

## ARAS-H: A RANKING-BASED DECISION AIDING METHOD FOR HIERARCHICALLY STRUCTURED CRITERIA

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**Abstract.** In most of real world problems, alternatives are evaluated according to a large set of criteria which confuses the Decision Maker (DM) in terms of allotting alternatives' assessments. So, to reduce the complexity of the presented problem, it is recommended to organize the criteria into a hierarchy tree to decompose the main problem into sub-ones. Therefore, the DM gains detailed insight on each level of the hierarchy instead of focusing only on one level. In this context, we propose an extension of the Additive Ratio ASsessment (ARAS) method to the case of hierarchically structured criteria. The proposed approach is called Hierarchical Additive Ratio ASsessment (ARAS-H) method. A major advantage of the ARAS-H method is that it enables the DM to analyze the partial pre-orders (the rankings of the alternatives) at each node of the criteria tree *i.e.*, according to each sub-criterion. The partial pre-orders present solutions of the problem with respect to each subset of criteria. In view of determining the criteria weights at each level of the hierarchy tree, we apply the AHP method. Finally, we apply the ARAS-H method on a case study related to tourism which aims to rank tourist destination websites brands in accordance with a four levels criteria hierarchy.

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### 1. INTRODUCTION

MCDA can be defined as a general framework for supporting complex decision-making situations with multiple and often conflicting objectives [17,20]. The International Society on Multiple Criteria Decision Making (MCDM) defines it as “The study of methods and procedures by which multiple and conflicting criteria can be incorporated into the decision process”. Belton and Stewart [5] defines MCDA as “an umbrella term to describe a collection of formal approaches which seek to take explicit account of multiple criteria in helping individuals or groups explore decisions that matter”. As a matter of fact, most of Multiple Criteria Decision Aiding (MCDA) methods deal with a flat structure of criteria. However, it is often the case that a practical application imposes a hierarchical structure of criteria. Actually, the hierarchy aims to decompose a complex decision making problem into smaller and manageable sub-ones to facilitate the task for the decision makers (DMs). In fact, for ranking problems, very few methods in the MCDA literature used a hierarchy of criteria for the decomposition of decision problems.

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*Keywords.* Multiple Criteria Decision Aid, Multiple Criteria Hierarchy Process (MCHP), ARAS-H, AHP method, Hierarchy of criteria.

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The ARAS method [31] aims to rank the alternatives from the best to the worst one evaluated according to one level of criteria. Nonetheless, most of case studies require a hierarchical structure of criteria namely in the field of water resource management, public healthcare management, business management, road safety problem and more. Indeed, the hierarchical structure decomposes the primary objective into separate components analyzed on their turn into sub-dimensions toward the lowest level of the hierarchy tree. In view of implementing the hierarchical ARAS (ARAS-H) method [18], we used the terminology defined by the Multiple Criteria Hierarchy Process (MCHP) methodology since the MCHP can be applied to any MCDA method [6]. Thus, the ARAS-H is considered as an extension of the ARAS method in the case of hierarchically structured criteria. The aim is to construct partial pre-orders at each node of the hierarchy tree so that the DM gains insight on all partial dimensions of the problem, instead of focusing uniquely on the comprehensive level. Indeed, structuring decision problems is needed in circumstances requiring a large set of criteria depicting the examined problem. In this way, employing a hierarchical decomposition facilitates the interpretation of the results as it permits DMs to explore feasible elementary dimensions of the whole problem. We adopted a bottom up approach to analyze criteria in different levels of the hierarchy tree. In effect, basing on partial pre-orders obtained at the level of elementary criteria, we proceed to construct partial pre-orders at the upper level of the hierarchy. As a matter of fact, most of multi-criteria methods require fixing criteria weights in order to be implemented. Henceforth, we opted for the AHP [25] being a popular MCDM method to elicit ARAS-H criteria weights at each level of the hierarchy tree by conducting pairwise comparisons of criteria with respect to their importance.

The paper is divided into six sections. Section 1 will give a brief state of the art survey on hierarchical MCDA methods. In Section 2, we will present the AHP method. In Section 3, we will state the steps of the ARAS method. In Section 4, we will explain the different steps of the ARAS-H method. In Section 5, a case study will be presented to discuss the feasibility of the proposed model. In Section 6, we will conclude and we will present our perspectives.

## 2. LITERATURE REVIEW

It is a fact that in the literature, very few authors dealt with the hierarchical structure of criteria in MCDA. To start with, Del Vasto-Terrientes *et al.* [13] proposed a method for ranking the alternatives using multiple and conflicting criteria organized in a hierarchical structure. Therefore, an extension of the ELECTRE-III method [22], called ELECTRE-III-H, was presented. Thus, using a bottom-up approach, the authors constructed outranking relations by propagating the partial pre-orders upwards in the hierarchy. In [12], the authors integrated the indicators of the Web Quality Index (WQI) into the ELECTRE-III-H method. As a matter of fact, MCHP methodology [6] has emerged in few works. In this context, Corrente *et al.* [11] applied the MCHP to the ELECTRE III ranking method. Also, they generalized the SRF (Simos–Roy–Figueira) method [16] and applied Stochastic Multiobjective Acceptability Analysis (SMAA) [4] so that the rankings and preference relations can be constructed at each level of the hierarchy tree. Moreover, Corrente *et al.* [9] applied MCHP to the ELECTRE-Tri methods [23, 30] as extensions of ELECTRE-Tri-B [2], ELECTRE-Tri-C [2] and ELECTRE-Tri-nC [3]. Actually, they considered the case of interaction between criteria (it could be a strengthening, weakening or antagonistic effect) and they extended the SRF method to determine criteria weights in a hierarchical structure of criteria. Nevertheless, they adopted a top-down procedure to infer criteria weights. Additionally, Corrente *et al.* [7] proposed an extension of ELECTRE [24, 28] and PROMETHEE [29] methods to the case of the hierarchy of criteria and adapted Robust Ordinal Regression (ROR) [8] to the hierarchical versions of ELECTRE and PROMETHEE methods. Indeed, ROR takes into account the preference information provided by the DM but its main specificity is that it gives recommendations taking into account not only one but the plurality of instances of the assumed preference model compatible with such preferences. Likewise, Corrente *et al.* [6] applied MCHP on ROR which permits to consider necessary and possible preference information at each node of the hierarchy tree. As a matter of fact, Podinovskaya and Podinovski [21] developed a new model with a hierarchical structure of criteria. They considered the qualitative and quantitative importance of criteria which enables the comparison of decision alternatives for different types of criteria scale. Moreover, Corrente *et al.* [10] proposed

TABLE 1. Saaty's pairwise comparison scale.

Option	Numerical values
Equal	1
Marginally strong	3
Strong	5
Very strong	7
Extremely strong	9
Intermediate values to reflect fuzzy inputs	2,4,6,8
Reflecting dominance of second alternative compared with the first	Reciprocals

an extension of the MCHP methodology to sorting problems within a hierarchical structure of criteria. They formulated a model allowing the inference of preference relations through preference disaggregation techniques based on an additive value function model (namely the UTADIS [14, 33] and UTADIS<sup>GMS</sup> [19] methods). The combination between MCHP and UTADIS, UTADIS<sup>GMS</sup> allows the consideration of global and partial preference judgments to add more flexibility to the specification of the input preference information required in the decision aiding process. Apart from this mentioned approaches, Saaty developed two well-known hierarchical methods: the AHP and the ANP. The application of Analytic Hierarchy Process (AHP) begins with a problem being decomposed into a hierarchy of criteria so as to be more easily analyzed and compared in an independent manner. After this logical hierarchy is constructed, the DM can systematically assess the alternatives by making pair-wise comparisons according to each of the chosen criteria. The Analytic Network Process (ANP) [26] is considered as a generalization of the AHP. The ANP can model complex decision problems, where a hierarchical model is not sufficient. However, the ANP allows for feedback connections and loops.

