JOINT PRICING DETERIORATING INVENTORY MODEL CONSIDERING PRODUCT LIFE CYCLE AND ADVANCE PAYMENT WITH A DISCOUNT FACILITY

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Abstract. The product life cycle of a deteriorating product is an important consideration in inventory management. This paper simultaneously investigates the optimum pricing and inventory decisions considering product life cycles under price-dependent demand and advance payment systems with a discount facility. A time-dependent holding cost is also introduced. The objective is to carefully balance the critical decision variables in order to maximize the total profit. Furthermore, the theoretical analysis validates the concavity of the profit function. A numerical example and sensitivity analysis are provided to show the characteristics of the model. The study shows that an advanced payment period, installment numbers, product's maximum life cycle, purchasing cost and demand function significantly influence the total profit. This inventory model with a known product lifetime and advance payments can provide management insights to inventory manager in his/her strategic planning.

1. Introduction and literature review

Inventory management plays an important role in balancing uncertain customer demand and supply. Economic order quantity (EOQ) is a basic formula that helps businesses to minimize the total inventory cost. In recent decades, supply chain management has become very popular, and various inventory models considering the mutual relationship among supply chain parties have been developed. Most inventory models assumed immediate payment upon the receipt of a product. Goyal [7] introduced allowable delay-in-payments by the supplier in his EOQ model. Aggarwal and Jaggi [1] pointed out that permissible delay might be financially sustainable for buyers. Recently, Mashud *et al.* [24] examined the delay-in-payments policy for non-instantaneous deteriorating items with a plan to invest in preservation technology, while Mashud *et al.* [25] examined an inventory model for two different types of deterioration rates. Panda *et al.* [30] examined a trade-credit approach for a two warehouse problem while Mashud *et al.* [26] provided a two-level trade-credit approach with shortages.

Advance payment is an alternative payment policy for suppliers. When a retailer places an order for the product, the supplier demands some money in advance. Next, the supplier provides some price discounts for advance payment. Lastly, a certain scheme allows the retailer to pay a part of the purchase cost by installments. Maiti *et al.* [22] incorporated an advance payment pattern, price-dependent demand and stochastic lead-time in their model. Gupta *et al.* [8] proposed a model with the retailers' advance payment to the wholesalers. Guria *et al.* [9] proposed an inventory procedure that allowed for an item's immediate payment for a random planning horizon. Taleizadeh [34] considered multiple prepayments for deteriorating items with shortages. Taleizadeh [35] developed an inventory pricing and replenishment model with advance payment and quality problems. Taleizadeh *et al.* [37] modified the previous model by considering complete backordering. Teng *et al.* [38] considered advance payment with a maximum products life cycle. Recently, Taleizadeh *et al.* [36] developed an incremental discount model while Khan *et al.* [19,20] provided a cash discount for full purchase cost payment. Khan *et al.* [18,21] and Shaikh *et al.* [33] considered advance payment for a two-warehouse problem with partial backordering. However, none of these models offered a discount on multiple prepayments or considered the products' maximum lifetime.

In general, customer demand depends largely on the relative price of the product. Using a discounted cash flow method, Hou and Lin [11] developed a replenishment decision model for deteriorating items with price dependent demand. Hasan and Mashud [10] formulated an EOQ model with price, continuous-time, and frequency of advertisement dependent demand under a shortage policy. Dye and Hsieh [6] modified the model by permitting the demand rates affected by both the selling price and the displayed stock level. Yang *et al.* [42] introduced time and selling price-dependent demand rates while allowing multiple production cycles. Alfares and Ghaithan [2] developed a pricing inventory model showing a time-varying unit holding cost with a discount facility and a rented warehouse agreement. Shaikh *et al.* [31] acquainted an inflationary inventory model, under which non-instantaneous deterioration and fully backlogged shortages were considered for a price and stock-dependent demand pattern. Mashud [23] considered different price-sensitive demand patterns and developed inventory models with shortages, while Mashud *et al.* [27] anticipated a model with the same demand pattern and controlling the deterioration rate through preservation technology investment under a credit policy. Jadidi *et al.* [16] considered price-dependent stochastic demand with all-unit quantity discounts and the effect of transportation. Chen *et al.* [5] optimized the discount strategy of an inventory model with an unknown customer demand distribution. Later, Shaikh *et al.* [32] framed a temporary discount EOQ model where shortages were allowed with stock-dependent demand and time-varying deterioration rates. They provided a discount facility for a backorder inventory model with time reliant deterioration, but did not consider an advance payment scheme, while Mashud *et al.* [28] considered advance payment with discount facility on prepayments.

In many situations, deterioration is a common occurrence. Deteriorating items, such as vegetables, fruits, flowers, and ice cream, are decayed, damaged, or expire over time. The lifetime of deteriorating items is limited, and these items have an expiration date. Specifically, the rate of deterioration rises over time and the items will totally degrade at the expiration date. Therefore, to develop an inventory strategy, the rate of deterioration should be taken into account. Researchers studied some extended models for declining items with an expiration date or maximum lifetime. Wu *et al.* [41] considered the expiration dates of declining products, in which the deterioration rate function approaches the full item value near the expiration date. Later, Chen and Teng [3] settled an EOQ model for a retailer where the product has a continuous deterioration, a maximum lifetime, and simultaneously, uses a trade-credit approach. Considering the seller's optimal credit period and the product's maximum lifetime, Wang *et al.* [40] examined the cycle time for deteriorating items. Extending the concept of Wu *et al.*'s [41], Tiwari *et al.* [39] delivered a supply chain inventory model with an expiration date to determine the optimum price and replenishment cycle. A new concept was introduced by Teng *et al.* [38] by simultaneously considering advance payments and a 100% decay rate at the product expiration date. Chen *et al.* [4] projected an EOQ model by considering demand for fresh products, which depend on their fresh-ending date and current inventory while Pal *et al.* [29] projected an inventory model with a stochastic production rate and production cost with a fixed lifetime. Many researchers took a price-sensitive demand pattern with a constant deterioration rate, but very few considered the product life cycle. Iqbal and Sarkar [12] considered a price-dependent demand that is influenced by the product's maximum lifetime. Iqbal and Sarkar [13] considered a deterioration rate that

TABLE 1. The distinction between previously established research and the current work.

Authors	Demand	Holding cost	Payment mode	Deterioration	Discount	Maximum lifetime
Aggarwal and Jaggi [1]	CN	CN	DL	CN	No	No
Hou and Lin [11]	PD & SD	CN	No	CN	No	No
Maiti *et al.* [22]	PD	CN	AV	No	No	No
Gupta *et al.* [8]	CN	CN	AV	No	Yes	No
Dye and Hsieh [6]	PD & SD	CN	No	CN	No	No
Chen and Teng [3]	CN	CN	DL	TV	No	Yes
Taleizadeh [34]	CN	CN	AV	CN	No	No
Wang *et al.* [40]	CPD	CN	DL	EXP	No	Yes
Wu *et al.* [41]	CN	CN	DL	EXP	No	Yes
Alfares and Ghaithan [2]	PD	TV	No	No	Yes	No
Chen *et al.* [4]	SD	CN	No	EXP	No	Yes
Teng *et al.* [38]	CN	CN	AV	TV	No	Yes
Shaikh *et al.* [31]	PD & SD	CN	No	CN	No	No
Tiwari *et al.* [39]	PD	CN	DL	EXP	No	Yes
Shaikh *et al.* [33]	PD & SD	TV	No	CN	No	No
Khan *et al.* [20]	PD & SD	CN	AV	CN	Yes	No
This article	PD	CN & TV	AV	EXP	Yes	Yes

Notes. Where AV: Advance, DL: Delay, CN: Constant, PD: Price-sensitive, CPD: Credit-period dependent, EXP: Expiration, TV: Time-varying. SD: Stock-dependent.

depends on time and its maximum lifetime. Recently, Iqbal and Sarkar [14] studied deteriorating items with lifetime-reliant demand rate and incorporated the consequence of preservation technology. Some of the main contributions of the projected model are as follows:

(i) This paper extends the idea of Teng *et al.* [38] by incorporating a realistic holding cost and discount incentive for multiple prepayments.
(ii) This model provides a new idea in discount policy with multiple prepayments which will extend the idea of instant cash discount on purchase cost anticipated in Khan *et al.* [20].
(iii) The proposed model will simultaneously examine pricing effects with an advance payment system and a product life cycle.

