SINGLE-ITEM LOT-BASED SUPPLYING AND BATCH PRODUCTION UNDER A BILATERAL CAPACITY RESERVATION: A PARTNERSHIP STRUCTURE

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Abstract. This paper studies a single-item lot-based supplying and batch production under a bilateral capacity reservation contract based on a partnership structure in which some suppliers, a manufacturer, and a third-party logistics provider are connected through long-term cooperation agreements. This configures a partnership supply chain and promotes the manufacturer's performance in a continuous improvement cycle. Since in this paper, it is assumed that the products are perishable, there is not any inventory storage. Moreover, due to a settled contract, the final products are delivered by third-party logistics corporate providing vehicles according to the products' features, their expiration time lengths, customers' demands and other considerations in each period. Thus, the delivery batch size, which is equal to the vehicle capacity in a full truckload mode, is targeted for the manufacturer operations resulting in a dynamic improvement environment. In this problem, respect to the performance level of the manufacturer, in each period, a certain capacity is reserved at each selected supplier level according to the appointed batch deliveries. Here, in addition to the ordinary costs incurred due to ordering and delivery processes, some penalties are imposed for orders/delivers exceeding or falling behind the reserved capacity. After investigating the computational complexity of the problem, we propose three dynamic programming algorithm as well. Finally, numerical results and some potential future researches are presented.

1. Introduction

A lot of researches that have been carried out in the field of the single-item lot-sizing problem (SILSP) and its corresponding issues, indicates the interesting aspects of SILSP when scheming the procurement of raw materials associated with ordering and transshipment charges [25]. Since manufacturing and remanufacturing capacities, production and delivery time-windows, bounded inventory, backlogging, and perishability encompass various dimensions of the SILSP, efficiently solving it is also critical [9].

The SILSP can be described as a problem representing variable demands for a single item during a planning horizon. The goal of planning is to specify periods in which manufacturing will occur and the sizes that have to be manufactured [4]. The manufacturing aims at meeting the entire demands, along with minimizing the

whole costs. The associated primary charges consist of the unit manufacturing cost, the setup cost, and the unit inventory holding cost [10]. In another point of view, there is always some waste in production processes resulted from supplying and/or manufacturing functions leading to a hierarchical change in decisions corresponding to different echelons of supply chains. Assuming that every product has a life cycle, Khan *et al.* [14] suggested a price discount strategy in the present business environment. Based on the cited concepts, they considered two supply chain models where shortages either are not allowed or may occur with partial backorders. As the holding cost per unit cannot be constant in the entire cycle length, they regarded it as a linearly increasing function of the storage time. Further, in spite of the most carried out researches in the corresponding scope, they presumed the unit purchasing cost as a decreasing function of the order size and provided a discount-based situation as well.

Since the majority of the holding charges are not always fixed due to uncertainty of today's competitive environment, Shaikh *et al.* [21] studied a two-warehouse model with interval-valued inventory cost and advance payment for deteriorating items under particle swarm optimization. In this regard, they took into account the inventory parameters as intervals and also assumed partially backlogged shortages in effect. To cope with the uncertainty arising from the real situations, they used some interval arithmetic and interval order relations and developed different variants of particle swarm optimization techniques to efficiently solve the problem. Pointing out the price and stock as two important factors in inventory analyses, Panda *et al.* [19] investigated a trade credit policy and provided a two-warehouse inventory formulation. In addition, they included the deteriorating factor with a price- and stock-dependent demand under partially backlogged shortage. The problem assumptions are: (i) price- and stock-dependent demand, (ii) price as a constant markup rate, (iii) backlogging rate as length of the waiting time of the customers, (iv) alternative trade credit policy, and (v) advertisement of the product. Due to the nonlinearity of the problem, they were not able to optimally solve the problem and only performed the sensitivity analyses in effect.

Advance payment is of highly impressive factor in SILSP schemes. The main target of each inventory control system is improving the total benefits of firms without decreasing the customers' satisfaction and also service levels [22]. On the other hand, usually, the cost of an order is spent at delivery time, while prepayment for some special products is commonly requested from the supplier ensuring that the purchaser never cancels the order. Khan *et al.* [15] investigated an advance payment scheme which is made by equal installment up to n times before receiving the products. They developed a two-warehouse inventory model for deteriorating items and assumed shortages with a constant partial backlogging rate. With the aim of encouraging buyers to increase their orders, vendors sometimes offer them to pay all or a proportion of the purchasing cost after receiving the items. In this regard, Taleizadeh *et al.* [23] considered the customer's inventory policy by considering two different conditions: (i) full prepayment with shortage, (ii) partial prepayment–partial delayed payment with shortage. As such, they investigated the effect of parameters such as price discounts linked to prepayment and length of prepayment on optimal periods of inventory and shortage quantity. Taking into cognizance the different situations of deterioration (and despite the ordinary assumption in the existing models), Khan *et al.* [16] discussed that the decay in owned warehouse begins earlier than that in the case of the rented warehouse. They formulated a two-storage inventory model with advance payment under three different situations according to different possibilities of starting times of deterioration in both warehouses. Observing that suppliers give different kinds of benefits to retailers due to advance payment, Khan *et al.* [17] studied a problem under both cases of advance payment (full or partial), an inventory model for deteriorating products where shortages are allowed and demand function is considered as price and stock dependent.

The SILSP with allowed shortages was first considered by Zangwill [26]. Indeed, it can be due that the cost of providing and warehousing the products is not beneficial or the available capacity is not adequate to meet entire demands on assigned schedules. In this regard, the backorders are formulated through the variables of newly manufactured products-flow which are at the contrary direction of the ones relating to the inventory flow [8]. In similar cases with the uncapacitated SILSP, the utmost optimal solutions make spanning-tree configurations. Zangwill [26] showed that the problem is optimally solvable in $\mathrm{O}(T^2)$ by generalizing the Wagner–Whitin algorithm. By applying similar techniques (the techniques similar to the ones utilized for the uncapacitated

SILSP), the complexity of the problem was decreased to $O(T \log T)$ or $O(T)$ [7]. As companies constantly need free cash flows to efficiently react to any uncertainty, Bian *et al.* [5] developed a two-level uncapacitated lot-sizing problem considering the financing cost of working capital requirement. They proposed sequential and centralized approaches to solve the two-level case with a serial chain structure and established a dynamic programming-based algorithm in effect.

Several studies considered the uncapacitated SILSP with backlogging and demonstrated that the problem can be often solved by polynomial-time algorithms. Absi *et al.* [1] studied the uncapacitated SILSP with backlogging costs and also production time windows, and indicated that the problem is optimally solvable through a dynamic programming algorithm with the complexity of $O(T^2)$. Chu *et al.* [11] generalized the uncapacitated SILSP (with backlogging) by incorporating outsourcing and inventory capacity and showed that the problem can be solved in $O(T^4 \log T)$. Van Vyve [24] considered two linear programming models in the lot-sizing problem with constant-capacity and backlogging. The first one presents a general cost function with the complexity of $O(T^3)$, and the second deals with the problem having $O(T^2)$ variables and $O(T^3)$ constraints. Hwang and Kang [13] investigated a two-level lot-sizing problem with stepwise transportation costs and backlogging within an integrated replenishment and dispatch plan. Assuming the capacitated and uncapacitated lot-sizing characteristics, they developed an improved $O(T^4)$ algorithm where T is the length of the planning horizon. Phouratsamay *et al.* [20] provided a two-level uncapacitated lot-sizing problem with inventory bounds that occurs in a supply chain composed of a supplier and a retailer. They assumed that the inventory bounds can be imposed at the retailer level, at the supplier level or at both levels and proposed a polynomial dynamic programming algorithm to solve the problem. Also, they proved that the uncapacitated lot-sizing problem with inventory bounds is strongly NP-hard and utilized a pseudo-polynomial algorithm that solves this problem when there are inventory bounds on both levels.

In this paper, we consider a novel and practical approach in the field of SILSP based on a lot-supplying and batch-production strategy. Indeed, as there is inevitably some waste in every production process, implementing an efficient mechanism is critical to achieve the intended goals. In this regard, the quantity of raw materials is delivered in lots by selected external suppliers with the best contract terms, and the production is fulfilled in batches (*e.g.*, pallet, truck, etc.). This procedure leads to a worthy competition between the suppliers for offering a preferable contract, and also makes the manufacturers to implement an advantageous maintenance and repair planning for keeping the consistency in the production conditions.

In this context, after choosing the qualified suppliers for long-term cooperation, short-term reservation contracts are conducted between the manufacturer and some of them based on an anticipated level of waste incurred in each period. Up to a certain level of the reserved capacity, the raw materials have a certain beneficial unit cost, and once the size of the order exceeds the predetermined level, the suppliers offer a second higher price.