### 3. THE AHP METHOD

The AHP method decomposes the main problem into a hierarchy of criteria to be easily analyzed by the DM. After the construction of the hierarchy tree, the DMs can systematically assess the alternatives by making pair-wise comparisons for each criterion. In fact, this comparison may use concrete data from the alternatives or human judgments. The steps of the AHP method are as follow:

- Step 1.** The first step consists in decomposing the problem into a hierarchy of goal, criteria, sub-criteria and alternatives.
- Step 2.** The second step of AHP method consists of transforming the empirical comparisons into numerical values to be compared for each criterion (Tab. 1).
- Step 3.** The pairwise comparisons of criteria are organized into a square matrix. The diagonal elements of the matrix are equal to 1. The criterion in the  $i$ th row is more important than criterion in the  $j$ th column if the value of element  $(i, j)$  is greater than 1. The  $(j, i)$  element of the matrix corresponds to the reciprocal of the  $(i, j)$  element.
- Step 4.** The contribution of each criterion to the organizational goal is determined by calculating the priority vector (or Eigenvector). The Eigenvector shows the relative weights between each criterion. It is obtained by calculating the mathematical average according to all criteria. The sum of all values from the vector is always equal to 1. The elements of the normalized eigenvector are known as weights for the criteria and ratings for the alternatives.
- Step 5.** The next step is to look for any data inconsistencies. The objective is to capture enough information to determine whether the DMs have been consistent in their choices or not. Thus, the consistency condition should verify this equality:  $a_{ij} = a_{ik} \times a_{kj}$  for all  $i, k, j$ . It expresses the transitivity of preferences. The consistency index (CI) is calculated as:  $CI = \frac{\lambda_{\max} - n}{n - 1}$  where  $\lambda_{\max}$  is the maximum eigenvalue of the judgment matrix. This consistency index (CI) can be compared with an average consistency index (RI) (Tab. 2). The

TABLE 2. The average consistency index (RI).

$N$	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

derived ratio,  $\frac{CI}{RI}$  is called the consistency ratio (CR). Saaty states that the value of CR should be less than 0,1.

**Step 6.** The last step of the AHP method is to produce the alternatives' priority values based on the judged importance of one alternative over another with respect to a common criterion.

#### 4. THE ARAS METHOD

**Step 1.** Constructing the decision matrix  $X$  for  $m$  alternatives and  $n$  criteria.

**Step 2.** Normalizing the decision making matrix to unify the incommensurable measures of the criteria so all the performances can be compared. The authors suggest these two normalization formula.

The criteria to be maximized, are normalized as follows.

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}.$$

The criteria to be minimized, are normalized as follows.

$$x_{ij} = \frac{1}{x_{ij}^*};$$

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$$

where,  $x_{ij}$  is the performance value of the alternative  $i$  with respect to the criterion  $j$ ;  $\bar{x}_{ij}$  denotes the normalized values of the normalized decision-making matrix  $\bar{X}$  and  $x_{ij}^*$  is the original value of minimized criteria.

**Step 3.** Forming the weighted-normalized matrix  $\hat{X}$  through this formula:

$$\hat{x}_{ij} = \bar{x}_{ij} w_j; \forall i = 1, \dots, m \text{ and } j = 1, \dots, n$$

where,  $\bar{x}_{ij}$  is the normalized value of the criterion  $j$ ;  $w_j$  is the weight of the criterion  $j$  such that  $\sum_{j=1}^n w_j = 1$  and  $w_j > 0$  for all  $j$ .

**Step 4.** Determining the values of optimality function denoted by  $S_i$  such that:

$$S_i = \sum_{j=1}^n \hat{x}_{ij}; \forall i = 1, \dots, m$$

**Step 5.** In ARAS method, the utility degree value  $K_i$  determines the relative efficiency of a feasible alternative  $i$ . That is to say,

$$K_i = \frac{S_i}{S_0}; \forall i = 1, \dots, m$$

where,  $S_0$  is the optimal value (i.e., the maximum value of  $S_i$ ) and the calculated values  $K_i$  are in the interval  $[0,1]$ .

**Step 6.** Ranking in an increasing order the values of the utility degrees  $K_i$  to rank the alternatives from the best one to the least important.

Nonetheless, the ARAS method deals with a flat structure of criteria. However, in most real world applications, criteria are organized naturally according to a hierarchical structure. For that, in the next section, we will define the MCHP methodology and present the ARAS-H algorithm.

## 5. THE PROPOSED ARAS-H ALGORITHM

### 5.1. The multiple criteria hierarchy process

In this section, we define the different concepts and notation with regard to a hierarchical structure of criteria. Therefore, we will distinguish between three types of criteria (Fig. 1).

$R$  is a set composed by a unique element that is the most general criterion. This corresponds to the root node, placed at the top of the tree. This criterion represents the main goal of the DM.

$E$  is the set of the most specific criteria, called elementary criteria. They are placed at the lowest level of the hierarchical tree (*i.e.*, the leaves). The performance of the alternatives is evaluated only in relation to these elementary criteria.

$I$  is the set of intermediate criteria (or sub-criteria). They are placed at intermediate levels of the tree, between the root criterion ( $R$ ) and the elementary criteria ( $E$ ).

$G$  is the set of all criteria such that  $G = R \cup E \cup I$

Taking Figure 1 as an example of a hierarchy of criteria, then:

- $R = \{G_0\}$
- $I = \{G_1, G_2\}$
- $E = \{G_{(1,1)}, G_{(1,2)}, G_{(2,1)}\}$
- $G = \{G_0, G_1, G_2, G_{(1,1)}, G_{(1,2)}, G_{(2,1)}\}$

**Definition 5.1.** The set of criteria is structured according to the following relations:

The root criterion in  $R$  does not have any parent.

Each intermediate criterion in  $I$  has a unique parent.

Each elementary criterion in  $E$  has a unique parent.

In what follows, we present the basic notations of the hierarchy of criteria introduced by Corrente *et al.* [6].

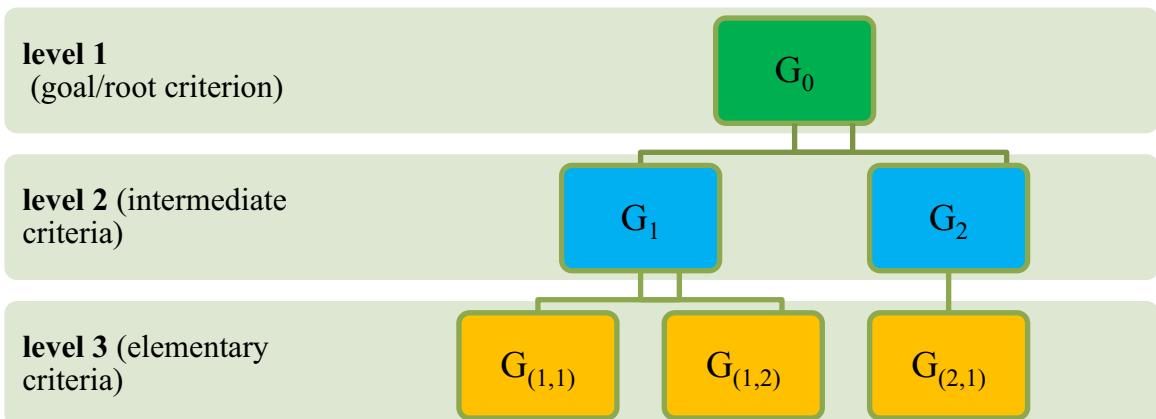


FIGURE 1. Example of a hierarchy of criteria.

