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WITHDRAWN: THE EFFECT OF CAPACITY CONSTRAINT ON PRICING DECISION AND COORDINATION CONTRACT IN RECYCLING CHANNELS

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Abstract. This article has been withdrawn at the request of the authors, who did not approve the publication in a supplementary issue. The Publisher apologizes for the inconvenience this is causing.

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1. Introduction

Sustainable production realizes the low consumption of resources and the reuse of end-of-life products, which plays an important role in economy, environment and sustainable development. At the same time, the recycling and remanufacturing of waste products has also been widely concerned by scholars [23, 26, 37]. For example, from 2017 to 2025, the remanufacturing industry in US is expected to grow at a compound annual growth rate (CAGR) of 6.6%. China's Ministry of Industry and Information Technology has issued a "high-end intelligent remanufacturing action plan (2018–2020)", which aims to reach 200 billion yuan by 2020. Many well-known firms utilize remanufacturing to improve their performance, such as Xerox [20,38]. In 2005, Foxconn announced to remanufacture scrapped iPhones for resale in China [42]. In addition, IBM, Kodak, HP and others also have success stories.

In CLSC, the original equipment manufacturer may delegate the recycling operation to retailers, thus there exists a conflict on the recycling channel. Some scholars studied a CLSC composed of one manufacturer and two symmetric retailers, where they assumed the collection channels were infinite, which was inconsistent with the actual warehouse capacities, number of employees, and collection processing capability. European Union, for

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instance, has issued the directives requiring the firms to recycle scrapped product and limit their abilities [10]. In this study, we design the four different scenarios: Neither retailer collects (NN), only a retailer collects (CN/NC) and both retailers collect (CC). Under this structure, the three cases are considered where the retailers compete with a Cournot game. In the first case, neither retailer is with constrained capacity to collect with the goals of maximizing their respective profit. On this basis, we further discuss only a retailer has capacity constraint. Finally, the case that both retailers have constrained capacity. For the game sequence, the manufacturer (Leader) first determines the wholesale price and transfer price, then two retailers (Followers) make decisions on collection rate and selling price later. Hence, we attempted to answer the following questions:

- In closed-loop supply chain, what is the equilibrium decision of supply chain members under different situations?
- How does capacity constraint affect the profitability of supply chain members in different situations?
- From the perspective of supply chain coordination, Can the supply chain be effectively coordinated when both retailers have constraint?

The rest of this article is described below: In Section 2, we combed the relevant literature from three perspectives and summarize our contributions. We describe and assume the model in Section 3. In Section 4, the optimal decision solution and profit of three different scenarios are obtained. Section 5 obtained the equilibrium results and coordination conditions, and analyzed and compared the differences in the optimal results in different situations. In Section 6, a summary of this paper, and prospects. The proof is in the Appendix A.

2. Literature review

The literature closely related to our work can be classified as the three main categories: closed-loop supply chain management, capability constraint in CLSC and coordination mechanism in CLSC. Further, we depict the research gap and contributions.

Nowadays, research focused on recycling and channel competition in CLSC. Among them, some scholars studied channel competition. Savaskan and Van Wassenhove [21] considered a manufacturer and two competitive retailers in consideration with deciding the decisions of pricing and collection. Ferrer and Swaminathan [8] explored the pricing decision between original equipment manufacturer and third-part remanufacturer in a competitive CLSC. Wu [31] proposed a supply chain composed of two manufacturers and one retailer to analyze the effects of price and service competition on the equilibrium. Jena and Sarmah [14] demonstrated the pricing and return rate decisions of CLSC in the competitive manufacturers. Örsdemir *et al.* [19] found that when the OEM was in a stronger competitive position, it relied on the quality strategy, otherwise it relied on the limited quantity. Wei and Zhao [30] analyzed the pricing and remanufacturing strategies of the CLSC in the situation of competition between manufacturer and retailer. Xu *et al.* [37] discussed the influences of low carbon and remanufactured preferences on retail prices, reduction rate and collection rate in a two-period CLSC.

Yet the previous researches ignored collection competition, some scholars explored the reverse channel competition in closed-loop supply chain [10,24], competition between a retailer and a third party [13], and competition between a manufacturer and a third party [16]. On this basis, Liu *et al.* [16] compared the impact of collection competition on equilibrium decision under collection modes (manufacturer and retailer, manufacturer and third party, retailer and third party). He *et al.* [10] designed the collection competition between a manufacturer and a retailer to reveal the influence of collection inconvenience on return rate.

Beyond that, Taleizadeh *et al.* [25] considered two channel structures, namely the CLSC with single channel forward dual channel reverse and the CLSC with forward and reverse dual channel to discussed a new coordination mechanism to increase the profits of supply chain members. Johari and Hosseini-Motlagh [15] illustrated the competitions in a closed-loop supply chain between the forward and reverse channel, found two-part tariff contract has a positive effect on whole supply chain. Based on product competition and collection competition, Wang *et al.* [27] studied the optimal pricing strategies of different individuals in CLSC and their impact on the optimal selection of manufacturer, and retailer/remanufacturer profits. Wang *et al.* [29] constructed a CLSC model of a manufacturer and two retailers to explore the optimal selection of competitive retailers, explored that the collusion of retailers helps to improve the overall profits of the supply chain. The difference is that we studied the order quantity and recovery rate of the two retailers under different constraints.

Capacity constraint is not only a common problem in many collection firms, but also has an important effect on the decision of CLSC. Hence, capacity constraint is becoming increasing crucial in actual circumstance. Hamdouch [9] studied the effect of capacity constraints on multi-cycle supply chain network. Wu *et al.* [32] applied the capacity constraint to supply chain replenishment system and improved the economic benefit of supply chain replenishment system. Ahmadi-Javid and Hoseinpour [1] demonstrates the location, inventory and pricing decisions of supply chain distribution network from the perspective of price-sensitive demand and inventory capacity constraint. However, in many studies, the production capacity of manufacturers is often considered to be infinite, which is unscientific in reality. Therefore, some scholars consider the capacity constraint of manufacturer. Based on the difference between new and remanufactured product, Bayındır *et al.* [4] analyzed the influence of the substitution policy on the profitability of remanufacturing with capacity constraint. Atamer *et al.* [2] illustrated the impact of optimal pricing and production decisions on the manufacturers' profit. Hsieh and Lai [12] established a two-stage supply chain model of a manufacturer with capacity constraints and two suppliers, and discussed the influence of capacity constraints on manufacturer's decision and profit. Wu *et al.* [33] established a supply chain model consisting of one supplier and two manufacturers with asymmetric capacity constraints, and analyzed the impact of supplier pricing strategies on incentives for manufacturer to share information. In addition, Wang *et al.* [28] analyzed the sustainable supply chain pricing, recycling and remanufacturing strategies, and found capacity constraints have a significant impact on recycling and remanufacturing. From the above, we found that they mainly focused on the impact of manufacturer's capacity constraint on supply chain member's strategies, but ignored the constraint of collection capacity.

The topic of coordinated mechanism in a closed-loop supply chain has been attention sharply in existing literature. From the view of revenue sharing. Bhattacharya *et al.* [5] compared the optimal order quantities under centralized and decentralized decision, and found that the order quantity and profit under centralized decision were the largest, and the coordination was realized through revenue sharing contract. Mitra and Webster [18] analyzed the impact of government subsidy on the benefits of members in the context of product competition and found that the subsidy sharing is conducive to improving the overall benefits of the supply chain. Xie *et al.* [34] realized the contract coordination of double-channel closed-loop supply chain through revenue sharing contract according to the characteristics of the recovery rate variation. Zhang *et al.* [40] constructed a vertical dual-channel CLSC model of return of defective products and return of rejected products, and adopted the revenue sharing contract to improve the total profit of the dual-channel CLSC.

Among them, two-part tariff contracts have been shown to be effective in improving performance. Choi *et al.* [7] proposed the optimal decision and member profits under different channel leadership to implement supply chain coordination by using revenue sharing contract and two-part tariff contract. Shi *et al.* [22] compared the influnces of revenue sharing contract and two-part tariff contract, thus the result showed that retailers were more willing to two-part tariff contract. Bai *et al.* [3] established a sustainable supply chain model with propose revenue- and cost-sharing with two-part tariff contract to coordinate. Therefore, we propose the method of two-part tariff to achieve a Pareto improvement.

According to the difference of previous literature in Table 1, the contributions of this research can be summarized as follows. First, we study the situation of both forward and reverse competitions between manufacturer and retailers in CLSC. Additionally, we also explore the influence of competition intensity on the collection channel. On this basis, we discuss the effect of capacity constraint on the equilibrium under the different situations from the conditions of retailers' capacity constraint. Thirdly, we try to design a two-part tariff contract to coordinate the CLSC and achieve a Pareto improvement.

TABLE 1. The difference between our work and the related literature.

Author(s)	Forward channel competition	Reverse channel competition	Capacity constraint	Coordination mechanism
Savaskan *et al.* [21]	√	×	×	×
Ferrer and Swaminathan [8]	√	×	×	×
Wu [31]	√	×	×	×
Jena and Sarmah [14]	√	×	×	√
Örsdemir *et al.* [19]	√	×	×	×
Wei and Zhao [30]	√	×	×	×
Xu *et al.* [36]	√	×	×	×
He *et al.* [10]	×	√	×	×
Huang *et al.* [13]	×	√	×	×
Taleizadeh and Sadeghi [24]	×	√	×	×
Liu *et al.* [16]	×	√	×	×
Zhao *et al.* [41]	×	√	×	×
Taleizadeh *et al.* [25]	√	√	×	√
Johari and Hosseini-Motlagh [15]	√	√	×	√
Wang *et al.* [27]	√	√	×	×
Wang *et al.* [29]	√	√	×	√
Our work	√	√	√	√

3. Problem description and assumption

In this paper, we analyze the effects of channel competition and capacity constraint on equilibrium in closed-loop supply chain including a manufacturer and two symmetric retailers. In the forward channel, the manufacturer provides a certain type of product to retailers, in turns, sells to consumers, while the two retailers are responsible for collecting the end-of-life products and then to the manufacturer for remanufacturing. We use the superscripts NN, CN and CC to distinguish the model and their corresponding variables. Further, we summarize the notation and explanation listed in Table 2.