In our problem, the production batch size must be the same as the vehicle capacity in a full truckload (FTL) shipment policy in each period, leading to an advantageous approach in fulfilling the customers' demands further in case the products are perishable or soon-expired. In addition to the classical SILSP problem, there exist two types of variable costs per order, respectively, for the orders above the reserved capacity and below it. In this strategy, once the order quantity is exceeded the specified level, the surplus numbers can be procured from other suppliers placed in the next ranks based on increasing total costs.

For the problem considered in this paper, the manufacturer accepts to pay a primary cost S_0 as the fixed ordering charge (also includes contract signing expenditures), and reserves a capacity V_t in the chosen supplier for each period. While the amount of order in period t is less than V_t, the manufacturer is committed to pay a unit cost of a_t and also a unit penalty cost of p_t for the difference up to the agreed level. But in case the order exceeds the reserved level, a cost of b_t is incurred for each unit of raw materials. As mentioned, these extra units may be procured from the second affordable supplier. Figure 1 shows the cost structure for a sample supplier according to the reserved capacity.

Similarly, as shown in Figure 2, it is set another contract with the third-party logistics provider based on batches reserved to deliver in each period (also again after signing the long-term agreement). In this context, as the products are soon-expired and need corresponding requirements of batching and delivering, the production

$$Z_t(x_t) = \begin{cases} 0 & \text{if} & x_t = 0 \\ S_t + a_t x_t + p_t(V_t - x_t) & \text{if} & 0 < x_t < V_t \\ S_t + a_t x_t & \text{if} & x_t = V_t \\ S_t + a_t V_t + S'_t + b_t(x_t - V_t) & \text{if} & x_t > V_t \end{cases}$$
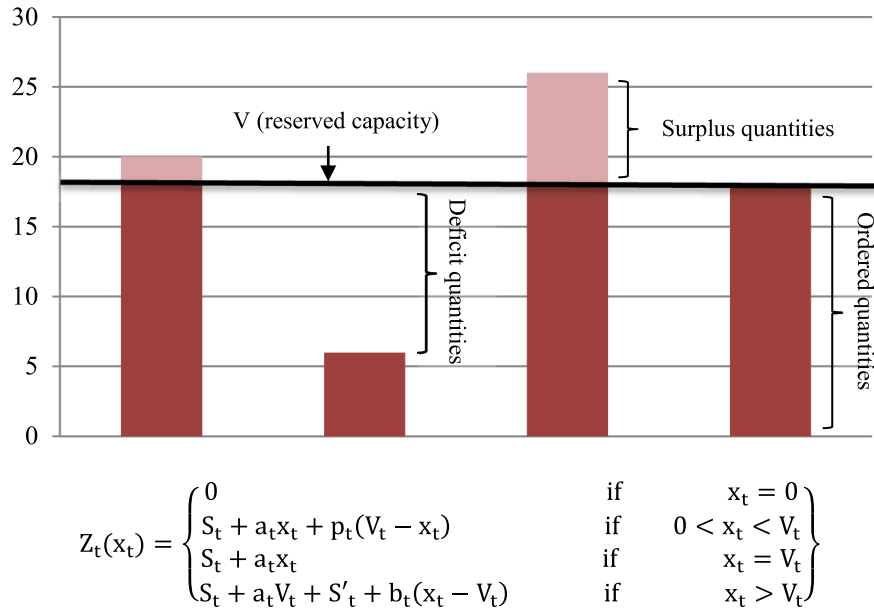
FIGURE 1. Structure of the problem in the supplier side with lot-based deliveries under a capacity reservation contract.

batches do not exceed the reserved number at the logistics corporate-level. Therefore, only a penalty charge is incurred by the logistics service in case the manufactured batches fall less than the reserved number.

Hence, compared with the most similar researches done in the corresponding area (as reviewed above), this paper contributes to the literature in the following ways:

– Developing single-item lot-based supplying and batch production structure (different from the common procedures in classical SILSPs).
– (Even more striking) utilizing a bilateral reservation mechanism (making the manufacturer proceed based on a dynamic improvement structure, providing a joint procedure in responding to the demands).
– Using a partnership supply strategy where the manufacturer is a focal point of the network (this results in an increased synergy in the network through which each member (level) is responsible for the customers' satisfaction).
– Considering manufacturing losses with the aim of providing a balanced production condition (this can extend a realistic and practical approach in the supply chain functions).
– Newly taking into account a batch-production formulation according to the perishability and other corresponding features of the products.
– Investigating the complexity of the problem.
– Proposing three DP-based algorithms in order to reduce the complexity of the problem.

The paper is further organized as follows. In Section 2, we state the problem in detail and formulate it based on MILP modeling. Section 3 is dedicated to the investigation of the problem complexity. Section 4 is assigned to proposing three dynamic programming algorithms for solving the problem. Finally, Section 5 concludes the paper and suggests some potential features for future studies.
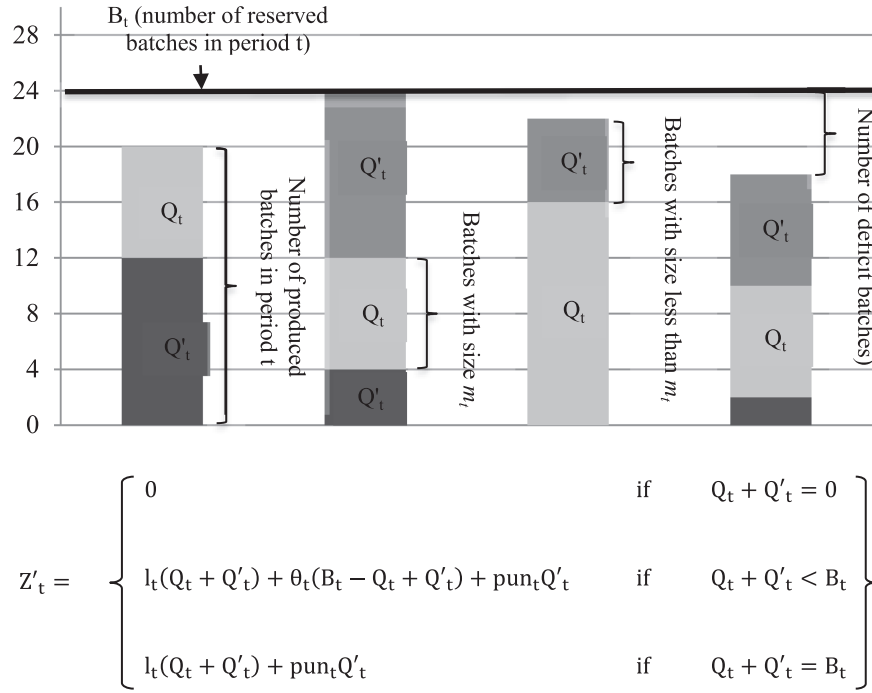
$$
Z'_t = \begin{cases} 0 & \text{if} \quad Q_t + Q'_t = 0 \\[1.5em] l_t(Q_t + Q'_t) + \theta_t(B_t - Q_t + Q'_t) + pun_t Q'_t & \text{if} \quad Q_t + Q'_t < B_t \\[1.5em] l_t(Q_t + Q'_t) + pun_t Q'_t & \text{if} \quad Q_t + Q'_t = B_t \end{cases}
$$

FIGURE 2. Structure of the problem in the distribution side with batch deliveries under a capacity reservation contract.

2. PROBLEM STATEMENT

As discussed previously, there always exists some waste in every production process stemming from incompatibility (or not meeting the desired features) in the corresponding operations. Here, we are to expand the think of developing a partnership supply mechanism in a particular supply chain leading to a progressive performance of the manufacturer. For this purpose, the manufacturer tries to improve its procedures and reduce the production losses; eventually, achieves an invariant operating condition. Hence, we offer a lot-supplying and batch-production strategy (LSBPS) aiming to obtain a balanced manufacturing process resulting in satisfying the taken demands of the customers entirely. Consider Figure 3, where a manufacturer is replenished by some suppliers in lot-based deliveries. The admitted demands of the product (determined by the manufacturer according to the customers' demands, products' requirements, carriers' capabilities, and other considerations) in each period is deterministic and known in advance. Each supplier proposes a capacity reservation contract and has a limited valence. In addition, each supplier tries to motivate the manufacturer by propounding a preferable contract. Raw materials could be shipped whether in an economical procedure (based on reserved numbers) or not (for exceeding numbers), implying distinct transportation charges for each order. According to pre-signed contracts, up to a certain level, the raw materials have an appointed beneficial unit cost. Once the size of the order exceeds the pre-specified level, the suppliers offer a second higher price and also a new ordering and contract costs. Actually, this second price is imposed due to setting up and again manufacturing the surplus demands. On the other hand, a penalty cost is also incurred in case the orders do not reach the size previously agreed through a contract. Hence, the manufacturer tries to keep stable production processes *via* developing an efficient maintenance & repair system, and eventually not to violate the contract terms.