This article presents an EOQ model considering the maximum lifetime of a decaying item. The deterioration rate increases over time and the products are totally deteriorated at the time of expiration. At the end of the product lifetime, the product has no salvage value. The retailer prepays a portion of his purchase cost as an advance through equal multiple installments to the supplier before receiving the products. The proposed model works under selling price-dependent demand, and the holding cost is a linear function consisting of constant and time-varying components. Table 1 summarizes the distinction between some related works.

Following the introduction in Section 1, Section 2 represents all the notations and model assumptions. Section 3 explains the development of the model. Section 4 presents some theoretical derivations from the basic model. Section 5 provides some numerical examples and sensitivity analysis in Section 6. Section 7 discusses some important managerial insights. In the end, Section 8 summarizes the findings and proposes some future works.

2. NOTATION AND ASSUMPTIONS

2.1. Notation

With the help of the following notations, the proposed model is constructed.

Notations	Units	Description
p_c	$/unit	purchasing cost of products per unit
h_c	$/unit/unit time	cost of holding product per unit per unit time
$\theta(t)$	per year	deterioration rate $(0 \le t \le m)$
$I(t)$	Units	inventory level
D	Units	demand rate
O	$/order	ordering cost per order, $O > 0$
H	$/cycle	holding cost per cycle
A	$/cycle	purchase cost per cycle
C	$/cycle	capital cost per cycle
S	Units	the initial level of inventory
Q	Units	quantity of products ordered by the retailer
L	Years	the time range for the retailer to pay in advance where, $L > 0$
r	$/unit	the rate of interest imposed on capital cost per year
n	Integer	number of equal prepayments
u	$/unit	constant factor of holding cost
v	$/unit	time reliant factor of holding cost
x	%	discount rate for a single installment of the prepayment
I_r	$/unit	rate of interest on the loan
m	years	the expiration date of products, $0 < m < 8$
β	%	discount on purchase cost for prepayments
γ	%	a portion that the retailer has to prepay from the purchase cost before receiving the products, $0 \le \gamma \le 1$
a	Constant	a constant factor in demand function, $a > 0$
b	Constant	price reliant factor in demand function, $b > 0$
Decision variables		
p	$/unit	the price of selling products
T	years	the cycle length of the proposed model

2.2. Assumptions

The succeeding assumptions have been considered for the advance of the model.

1. A linear price-sensitive demand $D = (a - bp)$, represents the customer demand that increases when the price of products decreases. Thus, D is a linearly decreasing function of the selling price, p; a and b are constants greater than zero. The demand D is non-negative with constraint $a > bp$ [39].
2. The deterioration rate of the perishable products is time dependent. All the deteriorating items deteriorate over time and salvage at the expiration date m, *i.e.*, $0 \le \theta(t) \le 1$ which satisfies $\theta'(t) < 0$ and $\theta(m) = 1$. Using the same assumption of Chen and Teng [3], Wang *et al.* [40] and Teng *et al.* [38], one has:

$$\theta(t) = \frac{1}{1 + m - t}, \quad 0 \le t \le T \le m.$$

3. No option for a replacement or repair is considered in the proposed model for the deteriorated items.
4. The lead time is constant.
5. There is no shortage allowed.
6. On behalf of the extremely seasonal and deteriorating items, the supplier demands a certain percentage of the retailer's purchase cost prepayments. The remaining balance is paid upon delivery of the purchase [34].
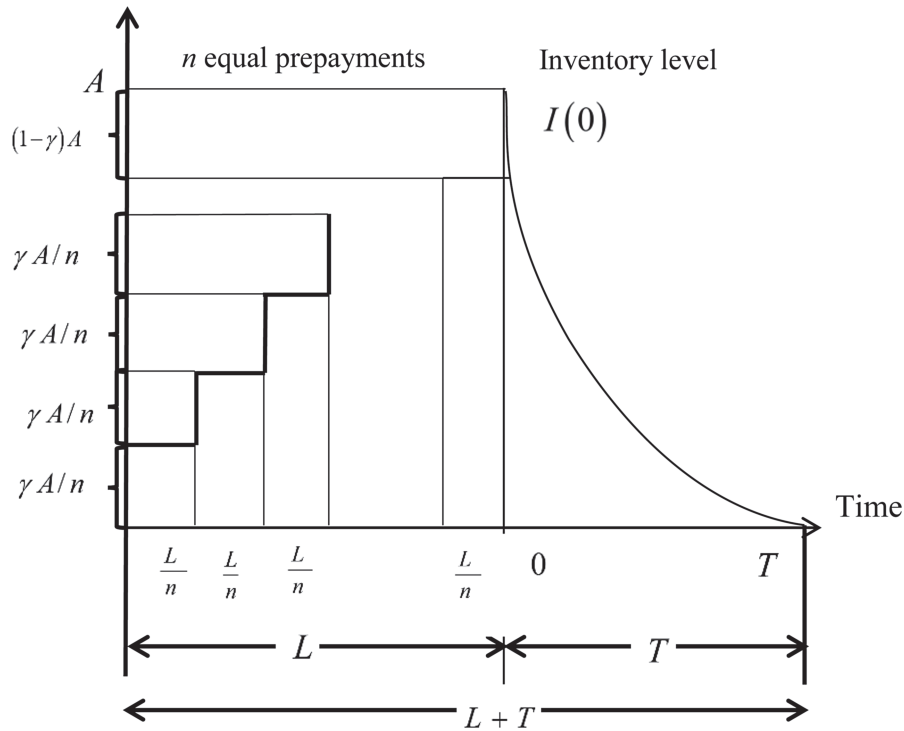
FIGURE 1. A graphical framework of the proposed model.

7. In terms of the prepayments, the supplier also offers a discount on purchased products. To receive the discount, the retailer accepts the condition and pays γA on n equal prepayments during L years before delivery (see [20]).

3. MATHEMATICAL FORMULATION

We have discussed two models with advance payments. The first case assumes that the payment is completed in several equal installments. Later, in the second case, the retailer borrows some money for his purchase cost from any financial institution to prepay the whole cost at a formerly determined time, receiving the products and enjoying a cash discount on that amount.

3.1. Case 1 (Discount when payment is paid in installments)

Figure 1 indicates the EOQ model where the retailer is bounded to n equal prepayments before the order delivery, and then the rest of the purchase cost must be completed at the time of delivery. When the full payments are done, the level of retailer's inventory $I(t)$ starts with S units, and then due to both demand and deterioration, it starts to decrease and gradually reaches zero at $t = T$.

Throughout the replenishment cycle $[0, T]$, the inventory level at time t is ascertained by the subsequent differential equation:

$$\frac{\mathrm{d}I}{\mathrm{d}t} + \theta(t)I(t) = -D, \qquad 0 \leq t \leq T \tag{3.1}$$

where $D = a - bp$ and $\theta(t) = \frac{1}{1+m-t}, 0 \leq t \leq m$.

Assuming the boundary conditions $I(0) = S$ and $I(T) = 0$.

By using the boundary conditions, we solve equation (3.1), and the obtained solution is

$$I(t) = D\left(1 + m - t\right)\ln\left(\frac{1 + m - t}{1 + m - T}\right). \tag{3.2}$$

At $t = 0, I(t) = S$. Then from equation (3.2), we get
$S = D\varphi$, where $\varphi = (1 + m)\ln\left(\frac{1+m}{1+m-T}\right)$.