In addition, the following assumptions are considered in our mathematical models:

Assumption 3.1. *Following the existing literature and many other ([11, 21, 23, 39]), we suppose that there is no significant difference between new and remanufactured product and the market demand is a linear function* $p_i = \alpha - q_i + \beta q_j, i, j \in \{1, 2\}, i \neq j$ *which shows a trend of monotonically decreasing and continuous with respect to the selling price.*

Assumption 3.2. *Consider the recycling competition between the two retailers, we introduce the competition intensity into our research, which reflects the competition level between two retailers in collecting end-of-life products. Therefore, the greater the competition, the more intense the collection. Consistent with Huang et al. [13], Liu et al. [16], Wang et al. [27] and Xu [35], the collection investments for both retailers are quadratic functions with the collection rate, where* τ_i *is the retailer i's collection rate, and* γ *represents the intensity of reverse competition. This function can be expressed as:* $k\left(\tau_1^2 + \gamma\tau_2^2\right)/2\left(1-\gamma^2\right)$ *and* $k\left(\gamma\tau_2^2 + \tau_1^2\right)/2\left(1-\gamma^2\right)$*, if* $\gamma = 0$*, this means that only one retailer collections its investment, the collection investment function of retailer becomes* $k\tau_i^2/2$*.*

Assumption 3.3. *In addition, the recycling quantities from retailers are set at a level those are much lower than the actual levels in collecting end-of-life products [6, 33]. To some extent, this's intuitively consistent with reality, we assume sufficiently large k to guarantee. Hence, we use the following relationship to characterize the collection rates of the two retailers as* $0 \leq \tau_1 \leq z_1$*,* $0 \leq \tau_2 \leq z_2$ *and* $0 \leq \tau_1 + \tau_2 \leq 1$*.*

TABLE 2. Notations and explanations.

Notation	Explanation
w	The manufacturer's wholesale price for unit product
b	The transfer price of end-of-life products from retailers to manufacturer
p_i	The retailer i's selling price for unit product, $i \in \{1, 2\}$
τ_i	The retailer i's collection rate for end-of-life product, $i \in \{1, 2\}$
α	The market size
q_i	The retailer i's order quantity
β	The intensity of demand competition between the two retailers, $\beta \in (0, 1)$
k	The scaling parameter for collection cost in recycling channel
γ	The competition intensity between the two retailers collection channels, $\gamma \in (0, 1)$
c_n	The production cost manufactured the new product *via* raw materials
c_r	The production cost manufactured the manufactured product *via* end-of-life products
s	The saving production cost provided between the new and remanufacturing product
z_i	The capacity constraint for retailer i's recycling channel, $i \in \{1, 2\}$
π_{sc}	The profit function for supply chain
π_m^M	The profit function for manufacturer in model M, where $M \in \{\mathrm{NN}, \mathrm{CN}, \mathrm{CC}\}$
π_{ri}^M	The profit function for retailer in model M, where $M \in \{\mathrm{NN}, \mathrm{CN}, \mathrm{CC}\}$ and $i \in \{1, 2\}$

In this paper, we considered the capacity constraints of recycling channels in the actual situation, established three models consisting of one manufacturer and two retailers, and discussed the impact of different recycling strategies on members of the supply chain. Details as follows: (1) Neither retailer collections (Model NN), the manufacturer (Leader) first determines the wholesale price. After that, two retailers (Followers) jointly decide the order quantity to maximize their respective profits. (2) Only one retailer collections (Model CN), as a leader, the manufacturer first determines the wholesale price and transfer price. Afterwards, the two retailers jointly decided on the order quantity and collection rate, and one of them had a capacity constraint. (3) Both retailers collection (Model CC), similar to Model CN, but now both retailers have capacity constraints. The three models are illustrated in Figure 1.

4. Model equilibrium

In this section, we analyze the equilibrium results of model NN, model CN and model CC. Furthermore, in the CC model, we establish the centralized and decentralized models and obtain the equilibrium results. We use superscripts NN, CN and CC to distinguish the model and their corresponding variables.

4.1. Neither retailer collections (Model NN)

First, we establish a supply chain model that neither retailer recycles. The manufacturer (Leader) first determines the wholesale price. After that, two retailers (Followers) jointly decide the order quantity. Under this structure, the manufacturer and retailers aim to maximize their respective profits. Therefore, the profit function of the supply chain is as follow

$$\pi_m^{\mathrm{NN}} = (w - c)(q_1 + q_2) \tag{4.1}$$

$$\pi_{r1}^{\mathrm{NN}} = (\alpha - q_1 + \beta q_2 - w) q_1 \tag{4.2}$$

$$\pi_{r2}^{\mathrm{NN}} = (\alpha - q_2 + \beta q_1 - w) q_2 \tag{4.3}$$

where $w(q_1 + q_2)$ is the revenue of selling products to retailers, $c(q_1 + q_2)$ is total production cost, $(\alpha - q_1 + \beta q_2 - w) q_1$ is represent the profit of Retailer 1 in the forward channel, $(\alpha - q_2 + \beta q_1 - w) q_2$ is represent the profit of Retailer 2 in the forward channel.

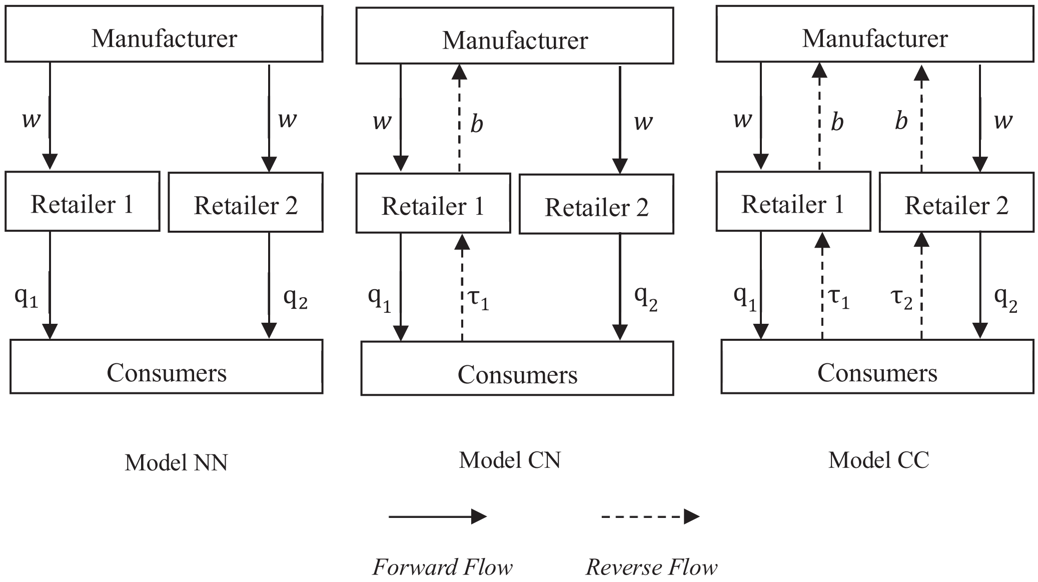

FIGURE 1. Supply chain models in different scenarios.

Proposition 4.1. *In Model NN, the manufacturer's equilibrium wholesale price is as follows* $w^{\mathrm{NN}^*} = \frac{a+c}{2}$. *The retailer's equilibrium decisions are* $q_1^{\mathrm{NN}^*} = q_2^{\mathrm{NN}^*} = \frac{a-c}{4+2\beta}$.

From Proposition 4.1, the manufacturer's wholesale price depends on the size of the market and the cost, the equilibrium result for both retailers is the same, and the selling quantity depends on the cost and the level of competition (β). By Proposition 4.1, we can also get the profit of the manufacturer and the retailers are $\pi_m^{\mathrm{NN}^*} = \frac{(a-c)^2}{2(2+\beta)}$ and $\pi_{r1}^{\mathrm{NN}^*} = \pi_{r2}^{\mathrm{NN}^*} = \frac{(a-c)^2}{4(2+\beta)^2}$ respectively.

4.2. Only one retailer collections (Model CN)

We construct a closed-loop supply chain model in which retailer 1 has capacity constraint. When the manufacturer makes the pricing decision, retailer 1 and retailer 2 make the decision simultaneously. The sequence of events is divided into two phases: (1) The manufacturer decides the wholesale price w and the transfer price b; (2) The retailer 1 decided to q_1 and τ_1, in order to maximize the π_{r1}^{CN}. Meanwhile, retailer 2 makes decisions about q_2 to maximize π_{r2}^{CN}. Therefore, the profit function of the supply chain is as follows

$$\pi_m^{\mathrm{CN}} = [w - c + (s-b)\,\tau_1]\,(q_1 + q_2) \tag{4.4}$$

$$\pi_{r1}^{\mathrm{CN}} = (\alpha - q_1 + \beta q_2 - w + b\tau_1)\,q_1 - \frac{k}{2}\tau_1^2 \tag{4.5}$$

$$\pi_{r2}^{\mathrm{CN}} = (\alpha - q_2 + \beta q_1 - w)\,q_2 \tag{4.6}$$

$$\text{s.t. } \tau_1 \le z_1,$$

Proposition 4.2. *In Model CN, the manufacturer's equilibrium wholesale price is as follows* $w^{\mathrm{CN}^*} = \left\{2a\left[2s^2 - k(2-\beta)\right]\left[s^2(2-\beta) + 4k\beta(2+\beta)\right] - (a-c)\,U_1\right\}/2T_2$, $b^{\mathrm{CN}^*} = [k(2-\beta) - U_2]/s$. *The retailer's equilibrium decisions are* $q_1^{\mathrm{CN}^*} = (a-c)\,s^2U_1/T_1T_2$, $q_2^{\mathrm{CN}^*} = (a-c)\left[3s^2 - 2k(2-\beta) + 2U_2\right]U_1/T_1T_2$, $\tau_1^{\mathrm{CN}^*} = (a-c)\,s\,[k(2-\beta) - A]\,U_1/T_1T_2$, *where* $U_1 = 2k^2(2-\beta)(2+\beta)^2 + s^2U_2(6+\beta) - 2k(2+\beta)\left[(2-\beta)U_2 + 2s^2(2+\beta)\right]$, $U_2 = \sqrt{k\left[2s^2 - k(2-\beta)\right](\beta-2)}$, $T_1 = 2\left[\left(4U_2 - 4k(2-\beta) + s^2(6+\beta)\right]\right.$, $T_2 = 2s^4(2-\beta) - 4k^2\beta\left(4-\beta^2\right) + ks^2\left(7\beta^2 + 20\beta - 4\right)$.

TABLE 3. The decisions with different strategies in model CC.

