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A PROFIT JUMP INVENTORY MODEL FOR IMPERFECT QUALITY ITEMS WITH RECEIVING REPARATIVE BATCH AND ORDER OVERLAPPING IN DENSE FUZZY ENVIRONMENT

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Abstract. This paper presents an economic order quantity (EOQ) inventory model for imperfect quality items with receiving a reparative batch and order overlapping in a dense fuzzy environment Here, the imperfect items are identified by screening and are divided into either scrap or reworkable items. The reworkable items are kept in store until the next items are received. Afterwards, the items are returned to the supplier to be reworked. Also, discount on the purchasing cost is employed as an offer of cooperation from a supplier to a buyer to compensate for all additional holding costs incurred to the buyer. The rework process is error free. An order overlapping scheme is employed so that the vendor is allowed to use the previous shipment to meet the demand by the inspection period. However, we assume the total monthly demand quantity as the dense fuzzy number because of learning effect. Moreover, first of all a profit maximization deterministic model is developed and solve by classical method. Fuzzifying the final optimized function *via* dense fuzzy demand quantity we have employed extended ranking index rule for its defuzzification. During the process of defuzzification we make an extensive study on the paradoxical unit square of the left and right deviations of dense fuzzy numbers. A comparative study is made after splitting the model into general fuzzy and dense fuzzy environment. Finally numerical and graphical illustrations and sensitivity analysis have been made for its global justifications.

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1. Introduction

The importance of inventory management systems is growing every day and many researchers are trying to solve management problems using mathematical models. The economic order quantity (EOQ) model is the basis of advanced inventory systems. By exploring the literature review on inventory systems, it is realized that many efforts have been conducted to provide inventory models in order to eliminate the limitations of the EOQ model. One of the assumptions in the EOQ models is that all the received items are perfect. However, this assumption is not comprehensive for several reasons that include faulty production process and failure in the process of transportation etc. So, the effect of imperfect items on inventory systems has become one of the interesting

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topics for many researchers to provide more practical models. Porteus [40] followed by Rosenblatt and Lee [41] presented the significant connection between imperfect quality and lot sizing. Schwaller [44] assumed that the imperfect items received in the lot would result in the inspection cost. In addition, Zhang and Gerchak [56] studied an EOQ model with the effect of a joint lot sizing and screening, in which the imperfect items were random variables. Further, Salameh and Jaber [43] investigated an economic production quantity model for defective items with a known probability distribution. Therefore, they assumed that, by the end of the inspection time, the imperfect items were sold as a single batch. Cardenas-Barron did not deviate from the main idea, but pointed out and rectified an existing error in the model, which was devised by Salameh and Jaber [43]. Subsequently, Papachristos and Konstantaras [38] examined the imperfect inventory model given that the imperfect items were random variable. Moreover, Moussawi-Haidar *et al.* [36] suggested an inventory model in which lot-sizing, defective items, quality control were combined. Karimi-Nasab and Sabri-Laghaie [25] formulated a new imperfect production inventory model in which the imperfect items were randomly produced.

Recently, Moussawi-Haidar *et al.* [37] considered the effect of imperfect items and deterioration. It should be noted that, in all the above-mentioned models, no shortage has been assumed during the inspection process, which was based on the model by Salameh and Jaber [43]. Since in several successive inspection processes, defective items may have been found to cause shortage, the supposed assumption could not be correct. This fault was discussed by Papachristos and Konstantaras [38] who concluded that the simple formula could not be found to prevent the occurrence of shortage during the inspection process. Luckily, Maddah *et al.* [30] developed a pragmatic method to overcome this fault. This method, called "an order overlapping scheme", lets the vendor use the previous order to meet the demand during the inspection process. This new approach can effectively prevent the occurrence of shortage during the inspection process. Therefore, this idea was incorporated into our model.

Another unrealistic assumption considered in the above models was that imperfect goods could just be sold at their salvage value and could not be reworked. However, many researchers have discovered this fault and incorporated the idea of reworking a part of imperfect items into their models. For example, Hayek and Salameh [24] proposed an economic production quantity model, in which the defective items were reworked by the end of the production time. Yu *et al.* [53] studied an EOQ model in which a part of defective items could be used as good items. It should be noted that the above inventory models consider that the reworkable items are sent back to be reworked and returned as the perfect items through the same period; however in our model we have assumed that reworkable items were kept in the buyer's warehouse until the next shipment arrived. Then, the supplier replaced the reworkable items with the perfect ones and sent them within the next order before the current lot was used up. In the present paper, it was assumed that the following lot was received from the supplier as "reparative batch". Also, in the previous papers, it was assumed that the perfect item holding costs and scrap item holding costs were the same. Moreover, Wahab and Jaber [48] presented an imperfect EOQ inventory model with different holding costs and learning in the inspection. Again transporting items is a major concern so we may point the works of D'Ambrosio *et al.* [7] and Cerulli *et al.* [3] in this field of research also.

In all the above models, researchers have only considered all the parameters and variables as crisp values. Although crisp models offer an overview of the approach of inventory systems under various assumptions, they are not able to provide factual terms. As a result, exerting crisp models in general can lead to errors in decision-making. Also, in crisp models, inventory managers must be flexible in determining the economic lot size to cause non random uncertainty based cost reduction.

Zadeh [55] drew the concepts of fuzzy sets among researchers. Since then, Bellman and Zadeh [2] applied it in decision making for industrial management problems. In the inventory management topics Sommer [47] developed a fuzzy scheduling inventory model considering a constraint in warehouse capacity. Park [39] presented an EOQ model for interpreting a fuzzy set theory. Chang *et al.* [5] developed an EOQ inventory model considering the backorder as a triangular fuzzy number. Chang [4], Mahata and Goswami [33] proposed an imperfect inventory model considering the fuzzy annual demand and fuzzy imperfect rate. Mahata and Mahata [34] studied a fuzzy EOQ inventory model with two phase trade credits for deteriorating items in the fuzzy sense.

In another study, Mahata [31] discussed an imperfect production model with partial backlogging of shortage quantity in fuzzy random environments. De and Sana [19] studied a hill type fuzzy stochastic model and got solutions *via* Bonferroni mean operator over score function of the fuzzy numbers. Furthermore, in the literature several ranking rules have been adopted by the researchers like Yager [52], Allahviranloo and Saneifard [1], Ezzati *et al.* [22], Deng [21] and Zhang *et al.* [57], etc. The concepts of deviation degree in fuzzy numbers were hosted by Wang *et al.* [49], Kumar *et al.* [29], Hajjari and Abbasbandy [23], Xu *et al.* [51], Yu *et al.* [54], etc. in developing the several inventory models. By this way numerous research articles along fuzzy environment have been studied ([8,11–20,32,35], etc.) yet. The main key factor of an inventory under smooth running is the supply of demand to the customers as quick as possible. But in reality we see due to lack of information the decision maker usually go for wrong decision in management system. So information gathering is one of the most essential part and parcel of any inventory process. In the literature, attempts have been taken by Kazemi *et al.* [26–28] to gain information through learning and forgetting process in fuzzy parameters for the backorder EOQ model with imperfect quality items. They applied Wright's [50] learning curve to gain knowledge in which the numbers of shipments are the vital factor. In another study, De and Mahata [11] discussed the learning effect on demand parameter in a backorder EOQ model through the duration of cycle time. Their basic notion is that, longer cycle time of an inventory can motivate customers in favour of that inventory spontaneously. They used the cloudy fuzzy approach on demand and compared the results with that of the general fuzzy model to justify their new approach. But in our study we have shown that customers/public interactions with the decision makers (DM) can change the motivation so that a catchment area over demand reaches very soon. In this study we have shown that adequate interaction could perform better goal in favour of inventory management system. However a reverse logic, more and more interactions of the DM to their learned customers can make a harmful situation (non favourable to DM) on the process itself. Also, such adequacy on interactions/negotiations/bargaining may vary from commodity to commodity, situations to situations even customer to customer or customer to DM implicitly. Thus we take the demand quantity as dense fuzzy number to estimate the actual learning outcomes in the inventory process itself. We have utilized De and Beg's [9, 10] ranking index rule to defuzzify the fuzzy objective function.

As it is obvious from the above-mentioned literature, none of the authors has presented an imperfect EOQ inventory model, either scrap or re-workable, along with receiving reparative batch considering various holding costs for perfect and scrap items under fuzzy conditions in the model parameters. Therefore, we tried to eliminate the gap in the literature. In this paper, scrap items were being sold for salvage value by the end of the inspection period. Upon the completion of the screening process, the buyer notifies the supplier the number of reworkable items; however, unlike some of the previous articles, here, it is assumed that reworkable items are stored in the buyer's warehouse until the next shipment arrives. Then, the supplier replaces the rework able items with the perfect ones and sends them within the next order before the current lot is exhausted. Totally, the major distinction between this paper and others lies in fuzziness in the demand parameter, the various assumptions on imperfect items, employing overlapping scheme to prevent shortages during the inspection period, discount rate provision of the purchasing cost to maintain a cooperative relationship, and considering receiving reparative. Moreover, we split the model into three different cases namely crisp, general fuzzy and dense fuzzy environments. Applying ranking index rule the new expressions for decision variables are developed. Numerical examples are also studied extensively. A comparative discussion along with sensitivity analysis and graphical illustrations are done to justify the new approach. At the end, a conclusion is made followed the scope of future work.