On the other hand, we consider another contract setting with the logistics corporate based on batches reserved to deliver in each period. Toward this end, as the products are perishable and need corresponding requirements
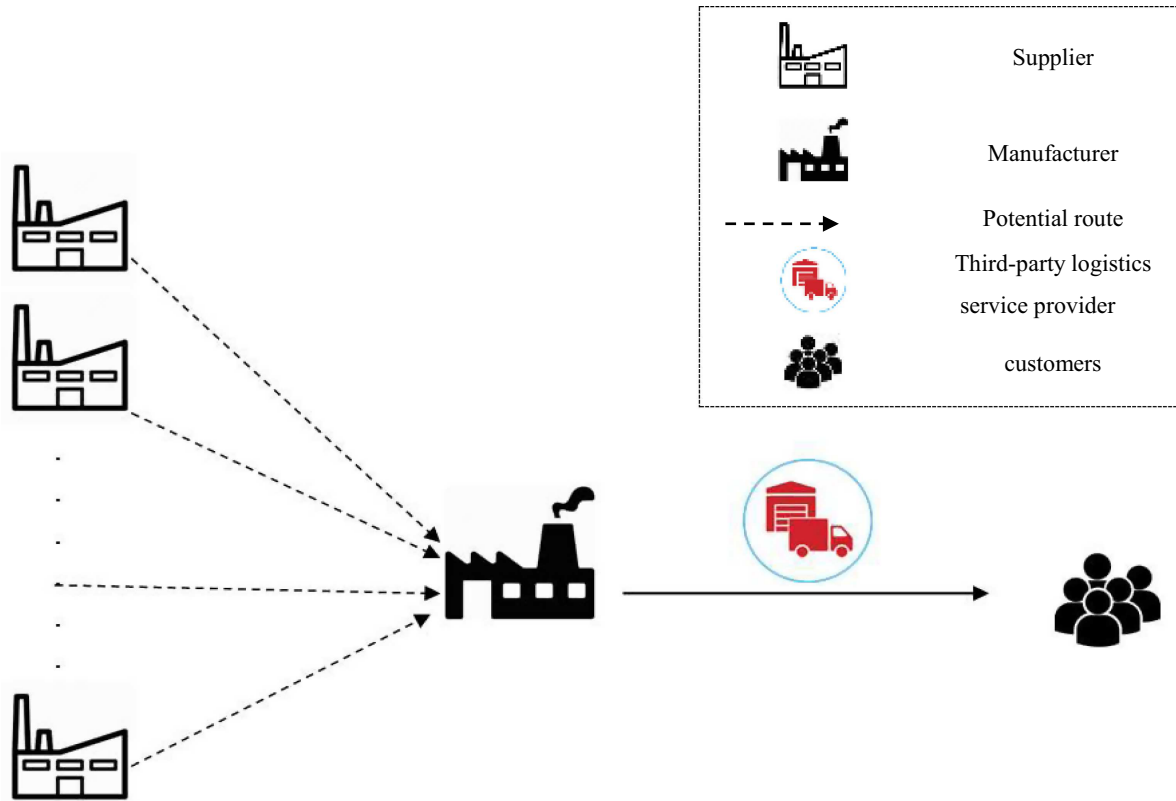
FIGURE 3. Schematic structure of the problem.

of batching and delivering, the production batches do not exceed the reserved number at the logistics corporate-level. Therefore, a penalty charge is incurred by the logistics service provider in case the manufactured batches fall less than the reserved number.

In addition, as discussed before, a partnership supply chain entails its members to try for common interests. It is also assumed that the manufacturer has established long-term cooperation (*i.e.*, strategic relationships) with both suppliers (the ones picked) and the logistics service provider. Therefore, we consider a situation where the taken suppliers (in a long-term relationships) and the logistics corporate do their works with no bug. Accordingly, we incorporate two kinds of penalties (imposed by the selected suppliers and the logistics provider) in the objective function.

In this formulation, we assume that the products are perishable, and so there is not any inventory storage. Moreover, the final products are delivered by a third-party logistics service which provides specified vehicles [with different capacities] with respect to the customers' demands, products features (such as expiration time lengths, shipping requirements), seasonal considerations, carbon emissions, capabilities of available transportation facilities, and other issues in each period. In order to keep the products fresh and incorporate the above requirements, each period is divided into several successive intervals through which the batches are manufactured. Thus, the whole demands may not be taken into account, and the vehicle capacity in a full truckload (FTL) shipment policy is targeted as the batch size for the manufacturer to achieve it. This leads to an advantageous approach in fulfilling the customers' demands, especially in the cited situation that the products are soon-expired. Over finite discrete-time periods, as the manufacturer's capacity is limited, backlogs are allowed. The manufacturer needs to select one or more suppliers (up to the maximum of two) based on the contract

terms and also shipment charges. These measures express some operational performance goals that must be met. Even more, we consider a missed opportunity cost – composed of different issues such as lost sales, missed reputation and dependability in suppliers and the third-party logistics provider opinions, etc. – resulted from manufacturing defective items. This promotes the notion of a partnership supplying approach and stimulates the triple actors of the above supply chain to cooperate in presenting the right products and finally satisfying the customers' needs. Eventually, the buyer determines the raw material lot-sizes in each period, and develops a balanced production condition through minimizing the losses arising from different roots.

With the above description, the notations and a mixed-integer linear programming model are presented.

2.1. Model parameters and decision variables

Indices

- i Set of suppliers (agreed for long-term cooperation), $i = \{1, 2, 3, \ldots, I\}$.
- j Set of intervals in each period, $j = \{1, 2, 3, \ldots, B_t\}$.
- t Set of discrete-time periods, $t = \{1, 2, 3, \ldots, T\}$.

Parameters

- d'_t Customers' demands of the product which are taken in period t ($d'_t = B_t m_t$).
- S_{it} Initial fixed cost of ordering and signing contract with supplier i in period t.
- S'_{it} Secondary fixed cost of ordering and signing contract with supplier i in period $t(S'_{it} > S_{it})$.
- a_{it} Cost of purchasing one unit of raw materials from supplier i in period t for the orders not exceeding the reserved capacity at supplier i.
- b_{it} Cost of purchasing one unit of raw materials from supplier i in period t for the orders exceeding the reserved capacity at supplier $i(b_{it} > a_{it})$.
- V_{it} Reserved capacity (of raw materials) at supplier i in period t.
- p_{it} Unit penalty cost of ordering for deficit quantity of raw materials out of V_{it}.
- C_{it} Total capacity of supplier i in procuring raw materials in period t.
- m_t Shipment capacity of the vehicle in period t (which is pre-specified the default size of each production batch in period t).
- B_t Number of reserved transportation vehicles at the third-party logistics provider in period t (which is pre-specified the default number of production batches in period t).
- $\bar{B}_t$ Minimum number of production batches (delivery vehicles) that must be carried out in period t.
- tr_{it} Unit shipment cost of raw materials from supplier i to the manufacturer in the first order at period t.
- tr'_{it} Unit shipment cost of raw materials from supplier i to the manufacturer in the second-order at period t ($tr'_{it} > tr_{it}$).
- K_t Production capacity of the manufacturer in period t.
- f^t_j Fixed production cost of each batch in interval j of period t.
- g_t Unit processing cost by the manufacturer in period t.
- l_t Fixed shipping cost paid to the third-party logistics provider for each batch in period t.
- θ_t Unit penalty cost incurred by the third-party logistics provider for deficit number of batches out of B_t.
- r_t Unit missed opportunity cost caused by defective items manufactured in period t (such as lost sales, missed reputation and dependability in suppliers and the third-party logistics provider opinions, etc.).
- pun_t Punishing cost imposed by the third-party logistics service provider per manufactured batch in less-than-truckload (m_t) mode in period t (composed of missed credit on customers' eyes, and also non-economic shipping cost (penalty) in less-than-truckload (LTL) mode due to environmental violations) in the partnership strategy.
- pen_{it} Unit penalty cost imposed by the supplier i in period t for the defective items manufactured from raw materials of supplier i [this penalty includes loss of dependability among customers, rivals, logistics service provider, etc.) in the partnership strategy.

M A large amount.

f_j^t Fixed production cost of each batch in interval j of period t.

g_t Unit processing cost by the manufacturer in period t.

l_t Fixed shipping cost paid to the third-party logistics provider for each batch in period t.

Decision variables

X_{it} Quantity of raw materials ordered to supplier i in price a_{it} (in period t).

X_{it}' Quantity of raw materials ordered to supplier i in price b_{it} (in period t).

Y_{it} Binary variable indicating whether supplier i gets an order in price a_{it} (1) or not (0) in period t.

Y_{it}' Binary variable indicating whether supplier i gets an order in price b_{it} (1) or not (0) in period t.

O_j^t Binary variable indicating whether the production batch in full-truckload (m_t) mode is done or not at interval j of period t.

$O_j'^t$ Binary variable indicating whether the production batch in less-than-truckload ($<m_t$) mode is done or not at interval j of period t.