Strategy	Optimal decisions (w, b, p_1, p_2, τ_1 and τ_2)
N–N–P	$q_1 = q_2 = \frac{(a-c)k}{2k(2+\beta)-4s^2\left(1-\gamma^2\right)}, \tau_1 = \tau_2 = \frac{(a-c)s\left(1-\gamma^2\right)}{k(2+\beta)-2s^2\left(1-\gamma^2\right)}, w = \frac{a+c}{2}, b = s$
Y–N–P	$q_1 = \frac{A+B_1z_1+C_1z_1^2}{D}, q_2 = \frac{A+B_2z_1+C_2z_1^2}{D}, \tau_1 = z_1, \tau_2 = \frac{2(a-c)s\left(1-\gamma^2\right)-\left[k(2+\beta)-2s^2\left(1-\gamma^2\right)\right]z_1}{2\left(k(2+\beta)-s^2\left(1-\gamma^2\right)\right)}$
	$w = \frac{4\left(a^2-c^2\right)\left(1-\gamma^2\right)+4cs\left(1-\gamma^2\right)z_1+k(2+\beta)z_1^2}{4\left(1-\gamma^2\right)(2a-2c+sz_1)}, b = \frac{2(a-c)s\left(1-\gamma^2\right)-\left(k(2+\beta)-2s^2\left(1-\gamma^2\right)\right)z_1}{\left(1-\gamma^2\right)(2a-2c+sz_1)}$
N–Y–P	$q_1 = \frac{A+B_2z_1+C_2z_1^2}{D}, q_2 = \frac{A+B_1z_1+C_1z_1^2}{D}, \tau_1 = \frac{2(a-c)s\left(1-\gamma^2\right)-\left[k(2+\beta)-2s^2\left(1-\gamma^2\right)\right]z_1}{2\left(k(2+\beta)-s^2\left(1-\gamma^2\right)\right)}, \tau_2 = z_2$
	$w = \frac{4\left(a^2-c^2\right)\left(1-\gamma^2\right)+4cs\left(1-\gamma^2\right)z_1+k(2+\beta)z_1^2}{4\left(1-\gamma^2\right)(2a-2c+sz_1)}, b = \frac{2(a-c)s\left(1-\gamma^2\right)-\left[k(2+\beta)-2s^2\left(1-\gamma^2\right)\right]z_1}{\left(1-\gamma^2\right)(2a-2c+sz_1)}$

Where $= 4(a-c)^2\left(-1+\gamma^2\right)\left(-k(-2+\beta)+s^2\left(-1+\gamma^2\right)\right)$; $B_1 = 4(a-c)s\left(-1+\gamma^2\right)\left(k(6+\beta)+3s^2\left(-1+\gamma^2\right)\right)$, $B_2 = 4(a-c)s\left(-1+\gamma^2\right)\left(k(2+3\beta)+3s^2\left(-1+\gamma^2\right)\right)$, $C_1 = \left(3k^2(2+\beta)^2+ks^2(22+9\beta)\left(-1+\gamma^2\right)+8s^4\left(-1+\gamma^2\right)^2\right)$, $C_2 = \left(3k^2(2+\beta)^2+ks^2(18+11\beta)\left(-1+\gamma^2\right)+8s^4\left(-1+\gamma^2\right)^2\right)$, $D = 4(-2+\beta)\left(-1+\gamma^2\right)\left(k(2+\beta)+s^2\left(-1+\gamma^2\right)\right)$ $(2a-2c+sz_1)$.

From Proposition 4.2, we can also get the profit of the manufacturer is
$\pi_m^{\mathrm{CN}^*} = (a-c)^2\left[2s^2+k(-2+\beta)+U_2\right]U_1/T_1T_2$.
The profit of the retailers are $\pi_{r1}^{\mathrm{CN}^*} = (a-c)^2 s^2\left[k(2-\beta)^2 - s^2\ (3-\beta)-(2-\beta)U_2\right]U_1^2/2T_1T_2$,
$\pi_{r2}^{\mathrm{CN}^*} = (a-c)^2\left[3s^2-2k(2-\beta)+2U_2\right]^2 U_1^2/2T_1T_2$, respectively.

4.3. Both retailers collection (Model CC)

In this section, we will discuss the order quantity and collection rate decisions of the closed-loop supply chain in both centralized and decentralized scenarios. Considering the capacity constraint, the Retailer 1 and Retailer 2 use "Y" and "N" options to indicate whether end-of-life products exceed capacity constraint. Further, we denote "P" or "F" to indicate that a part of end-of-life products or full of end-of-life products from two retailers turns remanufacturing.

4.3.1. Decentralized scenario

We establish a closed-loop supply chain model with capacity constraints for two retailers. Similar to 4.2, the sequence of events is divided into two phases: (1) The manufacturer decides the wholesale price w and the transfer price b; (2) The retailer 1 decided to q_1 and τ_1, in order to maximize the π_{r1}^{CC}. Meanwhile, Retailer 2 makes decisions about q_2 and τ_2 to maximize π_{r2}^{CC}. Therefore, the profit function of the supply chain is as follows

$$\pi_m^{\mathrm{CC}} = [w-c+(s-b)(\tau_1+\tau_2)](q_1+q_2) \tag{4.7}$$

$$\pi_{r1}^{\mathrm{CC}} = (\alpha-q_1+\beta q_2-w)q_1+b\tau_1(q_1+q_2)-\frac{k\left(\tau_1^2+\gamma\tau_2^2\right)}{2\left(1-\gamma^2\right)} \tag{4.8}$$

$$\pi_{r2}^{\mathrm{CC}} = (\alpha-q_2+\beta q_1-w)q_2+b\tau_2(q_1+q_2)-\frac{k\left(\gamma\tau_1^2+\tau_2^2\right)}{2\left(1-\gamma^2\right)} \tag{4.9}$$

$$\text{s.t. } \tau_1 \le z_1, \tau_2 \le z_2, \tau_1+\tau_2 \le 1$$

where $w(q_1+q_2)$ is the revenue of selling products to retailers, $[c-s)(\tau_1+\tau_2)](q_1+q_2)$ is total production cost, $b(\tau_1+\tau_2)(q_1+q_2)$ is the total transfer payment.

Proposition 4.3. *In model* CC*, the equilibriums can be expressed as following in Table 3.*

TABLE 4. The decisions with different strategies in the centralized scenario.

Strategy	Optimal decisions (q_1, q_2, τ_1 and τ_2)
N–N–P	$q_1 = \frac{(a-c)k}{2k(1+\beta)-4s^2(1-\gamma^2)}, q_2 = \frac{(a-c)k}{2k(1+\beta)-4s^2(1-\gamma^2)} \tau_1 = \frac{(a-c)s(1-\gamma^2)}{k(1+\beta)-2s^2(1-\gamma^2)}, \tau_2 = \frac{(a-c)s(1-\gamma^2)}{k(1+\beta)-2s^2(1-\gamma^2)}$
N–N–F	$q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = \frac{1}{2}, \tau_2 = \frac{1}{2}$
Y–N–P	$q_1 = \frac{k(a-c+sz_1)}{2[k(1+\beta)-s^2(1-\gamma^2)]}, q_2 = \frac{k(a-c+sz_1)}{2[k(1+\beta)-s^2(1-\gamma^2)]}, \tau_1 = z_1, \tau_2 = \frac{s(1-\gamma^2)(a-c+sz_1)}{k(1+\beta)-s^2(1-\gamma^2)}$
Y–N–F	$q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = z_1, \tau_2 = 1 - z_1$
N–Y–P	$q_1 = \frac{k(a-c+sz_2)}{2[k(1+\beta)-s^2(1-\gamma^2)]}, q_2 = \frac{k(a-c+sz_2)}{2[k(1+\beta)-s^2(1-\gamma^2)]}, \tau_1 = \frac{s(1-\gamma^2)(a-c+sz_2)}{k(1+\beta)-s^2(1-\gamma^2)}, \tau_2 = z_2$
N–Y–F	$q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = 1 - z_2, \tau_2 = z_2$
Y–Y–P	$q_1 = \frac{a-c+s(z_1+z_2)}{2+2\beta}, q_2 = \frac{a-c+s(z_1+z_2)}{2+2\beta}, \tau_1 = z_1, \tau_2 = z_2$
Y–Y–F	$q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = z_1, \tau_2 = z_2$

According to Proposition 4.3, in the N–N–P strategy, the equilibrium order quantity and collection rate of the two retailers are the same. At the same time, it can be found that the equilibrium result is related to the market size and the intensity of demand competition. By Proposition 4.3, we can also get the profit of the manufacturer and the retailers are $\pi_m^{\text{CC}^*} = \frac{(a-c)^2 k}{2k(2+\beta)+4s^2(-1+\gamma^2)'}, \pi_{r1}^{\text{CC}^*} = \pi_{r2}^{\text{CC}^*} = \frac{(a-c)^2 k(k+2s^2\gamma(-1+\gamma^2))}{4(k(2+\beta)+2s^2(-1+\gamma^2))^2}$.

4.3.2. Centralized scenario

We also establish a centralized model of the closed-loop supply chain in CC mode, and get the decisions of order quantity and collection rate with capacity constraint in recycling channel. Under this structure, as a system, the goal of manufacturer and retailers are to maximize the total profit. Therefore, the function of the profit supply chain is as follows:

$$\pi_{\text{SC}} = [a - c + s(\tau_1 + \tau_2)](q_1 + q_2) - q_1^2 - q_2^2 - 2\beta q_1 q_2 - \frac{k(\tau_1^2 + \tau_2^2)}{2(1-\gamma)} \tag{4.10}$$
$$\text{s.t. } \tau_1 \leq \text{z}_1, \tau_2 \leq \text{z}_2, \tau_1 + \tau_2 \leq 1.$$

The formula is obtained by simplifying the sum of formulas (4.7) to (4.9).

Proposition 4.4. *In the centralized scenario, the equilibriums can be expressed as following in Table 4.*

Proposition 4.4 demonstrates the following results: (1) neither the Retailer 1's or Retailer 2's collection rates are not affected by the capacity constraints under the condition $k < \min(k_1, k_2)$, where $k_1 = s(1-\gamma)(a - c + 2sz_1)/(1+\beta)z_1$ and $k_2 = s(1-\gamma)(a - c + 2sz_2)/(1+\beta)z_2$; (2) both the Retailer 1's and Retailer 2's collection rates are closely associated with the capacity constraints under the condition $k < \min(k_3, k_4)$, where $k_3 = s(1-\gamma)[\text{a} - \text{c} + s(\text{z}_1 + \text{z}_2)]/(1+\beta)\text{z}_1$ and $k_4 = s(1-\gamma)(\text{a} - \text{c} + s)/(1+\beta)\text{z}_2$; (3) only the Retailer 1's collection rate is affected by the capacity constraints under the condition $k < \min(\min(k_1, k_2), k_3)$; (4) only the Retailer 2's collection rate is affected by the capacity constraints under the conditions $k < \min(\min(k_1, k_2), k_4)$.