Let:

- $A$  is the finite set of alternatives;
- $l$  is the number of levels in the hierarchy of criteria;
- $G$  is the set of all criteria at all considered levels;
- $I_G$  is the set of indices presenting position of the specific criteria in the hierarchy tree;
- $G_r \in I_G$ , with  $r \in I_G$  denotes a subcriterion of the first level criterion  $G_f$  at level  $h$ ;
- $n(r)$  is the number of subcriteria of  $G_r$  in the subsequent level *i.e.*, the direct subcriteria of  $G_r$  are  $G_{(r,1)}, \dots, G_{(r,n(r))}$ ;
- $g_t: A \rightarrow \mathbb{R}$ , with  $t \in I_G$ , denotes an elementary subcriterion of the first level criterion  $g_{f1}$ , *i.e.*, a subcriterion at level  $l$ ;
- $EL$  is the set of indices of all elementary subcriteria,
- $E(G_r)$  is the set of indices of elementary subcriteria descending from  $G_r$ ,

$$E(G_r) = \{(r, i_{h+1}, \dots, i_l) \in I_G\} \text{ where } \begin{cases} i_{h+1} = 1, \dots, n(r) \\ \dots \\ i_l = 1, \dots, n(r, i_{h+1}, \dots, i_{l-1}) \end{cases}.$$

Thus,  $E(G_r) \in EL$  and, more precisely,  $E(G_r) = EL$  if all elementary subcriteria descend from criterion  $G_r$ .  
 LBO is the set of indices of all subcriteria of the last but one level;  
 LB ( $G_r$ ) is the set of indices of subcriteria of the last but one level descending from criterion/subcriterion  $G_r$ ;  
 When  $r = 0$ , then by  $G_r = G_0$ , then  $E(G_0) = EL$  and LB ( $G_0$ ) = LBO.

Within MCHP, in each node  $G_r \in G$  of the hierarchy tree, there exists a preference relation  $\succsim r$  on  $A$ , such that for all  $a, b \in A$ ,  $a \succsim r b$  means “ $a$  is at least as good as  $b$  on subcriterion  $G_r$ ”. The preference relations  $\succsim r$  have to satisfy a dominance principle for the hierarchy of criteria. In other words, if alternative  $a$  is at least as good as alternative  $b$  for all subcriteria  $G(r, j)$  of  $G_r$  of the level immediately below, then  $a$  is at least as good as  $b$  on  $G_r$ .

$w_j$ : be the criteria weight such that  $\sum_{j \in I_G} w_j = 1$  and  $w_j > 0$ .

We adopt a bottom-up approach for the construction of partial pre-orders *i.e.*, we start by ranking the alternatives from the lowest level of the hierarchy tree upward to the root criterion.

## 5.2. The ARAS-H algorithm

The first part of the proposed algorithm corresponds to the calculation of the utility degrees  $K_i$  of the alternatives according to the elementary criteria. In this case, the ARAS method is applied to aggregate performances of the alternatives at the lowest level of the criteria's hierarchy tree. The aim of the first stage of the ARAS-H algorithm is to construct, in order to interpret later on, the partial pre orders of intermediate criteria obtained from the calculation of the utility degrees  $K_i$  for criteria whose only descendants are elements in the set of elementary criteria ( $E$ ). Then, at the upper level of the criteria tree, we calculate the utility degrees  $K_i'$  of each alternative to rank them in a decreasing order. Therefore, we construct the partial pre-orders according to the 1st level of intermediate criteria. The last stage of the ARAS-H algorithm corresponds to the calculation of the utility degrees of the alternatives to rank them in a decreasing order at the first level of the hierarchy tree. The aim is to construct the complete pre order that is ranking the alternatives according to the root criterion.

**For**  $h = l$

**Step 1.** Fixing the hierarchical structure of the set of criteria using the MCHP, distinguishing the subsets of elementary criteria, and higher level criteria, up to the root criterion.

**Step 2.** Asking the DM to provide the performance matrix in which the alternatives are evaluated according to elementary criteria.

**Step 3.** Normalizing the performance matrix with respect to the min-max normalization technique.

$$\text{For criteria maximization : } \bar{x}_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \quad i \in A \text{ and } j \in \text{EL}$$

$$\text{For criteria minimization : } \bar{x}_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \quad i \in A \text{ and } j \in \text{EL}.$$

**Step 4.** Applying the AHP method to determine criteria weights and integrate them into ARAS-H to construct the weighted normalized decision matrix which consists in multiplying the normalized values by each elementary criterion weight.

$$\hat{x}_{ij} = \bar{x}_{ij} w_j; \quad i \in A; j \in \text{EL}.$$

**Step 5.** Calculating the optimality value  $S_i$  of each alternative  $i$  according to each intermediate criterion  $G_r$  from the weighted normalized values on elementary criteria descending from  $G_r$  such that:

$$S_i = \sum_{j \in E(G_r)} \hat{x}_{ij}; \quad \forall i \in A,$$

with,  $E(G_r)$  is the set of indices of elementary subcriteria descending from  $G_r$ .

**Step 6.** Calculating the utility degree  $K_i$  of each alternative according to each intermediate criterion  $G_r$  from the weighted normalized values on elementary criteria descending from  $G_r$  such that:

$$K_i = \frac{S_i}{S_0}; \quad \forall i \in A, \quad \forall G_r.$$

where,  $S_0$  is the optimal value (*i.e.*, the maximum value of  $S_i$ ) and the calculated values of  $K_i$  are in the interval  $[0,1]$ .

**Step 7.** Constructing the partial pre-orders of sub-criteria from ranking in a decreasing order the values of utility degrees  $K_i$ .

**For**  $h = l - 1$

**Step 8.** Calculating the utility degrees  $K_i$  for each alternative  $i \in A$  according to each first level intermediate criterion  $G_f$ .

$$K_i = \sum_{j \in I_G} K_i w_j; \quad \forall i \in A, \forall G_f.$$

Criteria weights are normalized such that

$$\sum_{j \in I_G} w_j = 1.$$

**Step 9.** Calculating the fraction  $K'_i = \frac{K_i}{\sum_{i \in A} K_i}$ .

**Step 10.** Ranking in a decreasing order the values of  $K'_i$ .

**Step 11.** Constructing the partial pre-orders at the upper level of the criteria's hierarchy tree.

For  $h = 1$  (*i.e.*, at the first level of the hierarchy tree).