H_j^t Total quantity of faultless products manufactured at interval j of period t.

W_{it} Quantity of defective items manufactured from raw materials of supplier i in period t

E_t Number of deficit batches out of B_t in period t.

2.2. Mathematical formulation

Regarding the mentioned assumptions, parameters and decision variables, a MILP optimization model is expressed as follows:

$$\text{Min } Z = \sum_{t=1}^{T} \sum_{i=1}^{I} \left[S_{it} Y_{it} + a_{it} X_{it} + S_{it}' Y_{it}' + b_{it} X_{it}' + p_{it} (V_{it} - X_{it}) \right]$$

$$+ \sum_{t=1}^{T} \sum_{i=1}^{I} \left(tr_{it} X_{it} + tr_{it}' X_{it}' \right) + \sum_{t=1}^{T} \sum_{j=1}^{J} f_j^t \left(O_j^t + O_j'^t \right)$$

$$+ \sum_{t=1}^{T} \sum_{i=1}^{J} g_t \left(X_{it} + X_{it}' \right) + \sum_{t=1}^{T} \sum_{i=1}^{I} \text{pen}_{it} W_{it} + \sum_{t=1}^{T} \sum_{j=1}^{J} l_t \left(O_j^t + O_j'^t \right)$$

$$+ \sum_{t=1}^{T} \theta_t E_t + \sum_{t=1}^{T} \sum_{j=1}^{J} \text{pun}_t O_j'^t + \sum_{t=1}^{T} r_t \left(B_t m_t - \sum_{j=1}^{J} H_j^t \right) \tag{2.1}$$

$$\text{s.t.} \quad \sum_{i=1}^{I} \left(X_{it} + X_{it}' \right) = B_t m_t \qquad\qquad \forall t = 1, 2, \ldots, T \tag{2.2}$$

$$\sum_{i=1}^{I} \left(X_{it} + X_{it}' - W_{it} \right) = \sum_{j=1}^{I} H_j^t \qquad\qquad \forall t = 1, 2, \ldots, T \tag{2.3}$$

$$H_j^t \leq m_t \qquad\qquad \forall j = 1, \ldots, J \quad \forall t = 1, 2, \ldots, T \tag{2.4}$$

$$\sum_{j=1}^{B_t} H_j^t \leq K_t \qquad\qquad \forall t = 1, 2, \ldots, T \tag{2.5}$$

$$\bar{B}_t \leq \sum_{j=1}^{J} \left(O_j^t + O_j'^t \right) \leq B_t \qquad\qquad \forall t = 1, 2, \ldots, T \tag{2.6}$$

$$E_t \leq B_t - \sum_{j=1}^{J} \left(O_j^t + O_j'^t \right) \qquad \forall t = 1, 2, \ldots, T \qquad (2.7)$$

$$X_{it} \leq V_{it} \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.8)$$

$$X_{it}' \leq C_{it} - V_{it} + 1 \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.9)$$

$$X_{it} \leq Y_{it} M \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.10)$$

$$X_{it}' \leq Y_{it}' M \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.11)$$

$$\sum_{i=1}^{I} Y_{it} \leq 2 \qquad \forall t = 1, 2, \ldots, T \qquad (2.12)$$

$$Y_{it}' \leq Y_{it} \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.13)$$

$$O_j^t + O_j'^t \leq 1 \qquad \forall j = 1, \ldots, J \quad \forall t = 1, 2, \ldots, T \qquad (2.14)$$

$$X_{it}, X_{it}', W_{it} \geq 0 \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.15)$$

$$H_j^t \geq 0 \qquad \forall j = 1, \ldots, J \quad \forall t = 1, 2, \ldots, T \qquad (2.16)$$

$$E_t \geq 0 \text{ and integer} \qquad \forall t = 1, 2, \ldots, T \qquad (2.17)$$

$$Y_{it}, Y_{it}' \in \{0, 1\} \qquad \forall i = 1, 2, \ldots, I \quad \forall t = 1, 2, \ldots, T \qquad (2.18)$$

$$O_j^t, O_j'^t \in \{0, 1\} \qquad \forall j = 1, \ldots, J \quad \forall t = 1, 2, \ldots, T. \qquad (2.19)$$

The objective function (Eq. (2.1)) minimizes the whole incurred costs including contract setting and ordering, purchasing, and penalties (first term), shipping raw materials from suppliers (second term), fixed manufacturing of batches(third term), processing charges of raw materials in manufacturing (fourth term), penalties imposed by selected suppliers (fifth term), dispatching batches to the customers (paid to the third-party logistics provider) (sixth term), reservation penalties incurred by the third-party logistics provider (seventh term), penalties imposed by the logistics corporate (such as losing credit on customers' viewpoints, non-economic shipping costs, etc.) (eighth term), and missed opportunity (ninth term). Equations (2.2) and (2.3) are the balancing restrictions connecting purchasing, manufacturing and delivery operations. Constraint (2.4) remarks that all manufactured products at each interval are bounded to the vehicle capacity in full-truckload mode (determining the batch size at each period]. Inequality (2.5) also indicates that the whole manufactured products do not exceed the operational capacity of the manufacturer. Constraint (2.6) ensures that all produced batches do not exceed the reserved number at the third-party logistics provider in each period (due to perishability requirements). The maximum number of batches in less-than-truckload is restricted by constraint (2.7). Inequalities (2.8) and (2.9) are related to the reservation and also capacity restrictions at each supplier, respectively. Constraints (2.10) and (2.11) are dependency limits showing that raw materials are purchased from a particular supplier only if it is elected for ordering. Constraint (2.12) ensures not exceeding the selected suppliers from a pre-defined number. Inequality (2.13) indicates another dependency restriction implying that a particular supplier is secondly ordered if it is chosen for the first time. Constraint (2.14) states the restriction corresponding

to whether a manufactured batch size is entirely fulfilled or not at each interval. Finally, Constraints (2.15) to (2.19) are for the feasibility domain of each decision variable.

3. THE COMPLEXITY OF THE PROBLEM

As we modified the classical SILSP and proposed a distinctive and novel configuration in satisfying the customers' demands, the problem is categorized in NP-hard class due to NP-hardness of simpler cases sited below. Following the matter, we use the notation offered by Akbalik *et al.* [3] in this way that $(K_t, a_t, b_t, p_t, R_t, V_t)$ specifies fixed ordering cost, per batch purchasing cost (when the number of batches ordered in period t is less than the reserved capacity), per batch purchasing cost (when the number of batches ordered in period t is more than the reserved capacity), unit purchasing cost, reserved batches in period t, and fixed size of each batch in period t), respectively. There are four classes of SILSP with batch ordering under the capacity reservation contract (SILSP-BOCRC) as NP-hard problems. For example, the problems where ($a_t = +\infty$ and V_t taking any optional quantity) or ($V_t = 0$ and a_t taking any optional quantity) are related to the uncapacitated SILSP-BOCRC. In these cases, there will be no reserved batch with the cost a_t in an optimal solution. It is noted that when the reserved capacity is equal to zero ($V_t = 0$), the unit purchasing cost a_t can take any optional quantity to be declined to the uncapacitated configuration. Another exceeding case is when $b_t \leq a_t \forall t \in \{1 \dots T\}$. But as we have logically considered the second price b_t more than the a_t, the mentioned case is ignored.

In this regard, Florian *et al.* [12] showed that the capacitated SILSP with fixed demands and zero backorder charges is NP-hard by making a reduction from the Knapsack Problem to a case of the problem (observe Florian *et al.* [12] Prop. 1, p. 671). Given that $b_t = +\infty$, the problem is changed into a capacitated SILSP. Moreover, by considering $K_t = 1$, it is gained a unitary fixed ordering cost; and through setting $V_t = 1$, per batch purchasing cost a_t will changed the unit procurement charge in the proof of Proposition 1 in Florian *et al.* [12]. Thus, since the configuration $(K/a_t/ + \infty/ - /R_t/V)$ – as a special case of $(K_t, a_t, b_t, p_t, R_t, V_t)$ – is NP-hard, our capacitated LSBPS problem also belongs to NP-hard classification.

Furthermore, in case the $V_t = 1$, the SILSP with only a fixed ordering cost and variable capacities is the same as the problem investigated by Bitran and Yanasse [6]. They considered a case of the capacitated SILSP with varying capacities and fixed ordering costs where either both of them are assumed NI (non-increasing) or both of them ND (non-decreasing). They illustrated that upon these assumptions which are equal to the notation of $(K_t, -, -, -, R_t, V)$, the problem is NP-hard even with no other charges (notice Bitran and Yanasse [6] Prop. 4, p. 1180).

As the same way, Akbalik and Rapine [2] indicated that the uncapacitated SILSP with batch production is NP-hard if per batch purchasing cost [when the number of batches ordered in period t is more than the capacity reserved] and fixed size of each batch in period t are variable, even with the whole other charges being zero. They reduced a case of the Unbounded Knapsack Problem (proven to be NP-complete by Lueker [18]) to the case of the uncapacitated SILSP stated above.