Next, we compare the equilibriums in the decentralized scenario and centralized scenarios in CC model, and investigate the differences on optimal performance.

Proposition 4.5. *From the above equilibriums, the following orders can be obtained:* $q_1^C \geq q_1^D$, $q_2^C \geq q_2^D$, $\pi_{\text{sc}}^C \geq \pi_{\text{sc}}^D$, *when* $\gamma < \frac{1}{1+\beta}$, *we have* $\tau_1^C \geq \tau_1^D$, $\tau_2^C \geq \tau_2^D$.

It can be seen from the above that the centralized scenario leads to a higher order quantity and a higher collection rate, while the total profit in the decentralized scenario is the smallest and the total profit in the centralized scenario is the largest. In addition, a higher cost savings and lower competition intensity created conditions for the two retailers' recycling, and considering the independence of manufacturers and retailers, centralized supply chains are difficult to achieve. Therefore, we will propose a contract to resolve profit conflicts and achieve supply chain coordination.

4.4. Coordinated contract

In reality, it's difficult to make an agreement possible to be accepted by both manufacturer and retailer, which is explained that the upstream and downstream as an independent entity cannot transfer their own decision authority and execute centralized scenario. In this section, we need to design corresponding contract coordination mechanism to make the equilibrium order quantity and equilibrium recovery rate of decentralized decision making reach the level of centralized decision making, and to meet the individual rational constraints and incentive compatibility constraints of all participants at the same time.

In the coordination contract, Two-part Tariff is priced by the manufacturer at a marginal cost and charged a fixed fee to the retailer [17]. The contract has been proven to coordinate to achieve Pareto improvement and eliminate double marginalization. Based on the above, we assume that the manufacturer signs and fulfills the agreement with the retailer: During the sales process, the manufacturer resells the product to the retailer at a relatively low wholesale price; in return, the retailer returns the manufacturer a fixed amount at the end of the period. Assume that the fixed fee charged by the manufacturer to the retailer in the closed-loop supply chain is "F", and the superscript of the decision variable under the coordination mechanism is "T". "θ" is the apportionment ratio. Therefore, the characteristics of the model are as follows

$$\pi_m^T = [w - c + (s - b)(\tau_1 + \tau_2)](q_1 + q_2) + F \tag{4.11}$$

$$\pi_{r1}^T = (\alpha - q_1 + \beta q_2 - w) q_1 + b\tau_1 (q_1 + q_2) - \frac{k\left(\tau_1^2 + \gamma\tau_2^2\right)}{2(1-\gamma^2)} - \theta F \tag{4.12}$$

$$\pi_{r2}^T = (\alpha - q_2 + \beta q_1 - w) q_2 + b\tau_2 (q_1 + q_2) - \frac{k\left(\gamma\tau_1^2 + \tau_2^2\right)}{2(1-\gamma^2)} - (1-\theta) F \tag{4.13}$$

$$\text{s.t. } \tau_1 \le z_1, \tau_2 \le z_2, \tau_1 + \tau_2 \le 1.$$

Proposition 4.6. *In this coordinated contract, the fixed fee should be satisfied as* $\frac{(a-c)^2k^2\left[k(1+\gamma)-2s^2(1-\gamma)\gamma(2-\beta\gamma)\right]}{2[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)[k(2+\beta)-2s^2(1-\gamma^2)]} \le F \le \min\left\{\frac{(a-c)^2k}{4\theta}X, \frac{(a-c)^2k}{4(1-\theta)}X\right\}$, *where,* $X = \frac{k(1+\gamma)-2s^2(1-\gamma)\gamma}{[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)} - \frac{k-2s^2\gamma\left(1-\gamma^2\right)}{[k(2+\beta)-2s^2(1-\gamma^2)]^2}$. *The two-part tariff contract can effectively coordinate the decentralized CLSC and achieves the performances to that in the centralized supply chain as following;* $q_1^T = q_1^C$, $q_2^T = q_2^C$, $\tau_1^T = \tau_1^C$, $\tau_2^T = \tau_2^C$, $\pi_{\text{sc}}^T = \pi_{\text{sc}}^C$.

Proposition 4.6 shows that the Two-part tariff contract can improve the performance of the closed-loop system to the performance of the centralized scenario. The manufacturer and the retailer negotiate a fixed fee F to ensure that the profits of the members are not lower than decentralized scenario. The increase in the number of new products purchased by consumers and the increase in the collection rate of end-of-life products will help retailers to increase their efficiency. In addition, it also illustrates that the sustainable operation of the closed-loop supply chain is facilitated by Two-part Tariff.

5. Comparative analysis

5.1. Analytic comparison

We first compare the manufacturer's equilibrium decisions among the three models. The main results are presented in the following theorem in Table 5.

TABLE 5. The comparison of the size relation of q_1 and q_2 under different modes.

The range of β	The range of γ	The range of k	Size relationship
$0<\beta<1$	$0<\gamma<\frac{1}{\sqrt{\frac{2-\beta}{1-\beta}}}$	$\frac{2s^2}{2-\beta}<k<T$	$q_1^{\mathrm{NN}^*}<q_1^{\mathrm{CC}^*}<q_1^{\mathrm{CN}^*}$
		$k>T$	$q_1^{\mathrm{NN}^*}<q_1^{\mathrm{CN}^*}<q_1^{\mathrm{CC}^*}$
	$\frac{1}{\sqrt{\frac{2-\beta}{1-\beta}}}<\gamma<1$	$k>\frac{2s^2}{2-\beta}$	$q_1^{\mathrm{NN}^*}<q_1^{\mathrm{CC}^*}<q_1^{\mathrm{CN}^*}$
	$0<\gamma<1$	$k>\frac{9s^2}{8-4\beta}$	$q_2^{\mathrm{CN}^*}<q_2^{\mathrm{NN}^*}<q_2^{\mathrm{CC}^*}$

where $T=\frac{s^2\left\{\beta\left(5-6\gamma^2\right)+\left(1-2\gamma^2\right)\left[-2+\sqrt{2-\beta}\sqrt{2+\beta\left(7-8\gamma^2\right)}\right]\right\}}{8\beta\left[1-2\gamma^2-\beta\left(1-\gamma^2\right)\right]}$.

Theorem 5.1. *We compare the equilibrium decisions of the manufacturer among the three models and find that: when* $k>\frac{2s^2}{2-\beta'}$*, we have* $w^{\mathrm{NN}^*}=w^{\mathrm{CC}^*}\le w^{\mathrm{CN}^*}, s=b^{\mathrm{CC}^*}\le b^{\mathrm{CN}^*}$.

Theorem 5.1 reveals the differences in manufacturer equilibrium decisions under different models. Interestingly, the manufacturers set the same wholesale price in both the model NN and model CC. It shows that when both retailers do not collection or both adopt a capacity constraint collection strategy, the manufacturer adopts the same strategy. In the CN model, the wholesale price is higher than the other two scenarios. In addition, in the CC model, the manufacturer transfers all the cost savings of remanufacturing to the retailer ($b^{\mathrm{CC}^*}=s$), and the manufacturer can only benefit directly from the forward channel. In the model CN, the manufacturer sets a higher transfer price ($b^{\mathrm{CN}^*}\ge s$), which is not conducive to the reverse channel.

Next, we compare the order volume of retailers in different models. Assuming $q_i^{\mathrm{NN}^*} q_i^{\mathrm{CN}^*}$ and $q_i^{\mathrm{CC}^*}$ are the order quantities of Retailer i in equilibrium state in model NN, CN and CC, we get the following theorem:

Theorem 5.2. *We compared the equilibrium decision between two retailers in three models and found that.*

Theorem 5.2 reveals the differences in retailers' equilibrium decisions under different models. In Retailer 1, we found that the order quantity in the NN mode is the lowest, and the difference in the degree of competition will affect the order quantity in the CN and CC modes. However, in Retailer 2, the order quantity in the CN mode is lower than in the other modes, and the CC mode is the highest, which indicates that when the Retailer 2 has the capacity constraint, it is beneficial to the increase of the order quantity. The deeper implication of Theorem 5.2 is that if both retailers adopt capacity constraints, they get more profit.

We further compare the profits of manufacturers under different models. Assuming that $\pi_m^{\mathrm{NN}^*}$, $\pi_m^{\mathrm{CN}^*}$ and $\pi_m^{\mathrm{CC}^*}$ are the profits of the manufacturer in the equilibrium state in the models NN, CN and CC, we get the following theorem:

Theorem 5.3. *We compared the profits of manufacturer in three models and found that*

(1) *When* $0<\gamma<\sqrt{3}/2$ *and* $k>\max\left(\frac{2s^2}{2-\beta},\frac{8s^2\left(1-\gamma^2\right)^2}{(2-\beta)(3-4\gamma^2)}\right)$*, we have* $\pi_m^{\mathrm{NN}^*}<\pi_m^{\mathrm{CN}^*}<\pi_m^{\mathrm{CC}^*}$.

(2) *When* $\sqrt{3}/2<\gamma<1$ *and* $k>\max\left(\frac{2s^2}{2-\beta},\frac{8s^2\left(1-\gamma^2\right)^2}{(2-\beta)(3-4\gamma^2)}\right)$*, we have* $\pi_m^{\mathrm{NN}^*}<\pi_m^{\mathrm{CC}^*}<\pi_m^{\mathrm{CN}^*}$.

Theorem 5.3 is similar to Theorem 5.2. We found that the intensity of competition has a great influence on the profit of manufacturer. In addition, the profit of the manufacturer is the lowest in the NN mode, indicating that when the retailer does not collect end-of-life products, the profit of the manufacturer is unfavorable. Under certain conditions, the manufacturer is most advantageous when both retailers have collection capacity constraint. Theorem 5.3 shows that when both retailers have capacity constraints, the manufacturer can make more profit.

TABLE 6. The comparison of wholesale price and transfer price under different models ($k = 500$).

Parameters			Decision variables				
a	s	β	w^{NN^*}	w^{CN^*}	w^{CC^*}	b^{CN^*}	b^{CC^*}
100	5	0.3	60	60.013	60	5.076	5
		0.5	60	60.014	60	5.086	5
		0.7	60	60.016	60	5.100	5
	10	0.3	60	60.279	60	10.669	10
		0.5	60	60.302	60	10.774	10
		0.7	60	60.340	60	10.917	10

TABLE 7. The comparison of retailers' profits under different models ($k = 500$).