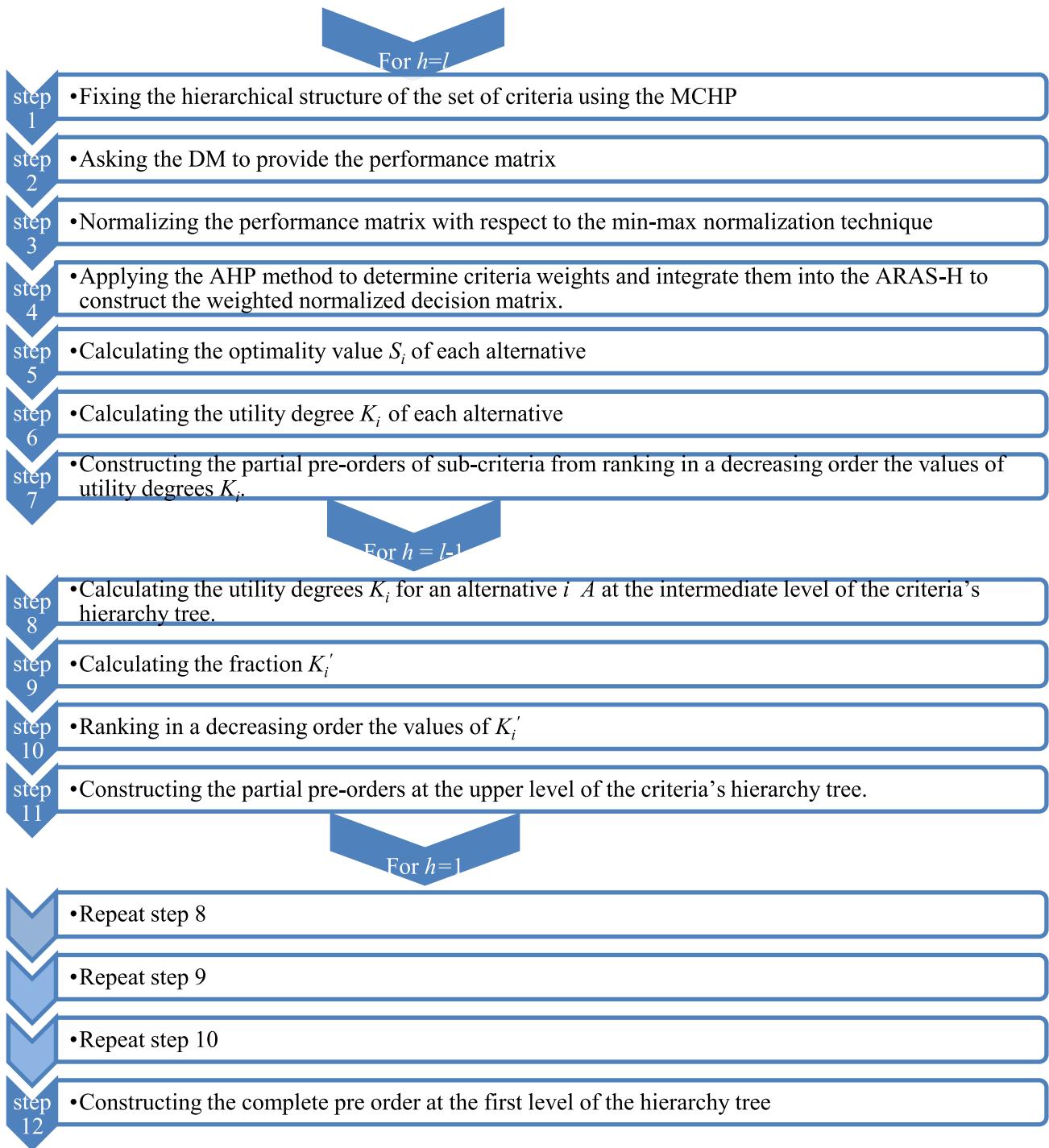
Go to step 8.

Go to step 9.

Go to step 10.

**Step 12** Constructing the complete pre order at the first level of the hierarchy tree (*i.e.*, ranking the alternatives according to the root criterion).

The proposed ARAS-H algorithm is illustrated in the below flow chart.



## A flow chart for the ARAS-H algorithm

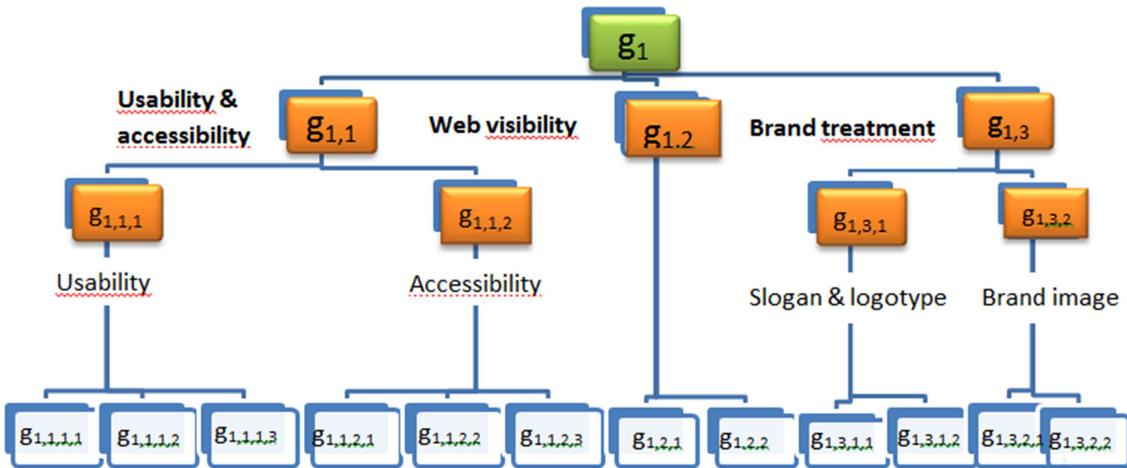


FIGURE 2. Hierarchy for website evaluation [13].

## 6. AN ILLUSTRATIVE EXAMPLE

The aim of this example is to present websites designed to promote tourist destination brands. In this section, we address the analysis of this dataset with the proposed ARAS-H method. Indeed, websites have become important tools for the communication of destination brands and the sale of a range of tourism services and related items [15].

Organizing criteria into a hierarchical structure will permit the DM to gain insight on rankings not only on the comprehensive level but at each node of the hierarchy. This will help him to identify the strengths and weaknesses of each website and therefore to develop appropriate strategies to deal with the discovered weaknesses. Due to the complexity of the problem, experts organized criteria into a hierarchical structure (Fig. 2). They are asked to identify the possible criteria to evaluate 10 tourist websites in different regions in the world (Andalusia, Catalonia, Barcelona, Madrid, Santiago de Compostela, Rias Baixas, Stockholm, Wales, Rome and Switzerland). The following dataset comes from a Spanish research project entitled “Online Communication for Destination Brands. Development of an Integrated Assessment Tool: Websites, Mobile Applications and Social Media (CODETUR)”, done on 2012. The main objective of this project was to define a website evaluation framework to help expert managers in order to boost and optimize the online communication of their brands. Once this framework has been defined, MCDA methods are therefore needed to support the decision making process.

$g_{1,1,1,1}$ : General indicators

$g_{1,1,1,2}$ : Multimedia

$g_{1,1,1,3}$ : Help

$g_{1,1,2,1}$ : Font

$g_{1,1,2,2}$ : Navigation

$g_{1,1,2,3}$ : Accessibility

$g_{1,2,1}$ : Internal factors

$g_{1,2,2}$ : External factors

$g_{1,3,1,1}$ : Slogan

$g_{1,3,1,2}$ : Logotype

$g_{1,3,2,1}$ : Brand in images

$g_{1,3,2,2}$ : Brand in text

TABLE 3. Criteria weights obtained by AHP.

	Criteria	Weights
	$g_{1,1,1,1}$	0.43
	$g_{1,1,1,2}$	0.49
	$g_{1,1,1,3}$	0.08
	$g_{1,1,2,1}$	0.06
	$g_{1,1,2,2}$	0.48
Elementary criteria	$g_{1,1,2,3}$	0.46
	$g_{1,2,1}$	0.2
	$g_{1,2,2}$	0.8
	$g_{1,3,1,1}$	0.12
	$g_{1,3,1,2}$	0.88
	$g_{1,3,2,1}$	0.8
	$g_{1,3,2,2}$	0.2
	$g_{1,1,1}$	0.67
Intermediate criteria	$g_{1,1,2}$	0.33
	$g_{1,3,1}$	0.83
	$g_{1,3,2}$	0.17
	$g_{1,1}$	0.73
1st level intermediate criteria	$g_{1,2}$	0.21
	$g_{1,3}$	0.06

TABLE 4. The performance matrix.