2. Problem statement

In this section, the problem is introduced with more details. An imperfect EOQ inventory model is presented. All the items received on a shipment are required to be inspected. The imperfect items that are identified through screening are divided into either scrap or rework able items. By the end of the inspection period, the scrap items are sold at a price of salvage value. Then, the buyer declares the number of rework able items; however unlike some of the previous articles in which rework able items are assumed to be sent back to the supplier and returned as the perfect items within the same period, the proposed model is assumed that rework able items are kept in a buyer's warehouse until the next shipment arrives. Then, the supplier replaces the rework able items with the perfect ones and sends them within the next order before the current lot is exhausted. By doing so, the supplier's costs (transportation costs) are reduced and, instead, the buyer's costs (holding costs) are raised. As a result, a coordinated policy should be employed so that economic benefits can be provided for both the buyer and the supplier. Discount on purchase costs can be used as an offer of cooperation from supplier to buyer (*i.e.*, the discount compensates for all additional holding cost incurred to the buyer). Moreover, to eliminate shortages within the inspection period, an "overlapping scheme" is employed: similar to Maddah *et al.*'s [30] idea that let the buyer to supply his/her needs from the previous order during the inspection process. Also, it is assumed that the holding costs for scrap items and perfect items are not the same. The main objective of this article is to develop a profit function of an imperfect production process over the market of flexible demand designed by learning experiences of the decision maker. To do this we assume the demand parameter D as triangular dense fuzzy number and to defuzzify the model we utilize the rule made by De and Beg [19].

Following are the assumptions and notations are considered in this paper:

Assumptions

- The input demand D is the triangular dense fuzzy number.
- Shortages are not allowed.
- The holding cost for re-workable items is higher than that of scrap items.
- A discount on the purchasing cost is applied to meet up the extra holding cost belonging to the buyer.
- An order overlapping scheme is considered.
- The demand (D) and screening processes (x) proceed concurrently with $D < x$.
- Reworking is done instantly and the process is error-free.

Notations

The following notations are used to develop the model.

D	Demand rate per month (units per month).
x	Inspection rate (units per month).
A	Ordering cost per cycle (\$).
r_s	Percentage rate of scrap items (random variable).
r_w	Percentage rate of rework able items (random variable).
$f(r_s)$	Probability density function of r_s.
$f(r_w)$	Probability density function of r_w.
s	Selling price per unit (\$).
w	Salvage value per unit (\$).
d_i	Unit inspection cost (\$).
h_w	Reworkable or perfect item holding cost rate per unit per cycle (\$).
h_s	Scrap item holding cost rate per unit per cycle (\$).
β	Discount rate for procurement cost (%).
c	Purchasing cost per unit (\$).
t_1	Screening time per cycle (months).
T	Cycle time (months) (decision variable).
Q	Order size per cycle (decision variable).
$H_s(Q)$	Scrap item holding cost per cycle (\$).
$H_w(Q)$	Perfect or rework able item holding costs per cycle (\$).
$\mathrm{TP}(Q)$	Total profit per cycle (\$).
$\mathrm{TPU}(Q)$	Net profit per unit time (\$) (decision variable).

3. Formulation of mathematical model (Extension of Maddah *et al.* [30])

Considering above assumptions and notations we formulate the model and it is shown in Figure 1. We assume 100% inspection is finished at time t_1. To avoid shortages, the overlapping scheme is used and it is supposed that the demand by the screening time is at least the same as the number of perfect quality items. It means that, for $0 \leq t \leq t_1$ we have,

$$xt_1\left(1-r_s-r_w\right) \geq Dt_1 \Rightarrow x \geq \frac{D}{1-r_s-r_w}\cdot \tag{3.1}$$

The goal is to obtain Q that maximizes the total profit per year, $\mathrm{TP}(Q)$, expressed by

$$\mathrm{TP}\left(Q\right)=\mathrm{TR}\left(Q\right)-\mathrm{TC}\left(Q\right) \tag{3.2}$$

where $\mathrm{TR}\left(Q\right)$ denotes the revenue per cycle and $\mathrm{TC}\left(Q\right)$ denotes the total cost per cycle which is obtained through the sale of good items and scrap items. Thus, they can be defined respectively as follows:

$$\mathrm{TR}\left(Q\right)=sQ\left(1-r_s\right)+wQr_s \tag{3.3}$$

and

$$\mathrm{TC}\left(Q\right)=\mathrm{OC}+\mathrm{SC}+\mathrm{PC}+\mathrm{HC} \tag{3.4}$$

where OC denotes the ordering cost per cycle ($\mathrm{OC}=A$), SC denotes the screening cost per cycle $(\mathrm{SC}=d_iQ)$, PC denotes the purchasing cost per cycle ($\mathrm{PC}=cQ\left(1-\beta\right)$), and HC denotes the holding cost per cycle, which includes the scrap item holding cost per cycle, $H_s\left(Q\right)$ and re-workable or perfect item holding cost per cycle, $H_w(Q)H_s\left(Q\right)$ can be obviously calculated using Figure 1 as shown in the shaded area:

$$H_s\left(Q\right)=h_s\left(\frac{Q^2r_s}{2x}\right)\cdot \tag{3.5}$$

To compute $H_w\left(Q\right)$, the total inventory quantity per cycle should be calculated. According to Figure 1, it is clear that the sum of the areas of ΔZBC, ΔBGR, GIJR, and ΔRJF minus ΔDEF can express the total inventory quantity per cycle. The area of ΔZBC is the same as that of ΔDEF; therefore, we have:

$$V=\underbrace{\frac{Q^2r_s}{2x}}_{\triangle\mathrm{BGR}}+\underbrace{\frac{Q^2\left(1-r_s\right)}{x}}_{\square\mathrm{GIJR}}+\underbrace{\frac{Q^2\left(1-r_s\right)^2}{2D}}_{\triangle\mathrm{RJF}}\cdot \tag{3.6}$$

Hence, the holding cost $H_w\left(Q\right)$ is given by

$$H_w\left(Q\right)=h_w\times V=h_w\left[\frac{Q^2r_s}{2x}+\frac{Q^2\left(1-r_s\right)}{x}+\frac{Q^2\left(1-r_s\right)}{2D}\right]\cdot \tag{3.7}$$

Thus,

$$\mathrm{TC}\left(Q\right)=A+d_iQ+cQ\left(1-\frac{r_wQ}{D}\right)+h_s\left(\frac{Q^2r_s}{2x}\right)+h_w\left[\frac{Q^2r_s}{2x}+\frac{Q^2\left(1-r_s\right)}{x}+\frac{Q^2\left(1-r_s\right)^2}{2D}\right]\cdot \tag{3.8}$$

Through items simplification, the expression for total cost per cycle can be calculated by:

$$\mathrm{TC}\left(Q\right)=A+d_iQ+cQ\left(1-\frac{r_wQ}{D}\right)+\left(h_s+h_w\right)\left(\frac{Q^2r_s}{2x}\right)+h_w\left[\frac{Q^2\left(1-r_s\right)}{x}+\frac{Q^2\left(1-r_s\right)^2}{2D}\right]\cdot \tag{3.9}$$

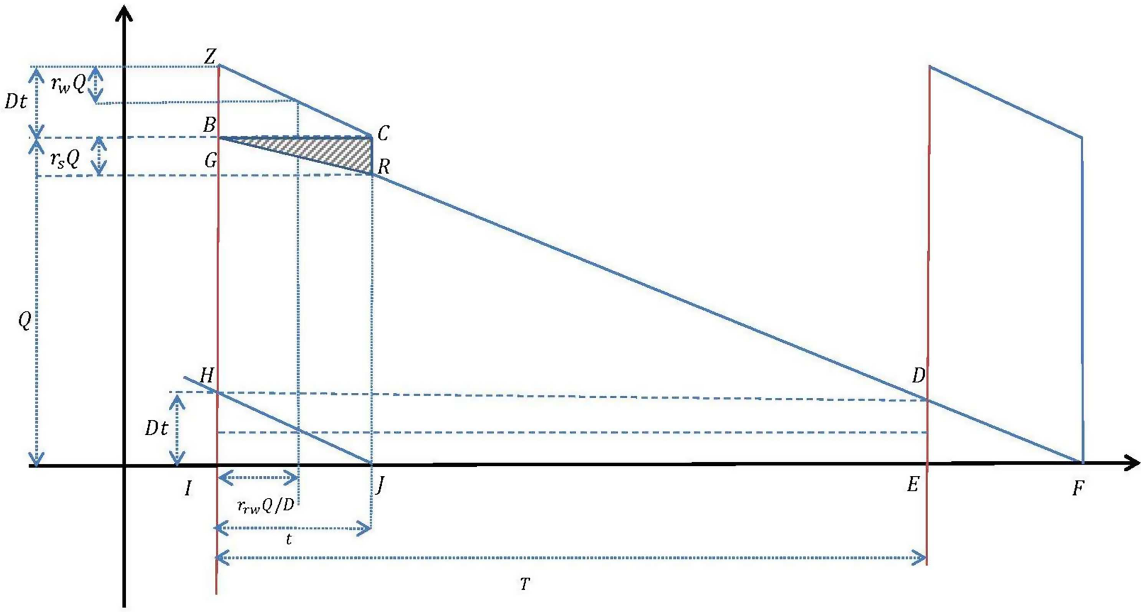

FIGURE 1. Imperfect EOQ model.

