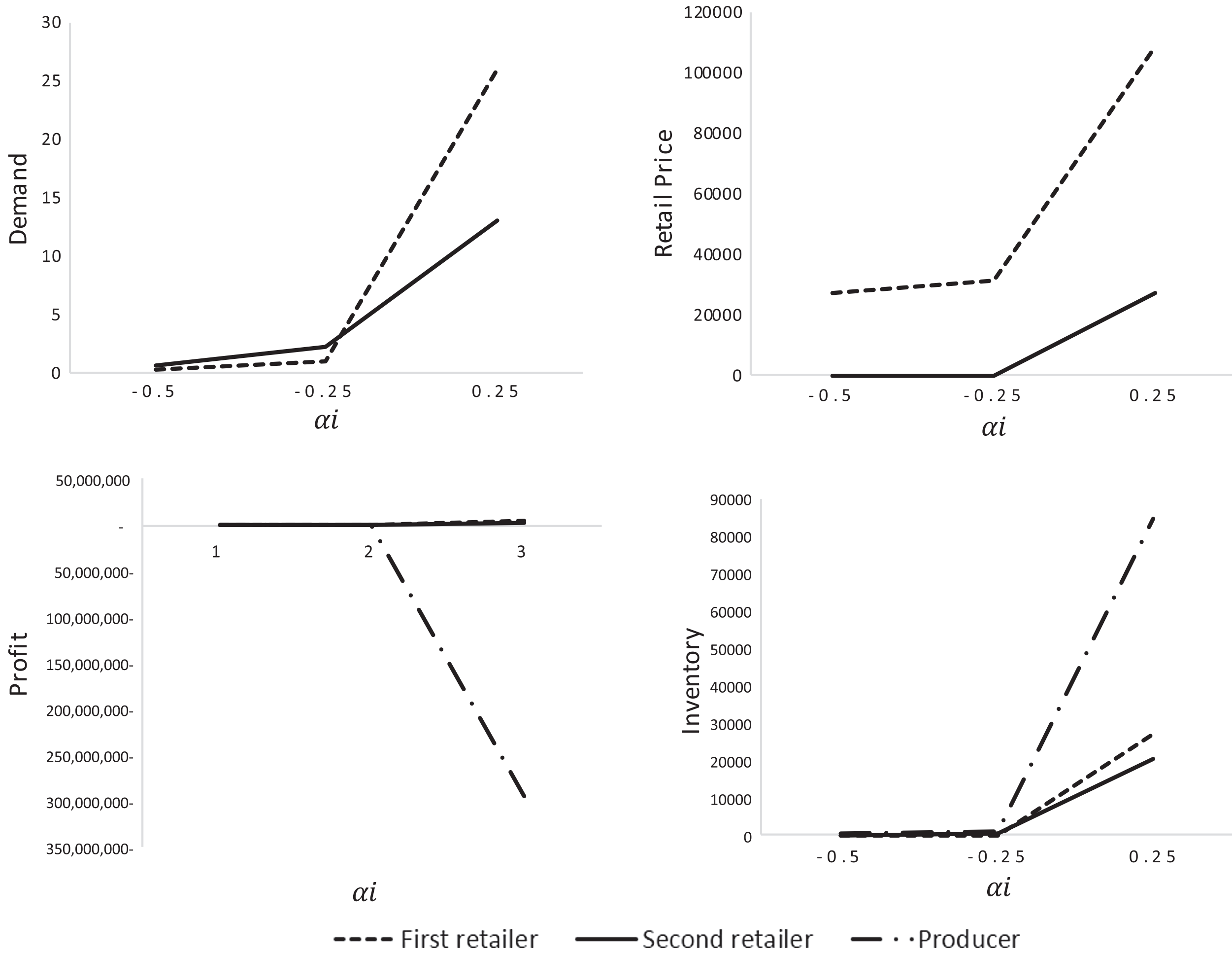

FIGURE 6. The impact of α_i.

product warehouse, and retail warehouses with exponential deterioration rates were considered. A Stackelberg–Nash–Cournot game model was developed and equilibrium wholesale and retail prices, order quantities, lead times and replenishment cycles were obtained using an exact methodology. The effectiveness of the modeling approach was evaluated through a numerical experiment, the sensitivity of the equilibrium is analyzed in terms of the deterioration rate in the producer's, and the retailers' warehouses, the retail and competitor price elasticities, and the market scale.