MAX	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$	$g_{1,3,2,2}$
Andalusia	100	100	60	25	62.5	16	72.2	75.53	100	58	100	60
Catalonia	74	30	0	75	87.5	38.67	33.3	59	0	78	70	30
Barcelona	100	60	80	75	62.5	100	74.1	85.93	0	74	100	70
Madrid	88	100	60	50	100	61.33	75.9	72.87	0	38	50	60
Santiago	100	80	20	50	50	32	100	68.6	100	48	80	60
Rias Baixas	50	30	0	75	62.5	16	46.3	59	100	38	50	30
Stockholm	100	100	100	100	62.5	38.67	74.1	82.47	100	54	60	70
Wales	88	70	0	100	87.5	0	92.6	65.93	100	58	80	30
Rome	100	100	100	100	62.5	16	74.1	65.93	0	38	70	60
Switzerland	76	100	100	100	62.5	16	83.3	90.2	100	68	100	100
$W_j$	0.44	0.49	0.08	0.06	0.48	0.46	0.2	0.8	0.12	0.88	0.8	0.2

All criteria are to maximize.

The application of the AHP method gave us criteria weights at all hierarchy levels (Tab. 3) (for more details see Appendix A).

#### Criteria weight determination

After obtaining the weights of all criteria in the hierarchy, we proceed to the application of the ARAS-H method to rank the alternatives at different levels of the hierarchy. To demonstrate the proposed approach, the DM is asked to supply us with the evaluation matrix of the alternatives according to elementary criteria (Tab. 4).

The next step consists in determining the normalized decision-making matrix (Tab. B.1, Appendix B) and the weighted normalized decision-making matrix (Tab. B.2, Appendix B).

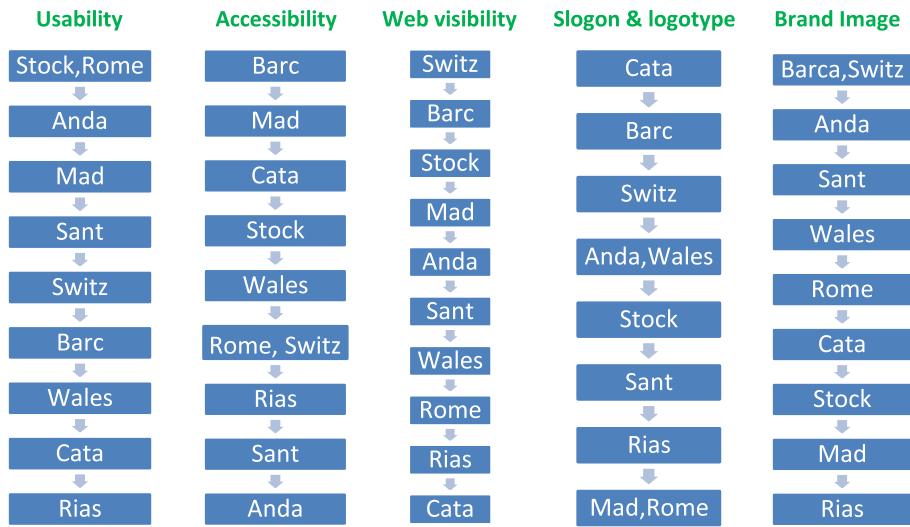


FIGURE 3. The partial pre-orders according the first level of intermediate criteria.

After constructing the weighted-normalized decision matrix, the ARAS method is applied for each subset of elementary criteria (following the organization of the criteria tree). The obtained partial pre-orders (Fig. 3) present the rankings of the alternatives with respect to the intermediate criteria, which are based only on elementary ones. In fact, the partial pre-orders are the results of the first stage of the ARAS-H algorithm at the level of intermediate criteria whose descendants are all elementary criteria. Within these classifications, the DM can observe the position of each website in the five lowest intermediate criteria. We proceed by propagating this information to the upper nodes of the hierarchy tree (for more details see Appendix B).

At this stage, the evaluations of partial pre-orders presented in Figure 3 are propagated to the upward level of the hierarchy tree as shown in Figure 4.

The alternative Rias Baixas is considered to be the worst alternative according to the sub-criterion “usability” together with the intermediate criterion “usability and accessibility”. Madrid takes the second position according to sub-criterion “accessibility”. For the intermediate criterion “usability and accessibility”, it outranks all of the other alternatives. Wales takes the 7th position in terms of “usability”. For the intermediate criterion “usability and accessibility”, it takes the 8th position. In other words, the results are quite similar from level to level with respect to the hierarchical structure of criteria which proves the robustness of the proposed ARAS-H algorithm.

For the root criterion, we calculate the utility degrees of all alternatives to obtain the final ranking as presented in Table 5.

The final ranking of the alternatives according to the root criterion as presented in Figure 5 is:

$$\text{Barc} \succ \text{Stock} \succ \text{Mad} \succ \text{Switz} \succ \text{Anda} \succ \text{Rome} \succ \text{Sant} \succ \text{Wales} \succ \text{Cata} \succ \text{Rias}.$$

As can be seen, Barcelona outranks all the other alternatives according to sub-criterion “brand treatment”. Also, it is considered as the best alternative for the root criterion. Together with Rias Baixas, which is considered to be the worst according to sub-criterion “usability and accessibility”, is also the worst alternative for the root criterion. The founded results are robust and therefore recommended.

However, the complete pre-order obtained by ARAS-H is slightly different from the one founded by ELECTRE-III-H (Fig. 6). As can be noticed from Figure 5 together with Figure 6, Barcelona and Stockholm maintain their position in both methods. They are considered to be the best alternatives. Catalonia and Rias Baixas maintain their position too. They are considered to be the worst alternatives in both ELECTRE-III-H and ARAS-H methods.



FIGURE 4. The partial pre-orders according the first level of intermediate criteria.

TABLE 5. The ranking of the alternatives according to the root criterion.

$g_1$	$K_i$	$K_i$	Rank
Andalusia	0.67	0.11	5
Catalonia	0.33	0.05	9
Barcelona	0.82	0.14	1
Madrid	0.74	0.12	3
Santiago	0.57	0.09	7
Rias Baixas	0.09	0.01	10
Stockholm	0.81	0.14	2
Wales	0.49	0.08	8
Rome	0.64	0.11	6
Switzerland	0.73	0.12	4



FIGURE 5. The complete pre-order of the alternatives from the ARAS-H method.

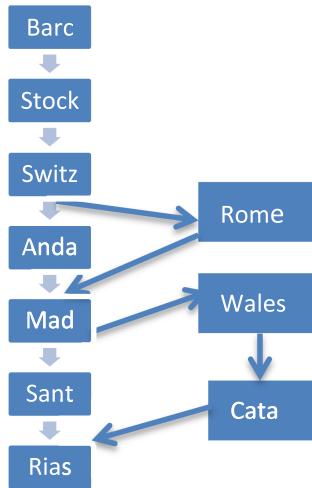


FIGURE 6. The complete pre-order obtained from ELECTRE-III-H [13].

TABLE 6. An illustration of criteria weight values.