Therefore

$$Q = S = D\varphi. \tag{3.3}$$

The retailer's sales revenue refers to the income derived from product sales. If p is the product's selling price, a is the constant part and b is the coefficient of price in demand, then the sales revenue can be written as

$$\text{SR} = p\int_0^T D\,\mathrm{d}t = pDT. \tag{3.4}$$

The cost of placing an order of products per cycle can be written as

$$\text{OC} = O. \tag{3.5}$$

If p_c is the purchasing cost per unit of product and Q be the ordered quantity, then the purchasing cost per cycle can be written as

$$A = p_c Q = p_c D\varphi. \tag{3.6}$$

After purchasing the products, the retailer has to store them for a certain period. The holding cost (H) includes the costs of storing products such as rent, labor, and utilities. The cost of holding some products varies over time as their freshness depends on time. If h_c is the cost of holding product per unit per unit time, u and v are the constant and time reliant factor of holding cost, then the H per cycle:

$$
\begin{aligned}
H &= h_c\int_0^T (u + vt)I(t)\,\mathrm{d}t \\
&= h_c D\left[
\begin{array}{l}
u\left\{\frac{(1+m)}{2}\varphi + \frac{1}{4}\left(-2T - 2mT + T^2\right)\right\} \\
+v\left\{\frac{1}{36}\left((1 + m - T)^2\left(5 + 5m + 4T\right) - 5(1 + m)^3\right) + \frac{(1+m)^2}{6}\varphi\right\}
\end{array}
\right].
\end{aligned} \tag{3.7}
$$

As the supplier requests the retailer to pay advance before delivery, the retailer prepays γ fraction of his purchase cost A in n number of equal installments before delivery. Agreeing to Taleizadeh [35], the capital cost for per cycle can be written as,

$$C = r\left[\frac{\gamma A}{n}\left(\frac{L}{n}\right)(1 + 2 + \ldots + n)\right] = \left(\frac{n+1}{2n}\right)\gamma L r A = \left(\frac{n+1}{2n}\right)\gamma L r p_c D\varphi. \tag{3.8}$$

To receive the maximum x percent discount on product purchases, the retailer must pay some portions of his purchase costs in n number of equal installments before delivery. The relationship between the discount rate β of the purchase cost and the number of prepayments n is

$$\beta = \frac{x}{n}, \qquad 0 \le x \le 100.$$

The discount on prepayments (DC) can be written as:

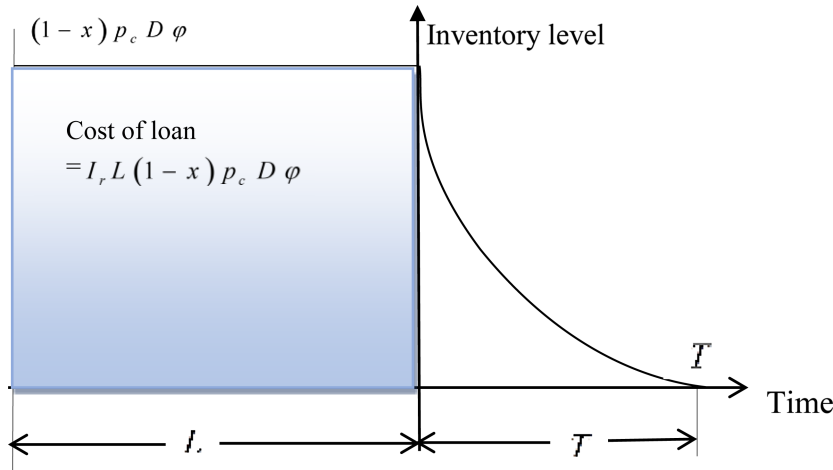$$\text{DC} = p_c Q\beta = \frac{xp_c}{n}D\varphi. \tag{3.9}$$

FIGURE 2. A pictorial representation when payment is done at a time.

Total profit per cycle written as a function of total cycle length T and selling price p is as follows:

$$
\begin{aligned}
\psi(p,T) &= \frac{1}{T}\left(\text{SR} - \text{OC} - A - C - H + \text{DC}\right) \\
&= \frac{1}{T}\left\{
\begin{array}{l}
pDT - O - p_c D\varphi\left[1 + \left(\frac{n+1}{2n}\right)\gamma Lr - \frac{x}{n}\right] \\
-h_c D\left[u\left\{\frac{(1+m)}{2}\varphi + \frac{1}{4}\left(-2T - 2mT + T^2\right)\right\}\right. \\
\left.+v\left\{\frac{1}{36}\left((1+m-T)^2(5+5m+4T) - 5(1+m)^3\right) + \frac{(1+m)^2}{6}\varphi\right\}\right]
\end{array}
\right\}.
\end{aligned}
\tag{3.10}
$$

3.2. Case 2 (Discount when payment is paid at a time)

Figure 2 indicates a situation where the retailer offers $x\%$ discount to the customer for complete advance payment before delivery [20]. When it is the buyer does not have enough fund during the time L, he may take a loan from a bank at an interest rate of $I_r\%$.

The purchased cost with discount can be written as

$$
A = (1-x)p_c D\varphi.
\tag{3.11}
$$

The cost of taking a loan can be written as

$$
G = I_r L(1-x)p_c D\varphi.
\tag{3.12}
$$

Then the total profit becomes

$$
\begin{aligned}
\psi(p,T) &= \frac{1}{T}\left(\text{SR} - \text{OC} - A - H - G\right) \\
&= \frac{1}{T}\left\{
\begin{array}{l}
pDT - O - (1 + I_r L)(1-x)p_c D\varphi \\
-h_c D\left[u\left\{\frac{(1+m)}{2}\varphi + \frac{1}{4}\left(-2T - 2mT + T^2\right)\right\}\right. \\
\left.+v\left\{\frac{1}{36}\left((1+m-T)^2(5+5m+4T) - 5(1+m)^3\right) + \frac{(1+m)^2}{6}\varphi\right\}\right]
\end{array}
\right\}.
\end{aligned}
\tag{3.13}
$$

4. Theoretical derivation

In this section, the theoretical derivations of the stated two cases are discussed. In Section 4.1, the theoretical derivation for Case 1 is derived whereas Section 4.2 stands for Case 2.

4.1. Case 1

To get the optimal solution T^*, performing the first and second-order partial derivatives of $\psi(p, T)$ in (3.10) with regard to T and re-arranging terms, one has

$$\frac{\partial \psi}{\partial T} = \frac{O}{T^2} + p_c D \mu_1 \left(1 + \frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right)$$
$$+ h_c D \left[\frac{u}{4}\left\{2(1+m)\mu_1 - 1\right\} + v\left\{\frac{1}{36T^2}\left(5\mu_2 - 5\mu_3 - 2\mu_4 T\right) - \frac{(1+m)^2}{6}\mu_1\right\}\right] \tag{4.1}$$

where $\mu_1 = (1+m)\left\{\frac{1}{T^2}\ln\left(\frac{1+m}{1+m-T}\right) - \frac{1}{T(1+m-T)}\right\}, \mu_2 = (1+m)^3, \mu_3 = (1+m)(1+m-T)^2$
$\mu_4 = (1+m-T)(5+5m+4T)$
and

$$\frac{\partial^2 \psi}{\partial T^2} = -\frac{2O}{T^3} - p_c D(1+m)\left(1 + \frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right)\left[\frac{2}{T^3}\ln\left(\frac{1+m}{1+m-T}\right) - \frac{2+2m-3T}{(1+m-T)^2 T^2}\right]$$
$$- h_c D \left[\begin{matrix} u(1+m)^2\left\{\frac{1}{T^3}\ln\left(\frac{1+m}{1+m-T}\right) - \frac{2+2m-3T}{2T^2(1+m-T)^2}\right\} \\ +v\left\{\frac{(1+m)^3}{6}\left(\frac{2}{T^3}\ln\left(\frac{1+m}{1+m-T}\right) - \frac{2+2m-3T}{(1+m-T)^2 T^2}\right) + \frac{10}{9}\frac{(1+m)^3}{T^3} - \frac{2}{9}\right\}\end{matrix}\right]. \tag{4.2}$$

From equations (4.1) and (4.2), we get the following theoretical results. By using (4.1), we define the following discrimination term:

$$W = 2O - p_c D(1+m)\eta_1\left(1 + \frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right) - \frac{h_c D}{2}\left[\frac{u\eta_2}{2} - \frac{v\eta_3}{3}\right] \tag{4.3}$$

where $\eta_1 = [m - \ln(1+m)], \eta_2 = \left[m^2 - 2(1+m)^2\ln(1+m) + 2m(1+m)^2\right], \eta_3 = (1+m)^3\left\{m - \ln(1+m)\right\} + \frac{1}{6}\left\{5(1+m)^3 - 5(1+m) - 2m(5+9m)\right\}$.