Summing up, as the simple cases investigated above are NP-hard, our capacitated LSBPS under two reservation contracts, along with supplier selection, is also classified as NP-hard in associated literature, implying that it is not possible to offer a solving algorithm with polynomial time complexity.

4. DYNAMIC PROGRAMMING ALGORITHM

We assume T periods and in each period $t \in \{1, 2, 3, \dots, T\}$, there are I suppliers in the joint supply chain (based on long-term agreements) suggesting different contracts.

4.1. Assumptions

To simplify the terminology of the main objective, we first consider some assumptions as follows:

– The transportation costs (incurred by shipping raw materials from suppliers to the manufacturer) are merged with the purchasing costs (a_{it}, b_{it}).

– Given the partnership supply approach, we consider the suppliers set as a whole, and thereby, the lost opportunity and penalty costs (imposed by the suppliers) are seamlessly included in pen_t.

– Following the simplicity requirements, we ignore the unit processing charges $g_t \sum_{i=1}^{I} (X_{it} + X'_{it})$ at each period.

– Less-than-truckload punishments are incurred in units of shortages instead of per batch, *i.e.*, we replace the term $\sum_{j=1}^{J} \text{pun}_t O'^t_j$ with $\sum_{j=1}^{B_t} \text{pun}_t (m_t - H^t_j)$ in each period.

– We develop two scenarios, respectively, based on whether there is not any restriction on the number of orderings (Scenario 1) (Fig. 4) or we are limited by at most twice a period (Scenario 2) (Fig. 5).

4.2. Scenario 1

We first obtain the function q_{it} by the following equation:

$$q_{it} = \min_i \left\{ S_{it} \boldsymbol{\lambda}_{0 < X_{it} \leq V_{it}} + a_{it} X_{it} + p_{it} (V_{it} - X_{it}) \boldsymbol{\lambda}_{X_{it} < V_{it}} \right\} \quad \forall t. \tag{4.1}$$

where $\boldsymbol{\lambda}_\delta$ is equal to 1 when δ is true, and 0, otherwise.

We assume that supplier i_j has been taken for the first time according to the calculation above. Then, it is conducted a comparison scheme based on the terms below in case we need to order for the second time:

$$q'_{it} = \min_i \left\{ \left(S'_{i_j t} \boldsymbol{\lambda}_{X_j t = V_{i_j t}} \boldsymbol{\lambda}_{0 < X'_{i_j t} \leq C_{i_j t} - V_{i_j t}} + b_{i_j t} X'_{i_j t} \right), \right.$$

$$\left. \min_{i \neq i_j} \left(S_{it} \boldsymbol{\lambda}_{C_{it} \geq B_t m_t - X_{i_j t}} \boldsymbol{\lambda}_{0 < X_{it} \leq V_{it}} + a_{it} X_{it} + p_{it} (V_{it} - X_{it}) \boldsymbol{\lambda}_{X_{it} < V_{it}} \right) \right\} \quad \forall t \tag{4.2}$$

where $\boldsymbol{\lambda}_\delta$ is equal to 1 when δ is true, and 0, otherwise.

Since the maximum number of suppliers captured must not exceed the pre-defined value 2, we are expected to do another assessment entailed due to distinctive outcomes from the previous step. In fact, this evaluation is incurred by the need for registering another order. It is carried out as follows:

(a) Supplier i_k has been selected for the second time

$$q''_{it} = \min_{i = i_j, i_k} \left\{ \left(S'_{i_j t} \boldsymbol{\lambda}_{X_{i_j t} = V_{i_j t}} \boldsymbol{\lambda}_{0 < X'_{i_j t} \leq C_{i_j t} - V_{i_j t}} + b_{i_j t} X'_{i_j t} \right), \right.$$

$$\left. \left(S'_{i_k t} \boldsymbol{\lambda}_{X_{i_k t} = V_{i_k t}} \boldsymbol{\lambda}_{0 < X'_{i_k t} \leq C_{i_k t} - V_{i_k t}} + b_{i_k t} X'_{i_k t} \right) \right\} \quad \forall t \tag{4.3}$$

where $\boldsymbol{\lambda}_\delta$ is equal to 1 when δ is true, and 0, otherwise.

(b) Supplier i_j has been again picked up for the second time

$$q'''_{it} = \min_{i \neq i_j} \left\{ S_{it} \boldsymbol{\lambda}_{C_{it} \geq B_t m_t - X_{i_j t}} \boldsymbol{\lambda}_{0 < X_{it} \leq V_{it}} + a_{it} X_{it} + p_{it} (V_{it} - X_{it}) \boldsymbol{\lambda}_{X_{it} < V_{it}} \right\} \quad \forall t \tag{4.4}$$

where $\boldsymbol{\lambda}_\delta$ is equal to 1 when δ is true, and 0, otherwise.

4.3. Scenario 2

Incorporating the constraint based upon the whole number of orderings (at each period) makes the problem more practical. Although unlimited ordering brings about an intensely competitive environment between the suppliers to attract the manufacturer through offering preferable contracts, considering such restriction yields a situation in which the manufacturer tries to develop efficient processes and have optimal decision-making. Thus, we site below corresponding equations to realize the scenario 2's requirements.
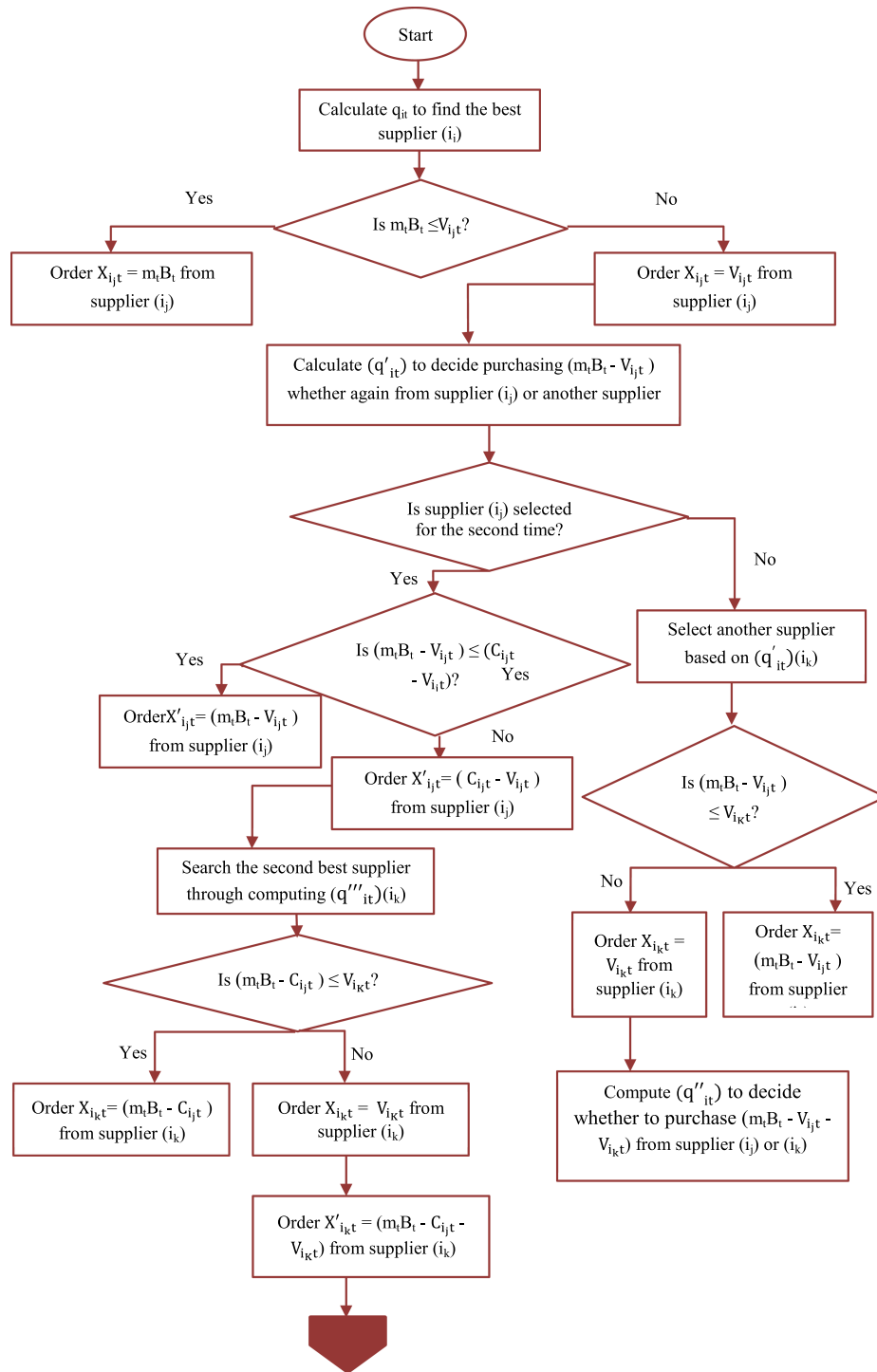
FIGURE 4. Flowchart of the DP algorithm for choosing the best supplier(s) and ordering the values needed based on scenario 1.
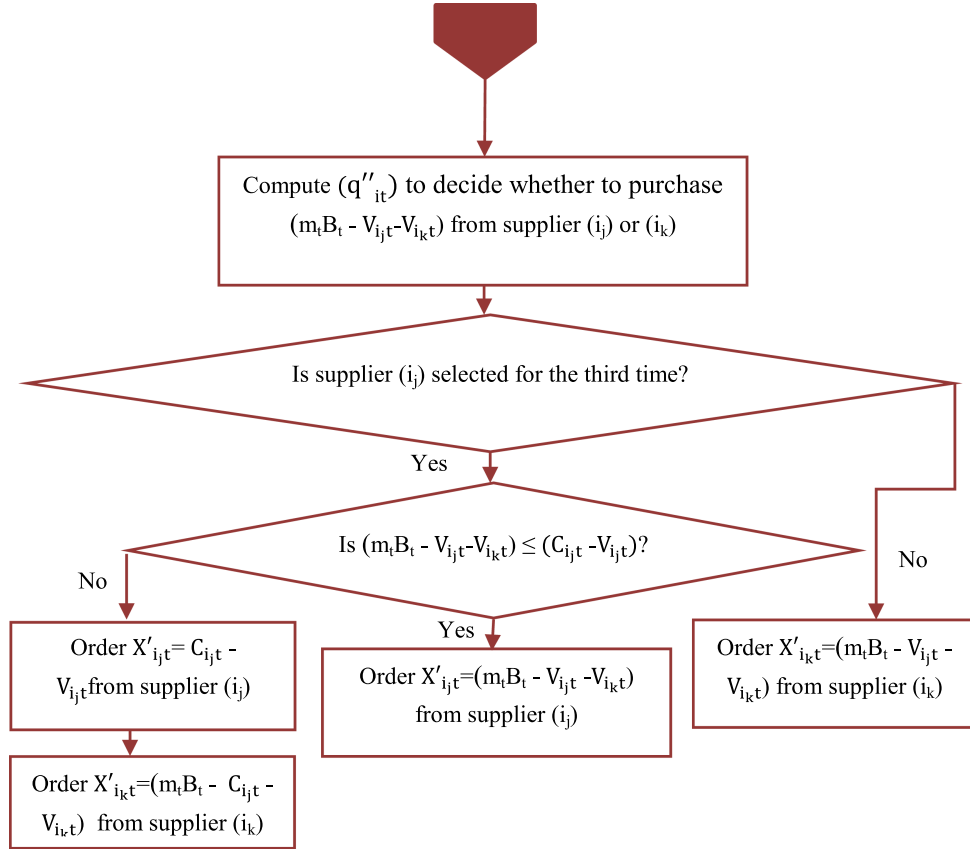
FIGURE 4. (Continued.)