Weights	Simos	AHP	Entropy	ANP
$w_{1,1,1,1}$	0.43	0.44	0.01	0.502
$w_{1,1,1,2}$	0.36	0.49	0.04	0.38
$w_{1,1,1,3}$	0.21	0.08	0.26	0.118
$w_{1,1,2,1}$	0.3	0.06	0.04	0.144
$w_{1,1,2,2}$	0.37	0.48	0.01	0.315
$w_{1,1,2,3}$	0.33	0.46	0.2	0.54
$w_{1,2,1}$	0.33	0.2	0.02	0.111
$w_{1,2,2}$	0.67	0.8	0.01	0.889
$w_{1,3,1,1}$	0.2	0.12	0.31	0.271
$w_{1,3,1,2}$	0.8	0.88	0.04	0.728
$w_{1,3,2,1}$	0.55	0.8	0.02	0.875
$w_{1,3,2,2}$	0.45	0.2	0.04	0.125
$w_{1,1,1}$	0.53	0.67	0.32	0.75
$w_{1,1,2}$	0.47	0.33	0.25	0.25
$w_{1,3,1}$	0.4	0.83	0.34	0.889
$w_{1,3,2}$	0.6	0.17	0.06	0.111
$w_{1,1}$	0.5	0.73	0.57	0.348
$w_{1,2}$	0.2	0.21	0.03	0.353
$w_{1,3}$	0.3	0.06	0.4	0.298

Indeed, a sensitivity analysis is crucial at this stage to study the effect of the criteria weight determination technique on the alternative ranking especially on the complete pre-order. As a matter of fact, very few methods in the literature dealt with criteria weight elicitation within a hierarchical structure. Among them, we cite the Simos method [27] which is considered to be a very simple and easy technique for the DMs. The procedure consists of ranking the subsets of cards from the least important to the most important while assigning a position to each card including the white ones. The procedure ends with the determination of the normalized weights. Another criteria weight elicitation method is the Entropy [32]. In this method, the DM interferes indirectly in

TABLE 7. The rankings of the alternatives according to the root criterion with respect to the ARAS-H method.

Alternatives	Simos	AHP	Entropy	ANP
Andalusia	5	5	5	4
Catalonia	9	9	10	9
Barcelona	2	1	6	2
Madrid	7	3	7	7
Santiago	6	7	1	5
Rias Baixas	10	10	8	10
Stockholm	8	2	4	3
Wales	4	8	2	6
Rome	3	6	9	8
Switzerland	1	4	3	1

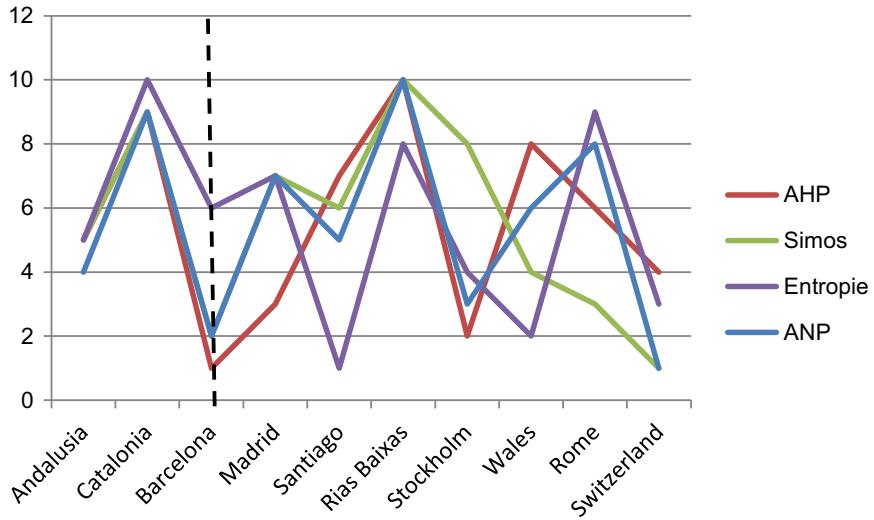


FIGURE 7. The curves of rankings according to different criteria weight determination methods.

the weight elicitation process. The criteria weights values are determined through the alternative performances from the decision making matrix. As a matter of fact, Abdullah and Otheman [1], extended the classical Entropy method to the case of hierarchically structured criteria. Besides these mentioned techniques, we used the ANP method. Therefore, Table 6 illustrates the obtained weights.

In Table 7, we present the rankings of the alternatives through the ARAS-H method according to the root criterion.

From a first view, the curves vary in a non-simultaneous way. But if we take the first part, the four methods take the same variation especially the AHP, ANP and Simos methods. Their variations are superimposed. This result is expected since the alternatives Andalusia, Catalonia and Barcelona take nearly the same position with respect to each of these criteria weight elicitation methods. After that, the curves fluctuate in different ways with a major disparity. So, we can conclude that the ARAS-H method is however sensitive to a change in different criteria weight determination techniques (Fig. 7).

## 7. CONCLUSION

In this paper, we proposed a ranking method which deals with a hierarchical structure of criteria called ARAS-H method. This method adopts a bottom-up approach. It constructs partial pre-orders from the lowest

level of the hierarchy tree and propagates them to the upward level until reaching the root criterion to construct the complete pre-order. Also, we presented a sensitivity analysis which aims to study the effect of some criteria weight elicitation methods on the complete pre-order of the ARAS-H method. The advantage of the proposed approach is that the DM gains insight on the ranking of the alternatives at each node of the hierarchy tree. Therefore, he will be able to detect the inconsistencies and analyzing them in a deepest way. Nevertheless, the proposed method deals with a small problem within a small set of alternatives as well as criteria. As a perspective, we intend to elicit ARAS-H criteria weights through mathematical programming at each level of the hierarchy tree. Moreover, we aim to deal with fuzzy data to allow the model to be applicable in the case of lack of certainty. Consequently, we will develop the fuzzy ARAS-H (F-ARAS-H) method.

## APPENDIX A.

Comparison matrix for usability group of criteria.

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$
$g_{1,1,1,1}$	1	1	5
$g_{1,1,1,2}$	1	1	7
$g_{1,1,1,3}$	0,2	0,14	1
Sum	2,2	2,14	13

Comparison matrix for usability group of criteria after normalization.

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	Sum	Eigenvector= $w_j$
$g_{1,1,1,1}$	0.455	0.467	0.385	1.306	0.435
$g_{1,1,1,2}$	0.455	0.467	0.538	1.46	0.487
$g_{1,1,1,3}$	0.091	0.065	0.077	0.233	0.078

Comparison matrix for accessibility group of criteria.

	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$
$g_{1,1,2,1}$	1	0.111	0.125
$g_{1,1,2,2}$	9	1	1
$g_{1,1,2,3}$	8	1	1
Sum	18	2.111	2.125

Comparison matrix for accessibility group of criteria after normalization.

	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	Sum	Eigenvector
$g_{1,1,2,1}$	0.056	0.053	0.059	0.167	0.056
$g_{1,1,2,2}$	0.5	0.474	0.471	1.444	0.481
$g_{1,1,2,3}$	0.444	0.474	0.471	1.389	0.463

Comparison matrix for web visibility group of criteria.

	$g_{1,2,1}$	$g_{1,2,2}$
$g_{1,2,1}$	1	0.25
$g_{1,2,2}$	4	1
Sum	5	1.25

Comparison matrix for web visibility group of criteria after normalization.

	$g_{1,2,1}$	$g_{1,2,2}$	Sum	Eigenvector
$g_{1,2,1}$	0.2	0.2	0.4	0.2
$g_{1,2,2}$	0.8	0.8	1.6	0.8

Comparison matrix for brand image group of criteria.

	$g_{1,3,2,1}$	$g_{1,3,2,2}$
$g_{1,3,2,1}$	1	4
$g_{1,3,2,2}$	0.25	1
Sum	1.25	5

Comparison matrix for brand image group of criteria after normalization.

	$g_{1,3,2,1}$	$g_{1,3,2,2}$	Sum	Eigenvector
$g_{1,3,2,1}$	0.8	0.8	1.6	0.8
$g_{1,3,2,2}$	0.2	0.2	0.4	0.2

Comparison matrix for usability & accessibility group of criteria.