Theorem 4.1. *When* $\frac{(1+m-2T)}{(1+m-T)^2 T}\left[p_c D\left(1 + \frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right) + \frac{h_c D(1+m)}{6}\left\{3u + v(1+m)\right\}\right] < 0, for any given p \geq 0,$
then the obtained results are:

(a) $\psi(p, T)$ *is maximized at* $T^* = m$ *when* $W \geq 0$.
(b) *When* $W < 0$, *there exists a unique* T^* *for which* $\psi(p, T)$ *is maximized at* $(0, m)$.

Proof. Please see Appendix A.   □

To obtain the optimal cycle time T^*, we perform the first-order partial derivative of $\psi(p, T)$ in (3.10) with regard to T. Then by determining the outcome to zero and rearranging the expressions,

$$\frac{O}{T^2} + p_c D \mu_1 \left(1 + \frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right)$$
$$+ h_c D \left[\frac{u}{4}\left\{2(1+m)\mu_1 - 1\right\} + v\left\{\frac{1}{36T^2}\left(5\mu_2 - 5\mu_3 - 2\mu_4 T\right) - \frac{(1+m)^2}{6}\mu_1\right\}\right] = 0. \tag{4.4}$$

Theorem 4.2. *For any fixed* $T > 0$, *the concavity of the function* $\psi(p, T)$ *exists in* p *and gives a unique maximum solution* p^*.

Proof. See Appendix B.   □

To attain the optimal selling price p^*, we perform the first-order partial derivative of $\psi(p, T)$ in (3.10) with regard to p. Then by determining the result to zero and rearranging the terms, we get

$$
\begin{aligned}
\frac{\partial \psi}{\partial p} = {} & (a - 2bp) + p_c b \left(1 + \frac{n+1}{2n} \gamma L r - \frac{x}{n} \right) \frac{1}{T} \varphi \\
& + h_c b \left[\begin{array}{l} u \left\{ \frac{(1+m)}{2T} \varphi + \frac{1}{4} \left(-2 - 2m + T \right) \right\} \\ + v \left\{ \frac{1}{36T} \left((1 + m - T)^2 \left(5 + 5m + 4T \right) - 5(1+m)^3 \right) + \frac{(1+m)^2}{6T} \varphi \right\} \end{array} \right] = 0.
\end{aligned}
\tag{4.5}
$$

Theorem 4.3. *For any fixed $p > 0$, the concavity of the function $\psi(p, T)$ exists in T and gives a unique maximum solution T^*.*

Proof. The proof is analogous to Theorem 4.2.   □

4.2. Case 2

To avoid the redundancy of the stated problem, we have omitted the theoretical derivation for Case 2. To understand the case's concavity, we have provided a three-dimensional graph (Fig. 6) and a numerical example (Example 2).

5. Numerical illustration

To demonstrate the model numerically, an algorithm together with some examples are given in this section and also gain some insights.

5.1. Algorithm

Step 1. For equation (4.3), use Lingo 17 software and input the initial parameters of O, a, b, m, r, L, n, p_c, h_c, u, v and γ.

Step 2. Set p to fixed.

Step 3. In equation (4.1) put $\frac{\partial \psi}{\partial T} = 0$ for a critical point $T = T^*$.

Step 4. From equation (4.1), determine the discrimination term W.

Step 5. The maximum value of the function depends on the following two cases:

    (a) If $W \geq 0$, then $T^* = m$.

    (b) If $W < 0$, then $T^* \in (0, m)$.

Step 6. Compute $\frac{\partial^2 \psi}{\partial T^2} < 0$ and then plug $T = T^*$ in this.

Step 7. If $\frac{\partial^2 \psi}{\partial T^2} < 0$, then maximum attained and Go to Step 8, otherwise no feasible solution found and Go to Step 9.

Step 8. Putting the critical point $T = T^*$ in equation (3.10) to compute the maximum value of $\psi(p, T)$.

Step 9. Stop.

5.2. Examples

Example 1. For case 1.

A case is considered in this projected model, where a retailer secures an item for consumption from a supplier. To get a discount, the retailer prepays some portion of the purchase cost in advance, and the remainder is paid for at delivery. Because of customer demand and product degradation, the retailer's inventory decreases over time. The following parameters are considered:

Ordering cost, $O = \$500$, $a = 60$, $b = 0.1$, maximum life time, $m = 5$ years, the rate of interest imposed on capital cost per year, $r = \$0.01$, lead-time of the chain, $L = 0.5$ years, no of installments, $n = 8$, purchase cost
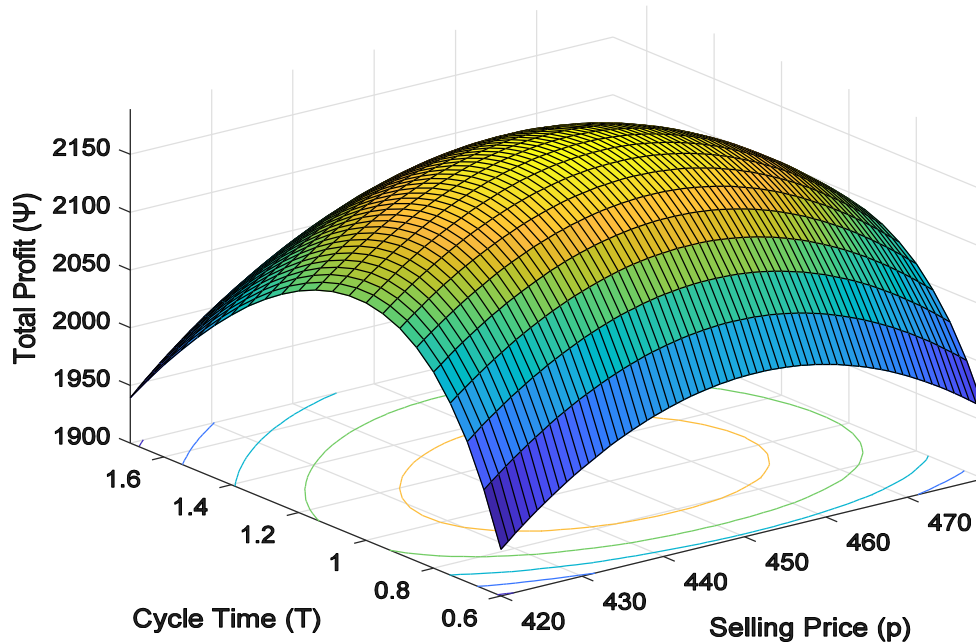
FIGURE 3. The concavity of $\psi(p, T)$ subject to decision variables.

to complete the order, $p_c = \$250$, holding cost for per unit products, $h_c = 1$, constant part in holding cost per unit products, $u = 2$, variable part in holding cost per unit products, $v = 1$, discount rate, $x = \frac{1}{10}$, the amount which needs to be prepaid, $\gamma = 0.2$.

By using these values we calculate: $\eta_1 = 3.208, \eta_2 = 255.993$ and $\eta_3 = 784.647$. Then, solving equation (4.3) by using Lingo 17 software, we get $W = -76693.958 < 0$. So, by Theorem 4.1, there exists a unique $T^* \in (0, m)$, in which $\psi(p, T)$ is maximized. That means T must belong to a range between 0 and 5.

Now we get the following optimal solution after a calculation with the help of Lingo 17 software: $p^* = \$436.495$, $T^* = 1.035$ years and $\psi^* = \$2190.435$.

Now, some graphs illustrate the concavity of the profit function. Figure 3 displays the 3D graph of $\psi(p, T)$ with regard to the variables p and T, whereas Figures 4 and 5 represent it in a two-dimensional model.

Example 2. For case 2.

In this case, all the parameters are considered same as Example 1 except the discount rate, $x = 0.33$, and the rate of interest imposed by a bank, $I_r = 0.3$, and so we obtain the following optimal solution using Lingo 17 software:

The optimal values are: $p^* = \$406.813$, $T^* = 1.075$ years and $\psi^* = \$3267.165$.