By substituting equations (3.8) and (3.3) into equation (3.2), the total profit per cycle is obtained by:

$$\mathrm{TP}\,(Q) = sQ\,(1-r_s) + wQr_s - A - d_iQ - cQ\left(1 - \frac{r_wQ}{D}\right) - (h_s + h_w)\left(\frac{Q^2 r_s}{2x}\right) - h_w\left[\frac{Q^2\,(1-r_s)}{x} + \frac{Q^2\,(1-r_s)^2}{2D}\right]. \tag{3.10}$$

Furthermore, it is considered that the expected value of TP (Q), means $E\,[\mathrm{TP}\,(Q)]$ is calculated, in which the expected values $\{E(1-r_s), E(r_s)$, and $E(r_w)\}$ are used instead of $1-r_s$, r_s and r_w, respectively. The expected net profit per unit time is calculated by applying the renewal reward theorem [42] (*i.e.*, dividing TP (Q) by the cycle length $T = \frac{(1-r_s)Q}{D}$ as follows:

$$E\,[\mathrm{TPU}\,(Q)] = \frac{D\left[s\,\{1-E\,(r_s)\} + wE\,(r_s) - c\left\{\frac{1-E(r_w)Q}{D}\right\} - d_i\right] - \frac{AD}{Q}}{1-E\,(r_s)} - \frac{Q}{2\,(1-E\,(r_s))}\left[\frac{D\,\{2h_w - h_wE\,(r_s) + h_sE\,(r_s)\}}{x} + h_wE\,(1-r_s)^2\right]. \tag{3.11}$$

Now, we consider:

$$\begin{cases} u = \frac{1}{1-E(r_s)}\left\{s\,\{1-E\,(r_s)\} + wE\,(r_s) - c - d_i - \frac{A}{Q}\right\} \\ W = \frac{Q}{2(1-E(r_s))}\left\{\frac{2h_w - h_wE(r_s) + h_sE(r_s)}{x}\right\} \\ X = \frac{h_wE(1-r_s)^2}{2(1-E(r_s))} \end{cases}. \tag{3.12}$$

Hence, by substituting equation (3.12) in equation (3.11), the expected annual net profit function reduced to

$$E\,[\mathrm{TPU}\,(Q)] = Du + \frac{cE\,(r_w)}{1-E\,(r_s)}Q - DW - QX = D\,(u-W) + Q\left\{\frac{cE\,(r_w)}{1-E\,(r_s)} - X\right\}. \tag{3.13}$$

Now to have the optimum value of the order quantity, we differentiate (3.13) with respect to Q and get the following:

$$\frac{\mathrm{d}\{E[\mathrm{TPU}(Q)]\}}{\mathrm{d}Q} = D\left(\frac{\mathrm{d}u}{\mathrm{d}Q} - \frac{\mathrm{d}W}{\mathrm{d}Q}\right) + \frac{cE(r_w)}{1-E(r_s)} - X = 0 \tag{3.14}$$

where,

$$\frac{\mathrm{d}u}{\mathrm{d}Q} = \frac{\mathrm{d}}{\mathrm{d}Q}\left\{\frac{1}{1-E(r_s)}\left\{s\{1-E(r_s)\} + wE(r_s) - c - d_i - \frac{A}{Q}\right\}\right\} = \frac{A}{Q\{1-E(r_s)\}} \tag{3.15}$$

and

$$\frac{\mathrm{d}W}{\mathrm{d}Q} = \frac{\mathrm{d}}{\mathrm{d}Q}\left\{\frac{Q}{2(1-E(r_s))}\left\{\frac{2h_w - h_wE(r_s) + h_sE(r_s)}{x}\right\}\right\} = \left\{\frac{2h_w - h_wE(r_s) + h_sE(r_s)}{2x(1-E(r_s))}\right\}. \tag{3.16}$$

Now, substituting (3.15) and (3.16) in (3.14), we have,

$$D\left[\frac{A}{Q^2\{1-E(r_s)\}} - \frac{2h_w - h_wE(r_s) + h_sE(r_s)}{2x(1-E(r_s))}\right] + \frac{cE(r_w)}{1-E(r_s)} - \frac{h_wE(1-r_s)^2}{2(1-E(r_s))} = 0. \tag{3.17}$$

Simplifying (3.17) we get $\frac{A}{Q^2} = -\frac{cE(r_w)}{D} + \frac{h_wE(1-r_s)^2}{2D} + \frac{2h_w - h_wE(r_s)+h_sE(r_s)}{2x}$ giving

$$Q^* = \sqrt{\frac{2AD}{D\left\{\frac{2h_w - h_wE(r_s)+h_sE(r_s)}{2x}\right\} - 2cE(r_w) + h_wE(1-r_s)^2}} \tag{3.18}$$

and this value yields

$$\frac{\mathrm{d}^2\{E[\mathrm{TPU}(Q)]\}}{\mathrm{d}Q^2} = \frac{-2AD}{Q^2\{1-E(r_s)\}} < 0. \tag{3.19}$$

This confirms that the objective function has global maximum and its value is given by

$$E[\mathrm{TPU}(Q)]^* = D\left[F_1 - \frac{F_2}{Q^*} - F_3Q^*\right] - F_4Q^* \tag{3.20}$$

where

$$\begin{cases} F_1 = \frac{s\{1-E(r_s)\}+wE(r_s)-c-d_i}{1-E(r_s)} \\ F_2 = \frac{A}{1-E(r_s)} \\ F_3 = \frac{2h_w - h_wE(r_s)+h_sE(r_s)}{2\{1-E(r_s)\}x} \\ F_4 = \frac{h_wE(1-r_s)^2}{2\{1-E(r_s)\}} - \frac{cE(r_w)}{\{1-E(r_s)\}} \end{cases}. \tag{3.21}$$

Note that, when the demand parameter D is finite and screening rate is large enough, that is the inspection process is finished simultaneously by the receiving an order, and finally when items are categorized as only perfect or imperfect (no rework able items so that no discount on purchasing cost), then Q^* in equation (3.18) is equivalent to

$$Q^* = \sqrt{\frac{2AD}{h_wE(1-r_s)^2}}. \tag{3.22}$$

This is the same as the results obtained by Shih [45] and Silver [46]. It shows that the proposed model is accurate. In addition, it should be noted that, when the demand parameter D is finite, and if all items are assumed to be perfect, our model becomes an equivalent to the EOQ inventory model.

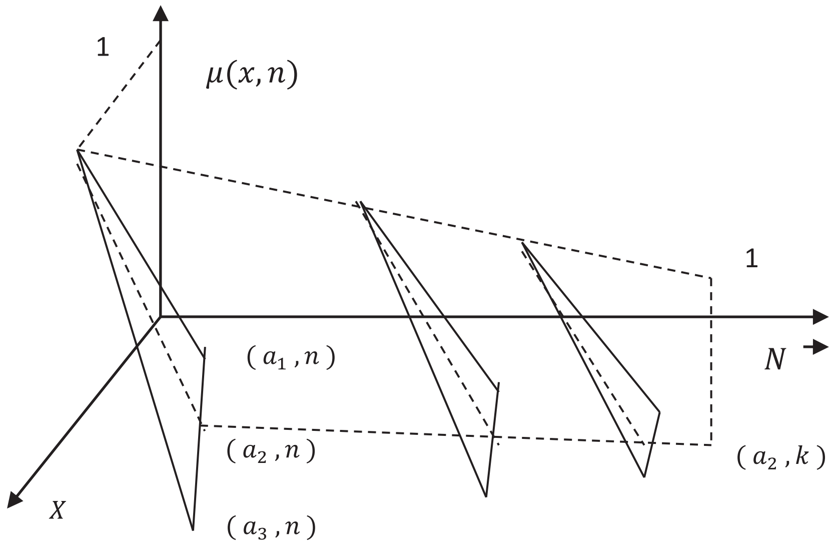

Figure 2. Membership function of TDFN.

4. Fuzzy mathematical model

Let the demand rate assumes flexible under dense fuzzy environment. Then the fuzzy problem (the tilt bar $\tilde{}$ represents the fuzzification of the parameters) corresponding to the crisp model (3.20) is given by

$$\widetilde{\text{Maximize}}\ \widetilde{Z_E} \cong \widetilde{D}\left[F_1 - \frac{F_2}{\widetilde{Q^*}} - F_3\widetilde{Q^*}\right] - F_4\widetilde{Q^*}. \tag{4.1}$$

$$\text{Subject to } \begin{cases} \widetilde{Q^*} = \sqrt{\frac{2A\widetilde{D}}{\widetilde{D}Y_1+Y_2}} \\ \widetilde{T^*} = \frac{(1-r_s)\widetilde{Q^*}}{\widetilde{D}} \end{cases} \tag{4.2}$$

$$\text{where } \begin{cases} Y_1 = \frac{2h_w - h_w E(r_s) + h_s E(r_s)}{x} \\ Y_2 = h_w E\left(1-r_s\right)^2 - 2cE\left(r_w\right) \end{cases} \tag{4.3}$$

and the other parameters are obtained from (3.21).