The results showed the following. An increase in deterioration rate at retail warehouses forces the retailers to offer lower prices to raise demand and prevent the product loss. As prices fall and deterioration costs increase, the chain's profit declines. Higher levels of demand can cause higher levels of production and inventory costs for the producer so that the producer's profit decreases. A growth in the price elasticities implies a reduction in pricing power of the retailer, which can consequently lead to a lower profit for the whole chain. Future studies can consider multiple complementary products and several producers. Different types of demand function such as inventory level-dependent demand or uncertain demand function can be also considered.

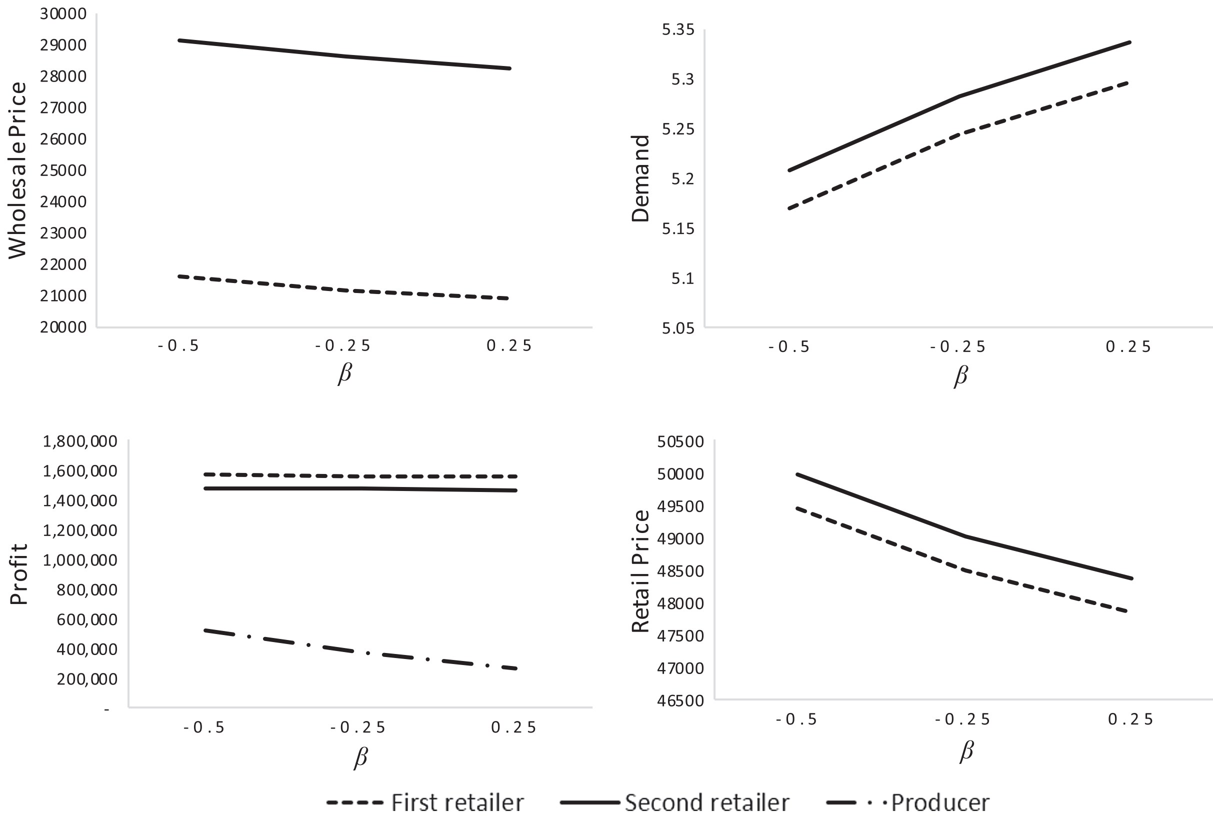

Figure 7. The impact of products deterioration rate for the producer (β).

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