6. SENSITIVITY ANALYSIS

The sensitivity investigation of the model presented in two sections; in the first section, we have introduced some graphs of some important parameters which have significant impacts on the models. In the second section, a table is given to show the elasticity of the model for the rest of the parameters. Taking the same values except for some variations in the time range for the retailer to pay in advance (prepayment time), Figures 7 and 8 show the variations of the profit function with the fluctuation of the value of prepayment time. It is noted from the figures that, with the increase in prepayment time, the profit of the system decreases significantly. This is
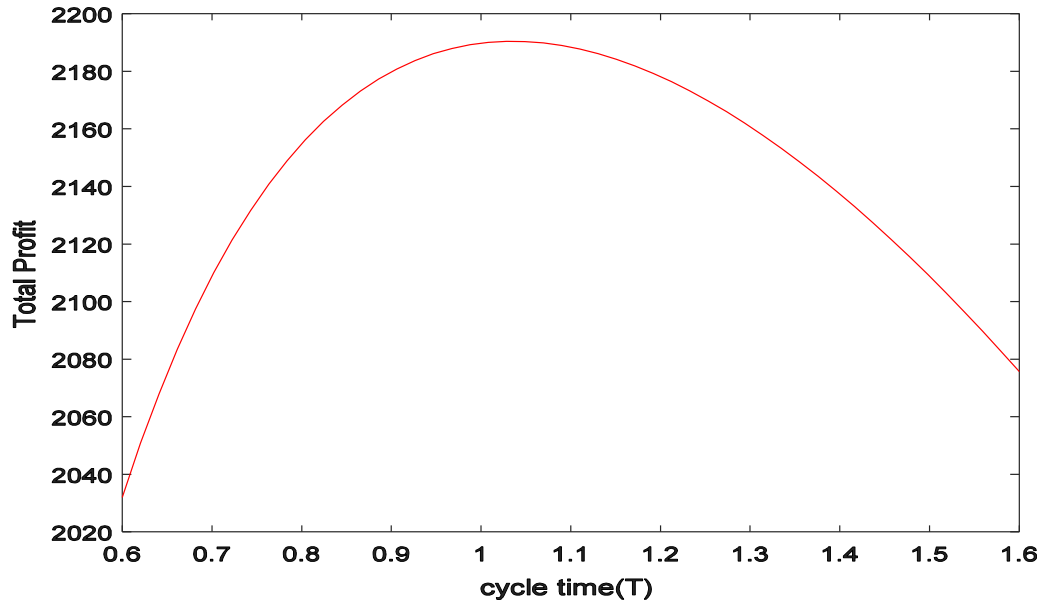
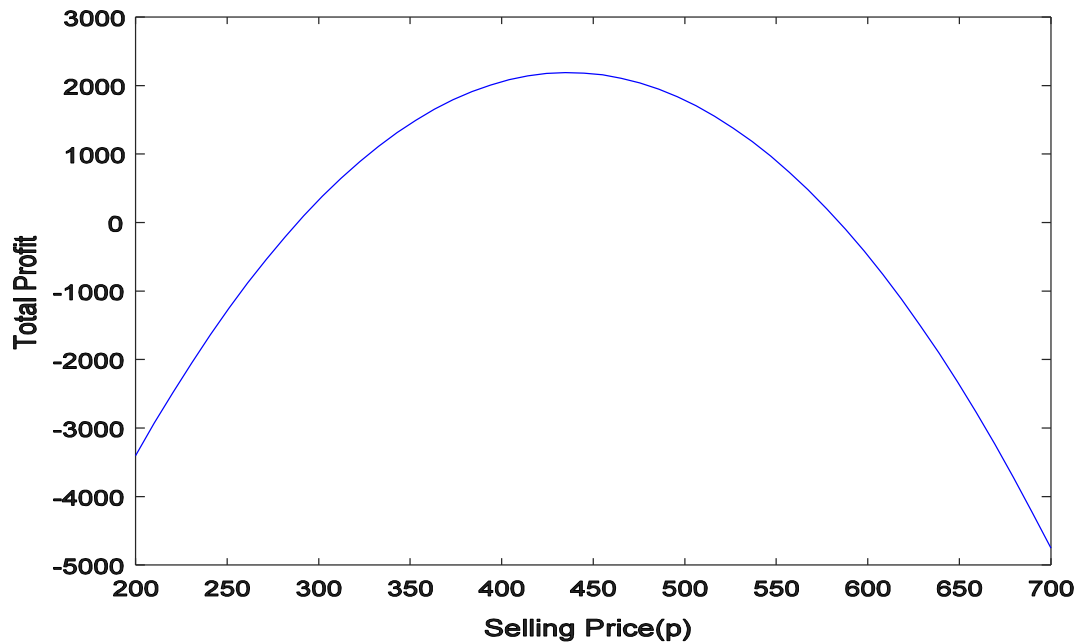FIGURE 4. Total profit function subject to the cycle time T.

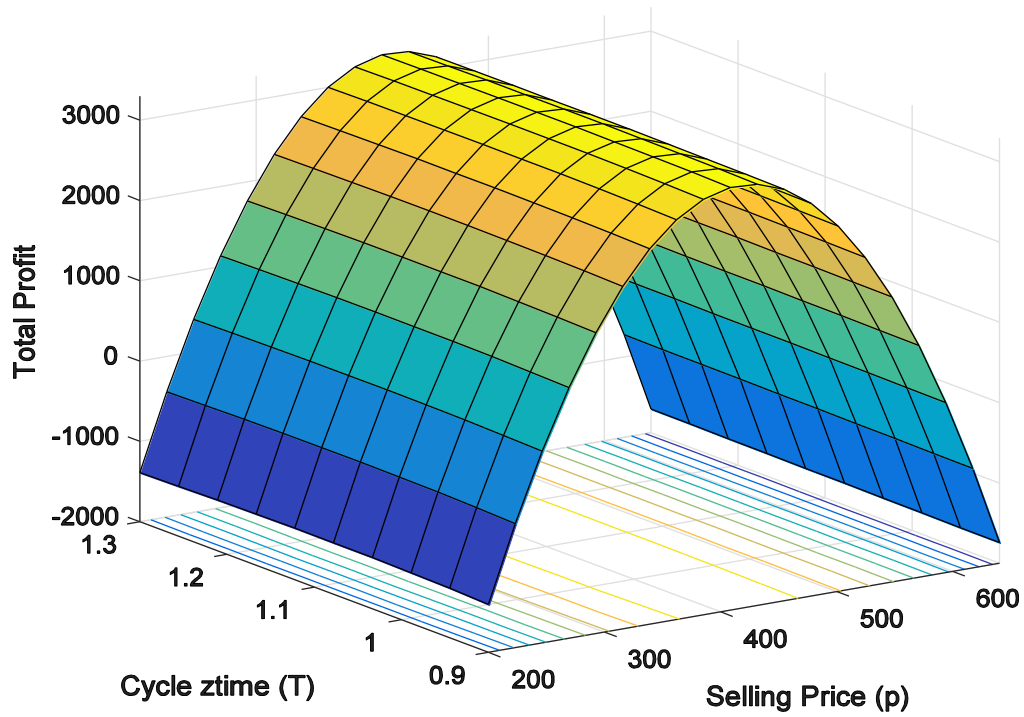FIGURE 5. Total profit function subject to the selling price p.

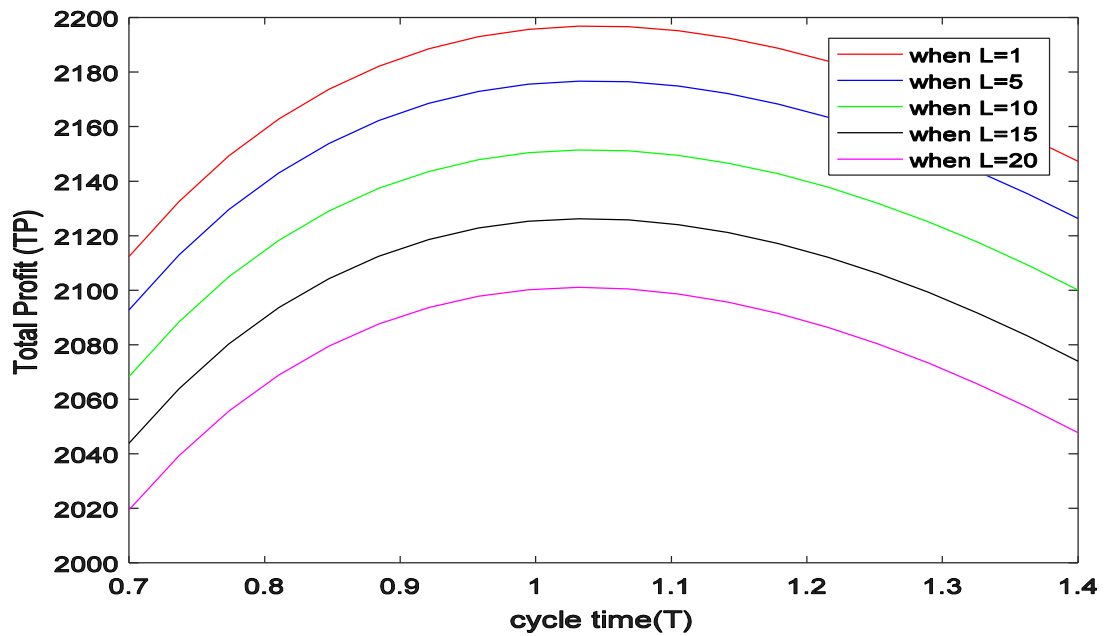FIGURE 6. The concavity of $\psi(p,T)$ subject to decision variables.