First, we probe the best supplier according to the equation primarily proposed in the previous scenario (q_{it}).

$$q_{it} = \min_{i} \left\{ S_{it}\boldsymbol{\lambda}_{0 < X_{it} \leq V_{it}} + a_{it}X_{it} + p_{it}\left(V_{it} - X_{it}\right)\boldsymbol{\lambda}_{X_{it} < V_{it}} \right\} \quad \forall t \tag{4.5}$$

where $\boldsymbol{\lambda}_{\delta}$ is equal to 1 when δ is true, and 0, otherwise.

It is assumed that supplier i_j has been selected for the first time. Afterward, we need to carry out another selection with the purpose of providing the intended size of raw materials at each period. Hence, we consider two conditions as follows:

(a) Quantities needed $(B_t m_t)$ is more than the reserved capacity at supplier i_j $\left(X_{i_j t} = V_{i_j t}\right)$

$$\tau_{ij} = \min_{i} \left\{ \left(S'_{i_j t}\boldsymbol{\lambda}_{X_{i_j t}=V_{i_j t}}\boldsymbol{\lambda}_{0 < X'_{i_j t} \leq C_{i_j t} - V_{i_j t}}\boldsymbol{\lambda}_{C_{i_j t} - X_{i_j t} \geq B_t m_t - X_{i_j t}} + b_{i_j t}X'_{i_j t} \right), \right.$$
$$\left. \min_{i \neq i_j}\left(S_{it}\boldsymbol{\lambda}_{V_{it} \geq B_t m_t - X_{i_j t}}\boldsymbol{\lambda}_{0 < X_{it} \leq V_{it}} + a_{it}X_{it} + p_{it}\left(V_{it} - X_{it}\right)\boldsymbol{\lambda}_{X_{it} < V_{it}} \right) \right\} \quad \forall t \tag{4.6}$$

where $\boldsymbol{\lambda}_{\delta}$ is equal to 1 when δ is true, and 0, otherwise.

(b) Quantities needed are less than the reserved capacity at the supplier $i_j(X_{i_j t} = B_t m_t)$

Under the above assumption, there is no need to register a new order.

To better understand, the flowcharts of the dynamic programming (DP) algorithms for the above-stated scenarios are respectively illustrated in Figures 4 and 5.
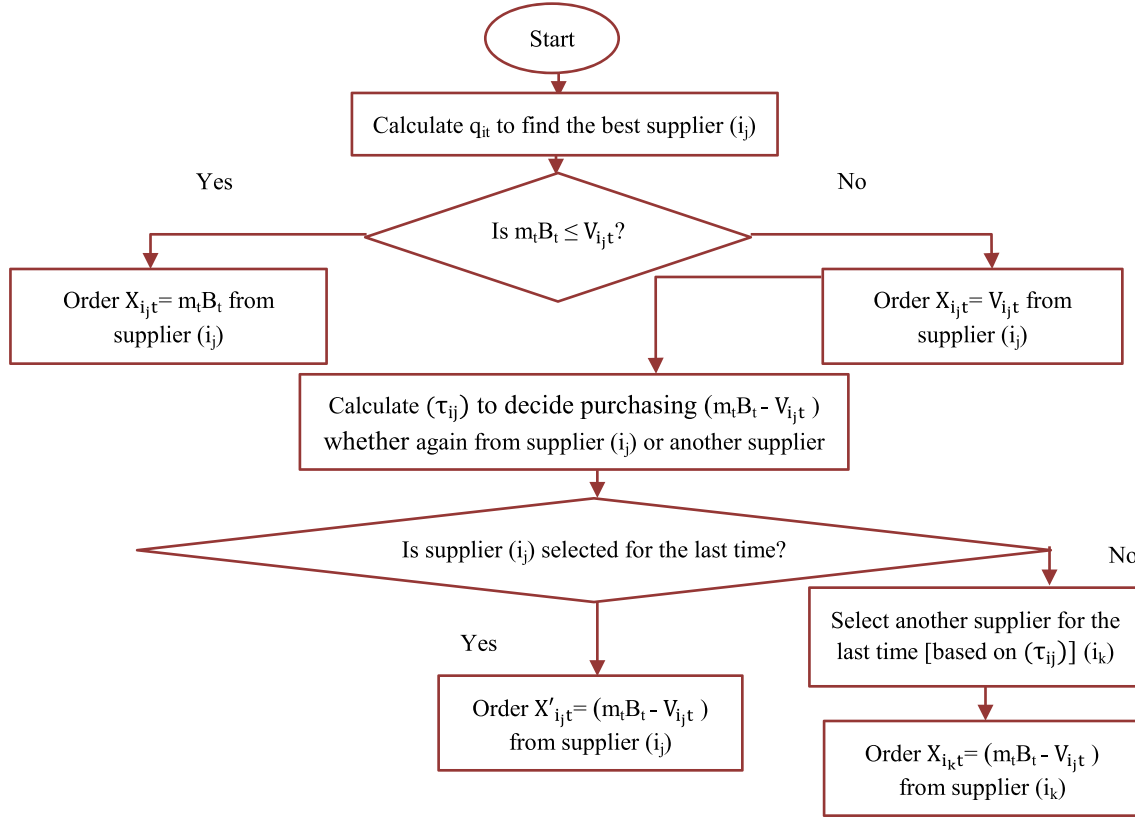
FIGURE 5. Flowchart of the DP algorithm for choosing the best supplier(s) and ordering the values needed based on scenario 2.