Now, as per De and Beg [9,10] we use the following membership function for dense fuzzy demand rate as

$$\mu(D,n) = \begin{cases} 0 & \text{if } D < D_2\left(1-\frac{\rho}{1+n}\right) \text{ and } D > D_2\left(1-\frac{\sigma}{1+n}\right) \\ \left\{\frac{D - D_2\left(1-\frac{\rho}{1+n}\right)}{\frac{\rho D_2}{1+n}}\right\} & \text{if } D_2\left(1-\frac{\rho}{1+n}\right) \le D \le D_2 \\ \left\{\frac{D_2\left(1+\frac{\sigma}{1+n}\right) - D}{\frac{\sigma D_2}{1+n}}\right\} & \text{if } D_2 \le D \le D_2\left(1+\frac{\sigma}{1+n}\right) \end{cases} \tag{4.4}$$

where, $0 < \rho, \sigma < 1$ and n being the natural number and the graphical representation of this triangular dense fuzzy number (TDFN) is given in Figure 2.

Therefore, to get the membership value of the optimum order quantity Q^* we need to proceed as follows: From (4.4), we write for all positive numbers as follows:

$$D_2\left(1-\frac{\rho}{1+n}\right) \leq D \leq D_2 \Rightarrow \sqrt{\frac{2AD_2\left(1-\frac{\rho}{1+n}\right)}{D_2Y_1+Y_2}} \leq \sqrt{\frac{2AD}{DY_1+Y_2}} \leq \sqrt{\frac{2AD_2}{D_2Y_1\left(1-\frac{\rho}{1+n}\right)+Y_2}}$$

and

$$D_2 \leq D \leq D_2\left(1+\frac{\sigma}{1+n}\right) \Rightarrow \sqrt{\frac{2AD_2}{D_2Y_1\left(1+\frac{\sigma}{1+n}\right)+Y_2}} \leq \sqrt{\frac{2AD}{DY_1+Y_2}} \leq \sqrt{\frac{2AD_2\left(1+\frac{\sigma}{1+n}\right)}{D_2Y_1+Y_2}}.$$

Thus,

$$\sqrt{\frac{2AD_2\left(1-\frac{\rho}{1+n}\right)}{DY_1+Y_2}} \leq Q^* \leq \sqrt{\frac{2AD_2}{D_2Y_1\left(1-\frac{\rho}{1+n}\right)+Y_2}} \quad \text{and}$$

$$\sqrt{\frac{2AD_2}{D_2Y_1\left(1-\frac{\rho}{1+n}\right)+Y_2}} \leq Q^* \leq \sqrt{\frac{2AD_2\left(1+\frac{\sigma}{1+n}\right)}{DY_1+Y_2}}$$

because $\sqrt{\frac{2AD_2}{D_2Y_1\left(1-\frac{\rho}{1+n}\right)+Y_2}} > \sqrt{\frac{2AD_2}{D_2Y_1\left(1+\frac{\sigma}{1+n}\right)+Y_2}}$.

Let us assume $Q_1 \leq Q^* \leq Q_2$ and $Q_2 \leq Q^* \leq Q_3$.

So

$$\begin{cases} Q_1 = \sqrt{\frac{2AD_2\left(1-\frac{\rho}{1+n}\right)}{DY_1+Y_2}} \\ Q_2 = \sqrt{\frac{2AD_2}{D_2Y_1\left(1+\frac{\sigma}{1+n}\right)+Y_2}} \\ Q_3 = \sqrt{\frac{2AD_2\left(1+\frac{\sigma}{1+n}\right)}{DY_1+Y_2}} \end{cases}. \tag{4.5}$$

And the corresponding membership function of the optimum order quantity is given by

$$\mu\left(Q^*, n\right) = \begin{cases} 0 & \text{if } Q^* < Q_2 \text{ and } Q^* > Q_3 \\ \frac{Q^*-Q_1}{Q_2-Q_1} & \text{if } Q_1 \leq Q^* \leq Q_2 \\ \frac{Q_3-Q^*}{Q_3-Q_2} & \text{if } Q_2 \leq Q^* \leq Q_3 \end{cases}. \tag{4.6}$$

Now, applying the fuzzy arithmetic (for details see Appendix A.1), the net membership of the optimal objective we get,

$$\mu\left(\widetilde{Z_E}\right) = \mu\left(F_1\widetilde{D}\right) - \mu\left(F_2\widetilde{D}/\widetilde{Q^*}\right) - \mu\left(F_3\widetilde{D}\widetilde{Q^*}\right) - \mu\left(F_4\widetilde{Q^*}\right) \tag{4.7}$$

$$\text{and} \quad \mu\left(\widetilde{T^*}\right) = \mu\left[\frac{(1-r_s)\,\widetilde{Q^*}}{\widetilde{D}}\right]. \tag{4.8}$$

Now, the $\alpha-$ *cuts* of $\mu\left(\widetilde{Z_E}\right)$ and $\mu\left(\widetilde{T^*}\right)$ are respectively given by,

$$\left[Z^*_{\mathrm{EL}}, Z^*_{\mathrm{ER}}\right] = F_1\left[D_1+\alpha\left(D_2-D_1\right), D_3-\alpha\left(D_3-D_2\right)\right] - F_2\left[\frac{D_1+\alpha\left(D_2-D_1\right)}{Q_3-\alpha\left(Q_3-Q_2\right)}, \frac{D_3-\alpha\left(D_3-D_2\right)}{Q_1+\alpha\left(Q_2-Q_1\right)}\right]$$

$$- F_3\left[\left\{D_1+\alpha\left(D_2-D_1\right)\right\}\left\{Q_1+\alpha\left(Q_2-Q_1\right)\right\},\left\{D_3-\alpha\left(D_3-D_2\right)\right\}\left\{Q_3-\alpha\left(Q_3-Q_2\right)\right\}\right]$$
$$- F_4\left[Q_1+\alpha\left(Q_2-Q_1\right),Q_3-\alpha\left(Q_3-Q_2\right)\right] \quad (4.9)$$

and
$$\left[T_L^*,T_R^*\right]=\left(1-r_s\right)\left[\frac{Q_1+\alpha\left(Q_2-Q_1\right)}{D_3-\alpha\left(D_3-D_2\right)},\frac{Q_3-\alpha\left(Q_3-Q_2\right)}{D_1+\alpha\left(D_2-D_1\right)}\right]. \quad (4.10)$$

Therefore, after little calculations, (4.9) and (4.10) reduces to

$$\begin{aligned}
I\left(Z_E^*\right)&=\frac{1}{2K}\sum_{n=0}^{K}\int_0^1\left[Z_{\mathrm{EL}}^*+Z_{\mathrm{ER}}^*\right]\mathrm{d}\alpha=\frac{F_1}{4K}\sum_{n=0}^{K}\left(D_1+2D_2+D_3\right)-\frac{F_4}{4K}\sum_{n=0}^{K}\left(Q_1+2Q_2+Q_3\right)\\
&\quad-\frac{F_2}{2K}\sum_{n=0}^{K}\int_0^1\left[\frac{D_1+\alpha\left(D_2-D_1\right)}{Q_3-\alpha\left(Q_3-Q_2\right)}+\frac{D_3-\alpha\left(D_3-D_2\right)}{Q_1+\alpha\left(Q_2-Q_1\right)}\right]\mathrm{d}\alpha\\
&\quad-\frac{F_3}{12K}\sum_{n=0}^{K}\left[Q_3D_2+Q_2D_3+2Q_3D_3+4Q_2D_2+2Q_1D_1+Q_1D_2+Q_2D_1\right]\\
&=\frac{F_1}{4K}\sum_{n=0}^{K}\left(D_1+2D_2+D_3\right)-\frac{F_4}{4K}\sum_{n=0}^{K}\left(Q_1+2Q_2+Q_3\right)\\
&\quad-\frac{F_3}{12K}\sum_{n=0}^{K}\left[Q_3D_2+Q_2D_3+2Q_3D_3+4Q_2D_2+2Q_1D_1+Q_1D_2+Q_2D_1\right]\\
&\quad-\frac{F_2}{2K}\sum_{n=0}^{K}\left[\frac{Q_2D_3-Q_1D_2}{\left(Q_2-Q_1\right)^2}\mathrm{Log}\left(\frac{Q_2}{Q_1}\right)-\frac{Q_3D_2-Q_2D_1}{\left(Q_3-Q_2\right)^2}\mathrm{Log}\left(\frac{Q_2}{Q_3}\right)\right.\\
&\quad\left.-\frac{Q_3D_3+2Q_2D_2+Q_1D_1-Q_1D_2-Q_2D_1-Q_3D_2-Q_2D_3}{Q_1Q_2+Q_3Q_2-Q_3Q_1-Q_2^2}\right]. \qquad (4.11)
\end{aligned}$$

And that for order quantity is given by

$$I\left(Q^*\right)=\frac{1}{4K}\sum_{n=0}^{K}\left(Q_1+2Q_2+Q_3\right) \quad (4.12)$$

and

$$\begin{aligned}
I\left(T^*\right)&=\frac{\left(1-r_s\right)}{2K}\sum_{n=0}^{K}\left[\frac{D_2Q_3-D_1Q_2}{\left(D_2-D_1\right)^2}\mathrm{Log}\left(\frac{D_2}{D_1}\right)-\frac{D_3Q_2-D_2Q_1}{\left(D_3-D_2\right)^2}\mathrm{Log}\left(\frac{D_2}{D_3}\right)\right.\\
&\quad\left.-\frac{D_3Q_3+2Q_2D_2+Q_1D_1-D_1Q_2-D_2Q_1-D_3Q_2-Q_2D_3}{D_1D_2+D_3D_2-D_3D_1-D_2^2}\right]. \qquad (4.13)
\end{aligned}$$