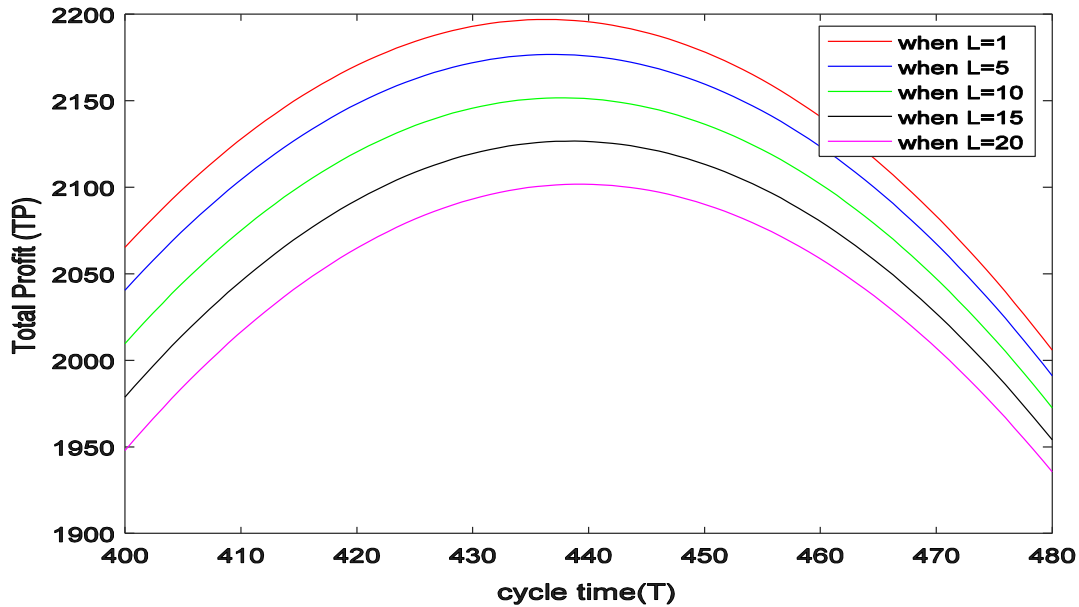FIGURE 7. $\psi(p,T)$ with the changing of L subject to cycle time T.

FIGURE 8. $\psi(p, T)$ with the changing of L subject to selling price p.

happening because when the prepayment time is increasing, the retailer needs to wait for the products for a longer time and needs to pay some interest in taking that time.

Now, taking the same values of the given numerical Example 1, except for some variations given in the number of installments, some dramatic changes are noted in the graph of the profit function with reliant to the changed decision variables presented in Figures 9 and 10. The graphs show that when the number of installments is one, that is, when the retailer pays all the purchase cost when ordering the products, then the total profit of the retailer becomes significantly higher than others. The profit will decline with the rise in the number of installments as he receives a smaller discount rate.

Other parameters that play important roles in the model are the maximum lifetime of the products. Figures 11 and 12 show that when the maximum lifetime is one, *i.e.* $m = 1$, the total profit for the retailer is minimum, while the total profit is maximum when m is large (*i.e.* $m = 20$). The result conforms to the study by Teng *et al.* [38] who observe that when the maximum lifetime increases, the total cost per unit time decreases and hence the total profit per unit time increases. A similar result is observed by Khan *et al.* [17].

The sensitivity analysis of the study is illustrated in Table 2.

Observations: The following observations have been constructed on the results of Table 2.

– The intensification of the parameter a increases both the values of p^* and total profit ψ^*. In contrast, T^* decreases while the reduction of T^* means that the retailer has to hold the products for a short period which will reduce holding costs. At the same time, the rise in selling price p^* generates more profit.

– With the escalations on the value of the parameter b, the values of p^* and total profit ψ^* decline, but T^* upsurges. The rise of T^* means that the retailer has to hold the product for a more extended period which increases the holding cost. Alternatively, the reduction of p^* indicates that the retailer has to sell products at lower prices. Hence, the total profit drops.

– The increase in u and p_c causes a decrease in T^* and total profit ψ^*, but an increase in p^*. On the other hand, the rise in parameter v and h_c causes an upsurge in p^* and T^* but a reduction in total profit ψ^*.

– If the parameter r upturns, then p^* rises, and total profit ψ^* falls while T^* remains steady. When the parameter γ increases, p^* decreases, and the total profit ψ^* increases, but T^* remains the same.
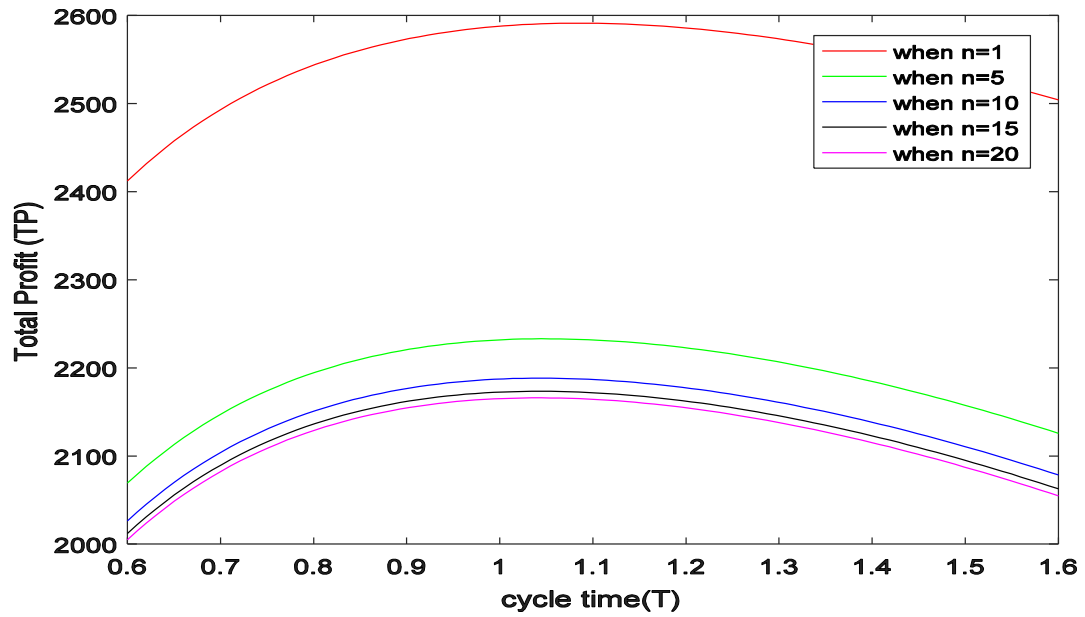
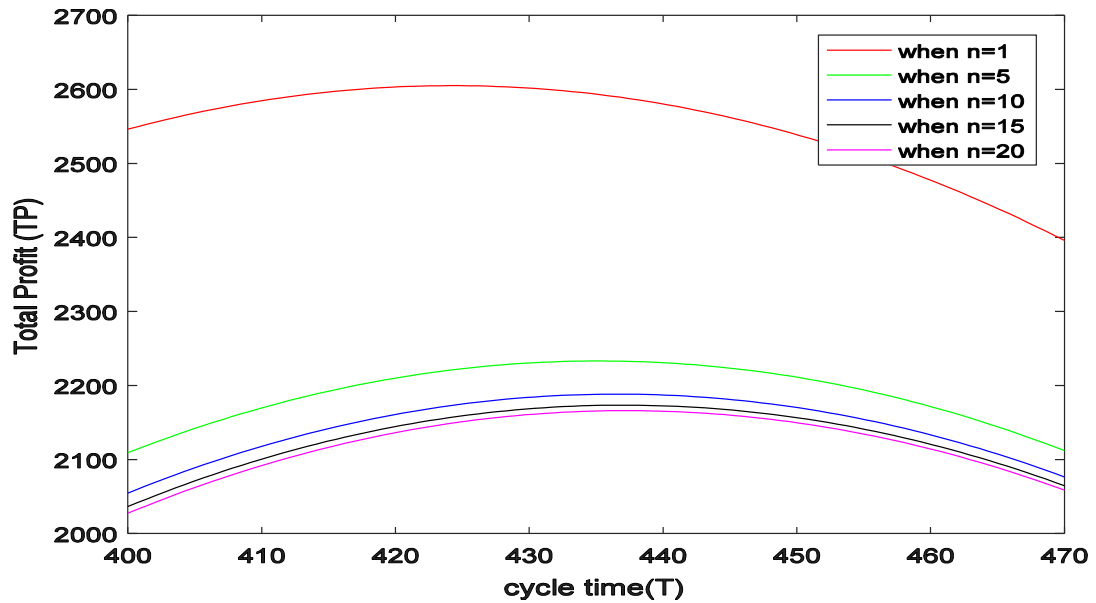FIGURE 9. $\psi(p, T)$ with the changing of n subject to cycle time T.