Parameters			Decision variables								
s	β	γ	$\pi_{r1}^{\mathrm{NN}^*}$	$\pi_{r2}^{\mathrm{NN}^*}$	$\pi_{r1+r2}^{\mathrm{NN}^*}$	$\pi_{r1}^{\mathrm{CN}^*}$	$\pi_{r2}^{\mathrm{CN}^*}$	$\pi_{r1+r2}^{\mathrm{CN}^*}$	$\pi_{r1}^{\mathrm{CC}^*}$	$\pi_{r2}^{\mathrm{CC}^*}$	$\pi_{r1+r2}^{\mathrm{CC}^*}$
5	0.3	0.3	302.46	302.46	604.92	310.64	299.80	610.44	318.94	318.94	637.88
	0.5	0.5	256.00	256.00	512.00	263.4	252.20	515.74	261.88	261.88	523.76
	0.7	0.7	219.48	219.48	438.96	226.85	214.64	441.49	219.87	219.87	439.74
10	0.3	0.3	302.46	302.46	604.92	338.6	286.57	625.17	380.27	380.27	760.54
	0.5	0.5	256.00	256.00	512.00	290.32	234.65	524.97	280.99	280.99	561.98
	0.7	0.7	219.48	219.48	438.96	254.48	192.68	447.16	220.15	220.15	440.30

5.2. Numerical experiments

In this section, we will use numerical experiments to examine and analyze the impact of relevant parameters on the optimal performance to gain more management insights. Referring to the relevant literature [21,27], the parameters in the models are defined as a $= 100, c_n = 20, s = 5,\ \beta = 0.3,\ \gamma = 0.3,\ z_1 = 0.5,\ z_2 = 0.455$.

5.3. The comparison of equilibrium wholesale and transfer prices

We compared the equilibrium wholesale price and the equilibrium transfer price between different models and designed two scenarios, where $s = 5$ represents a relatively low cost savings situation and $s = 10$ represents a relatively high cost savings situation. The results are shown in Table 6. As can be seen from Table 6, the cost saving (s) plays an important role in determining the transfer price and has a forward impact on the transfer price. In addition, we find that the wholesale price of the model CN is higher than the model NN and model CC, and the greater the competition, the higher the wholesale price in the model CN.

5.3.1. The comparison of retailers' profits

Next, we compare the profit of retailers under different models through numerical experiments. The results are shown in Table 7.

Similar to Table 6, we designed two scenarios ($s = 5$ or $s = 10$) and different competition strengths. According to Table 7, when the competition intensity is low, the profit sum of the two retailers under model CC is the largest, followed by model CN, and the smallest by model NN; When the degree of competition gradually increased, we found that the profits of the two retailers also began to decline gradually, with model CC experiencing the largest decline and model NN the smallest. In addition, we found that the cost savings had no effect on the model NN, but helped to increase the profit of the model CN and the model CC. In the model CN, it is obvious that retailers with capacity constraints will obtain higher profits.

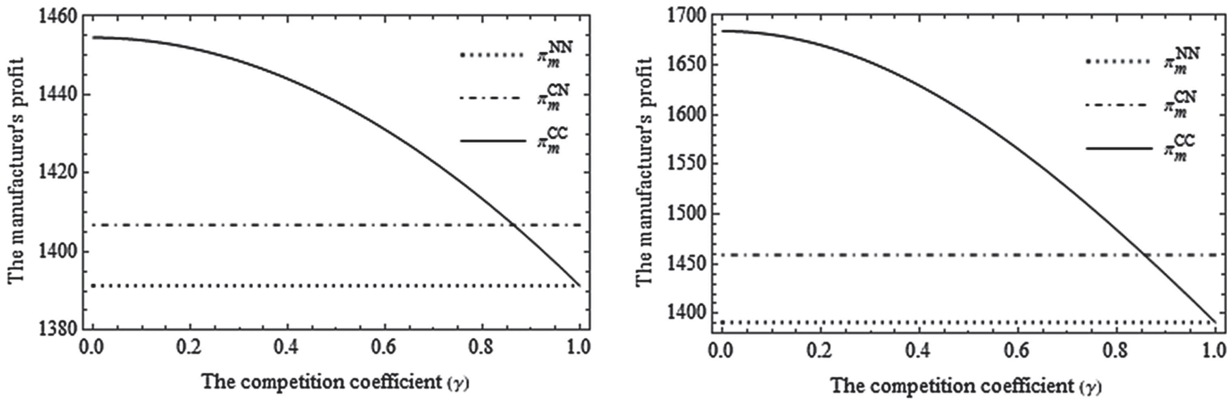

FIGURE 2. The manufacturer profit comparison under different scenarios. (a) $\beta = 0.3, s = 5$. (b) $\beta = 0.3, s = 10$.

TABLE 8. The comparison of supply chain coordination in model CC ($k = 500$).

	θ	F	w	q_1	q_2	τ_1	τ_2	π_m	π_{r1}	π_{r2}	π_{sc}
Centralized scenario		–	–	32.52	32.52	0.455	0.455	–	–	–	2601.626
Decentralized scenario			60	18.107	18.107	0.329	0.329	1448.62	318.939	318.939	2086.49
Coordination contract	0.5	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	576.503	576.503	2601.626
	0.6	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	483.706	669.3	2601.626
	0.7	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	390.91	762.097	2601.626
	0.8	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	298.113	854.894	2601.626
	0.9	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	205.316	947.691	2601.626
	1	927.969	26.954	32.52	32.52	0.455	0.455	1448.62	112.519	1040.49	2601.626

5.3.2. The manufacturer profit comparison under different modes

We tried to investigate whether channel competition and cost savings increased manufacturer's profits. To this end, we compared manufacturer's profits in three scenarios, as shown in Figure 2.

Figure 2a is the comparison result when $\beta = 0.3$ and $s = 5$. It can be found that the profit of the model CC manufacturer is gradually decreasing, and the profit of the manufacturer is the lowest in the NN mode. When the channel competition intensity is within a certain range, $\pi_m^{\text{NN}^*} < \pi_m^{\text{CN}^*} < \pi_m^{\text{CC}^*}$, and when the channel competition intensity is greater, $\pi_m^{\text{NN}^*} < \pi_m^{\text{CC}^*} < \pi_m^{\text{CN}^*}$. Figure 2b shows the comparison results when $\beta = 0.3$, $s = 10$, the results are similar to Figure 2a, but as the cost savings increase, the profit of the manufacturer has also improved. In addition, when $\gamma = 0.3$, manufacturers' profits increased by 1.1% (Model CN *vs.* Model NN) and 4.12% (Model CC *vs.* Model NN), it shows that when the intensity of channel competition is within a certain range, the model CC help manufacturer to increase profit. This result further validates Theorem 5.3.

5.3.3. The comparison of retailers' profits

According to the above analysis, in the centralized, decentralized and contract coordination scenarios, the optimal decision and profit of each effective case are shown in Table 8.

As can be seen from Table 8, compared with the decentralized scenario, the centralized supply chain improves the performance of the entire system. The order quantity of the centralized scenario is higher than that of the decentralized scenario, and the collection rate and profit are also higher than the decentralized scenario. In addition, under certain conditions, the contract can coordinate the position of the two retailers in the distribution of profits, and make the profits of the two retailers higher than in the decentralized scenario, and improve the order quantity and collection rate. The adoption of contracts is of great significance to improving economic and environmental benefits. Therefore, under the premise of ensuring the compatibility of incentive constraints, the optimal profit can be improved by coordinating the contract to achieve the performance of the concentrated scenario.

6. Conclusions

In this paper, We established a closed-loop supply chain composed of a manufacturer and two competing retailers, and proposed three collection models considering whether the two retailers have capacity constraints: such as Neither retailer collections (NN model), Only one retailer collections (CN model) as well as Both retailers collection (CC model) The equilibrium decisions of manufacturer and retailers under different models are obtained. The research results are as follows:

(1) The cost saving has different influence on manufacturer's wholesale price decision under different models.
(2) For retailers, the order quantity under the model NN is the lowest, and the difference in the competition intensity γ and the parameter k will affect the order quantity of the model CN and the model CC.
(3) The intensity of competition in different modes has a large impact on the profits of manufacturer and retailers. For the manufacturer, the manufacturer's profit under the NN model is the lowest; and under certain conditions, when both retailers have capacity constraints, it is the best for the manufacturer. For retailers, when the competition intensity is low, the sum of the profits of the two retailers under the model CC is the largest, and retailers with capacity constraints will obtain higher profits.
(4) Comparing the performance of the centralized scenario and the decentralized scenario in the model CC, we found that the order quantity and collection rate of the centralized scenario are higher, the minimum profit occurs in the decentralized scenario, and the maximum profit occurs in the centralized scenario. In addition, in order to coordinate the closed-loop supply chain, we proposed a two-part tariff contract to achieve Pareto improvement.

In addition, the numerical analysis discussed the following managerial insights:

(1) Facing the different strategies of retailers, the manufacturer should adopt differentiated pricing strategies to maximize their profits. When both retailers have capacity constraints, model CC is most beneficial to the manufacturer. At this time, the manufacturer should set the transfer price equal to the cost savings of remanufacturing, and the manufacturer directly benefit from the forward channel.
(2) From a profit maximization perspective, both retailers should adopt the same strategy. If only one retailer has the capacity constraint, it will earn a higher profit, while the other retailer will cause a loss of profit. If both retailers have capacity constraints, both retailers will achieve higher profits; as the level of cost savings increases, this increase is even more significant.
(3) The two-part contract is of great significance for improving economic and environmental benefits. Under the conditions of incentive compatibility, the optimal profit can be improved by coordinating the contract, and the performance under the centralized scenario can be achieved to ensure a win–win for members.

For future research, this paper can be expanded in the following aspects. In this study, we only consider the collection and competition strategies of two retailers, we can further study the cooperation between manufacturers and retailers or the cooperation between retailers. In this research, we focused on the impact of retailer collection strategies, and it would be more interesting if manufacturers or third-party collectors were involved in collection. In this research, we doesn't consider the uncertainty of market demand and the asymmetry of information, which is also the direction of future research on closed-loop supply chains with capacity constraints. In addition, introducing government insight into decision and coordination of closed-loop supply chains is also an important issue.

Appendix A.