However,

$$D_1+2D_2+D_3=D_2\left\{4+\frac{\sigma-\rho}{1+n}\right\} \quad (4.14)$$

and

$$Q_1+2Q_2+Q_3=\sqrt{\frac{2AD_2\left(1-\frac{\rho}{1+n}\right)}{D_2Y_1+Y_2}}+2\sqrt{\frac{2AD_2}{D_2Y_1\left(1+\frac{\sigma}{1+n}\right)+Y_2}}+\sqrt{\frac{2AD_2\left(1+\frac{\sigma}{1+n}\right)}{D_2Y_1+Y_2}}$$

$$= \lambda\left[\sqrt{1+\frac{\sigma}{1+n}}+\sqrt{1-\frac{\rho}{1+n}}+2\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}\right]. \tag{4.15}$$

Letting, $\lambda=\sqrt{\frac{2AD_2}{D_2Y_1+Y_2}}$ we have

$$\begin{aligned}
&I\left(Z_E^*\right)\\
&=\frac{F_1d_2}{4K}\sum_{n=0}^{K}\left\{4+\frac{\sigma-\rho}{1+n}\right\}-\frac{F_4\lambda}{4K}\sum_{n=0}^{K}\left[\sqrt{1+\frac{\sigma}{1+n}}+\sqrt{1-\frac{\rho}{1+n}}+2\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}\right]\\
&\quad-\frac{d_2F_2}{4\lambda K}\sum_{n=0}^{K}\left[\begin{aligned}
&+\sqrt{\left(1+\tfrac{\sigma}{1+n}\tfrac{Y_1\lambda^2}{2A}\right)}\left\{\frac{\left(1+\frac{\sigma}{1+n}-\sqrt{\left(1-\frac{\rho}{1+n}\right)\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}\right)}{\left(1-\sqrt{\left(1-\frac{\rho}{1+n}\right)\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}\right)^2}\right\}\mathrm{Log}\left|\left(1+\tfrac{\sigma}{1+n}\tfrac{Y_1\lambda^2}{2A}\right)\left(1-\tfrac{\rho}{1+n}\right)\right|\\
&+\sqrt{\left(1+\tfrac{\sigma}{1+n}\tfrac{Y_1\lambda^2}{2A}\right)}\left\{\frac{\left(\sqrt{\left(1+\frac{\sigma}{1+n}\right)\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}-1+\frac{\rho}{1+n}\right)}{\left(\sqrt{\left(1+\frac{\sigma}{1+n}\right)\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}-1\right)^2}\right\}\mathrm{Log}\left|\left(1+\tfrac{\sigma}{1+n}\tfrac{Y_1\lambda^2}{2A}\right)\left(1+\tfrac{\sigma}{1+n}\right)\right|\\
&-2\left\{\frac{\frac{\rho-\sigma}{1+n}\sqrt{\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}+\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)\left\{\frac{\sigma}{1+n}\sqrt{1+\frac{\sigma}{1+n}}-\frac{\rho}{1+n}\sqrt{\left(1-\frac{\rho}{1+n}\right)}\right\}}{\sqrt{\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}\left\{\sqrt{1+\frac{\sigma}{1+n}}+\sqrt{\left(1-\frac{\rho}{1+n}\right)}\right\}-1-\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)\sqrt{\left(1+\frac{\sigma}{1+n}\right)\left(1-\frac{\rho}{1+n}\right)}}\right\}
\end{aligned}\right]\\
&\quad-\frac{\lambda d_2F_3}{12K}\sum_{n=0}^{K}\left[\sqrt{1+\frac{\sigma}{1+n}}+\frac{\left(6+\frac{\sigma-\rho}{1+n}\right)}{\sqrt{\left(1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}\right)}}+2\left(1+\frac{\sigma}{1+n}\right)^{3/2}+2\left(1-\frac{\rho}{1+n}\right)^{3/2}+\sqrt{1-\frac{\rho}{1+n}}\right]
\end{aligned} \tag{4.16}$$

and

$$I\left(Q^*\right)=\frac{1}{4K}\sum_{n=0}^{K}\left(Q_1+2Q_2+Q_3\right)=\frac{\lambda}{4K}\sum_{n=0}^{K}\left[\sqrt{1+\frac{\sigma}{1+n}}+\sqrt{1-\frac{\rho}{1+n}}+2\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}\right] \tag{4.17}$$

$$\begin{aligned}
I\left(T^*\right)=\frac{(1-r_s)\lambda}{2KD}\sum_{n=0}^{K}\Bigg[&\frac{\sqrt{\left(1+\frac{\sigma}{1+n}\right)}-\left(1+\frac{\rho}{1+n}\right)\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}}{\left(\frac{\rho}{1+n}\right)^2}\mathrm{Log}\left|\frac{1}{1-\frac{\rho}{1+n}}\right|\\
&+\frac{\left(1+\frac{\sigma}{1+n}\right)\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}-\sqrt{\left(1+\frac{\rho}{1+n}\right)}}{\left(\frac{\sigma}{1+n}\right)^2}\mathrm{Log}\left|1+\frac{\sigma}{1+n}\right|\\
&-\frac{\left(\frac{\sigma}{1+n}\right)\sqrt{\left(1+\frac{\sigma}{1+n}\right)}-\frac{\rho}{1+n}\sqrt{\left(1+\frac{\rho}{1+n}\right)}+\left(\frac{\rho-\sigma}{1+n}\right)\sqrt{\frac{1}{1+\frac{\sigma}{1+n}\frac{Y_1\lambda^2}{2A}}}}{\frac{\sigma\rho}{(1+n)^2}}\Bigg].
\end{aligned} \tag{4.18}$$

Now we shall solve the above objective function (4.16) subject to the condition (4.17) and (4.18) in three different cases:

Case I: Crisp model.
Case II: General fuzzy model for $\rho>\sigma$, $\rho=\sigma$ and $\rho<\sigma$ (putting $n=0$ in dense fuzzy model).
Case III: Dense fuzzy model for $\rho>\sigma$, $\rho=\sigma$ and $\rho<\sigma$.

Table 1. Optimal solution for general fuzzy model.

Left and right deviations	$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$	$R_z = \left\{\frac{I(Z_E^*)-Z_E^*}{Z_E^*}\right\} \times 100\%$
$\rho(=.2) < \sigma(=.3)$	870.001	10.288	23 869.50	2.46
$\rho = \sigma = .2$	873.971	10.494	23 289.22	−0.03
$\rho(=.3) > \sigma(=.2)$	860.835	10.767	22 700.48	−2.56

Table 2. Optimal solution for dense fuzzy model.

Left and right deviations	n	$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$
$\rho < \sigma$		1758.71	20.69	1417.27
$\rho = \sigma$	1	1765.401	21.0	43 408.94
$\rho > \sigma$		1746.186	21.4	44 156.25

5. Numerical example 1

Suppose the inventory practitioner (DM) ordered a single item which is purchased and sold batch wise. These are combination of scrap rework able and good items. The monthly demand of this item is found to be 1000 units ($= D$) and the inspection rate is 10 000 units ($= x$) per month. The other information associated to the Inventory management is given below:

The set up cost per cycle $A = \$350$, unit holding cost for perfect items $h_w = \$3.5/\$/\text{month}$, unit holding cost for scrap items $h_s = \$1.5/\$/\text{month}$, unit selling price $s = \$50$, unit inspection cost $d_i = \$0.5$, unit salvage value $w = \$5$, unit purchasing cost $c = \$25$, Also, expected value of percentage rate of scrap items and reworkable items are $E(r_s) = 0.02, E(r_w) = 0.06$ respectively. The variance $E(1 - r_s)^2 = 0.9$. Now our problem is to find the best order quantity Q^* and the optimum cycle time T^* such that the DM will win maximum monthly total profit Z_E^* of the crisp case. This is obtained that: $Q^* = 909.628$ units, $= \$23\,296.40$ and the cycle time is $T^* = 10.697$ months. We also compute the numerical solutions of the fuzzy as well as dense fuzzy models and obtain the expected profit which is shown in Tables 1 and 2.

The Table 1 shows, in the case of general fuzzy solution if the left deviation assumes greater value than right deviation then the expected profit becomes \$22 700 but for its reverse it gets maximum value to \$23 869. If both the deviations are same then the profit value so obtained might lie in between them. Here also we see that, lower left fuzzy deviation (with respect to right deviation) giving higher profit value than crisp optimum, the other cases give lower profit optimum than crisp optimum as a whole. Table 2 shows that for the case of dense fuzzy model, the lower left fuzzy deviations giving a sudden downwards jump/suicidal jump of the expected profit function tremendously. But, for greater left fuzzy deviations we see the profit value reaches to maximum height. However, for equal fuzzy deviations, the profit value lies in between the above results.

5.1. Sensitivity analysis of the crisp and dense fuzzy model

For better justification of the proposed model we need to make the sensitivity analysis for the crisp as well as dense fuzzy model. To do this we take the changes from −50% to +50% of all the parametric values associated with the crisp model (shown in Tab. 3) and for the dense fuzzy model we perform that changes of the fuzzy deviations (ρ, σ) by means of vertices of a unit square only and taking the help of LINGO software, the numerical results are put in the Tables 4, 5, 6 and 7, respectively.