FIGURE 10. $\psi(p, T)$ with the changing of n subject to selling price p.

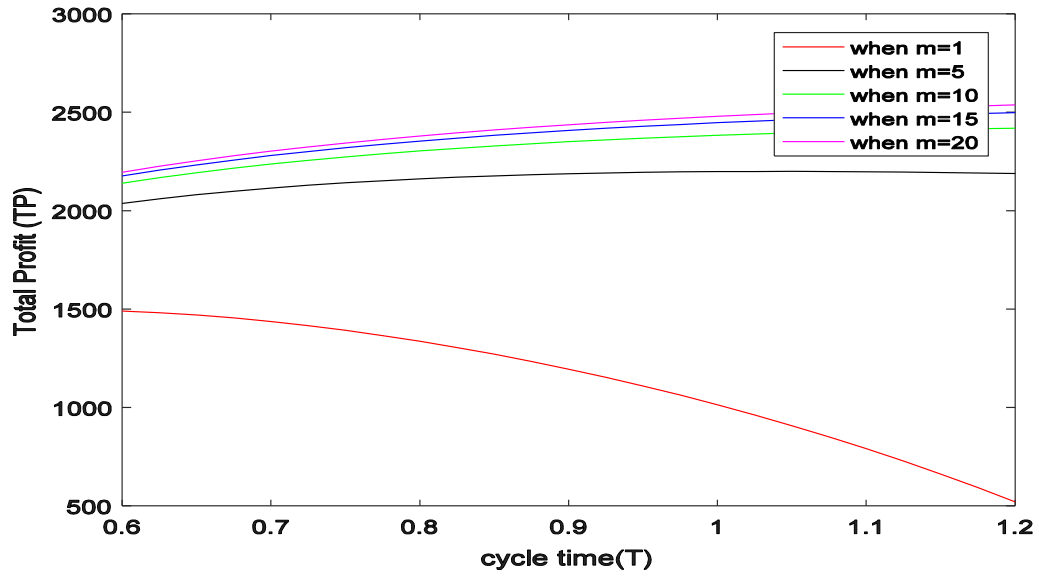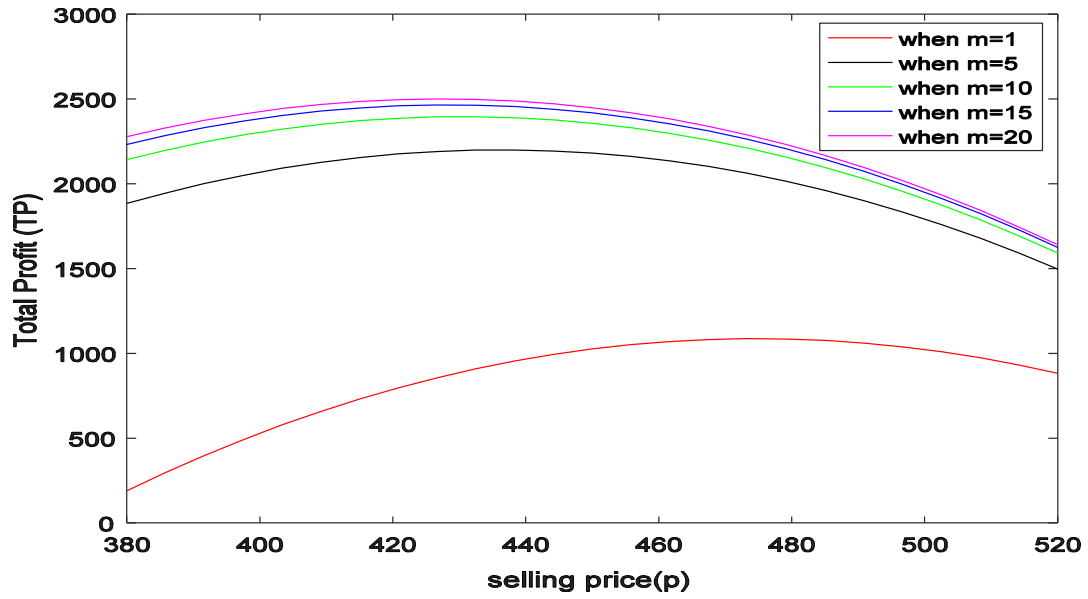FIGURE 11. $\psi(p, T)$ with the changing of m subject to cycle time T.

FIGURE 12. $\psi(p, T)$ with the changing of m subject to selling price p.

TABLE 2. Sensitivity analysis.

Parameter	% changes	p^*	T^*	ψ^*	% changes in ψ^*
a	−20	380.117	1.281	607.286	−66.38
	−10	407.957	1.136	1303.545	−37.34
	10	465.414	0.959	3264.856	45.47
	20	494.570	0.898	4525.002	99.22
b	−20	510.506	0.965	4070.627	82.45
	−10	469.302	0.998	3004.860	35.49
	10	409.830	1.078	1559.371	−27.29
	20	387.819	1.127	1066.479	−48.29
u	−20	436.403	1.041	2195.865	0.38
	−10	436.449	1.038	2193.146	0.16
	10	436.540	1.033	2187.731	−0.27
	20	436.586	1.030	2185.036	−0.48
v	−20	436.493	1.036	2191.049	3.29
	−10	436.494	1.036	2190.741	1.58
	10	436.496	1.035	2190.128	−1.62
	20	436.497	1.034	2189.822	−3.12
r	−20	436.479	1.035	2190.939	−0.03
	−10	436.487	1.035	2190.687	−0.04
	10	436.502	1.035	2190.182	−0.06
	20	436.510	1.035	2189.930	−0.07
γ	−20	436.479	1.035	2190.939	−0.07
	−10	436.487	1.035	2190.687	−0.06
	10	436.502	1.035	2190.182	−0.04
	20	436.510	1.035	2189.930	−0.03
p_c	−20	409.664	1.060	3150.914	40.91
	−10	423.045	1.044	2652.187	19.55
	10	450.037	1.034	1765.477	−17.92
	20	463.701	1.041	1377.292	−34.08
h_c	−20	436.401	1.042	2196.486	3.72
	−10	436.448	1.038	2193.454	1.80
	10	436.541	1.032	2187.427	−1.83
	20	436.588	1.029	2184.430	−3.56

7. MANAGERIAL INSIGHTS

Most products have an increasing deterioration rate over time and will be totally deteriorated at the end of their lifetime (*i.e.*, expiration date). This study provides important managerial insights when the effect of price-dependent demand and advance payments with discount are considered:

– The time range for the retailer for advance payment has a significant effect on the total profit. For a longer advanced payment time, the retailer needs to pay some interest. Therefore, negotiating a shorter payment time with the current supplier is beneficial for a retailer.
– The number of installments (payments) also has a significant effect on the total profit. The highest profit occurs when the number of installments is one, that is, the retailer will pay all of the purchase cost while ordering the products because he receives the highest discount rate, which depends on the number of the installments.
– The product's maximum lifetime also influences the total profit. A shorter lifetime means the customers need to buy those products more frequently, hence, more profitable for a retailer.