Proof of Proposition 4.1. The profit function in equations (4.2) and (4.3) is concave in q_1 and q_2. Therefore, for Retailer 1 and Retailer 2, the first-order conditions are given as follows:

$$\begin{cases} \frac{\partial \pi_{r1}^{\mathrm{NN}}}{\partial q_1^{\mathrm{NN}}} = a - w - 2q_1 - \beta q_2 = 0 \\ \frac{\partial \pi_{r2}^{\mathrm{NN}}}{\partial q_2^{\mathrm{NN}}} = a - w - \beta q_1 - 2q_2 = 0. \end{cases}$$

Further, by the first-order conditions, we can obtain optimal q_1 and q_2 under given w. The results are shown as follows. $q_1 = \frac{a-w}{2+\beta}, q_2 = \frac{a-w}{2+\beta}$. Substituting the result into the manufacturer's profit function, we obtain the following result: $\pi_m^{\mathrm{NN}} = \frac{2(a-w)(w-c)}{2+\beta}$. By the first-order condition, we obtain: $w = \frac{a+c}{2}$. The retailer's equilibrium decisions are $q_1^{\mathrm{NN}} = q_2^{\mathrm{NN}} = \frac{a-c}{4+2\beta}$. we can also get the profit of the manufacturer and the retailers are $\pi_m^{\mathrm{NN}} = \frac{(a-c)^2}{2(2+\beta)}$ and $\pi_{\mathrm{r}1}^{\mathrm{NN}} = \pi_{\mathrm{r}2}^{\mathrm{NN}} = \frac{(a-c)^2}{4(2+\beta)^2}$.

Therefore, the proof of Proposition 4.2 is completed. □

Proof of Proposition 4.2. In model CN, we introduce the backward induction to solve the model. From the second stage, the Retailer 1 determines the order quantity and collection rate to maximize profit. According to formula (4.5), the Hessian matrix can be obtained as following

$$\mathrm{H}_{r1}^{\mathrm{CN}} = \begin{bmatrix} \frac{\partial \pi_{r1}^{\mathrm{CN}}}{\partial q_1^2} & \frac{\partial \pi_{r1}^{\mathrm{CN}}}{\partial q_1 \partial \tau_1} \\ \frac{\partial \pi_{r1}^{\mathrm{CN}}}{\partial \tau_1 \partial q_1} & \frac{\partial \pi_{r1}^{\mathrm{CN}}}{\partial \tau_1^2} \end{bmatrix} = \begin{bmatrix} -2 \; b \\ b \quad -k \end{bmatrix}.$$

The second derivative of the Hessian matrix with respect to the order quantity and collection rate by Retailer 1 are $\left|\mathrm{H}_1^{\mathrm{CN}}\right| = -2 < 0$ and $\left|\mathrm{H}_2^{\mathrm{CN}}\right| = 2k - b^2 > 0$, we can get $k > \frac{b}{2}$. Therefore, the Hessian matrix for supply chain profit is a negative define function. Next, we introduce Karush–Kuhn–Tucker conditions to characterize the optimality condition and model a Lagrangian function of optimization problem in the profit for supply chain can be expressed as $\mathrm{L}_{r1}^{\mathrm{CN}} = \pi_{\mathrm{r}1} + \lambda_1 (z_1 - \tau_1)$, where λ_1 is the multipliers. Further, the optimal decisions with eight cases are discussed as following:

(a) When $\lambda_1 = 0$ and $\lambda_1 (z_1 - \tau_1) = 0$ indicates the collection quantity from retailer doesn't exceed the capacity constraint. Combining and solving the KKT conditions, we easily obtain

$$q_1 = \frac{k(a-w)(-2+\beta)}{2b^2 - k(4-\beta^2)}, q_2 = \frac{(a-w)\left[b^2 - k(2-\beta)\right]}{2b^2 - k(4-\beta^2)}, \tau_1 = \frac{b(a-w)(-2+\beta)}{2b^2 - k(4-\beta^2)}.$$

Case 1: $\lambda_1 = 0$, namely $z_1 > \tau_1$ and $\tau_1 < 1$. Under this situation, it means that the collection quantity from Retailer 1 doesn't exceed the capacity constraint; part of used product returns remanufacturing (N–N–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, we have $w = \left\{2a\left[2s^2 - k(2-\beta)\right]\left[s^2(2-\beta) + 4k\beta(2+\beta)\right] - (a-c)U_1\right\}/2T_2$, $\mathrm{b} = \left[k(2-\beta) - U_2\right]/s$.

Case 2: $\lambda_1 = 0$, namely $z_1 > \tau_1$ and $\tau_1 = 1$. Under this situation, it means that the collection quantity from manufacturer doesn't exceed the capacity constraint, full of used product returns remanufacturing (N–N–F Strategy). Equating first-order conditions to zero, we have $w = c - s + \sqrt{2k(2-\beta)}, b = \sqrt{2k(2-\beta)}$. $q_2 < 0$, discrepancy.

(b) When $\lambda_1 > 0$ and $\lambda_1 (\mathrm{z}_1 - \tau_1) = 0$ indicates the recycling quantity from Retailer 1 exceeds the capacity constraint. Combining and solving the KKT conditions, we can easily have $q_1 = \frac{(a-w)(2-\beta)+2bz_1}{4-\beta^2}$, $q_2 = \frac{(a-w)(2-\beta)-b\beta z_1}{4-\beta^2}$, $\tau_1 = z_1$. $q_2 < 0$, discrepancy.

Case 3: $\lambda_1 > 0$, namely $z_1 = \tau_1$ and $\tau_1 > 1$. Under this situation, it that the collection quantity from Retailer 1 exceeds the capacity constraint; part of used product returns remanufacturing (Y–Y–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, we have $w = 2a - c + sz_1, b = \frac{2(a-c+sz_1)}{z_1}$. $q_2 < 0$, discrepancy.

Case 4: $\lambda_1 > 0$, namely $z_1 = \tau_1$ and $\tau_1 = 1$. Under this situation, it means that the collection quantity from Retailer 1 exceeds the capacity constraint; full of used product returns remanufacturing (Y–Y–F Strategy). Equating the first-order conditions to zero and solving the KKT conditions, we have $\mathrm{w} = 2a - c + s, b = 2(a - c + s)$. $q_2 < 0$, discrepancy.

Therefore, the proof of Proposition 4.2 is completed. □

Proof of Proposition 4.3. In the decentralized scenario, we introduce the backward induction to solve the model. From the second stage, the Retailer 1 determines the order quantity and collection rate to maximize profit. According to formula (4.8), the Hessian matrix can be obtained as following

$$\mathrm{H}_{r1}^{\mathrm{CC}} = \begin{bmatrix} \frac{\partial^2 \pi_{r1}^{\mathrm{CN}}}{\partial q_1^2} & \frac{\partial^2 \pi_{r1}^{\mathrm{CN}}}{\partial q_1 \partial \tau_1} \\ \frac{\partial^2 \pi_{r1}^{\mathrm{CN}}}{\partial \tau_1 \partial q_1} & \frac{\partial^2 \pi_{r1}^{\mathrm{CN}}}{\partial \tau_1^2} \end{bmatrix} = \begin{bmatrix} -2 & b \\ b & \frac{-k}{1-\gamma^2} \end{bmatrix}.$$

The second derivate of Hessian matrix with respect to the order quantity and collection rate are $\left|\mathrm{H}_1^{\mathrm{CC}}\right| = -2 < 0$, $\left|\mathrm{H}_2^{\mathrm{CC}}\right| = \frac{2k}{1-\gamma^2} - b^2 > 0$, $k > \frac{b^2\{1-\gamma^2)}{2}$. For the profit of Retailer 1, the Hessian matrix is negative definite. Therefore, the Retailer 1's profit is a jointly concave function in the order quantity and collection rate. And then, we can introduce Karush–Kuhn–Tucker conditions to characterize the optimality condition. The results for Retailer 2 are similar to those for Retailer 1. Thus, the Lagrangian function of optimization problem in the two retailers profit function can be expressed as $\mathrm{L}_{r1}^{\mathrm{CC}} = \pi_{\mathrm{r}1} + \lambda_1(z_1 - \tau_1) + \lambda_2(z_2 - \tau_2) + \lambda_3(1 - \tau_1 - \tau_2)$, $\mathrm{L}_{r2}^{\mathrm{CC}} = \pi_{\mathrm{r}2} + \lambda_1(z_1 - \tau_1) + \lambda_2(z_2 - \tau_2) + \lambda_3(1 - \tau_1 - \tau_2)$, where λ_1, λ_2 and λ_3 is the multipliers. Next, we can discuss the optimal decisions with eight cases as following:

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$, namely $z_1 > \tau_1$, $z_2 > \tau_2$ and $\tau_1 + \tau_2 < 1$, indicates that the collection quantities from the Retailer 1 and Retailer 2 don't exceed capacity constraints, which part of end-of-life products returns remanufacturing (N–N–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, we have $q_1 = q_2 = \frac{(a-c)k}{2k(2+\beta)-4s^2(1-\gamma^2)}, \tau_1 = \tau_2 = \frac{(a-c)s(1-\gamma^2)}{k(2+\beta)-2s^2(1-\gamma^2)}$. Substituting the result into the manufacturer's profit function, by the first-order condition, we obtain the following result: $w = \frac{a+c}{2}, b = s$.

Case 2: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 > 0$, namely $z_1 > \tau_1$, $z_2 > \tau_2$ and $\tau_1 + \tau_2 = 1$, indicates that the collection quantities from the Retailer 1 and Retailer 2 don't exceed capacity constraints, which full of end-of-life products returns remanufacturing (N-N-F Strategy). $\lambda_3 = \frac{k}{2(-1+\gamma^2)} < 0$, discrepancy.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 = 0$, namely $z_1 > \tau_1$, $z_2 = \tau_2$ and $\tau_1 + \tau_2 > 1$, indicates that the collection quantity from the Retailer 1 doesn't exceed capacity constraint and that of Retailer 2 exceeds, which part of end-of-life products returns remanufacturing (N–Y–P Strategy).
When $\frac{b^2(1-\gamma^2)}{2} < k < \frac{2s(1-\gamma^2)(a-c+2sz_2)}{3(2+\beta)z_2}$, $\lambda_2 = \frac{k\left(2(a-c)s(-1+\gamma^2)+\left(3k(2+\beta)+4s^2(-1+\gamma^2)\right)z_2\right)}{2(-1+\gamma^2)(k(2+\beta)+s^2(-1+\gamma^2))} > 0$.
Equating the first-order conditions to zero and solving the KKT conditions, we have $q_1 = \frac{\mathrm{A}+B_2z_1+C_2z_1^2}{D}$, $q_2 = \frac{A+B_1z_1+C_1z_1^2}{D}, \tau_1 = \frac{2(a-c)s(1-\gamma^2)-\left(k(2+\beta)-2s^2(1-\gamma^2)\right)z_1}{2(k(2+\beta)-s^2(1-\gamma^2))}, \tau_2 = \mathrm{z}_2$. Substituting the result into the manufacturer's profit function, by the first-order condition, we obtain the following result: $w = \frac{4(a^2-c^2)(1-\gamma^2)+4cs(1-\gamma^2)z_1+k(2+\beta)z_1^2}{4(1-\gamma^2)(2a-2c+sz_1)}$, $b = \frac{2(a-c)s(1-\gamma^2)-\left(k(2+\beta)-2s^2(1-\gamma^2)\right)z_1}{(1-\gamma^2)(2a-2c+sz_1)}$.