Hereafter, we try to gain the function β_t as follows:

$$
\begin{aligned}
\min_{H_j^t} \beta_t = & \left[\left(\sum_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) \boldsymbol{\lambda'_{H_j^t > 0}} + \mathrm{pen}_t \left(B_t m_t - \sum_{j=1}^{B_t} H_j^t \right) \boldsymbol{\lambda'_{0 < \sum_{j=1}^{B_t} H_j^t < B_t m_t}} \right. \\
& \left. + \theta_t \left(B_t - \sum_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) \boldsymbol{\lambda'_{\sum_{j=1}^{B_t} \left\lceil \frac{H_j^t}{M_t} \right\rceil < B_t}} + \sum_{j=1}^{B_t} \mathrm{pun}_t \left(m_t - H_j^t \right) \boldsymbol{\lambda'_{0 < H_j' < m_t}} \right] \quad \forall t \qquad (4.7)
\end{aligned}
$$

where $\boldsymbol{\lambda'_\delta}$ is equal to 1 when δ is true, and 0, otherwise.

Drawing on the equation above, we develop another DP algorithm to address the manufacturing delivery operations as well. Therefore, the steps of the algorithm are illustrated in Table 1.

4.4. Numerical study

Note that the time complexity of the solving algorithm (in context of an overall procedure) is in $\mathrm{O}(TI \cdot T)$ and $\mathrm{O}(TI^2(I-1) \cdot TB_t J)$, illustrating the best and worst cases, respectively. Once the $\mathrm{O}(TI^2(I-1))$ or $\mathrm{O}(TI)$ values are gained [respectively in scenarios 1 and 2] for the optimal cost of the manufacturer, we can then compute the overall complexity with respect to the values captured in the second phase [according to the DP algorithm suggested for the manufacturing and delivery sections]. As stated, the complexity of DP is dependent on T, I, B_t,

TABLE 1. Pseudo-code of DP algorithm in the manufacturing-delivery operations.

Main steps of the algorithm for handling corresponding operations

(**Input** parameters): f_j^t, B_t, m_t, l_t, pen_t, θ_t, and pun_t

(**Initializations**): $d_t' = B_t m_t$

For $t = 1$ to T [the planning horizon]

While $d_t' > 0$

if $\sum\limits_{j=1}^{B_t} H_j^t = 0$ **then**

    stop; no delivery is carried out (number of shortages is equal to $B_t m_t$)

end if

if $H_j^t > 0$ **then**

compute $\min \beta_t = \left(\sum\limits_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right)$

if $0 < \sum\limits_{j=1}^{B_t} H_j^t < B_t m_t$ **then**

compute $\min \beta_t = \left(\sum\limits_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) + \text{pen}_t \left(B_t m_t - \sum\limits_{j=1}^{B_t} H_j^t \right)$

if $\sum\limits_{j=1}^{B_t} H_j^t = B_t m_t$ **then**

    do not change $\min \beta_t \left(\beta_t = \left(\sum\limits_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) \right)$

end if

if $\sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil < B_t$ **then**

compute $\min \beta_t = \left(\sum\limits_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) + \text{pen}_t \left(B_t m_t - \sum\limits_{j=1}^{B_t} H_j^t \right) + \theta_t \left(B_t - \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right)$

if $H_j^t < m_t$ **then**

compute $\min \beta_t = \left(\sum\limits_{j=1}^{B_t} f_j^t \left\lceil \frac{H_j^t}{m_t} \right\rceil + l_t \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right) + \text{pen}_t \left(B_t m_t - \sum\limits_{j=1}^{B_t} H_j^t \right) + \theta_t \left(B_t - \sum\limits_{j=1}^{B_t} \left\lceil \frac{H_j^t}{m_t} \right\rceil \right)$

    $+ \sum\limits_{j=1}^{B_t} \text{pun}_t \left(m_t - H_j^t \right)$

end if

end if

end if

end if

end while

end for

(**Output**): β_t, H_j^t

and J in the worst case, which respectively are the length of time horizon, the suppliers joint to the long-term agreement, the number of batches, and the number of intervals. On the other hand, the computational time of MILP formulation is thoroughly unpredictable and it is determined with respect to the whole parameters of the problem.

For experimental evaluation of the proposed approach, we consider below a particular illustration of the problem. As seen in Table 2, the instance consists of three suppliers that are to be assessed within three periods of time. Since there is no similar study with full coverage in the literature, we tried to determine the values of corresponding parameters with the help of some researches such as Akbalik *et al.* [3]. The cited parameters

TABLE 2. An illustration of the problem.

Supplier	Period	$d'_t = B_t m_t$	S_{it}	S'_{it}	a_{it}	b_{it}	V_{it}	p_{it}	C_{it}	tr_{it}	tr'_{it}	K_t	f_t	g_t	l_t	θ_t	r_t	pun_t	pen_{it}	Z
1	1	170	150	161	41	49	78	28	100	18	23	173	250	59	140	75	46	35	31	123 845
	2	183	160	172	40	48	136	30	150	19	24	190	266	68	155	79	49	43	30	175 248
	3	200	155	168	43	50	155	31	174	21	25	210	275	79	161	81	53	45	33	179 869
2	1	170	187	197	39	47	100	30	126	25	28	173	250	59	140	75	46	35	44	119 989
	2	183	199	210	45	51	170	33	185	17	20	190	266	68	155	79	49	43	45	199 899
	3	200	205	214	49	59	168	33	170	14	19	210	275	79	161	81	53	45	41	150 502
3	1	170	136	166	48	58	95	25	101	22	24	173	250	59	140	75	46	35	52	139 666
	2	183	155	175	55	61	99	28	123	24	26	190	266	68	155	79	49	43	50	133 665
	3	200	144	165	58	65	199	34	205	13	18	210	275	79	161	81	53	45	55	196 112
4	1	170	176	188	39	44	130	19	130	16	19	173	250	59	140	75	46	35	49	185 859
	2	183	169	179	39	47	110	29	118	23	29	190	266	68	155	79	49	43	49	178 965
	3	200	149	155	45	54	172	27	199	19	23	210	275	79	161	81	53	45	45	166 252

TABLE 3. A comparison scheme between MILP and the heuristic approaches.

	$T = 6$			$T = 15$		
	$I = 10$	$I = 15$	$I = 30$	$I = 10$	$I = 15$	$I = 25$
CPU time of heuristic algorithm based on scenario 1 (s)	0.062	0.16	2.66	4.09	8.89	10.02
CPU time of heuristic algorithm based on scenario 2 (s)	0.013	0.022	0.11	2.28	5.09	6.93
CPU time of MILP (s)	0.362	1.18	1.02	2.81	1.99	16.23
Gap in DP completion of scenario 1	0	0	0	0	0.009	0
Gap in DP completion of scenario 2	0	0	0	0	0	0.0015
Gap at MILP completion	0	0	0.023	0.04	0	0.89

are illustrated in different columns of the table. Noting the Z column illustrating the whole incurred charges, we can observe that suppliers 2 and 3 provide more preferable conditions comparing to the supplier 1. In this regard, supplier 2 is considered as the option with the highest priority for developing partnership relationships.

To test the efficiency of the heuristic algorithms offered in the corresponding sections, we now compare the performances of MILP and DP algorithms based on the measure of CPU time and the gaps in Table 3.
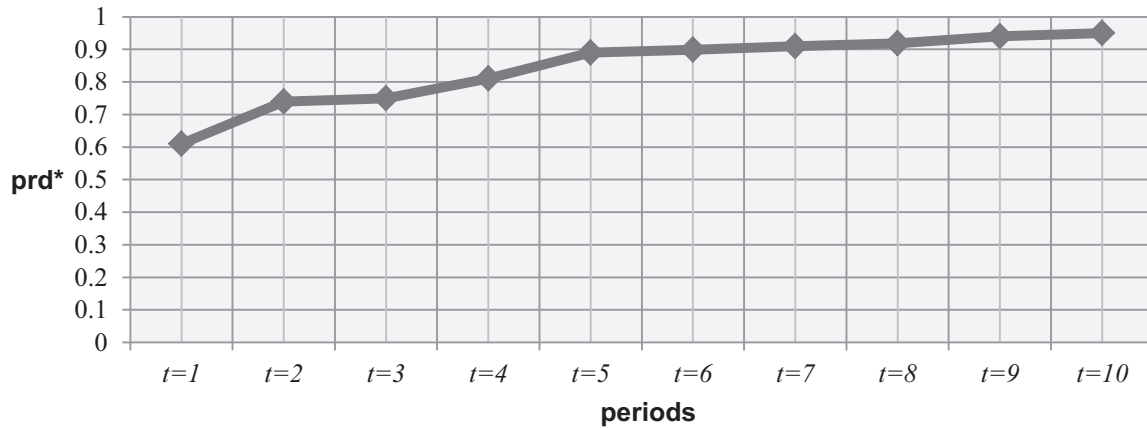
Figure 6 illustrates an analysis based on productivity measure to prove the preferable advantage of the problem in decreasing the manufacturing losses.