TABLE 3. Sensitivity analysis for crisp model.

Parametric change		T^*	Q^*	Z_E^*	$R_z = \left\{\frac{Z_E^* - Z_E^* - \text{crisp}}{Z_E^* - \text{crisp}}\right\} \times 100\%$
A	+50%	13.103	1114.062	23 119.90	−0.76
	+30%	12.197	1037.135	23 186.31	−0.47
	−30%	8.950	761.049	23 424.65	+0.55
	−50%	7.564	643.204	23 526.38	+0.98
D	+50%	7.352	937.762	34 979.91	+50.15
	+30%	8.402	928.828	30 306.40	+30.09
	−30%	14.732	876.920	16 286.97	−30.09
	−50%	19.718	838.338	11 614.81	−93.06
x	+50%	12.557	1067.738	23 412.66	+0.49
	+30%	11.885	1010.606	23 374.84	+0.34
	−30%	9.918	782.135	23 168.38	−0.55
	−50%	7.923	673.762	23 021.49	−1.18
s	+50%	10.697	909.628	48 296.38	+107.3
	+30%	10.697	909.628	38 296.38	+64.38
	−30%	10.697	909.628	8296.38	−64.38
	−50%	No feasible solution			
W	+50%	10.697	909.628	23 347.40	+0.22
	+30%	10.697	909.628	23 326.99	+0.13
	−30%	10.697	909.628	23 265.77	−0.13
	−50%	10.697	909.628	23 245.36	−0.22
d_i	+50%	10.697	909.628	23 041.30	−1.09
	+30%	10.697	909.628	23 143.32	−0.65
	−30%	10.697	909.628	23 449.44	+0.65
	−50%	10.697	909.628	23 551.48	+1.09
h_w	+50%	5.914	502.927	22 661.38	−2.73
	+30%	6.959	591.771	22 874.60	−1.81
	−30%	No feasible solution			
	−50%				
h_s	+50%	10.688	908.823	23 295.69	−0.0003
	+30%	10.691	909.144	23 295.96	−0.0001
	−30%	10.703	910.110	23 296.80	+0.002
	−50%	10.707	910.436	23 297.08	+0.003
c	+50%	No feasible solution			
	+30%				
	−30%	7.446	633.179	30 606.61	+31.38
	−50%	6.423	546.242	35 529.10	+52.50
$E(r_s)$	+50%	10.601	910.705	23 073.59	−0.95
	+30%	10.639	910.274	23 163.26	−0.57
	−30%	10.755	908.983	23 427.90	+0.56
	−50%	10.794	908.555	23 514.69	+0.94
$E(r_w)$	+50%	No feasible solution			
	+30%				
	−30%	7.446	633.179	22 953.54	−1.47
	−50%	6.423	546.242	22 774.00	−2.24
$E(1-r_s)$	+50%	6.323	537.714	22 753.26	−2.33
	+30%	7.352	625.174	22 939.09	−1.53;
	−30%	No feasible solution			
	−50%				

TABLE 4. Sensitivity analysis at $(\rho, \sigma) = (0.2, 0.3)$.

n	Parametric change		$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$	$R_z(\%)$	$R_Q(\%)$
	ρ	+50%	1739.498	21.1	43 833.32	88.15	91.23
		+30%	1747.310	21.9	42 206.69	81.17	92.09
		−30%	No feasible solution				
		−50%					
1	σ	+50%	No feasible solution				
		+30%					
		−30%	1764.669	21.0	42 958.33	84.4	93.99
		−50%	1769.291	21.2	44 323.28	90.25	94.51

Notes. Where, $R_z = \left\{\frac{I(Z_E^*)-Z_E^*}{Z_E^*}\right\} \times 100\%$ and $R_Q = \left\{\frac{I(Q^*)-Q^*}{Q^*}\right\} \times 100\%$.

TABLE 5. Sensitivity Analysis at $(\rho, \sigma) = (0.2, 0.2)$.

n	Parametric change		$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$	$R_z(\%)$	$R_Q(\%)$
	ρ	+50%	1746.186	21.4	44 156.25	89.54	91.96
		+30%	1754.000	21.2	44 250.37	89.95	92.82
		−30%	No feasible solution				
		−50%					
1	σ	+50%	1758.713	20.7	1417.28	−93.92	93.34
		+30%	1761.224	20.8	28 638.43	22.94	93.62
		−30%	1770.117	21.3	44 365.28	90.44	94.59
		−50%	1773.590	21.4	44 261.13	89.99	94.97

Notes. Where, $R_z = \left\{\frac{I(Z_E^*)-Z_E^*}{Z_E^*}\right\} \times 100\%$ and $R_Q = \left\{\frac{I(Q^*)-Q^*}{Q^*}\right\} \times 100\%$.

TABLE 6. Sensitivity analysis at $(\rho, \sigma) = (0.3, 0.2)$.

n	Parametric change		$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$	$R_z(\%)$	$R_Q(\%)$
	ρ	+50%	1715.110	22.1	43 004.79	84.59	88.55
		+30%	1727.909	21.8	43 534.84	86.87	89.95
		−30%	1763.526	21.1	43 750.55	87.79	93.87
		−50%	No feasible solution				
1	σ	+50%	1739.498	21.1	43 833.13	88.15	91.23
		+30%	1742.009	21.2	44 161.58	89.56	91.50
		−30%	1750.902	21.6	43 764.60	87.86	92.48
		−50%	1754.375	21.7	43 242.30	85.62	92.86

Notes. Where, $R_z = \left\{\frac{I(Z_E^*)-Z_E^*}{Z_E^*}\right\} \times 100\%$ and $R_Q = \left\{\frac{I(Q^*)-Q^*}{Q^*}\right\} \times 100\%$.

5.2. Discussion on sensitivity Tables 3–7

Sensitivity analysis Table 3 reveals that, among 12 several parameters, only the monthly demand D and unit selling price s are highly sensitive parameters. The other parameters are more or less insensitive with respect to the initial crisp value. At +50% change of D and s, the expected profits reach to \$34 979.91 and \$48 296.38, respectively. Moreover, at −50% changes of the parameter D the expected profit assumes value \$11 614.81 which is the reduction of 93.06% and that for $s$we get no feasible solution. Table 4 shows that for the case of ρ at

TABLE 7. Sensitivity analysis at $(\rho, \sigma) = (0.3, 0.3)$.

n	Parametric change		$I(Q^*)$	$I(T^*)$	$I(Z_E^*)$	$R_z(\%)$	$R_Q(\%)$
1	ρ	+50%	1708.422	21.84	43 811.82	88.06	87.89
		+30%	1721.220	21.50	44 124.23	89.41	89.33
		−30%	No feasible solution				
		−50%					
	σ	+50%	No feasible solution				
		+30%	1734.567	20.82	38 796.96	66.54	90.76
		−30%	1745.454	21.36	44 186.28	89.67	91.97
		−50%	1750.076	21.58	43 856.53	88.26	92.52

Notes. Where, $R_z = \left\{\frac{I(Z_E^*)-Z_E^*}{Z_E^*}\right\} \times 100\%$ and $R_Q = \left\{\frac{I(Q^*)-Q^*}{Q^*}\right\} \times 100\%$.

−30%, −50% and for the case of σ at +50%, +30% the objective function giving no feasible solutions, but for the other cases the expected profit getting the bounds ($42 206.69, $44 323.28) by reaching the enhancement +81–92% approx with respect to the increase of crisp order quantity and cycle time by +91–95% approx. In Table 5, we see a suicidal jump of the expected profit function occurs at +50% change of σ by reducing the value to −93.92% and at −30%, −50% changes of ρ no feasible solution occurs. For the other cases the profit lies within the bounds ($28 638.43, $44 365.28). Table 6 shows that at −50% change of ρ the objective function has no feasible solution, but for the other cases the profit value assumes the bounds ($43 004.79, $44 161.58) by the crisp enhancement +84–89% alone. Table 7 shows that at −50%, −30% changes of ρ and that for σ at +50% the objective function has no feasible solution, but beyond those the profit value assumes the bounds ($43 811.82, $44 186.28) by the crisp enhancement +66–89% alone. However, throughout the whole Tables 4–7 we see that the cycle time as well as order quantity jump tremendously to provide the maximum profit except the suicidal jump.

6. Graphical illustrations

Here we draw different graphs based on table data to justify the model. Figure 3 shows the specific overview of the profit function under dense fuzzy environment. It shows whenever we are considering the crisp as well as the fuzzy environment for optimizing the objective function we are getting very close solution but at the dense fuzzy environment the expected profit assumes almost double of that crisp/fuzzy solutions. Figure 3 reveals that, the model optimum exists and it follows the edges of a (ρ, σ) paradoxical square. At minimum $\sigma-$ path [referring to the coordinate (0.2, 0.1)] the highest profit value lies having coordinate (0.2, 0.14)] amounting $44 365.28. Then it goes towards right handed upper edges getting zigzag values. The top left vertex shows the suicidal [see Appendix A.4] point in which a great loss of profit occurs. Beyond the paradoxical path the profit values getting decreasing. Figure 4 shows the general overview of the expected profit function under (ρ, σ) paradoxical coordinates. The four coordinate points constitutes a small square unit called vertices of that unit square. If we go across the vertical line on σ-axis keeping ρ value at minimum then the expected profit quickly jumps down to minimum value. But if we would like to pass across the ρ-axis keeping the σ value at minimum then the expected profit reaches to the highest value. Again if we think of (ρ, σ) at lower and upper vertex of the unit square then we see fuzzy solutions began to decrease but the dense fuzzy solutions getting jumps to higher values respectively (Fig. 5).