Case 4: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, namely $z_1 > \tau_1$, $z_2 = \tau_2$ and $\tau_1 + \tau_2 = 1$, indicates that the collection quantity from the Retailer 1 doesn't exceed capacity constraint and that of Retailer 2 exceeds, which full of end-of-life products returns remanufacturing (N–Y–F Strategy). we have $\lambda_3 = \frac{k-kz_2}{-1+\gamma^2} < 0$, discrepancy.

Case 5: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$, namely $z_1 = \tau_1$, $z_2 > \tau_2$ and $\tau_1 + \tau_2 < 1$, indicates that the collection quantity from the Retailer 1 exceed capacity constraint and that of Retailer 2 doesn't exceeds, which part of end-of-life products returns remanufacturing (Y–N–P Strategy). When $\frac{b^2(1-\gamma^2)}{2} < k < \frac{2s(1-\gamma^2)(a-c+2sz_1)}{3(2+\beta)z_1}$, $\lambda_1 = \frac{k\left(2(a-c)s(-1+\gamma^2)+\left(3k(2+\beta)+4s^2(-1+\gamma^2)\right)z_1\right)}{2(-1+\gamma^2)(k(2+\beta)+s^2(-1+\gamma^2))} > 0$. Equating the first-order conditions to zero and solving the KKT condition, we have $q_1 = \frac{A+B_1z_1+C_1z_1^2}{D}, q_2 = \frac{A+B_2z_1+C_2z_1^2}{D}$, $\tau_1 = z_1, \tau_2 = \frac{2(a-c)s\left(1-\gamma^2\right)-\left(k(2+\beta)-2s^2\left(1-\gamma^2\right)\right)z_1}{2(k(2+\beta)-s^2(1-\gamma^2))}$. Substituting the result into the manufacturer's profit function, by the first-order condition, we obtain the following result $w = \frac{4\left(a^2-c^2\right)\left(1-\gamma^2\right)+4cs\left(1-\gamma^2\right)z_1+k(2+\beta)z_1^2}{4(1-\gamma^2)(2a-2c+sz_1)}$, $b = \frac{2(a-c)s\left(1-\gamma^2\right)-\left(k(2+\beta)-2s^2\left(1-\gamma^2\right)\right)z_1}{(1-\gamma^2)(2a-2c+sz_1)}$.

Case 6: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 > 0$, namely $z_1 = \tau_1$, $z_2 > \tau_2$ and $\tau_1 + \tau_2 = 1$, indicates that the collection quantity from the Retailer 1 exceed capacity constraint and that of Retailer 2 doesn't exceeds, which full of end-of-life products returns remanufacturing (Y–N–F Strategy). we have $\lambda_3 = \frac{k-kz_1}{-1+\gamma^2} < 0$, discrepancy.

Case 7: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$, namely $z_1 = \tau_1$, $z_2 = \tau_2$ and $\tau_1 + \tau_2 < 1$, indicates that the collection quantities from the Retailer 1 and the Retailer 2 exceed capacity constraint, which part of end-of-life products returns remanufacturing (Y–Y–P Strategy). we have $\lambda_1 = \frac{kz_1}{-1+\gamma^2} < 0, \lambda_2 = \frac{kz_2}{-1+\gamma^2} < 0$, discrepancy.

Case 8: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, namely $z_1 = \tau_1 \quad z_2 = \tau_2$ and $\tau_1 + \tau_2 = 1$, indicates that the collection quantities from the Retailer 1 and the Retailer 2 exceed capacity constraint, which full of end-of-life products returns remanufacturing (Y–Y–F Strategy). we have $\lambda_1 = \frac{kz_1}{-1+\gamma^2} < 0, \lambda_2 = \frac{kz_2}{-1+\gamma^2} < 0$, discrepancy.

Therefore, the proof of Proposition 4.3 is completed. □

Proof of Proposition 4.4 . Based on the second-order derivates of profit function for supply chain, we obtain the Hessian matrix with respect to the order quantity and collection rate is

$$\mathrm{H}_{\mathrm{sc}}^{\mathrm{CC}} = \begin{bmatrix} \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_2^2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_1 \partial q_2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_1 \partial \tau_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_1 \partial \tau_2} \\ \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_2 \partial q_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_2^2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_2 \partial \tau_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial q_2 \partial \tau_2} \\ \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_1 \partial q_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_1 \partial q_2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_1^2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_1 \partial \tau_2} \\ \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_2 \partial q_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_2 \partial q_2} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_2 \partial \tau_1} & \frac{\partial^2 \pi_{\mathrm{sc}}^{\mathrm{CC}}}{\partial \tau_2^2} \end{bmatrix} = \begin{bmatrix} -2 & -2\beta & s & s \\ -2\beta & -2 & s & s \\ s & s & \frac{-k}{1-\gamma} & 0 \\ s & s & 0 & \frac{-k}{1-\gamma} \end{bmatrix}.$$

According to the assumption, we get $|\mathrm{H}_1^{\mathrm{sc}}| = -2 < 0$, $|\mathrm{H}_2^{\mathrm{sc}}| = 4\left(1-\beta^2\right) > 0$, $|\mathrm{H}_3^{\mathrm{sc}}| = -\frac{4(1-\beta)\left[k(1+\beta)-s^2(1-\gamma)\right]}{1-\gamma} < 0$, $|\mathrm{H}_3^{\mathrm{sc}}| = \frac{4k(1-\beta)\left[k(1+\beta)-2s^2(1-\gamma)\right]}{(1-\gamma)^2} > 0$. $k > \frac{2s^2(1-\gamma)}{1+\beta}$. Therefore, the Hessian matrix for supply chain profit is a negative define function. Next, we introduce Karush–Kuhn–Tucker conditions to characterize the optimality condition and model a Lagrangian function of optimization problem in the profit for supply chain can be expressed as $\mathrm{L}_{\mathrm{sc}}^{\mathrm{CC}} = \pi_{\mathrm{SC}} + \lambda_1(z_1 - \tau_1) + \lambda_2(z_2 - \tau_2) + \lambda_3(1 - \tau_1 - \tau_2)$, where λ_1, λ_2 and λ_3 is the multipliers corresponding to slack variables. Further, the optimal decisions with eight cases are discussed as following:

Case 1: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ indicates that the collection quantities from the Retailer 1 and Retailer 2 don't exceed capacity constraints, which part of end-of-life products returns remanufacturing (N–N–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $k > \frac{2s^2(1-\gamma)}{1+\beta}$, we have $q_1 = \frac{(a-c)k}{2k(1+\beta)-4s^2(1-\gamma)}, q_2 = \frac{(a-c)k}{2k(1+\beta)-4s^2(1-\gamma)}, \tau_1 = \frac{(a-c)s(1-\gamma)}{k(1+\beta)-2s^2(1-\gamma)}, \tau_2 = \frac{(a-c)s(1-\gamma)}{k(1+\beta)-2s^2(1-\gamma)}$.

Case 2: $\lambda_1 = 0$, $\lambda_2 = 0$, $\lambda_3 > 0$ indicates that the collection quantities from the Retailer 1 and Retailer 2 don't exceed capacity constraints, which full of end-of-life products returns remanufacturing (N–N–F Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{2s(a-c+s)(1-\gamma)}{1+\beta}$, $\lambda_3 > 0$, we have $q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = \frac{1}{2}, \tau_2 = \frac{1}{2}$.

Case 3: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ indicates that the collection quantity from the Retailer 1 doesn't exceeds capacity constraint and that of Retailer 2 exceed, which part of end-of-life products returns remanufacturing (N–Y–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(1-\gamma)(a-c+2sz_2)}{(1+\beta)z_2}$, $\lambda_2 > 0$ we have $q_1 = \frac{k(a-c+sz_2)}{2[k(1+\beta)-s^2(1-\gamma)]}, q_2 = \frac{k(a-c+sz_2)}{2[k(1+\beta)-s^2(1-\gamma)]}, \tau_1 = \frac{s(1-\gamma)(a-c+sz_2)}{k(1+\beta)-s^2(1-\gamma)}, \tau_2 = z_2$.

Case 4: $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ indicates that the collection quantity from the Retailer 1 doesn't exceeds capacity constraint and that of Retailer 2 exceed, which part of end-of-life products returns remanufacturing (N–Y–F Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(a-c+s)(1-\gamma)}{(1+\beta)(1-z_2)}$, we have $q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = 1 - z_2, \tau_2 = z_2$.

Case 5: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$ indicates that the collection quantity from the Retailer 1 exceed capacity constraint and that of Retailer 2 doesn't exceeds, which part of end-of-life products returns remanufacturing (Y–N–P Strategy). Equating the first-order conditions to zero and solving the KKT condition, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(1-\gamma)(a-c+2sz_1)}{(1+\beta)z_1}$, $\lambda_1 > 0$, we have $q_1 = \frac{k(a-c+sz_1)}{2[k(1+\beta)-s^2(1-\gamma)]}, q_2 = \frac{k(a-c+sz_1)}{2[k(1+\beta)-s^2(1-\gamma)]}, \tau_1 = z_1$, $\tau_2 = \frac{s(1-\gamma)(a-c+sz_1)}{k(1+\beta)-s^2(1-\gamma)}$.

Case 6: $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 > 0$ indicates that the collection quantity from the Retailer 1 exceed capacity constraint and that of Retailer 2 doesn't exceeds, which full of end-of-life products returns remanufacturing (Y–N–F Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(a-c+s)(1-\gamma)}{(1+\beta)(1-z_2)}$, $\lambda_3 > 0$, we have $q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = z_1, \tau_2 = 1 - z_1$.

Case 7: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$ indicates that the collection quantities from the Retailer 1 and the Retailer 2 exceed capacity constraint, which part of end-of-life products returns remanufacturing (Y–Y–P Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(1-\gamma)[a-c+s(z_1+z_2)]}{(1+\beta)z_2}$, $\lambda_1 > 0$, $\lambda_2 > 0$, we have $q_1 = \frac{a-c+s(z_1+z_2)}{2+2\beta}, q_2 = \frac{a-c+s(z_1+z_2)}{2+2\beta}, \tau_1 = z_1, \tau_2 = z_2$.

Case 8: $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$ indicates that the collection quantities from the Retailer 1 and the Retailer 2 exceed capacity constraint, which full of end-of-life products returns remanufacturing (Y–Y–F Strategy). Equating the first-order conditions to zero and solving the KKT conditions, when $\frac{2s^2(1-\gamma)}{1+\beta} < k < \frac{s(1-\gamma)(a-c+s)}{(1+\beta)z_2}$, we have $q_1 = \frac{a-c+s}{2+2\beta}, q_2 = \frac{a-c+s}{2+2\beta}, \tau_1 = z_1, \tau_2 = z_2$.