– Finally, the purchasing cost per unit product and the demand function significantly influences the total profit. Therefore, a retailer should search for the product with a lower purchasing cost and seek to set an optimal selling price to attract more demand.

8. Conclusion

In this study, a deteriorating item EOQ model is developed considering product life cycle and time reliant deterioration. At the maximum lifetime, the items fully deteriorate and have no value. The concept of advance payments with equal installments is also introduced. The supplier also provides a price discount when the retailer prepays some portion of the purchase cost before the delivery time, while another discount approach is applied with a single payment before receiving the products. In this case, the second discount approach provides a better result because the retailer does not need to pay interest for a long time. Yet at the same time, the first approach of the proposed model also exhibits some managerial insights for the retailer who cannot pay in single installments before receiving products. Selling price-sensitive demand with a realistic linear holding cost is considered where the holding cost comprises a constant part and a variable part, and this idea of holding cost extends that of Teng *et al.* [38], who considered a constant holding cost. The elasticity of some important factors is studied in the sensitivity analysis section. The effect of some parameters is carefully examined, and some valuable theoretical derivations have been found from the model. The total profit is significantly influenced by the prepayment time, the number of prepaying installments, and the product life cycle.

This study is limited by assuming product price as the sole influence of demand. Therefore, future research direction can incorporate other influencing factors such as stock level (see [20, 31]) and advertisement policy (see [24]). In addition, further studies can incorporate the effect of wastes disposal (see [15]) or greenhouse gas emissions from its disposal (see [13]).

Appendix A.

Proof of Theorem 4.1. Considering equation (4.1) to state

$$F(T) = \frac{O}{T^2} + p_c D \mu_1 \left(1 + \frac{n+1}{2n} \gamma Lr - \frac{x}{n} \right)$$
$$+ h_c D \left[\frac{u}{4} \left\{ 2(1+m)\mu_1 - 1 \right\} + v \left\{ \frac{1}{36T^2} \left(5\mu_2 - 5\mu_3 - 2\mu_4 T \right) - \frac{(1+m)^2}{6} \mu_1 \right\} \right]. \tag{A.1}$$

Then

$$F(m) = \frac{1}{m^2} \left[O - p_c D(1+m)\eta_1 \left(1 + \frac{n+1}{2n} \gamma Lr - \frac{x}{n} \right) - \frac{h_c D}{2} \left\{ \frac{u\eta_2}{2} - \frac{v\eta_3}{3} \right\} \right]. \tag{A.2}$$

To find the value of μ_1, here we use L'Hospital's Rule. Let us calculate

$$\lim_{T \to 0} \mu_1$$
$$\lim_{T \to 0} \left[(1+m) \left\{ \frac{1}{T^2} \ln \left(\frac{1+m}{1+m-T} \right) - \frac{1}{(1+m-T)T} \right\} \right]$$
$$= (1+m) \lim_{T \to 0} \left[\frac{(1+m-T) \ln \left(\frac{1+m}{1+m-T} \right) - T}{(1+m-T)T^2} \right]$$
$$= (1+m) \lim_{T \to 0} \left[\frac{-1}{2T + 2mT - 3T^2} \ln \left((1+m)(1+m-T)^{-1} \right) \right]$$

$$= (1+m) \lim_{T\to 0} \left[\frac{-1}{(2+2m-6T)(1+m-T)} \right]$$

$$= \frac{-1}{2(1+m)}. \tag{A.3}$$

Using (A.3) in (A.1), we get

$$F(0) = \lim_{T\to 0} \left[\begin{array}{l} \frac{-p_c D}{2(1+m)}\left(1+\frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right) - \frac{h_c uD}{2} + \frac{h_c vD}{12}(1+m) \\ + \frac{h_c vD}{36T^2}\left\{5(1+m)^3 - 5(1+m)\frac{1}{(1+m-T)^{-2}}\right\} \\ - \frac{h_c vD}{18T}(1+m-T)(5+5m-4T) \end{array} \right] = \infty. \tag{A.4}$$

From (4.2), let us consider

$$\rho(T) = (1+m) \ln\left(\frac{1+m}{1+m-T}\right) - T.$$

It is obvious that $\rho(0) = 0$.

Now $\frac{d\rho(T)}{dT} = \frac{1+m}{1+m-T} - 1$, which is greater than zero, for all values of $T > 0$.

It indicates that despite $T > 0, \rho(T) > 0$.

As a result,

$$\frac{(1+m)}{T} \ln\left(\frac{1+m}{1+m-T}\right) > 1, \quad \text{for all } T > 0. \tag{A.5}$$

Simplifying (4.2) and using (A.5), we get

$$\frac{dF(T)}{dT} \leq -p_c D\left(1+\frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right)\left[\frac{2}{T^2} - \frac{(1+m)(2+2m-3T)}{(1+m-T)^2 T^2}\right]$$

$$- \frac{h_c uD(1+m)}{2}\left[\frac{2}{T^2} - \frac{(1+m)(2+2m-3T)}{(1+m-T)^2 T^2}\right] - \frac{h_c vD(1+m)^2}{6}\left[\frac{2}{T^2} - \frac{(1+m)(2+2m-3T)}{(1+m-T)^2 T^2}\right]$$

$$+ \frac{2h_c vD}{9} - \frac{10h_c vD(1+m)^3}{9T^3}$$

$$= \frac{(1+m-2T)}{(1+m-T)^2 T}\left[p_c D\left(1+\frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right) + \frac{h_c D(1+m)}{6}\left\{3u + v(1+m)\right\}\right]$$

$$- \frac{2h_c vD}{9}\left\{\frac{5(1+m)^3}{T^3} - 1\right\}$$

$$< 0 \text{ if } \frac{(1+m-2T)}{(1+m-T)^2 T}\left[p_c D\left(1+\frac{n+1}{2n}\gamma Lr - \frac{x}{n}\right) + \frac{h_c D(1+m)}{6}\left\{3u + v(1+m)\right\}\right] < 0. \tag{A.6}$$

If $W \geq 0$, then $F(m) \geq 0$. For all $T < m$ with the help of equation (A.4) and (A.6) one can write $F(T) > 0$. Hence, the function $\psi(p, T)$ is increasing in T, and at the same time maximized at $T^* = m$. Finally, when $W < 0$, then $F(m) < 0$. By putting on the Mean Value Theorem into $F(0) = \infty$ and $F(m) < 0$, there occurs a unique value of T say, T^* for which $F(T^*) = 0$ and consequently $\psi(p, T)$ is maximized at unique $T^* \in (0, m)$, which concludes the proof. □

APPENDIX B.

Proof of Theorem 4.2. Performing the first-order and second-order partial derivatives of $\psi(p, T)$ in (3.10) with regard to p, we get

$$
\begin{aligned}
\frac{\partial \psi}{\partial p} = {}& (a - 2bp) + p_c b(1 + m)\left(1 + \frac{n+1}{2n}\gamma L r - \frac{x}{n}\right)\frac{1}{T}\varphi \\
& + h_c b \left[\begin{array}{l} u\left\{\frac{(1+m)}{2T}\varphi + \frac{1}{4}\left(-2 - 2m + T\right)\right\} \\ +v\left\{\frac{1}{36T}\left((1 + m - T)^2\left(5 + 5m + 4T\right) - 5(1+m)^3\right) + \frac{(1+m)^2}{6T}\varphi\right\} \end{array}\right]
\end{aligned}
\tag{B.1}
$$

and

$$
\frac{\partial^2 \psi}{\partial p^2} = -2b < 0.
\tag{B.2}
$$

Therefore the concavity of the function $\psi(p, T)$ exists in p and gives a unique maximum solution p^*. This completes the proof. $\qquad\square$

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