4.4.1. Sensitivity analysis

In this section, some beneficial sensitivity analyses are carried out according to the penalty measures imposed by different echelons of the under-discussion supply chain. Accordingly, we investigate the impacts of parameters θ_t, pun_t and pen_t on Z (objective function) within six three-dimensional diagrams. As seen in Figure 7, there exists an ascending relation between the Z and the triple penalties (θ_t, pun_t, pen_t). In other words, increasing values of (θ_t, pun_t, pen_t) result in rising costs as well. In this regard, the objective function of scenario 2 illustrates more growth comparing to that of scenario 1.

4.5. Managerial insights

Table 2 provides a combination of parameters (*e.g.*, customers' demands, reserved capacity, penalty costs and etc.) within a particular instance. As shown in the numerical example, it is highly important to observe the overall performances of the included suppliers. Accordingly, pointing out the whole incurred charges (the Z column), we can acknowledge that the suppliers 2 and 3 bring forward more affordable conditions comparing to

$$* \text{prd} = \sum_{i=1}^{I}(X_{it} + X'_{it}) / \sum_{j=1}^{J} H_j^t$$

FIGURE 6. Productivity analysis at each period illustrating a progressive approach.

the supplier 1. In other words, when analyzing the suppliers' performances through a three-period horizon, the choices will be supplier 2, 3 and gain 2, respectively. This specifies how to develop a partnership structure in supply chains. Therefore, supplier 2 is considered as the most convincing option for developing such partnerships.

A comparison scheme has been presented within Table 3. In this table, CPU time and the absolute optimization gap are considered as the evaluation criteria. Also, two horizons of 6 and 15 time-periods have been assigned for the assessment wherein the problem is solved based on the MILP formulation and the developed scenarios as well. As observed, in most cases, the cited scenarios provide more efficient performances. Further, it is noteworthy that the solving procedure developed based on scenario 2 and the manufacturing-delivery algorithm is mostly preferred.

A productivity analysis has been also developed in Figure 6. It illustrates the preferable advantage of the suggested approach – single-item lot-based supplying and batch production under a bilateral capacity reservation contract – in decreasing the manufacturing losses. In this regard, the relation $\sum_{i=1}^{I}(X_{it} + X'_{it}) / \sum_{j=1}^{J} H_j^t$ indicates the ratio of items purchased from suppliers to the number of faultless products in different periods. The ascending trend of the diagram designates an uninterrupted improvement in the under-investigation time horizon. Hence, establishing the supply and manufacturing communications (in supply chains) based on the proposed approach can lead to the further development of partnership structures and waste reductions as well.

5. CONCLUSION AND FUTURE REMARKS

Lot-sizing and batch-production of perishable products, along with the supplier selection process simultaneously address the sourcing and delivery operations. It is always worth providing a balanced production scheme and developing a partnership supply procedure in all kinds of enterprises. Indeed, establishing a coordination structure in the context of supplying, manufacturing, and deliveries supports the notion of decreased undesirability. Taking the right suppliers and dividing lot-sizes among them become a major challenge for a decision-maker when suppliers offer non-negligible contracts with certain delivery performance. Our work opens appealing research avenues through considering a progressive improvement approach in a particular supply chain. In this regard, we study a single-item lot-based supplying and batch production under a bilateral capacity reservation contract based on a partnership supply strategy. Further, the products are perishable and delivered by a third-party logistics provider specifying the batch-size of the production with respect to the products' delivery requirements, customers' demands, and other considerations. This implies a mechanism in which two short-term contracts are signed with the selected suppliers and a third-party logistics provider, respectively. In fact, we
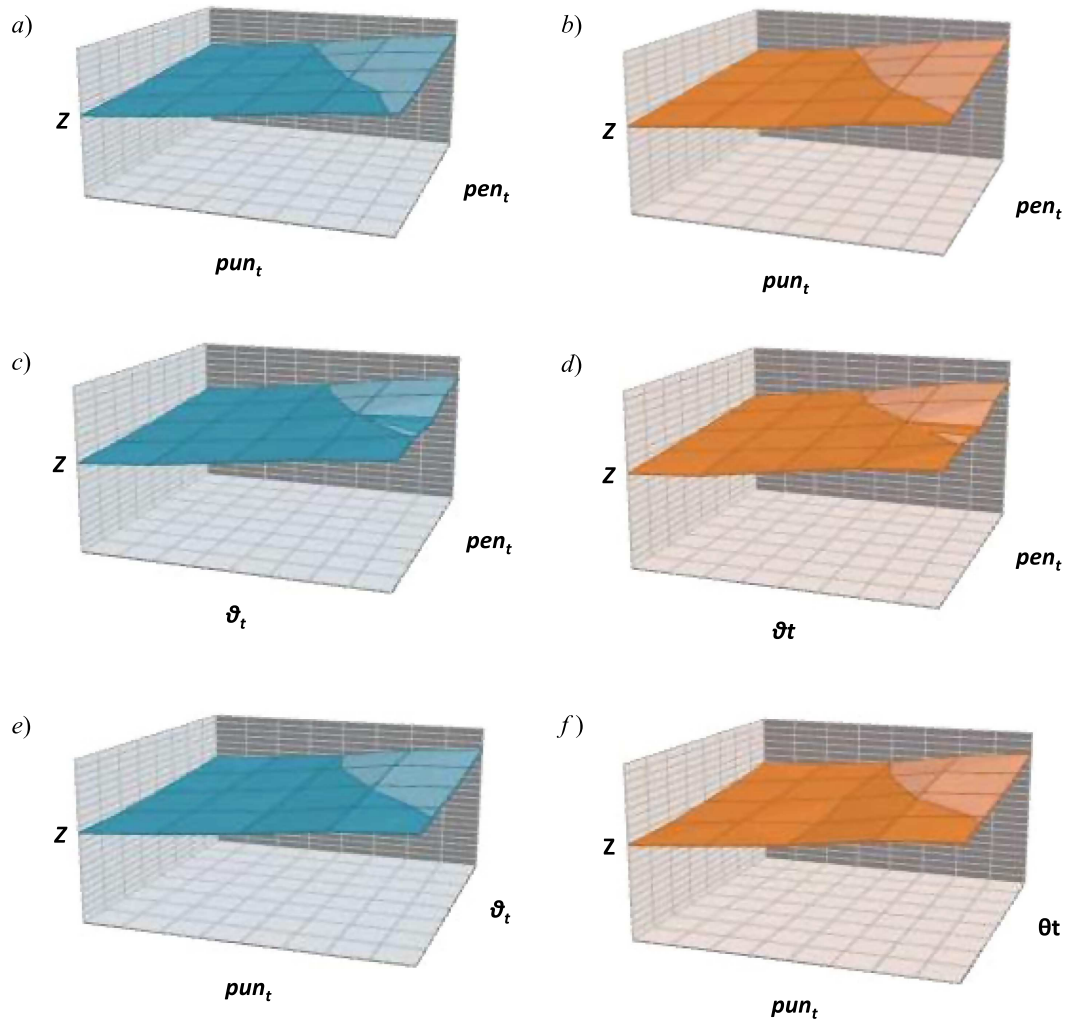
FIGURE 7. The effects of $(\theta_t, \text{pun}_t, \text{pen}_t)$ on the total cost. (a) The effects of $(\text{pun}_t, \text{pen}_t)$ on the whole cost (Z) in scenario 1. (b) The effects of $(\text{pun}_t, \text{pen}_t)$ on the whole cost (Z) in scenario 2. (c) The effects of (θ_t, pen_t) on the whole cost (Z) in scenario 1. (d) The effects of (θ_t, pen_t) on the whole cost (Z) in scenario 2. (e) The effects of (θ_t, pun_t) on the whole cost (Z) in scenario 1. (f) The effects of (θ_t, pun_t) on the whole cost (Z) in scenario 2.

develop a partnership supply chain where the manufacturer is a focal point of the network. There are strategic (long-term) cooperation agreements among the manufacturer, suppliers (the ones taken), and the logistics provider. Accordingly, in addition to the ordinary costs incurred due to ordering and delivery processes, some penalties are imposed for orders/delivers exceeding or falling behind the reserved capacity. In another point of view, although it is straightforward to observe that the capacity reservation models (in general formulations) are more difficult than the capacitated lot-sizing problem, we investigated the complexity of the problem in the corresponding section, showing the NP-hardness of solving its model. Hence, we proposed three dynamic programming (DP) algorithms to reduce the problem complexity. We computed the overall complexity of the algorithms with time $O(TI \cdot T)$ and $O(TI^2(I-1) \cdot TB_t J)$ in the best and worst cases, respectively.

Among the possible research perspectives of this work, one can consider the uncertainty of the parameters required in practical issues in the first extension. Second, incorporating scheduling decisions such as due dates of the products and delivery time windows are also striking. Further, the problem becomes more interesting when investigating other reservation contracts with more complex cost structures, which can also vary with time. Another perspective is to study the current model in the multi-item cases. Another direction is to propose the metaheuristic algorithm to solve the problem, especially for large-size instances.

Conflicts of interest

No potential conflict of interest was reported by the authors.

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