	$g_{1,1,1}$	$g_{1,1,2}$
$g_{1,1,1}$	1	2
$g_{1,1,2}$	0.5	1
Sum	1.5	3

Comparison matrix for usability & accessibility group of criteria after normalization.

	$g_{1,1,1}$	$g_{1,1,2}$	Sum	Eigenvector
$g_{1,1,1}$	0.667	0.667	1.333	0.667
$g_{1,1,2}$	0.333	0.333	0.667	0.333

Comparison matrix for brand treatment group of criteria.

	$g_{1,3,1}$	$g_{1,3,2}$
$g_{1,3,1}$	1	5
$g_{1,3,2}$	0.2	1
Sum	1.2	6

Comparison matrix for brand treatment group of criteria after normalization.

	$g_{1,3,1}$	$g_{1,3,2}$	Eigenvector
$g_{1,3,1}$	0.833	0.833	0.833
$g_{1,3,2}$	0.167	0.167	0.167

Comparison matrix for the root criterion.

	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$
$g_{1,1}$	1	9	8
$g_{1,2}$	0.111	1	7
$g_{1,3}$	0.125	0.143	1
Sum	1.236	10.143	16

Comparison matrix for the root criterion after normalization.

	$g_{1,1}$	$g_{1,2}$	$g_{1,3}$	Sum	Eigenvector
$g_{1,1}$	0.809	0.887	0.5	2.196	0.732
$g_{1,2}$	0.09	0.099	0.438	0.626	0.209
$g_{1,3}$	0.101	0.014	0.063	0.178	0.059

## APPENDIX B.

TABLE B.1. Normalized decision-making matrix.

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$
Andalusia	1	1	0.6	0	0.125	0.16	0.58	0.42	1	0.5	1
Catalonia	0.48	0	0	0.67	0.375	0.3867	0	0	0	1	0.4
Barcelona	1	0.43	0.8	0.67	0.125	1	0.61	0.86	0	0.9	1
Madrid	0.66	1	0.6	0.33	0.5	0.6133	0.64	0.44	0	0	0
Santiago	1	0.71	0.2	0.33	0	0.32	1	0.31	1	0.25	0.6
Rias Baixas	0	0	0	0.67	0.125	0.16	0.19	0	1	0	0
Stockholm	1	1	1	1	0.125	0.3867	0.61	0.75	1	0.4	0.2
Wales	0.66	0.57	0	1	0.375	0	0.89	0.22	1	0.5	0.6
Rome	1	1	1	1	0.125	0.16	0.61	0.22	0	0	0.4
Switzerland	0.52	1	1	1	0.125	0.16	0.75	1	1	0.75	1

TABLE B.2. Weighted normalized decision-making matrix.

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$g_{1,2,1}$	$g_{1,2,2}$	$g_{1,3,1,1}$	$g_{1,3,1,2}$	$g_{1,3,2,1}$
Andalusia	0.44	0.49	0.048	0	0.06	0.074	0.116	0.336	0.12	0.44	0.8
Catalonia	0.211	0	0	0.04	0.18	0.178	0	0	0	0.88	0.32
Barcelona	0.44	0.211	0.064	0.04	0.06	0.46	0.122	0.688	0	0.792	0.8
Madrid	0.29	0.49	0.048	0.02	0.24	0.282	0.128	0.352	0	0	0
Santiago	0.44	0.348	0.016	0.02	0	0.147	0.2	0.248	0.12	0.22	0.48
Rias Baixas	0	0	0	0.04	0.06	0.074	0.038	0	0.12	0	0
Stockholm	0.44	0.49	0.08	0.06	0.06	0.178	0.122	0.6	0.12	0.352	0.16
Wales	0.29	0.279	0	0.06	0.18	0	0.178	0.176	0.12	0.44	0.48
Rome	0.44	0.49	0.08	0.06	0.06	0.074	0.122	0.176	0	0	0.32
Switzerland	0.229	0.49	0.08	0.06	0.06	0.074	0.15	0.8	0.12	0.66	0.8

Usability.

	$g_{1,1,1,1}$	$g_{1,1,1,2}$	$g_{1,1,1,3}$	$S_i$	$K_i$	Rank
Andalusia	0.44	0.49	0.05	0.98	0.97	2
Catalonia	0.21	0	0	0.21	0.21	8
Barcelona	0.44	0.21	0.06	0.71	0.71	6
Madrid	0.29	0.49	0.05	0.83	0.82	3
Santiago	0.44	0.35	0.02	0.80	0.79	4
Rias Baixas	0	0	0	0	0	9
Stockholm	0.44	0.49	0.08	1.01	1	1
Wales	0.29	0.28	0	0.57	0.56	7
Rome	0.44	0.49	0.08	1.01	1	1
Switzerland	0.23	0.49	0.08	0.8	0.79	5

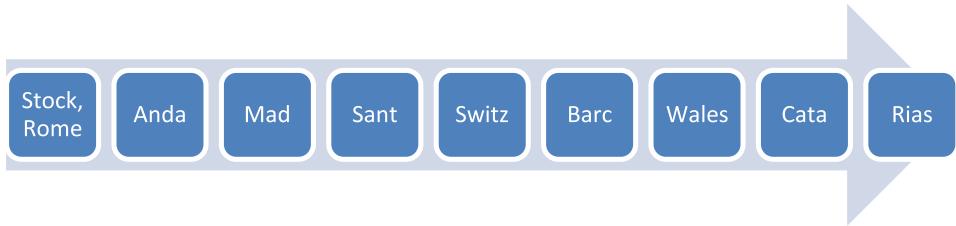


FIGURE B.1. Example of a partial a pre-order generated from elementary criteria ( $g_{1,1,1,1}$ ,  $g_{1,1,1,2}$ ,  $g_{1,1,1,3}$ ).

Accessibility.

	$g_{1,1,2,1}$	$g_{1,1,2,2}$	$g_{1,1,2,3}$	$S_i$	$K_i$	Rank
Andalusia	0	0.06	0.074	0.13	0.24	9
Catalonia	0.04	0.18	0.18	0.4	0.71	3
Barcelona	0.04	0.06	0.46	0.56	1	1
Madrid	0.02	0.24	0.28	0.54	0.97	2
Santiago	0.02	0	0.15	0.167	0.298	8
Rias Baixas	0.04	0.06	0.07	0.17	0.31	7
Stockholm	0.06	0.06	0.18	0.3	0.53	4
Wales	0.06	0.18	0	0.24	0.43	5
Rome	0.06	0.06	0.07	0.19	0.34	6
Switzerland	0.06	0.06	0.07	0.19	0.34	6

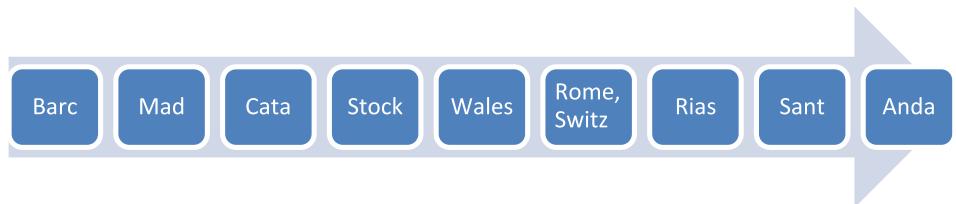


FIGURE B.2. Example of a partial a pre-order generated from elementary criteria ( $g_{1,1,2,1}$ ,  $g_{1,1,2,2}$ ,  $g_{1,1,2,3}$ ).

Usability & Accessibility.