Therefore, the proof of Proposition 4.4 is completed. □

Proof of Proposition 4.5. This relationship can be derived through the algebraic comparison. □

Proof of Proposition 4.6. The procedure is similar to that of Proposition 4.3 we introduce the backward induction to solve the model. From the second stage, the Retailer determines the order quantity and collection rate to maximize profit. According to formulas (4.12) and (4.13), The retailer's optimal responses are q_1^T, q_2^T, τ_1^T and τ_2^T respectively. In model CC, in order to achieve the centralized strategy level of the closed-loop supply chain under the two-part tariff contract, $q_1^T = q_1^C$, $q_2^T = q_2^C$, $\tau_1^T = \tau_1^C$, $\tau_2^T = \tau_2^C$ should be set. Then, we can get $w^T = \frac{2cs^2(-1+\gamma)+ak\beta(1+\gamma)+ck(2+\beta)(1+\gamma)+as^2(-2-2\gamma+4\gamma^2)}{2(k(1+\beta)+2s^2(-1+\gamma))(1+\gamma)}$, $b^T = \frac{s}{1+\gamma}$. Substituting w^T and b^T into formulas (4.11) to (4.13) gives $\pi_m^T = \frac{(a-c)^2 k\left[k\beta(1+\gamma)-2s^2(1-\gamma)\right]}{2[k(1+\beta)+2s^2(-1+\gamma)]^2(1+\gamma)} + F$, $\pi_{r1}^T = \frac{(a-c)^2 k\left[k(1+\gamma)-2s^2(1-\gamma)\gamma\right]}{4[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)} - \theta F$, $\pi_{r2}^T = \frac{(a-c)^2 k\left[k(1+\gamma)-2s^2(1-\gamma)\gamma\right]}{4[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)} - (1-\theta)F$.

In order for both the manufacturer and the two retailers to accept the two-part contract, personal rational constraints must be satisfied, that is, $\pi_m^T \geq \pi_m^D$, $\pi_{r1}^T \geq \pi_{r1}^D$, $\pi_{r2}^T \geq \pi_{r2}^D$ are satisfied, so the effective value range of F can be obtained as $\frac{(a-c)^2k^2[k(1+\gamma)-2s^2(1-\gamma)\gamma(2-\beta\gamma)]}{2[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)[k(2+\beta)-2s^2(1-\gamma^2)]} \leq F \leq \min\left\{\frac{((a-c)^2k)}{4\theta}\left(\frac{k(1+\gamma)-2s^2(1-\gamma)\gamma}{[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)} - \frac{k-2s^2\gamma(1-\gamma^2)}{[k(2+\beta)-2s^2(1-\gamma^2)]^2}\right),\right.$ $\left.\frac{(a-c)^2k}{4(1-\theta)}\left(\frac{k(1+\gamma)-2s^2(1-\gamma)\gamma}{[k(1+\beta)-2s^2(1-\gamma)]^2(1+\gamma)} - \frac{k-2s^2\gamma(1-\gamma^2)}{[k(2+\beta)-2s^2(1-\gamma^2)]^2}\right)\right\}$.

Therefore, the proof of Proposition 4.6 is completed. □

Proof of Theorem 5.1. When $k > \frac{2s^2}{2-\beta}$, we can get $w^{\mathrm{CN}^*} > 0$. By subtracting between w^{NN^*} and w^{CN^*}, When $k > \frac{2s^2}{2-\beta}$, and k is large enough according to Assumption 3.3, we can get $w^{\mathrm{CN}^*} \geq w^{\mathrm{CC}^*}$, Similarly, we can get $b^{\mathrm{CC}^*} \leq b^{\mathrm{CN}^*}$. Therefore, the proof of Theorem 5.1 is completed. □

Proof of Theorem 5.2. When $k > \frac{2s^2(1-\gamma^2)}{2+\beta}$, we can get $q_1^{\mathrm{CC}} > 0$, $q_1^{\mathrm{CC}} - q_1^{\mathrm{NN}} = \frac{(a-c)s^2(1-\gamma^2)}{(2+\beta)[k(2+\beta)-2s^2(1-\gamma^2)]} > 0$, so we can get $q_1^{\mathrm{NN}} < q_1^{\mathrm{CC}}$. Similar, when $k > \frac{2s^2}{2-\beta}$, we can get $q_1^{\mathrm{NN}} < q_1^{\mathrm{CN}}$. And because of $\frac{2s^2}{2-\beta} - \frac{2s^2(1-\gamma^2)}{2+\beta} = \frac{2s^2[2\gamma^2+\beta(2-\gamma^2)]}{4-\beta^2} > 0$, so we can get, when $k > \frac{2s^2}{2-\beta}$, the q_1^{NN} is the smallest. Next, we compare the size relationship between q_1^{CC} and q_1^{CN}. Similarly, by subtracting between q_1^{CC} and q_1^{CN}: When $0 < \gamma < \frac{1}{\sqrt{\frac{2-\beta}{1-\beta}}}$ and $\frac{2s^2}{2-\beta} < k < \frac{s^2\left\{\beta(5-6\gamma^2)+(1-2\gamma^2)\left[-2+\sqrt{2-\beta}\sqrt{2+\beta(7-8\gamma^2)}\right]\right\}}{8\beta[1-2\gamma^2-\beta(1-\gamma^2)]}$, we can get $q_1^{\mathrm{CN}} > q_1^{\mathrm{CC}}$; When $0 < \gamma < \frac{1}{\sqrt{\frac{2-\beta}{1-\beta}}}$ and $k > \frac{s^2\left\{\beta(5-6\gamma^2)+(1-2\gamma^2)\left[-2+\sqrt{2-\beta}\sqrt{2+\beta(7-8\gamma^2)}\right]\right\}}{8\beta[1-2\gamma^2-\beta(1-\gamma^2)]}$, we can get $q_1^{\mathrm{CN}} < q_1^{\mathrm{CC}}$; When $\frac{1}{\sqrt{\frac{2-\beta}{1-\beta}}} < \gamma < 1$ and $k > \frac{2s^2}{2-\beta}$, we can get $q_1^{\mathrm{CN}} > q_1^{\mathrm{CC}}$.

When $k > \frac{2s^2(1-\gamma^2)}{2+\beta}$, we can get $q_2^{\mathrm{CC}} > 0$, $q_2^{\mathrm{CC}} - q_2^{\mathrm{NN}} = \frac{(a-c)s^2(1-\gamma^2)}{(2+\beta)[k(2+\beta)-2s^2(1-\gamma^2)]} > 0$, so we can get $q_2^{\mathrm{NN}} < q_2^{\mathrm{CC}}$. When $k > \frac{9s^2}{8-4\beta}$, we can get $q_2^{\mathrm{CN}} > 0$, by subtracting between q_2^{NN} and q_2^{CN}, we can get $q_2^{\mathrm{NN}} > q_2^{\mathrm{CN}}$. Similar, when $k > \max\left\{\frac{9s^2}{8-4\beta}, \frac{2s^2}{2-\beta}\right\}$, we can get $q_2^{\mathrm{CN}} < q_2^{\mathrm{CC}}$. And because of $\frac{9s^2}{8-4\beta} - \frac{2s^2}{2-\beta} = \frac{s^2}{8-4\beta} > 0$. Hence, when $k > \frac{9s^2}{8-4\beta}$, we have $q_2^{\mathrm{CN}} < q_2^{\mathrm{NN}} < q_2^{\mathrm{CN}}$.

Therefore, the proof of Theorem 5.2 is completed. □

Proof of Theorem 5.3. Because $\pi_m^{\mathrm{CC}} - \pi_m^{\mathrm{NN}} = \frac{(a-c)^2 s^2(1-\gamma^2)}{(2+\beta)[k(2+\beta)-2s^2(1-\gamma^2)]} > 0$, so when $k > \frac{2s^2(1-\gamma^2)}{2+\beta}$, we can get $\pi_m^{\mathrm{CC}} > \pi_m^{\mathrm{NN}}$. Similarly, by subtracting between π_m^{CN} and π_m^{NN}: When $k > \frac{s^2(6+\beta)^2}{8(4-\beta^2)}$, we can get $\pi_m^{\mathrm{CN}} > \pi_m^{\mathrm{NN}}$. $\pi_m^{\mathrm{CC}} - \pi_m^{\mathrm{CN}} = \frac{1}{2}(a-c)^2\left[\frac{\sqrt{k(2s^2-k(2-\beta))(-2+\beta)}}{s^2(2-\beta)+4k\beta(2+\beta)} + k\left(\frac{-2-3\beta}{s^2(2-\beta)+4k\beta(2+\beta)} + \frac{1}{k(2+\beta)-2s^2(1-\gamma^2)}\right)\right]$. To solve the available, when $\gamma\left\langle\frac{\sqrt{3}}{2}, k\right\rangle \max\left(\frac{2s^2}{2-\beta}, \frac{8s^2(1-\gamma^2)^2}{(2-\beta)(3-4\gamma^2)}\right)$, $\pi_m^{\mathrm{CC}} - \pi_m^{\mathrm{CN}} > 0$; when $\gamma > \frac{\sqrt{3}}{2}, k > \max\left(\frac{2s^2}{2-\beta}, \frac{8s^2(1-\gamma^2)^2}{(2-\beta)(3-4\gamma^2)}\right)$, $\pi_m^{\mathrm{CC}} - \pi_m^{\mathrm{CN}} < 0$. Hence, When $0 < \gamma < \sqrt{3}/2$ and $k > \max\left(\frac{2s^2}{2-\beta}, \frac{8s^2(1-\gamma^2)^2}{(2-\beta)(3-4\gamma^2)}\right)$ we have $\pi_{\mathrm{m}}^{\mathrm{NN}^*} < \pi_{\mathrm{m}}^{\mathrm{CN}^*} < \pi_{\mathrm{m}}^{\mathrm{CC}^*}$; When $\sqrt{3}/2 < \gamma < 1$ and $k > \max\left(\frac{2s^2}{2-\beta}, \frac{8s^2(1-\gamma^2)^2}{(2-\beta)(3-4\gamma^2)}\right)$, we have $\pi_{\mathrm{m}}^{\mathrm{NN}^*} < \pi_{\mathrm{m}}^{\mathrm{CC}^*} < \pi_{\mathrm{m}}^{\mathrm{CN}^*}$.

Therefore, the proof of Theorem 5.3 is completed. □

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