7. Conclusion

In this article we have developed a profit seeking EOQ model for imperfect quality items with receiving reparative batch and order overlapping under dense fuzzy environment. To avoid the shortages, the orders are

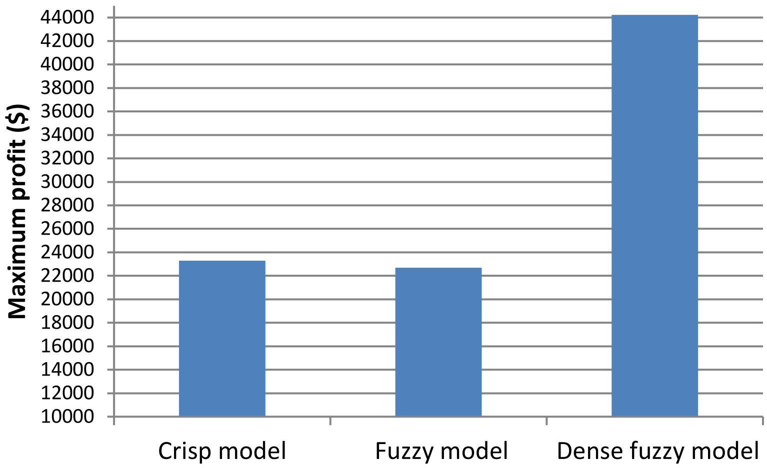

Figure 3. Optimal profit under several environments.

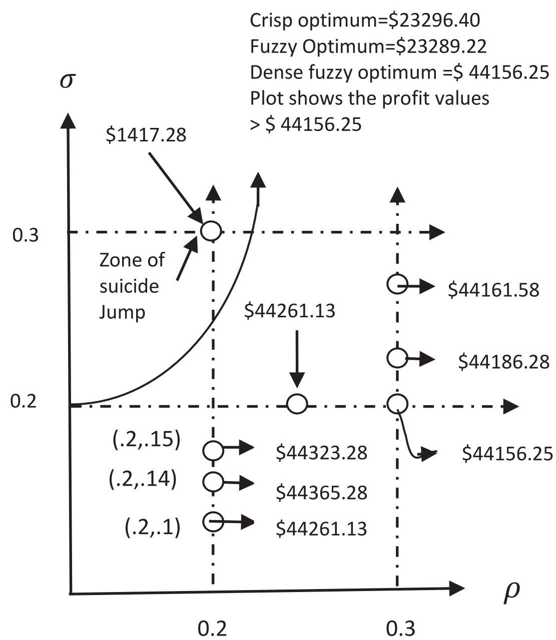

Figure 4. Optimum profit path at paradoxical square.

overlapped for customers' satisfactions, scrap items are separated through screening process and a salvage value has been taking place for that scrap items. At the end of the cycle time a profit function is developed in which the order quantity is the decision variable. As per real situations, since the demand quantity per month may vary with learning experiences of the decision maker (DM) over interaction/negotiations with the learned customers, so it is quiet natural to consider the monthly demand quantity as dense fuzzy number. The basic overview of the dense fuzzy number is to reduce the uncertainty on demand per cycle. By this way, it will be quite easier to any DM so that (s)he could order the specific requirements of the customers without hesitation. Defuzzification is done with the help of ranking index rule for both fuzzy and dense fuzzy models. This study explores that

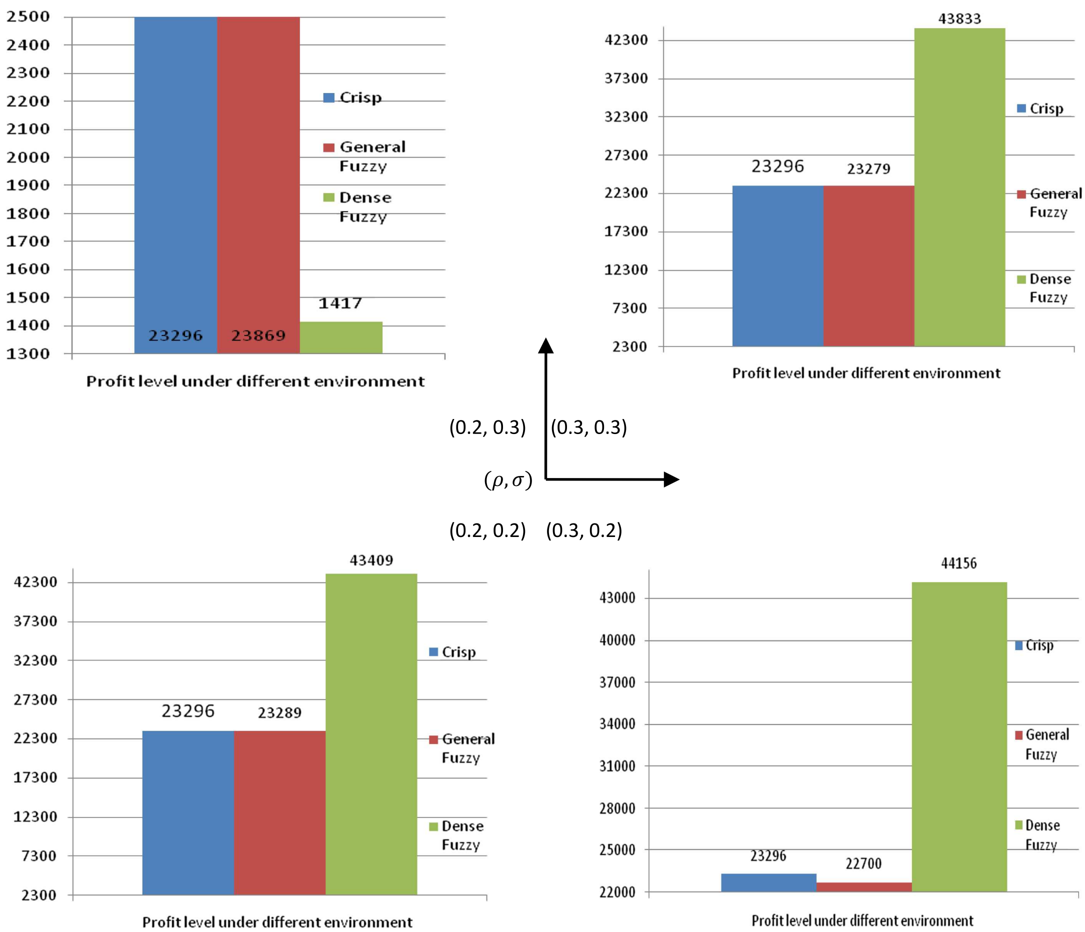

Figure 5. Profit Jump under (ρ, σ) paradoxical coordinates.

a tremendous situation exists to reach into a golden profit which is beyond the imagination of crisp, even the general fuzzy environment also. However, a situation may come where huge amount of profit loss arises. We rename this situation as the suicide zone. Moreover, we call the profit jump against the (ρ, σ)-coordinate as paradoxical because, based on four vertical co-ordinates of the unit square if we wish to get a profit sensitivity then we might have seen that the zone of suicide point and the zone of golden point must lie on the boundary of the same square or any one of its foot. To overcome this zone, the DM might have to follow the unit square paradoxical alert during experiencing the inventory management process exclusively.

Scope of future work

Considering all the cost coefficients as dense fuzzy or intuitionistic dense fuzzy numbers to the related models of this study anyone can develop more new research in these directions.

Appendix A.

A.1. Basic arithmetic operations on triangular fuzzy number (TFN)

Let $A = \langle a_1, a_2, a_3\rangle$ and $B = \langle b_1, b_2, b_3\rangle$ be two TFN, then for usual Arithmetic operations $\{+, -, \times, \div\}$, namely addition, subtraction, multiplication between A and B are given below

(i) $A + B = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3\rangle$,

(ii) $A - B = \langle a_1 - b_3, a_2 - b_2, a_3 - b_1\rangle$,

(iii) $\mathrm{A} \times B = \langle \mathrm{Min}(a_i b_j), \mathrm{Max}(a_i b_j)\rangle\ \forall i, j = 1, 2, 3$,

(iv) $A/B = \langle \mathrm{Min}(a_\mathrm{i}/b_\mathrm{j}), \mathrm{Max}(a_\mathrm{i}/b_\mathrm{j})\rangle$ for $b_j \neq 0,\ \forall i, j = 1, 2, 3$,

(v) $\begin{array}{ll} \delta A = \langle \delta a_1, \delta a_2, \delta a_3\rangle & \text{if } \delta \geq 0 \\ \text{and } \delta A = \langle \delta a_3, \delta a_2, \delta a_1\rangle & \text{if } \delta < 0 \end{array}.$

A.2. Crisp convergence for the objective function

First of all we shall transfer the dense fuzzy into general fuzzy by putting $n = 0$ throughout, then taking limit as under $(\rho \to 0 \leftarrow \sigma)$. It is seen that, as $\rho \to 0 \leftarrow \sigma$ then

$$D_1 = D_2\left(1 - \frac{\rho}{1+n}\right) \to D_2 \quad \text{and} \quad D_3 = D_2\left(1 + \frac{\sigma}{1+n}\right) \to D_2.$$

Moreover we have $Q_1 = \sqrt{\frac{2AD_2\left(1-\frac{\rho}{1+n}\right)}{D_2Y_1+Y_2}} \to \sqrt{\frac{2AD_2}{D_2Y_1+Y_2}}$; $\quad Q_2 = \sqrt{\frac{2AD_2}{D_2Y_1\left(1+\frac{\sigma}{1+n}\right)+Y_2}} \to \sqrt{\frac{2AD_2}{D_2Y_1+Y_2}}$ and $Q_3 = \sqrt{\frac{2AD_2\left(1+\frac{\sigma}{1+n}\right)}{D_2Y_1+Y_2}} \to \sqrt{\frac{2AD_2}{D_2Y_1+Y_2}}$. Thus from above, $D_1 \to D_2 \leftarrow D_3 \Rightarrow D_1 \to D \leftarrow D_3$ and $Q_3 \to Q_1 \leftarrow Q_2 \Rightarrow \sqrt{\frac{2AD}{DY_1+Y_2}} = Q$.