	$K_i$	Rank
Andalusia	0.73	5
Catalonia	0.37	9
Barcelona	0.80	3
Madrid	0.87	1
Santiago	0.63	7
Rias Baixas	0.10	10
Stockholm	0.84	2
Wales	0.52	8
Rome	0.78	4
Switzerland	0.64	6

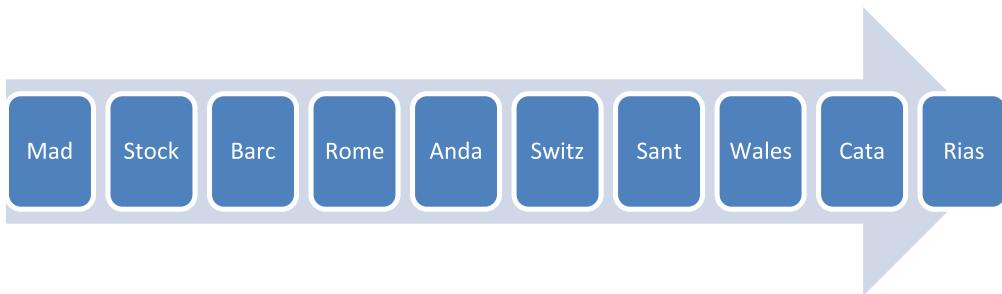


FIGURE B.3. The partial pre-order generated from the aggregation of “usability” and “accessibility” criteria.

## REFERENCES

- [1] L. Abdullah and A. Otheman, A new entropy weight for sub-criteria in interval type-2 fuzzy TOPSIS and its application. *Int. J. Intell. Syst. Appl.* **2** (2013) 25.
- [2] J. Almeida-Dias, J.R. Figueira and B. Roy, Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions. *Eur. J. Oper. Res.* **3** (2010) 565–580.
- [3] J. Almeida-Dias, J.R. Figueira and B. Roy, A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-nC method. *Eur. J. Oper. Res.* **3** (2012) 567–579.
- [4] S. Angilella, S. Corrente and S. Greco, Stochastic multiobjective acceptability analysis for the Choquet integral preference model and the scale construction problem. *Eur. J. Oper. Res.* **1** (2015) 172–182.
- [5] V. Belton and T. Stewart, *Multiple Criteria Decision Analysis: An Integrated Approach*. Springer Science & Business Media (2002).
- [6] S. Corrente, S. Greco and R. Słowiński, Multiple criteria hierarchy process in robust ordinal regression. *Decis. Support Syst.* **3** (2012) 660674.
- [7] S. Corrente, S. Greco and R. Słowiński, Multiple criteria hierarchy process with ELECTRE and PROMETHEE. *Omega* **5** (2013) 820846.
- [8] S. Corrente, S. Greco, M. Kadziński and R. Słowiński, Robust ordinal regression in preference learning and ranking. *Mach. Learn.* **23** (2013) 381–422.
- [9] S. Corrente, S. Greco and R. Słowiński, Multiple criteria hierarchy process for ELECTRE tri methods. *Eur. J. Oper. Res.* **1** (2016) 191203.
- [10] S. Corrente, M. Doumpos, S. Greco, R. Słowiński and C. Zopounidis, Multiple criteria hierarchy process for sorting problems based on ordinal regression with additive value functions. *Ann. Oper. Res.* **12** (2017) 117139.
- [11] S. Corrente, J.R. Figueira, S. Greco and R. Słowiński, A robust ranking method extending ELECTRE III to hierarchy of interacting criteria, imprecise weights and stochastic analysis. *Omega* **73** (2017) 117.
- [12] L. Del Vasto-Terrientes, J. Fernández-Cavia, A. Huertas, A. Moreno and A. Valls, Official tourist destination websites: hierarchical analysis and assessment with ELECTRE-III-H. *Tour. Manag. Perspect.* **15** (2015) 1628.

- [13] L. Del Vasto-Terriente, A. Valls, R. Slowinski and P. Zielińiewicz, ELECTRE-III-H: an outranking-based decision aiding method for hierarchically structured criteria. *Expert Syst. Appl.* **11** (2015) 49104926.
- [14] J.M. Devaud, G. Groussaud and E. Jacquet-Lagreze, UTADIS: Une méthode de construction de fonctions d'utilité additives rendant compte de jugements globaux. *Eur. Work. Group Multicriteria Decis. Aid Boch.* (1980) 94.
- [15] J. Fernández-Cavia and A. Huertas-Roig, City brands and their communication through web sites: Identification of problems and proposals for improvement. In: *Web Technologies: Concepts, Methodologies, Tools, and Applications*. IGI Global (2010) 1174–1297.
- [16] J. Figueira and B. Roy, Determining the weights of criteria in the ELECTRE type methods with a revised Simos' procedure. *Eur. J. Oper. Res.* **2** (2002) 317–326.
- [17] S. Greco, J. Figueira and M. Ehrgott, *Multiple Criteria Decision Analysis*. Springer (2016).
- [18] M. Ghram and H.M. Frikha, Multiple Criteria Hierarchy Process within ARAS method. In 2019 6th International Conference on Control, Decision and Information Technologies (CoDIT) (2019) 995–1000.
- [19] S. Greco, V. Mousseau and R. Slowiński, Multiple criteria sorting with a set of additive value functions". *Eur. J. Oper. Res.* **3** (2010) 1455–1470.
- [20] R.L. Keeney and H. Raiffa, *Decisions with Multiple Objectives: Preferences and Value Tradeoffs*. J. Wiley New York (1976).
- [21] O.V. Podinovskaya and V.V. Podinovski, Criteria importance theory for multicriterial decision making problems with a hierarchical structure. *Eur. J. Oper. Res.* **3** (2017) 983992.
- [22] B. Roy, ELECTRE III: un algorithme de classements fondé sur une représentation floue des préférences en présence de critères multiples. *Cahiers du CERO* **1** (1978) 3–24.
- [23] B. Roy and D. Bouyssou, Aide multicritère à la décision: méthodes et cas. *Economica Paris* (1993).
- [24] B. Roy, *Multicriteria Methodology for Decision Aiding*. Springer Science & Business Media (1996).
- [25] T.L. Saaty, *The Analytic Hierarchy Process: Planning, Priority Setting, Resource Allocation*. McGraw-Hill. Inc New York, NY (1980).
- [26] T.L. Saaty, *Decision Making with Dependence and Feedback: The Analytic Network Process: The Organization and Prioritization of Complexity*, 1st edition. Pittsburgh, PA: RWS Publications (1996).
- [27] J. Simos, Évaluer l'impact sur l'environnement: Une approche originale par l'analyse multicritère et la négociation. In: *Évaluer l'impact sur l'environnement: une approche originale par l'analyse multicritère et la négociation*. Presses polytechniques et universitaires romandes (1990).
- [28] P. Vincke, *Multicriteria Decision-aid*. John Wiley & Sons (1992).
- [29] J.P. Vincke and P. Brans, A preference ranking organization method: the PROMETHEE method for MCDM. *Manag. Sci.* **6** (1985) 647–656.
- [30] W. Yu, ELECTRE TRI: Aspects méthodologiques et manuel d'utilisation. Document du LAMSADE. Paris: Université de Paris-Dauphine, Working Paper (1992).
- [31] E.K. Zavadskas and Z. Turskis, A new Additive Ratio ASsessment (ARAS) method in multicriteria decision-making. *Technol. Econ. Dev. Econ.* **2** (2010) 159172.
- [32] M. Zeleny, *Multiple Criteria Decision Making Kyoto 1975*. Springer Science & Business Media (2012).
- [33] C. Zopounidis and M. Doumpos, Multicriteria classification and sorting methods: a literature review. *Eur. J. Oper. Res.* **2** (2002) 229–246.