So,

$$\lim_{\substack{D_1 \to D_2 \\ D_3 \to D_2}} I\left(Z_E^*\right) = \frac{4D_2F_1}{4} - \frac{F_4}{4}\left(Q_1 + 2Q_2 + Q_3\right) - \frac{D_2F_3}{12}\left[Q_3 + Q_2 + 2Q_3 + 4Q_2 + 2Q_1 + Q_1 + Q_2\right]$$
$$- \frac{D_2F_2}{2}\left[\frac{Q_2 - Q_1}{(Q_2 - Q_1)}\mathrm{Log}\left(\frac{Q_2}{Q_1}\right) - \frac{Q_3 - Q_2}{(Q_3 - Q_2)}\mathrm{Log}\left(\frac{Q_2}{Q_3}\right) - \frac{Q_3 + 2Q_2 + Q_1 - Q_1 - Q_2 - Q_3 - Q_2}{Q_1Q_2 + Q_3Q_2 - Q_3Q_1 - Q_2^2}\right]$$

and that for order quantity is given by $I\left(Q^*\right) = \frac{1}{4}\left(Q_1 + 2Q_2 + Q_3\right)$

$$\Rightarrow \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_2}} I\left(Z_E^*\right) = D_2F_1 - Q_2F_4 - D_2Q_2F_3$$
$$- \frac{D_2F_2}{2} \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_2}} \left[\frac{\mathrm{Log}\left(\frac{Q_2}{Q_1}\right)}{(Q_2 - Q_1)} - \frac{\mathrm{Log}\left(\frac{Q_2}{Q_3}\right)}{(Q_3 - Q_2)} - \frac{Q_3 + 2Q_2 + Q_1 - Q_1 - Q_2 - Q_3 - Q_2}{Q_1Q_2 + Q_3Q_2 - Q_3Q_1 - Q_2^2}\right]$$

$$\Rightarrow \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_1}} I\left(Z_E^*\right) = D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{2} \lim_{\substack{Q_1 \to Q_1 \\ Q_3 \to Q_2}} \left[\frac{\operatorname{Log}\left(\frac{Q_2}{Q_1}\right)}{(Q_2 - Q_1)} - \frac{\operatorname{Log}\left(\frac{Q_2}{Q_3}\right)}{(Q_3 - Q_2)}\right]$$

$$\Rightarrow \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_1}} I\left(Z_E^*\right) = D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{2Q_2} \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_1}} \left[\frac{\operatorname{Log}\left(\frac{Q_2}{Q_1}\right)}{\left(1 - \frac{Q_1}{Q_2}\right)} - \frac{\operatorname{Log}\left(\frac{Q_2}{Q_3}\right)}{\left(\frac{Q_3}{Q_2} - 1\right)}\right]$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{2Q_2} \lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_1}} \left[\frac{\operatorname{Log}\left(\frac{Q_2}{Q_1}\right)}{\left(1 - \frac{Q_1}{Q_2}\right)} + \frac{\operatorname{Log}\left(\frac{Q_2}{Q_1}\right)}{\left(1 - \frac{Q_1}{Q_2}\right)}\right]$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{Q_2} \lim_{Q_1 \to Q_2} \left[\frac{\operatorname{Log}\left(\frac{Q_2}{Q_1}\right)}{\left(1 - \frac{Q_1}{Q_2}\right)}\right]$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{Q_2} \lim_{Q_1 \to Q_2} \left[\frac{\operatorname{Log}\left(\frac{Q_1}{Q_2}\right)}{\left(\frac{Q_1}{Q_2} - 1\right)}\right]$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{Q_2} \lim_{\frac{Q_1}{Q_2} \to 1} \left[\frac{\operatorname{Log}\left(\frac{Q_1}{Q_2} - 1 + 1\right)}{\left(\frac{Q_1}{Q_2} - 1\right)}\right]$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{Q_2} \lim_{k \to 0} \left[\frac{\operatorname{Log}(k+1)}{k}\right], \text{ for } \frac{Q_1}{Q_2} - 1 = k \text{ as } \frac{Q_1}{Q_2} \to 1,\ k \to 0$$

$$= D_2F_1 - Q_2F_4 - D_2Q_2F_3 - \frac{D_2F_2}{Q_2}$$

$$\Rightarrow I\left(Z_E^*\right) = DF_1 - Q^*F_4 - DQ^*F_3 - \frac{DF_2}{Q^*}, [\text{Since, } D_2 = D, Q_2 \to Q^*].$$

Because, $Q_2 = \sqrt{\frac{2AD_2}{D_2Y_1(1+\sigma)+Y_2}} \to \sqrt{\frac{2AD}{DY_1+Y_2}} \to Q^*$ as $\sigma \to 0$.

Hence we arrive at the crisp objectives function.

A.3. Crisp convergence for the cycle time T

We consider the expected time equation (4.18) and taking limits on it as follows:

$$\lim_{\substack{Q_1 \to Q_2 \\ Q_3 \to Q_2}} I\left(T^*\right) = \frac{(1 - r_s)}{2} \left[\frac{D_2Q_3 - D_1Q_2}{(D_2 - D_1)^2} \operatorname{Log}\left(\frac{D_2}{D_1}\right) - \frac{D_3Q_2 - D_2Q_1}{(D_3 - D_2)^2} \operatorname{Log}\left(\frac{D_2}{D_3}\right)\right.$$
$$\left. - \frac{D_3Q_3 + 2Q_2D_2 + Q_1D_1 - D_1Q_2 - D_2Q_1 - D_3Q_2 - Q_2D_3}{D_1D_2 + D_3D_2 - D_3D_1 - D_2^2}\right]$$
$$= \lim_{\substack{D_1 \to D_2 \\ D_3 \to D_1}} \frac{(1 - r_s)Q_2}{2} \left[\frac{D_2 - D_1}{(D_2 - D_1)} \operatorname{Log}\left(\frac{D_2}{D_1}\right) - \frac{D_3 - D_2}{(D_3 - D_2)} \operatorname{Log}\left(\frac{D_2}{D_3}\right)\right]$$

$$= \lim_{\substack{D_1 \to D_2 \\ D_3 \to D_1}} \frac{(1-r_s)\,Q_2}{2}\left[\frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{(D_2-D_1)} - \frac{\operatorname{Log}\left(\frac{D_2}{D_3}\right)}{(D_3-D_2)}\right]$$

$$= \lim_{\substack{D_1 \to D_2 \\ D_3 \to D_1}} \frac{(1-r_s)\,Q_2}{2}\left[\frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{(D_2-D_1)} + \frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{(D_2-D_1)}\right]$$

$$= \lim_{\substack{D_1 \to D_2 \\ D_3 \to D_1}} \frac{(1-r_s)\,Q_2}{D_2}\left[\frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{\left(1-\frac{D_1}{D_2}\right)}\right] = \lim_{D_1 \to D_2} \frac{-(1-r_s)\,Q_2}{D_2}\left[\frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{\left(\frac{D_1}{D_2}-1\right)}\right]$$

$$= \lim_{D_1 \to D_2} \frac{(1-r_s)\,Q_2}{D_2}\left[\frac{\operatorname{Log}\left(\frac{D_2}{D_1}\right)}{\left(\frac{D_1}{D_2}-1\right)}\right]$$

$$= \lim_{\frac{D_1}{D_2}-1\to 0} \frac{(1-r_s)\,Q_2}{D_2}\left[\frac{\operatorname{Log}\left(1+\frac{D_1}{D_2}-1\right)}{\left(\frac{D_1}{D_2}-1\right)}\right] = \frac{(1-r_s)\,Q_2}{D_2} \Rightarrow T^* = \frac{(1-r_s)\,Q^*}{D}.$$

A.4. Suicidal point

A tremendous loss of profit occurs due to careless attempt/drive or decision of inventory practitioner (decision maker). A story behind it, in Bidarva, the district of Maharastra, India, whenever the cotton producing farmers fall into great loss of their production due to natural calamity or careless use of pesticides in the cotton field they usually get suicide in that field to escape themselves from the larger amount of bank loans and from the pressures of livelihoods of their family.

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